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Heterotic $E_8 \times E_8$ ' string theory

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1. Introduction

1.1 String theory in modern physics

String theory is an attempt to explain all the particles and fundamental forces of nature within a single theory, modeling them as vibrations of supersymmetric strings. According to the theory, the fundamental components of reality are extended objects - strings (Interaction doesn't take place in one point, as in quantum field theory). Size of compactified strings is Planck length. That makes it impossible to observe them experimentally. Different frequencies determine the masses of individual fields, both observed by the standard model and hypothetical (gravitons, leptiquarks).

The considered superstring theory based on the group $E_8 \times E_8'$ is a theory with only has closed heterotic strings. It does not contain gravity and gauge anomalies [1]. That allows using it to describe quantum gravity and obtain the masses and properties of particles of the Standard Model. Also, superstring theory can be a solution to the problem of hidden mass and divergence in Feynman diagram in QFT.

Despite the advantages of the theory, it has a number of significant defects: a large number of free parameters, weak predictive power, and difficulty in experimental verification.

1.2 Brief history of string theory

Historical background on the theory of mirror particles and string theories is based on review articles [2–4]. String theory emerged in the 1970s from the work of Gabriele Veneziano on string models of hadrons. The 1980s and 1990s saw a rapid development of string theory. But there were many contradictions in the theory: unobservable particles, a huge number of false vacuums, computational problems (in most cases, the theory is perturbative).

In 1984, Michael Green and John Schwarz [5] showed that in the presence of 9 spatial dimensions: 3 ordinary and 6 compact (Calabi-Yau manifolds). It turns out to conform QFT and GR within the same theory. Also, different diagrams for fermionic and bosonic fields can cancel each other and cause "Anomaly Cancellation". This study has been called the first superstring revolution.

In the mid-1990s, Edward Witten, Joseph Polchinski, and other physicists found strong evidence that the various superstring theories represent various extreme cases of the 11-dimensional M-theory. This discovery marked the second superstring revolution. The main element of M-theory are «branes» - multidimensional objects like one-dimensional strings. So far, there is no experimental evidence that would unequivocally indicate that any of the superstring models or M-theory is the correct fundamental description of the Universe.

In 1966 Pomeranchuk, Okun and Kobzarev [2] introduce a new symmetry to replace the broken one CP: CPA -, where A - is responsible for the belonging of the particle to the «mirror world». A-symmetry is necessary for conservation of independence of P-transforms and T-transforms when CP is violated. In another words, to preserve the symmetry between right and left in nature. The mirror world has its own strong and electro-weak interactions and can interact with the ordinary world solely through oscillations with their non-mirror partners.

In the 1980s, various mirror theories were built, and experiments were also carried out on accelerators to search for particle decays into «nothing». The first works appear linking the mirror and ordinary worlds with the help of one symmetry group [6].

At the moment, there are many different theories, including different types of dark symmetry [7–11]. Many models of the superstring mirror world are «toy models», but $E8 \times E8'$ is an interesting candidate for Theory of Everything.

2. Theory of the group $E8 \times E8'$

$E8 \times E8'$ theory has historically been seen as the most promising theory for describing physics beyond the Standard Model. It was created in 1987 by Gross, Harvey, Martinez, and Rom [1] and for a long time was the only string theory with symmetry groups of the standard model. $E8$ – contains as a subgroup the standard model group $SU(3) \times SU(2) \times U(1)$. $E8'$ – does not break symmetry and forms «shadow world».

2.1 Why 10 dimensions?

We observe a four-dimensional space-time (4 dimensions), in which the particles of the standard model are located (4 vector bosons + 8 gluons + Higgs field = 12 generators). $E8$ group has enough generators to get the fields of the standard symmetry model in a «natural» way with the help of additional spaces.

In addition, 10 measurements allow you to remove calibration and gravitational anomalies **Maybe better UV and Gravitational anom** [4]. Anomalies in quantum field models are associated with violation of classical symmetries (Lorentz invariance, gauge symmetry, etc.) at the quantum level. Gravitational anomalies indicate a violation at the quantum level of the general covariance of the theory or local Lorentz invariance. In works on superstring theory, it was shown that cancellation of anomalies at the quantum field level takes place for the group $E8 \times E8'$ and $SO(32)$.

2.2 Generators and symmetries

Group $E8$ contains as a subgroup $SU(3) \times E6$. We can strongly restrict the necessary form of the $E6$ manifold of complementary spaces from the condition of the form of 4-dimensional space-time. These conditions are met by a 6-dimensional Calabi-Yao manifold. Next comes the violation $E6$: quarks and leptons are massless excitations of the superstring. Interaction constants turn out to be related to topological characteristics of the Calabi-Yao manifold. Then there is a symmetry breaking up to the standard model group.

The symmetry group of the Standard Model contains 12 generators and 12 gauge bosons corresponding to them. The $E6$ group has 78 generators, that is, it must contain 66 unknown gauge bosons, in other words, several new interactions.

2.3 Symmetry breaking

The process of gradual symmetry breaking is well shown in the works [7; 10]. Integration over 6 compactified dimensions of $E8$ superstring theory leads to an effective theory with $E6$ symmetry in in four dimensions. The further process can be defined ambiguously (the number of options is very large and creates a «landscape problem»). For all variants, this is a gradual spontaneous symmetry breaking as the fields roll down to nonzero vacuum expectation values.

$$\begin{aligned}
 E8 \rightarrow E6 \rightarrow SO(3, 1) \times SU(3) \times SU_L(2) \times SU_R(2) \times U_X(1) \rightarrow \\
 \rightarrow SO(3, 1) \times SU(3) \times SU_L(2) \times U_Y(1) \times U_X(1). \quad (1)
 \end{aligned}$$

E8		$\frac{1}{\sqrt{2}}\omega_7^3$	$\frac{1}{2}\omega_8^3$	U^3	V^3	w	x	y	z	$F4$	$G2$	#
 	$\omega_L^{\wedge V} \omega_R^{\wedge V}$	± 1	± 1	0	0	0	0	0	0	$D2_G$	1	4
 	$W^\pm B_1^\pm$	0	$\pm 1 \pm 1$	0	0	0	0	0	0	$D2_{ew}$	1	4
   	$e\phi_+ e\phi_- e\phi_1 e\phi_0$	± 1	± 1	0	0	0	0	0	0	4×4	1	16
   	$\nu_{eL} e_L \nu_{eR} e_R$	$\pm 1/2 \dots$	$\text{even}\#>0$	0	0	$-1/2$	$-1/2$	$-1/2$	$-1/2$	8_{S+}	l	8
   	$\bar{\nu}_{eL} \bar{e}_L \bar{\nu}_{eR} \bar{e}_R$	$\pm 1/2 \dots$	$\text{even}\#>0$	0	0	$1/2$	$1/2$	$1/2$	$1/2$	8_{S+}	\bar{l}	8
   	$u_L d_L u_R d_R$	$\pm 1/2 \dots$	$\text{even}\#>0$	0	0	$-1/2$	$\pm 1/2 \dots$	$\text{two}>0$	0	8_{S+}	q_I	24
   	$\bar{u}_L \bar{d}_L \bar{u}_R \bar{d}_R$	$\pm 1/2 \dots$	$\text{even}\#>0$	0	0	$1/2$	$\pm 1/2 \dots$	$\text{one}>0$	0	8_{S+}	\bar{q}_I	24
   	$\nu_{\mu L} \mu_L \nu_{\mu R} \mu_R$	$\pm 1/2 \dots$	$\text{odd}\#>0$	0	0	$-1/2$	$1/2$	$1/2$	$1/2$	8_{S-}	l	8
   	$\bar{\nu}_{\mu L} \bar{\mu}_L \bar{\nu}_{\mu R} \bar{\mu}_R$	$\pm 1/2 \dots$	$\text{odd}\#>0$	0	0	$1/2$	$-1/2$	$-1/2$	$-1/2$	8_{S-}	\bar{l}	8
   	$c_L s_L c_R s_R$	$\pm 1/2 \dots$	$\text{odd}\#>0$	0	0	$1/2$	$\pm 1/2 \dots$	$\text{two}>0$	0	8_{S-}	q_I	24
   	$\bar{c}_L \bar{s}_L \bar{c}_R \bar{s}_R$	$\pm 1/2 \dots$	$\text{odd}\#>0$	0	0	$-1/2$	$\pm 1/2 \dots$	$\text{one}>0$	0	8_{S-}	\bar{q}_I	24
   	$\nu_{\tau L} \tau_L \nu_{\tau R} \tau_R$	± 1	± 1	0	0	1	0	0	0	8_V	1	8
   	$\bar{\nu}_{\tau L} \bar{\tau}_L \bar{\nu}_{\tau R} \bar{\tau}_R$	± 1	± 1	0	0	-1	0	0	0	8_V	1	8
   	$t_L b_L t_R b_R$	± 1	± 1	0	0	0	-1	0	0	8_V	q_{II}	24
   	$\bar{t}_L \bar{b}_L \bar{t}_R \bar{b}_R$	± 1	± 1	0	0	0	1	0	0	8_V	\bar{q}_{II}	24
	g	0	0	0	0	0	1	-1	0	1	$A2$	6
   	$x_1\Phi$	0	0	0	0	-1	± 1	0	0	1	q_{III}	6
   	$x_2\Phi$	0	0	0	0	1	± 1	0	0	1	q_{III}	6
   	$x_3\Phi$	0	0	0	0	0	$\pm(1 \ 1)$	0	0	1	q_{III}	6

Pic. 1 - - Group E8 fields based on one of the subgroup types [12].

3. E8×E8' in cosmology

Group E8 describe «ordinary» world, E8' «mirror», which interact exclusively gravitationally. As a result, both worlds, existing in parallel, practically do not feel each other after the Planck era. Each of them establishes its own thermodynamic equilibrium due to its internal non-gravitational interactions. In principle, galaxies can exist in which either ordinary or shadow matter predominates.

3.1 Baryon asymmetry

Model E8×E8' is supersymmetric model, which makes it possible to describe the baryon asymmetry with Affleck-Dine-Linde mechanism [13]. In the supersymmetry theory, ordinary quarks and leptons have scalar partners that carry baryon and lepton numbers. Since the latter decay into fermions in the early universe, the net baryon number they carry may then form the currently observed excess of ordinary baryons. This is due to the interaction of scalars with the inflaton field.

3.2 Dark matter

The mirror sector E8' contains 248 unknown particles and 248 unknown interaction fields, which can contribute to the dark mass.

The contribution can be estimated as an additional density in the parameters of the Hubble parameter:

$$H(z) = H_0 \left[\Omega_r (1+z)^4 + \Omega_m (1+z)^3 \Omega_\Lambda \right]. \quad (2)$$

For example, in the model reviewed by Zurab Berezhiani ($SU(3) \times SU(2) \times U(1)$) [8] Ω_r consists of ordinary and dark photons and neutrinos, Ω_m consists of ordinary and dark baryons, and the contribution of another dark matter not described by the model is not excluded. Accounting for this kind of additional terms affects the co-radiation. When using the parameters described in the article, it is possible to describe the observed spectrum of relic radiation with high accuracy.

Another available version of the contribution to the dark matter in theory E8×E8' was shown in the article [14]. Nontrivial homotopy group causes the appearance of new states of a closed string. The mass of string is determined by the dimensions of the complementary spaces. You can choose the masses such

that such structures turn into black holes. In this case, with different model parameters. It is possible to estimate the contribution of such black holes to the dark matter of the Universe and obtain restrictions on this theory from cosmology.

In the article [15] it has been shown that the scalar field involved in supersymmetric theories can make a large contribution to the dark matter density. Also, stable neutral particles that appear as supersymmetric partners (neutralinos, heavy neutrinos, photinos, etc.) are possible candidates for a weakly interacting matter particle (WIMP). Accelerators are actively searching for such candidates [16], but so far without success.

4. Problems in string theory

Landscape Problem with a finite form of Calabi-Yau space is still unresolved. On macroscopic scales, the theory should be reduced to the well-known and very well tested particle physics. But, as it turns out, there are at least ways to do this. At the same time, each of the resulting four-dimensional theories describes its own world, which may be similar to reality, or may be fundamentally different from it. This kind of uncertainty gave rise to the **anthropic principle**, the idea that we live in a world with such parameters of physics, because life could not have arisen with others. This concept makes the theory difficult to test.

The problem of the possibility of testing the theory. It is impossible to conduct an experiment under terrestrial conditions to test string theory in the foreseeable time, due to technological imperfection, which lowers the credibility of the theory. Constraints from cosmology are difficult to draw because of the model-dependent results of different theories.

5. Toy model

The main property of string theory is the absence of divergences. In order to show this process, consider the following toy model. We will have two kinds of particles, Majorana fermion field and complex boson field. And the corresponding terms of the Yukawa interaction and the 4-boson interaction. The Lagrangian of such a model can be written as follows:

$$L = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi \phi^* + \quad (3)$$

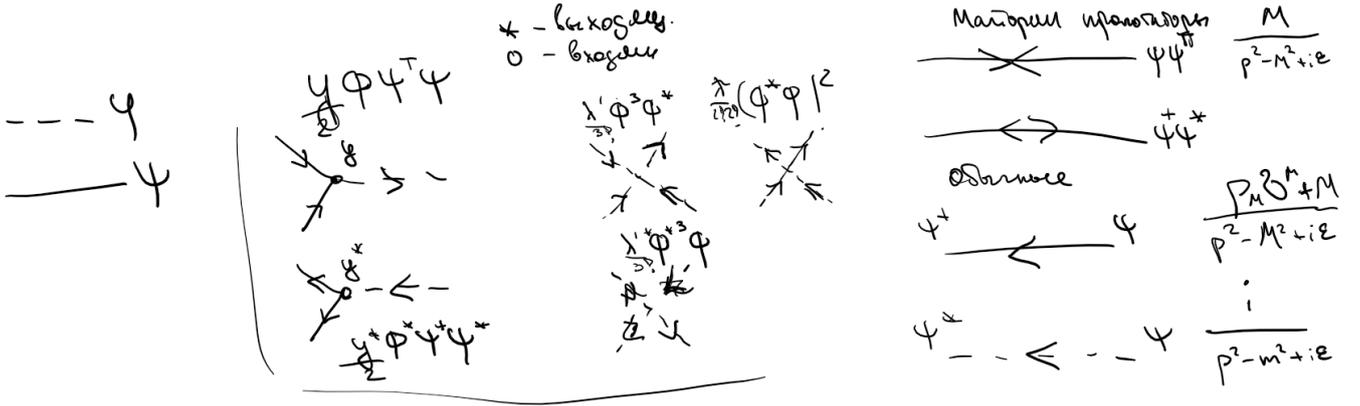
$$+ \psi^T \sigma^m u \partial_m u \psi - \frac{M}{2} (\psi^T \psi + \psi^+ \psi^*) + \quad (4)$$

$$+ \frac{1}{3!} \phi \phi^* (\lambda' \phi^2 + \lambda^* \phi^{*2}) + \frac{1}{2!2!} \tilde{\lambda} (\phi \phi^*)^2 + \quad (5)$$

$$+ \frac{y}{2} \phi \psi^T \psi + \frac{y^*}{2} \phi \psi^+ \psi^*. \quad (6)$$

We get a kind of SUSY toy model. Let us show that loop diagrams of interactions can cancel each other if the model parameters are chosen correctly.

Firstful, we find Feinman rules (2) of this model. Than we look for all one loop diagrams in this theory.



Pic. 2 - - Fainman rules for our toy model

we will look for easy process of scalar field propogation. So our loop diagrams will be radiative corrections to mass of scalar(boson) field. We can find only 6 one-loop diagrams: 3 with boson propogator and 3 with fermion (they shown in (3)). Now we can estimate amplitudes of all this diagrams:

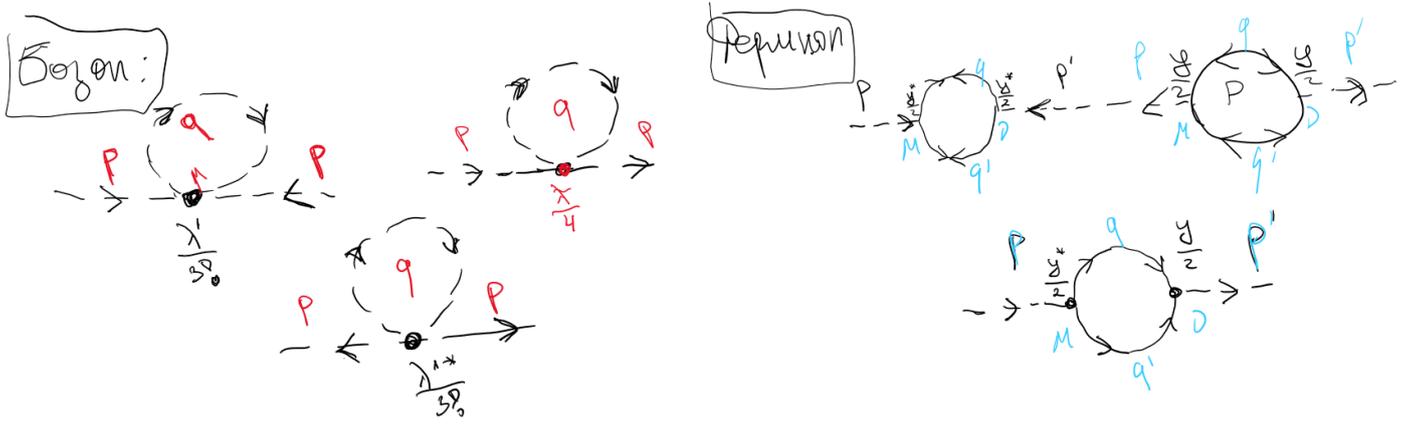
$$A_{f1} = -yy^* \int \frac{d^4q}{(2\pi)^4} \gamma_\nu \frac{q_\rho \gamma^\rho - M}{q^2 - M^2 + i\epsilon} \cdot \frac{(p-q)_\sigma \gamma^\sigma - M}{(p-q)^2 - M^2 + i\epsilon} \gamma^\mu,$$

$$A_{f2} = y^* y^* \int \frac{d^4q}{(2\pi)^4} \gamma_\nu \frac{M^2}{(q^2 - M^2 + i\epsilon)((p-q)^2 - M^2 + i\epsilon)} \gamma^\mu,$$

$$A_{f3} = yy \int \frac{d^4q}{(2\pi)^4} \gamma_\nu \frac{M^2}{(q^2 - M^2 + i\epsilon)((p-q)^2 - M^2 + i\epsilon)} \gamma^\mu$$

$$A_{b1} = i\tilde{\lambda}^2 \int \frac{d^4q}{(2\pi)^4} \frac{4 - q^2/m^2}{q^2 - m^2 + i\epsilon},$$

$$A_{b2,b3} = i\lambda\lambda^* \int \frac{d^4q}{(2\pi)^4} \frac{4 - q^2/m^2}{q^2 - m^2 + i\epsilon}.$$



Pic. 3 - - All one-loop diagrams

Calculation of these amplitudes is reduced to the calculation of loop integrals. At the moment, the question of methods for calculating these integrals is being studied.

After their calculation, the amplitudes will be functions of the initial momentum of the system, particle masses and interaction constants:

$$A_f(y, y^*, M, m) = A_b(\lambda, \lambda^*, \tilde{\lambda}, m, M). \quad (7)$$

By solving this equation, we can obtain the condition of the model parameters, under which we obtain the reduction of divergences for our process.

6. Conclusion

Superstring theory fits perfectly into the modern theory of elementary particles. They pass in the low-energy limit into supersymmetric grand unified theories. The main advantages of the superstring approach are the following:

- Allows you to combine all fundamental interactions.
- Anomaly Cancellation
- Allows in a natural way to obtain the parameters of gauge theories from first principles.
- Explains some of the cosmological problems.

Also noteworthy are the fundamental questions in superstring theory, the solution of which is necessary to substantiate it:

- landscape problem
- theory testing problem
- dynamic substantiation of the compactification process

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