Inflation

Lecture from the course
« Introduction to
cosmoparticle physics »

Friedman's equations

$$\begin{cases} \dfrac{\ddot a}{a} = -\dfrac{4\pi G}{3}\left(\varepsilon + \dfrac{3p}{c^2}\right) + \dfrac{\Lambda c^2}{3} \\ \left(\dfrac{\dot a}{a}\right)^2 - \dfrac{8\pi G\varepsilon}{3} = -\dfrac{Kc^2}{a^2} + \dfrac{2\Lambda c^2}{3} \end{cases}$$

Λ-term is equivalent to the matter with vacuum-like equation of state (E.Glinner 1965, Ya.Zeldovich 1968):

$$p = -\varepsilon = -\frac{\Lambda}{8\pi G}$$

$$m_{\mathrm{Pl}} = \sqrt{\frac{\hbar c}{G}} \approx 2 \cdot 10^{-5}\,\mathrm{g}$$

$$l_{\mathrm{Pl}} = \sqrt{\frac{G\hbar}{c^3}} \approx 1{,}6 \cdot 10^{-33}\,\mathrm{cm}$$

$$t_{\mathrm{Pl}} = \sqrt{\frac{G\hbar}{c^5}} \approx 0{,}5 \cdot 10^{-43}\,\mathrm{s}$$

$$\hbar = c = 1$$

$$\varepsilon_{cr} = \frac{3H^2}{8\pi G} \qquad H \equiv \frac{\dot a}{a}$$

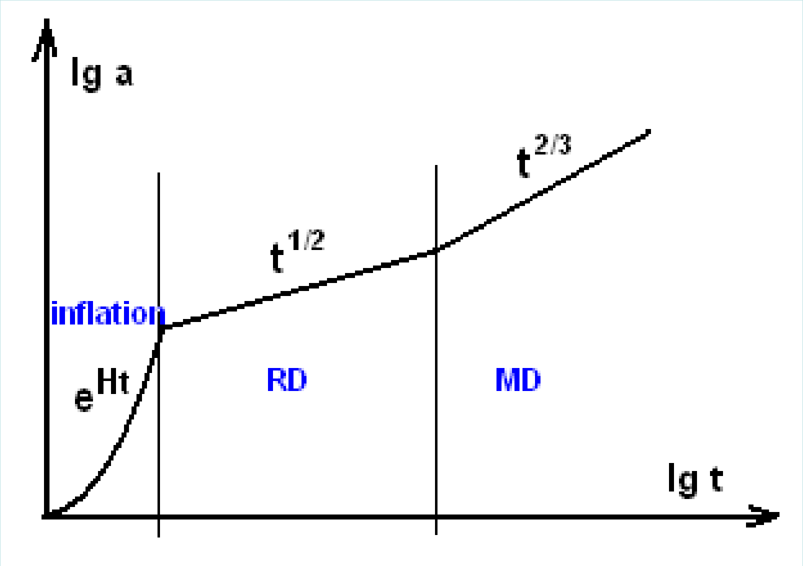
$$\text{If } p < -\frac{\varepsilon}{3} \quad \Rightarrow \quad \ddot a > 0$$

Prerequisites for inflation

From Friedman's equations we have:

$$a(t) \propto \exp\left(\int H dt\right)$$

$$H = \sqrt{8\pi G\varepsilon / 3}$$



E.Gliner, I.Dymnikova 1965, 1975: vacuum-like state of matter (p=-ε)

A.Starobinsky 1979, 1980: realization due to quantum corrections in *R*

A.Guth 1981: realization due to scalar field and solution of cosmological problems (introduction of term "inflation")

Solution for cosmological problems

Exponential expansion provides solution for the problems of

singularity:

In the period of inflation $\varepsilon \neq \infty$

initial state:

$H_0 = (8\pi G\varepsilon/3)^{1/2}$

For instance, if inflation began at Planck time and finished in GUT era

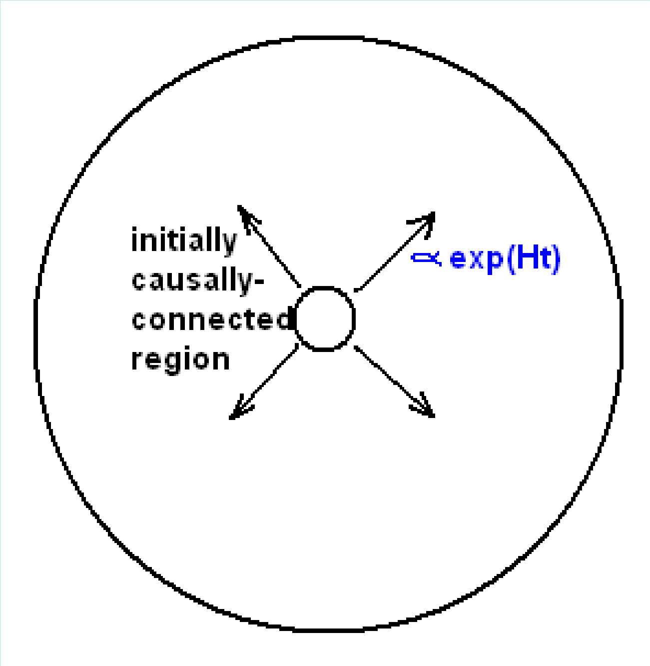
$$\varepsilon \sim m_{\mathrm{Pl}}^4 \qquad\qquad t_{\text{оконч.инфляц.}} \sim t_{\mathrm{GUT}} \sim \Lambda_{\mathrm{GUT}}^{-1}$$

$$a(t_{\mathrm{GUT}})/a(t_{\mathrm{Pl}}) \propto \exp\left(H \cdot \Delta t\right) \sim \exp\left(\sqrt{8\pi G\varepsilon/3} \cdot t_{\mathrm{GUT}}\right) \sim \exp\left(m_{\mathrm{Pl}}/\Lambda_{\mathrm{GUT}}\right) \sim e^{10000}$$

Solution for problem of magnetic monopoles

Problem of magnetic monopole overproduction:

if monopoles are produced before the end of inflational stage => $n \sim 1/a^3 \sim \exp(-3Ht) \rightarrow 0$



Inflation

Scalar field

$$L = \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - V(\varphi)$$

$$\begin{cases} \ddot{\varphi} + 3H\dot{\varphi} + \dfrac{dV}{d\varphi} = 0 \\ H^2 = \dfrac{8\pi G}{3}\left(\dfrac{\dot{\varphi}^2}{2} + V\right) - \dfrac{K}{a^2} \end{cases}$$

$$\varepsilon = \frac{\dot{\varphi}^2}{2} + V, \qquad p = \frac{\dot{\varphi}^2}{2} - V$$

$$\text{If} \quad \dot{\varphi}^2 << V \quad \Rightarrow \quad \boxed{p \cong -\varepsilon}$$

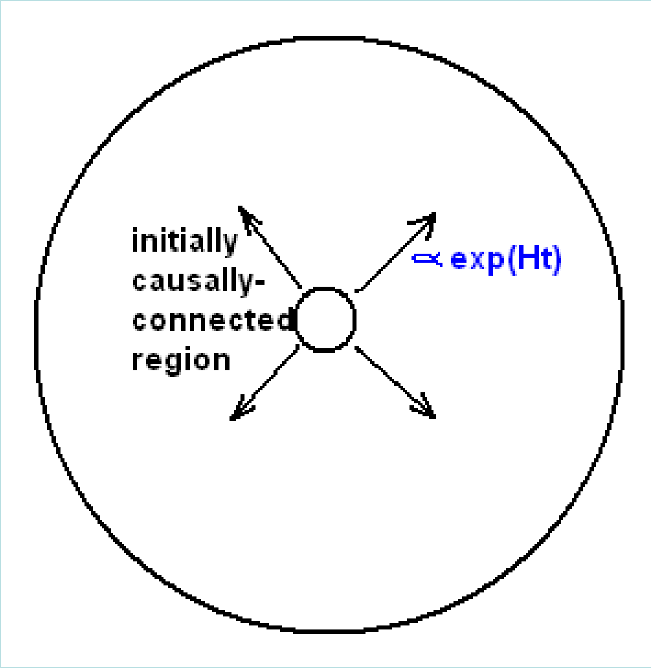
Exponential expansion, accounted for by this equation of state, would resolve the problems of

horizon

flatness.

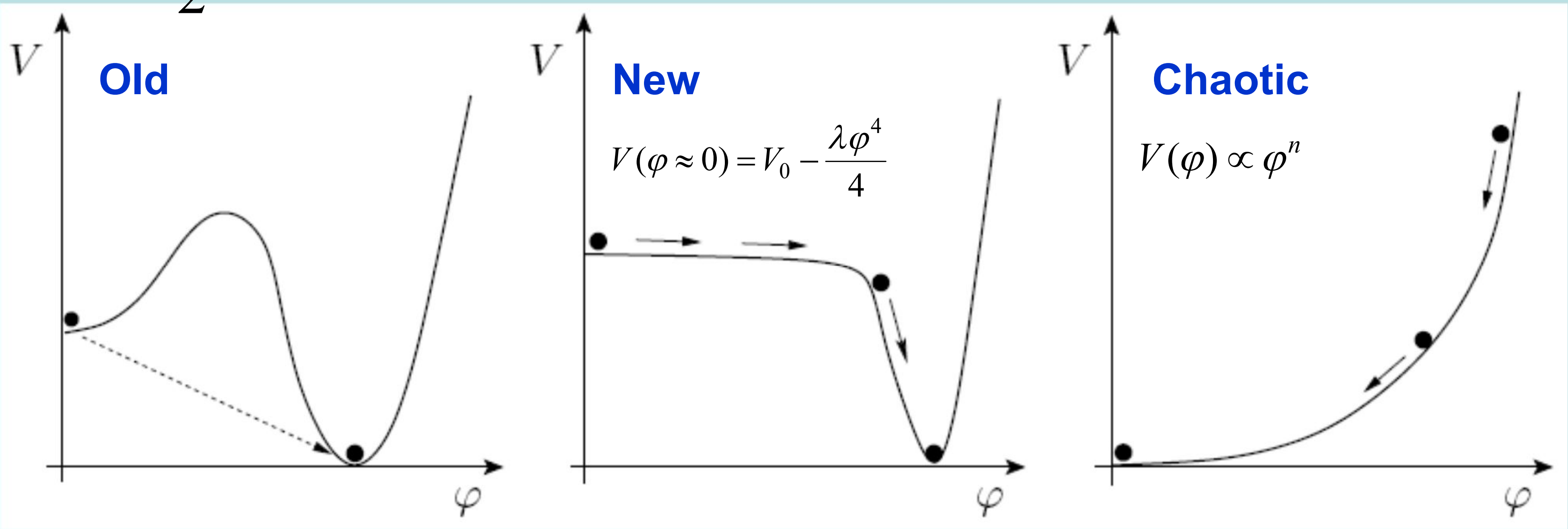
$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{8\pi G\varepsilon}{3} = -\frac{Kc^2}{a^2}$$

(or $\Omega - 1 = \frac{K}{\dot{a}^2} \rightarrow 0$)



"Old", "New" and Chaotic Inflation

$$L = \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - V(\varphi)$$



A.Guth (1981)

•Phase transition of GUT (**due to tunneling**)

•Too large inhomogeneities

•Problem of before-inflation conditions

A.Albrecht, P.Steinhardt (1982), A.Linde (1982)

•Phase transition of GUT (**slow rolling down**)

•Fine-tuning of parameters: $\lambda \sim 10^{-12}$

•Problem of before-inflation conditions

A.Linde (1983)

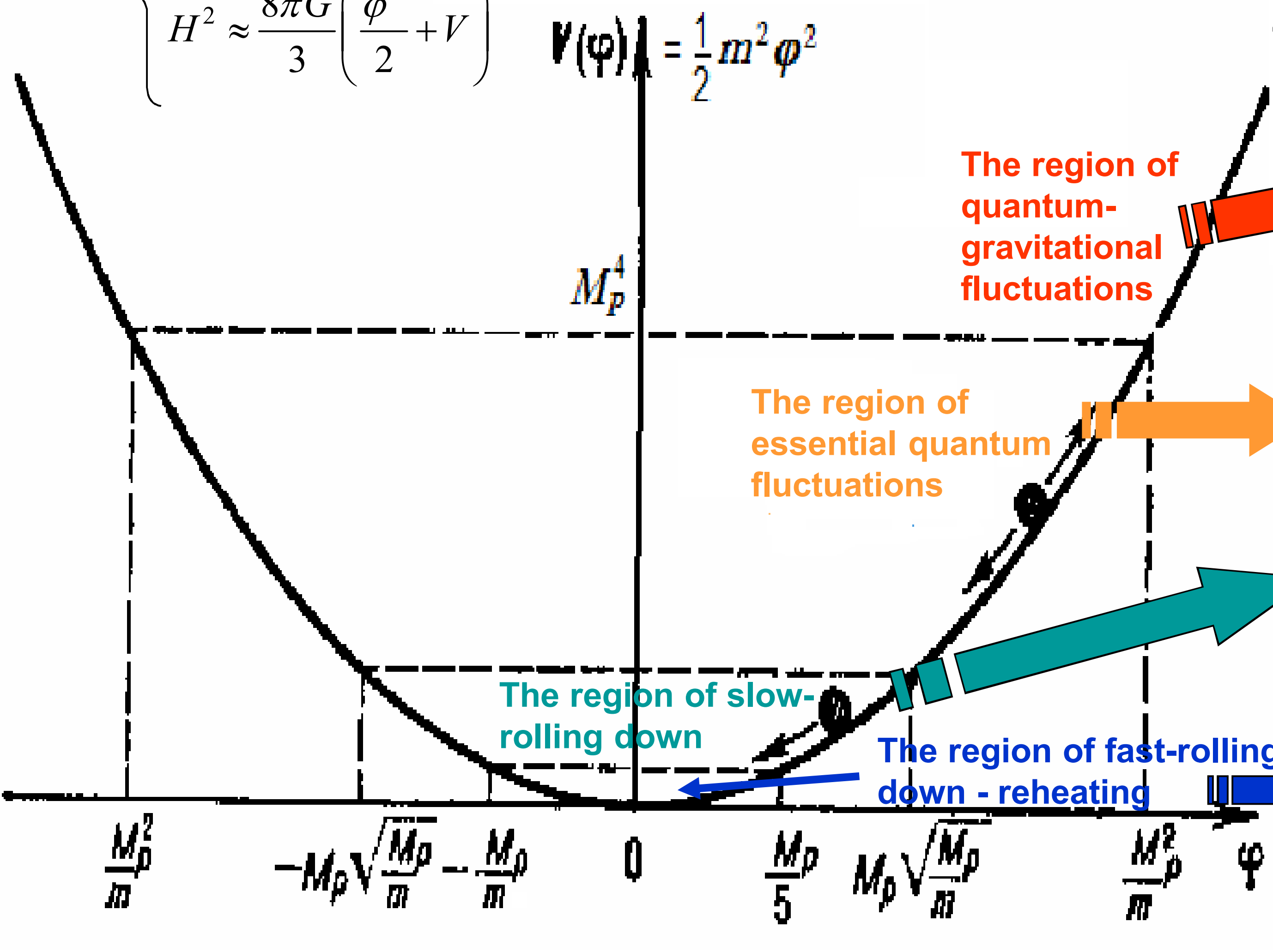
*Quantum fluctuations

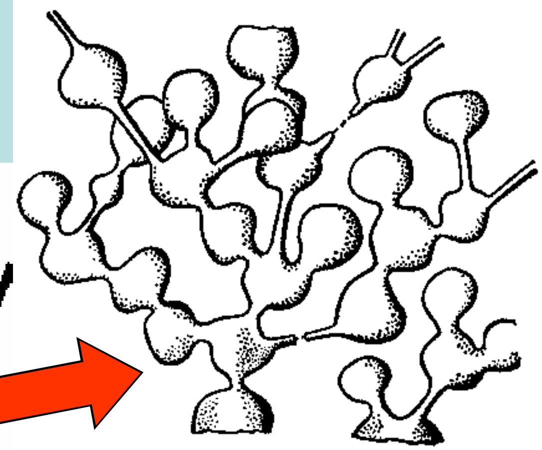
Chaotic/eternal inflation

"friction" provides slow rolling down

$$\begin{cases} \ddot{\varphi} + 3H\dot{\varphi} + \dfrac{dV}{d\varphi} = 0 \\ H^2 \approx \dfrac{8\pi G}{3}\left(\dfrac{\dot{\varphi}^2}{2} + V\right) \end{cases}$$

$$a \propto \exp\left(2\pi\varphi^2 / m_{\rm Pl}^2\right)$$





"Multiverse"

"Eternal" inflation

LSS requires:

$$m \sim 10^{-6} m_{\rm Pl}$$

$$\kappa\sigma T_R^4 = V(\varphi \sim m_{\rm Pl}/5)$$

$$T_R \sim 10^{10-16}\,\text{GeV}$$

Conclusions

- Magnetic Monopole overproduction has instigated critical analysis of the old Big Bang scenario. It found solution in the framework of inflational models.
- The problems of initial state, horizon, flattness, of the origin of primordial fluctuations also find solutions in inflational scenario of very early Universe.
- Inflation involves hypothetical inflaton field.
- Physics of inflation is beyond the Standard model and implies methods of cosmoparticle physics for its study.