Fermion masses and mixing within a gauged $SU(3)_F$ family symmetry model

Albino Hernández Galeana

ESFM - Instituto Politécnico Nacional, México

BLED Workshop, 4 - 10 July, 2022

Open questions:

There are several open questions, motivations, to look for physics "Beyond the Standard Model" (BSM):

- Neutrino masses and mixing, oscillation
- Hierarchy of masses
- Dart Matter (DM), Dart Energy (DE), Matter-Antimatter Asymmetry.
- Flavor Anomalies: Lepton flavor universality violation in rare B decays, $g_\mu - 2$ at Fermilab, violation of unitarity in quark mixing matrix V_{CKM}.

Main goal of this BSM: To account for the hierarchy of fermion masses:
$m_t \gg m_c \gg m_u \quad , \quad m_b \gg m_s \gg m_d \quad , \quad m_\tau \gg m_\mu \gg m_e$,
and the quark and lepton mixing matrices: V_{CKM} and U_{PMNS}.

Standard Model (SM)

Ordinary Fermions: $Q = T_{3L} + \frac{1}{2} Y$

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$q^o_{jL} = \begin{pmatrix} u^o_{jL} \\ d^o_{jL} \end{pmatrix}$	3	2	$\frac{1}{3}$
u^o_{jR}	3	1	$\frac{4}{3}$
d^o_{jR}	3	1	$-\frac{2}{3}$
$l^o_{jL} = \begin{pmatrix} \nu^o_{jL} \\ e^o_{jL} \end{pmatrix}$	1	2	-1
e^o_{jR}	1	1	-2
ν^o_{jR}	1	1	0
$\phi_{SM} = \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}$	1	2	+1
$\tilde{\phi}_{SM} = i\,\sigma_2\,\phi^*_{SM}$	1	2	-1

Table: SM fermion content and charges, $j = 1, 2, 3$ family index

Comments:

- $G_{SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$: Standard Model Group
- Neutrinos are massless
- Anomalies cancel for each family, $j = 1, 2, 3$.
- $\overline{q^o_{iL}}\,\phi_{SM}\,u^o_{jR}$, $i, j = 1, 2, 3$ is gauge invariant. So, the Yukawa couplings $Y_{i,j}\,\overline{q^o_{iL}}\,\phi_{SM}\,u^o_{jR}$, $i, j = 1, 2, 3$ are gauge invariant, with $Y_{i,j}$ completely arbitrary.
- SM gauge bosons: Z, A(photon), and H(Higgs) couplings to fermions do not change flavor.
- $\frac{g}{\sqrt{2}}\,\bar{\Psi}_{uL}\,V_{CKM}\,\gamma_\mu \Psi_{dL}\,W^{+\mu}$

MODEL WITH LOCAL $SU(3)_F$ VECTOR-LIKE FAMILY SYMMETRY

	$SU(3)_F$	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
ψ_q^o	3	3	2	$\frac{1}{3}$
ψ_{uR}^o	3	3	1	$\frac{4}{3}$
ψ_{dR}^o	3	3	1	$-\frac{2}{3}$
ψ_l^o	3	1	2	-1
ψ_{eR}^o	3	1	1	-2

Table: SM Fermions and charges

NEW FERMIONS

	$SU(3)_F$	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$\psi^o_{\nu R}$	3	1	1	0
$U^o_{L,R}$	1	3	1	$\frac{4}{3}$
$D^o_{L,R}$	1	3	1	$-\frac{2}{3}$
$E^o_{L,R}$	1	1	1	-2
$N^o_{L,R}$	1	1	1	0

- Right handed neutrinos are needed to cancel anomalies.
- U,D,E,N: $SU(2)_L$ weak singlets vector-like fermions, with N a neutral lepton, are needed to generate SM fermion masses and mixing.

3+5 Neutrino scenario

3 Active L-handed SM neutrinos

5 Sterile neutrinos: (3 Right handed & N_L, N_R vector like neutrinos)

SCALAR FIELDS

	$SU(3)_F$	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
η_1, η_2	3	1	1	0
Φ^u	3	1	2	-1
Φ^d	3	1	2	+1

- η_1, η_2 are introduced to break spontaneously $SU(3)_F$.
- Φ^u, Φ^d are introduced to break spontaneously the $SU(2)_L \times U(1)_Y$.
- u-quarks and neutrinos coupled only to Φ^u
- d-quarks and charged leptons couple only to Φ^d
- $\overline{\psi_q^o}\, \Phi^u\, \psi_{uR}^o$ **and** $\overline{\psi_q^o}\, \Phi^d\, \psi_{dR}^o$ **are forbidden by the** $SU(3)_F$ **gauge family symmetry.**
- $\bar{3} \times 3 = 1 + 8\ ,\ 3 \times 3 = \bar{3} + 6\ ,\ \bar{3} \times 3 \times 3 = (1+8) \times 3 = 3 + \bar{6} + 3 + 15$

$SU(3)_F$ GAUGE BOSONS

$$i\mathcal{L}_{int,SU(3)} =$$
$$\frac{g_H}{2}\left(\bar{f}^o_1\gamma_\mu f^o_1 - \bar{f}^o_3\gamma_\mu f^o_3\right) Z^\mu_1 + \frac{g_H}{2\sqrt{3}}\left(\bar{f}^o_1\gamma_\mu f^o_1 + \bar{f}^o_3\gamma_\mu f^o_3 - 2\bar{f}^o_2\gamma_\mu f^o_2\right) Z^\mu_2$$
$$+ \frac{g_H}{\sqrt{2}}\left(\bar{f}^o_1\gamma_\mu f^o_2\, Y^+_1 + \bar{f}^o_1\gamma_\mu f^o_3\, Y^+_2 + \bar{f}^o_2\gamma_\mu f^o_3\, Y^+_3 + h.c.\right)$$

g_H is the $SU(3)_F$ coupling constant, Z_1, Z_2 and $Y^\pm_i$, $i = 1, 2, 3$, $j = 1, 2$ are the eight gauge bosons, which have **flavor changing couplings to both left and right handed SM fermions**

$$g_H \begin{pmatrix} \bar{f}^o_1 & \bar{f}^o_2 & \bar{f}^o_3 \end{pmatrix} \gamma_\mu \begin{pmatrix} \frac{Z_1}{2} + \frac{Z_2}{2\sqrt{3}} & \frac{Y^+_1}{\sqrt{2}} & \frac{Y^+_2}{\sqrt{2}} \\ \frac{Y^-_1}{\sqrt{2}} & -\frac{Z_2}{\sqrt{3}} & \frac{Y^+_3}{\sqrt{2}} \\ \frac{Y^-_2}{\sqrt{2}} & \frac{Y^-_3}{\sqrt{2}} & -\frac{Z_1}{2} + \frac{Z_2}{2\sqrt{3}} \end{pmatrix}^\mu \begin{pmatrix} f^o_1 \\ f^o_2 \\ f^o_3 \end{pmatrix}$$

Spontaneous Symmetry breaking (SSB)

We would like to be consistent with low energy Standard Model(SM) and simultaneously generate and account for the hierarchy of quark and lepton masses and mixing

$$SU(3)_F \times G_{SM} \xrightarrow{\langle\eta_2\rangle,\langle\eta_1\rangle} G_{SM} \xrightarrow{\langle\Phi^u\rangle,\langle\Phi^d\rangle} SU(3)_C \times U(1)_Q$$

SM fermions are massless	SM fermions still massless	SM fermions become massive (PDG known values)

$SU(3)_F$ family symmetry breaking

Two SM singlet scalars are introduced in the fundamental representation of $SU(3)_F$:

$$\langle\eta_1\rangle = \begin{pmatrix} \Lambda_1 \\ 0 \\ 0 \end{pmatrix} \quad , \quad \langle\eta_2\rangle = \begin{pmatrix} 0 \\ \Lambda_2 \\ 0 \end{pmatrix}$$

$$SU(3)_F \times G_{SM} \quad \xrightarrow{\langle\eta_2\rangle} \quad \underset{Z_1,\, Y_2^{\pm}}{SU(2)_F \times G_{SM}} \quad \xrightarrow{\langle\eta_1\rangle} \quad G_{SM}$$

Λ_2: 5 very heavy boson masses of order M_2 ($M_2^2 = \frac{g_H^2 \Lambda_2^2}{2}$) $\gtrsim 10^6\ TeV$ ($Z_2, Y_1^{\pm}, Y_3^{\pm}$)

Λ_1: 3 boson masses of order M_1 ($M_1^2 = \frac{g_H^2 \Lambda_1^2}{2}$) $\gtrsim 10^3\ TeV$ ($Z_1, Y_2^{\pm}$)

To suppress properly FCNC like, for instance: $\mu \to e\gamma$ **(**$Br < 5.7 \times 10^{-13}$**) ,** $\mu \to e\, e\, e$ **(**$Br < 1 \times 10^{-12}$**) ,** $K^o - \bar{K^o}$**,** $D^o - \bar{D^o}$**,** $M_1 \gtrsim 10^3$ **TeV's**

$SU(3)_F$ boson masses

The above scalar fields and VEV's break completely the $SU(3)$ family symmetry, generating the mass terms

- $\langle \eta_1 \rangle :\quad \frac{g_H^2 \Lambda_1^2}{2} \left(Y_1^+ Y_1^- + Y_2^+ Y_2^- \right) + \frac{g_H^2 \Lambda_1^2}{4} \left(Z_1^2 + \frac{Z_2^2}{3} + 2\, Z_1 \frac{Z_2}{\sqrt{3}} \right)$
- $\langle \eta_2 \rangle :\quad \frac{g_H^2 \Lambda_2^2}{2} \left(Y_1^+ Y_1^- + Y_3^+ Y_3^- \right) + g_H^2 \Lambda_2^2 \frac{Z_2^2}{3}$

Electroweak Symmetry Breaking

In this scenario we introduce two triplets of $SU(2)_L$ Higgs doublets;
$\Phi^u = (3, 1, 2, -1), \Phi^d = (3, 1, 2, +1)$:

$$\Phi^u = \begin{pmatrix} \begin{pmatrix} \phi^o \\ \phi^- \end{pmatrix}_1^u \\ \begin{pmatrix} \phi^o \\ \phi^- \end{pmatrix}_2^u \\ \begin{pmatrix} \phi^o \\ \phi^- \end{pmatrix}_3^u \end{pmatrix}, \langle \Phi^u \rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} v_{u1} \\ 0 \end{pmatrix} \\ \frac{1}{\sqrt{2}} \begin{pmatrix} v_{u2} \\ 0 \end{pmatrix} \\ \frac{1}{\sqrt{2}} \begin{pmatrix} v_{u3} \\ 0 \end{pmatrix} \end{pmatrix} ; \ \Phi^d = \begin{pmatrix} \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}_1^d \\ \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}_2^d \\ \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}_3^d \end{pmatrix}, \langle \Phi^d \rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{d1} \end{pmatrix} \\ \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{d2} \end{pmatrix} \\ \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{d3} \end{pmatrix} \end{pmatrix}$$

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \xrightarrow{\langle \Phi^u \rangle, \langle \Phi^d \rangle} SU(3)_C \times U(1)_Q$$

W and Z boson masses:

$$\frac{g^2}{4}\,(v_u^2+v_d^2)\,W^+W^- + \frac{(g^2+g'^2)}{8}\,(v_u^2+v_d^2)\,Z_o^2$$

$$+\frac{1}{4}\,g_H\,\sqrt{g^2+g'^2}$$

$$Z_o\Bigg[\,(v_{1u}^2-v_{2u}^2-v_{1d}^2+v_{2d}^2)\,Z_1+(v_{1u}^2+v_{2u}^2-2v_{3u}^2-v_{1d}^2-v_{2d}^2+2v_{3d}^2)\,\frac{Z_2}{\sqrt{3}}$$

$$+2\,(v_{1u}v_{2u}-v_{1d}v_{2d})\,\frac{Y_1^+ + Y_1^-}{\sqrt{2}} + 2\,(v_{1u}v_{3u}-v_{1d}v_{3d})\,\frac{Y_2^+ + Y_2^-}{\sqrt{2}}$$

$$+2\,(v_{2u}v_{3u}-v_{2d}v_{3d})\,\frac{Y_3^+ + Y_3^-}{\sqrt{2}}\Bigg]$$

$+$ **tiny contributions to the** $SU(3)$ **gauge boson masses**

$v_u^2 = v_{u1}^2+v_{u2}^2+v_{u3}^2$, $v_d^2 = v_{d1}^2+v_{d2}^2+v_{d3}^2$. Hence, if we define $M_W=\frac{1}{2}g\,v$, we may write $v=\sqrt{v_u^2+v_d^2}\approx 246$ GeV.

Yukawa couplings:

Dirac Yukawa couplings:

$$
\begin{aligned}
&h_u\, \overline{\psi_q^o}\, \Phi^u\, U_R^o \;+\; h_{1u}\, \overline{\psi_{uR}^o}\, \eta_1\, U_L^o \;+\; h_{2u}\, \overline{\psi_{uR}^o}\, \eta_2\, U_L^o \;+\; M_U\, \overline{U_L^o}\, U_R^o \\
&h_d\, \overline{\psi_q^o}\, \Phi^d\, D_R^o \;+\; h_{1d}\, \overline{\psi_{dR}^o}\, \eta_1\, D_L^o \;+\; h_{2d}\, \overline{\psi_{dR}^o}\, \eta_2\, D_L^o \;+\; M_D\, \overline{D_L^o}\, D_R^o \\
&h_\nu\, \overline{\psi_l^o}\, \Phi^u\, N_R^o \;+\; h_{1\nu}\, \overline{\psi_{\nu R}^o}\, \eta_1\, N_L^o \;+\; h_{2\nu}\, \overline{\psi_{\nu R}^o}\, \eta_2\, N_L^o \;+\; m_D\, \overline{N_L^o}\, N_R^o \\
&h_e\, \overline{\psi_l^o}\, \Phi^d\, E_R^o \;+\; h_{1e}\, \overline{\psi_{eR}^o}\, \eta_1\, E_L^o \;+\; h_{2e}\, \overline{\psi_{eR}^o}\, \eta_2\, E_L^o \;+\; M_E\, \overline{E_L^o}\, E_R^o \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad +h.c
\end{aligned}
$$

Majorana Yukawa couplings:

$$
h_L\, \overline{\psi_l^o}\, \Phi^u\, (N_L^o)^c \;+\; m_L\, \overline{N_L^o}\, (N_L^o)^c
$$

$$
h_{1R}\, \overline{\psi_{\nu R}^o}\, \eta_1\, (N_R^o)^c \;+\; h_{2R}\, \overline{\psi_{\nu R}^o}\, \eta_2\, (N_R^o)^c \;+\; m_R\, \overline{N_R^o}\, (N_R^o)^c \;+h.c
$$

Tree level Dirac Fermion Masses

$\bar{\psi}^o_{SM,L}\,\Phi^{u,d}\,\psi^o_{SM,R}$ **are not $SU(3)_F$ gauge invariant**

Allowed Tree Level Yukawa couplings:

$$h_e\,\bar{\psi}^o_l\,\Phi^d\,E^o_R \;+\; h_{1e}\,\bar{\psi}^o_e\,\eta_1\,E^o_L \;+\; h_{2e}\,\bar{\psi}^o_e\,\eta_2\,E^o_L \;+\; M_E\,\bar{E}^o_L\,E^o_R \;+ h.c$$

Dirac See-Saw Mechanisms

$$\begin{array}{c|cccc} & e^o_R & \mu^o_R & \tau^o_R & E^o_R \\ \hline \overline{e^o_L} & 0 & 0 & 0 & h_e\,v_{d1} \\ \overline{\mu^o_L} & 0 & 0 & 0 & h_e\,v_{d2} \\ \overline{\tau^o_L} & 0 & 0 & 0 & h_e\,v_{d3} \\ \overline{E^o_L} & h_{1e}\,\Lambda_1 & h_{2e}\,\Lambda_2 & 0 & M_E \end{array} \equiv \begin{array}{c|cccc} & e^o_R & \mu^o_R & \tau^o_R & E^o_R \\ \hline \overline{e^o_L} & 0 & 0 & 0 & a_1 \\ \overline{\mu^o_L} & 0 & 0 & 0 & a_2 \\ \overline{\tau^o_L} & 0 & 0 & 0 & a_3 \\ \overline{E^o_L} & b_1 & b_2 & 0 & \mathrm{M} \end{array}$$

Table: Generic tree level Dirac mass matrix M^o for u and d quarks, charged leptons and Dirac neutrinos

Free Parameters:

- $\{v_{u1}, v_{u2}, v_{u3}\}$ and $\{v_{d1}, v_{d2}, v_{d3}\}$ are constrained by EWSB (W and Z^o boson masses).
- Λ_1 , Λ_2 are the scales of $SU(3)_F$ SSB.

Free parameters to fit masses for quarks, charged leptons, and neutrinos:

- h_u , h_{1u} , h_{2u} and M_U; u-quarks..
- h_d , h_{1d} , h_{2d} and M_D; d-quarks.
- h_e , h_{1e} , h_{2e} and M_E; charged leptons.
- h_ν , $h_{1\nu}$, $h_{2\nu}$, m_D (Dirac) and h_L , m_L and h_{1R} , h_{2R} , $h_{2\nu}$, m_R (Majorana); neutrinos

V_L^o , V_R^o mixing matrices

$$V_L^o = \begin{pmatrix} c_1 & s_1\, c_2 & s_1\, s_2\, c_\alpha & s_1\, s_2\, s_\alpha \\ -s_1 & c_1\, c_2 & c_1\, s_2\, c_\alpha & c_1\, s_2\, s_\alpha \\ 0 & -s_2 & c_2\, c_\alpha & c_2\, s_\alpha \\ 0 & 0 & -s_\alpha & c_\alpha \end{pmatrix} \;,\; V_R^o = \begin{pmatrix} 0 & c_r & s_r\, c_\beta & s_r\, s_\beta \\ 0 & -s_r & c_r\, c_\beta & c_r\, s_\beta \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -s_\beta & c_\beta \end{pmatrix}$$

$$V_L^{o\,T} M^o M^{o\,T}\; V_L^o = V_R^{o\,T} M^{o\,T} M^o\; V_R^o = Diag(0, 0, \lambda_3^2, \lambda_4^2)$$

$$V_L^{o\,T} M^o\; V_R^o = Diag(0, 0, -\lambda_3, \lambda_4)$$

$$s_1 = \frac{v_1}{\sqrt{v_1^2+v_2^2}} \;,\; c_1 = \frac{v_2}{\sqrt{v_1^2+v_2^2}} \;,\; s_2 = \frac{\sqrt{v_1^2+v_2^2}}{\sqrt{v_1^2+v_2^2+v_3^2}} \;,\; c_2 = \frac{v_3}{\sqrt{v_1^2+v_2^2+v_3^2}}$$

$$s_r = \frac{h_1\, \Lambda_1}{\sqrt{h_1^2\, \Lambda_1^2+h_2^2\, \Lambda_2^2}} \;,\; c_r = \frac{h_2\, \Lambda_2}{\sqrt{h_1^2\, \Lambda_1^2+h_2^2\, \Lambda_2^2}} \;,\; \frac{s_1}{c_1} = \frac{v_1}{v_2} \;,\; \frac{s_2}{c_2} = \frac{\sqrt{v_1^2+v_2^2}}{v_3}$$

$$s_R = \frac{h_{1R}\, \Lambda_{1R}}{\sqrt{h_{1R}^2\, \Lambda_{1R}^2+h_{2R}^2\, \Lambda_{2R}^2}} \;,\; c_R = \frac{h_{2R}\, \Lambda_{2R}}{\sqrt{h_{1R}^2\, \Lambda_{1R}^2+h_{2R}^2\, \Lambda_{2R}^2}}$$

Non-zero tree level eigenvalues

The non-zero eigenvalues may be obtained from diagonalization of the 2*x*2 matrix

$$m_o = \begin{pmatrix} 0 & a \\ b & M \end{pmatrix} \ , \ a = \sqrt{a_1^2 + a_2^2 + a_3^2} \ , \ b = \sqrt{b_1^2 + b_2^2 + b_3^2}$$

$$v_L = \begin{pmatrix} c_\alpha & s_\alpha \\ & \\ -s_\alpha & c_\alpha \end{pmatrix} \quad , \quad v_R = \begin{pmatrix} c_\beta & s_\beta \\ & \\ -s_\beta & c_\beta \end{pmatrix}$$

$$\lambda_3 \ c_\alpha \ c_\beta = \lambda_4 \ s_\alpha \ s_\beta$$

$$v_L{}^T \, m_o \, v_R = Diag(0, 0, -\lambda_3, \lambda_4)$$

$$v_L{}^T \, m_o \, m_o^T \, v_L = v_R{}^T \, m_o^T \, m_o \, v_R = Diag(\lambda_3^2, \lambda_4^2)$$

One loop contribution to fermion masses

After tree level contributions the fermion global symmetry is broken down to:

$$SU(2)_{q_L} \otimes SU(2)_{u_R} \otimes SU(2)_{d_R} \otimes SU(2)_{l_L} \otimes SU(2)_{\nu_R} \otimes SU(2)_{e_R}$$

Therefore, in this scenario light fermion masses, including neutrinos, may get extremely small masses from radiative corrections mediated by the $SU(3)_F$ heavy gauge bosons.

$$c_Y \frac{\alpha_H}{\pi} \sum_{k=3,4} m_k^o \, (V_L^o)_{ik} (V_R^o)_{jk} f(M_Y, m_k^o) \qquad , \qquad \alpha_H \equiv \frac{g_H^2}{4\pi}$$

M_Y is the gauge boson mass, c_Y is coupling constant, $m_3^o = -\lambda_3$, $m_4^o = \lambda_4$, and $f(x,y) = \frac{x^2}{x^2 - y^2} \ln \frac{x^2}{y^2}$.

$$\sum_{k=3,4} m_k^o \, (V_L^o)_{ik} (V_R^o)_{jk} f(M_Y, m_k^o) = \frac{a_i \, b_j \, M}{\lambda_4^2 - \lambda_3^2} \, F(M_Y) \quad i, j = 1, 2, 3,$$

$$F(M_Y) \equiv \frac{M_Y^2}{M_Y^2 - \lambda_4^2} \ln \frac{M_Y^2}{\lambda_4^2} - \frac{M_Y^2}{M_Y^2 - \lambda_3^2} \ln \frac{M_Y^2}{\lambda_3^2}$$

One loop Dirac fermion masses

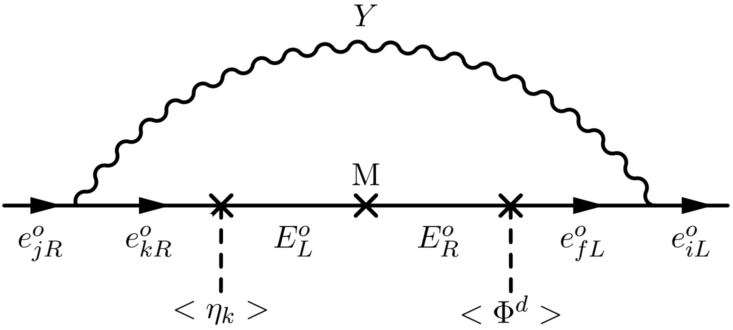


Figure: Generic one loop diagram contribution to the mass term $m_{ij}\ \bar{e}^o_{iL} e^o_{jR}$

One loop mass matrix M_1^o

	e_R^o	μ_R^o	τ_R^o	E_R^o
$\overline{e_L^o}$	D_{11}	D_{12}	0	0
$\overline{\mu_L^o}$	D_{21}	D_{22}	0	0
$\overline{\tau_L^o}$	D_{31}	D_{32}	D_{33}	0
$\overline{E_L^o}$	0	0	0	0

Table: One loop diagram contribution to the mass term $m_{ij}\, \bar{e}_{iL}^o e_{jR}^o$

Mass matrix up to one loop for quarks and charged leptons

$$M = Diag(0, 0, -\lambda_3, \lambda_4) + {V_L^o}^T M_1^o \, V_R^o$$

$$M = \begin{pmatrix} 0 & m_{12} & c_\beta \, m_{13} & s_\beta \, m_{13} \\ m_{21} & m_{22} & c_\beta \, m_{23} & s_\beta \, m_{23} \\ c_\alpha \, m_{31} & c_\alpha \, m_{32} & -\lambda_3 + c_\alpha c_\beta \, m_{33} & c_\alpha s_\beta \, m_{33} \\ s_\alpha \, m_{31} & s_\alpha \, m_{32} & s_\alpha c_\beta \, m_{33} & \lambda_4 + s_\alpha s_\beta \, m_{33} \end{pmatrix}$$

The diagonalization of M yields the physical masses for u, d, e and ν fermions. Using a new biunitary transformation $\chi_{L,R} = V^{(1)}_{L,R}\, \Psi_{L,R};\ \bar{\chi}_L\, M\, \chi_R = \bar{\Psi}_L\, {V^{(1)}_L}^T M\, V^{(1)}_R\, \Psi_R$, with ${\Psi_{L,R}}^T = (f_1, f_2, f_3, F)_{L,R}$ the mass eigenfields, that is

$$ {V^{(1)}_L}^T M\, M^T\, V^{(1)}_L = {V^{(1)}_R}^T M^T\, M\, V^{(1)}_R = Diag(m_1^2, m_2^2, m_3^2, M_F^2)\ , $$

$m_1^2 = m_e^2$, $m_2^2 = m_\mu^2$, $m_3^2 = m_\tau^2$ and $M_F^2 = M_E^2$ for charged leptons.

Therefore, the transformation from massless to mass fermion eigenfields reads

$$ \psi_L^o = V_L^o\, V_L^{(1)}\, \Psi_L \qquad \text{and} \qquad \psi_R^o = V_R^o\, V_R^{(1)}\, \Psi_R $$

and for neutrinos $\Psi_\nu^o = U_\nu^o\, U_\nu\, \Psi_\nu$.

Quark $(V_{CKM})_{4\times4}$ and Lepton $(U_{PMNS})_{4\times8}$ mixing matrices

Vector like quarks are $SU(2)_L$ weak singlets, and hence, the interaction of L-handed up and down quarks; ${f^o_{uL}}^T = (u^o, c^o, t^o)_L$ and ${f^o_{dL}}^T = (d^o, s^o, b^o)_L$, to the W charged gauge boson is

$$\frac{g}{\sqrt{2}}\, \bar{f^o}_{uL}\gamma_\mu f^o_{dL} W^{+\mu} = \frac{g}{\sqrt{2}}\, \bar{\Psi}_{uL}\, [(V^o_{uL}\, V^{(1)}_{uL})_{3\times4}]^T\, (V^o_{dL}\, V^{(1)}_{dL})_{3\times4}\, \gamma_\mu \Psi_{dL}\, W^{+\mu},$$

g is the $SU(2)_L$ gauge coupling. Hence, the non-unitary V_{CKM} of dimension 4×4 is identified as

$$(V_{CKM})_{4\times4} = [(V^o_{uL}\, V^{(1)}_{uL})_{3\times4}]^T\, (V^o_{dL}\, V^{(1)}_{dL})_{3\times4}$$

Similar analysis of the couplings of active L-handed neutrinos and L-handed charged leptons to W boson, leads to the lepton mixing matrix

$$(U_{PMNS})_{4\times8} = [(V^o_{eL}\, V^{(1)}_{eL})_{3\times4}]^T\, (U^o_\nu\, U_\nu)_{3\times8}$$

Neutrino masses: $3+5$ Scenario

Tree level Dirac Neutrino masses

$$h_D\, \overline{\Psi^o_l}\, \Phi^u\, N^o_R \;+\; h_{\nu 1}\, \overline{\Psi^o_{\nu_R}}\, \eta_1\, N^o_L \;+\; h_{\nu 2}\, \overline{\Psi^o_{\nu_R}}\, \eta_2\, N^o_L \;+\; m_D\, \overline{N^o_L}\, N^o_R \;+ h.c$$

h_D, h_1, h_2, and h_3 are Yukawa couplings, and m_D a Dirac type invariant neutrino mass for the sterile neutrino $N^o_{L,R}$. After electroweak symmetry breaking, we obtain in the interaction basis $\Psi^{o\,T}_{\nu\,L,R} = (\nu^o_e, \nu^o_\mu, \nu^o_\tau, N^o)_{L,R}$, the mass terms

$$h_D\, \left[v_{u1}\, \bar{\nu}^o_{eL} + v_{u2}\, \bar{\nu}^o_{\mu L} + v_{u3}\, \bar{\nu}^o_{\tau L} \right]\, N^o_R + \left[h_{\nu 1}\, \Lambda_1\, \bar{\nu}^o_{eR} + h_{\nu 2}\, \Lambda_2\, \bar{\nu}^o_{\mu R} \right]\, N^o_L$$
$$+ m_D\, \bar{N}^o_L\, N^o_R + h.c.$$

Tree level Neutrino Dirac masses

	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
$\overline{\nu^o_{eL}}$	0	0	0	$h_D\, v_{u1}$
$\overline{\nu^o_{\mu L}}$	0	0	0	$h_D\, v_{u2}$
$\overline{\nu^o_{\tau L}}$	0	0	0	$h_D\, v_{u3}$
$\overline{N^o_L}$	$h_{\nu 1}\, \Lambda_1$	$h_{\nu 2}\, \Lambda_2$	0	m_D

Table: Tree level Dirac mass terms $m_{ij}\, \bar{\nu}^o_{iL}\, \nu^o_{jR}$

Tree level Majorana masses

Since $N^o_{L,R}$ are completely sterile neutrinos, we may also write the left and right handed Majorana type couplings

$$h_L \, \bar{\Psi}^o_l \, \Phi^u (N^o_L)^c \quad + \quad m_L \, \bar{N}^o_L \, (N^o_L)^c + h.c$$

$$h_{1R} \, \bar{\Psi}^o_{\nu R} \, \eta_1 \, (N^o_R)^c \; + \; h_{2R} \, \bar{\Psi}^o_{\nu R} \, \eta_2 \, (N^o_R)^c \; + \; m_R \, \bar{N}^o_R \, (N^o_R)^c + h.c.$$

respectively. After spontaneous symmetry breaking, we also get the left handed and right handed Majorana mass terms

$$h_L \, \left[v_{u1} \, \bar{\nu}^o_{eL} + v_{u2} \, \bar{\nu}^o_{\mu L} + v_{u3} \, \bar{\nu}^o_{\tau L} \right] \, (N^o_L)^c \quad + \quad m_L \, \bar{N}^o_L \, (N^o_L)^c$$

$$+ \left[h_{1R} \, \Lambda_1 \, \bar{\nu}^o_{eR} + h_{2R} \, \Lambda_2 \, \bar{\nu}^o_{\mu R} \right] \, (N^o_R)^c \; + \; m_R \, \bar{N}^o_R \, (N^o_R)^c + h.c.$$

	ν^o_{eL}	$\nu^o_{\mu L}$	$\nu^o_{\tau L}$	N^o_L
ν^o_{eL}	0	0	0	$h_L v_{u1}$
$\nu^o_{\mu L}$	0	0	0	$h_L v_{u2}$
$\nu^o_{\tau L}$	0	0	0	$h_L v_{u3}$
N^o_L	$h_L v_{u1}$	$h_L v_{u2}$	$h_L v_{u3}$	m_L

Table: Tree level L-handed Majorana mass terms $m_{ij}\, \bar{\nu}^o_{iL}\, (\nu^o_{jL})^c$

	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
ν^o_{eR}	0	0	0	$h_{1R}\Lambda_1$
$\nu^o_{\mu R}$	0	0	0	$h_{2R}\Lambda_2$
$\nu^o_{\tau R}$	0	0	0	0
N^o_R	$h_{1R}\Lambda_1$	$h_{2R}\Lambda_2$	0	m_R

Table: Tree level R-handed Majorana mass terms $m_{ij}\, \bar{\nu}^o_{iR}\, (\nu^o_{jR})^T$

Tree Level Majorana mass matrix

	$(\nu^o_{eL})^c$	$(\nu^o_{\mu L})^c$	$(\nu^o_{\tau L})^c$	$(N^o_L)^c$	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
$\overline{\nu^o_{eL}}$	0	0	0	$h_L v_{u1}$	0	0	0	$h_D v_{u1}$
$\overline{\nu^o_{\mu L}}$	0	0	0	$h_L v_{u2}$	0	0	0	$h_D v_{u2}$
$\overline{\nu^o_{\tau L}}$	0	0	0	$h_L v_{u3}$	0	0	0	$h_D v_{u3}$
$\overline{N^o_L}$	$h_L v_{u1}$	$h_L v_{u2}$	$h_L v_{u3}$	m_L	$h_1 \Lambda_1$	$h_2 \Lambda_2$	0	M_D
$\overline{(\nu^o_{eR})^c}$	0	0	0	$h_1 \Lambda_1$	0	0	0	$h_{1R} \Lambda_1$
$\overline{(\nu^o_{\mu R})^c}$	0	0	0	$h_2 \Lambda_2$	0	0	0	$h_{2R} \Lambda_2$
$\overline{(\nu^o_{\tau R})^c}$	0	0	0	0	0	0	0	0
$\overline{(N^o_R)^c}$	$h_D v_{u1}$	$h_D v_{u2}$	$h_D v_{u3}$	M_D	$h_{1R} \Lambda_1$	$h_{2R} \Lambda_2$	0	m_R

Tree level U_ν^o mixing matrix

$$U_\nu^o = \begin{pmatrix} c_1 & s_1\, c_2 & \frac{s_1\, s_2}{c_2}\, \nu_{33} & 0 & \frac{s_1\, s_2}{c_2}\, \nu_{34} & \frac{s_1\, s_2}{c_2}\, \nu_{35} & \frac{s_1\, s_2}{c_2}\, \nu_{36} & \frac{s_1\, s_2}{c_2}\, \nu_{38} \\ -s_1 & c_1\, c_2 & \frac{c_1\, s_2}{c_2}\, \nu_{33} & 0 & \frac{c_1\, s_2}{c_2}\, \nu_{34} & \frac{c_1\, s_2}{c_2}\, \nu_{35} & \frac{c_1\, s_2}{c_2}\, \nu_{36} & \frac{c_1\, s_2}{c_2}\, \nu_{38} \\ 0 & -s_2 & \nu_{33} & 0 & \nu_{34} & \nu_{35} & \nu_{36} & \nu_{38} \\ 0 & 0 & 0 & 0 & \nu_{44} & \nu_{45} & \nu_{46} & \nu_{48} \\ 0 & 0 & \nu_{53} & 0 & \nu_{54} & \nu_{55} & \nu_{56} & \nu_{58} \\ 0 & 0 & \nu_{63} & 0 & \nu_{64} & \nu_{65} & \nu_{66} & \nu_{68} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \nu_{84} & \nu_{85} & \nu_{86} & \nu_{88} \end{pmatrix}$$

$$U_\nu^{o\,T}\, M_\nu^o\, U_\nu^o = \mathrm{DiagonalMatrix}[\{0, 0, 0, 0, m_5^o, m_6^o, m_7^o, m_8^o\}]$$

One loop Dirac neutrino masses

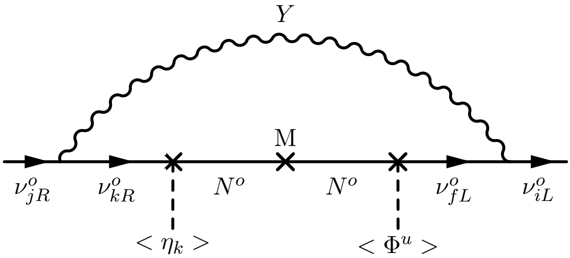


Figure: Generic one loop diagram contribution to the Dirac mass term $m_{ij}\, \bar{\nu}^o_{iL} \nu^o_{jR}$. M $= m_D, m_L, m_R$

One loop L-handed Majorana masses

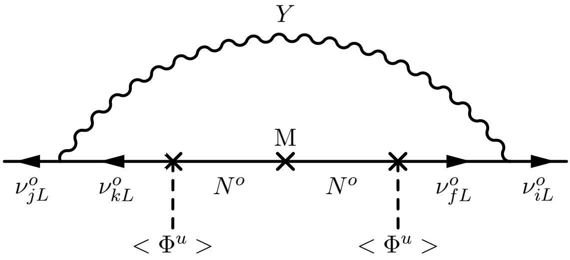


Figure: Generic one loop diagram contribution to the L-handed Majorana mass term $m_{ij}\, \bar{\nu}^o_{iL}(\nu^o_{jL})^T$. M $= m_D, m_L, m_R$

One loop R-handed Majorana masses

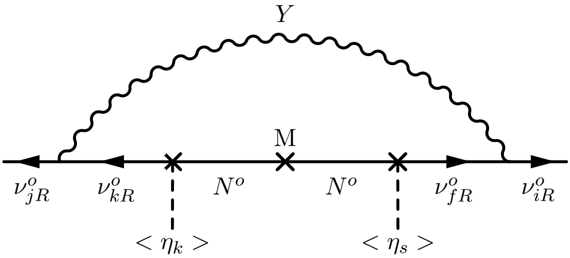


Figure: Generic one loop diagram contribution to the R-handed Majorana mass term $m_{ij}\, \bar{\nu}^o_{iR} (\nu^o_{jR})^T$. $\mathrm{M} = m_D, m_L, m_R$

General One Loop Neutrino Masses

	$(\nu^o_{eL})^c$	$(\nu^o_{\mu L})^c$	$(\nu^o_{\tau L})^c$	$(N^o_L)^c$	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
$\overline{\nu^o_{eL}}$	$L_{\nu 11}$	$L_{\nu 12}$	$L_{\nu 13}$	0	$D_{\nu 15}$	$D_{\nu 16}$	0	0
$\overline{\nu^o_{\mu L}}$	$L_{\nu 12}$	$L_{\nu 22}$	$L_{\nu 23}$	0	$D_{\nu 25}$	$D_{\nu 26}$	0	0
$\overline{\nu^o_{\tau L}}$	$L_{\nu 13}$	$L_{\nu 23}$	$L_{\nu 33}$	0	$D_{\nu 35}$	$D_{\nu 36}$	$D_{\nu 37}$	0
$\overline{N^o_L}$	0	0	0	0	0	0	0	0
$\overline{(\nu^o_{eR})^c}$	$D_{\nu 15}$	$D_{\nu 25}$	$D_{\nu 35}$	0	$R_{\nu 55}$	$R_{\nu 56}$	0	0
$\overline{(\nu^o_{\mu R})^c}$	$D_{\nu 16}$	$D_{\nu 26}$	$D_{\nu 36}$	0	$R_{\nu 56}$	$R_{\nu 66}$	0	0
$\overline{(\nu^o_{\tau R})^c}$	0	0	$D_{\nu 37}$	0	0	0	0	0
$\overline{(N^o_R)^c}$	0	0	0	0	0	0	0	0

Neutrino Majorana mass matrix up to one loop

$$M_\nu = {U^o_\nu}^T \, M^o_{1\,\nu} \, U^o_\nu + Diag(0,0,0,0,m^o_5,m^o_6,m^o_7,m^o_8)$$

$$M_\nu = \begin{pmatrix} 0 & 0 & 0 & \mu_{14} & \tau_{15} & \tau_{16} & \tau_{17} & \tau_{18} \\ 0 & 0 & \mu_{23} & \mu_{24} & \tau_{25} & \tau_{26} & \tau_{27} & \tau_{28} \\ 0 & \mu_{23} & 0 & \mu_{34} & \tau_{35} & \tau_{36} & \tau_{37} & \tau_{38} \\ \mu_{14} & \mu_{24} & \mu_{34} & \mu_{44} & \tau_{45} & \tau_{46} & \tau_{47} & \tau_{48} \\ \tau_{15} & \tau_{25} & \tau_{35} & \tau_{45} & \rho_{55}+m^o_5 & \rho_{56} & \rho_{57} & \rho_{58} \\ \tau_{16} & \tau_{26} & \tau_{36} & \tau_{46} & \rho_{56} & \rho_{66}+m^o_6 & \rho_{67} & \rho_{68} \\ \tau_{17} & \tau_{27} & \tau_{37} & \tau_{47} & \rho_{57} & \rho_{67} & \rho_{77}+m^o_7 & \rho_{78} \\ \tau_{18} & \tau_{28} & \tau_{38} & \tau_{48} & \rho_{58} & \rho_{68} & \rho_{78} & \rho_{88}+m^o_8 \end{pmatrix}$$

$$M_\nu = \begin{pmatrix} M_\mu & M_\tau \\ M_\tau^T & M_\rho \end{pmatrix}$$

where

$$M_\mu = \begin{pmatrix} 0 & 0 & 0 & \mu_{14} \\ 0 & 0 & \mu_{23} & \mu_{24} \\ 0 & \mu_{23} & 0 & \mu_{34} \\ \mu_{14} & \mu_{24} & \mu_{34} & \mu_{44} \end{pmatrix} \quad , \quad M_\tau = \begin{pmatrix} \tau_{15} & \tau_{16} & \tau_{17} & \tau_{18} \\ \tau_{25} & \tau_{26} & \tau_{27} & \tau_{28} \\ \tau_{35} & \tau_{36} & \tau_{37} & \tau_{38} \\ \tau_{45} & \tau_{46} & \tau_{47} & \tau_{48} \end{pmatrix}$$

See-saw approximation

$$M_\rho = \begin{pmatrix} \rho_{55} + m_5^o & \rho_{56} & \rho_{57} & \rho_{58} \\ \rho_{56} & \rho_{66} + m_6^o & \rho_{67} & \rho_{68} \\ \rho_{57} & \rho_{67} & \rho_{77} + m_7^o & \rho_{78} \\ \rho_{58} & \rho_{68} & \rho_{78} & \rho_{88} + m_8^o \end{pmatrix}$$

By assuming the natural hierarchy $|(M_\mu)_{ij}| \ll |(M_\tau)_{ij}| \ll |(M_\rho)_{ij}|$ for the mass terms, the dominant contribution for the active neutrinos and a heavy sterile neutrino, **the 3+1 scenario**, comes from the See-saw approximation:

$$M_\nu^{\text{light}} \approx M_\mu - M_\tau \, M_\rho^{-1} \, M_\tau^T \; .$$

Numerical results

Particular parameter space region at the M_Z scale:

Input values for the $SU(3)_F$ family symmetry:

$$M_1 = 5300\,\text{TeV's} \quad , \quad M_2 = 10^3\ M_1 \quad , \quad \frac{\alpha_H}{\pi} = 0.1266$$

with M_1, M_2 the horizontal boson masses, and the coupling constant, respectively,

$$M_1^2 = \frac{g_H^2\,\Lambda_1^2}{2} \qquad , \qquad M_2^2 = \frac{g_H^2\,\Lambda_2^2}{2}$$

$$g_H = 2.235\ ,\ \Lambda_1 = 3352.7\ TeV\ ,\ \Lambda_2 = 10^3\,\Lambda_1$$

Quark masses and $(V_{CKM})_{4\times 4}$ mixing

u-quarks:

Tree level see-saw mass matrix:

$$M_u^o = \begin{pmatrix} 0 & 0 & 0 & 32445.2 \\ 0 & 0 & 0 & 153081 \\ 0 & 0 & 0 & 188414. \\ 7.09\times 10^9 & -1.3905\times 10^9 & 0 & 5.04313\times 10^9 \end{pmatrix} \text{MeV}\,,$$

the mass matrix up to one loop corrections:

$$M_u = \begin{pmatrix} 0 & -164.78 & -450.207 & -644.992 \\ 2.05472 & 600.381 & 1627.36 & 2331.45 \\ -2.47402 & 1593.14 & -173000. & 39878.4 \\ -0.000039 & 0.025346 & 0.442859 & 8.811\times 10^9 \end{pmatrix} \text{MeV}$$

the u-quark masses

$$(\, m_u\,,\ m_c\,,\ m_t\,,\ M_U\,) = (\, 1.38049\,,\ 638.077\,,\ 173016\,,\ 8.81\times 10^9\,)\ \text{MeV}$$

the mixing matrices:
$V_{u\,L} = V^{o}_{u\,L}\, V^{(1)}_{u\,L}$:

$$\begin{pmatrix} 0.985383 & 0.105814 & -0.133508 & 2.677 \times 10^{-6} \\ 0.000823 & -0.78665 & -0.617398 & 0.000012 \\ -0.170353 & 0.608264 & -0.775239 & 0.000015 \\ 0 & 4.732 \times 10^{-7} & 0.000020 & 1. \end{pmatrix}$$

$V_{u\,R} = V^{o}_{u\,R}\, V^{(1)}_{u\,R}$:

$$\begin{pmatrix} -0.000293 & 0.187295 & -0.563405 & 0.804671. \\ -0.001495 & 0.982274 & 0.101149 & -0.157814 \\ 0.999999 & 0.001523 & -0.000014 & 0. \\ 3.507 \times 10^{-7} & 0.007521 & 0.819966 & 0.572364 \end{pmatrix}$$

d-quarks:

$$M_d^o = \begin{pmatrix} 0 & 0 & 0 & 478.509. \\ 0 & 0 & 0 & 2132.96. \\ 0 & 0 & 0 & 2914.63. \\ 3.198\times 10^6 & -501085. & 0 & 1.972\times 10^6 \end{pmatrix} \text{ MeV}$$

$$M_d = \begin{pmatrix} 0 & -3.7187 & -29.5924 & -48.5743 \\ 34.2 & 45.5029 & -1.54773 & -2.54051 \\ -45.6 & 34.1272 & -2860. & 412.606. \\ -0.0228 & 0.017063 & 0.125684 & 3.791\times 10^6 \end{pmatrix} \text{ MeV}$$

$$(\,m_d\,,\,m_s\,,\,m_b\,,\,M_D\,) = (\,2.8206\,,\,57.\,,\,2860.72\,,\,3.79\times 10^6\,)\text{ MeV}$$

Mixing matrices:

$V_{d\,L} = V^{o}_{d\,L}\, V^{(1)}_{d\,L}$:

$$\begin{pmatrix} -0.982152 & -0.123914 & -0.141504 & 0.000067 \\ 0.183872 & -0.790966 & -0.583578 & 0.000358 \\ 0.039611 & 0.599181 & -0.799633 & 0.000487 \\ -0.000019 & 0 & 0.000608 & 1. \end{pmatrix}$$

$V_{d\,R} = V^{o}_{d\,R}\, V^{(1)}_{d\,R}$:

$$\begin{pmatrix} 0.082490 & -0.123915 & -0.515747 & 0.843709 \\ 0.593446 & -0.790966 & 0.068728 & -0.132177 \\ 0.800455 & 0.59918 & -0.015932 & -7.329\times 10^{-9} \\ 0.016995 & -3.184\times 10^{-8} & 0.853831 & 0.520273 \end{pmatrix}$$

Quark mixing matrix

$$V_{CKM} = \begin{pmatrix} -0.974392 & -0.224827 & -0.003696 & -0.000016 \\ -0.224475 & 0.973562 & -0.042288 & 0.000021 \\ -0.013106 & 0.040376 & 0.999098 & -0.000608 \\ 3.74 \times 10^{-7} & -1.28 \times 10^{-6} & -0.000020 & 1.24 \times 10^{-8} \end{pmatrix}$$

Lepton masses and $(U_{PMNS})_{4\times 8}$ mixing

Charged leptons:

Tree level:

$$M_e^o = \begin{pmatrix} 0 & 0 & 0 & 2670.25 \\ 0 & 0 & 0 & 11902.6 \\ 0 & 0 & 0 & 16264.7 \\ 1.218\times 10^{10} & -2.322\times 10^{9} & 0 & 6.078\times 10^{10} \end{pmatrix} \mathrm{MeV}\,,$$

up to one loop corrections:

$$M_e = \begin{pmatrix} 0 & -19.9797 & -83.226 & -16.9884 \\ 0.6408 & 71.9782 & 293.027 & 59.814 \\ -0.8544 & 168.853 & -1712.54 & 480.432 \\ -2.74\times 10^{-7} & 0.000054 & 0.000755 & 6.20\times 10^{10} \end{pmatrix} \mathrm{MeV}$$

the charged lepton masses

$$(\, m_e\,,\, m_\mu\,,\, m_\tau\,,\, M_E\,) = (\, 0.4860\,,\, 102.717\,,\, 1746.17\,,\, 6.20\times 10^{10}\,)\ \mathrm{MeV}$$

Mixing matrices:

$V_{eL} = V^{o}_{eL}\, V^{(1)}_{eL}$:

$$\begin{pmatrix} 0.986458 & 0.074461 & -0.146138 & 4.309 \times 10^{-8} \\ 0.002766 & -0.898433 & -0.439101 & 1.933 \times 10^{-7} \\ -0.163991 & 0.43275 & -0.886473 & 2.624 \times 10^{-7} \\ 0 & 5.689 \times 10^{-8} & 3.238 \times 10^{-7} & 1 \end{pmatrix}$$

$V_{eR} = V^{o}_{eR}\, V^{(1)}_{eR}$:

$$\begin{pmatrix} -0.001109 & 0.258469 & -0.945829 & 0.196466 \\ -0.005573 & 0.965887 & 0.256182 & -0.037429 \\ 0.999984 & 0.005670 & 0.000376 & 0 \\ 9.591 \times 10^{-6} & -0.014929 & 0.199442 & 0.979796 \end{pmatrix}$$

Neutrino masses and Lepton $(U_{PMNS})_{4\times 8}$ mixing:

	$(\nu^o_{eL})^c$	$(\nu^o_{\mu L})^c$	$(\nu^o_{\tau L})^c$	$(N^o_L)^c$	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
$\overline{\nu^o_{eL}}$	0	0	0	975.261	0	0	0	13.247
$\overline{\nu^o_{\mu L}}$	0	0	0	4601.39	0	0	0	62.502
$\overline{\nu^o_{\tau L}}$	0	0	0	5663.49	0	0	0	76.928
$\overline{N^o_L}$	975.261	4601.39	5663.49	800.	1404.	2188.33	0	22500.
$\overline{(\nu^o_{eR})^c}$	0	0	0	1404.	0	0	0	2.73×10^7
$\overline{(\nu^o_{\mu R})^c}$	0	0	0	2188.33	0	0	0	1.24×10^7
$\overline{(\nu^o_{\tau R})^c}$	0	0	0	0	0	0	0	0
$\overline{(N^o_R)^c}$	13.247	62.502	76.928	22500.	2.73×10^7	1.24×10^7	0	1.81×10^9

Table: Tree Level Majorana masses (eV)

One loop Majorana masses (eV)

	$(\nu^o_{eL})^c$	$(\nu^o_{\mu L})^c$	$(\nu^o_{\tau L})^c$	$(N^o_L)^c$	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
$\overline{\nu^o_{eL}}$	-0.5749	-2.7124	-3.3385	0	-0.7559	0.1488	0	0
$\overline{\nu^o_{\mu L}}$	-2.7124	-12.7979	-15.7519	0	-3.1858	0.6214	0	0
$\overline{\nu^o_{\tau L}}$	-3.3385	-15.7519	-19.3877	0	-3.9212	0.8641	-0.0806	0
$\overline{N^o_L}$	0	0	0	0	0	0	0	0
$\overline{(\nu^o_{eR})^c}$	-0.7559	-3.1858	-3.9212	0	-284546.	-129640.	0	0
$\overline{(\nu^o_{\mu R})^c}$	0.1488	0.6214	0.8641	0	-129640.	-59065.7	0	0
$\overline{(\nu^o_{\tau R})^c}$	0	0	-0.0806	0	0	0	0	0
$\overline{(N^o_R)^c}$	0	0	0	0	0	0	0	0

Neutrino Majorana mass matrix up to one loop (eV)

$$M_\nu = {U^o_\nu}^T \, M^o_{1\,\nu} \, U^o_\nu + Diag(0,0,0,0,m^o_5,m^o_6,m^o_7,m^o_8)$$

$$\begin{pmatrix}
0 & 0 & 0 & -0.0470 & 0.0067 & 0.0060 & 0.0648 & -0.0010 \\
0 & 0 & 0.0515 & 0.0490 & -0.0067 & -0.0065 & 0.0368 & -0.0006 \\
0 & 0.0515 & 0 & -0.0116 & -0.0442 & -0.0419 & 3.56 \times 10^{-6} & -3.42 \times 10^{-9} \\
-0.0470 & 0.0490 & -0.0116 & -3.4587 & -6.3112 & -5.9927 & 2.3594 & -0.0390 \\
0.0067 & -0.0067 & -0.0442 & -6.3112 & -7121.58 & -12.9471 & -873.148 & 14.4492 \\
0.0060 & -0.0065 & -0.0419 & -5.992 & -12.947 & 7892.83 & 897.75 & -14.856 \\
0.0648 & 0.0368 & 3.56 \times 10^{-6} & 2.359 & -873.148 & 897.758 & -839967. & 5684.58 \\
-0.0010 & -0.0006 & -3.42 \times 10^{-9} & -0.0390 & 14.449 & -14.856 & 5684.58 & 1.81 \times 10^{9}
\end{pmatrix}$$

Neutrino masses:

$$(m_1 = 0.00064, m_2 = 0.05101, m_3 = 0.05174, m_4 = 3.45909,$$

$$m_5 = 7120.68, m_6 = 7893.8, m_7 = 83996, m_8 = 1.8128 \times 10^9) \text{ eV}$$

Squared neutrino mass differences:

$$m_2^2 - m_1^2 = 2.602 \times 10^{-3}\ eV^2$$

$$m_3^2 - m_2^2 = 7.555 \times 10^{-5}\ eV^2$$

$$m_4^2 - m_1^2 = 11.965\ eV^2$$

Neutrino mixing: $U_\nu = U_\nu^o\, U_\nu^1$

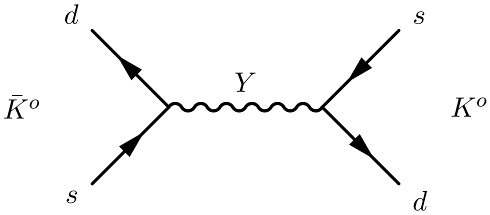
$$
\begin{pmatrix}
0.97728 & 0.12392 & -0.10651 & -0.03593 & 0.09454 & -0.08939 & 3.50\times 10^{-6} & 7.30\times 10^{-9} \\
-0.21112 & 0.52944 & -0.53615 & -0.10388 & 0.44607 & -0.42175 & 0.00001 & 3.44\times 10^{-8} \\
-0.00010 & -0.44846 & 0.45207 & -0.1536 & 0.54903 & -0.51910 & 0.00001 & 4.24\times 10^{-8} \\
-0.00001 & 9.50\times 10^{-6} & -0.00001 & -0.00116 & -0.68718 & -0.72648 & -0.00215 & 0.00001 \\
-0.00557 & 0.00505 & -0.00311 & -0.40713 & -0.05763 & 0.05246 & 0.90987 & 0.01505 \\
0.01224 & -0.01109 & 0.00684 & 0.89361 & 0.12287 & -0.11889 & 0.41455 & 0.00685 \\
0.01299 & -0.70926 & -0.70481 & -0.00358 & 6.22\times 10^{-6} & 5.30\times 10^{-6} & 0 & 0 \\
1.45\times 10^{-9} & 6.46\times 10^{-10} & -1.15\times 10^{-10} & 9.37\times 10^{-9} & 0.00003 & 0.00003 & -0.01654 & 0.99986
\end{pmatrix}
$$

U_{PMNS} lepton mixing matrix :

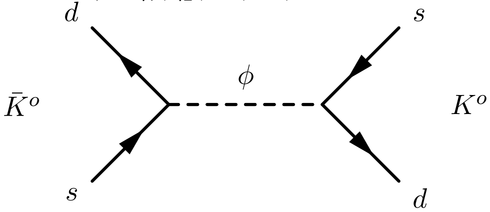
$$
\left(\begin{array}{cccc}
0.96347 & 0.19726 & -0.180688 & -0.01054 \\
0.26240 & -0.66052 & 0.66940 & 0.02418 \\
-0.05002 & 0.14696 & -0.14976 & 0.18702 \\
1.26 \times 10^{-9} & -1.002 \times 10^{-8} & 1.04 \times 10^{-8} & -6.19 \times 10^{-8}
\end{array}\right.
$$

$$
\left.\begin{array}{cccc}
0.00446 & -0.00421 & 2.47 \times 10^{-7} & 3.44 \times 10^{-10} \\
-0.15613 & 0.14761 & -5.68 \times 10^{-6} & -1.20 \times 10^{-8} \\
-0.69639 & 0.65842 & -0.00002 & -5.38 \times 10^{-8} \\
2.34 \times 10^{-7} & -2.21 \times 10^{-7} & 0 & 0
\end{array}\right)
$$

FCNC in Neutral Mesons



Generic tree level contribution to $K^o - \bar{K}^o$ from the $SU(3)_F$ horizontal gauge bosons and scalar fields $\phi = \eta_1 , \eta_2 , \Phi^u , \Phi^d ,$



FCNC in Neutral Mesons

PDG2018: CKM quark-mixing matrix
To illustrate the level of suppression required for BSM contributions, consider a class of models in which the unitarity of the CKM is maintained, and the dominant effect of the new physics is to modify the neutral meson amplitudes by

$$\frac{z_{ij}}{\Lambda^2}\,(\bar{q}_i\gamma^\mu P_L q_j)^2$$

The existent data imply that

$$\frac{\Lambda}{\sqrt{|z_{ij}|}} \gtrsim \begin{cases} 10^4 \text{ TeV for } K^o - \bar{K}^o \\ 10^3 \text{ TeV for } D^o - \bar{D}^o \\ 500 \text{ TeV for } B^o - \bar{B}^o \\ 100 \text{ TeV for } B^o_s - \bar{B}^o_s \end{cases}$$

$K^o - \bar{K}^o$ and $D^o - \bar{D}^o$ meson mixing

$K^o - \bar{K}^o$:

$$\delta_L = 0.145437 \quad , \quad \frac{M_1}{\frac{g_H}{2}\,|\delta_L|} = 32601.3 \text{ TeV's}$$

$$\delta_R = 0.469396 \quad , \quad \frac{M_1}{\frac{g_H}{2}\,|\delta_R|} = 10101.1 \text{ TeV's}$$

$$\sqrt{|\delta_{LR}|} = 0.369507 \quad , \quad \frac{M_1}{\frac{g_H}{2}\,\sqrt{|\delta_{LR}|}} = 12831.8 \text{ TeV's}$$

$D^o - \bar{D}^o$:

$$\delta_L = 0.000647997 \quad , \quad \frac{M_1}{\frac{g_H}{2}\,|\delta_L|} = 7.31705 \times 10^6 \text{ TeV's}$$

$$\delta_R = 0.00146872 \quad , \quad \frac{M_1}{\frac{g_H}{2}\,|\delta_R|} = 3.22828 \times 10^6 \text{ TeV's}$$

$$\sqrt{|\delta_{LR}|} = 0.669613 \quad , \quad \frac{M_1}{\frac{g_H}{2}\,\sqrt{|\delta_{LR}|}} = 7080.85 \text{ TeV's}$$

Conclusions:

- Particle model with local $SU(3)_F$ family symmetry can account for the hierarchical spectrum of quark masses and mixing, and charged lepton masses
- FCNC in $K^o - \bar{K^o}$ and $D^o - \bar{D^o}$ Neutral Mesons may be properly suppressed
- The mass of the $SU(2)_L$ weak singlet vector-like D quark, M_D may lie within a few TeV's region
- Simultaneously this scenario can fit the squared neutrino mass differences: $m_2^2 - m_1^2 \approx 2.60 \times 10^{-3}\ eV^2$, $m_3^2 - m_2^2 \approx 7.55 \times 10^{-5}\ eV^2$, $m_4^2 - m_1^2 \approx 11.96\ eV^2$, as well as the majority of the corresponding entries of the U_{PMNS} lepton mixing matrix.

THANK YOU