Mechanisms of producing PBHs and their evolution

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Abstract

PBHs are nowadays one of the most attractive and fascinating research areas in cosmology for their possible theoretical and observational implications. In this paper we present a review of different mechanisms for generating PBHs and approasches to study their evolution via mass accretion.

Keywords Accretion, phase transition, $f(R)$-gravity, primordial black holes, cosmology, domain wall.

Contents

1 Introduction

The idea of primordial black holes (PBH) formation in early universe was suggested in 1967 by Zeldovich and Novikov [1], and also by Hawking and Carr [2–4]. PBHs have been a subject of interest for fifty years. In particular, only PBHs by construction can be light enough for the so-called Hawking radiation to be essential to change the mass of a black hole. Many different mechanisms for the formation of PBHs have been proposed: density fluctuations [2, 3], first order phase transitions [4, 5], cosmic string collapse [6], appearance of PBHs in hybrid inflation models [7], second order phase transitions [8, 9], reviews of PBH formation mechanisms are given in [10, 11]. Recently a new mechanism of formation of PBHs discovered in the multidimensional modified gravity model [12, 13], containing tensor and quadratic to scalar curvature corrections, will be considered. The possibilities of f(R)-gravity are widely studied [14, 15], they offer solutions to many cosmological problems [16–19]. Today, the primordial origin of some discovered black holes (quasars at small z [20, 21], BHs of intermediate masses detected by gravitational-wave observatories [22]) is hotly discussed [23, 24].

The next aspect to be discussed is accretion process. Accretion is the process in which a black hole can capture particles from a nearby source of fluid and there is increase in mass as well as angular momentum of the accreting object, see e.g. [25]. There are different approaches to study this process and some of them are reviewed here.

It is also worth noting that black holes of primordial origin can at least partially fill the hidden mass [26]. At the moment, restrictions are imposed on a wide range of masses of PBH as a dark matter candidate [27].

2 Production mechanisms

2.1 First order phase transitions

Let us consider the mechanism for producing PBHs by means of scalar field dynamics. For realization of this mechanism it is necessary that the field potential has at least two minima, one of them must be false. Let the field be in a false vacuum ϕ_1 at the initial moment of time, and denote the true vacuum by ϕ_0 (Fig. 1). As a result of quantum tunneling, in one region of space the field will have a value of ϕ_1 and in another region of space– ϕ_0. These regions are called bubbles. In this formulation, the free energy of a bubble consists of two parts — volumetric and surface energy. We denote the surface energy density μ and the difference of potential values at minima $\Delta V = E(\phi_0) - E(\phi_1)$. In this case the free energy of a bubble with radius R can be written as

$$F(R) = 4\pi R^2 \mu - \frac{4\pi}{3} R^3 \Delta V. \tag{1}$$

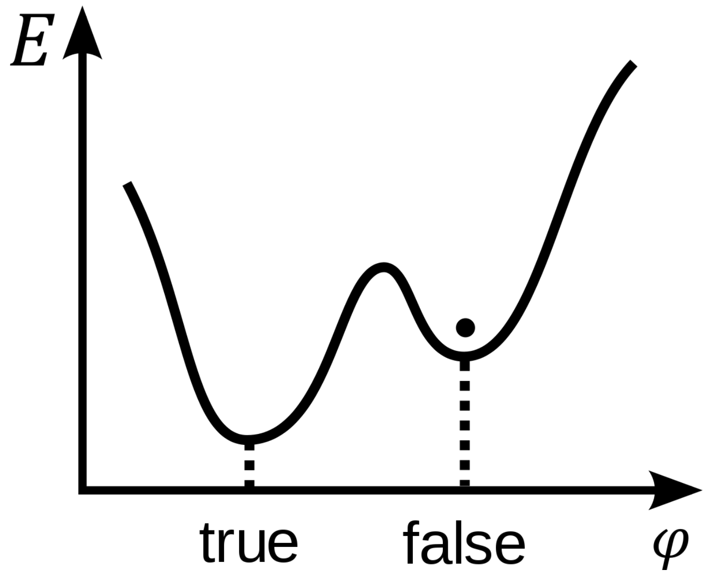

Figure 1: Schematic representation of a scalar field potential in which phase transitions of the first kind are possible.

Obviously, the dependence (1) has a maximum at the point $R_{cr} = 2\mu/\Delta V$, after which it becomes energetically advantageous for the bubble to expand infinitely. Then expansion of bubbles of true vacuum in the region of false vacuum becomes possible, while potential energy of false vacuum is converted into kinetic energy of the walls, which leads to ultra-relativistic speed of expansion in short time.

When a pair of bubbles of true vacuum collide, a new bubble of false vacuum can arise in the region of false vacuum. If its size is smaller than its gravitational radius, it becomes a black hole for a distant observer. However, if the bubble shrinks to a size comparable to the wall thickness $d \sim \delta > r_g$, the collapse in the PBH will not occur, since the bubble will oscillate and lose energy and finally will decay by tunneling.

2.2 Second order phase transitions

In contrast to first order phase transitions, the medium's parameters change continuously, instead of jumping. These phase transitions involve cosmological inflation. For realization of this mechanism it is necessary that the field potential possesses at least one nodal point.

Let the field be near the maximum of the potential at the initial moment of time. Classical motion of the field would lead to the field rolling into one of the minima, however, as a result of quantum fluctuations at the inflationary stage and "freezing" of the classical motion, the field can tunnel through the barrier and eventually in one region of space rolls into one minimum and in another region, respectively, into another minimum. These two minima will be connected by a domain wall.

Characteristic scale of non-vanishing fluctuation on inflationary stage is H_{inf}^{-1}. If it is formed at the moment t' during inflation so by the end of inflaton domination it would be $e^{N_{\text{inf}} - H_{\text{inf}} t'}$ times bigger. Its further evolution depends on the relation between two timescales — $t_\sigma = 1/2\pi G\sigma$, where σ is surface energy density of the wall, and moment at which wall crosses the cosmological horizon — t_H.

If $t_\sigma \gg t_H$ the wall is called subcritical and black hole forms much earlier before wall could become dominant in the Universe. But in case $t_\sigma \lesssim t_H$ wall called supercritical and there is a wormhole formation as a way to export problem of wall domination to baby universe.

See [28–31] and references therein for detailed analysis.

2.3 PBH production in $f(R)-$gravity

The idea of the proposed mechanism is based on the known possibility of formation of domain walls during cosmological inflation followed by their collapse into primordial black holes [31, 32]. The formation of such domain walls requires a scalar field with a nontrivial potential containing several vacuums. It is this effective scalar field that arises in multidimensional f(R)-models in Einstein frame [13, 33, 34]. This field controls the size of compact additional space, and its different vacuums correspond to different universes. In the paper the parameters of the domain walls formed by the field are calculated and we conclude that, having appeared at the stage of inflation, they will immediately collapse into PBH during rehitting. For a distant observer in the Jordan frame the appearance of such BHPs is interpreted as a manifestation of nontrivial $f(R)-$gravitational dynamics of multidimensional space.

The model considered contains quadratic and tensor corrections to scalar curvature:

$$S[g_{\mu\nu}] = \frac{m_D^{D-2}}{2} \int d^{4+n}x \sqrt{|g_D|} \left[f(R) + c_1 R_{AB} R^{AB} + c_2 R_{ABCD} R^{ABCD} \right] ,$$

$$f(R) = a_2 R^2 + R - 2\Lambda_D \,, \tag{2}$$

Multidimensional space is represented as a direct product $\mathbb{M} = \mathbb{M}_4 \times \mathbb{M}_n$, where $\mathbb{M}_4$ is a four-dimensional space, $\mathbb{M}_n$ is a compact extra space with n dimensions:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu - e^{2\beta(t)} d\Omega_n^2 \,. \tag{3}$$

One can obtain scalar curvature:

$$R = R_4 + R_n + P_k \,, \quad P_k = 2n\,\partial^2\beta + n(n+1)(\partial\beta)^2 \,,$$

where R_4, R_n — scalar curvature for $\mathbb{M}_4, \mathbb{M}_n$. As shown in [13] in the limit of effective field theory:

$$R_4, P_k \ll R_n \,, \tag{4}$$

one obrain effective field theory where scalar curvature of extra space is considered as scalar field.

$$S = \frac{m_4^2}{2} \int d^4x \sqrt{-g_4}\, \mathrm{sign}(f') \left[R_4 + K(\phi)(\partial\phi)^2 - 2V(\phi) \right] , \tag{5}$$

where effective 4-dimensional Planck mass in Einstein frame: $m_4 = \sqrt{2\pi^{\frac{n+1}{2}}/\Gamma(\frac{n+1}{2})}$ and $g_4^{\mu\nu}$ — observable 4-dimensional metric.

Effective potential $V(\phi)$ occured to be appropriate for domain wall formation. Kinetic term and potential are given by expressions [34]):

$$K(\phi) = \frac{1}{4\phi^2} \left[6\phi^2 \left(\frac{f''}{f'} \right)^2 - 2n\phi \left(\frac{f''}{f'} \right) + \frac{n(n+2)}{2} \right] + \frac{c_1 + c_2}{f'\phi} \,, \tag{6}$$

$$V(\phi) = -\frac{\mathrm{sign}\,(f')}{2(f')^2} \left[\frac{|\phi|}{n(n-1)} \right]^{n/2} \left[f(\phi) + \frac{c_1 + 2c_2/(n-1)}{n} \phi^2 \right] . \tag{7}$$

By the end of cosmological inflation field dynamic is no more "freezed" so walls will arise at some point and BHs would appear. For details see [12] and references therein. Although the mass spectrum is yet to be analyzed in this model.

3 Mass accretion mechanisms

3.1 Bondi accretion

One of the first solutions to the accretion problem was that of Hoyle and Littleton [35], but the analytical formula was derived by Bondi [36]. Below is an overview of Bondi's solution.

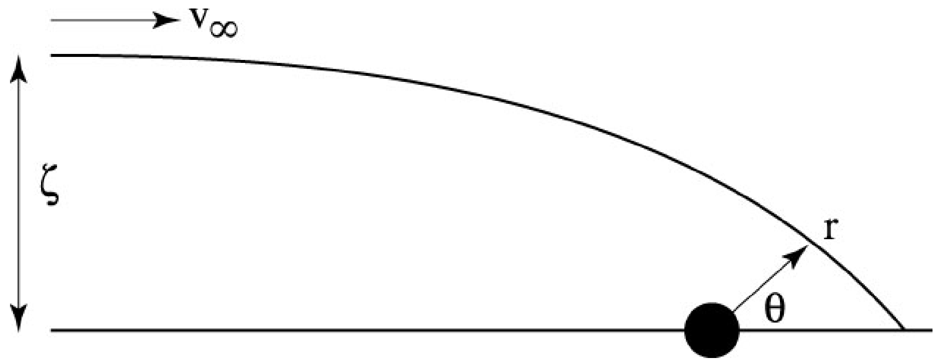

Figure 2: Schematic representation of the problem statement. ζ — impact parameter, v_∞ — velocity of the test particle at large distance from the accreting object.

Bondi accretion is a spherically symmetric accretion on a compact object. The accretion rate is assumed to be $\dot{M} \approx \pi R^2 \rho v$, where R — the capture radius or impact parameter, ρ — the density of the surrounding matter, and v — the relative speed. The capture radius can be determined from the equality of the escape velocity and some characteristic velocity of matter. It is usually assumed to be equal to the speed of sound in the surrounding matter and then the accretion rate is obtained: $\dot{M} \approx \pi \rho G^2 M^2 / c_s^3$, where c_s is the speed of sound in the matter surrounding the compact object.

The obvious drawback of the model is that it does not take into account possible relativistic effects, it (the model) is Newtonian. However, it is possible in this approach to take into account the expansion of the Universe [37]. The Bondi problem can be posed as follows:

$$\dot{M} = 4\pi r^2 \rho v, \tag{8}$$

$$v\,\frac{dv}{dr} = -\frac{1}{\rho}\frac{dp}{dr} - \frac{GM(<r)}{r} - \beta(z)v, \tag{9}$$

$$p = K\rho^\gamma \tag{10}$$

where $\beta(z)$ — the viscosity coefficient of the plasma around the accretor due to the interaction of electrons with photons, basically the Compton effect. The viscosity is given by the expression $\beta(z) = 2.06 \cdot 10^{-23}\, x_e (1+z)^4\, c^{-1}$, where x_e — the electron fraction in the plasma and z — the redshift. If we go to coordinates $r = a(t)x$, the Hubble expansion is added to the viscosity and the effective viscosity is $\beta_{eff} = \beta(z) + H$. Thus, the rapid expansion of the Universe reduces the accretion rate and hence the accretor cannot significantly increase its mass.

3.2 Accretion inside neutron star

In [38] considered accretion by a small black hole trapped inside a neutron star. With this mechanism it is possible to give constraints on the population of small-mass black holes, i.e. to use neutron stars as "dark matter detectors". The observed population of neutron stars imposes constraints on the mass range of PBHs: $10^{-15} M_\odot \lesssim M_{PBH} \lesssim 10^{-9} M_\odot$.

As stated above, it is assumed that a black hole is trapped inside a neutron star and begins to absorb it. The equation of state of matter inside the star is chosen in the form $P = K\rho^{\Gamma}$, which gives the speed of sound near the centre of the neutron star $a_c \approx \left(\frac{\Gamma P_c}{\rho_c}\right)^{1/2}$. Then the trapping radius of the black hole can generally be defined as:

$$r_c = \frac{m(r_c) + M_{BH}}{a_c^2}, \tag{11}$$

If we put $m(r_c) \approx \frac{4\pi}{3} r_c^3 \rho_c$, then we can write the accretion rate as:

$$\dot{M} = 4\pi r_c^2 \rho_c a_c = 3a_c^3 \left(1 + \frac{M_{BH}}{m(r_c)}\right)^{-1}. \tag{12}$$

Using (11) and (12) we can write the characteristic accretion time:

$$\tau_{acc} = \frac{M_{BH}}{\dot{M}} = \frac{M_{BH}}{3a_c^3}\left(1 + \frac{M_{BH}}{m(r_c)}\right). \tag{13}$$

As a conclusion to this point, it is worth noting that with $m(r_c) \ll M_{BH}$ it is reduced to Bondi accretion.

3.3 Accretion in Schwarzschild spacetime

An approach related to the GTR equations. The Schwarzschild metric is given by the expression:

$$ds^2 = -\left(1 - \frac{2GM}{r}\right)dt^2 + \frac{dr^2}{\left(1 - \frac{2GM}{r}\right)} + r^2 d\Omega^2. \tag{14}$$

In [39, 40] the (14) metric and the associated equality to zero of the covariant derivative of the energy-momentum tensor (EMT) are considered. The EMT is chosen as the EMT of an ideal fluid:

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p g_{\mu\nu}. \tag{15}$$

Omitting the details of the derivation given in [39, 40], let us write down the expression for the accretion rate:

$$\dot{M} = 4\pi A G^2 M^2 (\rho_\infty + p_\infty), \tag{16}$$

where $A = \text{const}$, which generally speaking depends on the parameter of the equation of state ω. The equation (16) does not change its form for arbitrary ω (only the constant A changes). For example, in [41] consider the RD stage and obtain the same equation, but with a different constant A.

The equation (16) is easily integrated if we put $\rho(t) = \rho_0 \left(t_0/t\right)^2$, we can get:

$$1/M = 1/M_0 - 4\pi G^2 A \rho_0 t_0 (1 + \omega)\left(1 - \frac{t_0}{t}\right). \tag{17}$$

The obvious drawback of this model is that the Schwarzschild metric is not asymptotically FRW, which casts doubt on the effectiveness of the resulting formula in the early stages of the evolution of the universe, when the characteristic accretion time will be comparable to or greater than the Hubble time, i.e. the characteristic expansion time of the universe.

3.4 McVittie solution

The proposed in [12] mechanism of generation of PBH within the modified gravity framework possibly allows one to produce them practically immediately after the cosmological inflation. The question about growth of mass of PBH with evolution of the Universe naturally arises. Since there are no observational data on this distant period of evolution of the Universe, it is necessary to estimate the influence to the final result of the free parameters of the problem, namely, the parameter ω, the moment of the end of reheating t_{reh}. Of course, the initial mass of a black hole is also a free parameter, however, taking into account the fact that the early Universe is still very rapidly expanding, it is necessary to consider an accretion model of matter taking into account the expansion of the Universe. In [42, 43] exact solutions to the accretion problem have been considered. The solution with the McVittie metric and the energy-momentum tensor of a non-ideal fluid with radial flow is of most interest for the evaluation of accretion in this case. In this model it is possible to calculate how many times the mass of the PBH will change, the mass of the PBH grows with the Universe, depending on the state parameter ω.

In [42] considered the metric

$$ds^2 = -\frac{B^2}{A^2}dt^2 + a^2(t)A^4\left(d\bar{r}^2 + \bar{r}^2 d\Omega^2\right), \tag{18}$$

where $A = 1 + \dfrac{Gm(t)}{2\bar{r}}$, $B = 1 - \dfrac{Gm(t)}{2\bar{r}}$. This metric is asymptotically FRW, and when the Universe "stops" expanding, it passes into the Schwarzschild metric via radial coordinate replacement. Obviously, it describes a strongly gravitational object. In this spacetime, the physically relevant [44–46] mass will be the quasi-local mass $m_H(t) = m(t)a(t)$, and $m(t)$ is only the coefficient of the metric.

the EMT of matter around a black hole:

$$T_{ab} = (p+\rho)u_a u_b + p g_{ab} + q_a u_b + q_b u_a, \tag{19}$$

in which

$$u^a = \left(\frac{A}{B}\sqrt{1 + a^2A^4u^2}, u, 0, 0\right),\ q^c = (0, q, 0, 0), \tag{20}$$

where q^c describes the radial energy flux and u^a — 4-velocity of the surrounding fluid. Then the field equations are as follows:

$$\dot{m}_H = -aB^2\mathcal{A}\sqrt{1 + a^2A^4u^2}\left[(p+\rho)u + q\right], \tag{21}$$

$$-3\left(\frac{AC}{B}\right)^2 = -8\pi\left[(p+\rho)a^2A^4u^2 + \rho\right], \tag{22}$$

$$-\left(\frac{A}{B}\right)^2\left(2\dot{C} + 3C^2 + \frac{2\dot{m}C}{\bar{r}AB}\right) = 8\pi\left[(p+\rho)a^2A^4u^2 + p + 2a^2A^4qu\right], \tag{23}$$

$$-\left(\frac{A}{B}\right)^2\left(2\dot{C} + 3C^2 + \frac{2\dot{m}C}{\bar{r}AB}\right) = 8\pi p, \tag{24}$$

where $\mathcal{A} = \int\int d\theta d\varphi\sqrt{g_\Sigma} = 4\pi a^2A^4\bar{r}^2$ and $C = \dfrac{\dot{a}}{a} + \dfrac{\dot{m}}{A\bar{r}}$. From the last two equations of the system we get:

$$q = -(p+\rho)\,\frac{u}{2}. \tag{25}$$

So we get the accretion rate:

$$\dot{m}_H = -\frac{1}{2}aB^2\sqrt{1 + a^2A^4u^2}(p+\rho)\mathcal{A}u. \tag{26}$$

Consider the formula (26). It will be useful to find the dependence of the accretion rate on cosmological parameters, to do so, following [43] let us take the $\bar{r} \to \infty$ limit for the expression (26):

$$\dot{m}_H = -2\pi a^3 (p_\infty + \rho_\infty) \lim_{r\to\infty} (ur^2). \tag{27}$$

It is interesting to compare this formula with others, such as the Babichev-Eroshenko-Dokuchaev formula [39–41]:

$$\frac{dM}{dt} = 4\pi G^2 A M^2 \left[p_\infty + \rho_\infty\right]. \tag{28}$$

The formula (28) can be derived using the stationary Schwarzschild metric or the non-stationary Schwarzschild metric [47]:

$$ds^2 = -\left(1 - \frac{2GM(t)}{r}\right) dt^2 + \left(1 - \frac{2GM(t)}{r}\right)^{-1} dr^2 + r^2 d\Omega^2. \tag{29}$$

Since the metric (18) changes into (29) at "stop" expansion of the Universe (in other words put $\dot{a} = 0$ in the field equations) and replace the radial coordinate, one would expect that the accretion rate (26) would change into the formula (28), but in general this is not the case. The reason is that in deriving this formula the assumption $\lim_{r\to\infty}(ur^2) = -2AG^2M^2$ is made.

We continue to consider the limit $r \to \infty$ for the equation (26). For large $\bar{r}$, we obviously have:

$$p(r;t) = p_\infty(t) + p_1(t)/r + \mathcal{O}(1/r^2), \tag{30}$$

$$\rho(r;t) = \rho_\infty(t) + \rho_1(t)/r + \mathcal{O}(1/r^2), \tag{31}$$

$$u(r;t) = u_\infty(t)/r^2 + \mathcal{O}(1/r^3). \tag{32}$$

Then, by substituting these approximations into the field equations and by combining the conservation laws, one can eventually obtain the dependence of $m_H(t)$ on the scale factor. See [43] for details of the derivation.

For the quasi-local mass we obtain a second order differential equation:

$$\dot{m}_H + 2\frac{\dot{a}}{a}\dot{m}_H - 4\pi G\left[(3\omega+1)m_H - 3\omega m_0 a\right](p_\infty + \rho_\infty) = 0. \tag{33}$$

The last expression contains a constant of integration m_0, which has not yet been assigned a definite meaning. The origin of this term is as follows: the differential equation for the first coefficient of the main part of the Laurent series for the density (30) can be derived from conservation laws. The equation for ρ_1 is as follows:

$$\dot{\rho}_1 + 3\frac{\dot{a}}{a}(p_1 + \rho_1) + 3G\dot{m}(p_\infty + \rho_\infty) = 0, \tag{34}$$

from which we get

$$\rho_1(t) = 3G(m_0 - m)(p_\infty + \rho_\infty). \tag{35}$$

Thus, an additional assumption about the value of m_0 at some initial point in time is required. Hence, it follows that (34) requires not two but three initial conditions to find a partial solution.

So, the dependence of the quasi-local mass on the scale factor is as follows:

$$m_H(t) = C_1 a^{1+3\omega}(t) - C_2 a^{-3(1+\omega)/2}(t) + \frac{3(1+\omega)}{3\omega+5} m_0 a(t), \tag{36}$$

in which C_1, C_2 are determined from the initial mass $m_H(t_0)$ and the initial accretion rate $\dot{m}_H(t_0)$.

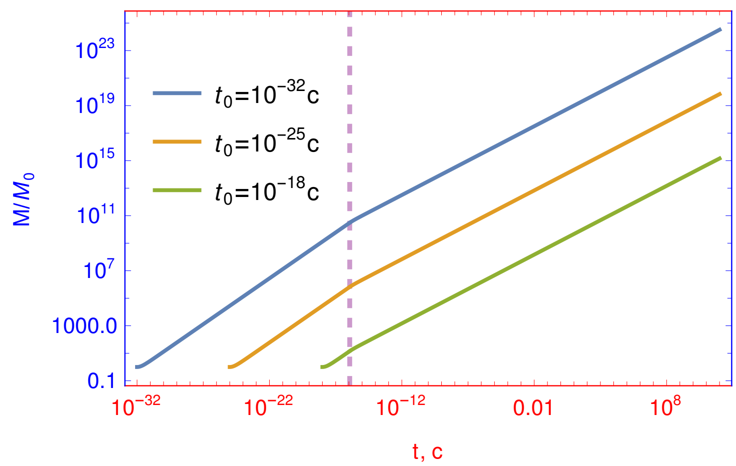

Figure 3: Mass of PBH at time t, derived from the formula (36). The dotted line shows the moment when the reheating ends (10^{-16}s, chosen for demonstration), the state parameter at the reheating stage is chosen $\omega = 0$.

3.5 Eddington limit

The Eddington luminosity or Eddington limit is the maximum luminosity that an object can achieve given with a balance between the gravitational force and the radiative pressure force. This state is called hydrostatic equilibrium 4.

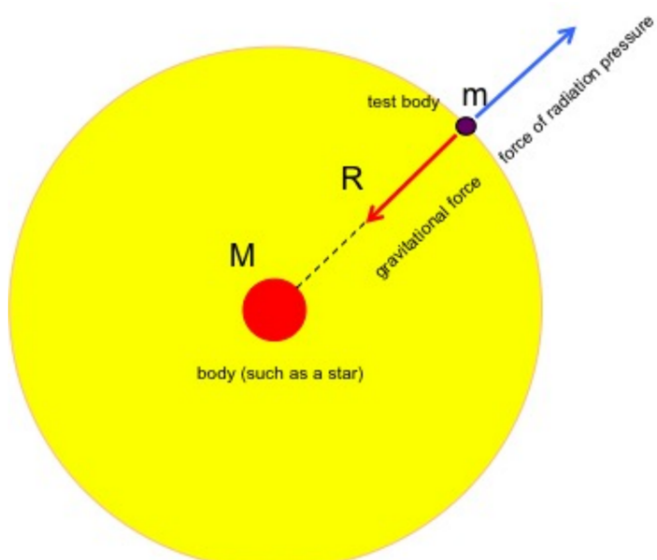

Figure 4: Eddington limit schematic representation.

The gravitational force acting on the test particle $F_{GRAV} = \dfrac{GMm}{R^2}$, and the radiation pressure is given by the formula $P_{RAD} = \dfrac{\Phi}{c} = \dfrac{1}{c}\dfrac{L}{f(R)}$, where Φ — flux and denominator:

$$f(R) = \int_{\phi=0}^{2\pi}\int_{\theta=-\pi/2}^{\pi/2}\int_{r=0}^{R} g(r,\theta,\phi)\,drd\theta d\phi, \tag{37}$$

where the integrand function characterises the distribution of matter within the object. Denote plasma transparency — κ, then the radiation pressure force on the test object $F_{RAD} = P_{RAD}\kappa m$. Equating the radiation pressure force to the gravitational force and assuming spherical symmetry and that the main component of the plasma — protons and the dominant process —Thomson electron scattering, we obtain

$$L_{edd} = \frac{4\pi cGMm_p}{\sigma_{th}} \tag{38}$$

The luminosity created by accretion of matter can be represented in the following form $L_{acc} = \varepsilon\dot{M}c^2$, where ε — radiative efficiency. Equating the Eddington limit to the luminosity from accretion of matter, we obtain:

$$L_{acc} = \varepsilon\dot{M}c^2 = L_{edd} = \frac{4\pi cGMm_p}{\sigma_{th}} \tag{39}$$

It's easy to get the dependence of mass on time:

$$M(t) = M_0\,\exp\left(\frac{4\pi Gm_p t}{\varepsilon c\sigma_{th}}\right). \tag{40}$$

The formula (40) is usually used as a marginal estimate of the mass change due to accretion, but in reality this limit can be circumvented by the lack of spherical symmetry [48, 49]. It is also worth noting that the radiative efficiency ε is generally speaking a function of the angular momentum of the accreting object. This dependence can significantly affect the black hole mass growth [50], this paper has considered in some detail the influence of the angular momentum on the accretion rate.

4 Conclusion

In the present paper we consider some mechanisms of the formation of PBH, including the mechanism of the formation of BHP without involvement of the matter fields — by the dynamics of compact extra dimensions in the model of modified gravity. Cosmological consequences of models with multidimensional f(R)-gravity are rather rich and lead to various exotic observational manifestations. Therefore, the confirmation of the primordial origin of some classes of black holes may be evidence in favour of the multiverse.

The best known models of accretion of matter by black holes are also considered, among which the black hole model in the expanding Universe — the McVittie model, which requires, however, the introduction of a quasi-local mass, which is not a generally accepted notion, is given. Accretion in the McVittie metric shows stepwise growth at any stage of the evolution of the Universe, it indicates that the growth of mass in the early stages of Universe evolution can be very substantial.

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