

# Brane world cosmology

Lecture from the course  
“Cosmoparticle Physics”

# Different approaches of investigation

## Fundamentally theoretic approach.

superstring  $\cup$  supergravity  $\subset$  M-theory (?)

D=10

D $\leq$ 11

## Phenomenological approach.

D=4+d

Studying possible manifestations of extra dimensions and possibilities of solutions of basic problems facing the M-theory (hierarchy of electroweak and Planck scales:  $200\text{GeV} \leftrightarrow 10^{19}\text{GeV}$ , and others) by the example of simple (maybe inconsistent with fundamental theory) models.

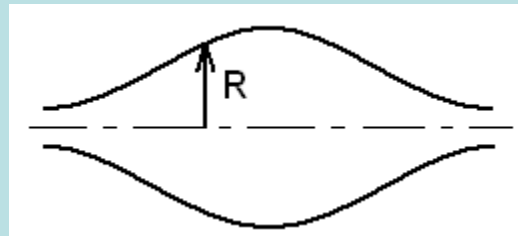
# 1. Kaluza-Klein (KK) models

$$g_{MN} = \begin{pmatrix} g_{\mu\nu} & g_{\mu n} \\ g_{m\nu} & g_{mn} \end{pmatrix}$$

$M, N = 0, \dots, D$   
 $\mu, \nu = 0, 1, \dots, 3$   
 $m, n = 4, \dots, D$   
⏟  
 $d$

Describes other interactions

If extra  $d$  dimensions are compactified with  $R = \text{const}$ , then they are manifested in form of the **KK-mass states** for all particles, if  $R \neq \text{const}$  – in form of **interaction**.



# Superstring theory, M-theory

*Each species of particles is a certain vibration mode of a string – fundamental element of zero thickness and of Planckian length. Strings may be closed and open.*

Most popular string model is a so called *heterotic string* (closed) – combines the models of *superstring* and of *bosonic string*.

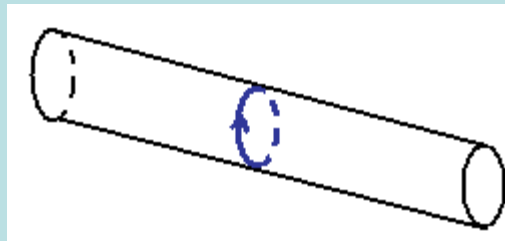


Fixed gauge groups:  $E_8 \times E_8$ ,  $SO(32)$



Each contains 248 bosons  
and 248 fermion fields

Possible *consequence*: existence of homotopically stable particle



Their mass is proportional to  
the radius of compactification

Superstring  $\cup$  supergravity  $\subset$  M-theory (?)

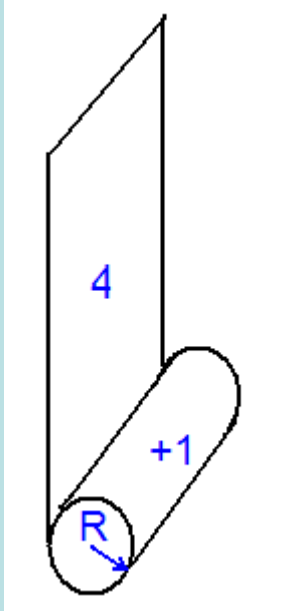
D=10

D $\leq$ 11

# 2. Progress of phenomenological approach

## 1) Kaluza-Klein type models.

$$M_4 \times K_d$$



Solution of Klein-Gordon equation in 5 dimensions  
(4 ordinary ( $x_\mu$ ,  $\mu = 0, \dots, 3$ ) + 1 cylindrical ( $z$ )) for massless field

$$\partial_{(5)}^2 \phi = 0$$

gives

$$\phi = \exp(ip_\mu x^\mu) \exp\left(\frac{inz}{R}\right) \quad n = 0, \pm 1, \pm 2, \dots$$

Whence we have

$$p^2 = \frac{n^2}{R^2} \quad n = 0, \pm 1, \pm 2, \dots$$

From point of view 4d space-time a particle has got mass states – “KK-states”

$$m_n = \frac{|n|}{R}$$

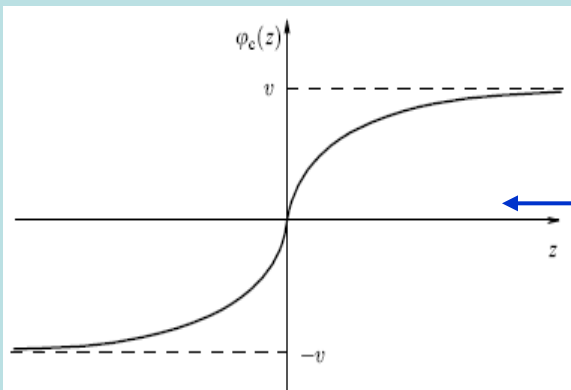
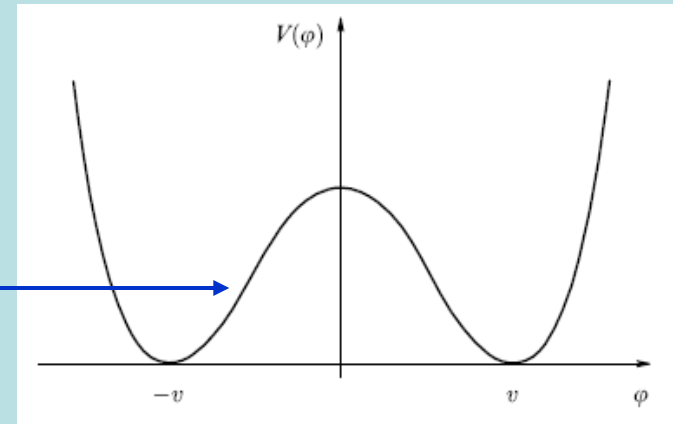
Modern experiment restricts extra dimension manifestation so as  
 $1/R > \text{a few} \times 100 \text{ GeV} \Rightarrow R < 10^{-17} \text{ cm}$ , i.e. extra  
dimension(s) should be very compact (with microscopic size).

# 2. Progress of phenomenological approach

## 2) Capture of the fields by 4d manifold (brane)

The matter fields can be localized (“captured”) within narrow hyper-surface (“brane”), while extra dimension(s) can be **large**. This can be realized with the help of scalar field interacting with usual matter field as demonstrated below.

Let a scalar field ( $\phi$ ) in 5 (infinite) dimensions has potential as shown.



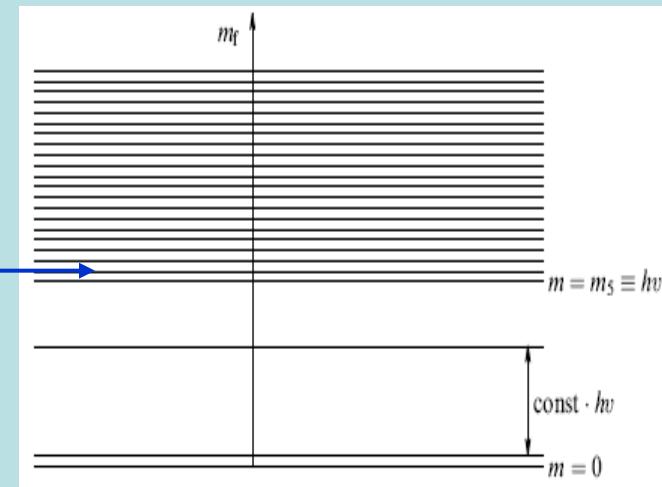
It leads to domain wall dividing 5d space-time on two parts with respect to one ( $z$ ) of the dimensions.

# 2. Progress of phenomenological approach

## 2) Capture of the fields by 4d manifold (brane)

Let a fermion ( $\psi$ ) has a Yukawa coupling with given scalar field ( $h\phi\psi\psi$ ).

Then fermion gets mass states as shown on the picture, where zero-state turns to be localized (captured) near  $z=0$  (domain wall), damping as  $\exp(-h\nu|z|)$ .



Mechanism of the capture of gauge fields must be more complicated.

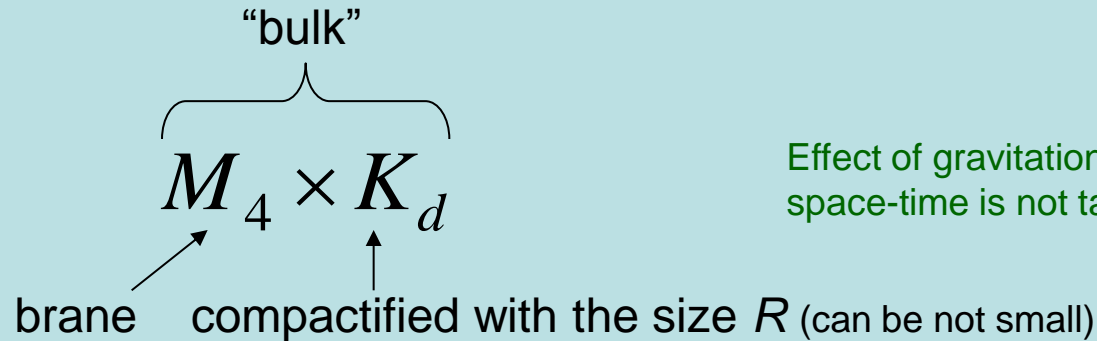
### Possible consequences:

- 1)  $E \sim h\nu \Rightarrow e^+e^- \rightarrow \text{nothing}$  (products leave 4d space-time)
- 2) Decay of massive particles, e.g. DM.

## 2. Progress of phenomenological approach

### 3) ADD type models (or low scale gravity).

ADD – Arkani-Hamed, Dimopoulos, Dvali



Effect of gravitation of brane in  $4+d$  space-time is not taken into account.


Ordinary matter is considered to be captured on 4d brane ( $M_4$ ) of the width  $\Delta z \ll R$ . Gravity, contrary to ordinary matter, propagates over all  $D=4+d$  space-time ( $M_4 \times K_d$  – "bulk").



## 2. Progress of phenomenological approach


### 3) ADD type models (or low scale gravity).

In given case the law of gravity has the form

$$V(r) \cong \begin{cases} \frac{m_1 m_2}{M_{\text{Pl}}^{d+2}} \frac{1}{r^{d+1}}, & r \ll R \\ \frac{m_1 m_2}{M_{\text{Pl}}^{d+2}} \frac{1}{R^d r}, & r \gg R \end{cases} \quad \longrightarrow \quad m_{\text{Pl}}^2 = M_{\text{Pl}}^{2+d} R^d$$


Effective 4d Planck mass ( $m_{\text{Pl}}$ ) can be much larger than *true* “bulk” Planck mass ( $M_{\text{Pl}}$ ) and the last one can be as small as electroweak scale ( $\sim 1\text{TeV}$ ) provided large enough  $R$ . That is hierarchy problem of particle physics is *re-formulated* in terms of large extra dimensions.

We have

$$R \sim M_{\text{Pl}}^{-1} \left( \frac{m_{\text{Pl}}}{M_{\text{Pl}}} \right)^{2/d} \sim (\text{for } M_{\text{Pl}} \sim 1\text{TeV}) \sim 10^{-17+32/d} \text{cm} \leq R^{\text{exper}} < 0.2\text{mm}$$


( $0.2\text{mm} \approx \text{meV}^{-1}$ )

$d=1$  is excluded, while  $d \geq 2$  is allowed.

## 2. Progress of phenomenological approach

### 3) ADD type models (or low scale gravity). Possible consequences

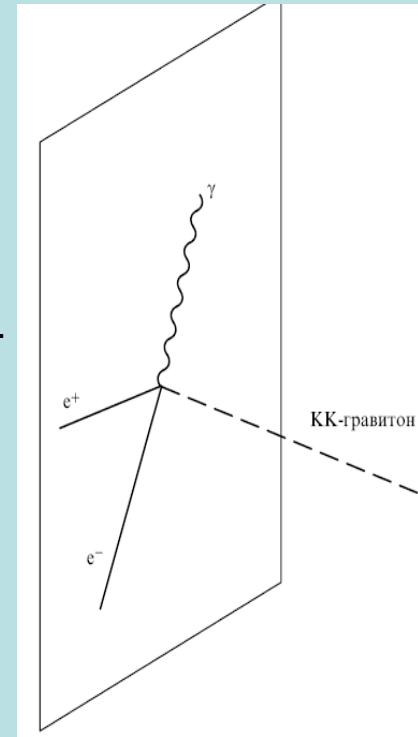
At  $E \sim M_{\text{Pl}}$  there should be **copious production of KK-gravitons** with  $m_n = n/R < E$  ( $\Delta m = 1/R < 1 \text{ meV}$ )

$$e^+ e^- \rightarrow \gamma + \text{nothing}$$

$$\sigma \sim \frac{\alpha}{m_{\text{Pl}}^2} N_{\text{KK}}(E), \quad N_{\text{KK}}(E) = \left( \frac{E}{\Delta m} \right)^d - \text{number of KK-states with } m < E.$$

Large  $N_{\text{KK}}(E \sim M_{\text{Pl}})$  compensates small  $1/m_{\text{Pl}}^2$ , so  $\sigma \sim \sigma_{\text{e/m}}$ .

**Miniblack hole formation** at  $E \sim M_{\text{Pl}}$ .



## 2. Progress of phenomenological approach

### 3) ADD type models (or low scale gravity): Possible consequences

Problem of overproduction of KK-gravitons in the early Universe.

At  $T \gg 1/R$  KK-gravitons are produced with the rate

$$\dot{N}_{G_{KK}} = n_a n_b \left\langle \sigma_{ab \rightarrow G_{KK}} v_{ab} \right\rangle V \Rightarrow \left\{ \begin{array}{l} V = N_\gamma / n_\gamma \\ n_{a,b,\gamma} \sim T^3 \\ dt \sim m_{\text{Pl}} dT / T^3 \\ \langle \sigma v \rangle \sim N_{KK} m_{\text{Pl}}^{-2} \end{array} \right\} \Rightarrow$$

$$n_{G_{KK}} = n_\gamma \int_{t(T=T_R)}^{t(T \sim m_{G_{KK}})} \frac{\dot{N}_{G_{KK}}}{N_\gamma} dt \sim n_\gamma \int_0^{T_R} T^6 m_{\text{Pl}}^{-2} \left( \frac{T}{\Delta m} \right)^d T^{-3} \frac{m_{\text{Pl}} dT}{T^3} \sim \frac{T_R}{m_{\text{Pl}}} \left( \frac{T_R}{\Delta m} \right)^d n_\gamma$$

$$\varepsilon_{G_{KK}} \sim (m_{KK} \sim T) \sim \frac{T_R^2}{m_{\text{Pl}}} \left( \frac{T_R}{\Delta m} \right)^d n_\gamma$$

## 2. Progress of phenomenological approach

### 3) ADD type models (or low scale gravity): Possible consequences

For agreement with BBN one requires

$$\varepsilon_{G_{KK}} \sim \frac{T_R^2}{m_{Pl}} \left( \frac{T_R}{\Delta m} \right)^d n_\gamma < T_{n/p} n_\gamma$$
$$T_R \leq M_{Pl} \left( \frac{T_{n/p}}{0.1 m_{Pl}} \right)^{\frac{1}{2+d}} \sim (0.6 \cdot 10^{-6} \div 2 \cdot 10^{-3}) M_{Pl} \quad \text{for } d=2 \div 6 \quad > T_{n/p} \approx 1 \text{ MeV}$$

Conclusion is not changed in principle, if KK-gravitons are decayed before BBN.

Low  $T_R$  ( **$\sim \text{MeV} \div \text{GeV}$** ), as obtained for  $M_{Pl} \sim 1 \text{ TeV}$ , puts forward the questions of baryosynthesis, inflation, dark matter origin in the early Universe.

$M_{Pl} \gg 1 \text{ TeV}$  ( $> 10 \div 30 \text{ TeV}$ ) is preferable.

Satisfying to  $\Omega(\text{modern})$  in case of stable KK-gravitons intensifies constraints.

**Constraint from SN1987:** agreement with an observed neutrino signal requires

$$M_{Pl}(d=2) \geq 30 \text{ TeV} \quad \text{Generally it agrees with constraints from cosmology.}$$

## 2. Progress of phenomenological approach

### 3) ADD type models (or low scale gravity): Possible consequences

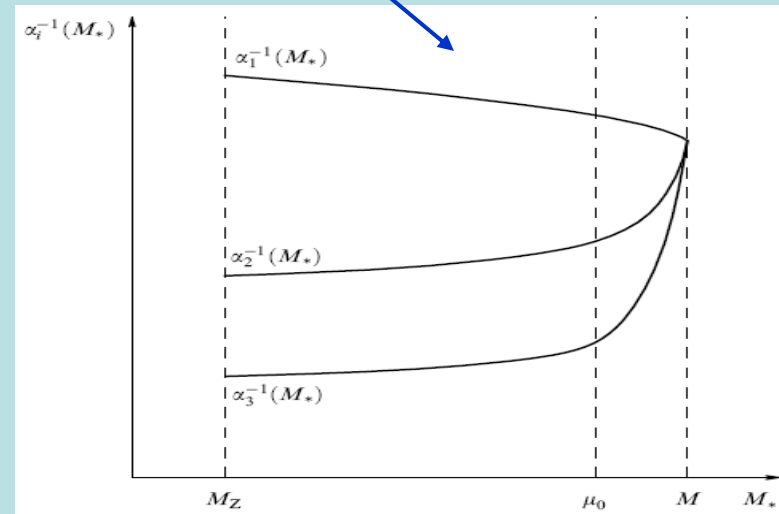
**Solution of small masses of known neutrinos** due to sterile neutrino with KK-states and mixed with ordinary ones. It can be provided, as well as for a bulk Plank mass, by large extra dimensions.

$$m_\nu \sim \frac{vev}{(M_{Pl}R)^{d/2}} \sim \frac{vev \cdot M_{Pl}}{m_{Pl}}, \quad vev \sim m_W$$

All possible **physics of Grand Unification** should take place at  $E \leq M_{Pl}$ . In this case gauge constants should run following another law: a power-like law can be suggested (logarithmic law changes onto power-like starting from some  $\mu$  until  $M_{Pl}$ ).

As a consequence of low energetic GUT physics, a **problem of fast proton decay** appears.

$$\tau_p \sim \frac{1}{\alpha_{GUT}^2 m_p} \left( \frac{M_{GUT}}{m_p} \right)^4$$

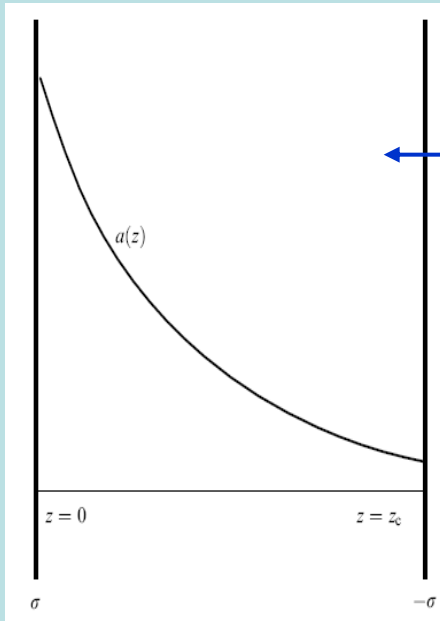


# 2. Progress of phenomenological approach

## 3) RS type models (or low scale gravity).

RS – Randall, Sundrum

There are attempts to realize idea of solving hierarchy problem due to extra dimensions taking properly into account effect of gravity in a bulk space.



← In such configuration 4d gravity constant on the brane at  $Z=Z_c$  (with negative tension, RS-1) turns out to be reduced exponentially as compared to a bulk (5d) gravity constant.

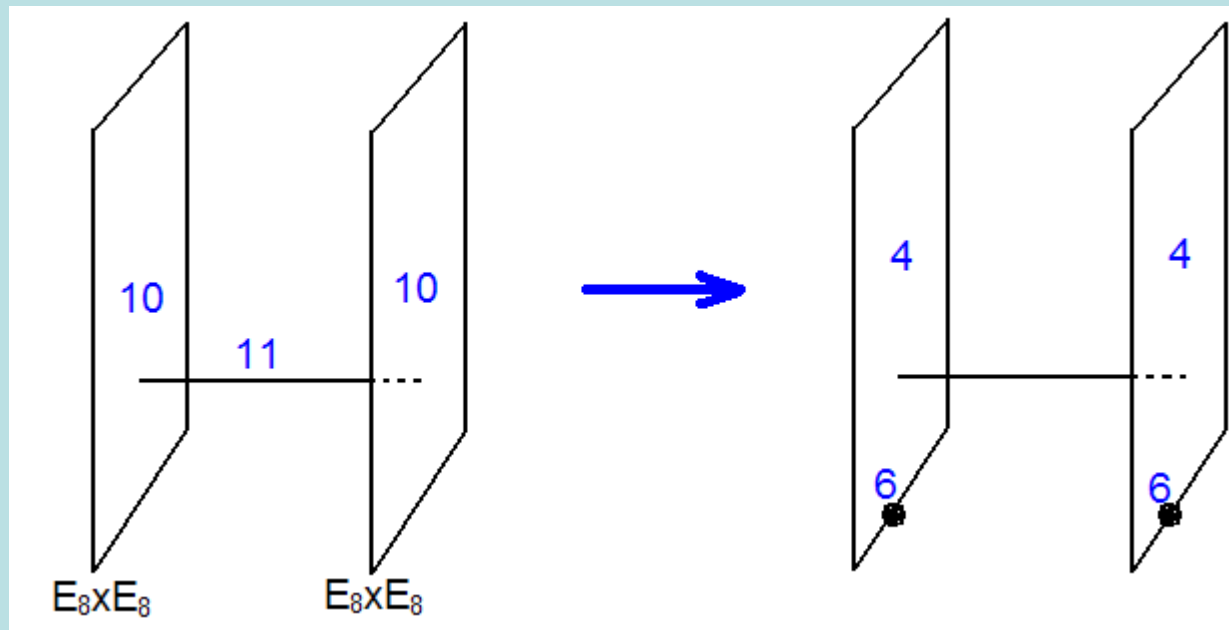
$$m_{\text{Pl}} \sim \exp(kz_c) M_{\text{Pl}}$$

$k$  – curvature of the bulk space, can be  $\sim M_{\text{Pl}}$ .  
 Note, 5d gravity constant has dimension here  $M_{\text{Pl}}^{-3}$ .

Lightest KK-gravitons has  $m \sim k$  (from point of view of our brane) and interaction strength  $\sim \sqrt{k/M_{\text{Pl}}^3}$ .

There are different variations of such kind model: model with our world on the  $z=0$  brane (RS-2: in this case there is no exponential suppression of our 4d gravity constant with respect to 5d one, but electroweak symmetry breaking can be connected with the second brane ( $z=z_c$ ) and in this manner relative exponential suppression can be reached), model with 3 branes, with intersecting branes.

### 3. Progress of fundamentally theoretic approach



Horava P, Witten E 1996 Nucl.Phys.B460 506; Nucl.Phys.B475 94

Remark: In principle, in string theory the tips of open strings can belong to the only brane – D-brane, which, in turn, could be 3-dimensional. However, it is not most likely to correspond to a realistic theory, and both different D-branes and closed strings (as in the case of heterotic string shown above) are considered.

## 4. Birth of brane world

There is no definite way of elaboration of brane cosmological model (including description of early Universe).

One of the scenario discussed is a *cyclic model* with colliding branes (Steinhardt P. J., Turok N. 2002 *Science* 296 1436; *ibid* 2002 *Phys.Rev.D* 65 126003) – alternative to inflationary models.

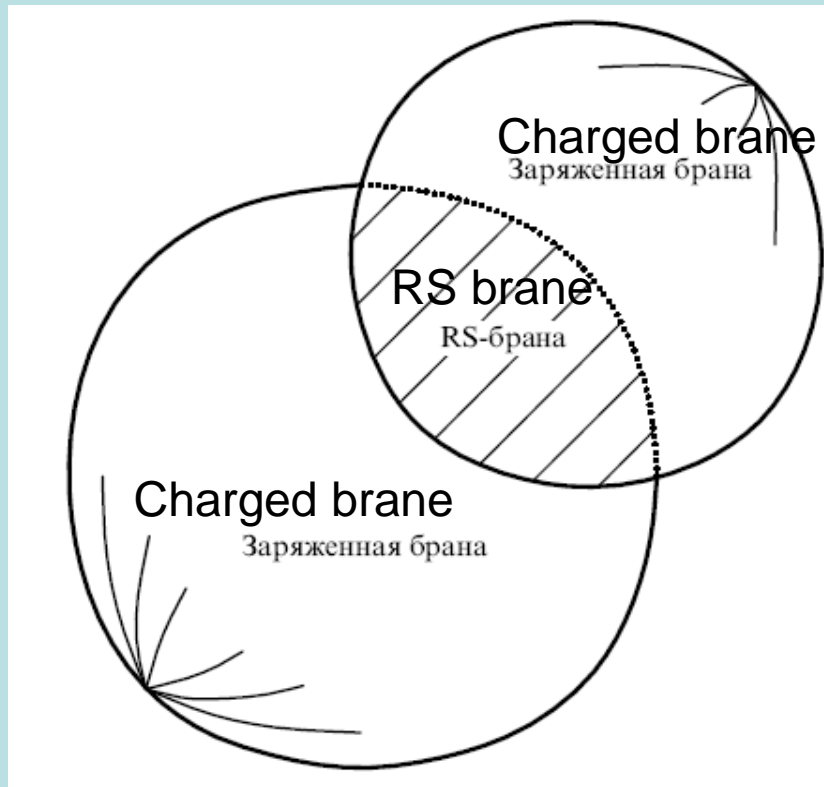
However, physics of brane collision is unclear.



## 4. Birth of brane world

A birth of brane(s) filled with matter and an empty interior bulk is a greater problem.

One possibility is illustrated:



# Conclusions

- The extensions of Standard Model involve extra dimensions, which may not be small (compactified).
- In the case of infinite extra dimensions physics of the standard model can be localized on « branes », while gravity comes from the bulk space.
- Large extra dimensions can lead to significant effects of gravity at scales, much smaller, than 4D Planck scale.
- The test for extra dimensions is a challenge for cosmoparticle physics. Analysis of their signatures in astrophysical data and at accelerators can shed light on their existence.