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STUDY OF THE FORMATION AND EVOLUTION OF PBHS BINARIES IN THE EARLY UNIVERSE

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Introduction

Almost 8 years have passed since the first direct detection of *gravitational waves* (GW) and a little more than 5 years since the Nobel Prize was awarded for this discovery [1]. As a result, the use of interferometers to study GW has created a new field of experimental cosmology — gravitational-wave astronomy. And even the very first recorded signal created a lot of space for all sorts of discussions. In particular, the masses of black holes (BHs) that generated this signal are estimated at about 30 $M_\odot$ [2], which leads to obvious questions about the nature of such objects (see [3] and sources in this paper). One of the possible explanations is the primary nature of black holes. The mechanisms of formation of such *primordial black holes* (PBHs) are discussed in more detail in the 1 section. The increased cosmological energy density at early times is very characteristic of most of these models, which gives a rough relationship between the PBH mass and the horizon mass during formation [4]:

$$t \approx \frac{GM}{c^3} \approx \frac{30M_\odot}{10^{-33}M_\odot \cdot 10^{38}} s \approx 10^{-4} s \tag{1}$$

Thus, PBHs of such masses should form at the radiation-dominated (RD) stage. [5; 6] shows how, by considering the evolution of binary PBHs in the early universe, one can estimate their contribution to the merger rate observed by the gravitational wave interferometers of the LIGO, Virgo, KaGra collaborations ($2 - 53$ Gpc^{-3} year^{-1}, see [7]).

In this work, we will carry out a similar work, as in [5; 6], for PBHs surrounded by clusters of smaller BHs, and also touch upon the existence of such systems in the early universe, primary black holes (PSHs) in general, and *N-body* simulation.

1 Generalization of PBH models

PBHs are hypothetical objects similar in properties to black holes (BHs) that could presumably form in the early Universe. Their existence can shed light on such phenomena as:

1. Supermassive black holes ($10^6 - 10^{10} M_{\odot}$) at the centers of galactic nuclei. The origin of such objects can be explained by the cumulative effect due to the long evolution of the galaxy. However, such objects have also been found at high redshifts $z > 6$, which is much more difficult to explain.

2. Merging black holes binaries. A gravitationally bound pair of BHs, which, due to the emission of gravitational waves (GW), gradually approach each other, eventually forming a new BH of a larger mass. Since 2015, such events have been recorded using the gravitational-wave interferometers of the `LIGO/Virgo` collaborations. Restoring the origin of these waves, we can calculate the properties of the original binary. In some detected events, these properties indicate their primary origin [5].

3. The reionization of the Universe can be associated with radiation from low-mass PBHs with neutral gas or as a result of accretion of matter onto massive PBHs.

4. The formation of structures at high redshifts $z > 10$ can be related to the influence of PBHs in the early universe.

5. Hidden mass. If PBHs exist, then they make a measurable contribution to the hidden mass of the Universe.

In other words, in the early universe there must be special inhomogeneities of matter, leading to the formation of massive objects that affect star formation and the anisotropy of matter in general.

After the discovery of the cosmic microwave background with a temperature of 2.7K [8] *the cold universe model* was refuted. One of its supporters was Ya.B. Zel'dovich. He and many other scientists were offered a new model consistent with this experiment — *hot universe model* [9]. In this regard, Ya.B. Zel'dovich and I.D. Novikov revised the concept of PBHs and proved analytically that if such objects exist, then the accretion of matter around them should cause strong radiation [10]. Further development of the study of the possibility of the existence of black hole, in particular Hawking, Karom, revealed the connection between their formation and the conditions of strong inhomogeneities in the very early Universe.

In the context of the modern description of cosmology based on the inflationary cosmological model with baryosynthesis and hidden mass, the study of PBHs is becoming increasingly relevant. The developing gravitational-wave astronomy and the search for gravitational lensing phenomena significantly expand the experimental possibilities for detecting PBHs or confirming their absence.

1.1 Mechanisms of formation

As noted earlier, the field of cosmology that studies PBHs is incredibly vast. This is due, firstly, to the fact that earlier, only by indirect indications of the mass of the object reconstructed from the X-ray signal and the absence of radiation in other ranges, objects were classified as black holes. Secondly, the origin of these objects can be explained in a variety of ways, and at the moment, in most cases, at least some of the theories cannot be discarded with sufficient certainty.

In connection with the breakthrough of gravitational-wave astronomy technologies, the sensitivity of interferometers to gravitational waves has greatly increased, making it possible to statistically reliably confirm the detection of gravitational waves and investigate the properties of their sources (which solved the first aspect noted earlier). In 2014, the `LIGO` collaboration recorded the gravitational wave signal GW150914 corresponding to a BH merger of 10 solar masses with a confidence of 5σ. As noted earlier in [3; 5; 6], hypotheses explaining the nature of such objects are presented. One such hypothesis is a primordial origin in the early universe. This has greatly increased interest in the topic of PBH in recent years.

This work does not aim to consider the issue of formation mechanisms from all sides, so below is a list with some of them.

1. Density fluctuations - This model is based on quantum density fluctuations that occurred in the early Universe (mainly the epoch of cosmological

inflation is considered). But anisotropy of CMB predicts small fluctuations that are not enough for PBHs formation. So some inflaton models define methods where them are possible (examples of such works [11; 12]).

2. Phase transitions of the first and second kind - Theories take a look at scalar fields with contrast properties of vacuum and search transitions in them.

3. The evolution of gravitational instability at the stage of dominance of superheavy nonrelativistic particles, which are predicted in the framework of grand unification theories.

4. Collapse of cosmic strings

5. Baryon charge fluctuations explained by the modified Affleck-Dine mechanism of baryogenesis.

1.2 Accumulations of PCH. Models of formation and evolution.

1.2.1 Clustering and mass distribution

Some models of the formation of black hole have evolved partly on the ability to recreate their clusters. This is especially fully described in [13].

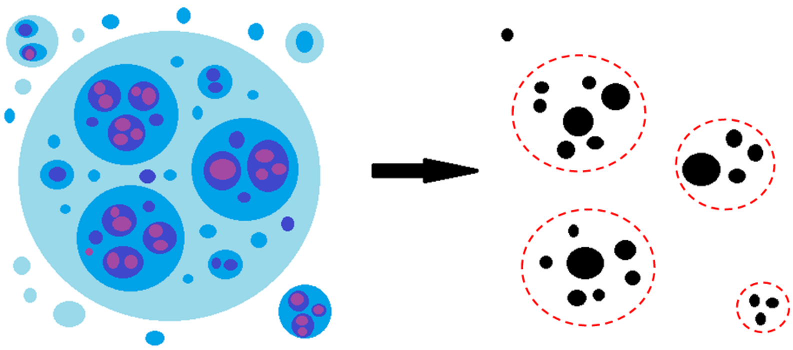

Рисунок 1.1 – Multiple fluctuations expanding and overlapping each other naturally provide a self-similar structure. Clusters of PBHs of various masses and sizes are formed [14].

An interesting perspective on clustering and how it evolves are real clusters such as globular clusters. [15] shows how *N-body* modeling yielded a globular cluster similar to *Palomar 5*. The resulting cluster gradually increased the proportion of BHs among the stars in the cluster, and after a billion years led to the complete «weathering» of stars, leaving only a cluster of BHs. One of the conclusions of the work says that such clusters can be a frequent occurrence. Perhaps a similar mechanism can be applied to PBH clusters that have survived to our epochs.

There are two ways to create clusters. The natural method implies that in the Universe there is a distribution of the mass spectrum of PBHs, so that at the stage of formation of structures they have already begun to contribute to the subsequent formation of the Universe and, in particular, form clusters. A narrower way to model specific clusters, study their evolution and calculate the parameters of long-lived ones.

1.2.2 Gravitational-wave evaporation of PBH binaries

The rate of black hole mergers measured by the `LIGO`, `Virgo` and `KAGRA` collaboration detectors. This value is still only a statistical parameter and does not in any way indicate any new physics, but there should be some correlation between this value and the density of binary BHs in the observed region.

On the other hand, this value in [5; 6] can be used to test the hypothesis that explains the origin of the GW150914 signal sources as primary.

2 N-body simulation

2.1 Methods of N-body simulation

The study of the evolution and nature of celestial objects is possible in 2 ways — external observations and modeling. Modeling makes it possible to study individual cosmological phenomena in more detail and selectively. However, this method also has limitations related to the accuracy, scale and speed of calculations.

Depending on what phenomena and how we want to investigate, we can use different computational models, each of which has its pros and cons.

One of the easiest to implement i is the *N-body modeling* method. Its essence is to use classical mechanics and Newtonian gravity. Each object in a short moment of time is given the total acceleration from the force of attraction to the surrounding massive objects.

2.2 Software implementation of PBH cluster dynamics

This section discusses a program written for the first evaluation of the phenomena occurring in PBH clusters. For such a task, an easy-to-implement concept in the Python programming language was chosen using the classic N-body model.

The program itself consists of three parts.

1. Initial condition data generation

2. Modeling: Based on classical mechanics and Newtonian gravity, the dynamics of objects is predicted

3. Data visualization: dynamic 2D images, essential graphics.

3 Simulation of PBH Clusters and Study of Their Evolution

3.1 Calculation of system parameters: primary black hole cluster - supermassive black hole

In this section, in order to further study the evolution of PBH clusters and the possible mass distributions of PBHs in them, some calculations are given for a system consisting of a cluster of PBHs in the amount of 100 pieces with a total mass of $100M_{\odot}$ and one super-massive BH, also with a mass of $100M_{\odot}$. We will solve in the classical Newtonian N-body formalism.

First of all, we obtain the cluster relaxation time:

$$T_E = \frac{1}{25.5 \lg N/2} \cdot \sqrt{\frac{NR_1^3}{Gm_1}} \tag{2}$$

Knowing the relaxation time, for a stable location of the cluster in orbit, we take the orbit period much larger than this time, let's say 1000 times.

Consider the problem as two rotating bodies around a center of mass equidistant at a distance R. Force that creates centripetal acceleration $F_g = \frac{Gm_1m_2}{(2R)^2} = ma$. Then from Newton's second law we can find the speed of bodies and derive the radius from the period.

$$m_1\frac{v^2}{R} = \frac{Gm_1m_2}{(2R)^2} \Rightarrow v = \sqrt{\frac{Gm_2}{4R}} \tag{3}$$

$$m_1\omega^2R = m_1\left(\frac{2\pi}{T}\right)^2 \cdot R = \frac{Gm_1m_2}{(2R)^2} \Rightarrow R^3 = \left(\frac{T}{2\pi}\right)^2 \cdot \frac{Gm_2}{4} \tag{4}$$

Having received complete information about the initial data, we use them to fill in arrays of masses, positions and velocities of objects (see section ??). We pass to a system where the cluster moves at a speed 3, for example, along the Y axis, and the opposite BH $100M_{\odot}$ is against the Y axis. Then their positions should be in the extreme positions along the X axis in the orbit with the calculated radius ??.

Further, having tested for a short period of time and roughly estimating the complexity of calculations from an increase in this period, we can choose the optimal time step to simulate at least one period of rotation of these objects.

3.2 Further research

The N-body simulation of black hole clusters and the coalescence of its pairs is a fascinating area of research in astrophysics. While I have developed a program for such a simulation and have conducted some initial simulations, there is still much work to be done to validate my findings and demonstrate that they are consistent with real-world processes.

Further research in this area will require simulation more data to analyze and compare with observations made by the LIGO and Virgo experiments. This will involve refining the simulation model to more accurately represent the physics of black hole clusters and their coalescence. Additionally, it will require exploring the impact of various factors, such as black hole mass, velocity, and the distribution of the clusters parameters, on the coalescence of black hole binaries.

Through this research, we will gain a deeper understanding of the properties of black hole clusters and their binaries. This will help answer the question of whether such clustered black hole binaries can produce the BH binaries observed by LIGO and Virgo. Additionally, this research will contribute to our knowledge of the universe and the physical processes that govern its evolution. Ultimately, the potential applications of this research extend beyond astrophysics, as it may provide insight into the behavior of other complex systems in nature.

4 Conclusion

In this survey, we discussed issues of modern cosmology, the successes of gravitational-wave astronomy and the concept of primary black holes (PBHs). The gravitational wave signal GW150914 and hypotheses about the nature of the black holes that gave rise to it are considered. One of the hypotheses noted the possibility of the existence of PBH clusters with central massive BHs at the epoch of the RD-MD transition, the merging of which will also affect the rate of mergers observed by the experiments of the LIGO, Virgo, and KaGra collaborations.

A Python script for N-body simulations was developed. Clusters simulations was provided and next step will be estimating of merger rate for cluster coalescence.

As result with this work we will understand more about nature of binaries systems and maybe give more gravitational wave samples for collaborations like LIGO, Virgo and KaGra.

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