Double Exponential fractal structure of H_n quasi-crystals

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July 10, 2022

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Quantum Gravity Research https://quantumgravityresearch.org/ is working on a graph-theoretic approach to quantum gravity and particle physics operated on quasicrystalline point space.

The **Quasi-crystal** is a projection of a higher dimensional crystal slice to a lower dimension via an irrational angle.

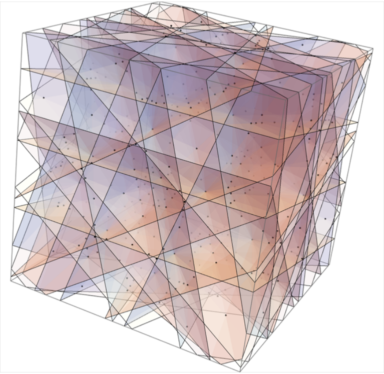
We use the largest exceptional Lie group, **E8**, as our hyper-lattice.

The **Emergence Theory** is a theory of pixelated spacetime and of reality as a quasicrystalline point space projected from the E8 crystal.

Producing quasicrystal lattices

There are several **methods of calculating quasicrystal lattices**:

- **Cut and project** high-dimensional lattices;
- String **substitutions**;
- Considering points of **multy-grids** intersections:



Terms

Terms to be familiar with:

- Golden ratio,
- Fibonacci chain,
- Inflation

Golden Ratio

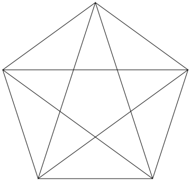
The solution of equation

$$\Phi(\Phi - 1) = 1 \tag{1}$$

is called the Golden Ratio:

$$\Phi = \frac{\sqrt{5}+1}{2} \approx 1.61803 \tag{2}$$

It appears in relations between elements of uniform pentagon that possesses the symmetry of H_2 Coxeter group.



Fibonacci chain and inflation

Consider the symbolic substitutions of two symbols L (Long) and S (Short) with two rules:

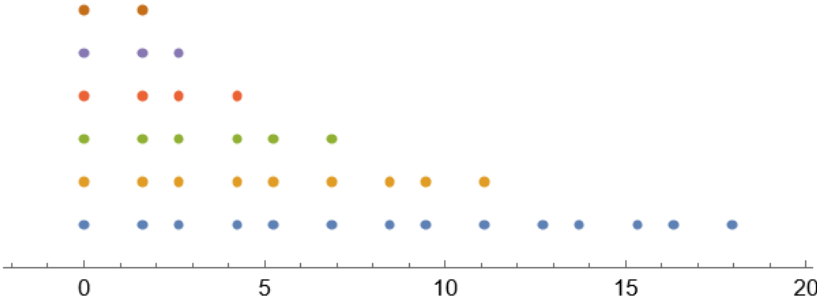
- $S \to L$,
- $L \to LS$.

Starting with either S or L, and applying the rule set repeatedly, we have an **inflation** of the **Fibonacci chain**:

- S;
- L;
- LS;
- LSL;
- $LSLLS$;
- $LSLLSLSL$;
- $LSLLSLSLLSLLS$;
- $LSLLSLSLLSLLSLSLLSLSL$;
- $\ldots$

Fibonacci chain point set on the number line

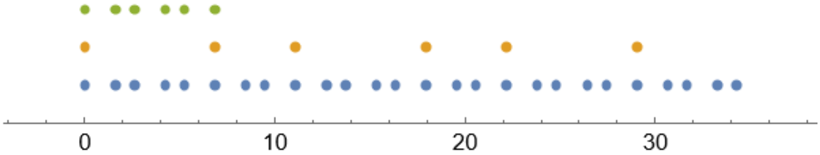
Treating L as Long and S as Short segments, with the *golden ratio* $L/S = \Phi$, one can place the *Fibonacci chain* onto numeric line:



The chain is non-periodic, so it has **no translation symmetry**.

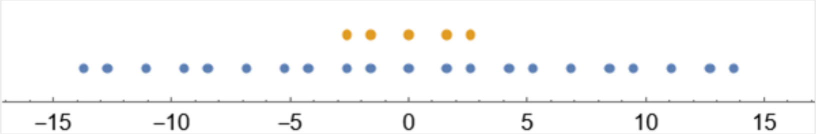
Long-Long spaced nodes also form the Fibonacci chain

All Long-Long spaced nodes form a subset of the original Fibonacci chain that is the Fibonacci chain too, that is scaled by $\Phi^3 \approx 4.23607$.



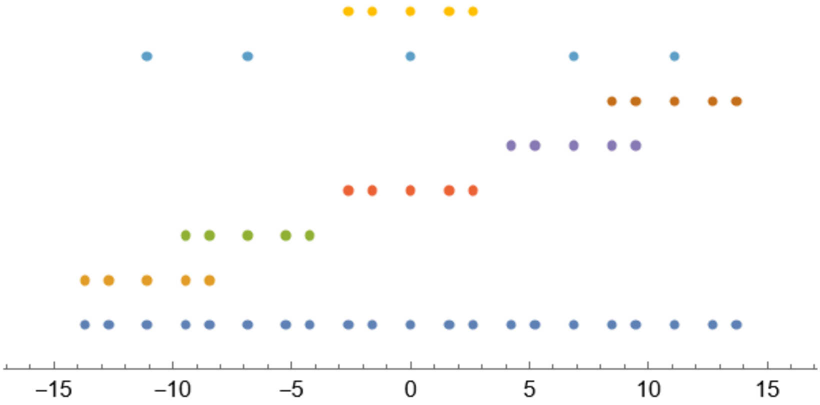
Kernel

The LL-spaced node together with two S segments ($SLLS$) is a minimal pattern (**kernel**) that fills the whole Fibonacci chain, being repeated.
The node in the origin also fulfills this condition if one considers the Fibonacci chain together with its negative S^0-**symmetrical** part (reflection):



Double-exponential inflation

Using this *self-similarity*, one can easily find new nodes of the Fibonacci chain by putting a copy of the **whole kernel** in the place of **each node** of the same **kernel scaled** by the factor Φ^3:



Node count and size

The result of one *inflation* can be inflated again. The count of nodes grows more quickly than exponentially, being multiplied by itself on the each step:

$$N_n = N_0^{2^n} \tag{3}$$

There is a correction of distinct node count caused by overlapping:

- In case of L **spacing** between nodes in the scaled copy, non-scaled copies are aligned so that the L **space** between them is left;
- S-**spaced** nodes do overlap so that their S-spaced segments coincide completely. This effect reduces the count of non-coinciding nodes by the fraction of S nodes that has an asymptotic of $2 - \Phi \approx 0.381966$.

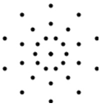
The radius of the calculated part of chain increases the same very quick way:

$$R_n = R_0 \Phi^{3 \cdot 2^n} \tag{4}$$

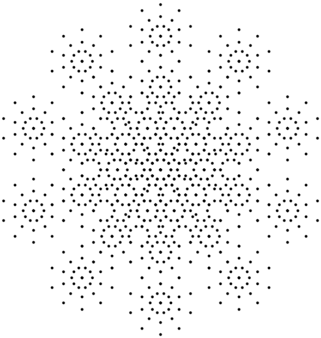
The iteration number n stays inside two nested exponents, so N_n

2D Kernel and its 0th inflation

The 2D analog of 5-node 1D *SLLS* kernel can be obtained by applying the H_2 symmetry (5-fold rotations) to the 1D kernel.

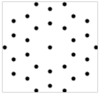


However, this structure has voids after Φ^3-inflation:

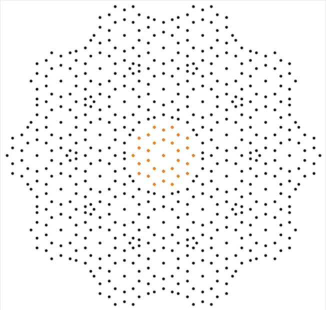


2D Kernel with the 3^{rd} shell added

Adding *the* 3^{rd} *shell* containing 10 nodes of the larger $(L \times \sqrt{4 - \Phi^{-2}})$ decagon to the kernel fixes this void problem:



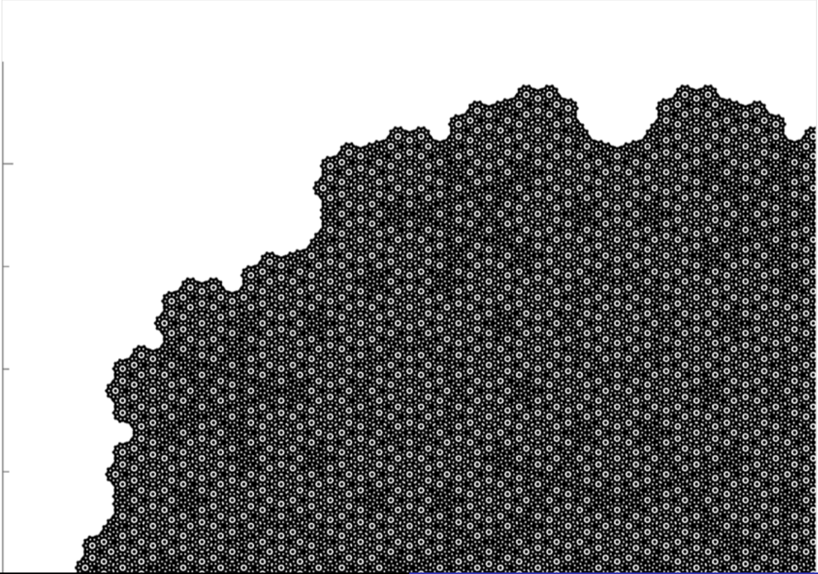
Than, it produces the 0th inflation without voids:



The 1D kernel is still the equatorial cross-section of 2D kernel.

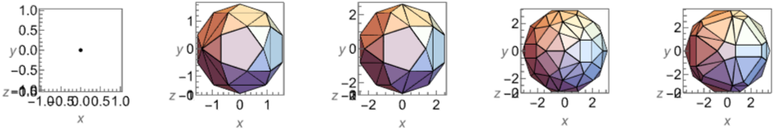
The 1st inflation

After the 1^{st} step the result (shown partially) is as follows:



3D kernel

The 3D kernel is produced via applying H_3 (dodecahedral) rotation to the 2D kernel with **5 points added** to produce the 4^{th} shell.
It contains 193 nodes in the origin and 4 shells:



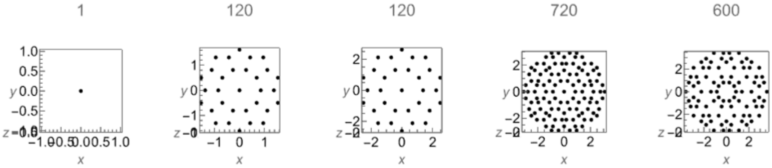
4D kernel

To produce 4D kernel, we identified H_4 set of rotations and applied them to 3D kernel. No new nodes was needed to add, so probably there is no 5th shell.

This supported by analyse of point density distribution that did not show obvious differences from 3D lattice that is confirmed having no voids.

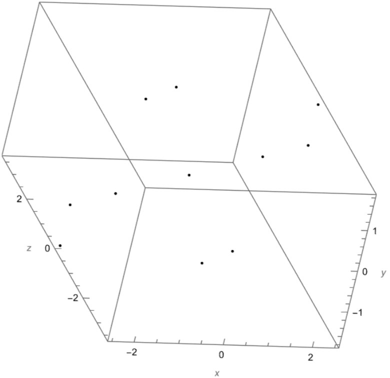
The 4D kernel contains 1561 nodes and also 4 shells that have shape of two Φ-scaled 600-cell polytopes, one 720-vertice polytope and 120-cell uniform polytope.

They are shown in their projection onto the 2D xy plane:



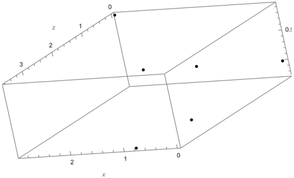
1D kernel with all 3D points added

Adding all extra points immediately into 1D kernel on the most first step allows us to separate point coordinate data from rotational operations completely.



Asymmetrical initial set

Eliminating duplication that comes from S^0 rotation, we get the minimal *initial set* containing 6 points. All four kernels can be obtained from this initial set by applying symmetry operations; the lattices can be produced from these kernels by the $\Phi^{3\cdot 2^n}$-scaling inflation. This initial set in numeric form is as follows:
$((0,0,0,0),(1.61803,0,0,0),(2.61803,0,0,0),$
$(2.92705,0.951057,0,0),(0,0,3.07768,0),$
$(0.809017,-0.262866,3.60341,0)$



In the table the brief description of shapes of four shells of four kernels as well as of the initial set are provided:

	0th	1st shell	2nd shell	3rd shell	4th shell	Vertex count
radius	0	Φ	Φ^2	$\Phi\sqrt{4-\Phi^{-2}}$	$\Phi^2\sqrt{2}$	
Initial set	Point	Point	Point	(Two Points)	(Point)	6
1D	Point	Two points	Two points	(Four points)	(Two points)	5+6
2D	Point	Decagon	Decagon	Dec.(+2 points)	(2 5-gons)	31+12
3D (FIG)	Point	Icosidodecahedron	Icosidodecahedron	[72-vertices]	C5C/20G	193
4D	Point	600-cell	600-cell	[720-vertices]	120-cell	1561

The parenthesis () mean that points in them appear out of corresponding space dimension, in case of using 6-point initial set.

Speculation on iterations and scale levels

There are 40 decimal orders of magnitude between classical electron radius and the size of visible Universe. This gap is of the similar order as the difference between radii of quasi-crystals on the 6th and the 7th iteration.
Mapping them together, one finds Planck scale on the 5th iteration, having 4 sub-Planck levels beyond it.

n	$\Phi^{3\cdot 2^n}$	Scale, m	
0	4.23607	$8.93856 \cdot 10^{-55}$	
1	17.9443	$3.78644 \cdot 10^{-54}$	Sub-Planck levels
2	321.997	$6.79448 \cdot 10^{-53}$	
3	103682	$2.1878 \cdot 10^{-50}$	
4	$1.075 \cdot 10^{10}$	$2.26836 \cdot 10^{-45}$	
5	$1.15562 \cdot 10^{20}$	$2.43847 \cdot 10^{-35}$	Planck scale
6	$1.33545 \cdot 10^{40}$	$2.81794 \cdot 10^{-15}$	Classical electron radius (fixed)
7	$1.78342 \cdot 10^{80}$	$3.76321 \cdot 10^{25}$	Universe scale
8	$3.18059 \cdot 10^{160}$	$6.71139 \cdot 10^{105}$	"Hyperverse" scale

The 8th iteration is probably the last one in this game: it corresponds to the "Hyperverse" in which our Universe plays the role that a point does in out Universe.

Conclusion

- The new effective way of calculating the family of four H_n quasi-crystals is found. The performance is now limited by storage memory but not by CPU.
- We see the curious hierarchy of scales that allows us to say that even the Universe has fractal structure it can be non-distinguishable on wide range of scales from electron radius up to the Universe size.