REPORT
BY TOPIC
"DESCRIPTION OF MASSLESS PARTICLES IN THE TWISTOR THEORY"

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CONTENT

INTRODUCTION

The theory of twistors was proposed by Roger Penrose in 1967. It is based on the correspondence between the complex space of twistors and light rays in the usual Minkowski space. The name of the theory comes from the beautiful Robinson correspondence, which is an implementation of a (non-zero) twistor and is described in Section 1.4.

The basic space in the theory of twistors is not the Minkowski space, but the space of twistors, the elements of which (roughly speaking) are light rays. A non-local theory is obtained automatically. Non-locality is one of the advantages of the theory of twistors, which makes it possible to remove the gravity quantization problem known in QFT.

Twistors are needed in order to use them to describe solutions of conformally invariant equations of field theory on Minkowski space. [1] "Twistor program" [2] Penrose's idea was to use the twistor correspondence constructed by him to compare the solutions of equations of the specified type to objects of complex analytical geometry on the space of twistors. When moving to the twistor description, conformally invariant equations "disappear and only complex geometry remains.

The theory of twistors has had a serious impact on differential and integral geometry, the theory of nonlinear differential equations and representation theory, and in physics - on general relativity and quantum field theory.

1. GEOMETRY OF TWISTORS

1.1. TWISTOR CORRESPONDENCE

The Minkowski space M is a four-dimensional real space $\mathbb{R}^4$. Its points are called events. On the space M there is a quadratic form with the signature $(+---)$: $beta(x, x) = x_0^2 - x_1^2 - x_2^2 - x_3^2$. A nonzero vector x is called isotropic if $\beta(x, x) = 0$. An affine line in M is called light if it is parallel to a line generated by some light vector. A light cone centered at the point x^* is the union of all light lines passing through the point x^*. An equivalent definition can be given:

$$C_{x^*} = \{x \in M : \beta(x - x^*, x - x^*) = 0\}. \tag{1.1}$$

The projective space of zero twistors $\mathbb{PN}$ is a set whose elements are the light rays of the space M. We will call the space $\mathbb{PN}$ also the space of light rays. The geometric structure of this space will be discussed below. For now, we note that it is five-dimensional. Indeed, the direction of the light line is parametrized by the points of the two-dimensional sphere $\mathbb{S}^2$. When the direction is set, the position of the line is set by the intersection point of this line with a three-dimensional hyperplane perpendicular to the selected direction. Thus, there are $2 + 3 = 5$ degrees of freedom of the light beam [3].

So, let $\mathbb{M}$ be a four-dimensional Minkowski space, and $R = (t, x, y, z)$ be an arbitrary point of it. The twistor space $\mathbb{T}$ for $\mathbb{M}$ is called a four-dimensional vector space over a field of complex numbers. The points of this space are called twisters. Thus, a twistor is a four of complex numbers $Z^\alpha = (Z^0, Z^1, Z^2, Z^3)$. A twister Z^α is called an incident event R if the relation holds

$$\begin{pmatrix} Z^0 \\ Z^1 \end{pmatrix} = \frac{i}{\sqrt{2}} \begin{pmatrix} t + z & x + iy \\ x - iy & t - z \end{pmatrix} \begin{pmatrix} Z^2 \\ Z^3 \end{pmatrix} \tag{1.2}$$

The twistor space corresponds to the dual (or dual) space $\mathbb{T}^*$. The elements of the dual space $\overline{Z}_\alpha$ are expressed in terms of the elements of the space $\mathbb{T}$ by

the formula

$$(\overline{Z}_0, \overline{Z}_1, \overline{Z}_2, \overline{Z}_3) = (\overline{Z^2}, \overline{Z^3}, \overline{Z^0}, \overline{Z^1}). \tag{1.3}$$

Equality (1.3), defines the Hermitian product on the space $\mathbb{T}$. The norm of the Z twistor is the value

$$\overline{Z}Z = \overline{Z}_0 Z^0 + \overline{Z}_1 Z^1 + \overline{Z}_2 Z^2 + \overline{Z}_3 Z^3 =$$
$$= \overline{Z^2} Z^0 + \overline{Z^3} Z^1 + \overline{Z^0} Z^2 + \overline{Z^1} Z^3 = \tag{1.4}$$
$$= \frac{1}{2}(|Z^0 + Z^2|^2 + |Z^1 + Z^3|^2 - |Z^0 - Z^2|^2 - |Z^1 - Z^3|^2).$$

A Z twistor is called null if $\overline{Z}Z = 0$.

The theorem. A Z twistor is incident to some event in $\mathbb{M}$ if and only if it is zero.

The implication of $\rightarrow$ is proved by a simple calculation.

The projective space of twistors is called the projectivization of the space of twistors $\mathbb{PT}$. A projective space is a set of lines passing through 0, in $\mathbb{T}$. Homogeneous coordinates are introduced on the space $\mathbb{PT}$ (coordinates defined up to multiplication by a nonzero constant). That is , an arbitrary point in the projective space has the form:

$$[Z^0 : Z^1 : Z^2 : Z^3]. \tag{1.5}$$

$\mathbb{PN}$ denotes the space of projective zero twistors. This space has 5 real dimensions. On the other hand, in the space $\mathbb{T}$ zero twistors form a 7-dimensional real subspace. It divides the original space into two parts. Twistors with $\overline{Z}Z > 0$ are called positive and form the space $\mathbb{T}^+$. Negative twistors $\overline{Z}Z < 0$ form the space $\mathbb{T}^-$. Similarly, $\mathbb{PN}$ divides $\mathbb{PT}$ into two parts: $\mathbb{TP}^+$ and $\mathbb{TP}^-$.

How are the Minkowski space M and $\mathbb{PN}$ related? If two points P and Q of the Minkowski space are incident to the same twistor Z (zero), then they are separated by a zero interval, that is, they are separated by a zero interval. In addition, the twistors Z and λZ with non-zero λ are responsible for one light beam. Thus, there is a mapping that translates the space $\mathbb{PN}$ into the space of light rays of Minkowski space. Note that for $Z^2 = Z^3 = 0$, the matrix (1.2) must have infinite elements. I.e. such a zero twistor should be answered by a

light beam at infinity. Such a ray lies on an infinitely distant light cone $\mathbb{J}$ of a compactified Minkowski space $M^{\#}$.

On the contrary, let P be an arbitrary point M. It follows from formula (1.2) that two conditions are imposed on the components Z^0, Z^1, Z^2, Z^3. Each relation defines a three-dimensional subspace in $\mathbb{T}$. Hence, it defines a two-dimensional subspace (a projective plane) in the projectivization of $\mathbb{PT}$. These two planes intersect along the projective line $\mathbb{CP}^1$, which is homeomorphic to the sphere $\mathbb{S}^2$. This sphere (or projective line) lies in $\mathbb{PN}$. Let j be the vertex of the cone $\mathbb{J}$. It corresponds to the projective line (sphere) $\mathbb{I}$ in $\mathbb{PN}$. Any other point of the cone $\mathbb{J}$ corresponds to a sphere intersecting with $\mathbb{I}$.

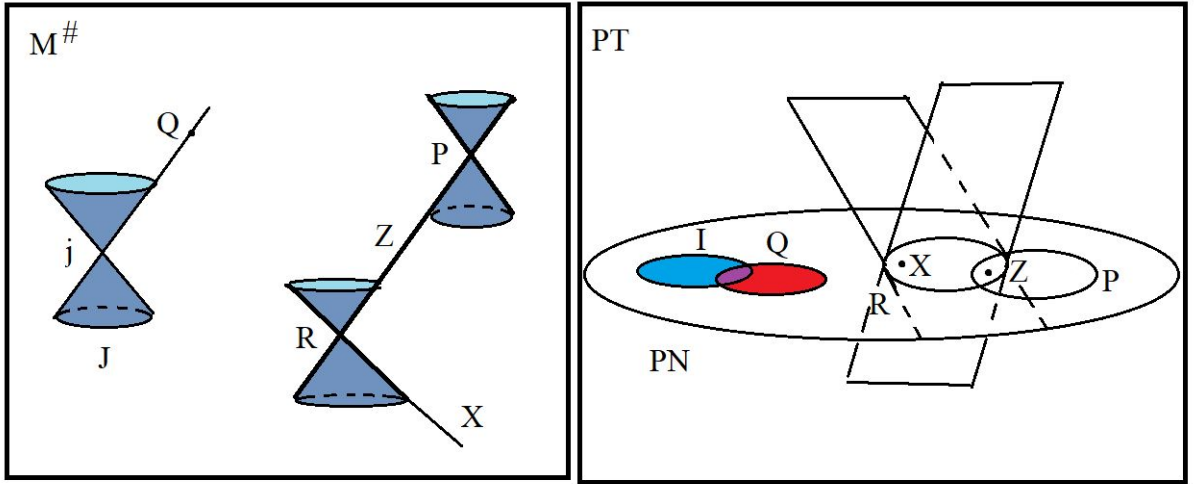All elements of twistor matching are shown in the following figure.

Рисунок 1.1 — Twistor matching scheme

1.2. COMPACTIFIED MINKOWSKI SPACE

The compactified Minkowski space $\mathbb{M}^{\#}$ is obtained from the usual Minkowski space $\mathbb{M}$ by adding an infinitely distant light cone $\mathbb{J}$. The resulting space has a higher symmetry than the Minkowski space itself. However, this definition does not reveal this symmetry.

Let's give a more "symmetric definition". Consider the six-dimensional space $\mathbb{E}^{2,4}$ with the metric

$$ds^2 = dw^2 + dt^2 - dx^2 - dy^2 - dz^2 - dv^2. \tag{1.6}$$

In this space, consider a five-dimensional cone K, given by the equation

$$w^2 + t^2 - x^2 - y^2 - z^2 - v^2 = 0. \tag{1.7}$$

Рисунок 1.2 — The intersection of the cone $w^2 + t^2 - x^2 - y^2 - z^2 - v^2 = 0$ and the hyperplane $w - v = 1$ is a Minkowski space

1.3. MINKOWSKI SPACE COMPLEXIFICATION

The complex Minkowski space $\mathbb{C}M$ is a complexification of the Minkowski space M coinciding with a 4-dimensional complex vector space consisting of vectors $z = \left(z^0, z^1, z^2, z^3\right) \in \mathbb{C}^4$. Also, as in the real case, the vector $z \in \mathbb{C}M$ is called a complex light vector if

$$|z|^2 = \left(z^0\right)^2 - \left(z^1\right)^2 - \left(z^2\right)^2 - \left(z^3\right)^2 = 0. \tag{1.8}$$

A complex light cone with a vertex at the point $z_0 \in \mathbb{C}M$ is given by the equation: $(z - z_0)^2 = 0$. The analogues of the cones of the future and the past in the complex case are the pipe of the future

$$\mathbb{C}M_+ = \left\{ z = x + iy \in \mathbb{C}M : |y|^2 > 0, y^0 > 0 \right\} \tag{1.9}$$

and the pipe of the past

$$\mathbb{C}M_- = \left\{ z = x + iy \in \mathbb{C}M : |y|^2 > 0, y^0 < 0 \right\} \tag{1.10}$$

The Euclidean space E is a 4-dimensional real vector subspace in $\mathbb{C}M$, given by the equations

$$z^0 = x^0, z^1 = ix^1, z^2 = ix^2, z^3 = ix^3, \tag{1.11}$$

where x^0, x^1, x^2, x^3 are arbitrary real numbers.

1.4. NON-ZERO TWISTORS

Note that the first two components Z^0, Z^1 of the Z twistor are two components of the ω 2-spinor. $\omega^0 = Z^0, \omega^1 = Z^1$. The last two components Z^2, Z^3 are components of the hatched (dual) spinor π. $\pi_{0'} = Z^2, \pi_{1'} = Z^3$ [4]. Thus,

$$Z = (\omega, \pi). \tag{1.12}$$

For the conjugate spinor:

$$\overline{Z} = (\overline{\omega}, \overline{\pi}). \tag{1.13}$$

The incidence ratio between a twistor and an event in Minkowski space is now written as

$$\omega = ir\pi, \tag{1.14}$$

where r is the matrix

$$\frac{1}{\sqrt{2}} \begin{pmatrix} t + z & x + iy \\ x - iy & t - z \end{pmatrix} \tag{1.15}$$

The outer product $\overline{\pi}\pi$ is interpreted as the 4-momentum of some particle. The symmetrized occurrences $\omega\overline{\pi}$ and $\overline{\omega}\pi$ describe parts of the angular momentum of a particle.

From the momentum and the moment of momentum, it is possible to

construct a helicity. It will be equal to half the norm of the twistor:

$$s = \frac{1}{2}\overline{Z}_\alpha Z^\alpha. \tag{1.16}$$

From the last formula, we can conclude that non-zero twistors correspond to particles with non-zero helicity. That is, in classical language, non-zero twistors correspond to rotating massless particles (hence the name).

The Penrose correspondence is the correspondence between the space of twistors (including nonzero ones) and the space $\mathbb{C}M^{\#}$. A set of incident events from $\mathbb{C}M^{\#}$ corresponds to the Z twistor. Now the coordinates t, x, y, z are complex numbers. The set of points incident to the Z twistor forms a plane called the α-plane. The dual twistor $\overline{Z}$ defines a β-plane in $\mathbb{C}M^{\#}$.

The material picture is as follows. The plane corresponds to the conjugate twistor $\overline{Z}$ in the projective twistor space $\mathbb{PT}$ (points and planes are projectively dual). This plane intersects with $\mathbb{PN}$ on some 3-dimensional set. This set corresponds to a 3-dimensional family of light rays in the usual Minkowski space.

$\mathbb{M}^{\#}$ Конгруэнция Робинсона $\mathbb{PT}$

Рисунок 1.3 — A nonzero twistor Z corresponds to a 3-dimensional system of light rays in Minkowski space

A 3-dimensional family of light rays corresponding to a nonzero twistor Z is called the Robinson congruence [5]. At a fixed point in time, it can be depicted (Figure 1.4). Each point in this picture corresponds to a light beam. The arrows show the directions of the light rays. passing through this point. Over time, this entire configuration moves as a whole at the speed of light in the direction of one straight line in this picture, and this movement represents the motion of a rotating massless particle described by the twistor [6].
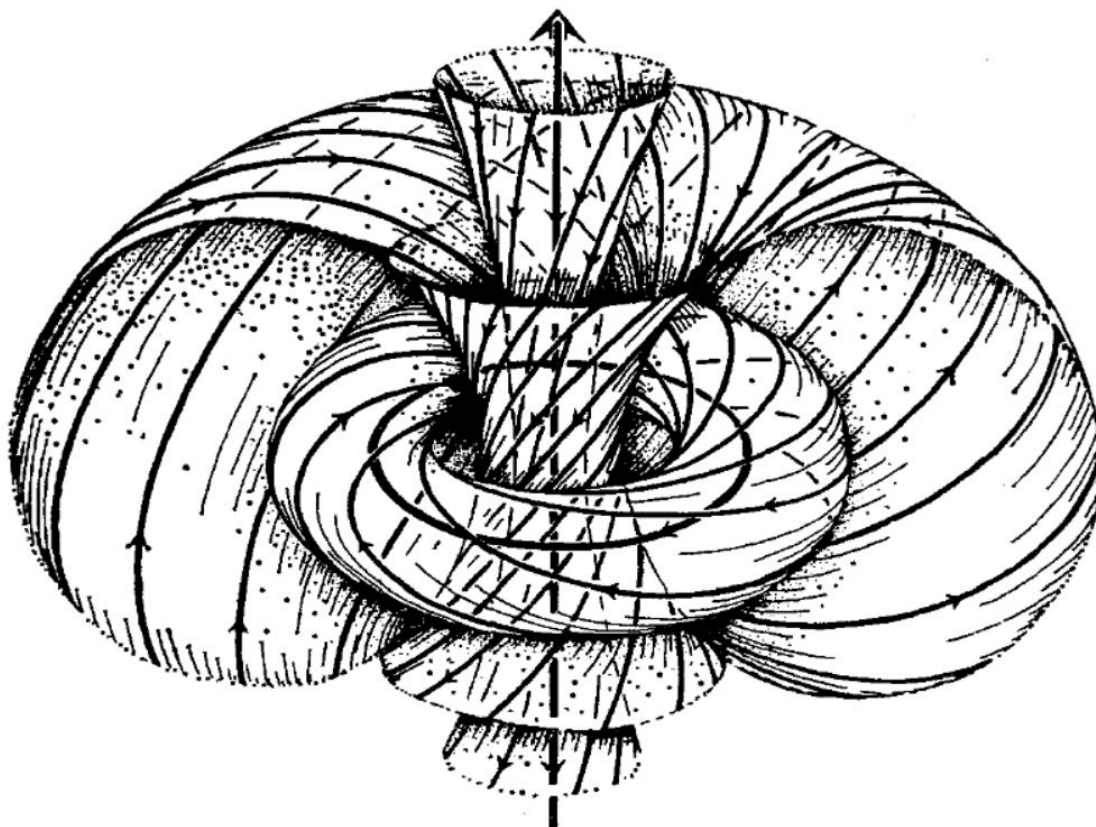
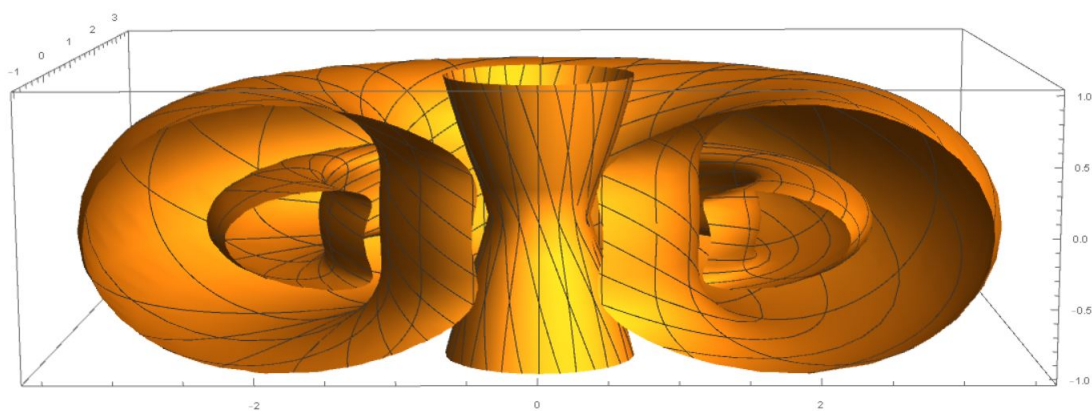Рисунок 1.4 — Robinson congruence at a fixed point in time

Рисунок 1.5 — Construction of the Robinson congruence in the Mathematica
package

2. QUANTIZATION AND MASSLESS PARTICLES

2.1. QUANTIZATION

Quantum twistor theory is constructed using non-local variables Z^α. Instead of the usual coordinates of the Minkowski space, twistors are used.

Quantization takes place according to the standard canonical scheme. The twistors Z^α and $\overline{Z}_\alpha$ turn into operators with the following switching relations:

$$\left[Z^\alpha, \overline{Z}_\beta\right] = \hbar\delta_\beta^\alpha; \tag{2.1}$$

$$\left[Z^\alpha, Z^\beta\right] = 0; \left[\overline{Z}_\alpha, \overline{Z}_\beta\right] = 0. \tag{2.2}$$

The twistor function $f(Z)$ is introduced. This is a twistor function in the Z representation. It should not depend on $\overline{Z}$. That is, $\frac{\partial f(Z)}{\partial \overline{Z}} = 0$. The twistor function $f(Z)$ must be holomorphic to [7].

In the Z representation, the conjugate twistor $\overline{Z}$ is answered by the $-\hbar\frac{\partial}{\partial Z}$. The helicity operator is written as:

$$s = \frac{1}{4}\left(Z\overline{Z} + \overline{Z}Z\right) = -\frac{1}{2}\hbar\left(2 + Z\frac{\partial}{\partial Z}\right). \tag{2.3}$$

As is known, the eigenfunctions of the operator $Z\frac{\partial}{\partial Z}$ are homogeneous, while the eigenvalues are the degrees of uniformity. Therefore, the twistor function $f(Z)$ of a massless particle with a certain helicity value S must be of homogeneous degree $-2S - 2$. This follows from equation (2.3).

So, in particular, the photon's twistor function ($S = \pm1$) will be the sum of two parts, one of which, homogeneous of degree 0, describes a left-polarized component ($S = -1$), and the other, of degree -4, describes a right-polarized component ($S = 1$). A neutrino, considered as a massless particle, has a wave function with a degree of uniformity -1 (since the helicity is equal to $-\frac{1}{2}$).

The wave function of a massless scalar particle has a degree of uniformity -2. The graviton has $S = \pm 2$. Its left-polarized part $(S = -2)$ corresponds to a twistor wave function, homogeneous of degree 2, and the right-polarized part $(S = 2)$ corresponds to a twistor wave function, homogeneous of degree -6.

2.2. MATHEMATICAL DIGRESSION: SHEAVES AND THEIR COHOMOLOGY

Let X be a complex manifold (for example, a Riemann sphere or a twistor space) and (U_i) be a covering of X by open sets. We will say that a bundle of holomorphic functions P is given on X if each open set of (U_i) a class of holomorphic functions on this set is mapped. More generally, open sets can be mapped to any Abelian group [8].

If a bundle P is given on X, then Cech cohomology can be constructed on X. To do this, we define p-cocains. 0-a chain is a set of functions f_i defined on each set U_i; 1-a chain is a set of functions f_{ij} defined on double intersections $U_i \cap U_j$, so that the relation $f_{ij} = -f_{ji}$; a 2-chain is a set of functions f_{ijk} defined at triple intersections $U_i \cap U_j \cap U_k$, and f_{ijk} must be skew-symmetric in indices; and so on. The group of p-chains is this is the set of all p-cocains.
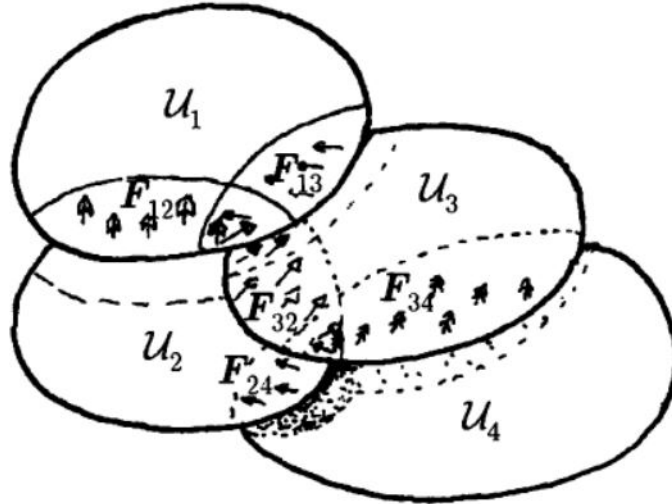

Рисунок 2.1 — A 1-cochain is a set of holomorphic functions on double intersections

The boundary operator (or codifferentiation operator) δ maps p-cocains

$\alpha = \{f_{i...k}\}$ $(p+1)$ is the chain $\delta\alpha = \{g_{i...kl}\}$. If $p = 0$, then $g_{ij} = f_j - f_i$; if $p = 1$, then $g_{ijk} = f_{jk} - f_{ik} + f_{ij}$; if $p = 2$, then $g_{ijkl} = f_{jkl} - f_{ikl} + f_{ijl} - f_{ijk}$; etc.

a p-chain α is called a cocycle if $\delta\alpha = 0$, and a co-boundary if it has the form $\alpha = \delta\beta$. It is easy to check that all the co-boundaries are cocycles, i.e. $\delta^2 = 0$ (this is called the Poincare identity).

Denote by $H^p_U(X, P)$ the factor group of the group of p-cocycles over the subgroup of p-co-boundaries. Passing to the limit over ever smaller covers of U of the manifold X, we obtain in the limit the space $H^p(X, P)$, called the p-th cohomology group X with coefficients in the bundle P.

2.3. MASSLESS FIELDS

A particle with negative helicity $S = -\frac{1}{2}n$ is described by a field function of the form $\phi_{AB...L}$. A particle with positive helicity $S = \frac{1}{2}n$ is described by a field function with hatched indices. $\phi_{A'B'...L'}$. Each function has n indexes.

Each of them is completely symmetric over all n indices and has a positive frequency, satisfying the corresponding equations

$$\nabla^{AA'}\phi_{AB...L} = 0, \tag{2.4}$$

$$\nabla^{AA'}\phi_{A'B'...L'} = 0. \tag{2.5}$$

For $S = 0$ we have the D'Alembert equation

$$\Box\phi = 0. \tag{2.6}$$

For $S = +\frac{1}{2}$ we obtain the Weyl equation for neutrinos; for $S = \pm 1$ we have a 2-spinor version of Maxwell's equations for a free field; for $S = \pm 2$; - gauge invariant spinor form of a free linearized Einstein field.

Solutions of the above equations can be obtained from the twistor function $f(z)$ by the following formulas [9]:

$$\phi_{AB...L} = \frac{1}{(2\pi i)^2} \oint \frac{\partial}{\partial\omega^A} \frac{\partial}{\partial\omega^B} ... \frac{\partial}{\partial\omega^L} f(Z) d\pi_{0'} \wedge d\pi_{1'}, \tag{2.7}$$

$$\phi_{A'B'...L'} = \frac{1}{(2\pi i)^2} \oint \pi_{A'}\pi_{B'}...\pi_{L'}f(Z)d\pi_{0'} \wedge d\pi_{1'} \tag{2.8}$$

In the case of positive helicity, the function $f(Z)$ is first multiplied n times by π, which gives n hatched indices; in the case of negative helicity , the operation $\frac{\partial}{\partial\omega}$ is applied first n times, which gives n of non-shaded spinor indices. Then multiplication by the 2-form $d\pi_{0'} \wedge d\pi_{1'}$ is performed and integration by a suitable 2-dimensional In this case, the incidence ratio $\omega = ir\pi$ is first used to exclude the value of ω by expressing it in terms of π and r. Integration eliminates π, and the result is an indexed value of $\phi...$ at any chosen space-time point R (so $\phi...$ depends only on r).

Integration takes place along a two-dimensional contour in the twistor space. This circuit lies in a set of twistors incident to this event. In a projective picture, a two-dimensional contour turns into a one-dimensional one. It should lie in a set of twistors incident to the event. That is, it must lie on the projective complex line (or on the Riemann sphere) in the space of projective twistors. Note that this line lies in the space $\mathbb{PN}$.

The condition of positive frequency of the field function (a function is called positively partial if only exponents with negative exponents $exp(-ikx)$ are present in its expansion into the Fourier integral) is provided by the requirement that the contour integrals (2.7) and (2.8) retain meaning when the projective line (Riemann sphere) falls into the region of positive twistors $\mathbb{PT}^+$.
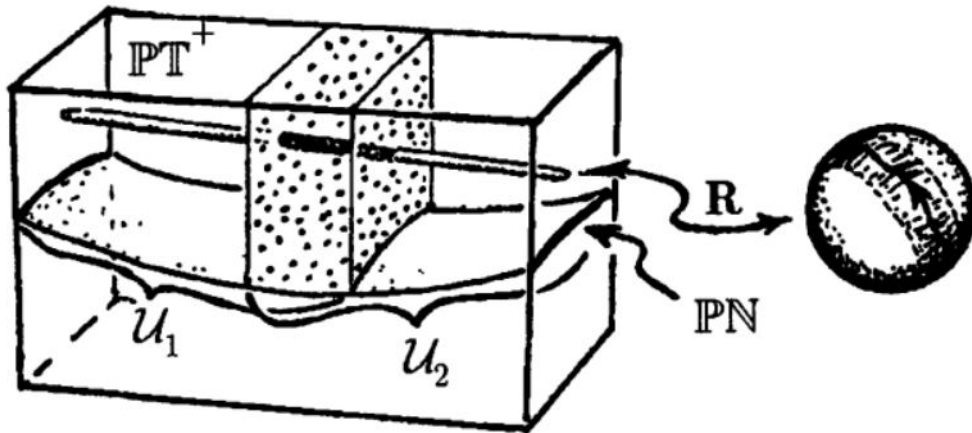

Рисунок 2.2 — Integration is performed along a two-dimensional (in the projective case, one-dimensional) contour, which lies entirely in the set of twistors incident to this event.

2.4. TWISTOR FUNCTION

Now let's give an exact definition of the twistor function $f(Z)$.

Since it must be holomorphic, its domain of definition cannot coincide with the entire twistor space $\mathbb{T}$ or even with the entire $\mathbb{T}^+$. You need to set $f(Z)$ on a smaller set.

Let's cover the space $\mathbb{PT}^+$ with two open sets U_1 and U_2. They intersect along the area of the ring type $U_1 \cap U_2$ (Fig.2.2). The function $f = f_{12} = -f_{21}$ is defined on this area. This function is holomorphic in $U_1 \cap U_2$, but when continued, all $\mathbb{PT}^+$ has features lying in $U_1 \backslash U_2$ or in $U_2 \backslash U_1$. In more general cases, more open sets and, accordingly, more functions may be needed. But in any case, the correct description of the twistor function is The massless particle of helicity S consists in the fact that it is an element of the cohomology group over a bundle of holomorphic functions with the degree of uniformity $-2S - 2$:

$$H^1(\overline{\mathbb{PT}^+}, O(-2S - 2)). \tag{2.9}$$

The use of the space $\mathbb{PT}^+$ in this definition ensures the fulfillment of the condition of the positive frequency of the field function (one could consider the space $\mathbb{PT}^-$). The closure continuity $\overline{\mathbb{PT}^+} = \mathbb{PT}^- \cup \mathbb{PN}$ ensures the normalizability of the field function. For $\mathbb{PT}^+$, it is quite enough to consider covers with two sets, but the construction itself can be generalized and in case $\mathbb{PT}^+$ is replaced by some other subset of X in $\mathbb{PT}$. Then a more complex coating may be needed.

So, the twistor function is defined at the intersection of two regions up to the addition of a function of the form $h_1 - h_2$, where h_1 and h_2 are defined on the sets U_1 and U_2. It can be proved that the contour integrals (2.7) and (2.8) do not depend on the choice of the function $f(Z)$ from the equivalence class.

2.5. SPLITTING

Consider the Riemann sphere $\mathbb{S}^2$ with the equator $\mathbb{R}$ and the poles i and $-i$. The complex function (holomorphic) defined on the real axis $\mathbb{R}$ (at the equator) is split into a positive-frequency part holomorphically extended to the northern hemisphere, and a negative-frequency part holomorphically extended

to the southern hemisphere.

Approximately the same thing happens in the twistor case. A twistor function defined on $\mathbb{PN}$ (an element of the 1st cohomology representing a massless field) is split into a positive-frequency part holomorphically continued on $\mathbb{PT}^+$, and a negative-frequency part holomorphically continued in $\mathbb{PT}^-$.
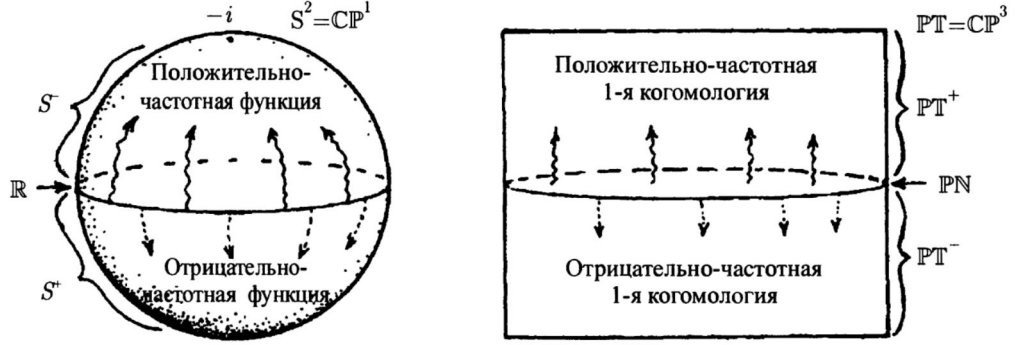

Рисунок 2.3 — The twistor function defined on $\mathbb{PN}$ is split into positive-frequency and negative-frequency parts.

So, massless fields in the Minkowski space $M^\#$ are represented by elements of the first cohomology on $\mathbb{PN}$. Each of them can be represented as the sum of an element continued in $\mathbb{PT}^+$ and an element continued in $\mathbb{PT}^-$. The first term describes a positive-frequency massless field, the second - a negative-frequency massless field. In the language of space-time terms, the positive-frequency part of the field, when continued, forms a pipe of the future in the space $\mathbb{CM}$. The negative frequency part of the field, when continued, forms a pipe of the past in the space $\mathbb{CM}$.

2.6. NONLINEAR GRAVITON

First, consider the case when the twistor function $f(Z)$ is homogeneous of degree o (corresponds to a photon with $S = -1$). The twistor space $\mathbb{T}^+$ can be considered as a bundle over the space $\mathbb{PT}^+$. The function f can be used to deform this bundle in order to obtain a curved twistor structure τ^+. Let $\mathbb{T}^+$ be covered by two sets U_1 and U_2 and the function f is defined at the intersection of $U_1 \cap U_2$. Thus, $\mathbb{T}^+$ is the union of the sets $\mathbb{P}U_1$ and $\mathbb{P}U_2$, whose elevations in the bundle space coincide with U_1 and U_2.

To get a curved space of τ^+ twistors, we glue the sets U_1 and U_2 in a

different way, namely, for $\hat{Z}^\alpha \in U_1$ and $Z^\alpha \in U_2$, the transition function in τ^+ at the intersection of $U_1 \cap U_2$:

$$\hat{Z}^\alpha = e^{f(Z)} Z. \tag{2.10}$$

Here it is necessary that f has uniformity of degree zero, since the transformation $Z \to \lambda Z$ must correspond to the transformation $\hat{Z} \to \lambda \hat{Z}$.

Since the geometry of space-time is completely determined by the structure of the projective twistor space, the space-time geometry corresponding to τ^+ remains the same as for $\mathbb{T}^+$, i.e. by Minkowski geometry. However, the change in the phases of the twistors that occurs in this case leads to the appearance of "connectivity" at M induced by the electromagnetic potential. It turns out that this connectivity corresponds to the same electromagnetic field that is obtained by contour integration, but now it describes the photon in the active function, since the description in the twistor space includes the form of interaction of the photon.

Using the $\mathbb{PT}^+$ subset of the $\mathbb{PT}$ space as the bundle base in the above construction is related to with the fact that we are considering a free photon of positive frequency. However, this construction can also be carried out when other subsets of X in $\mathbb{PT}$ are selected as the base.

A similar construction can be carried out in the case of gravity, when the degree of uniformity of f is 2 (i.e., the helicity is -2). Now the undisturbed space is $\mathbb{T}^+$ It is considered as a bundle in another sense: the role of the base is played by the spinor space $\pi_{A'}$, and the projection mapping is the mapping $(\omega^A, \pi_{A'}) \to \pi_{A'}$.

We will construct a deformed space τ using two open sets U_1 and U_2 from $\mathbb{T}$ and a function f (degree of uniformity 2) defined at the intersection of $U_1 \cap U_2$. To get τU_1 and U_2 are glued together in a different way, namely for $\hat{Z}^\alpha \in U_1$ and $Z^\alpha \in U_2$ at the intersection of $U_1 \cap U_2$, the transition function is defined as follows how:

$$\hat{Z}^\alpha = exp\left\{ \epsilon^{AB} \frac{\partial f(Z)}{\partial \omega^A} \frac{\partial}{\partial \omega^B} \right\} Z. \tag{2.11}$$

The absence of π derivatives in the above ratio means that the twistor on one flap must have the same π part as the twistor matched with it on the

neighboring flap. It follows that the operation of "projecting" the spinor π from the space τ has a consistent character throughout this space. That is, there is a global projection of the space τ on the space of spinors π. Thus, τ is a bundle over the space π. Each layer turns out to be a complex 2-manifold with a symplectic structure, as is the π space itself.
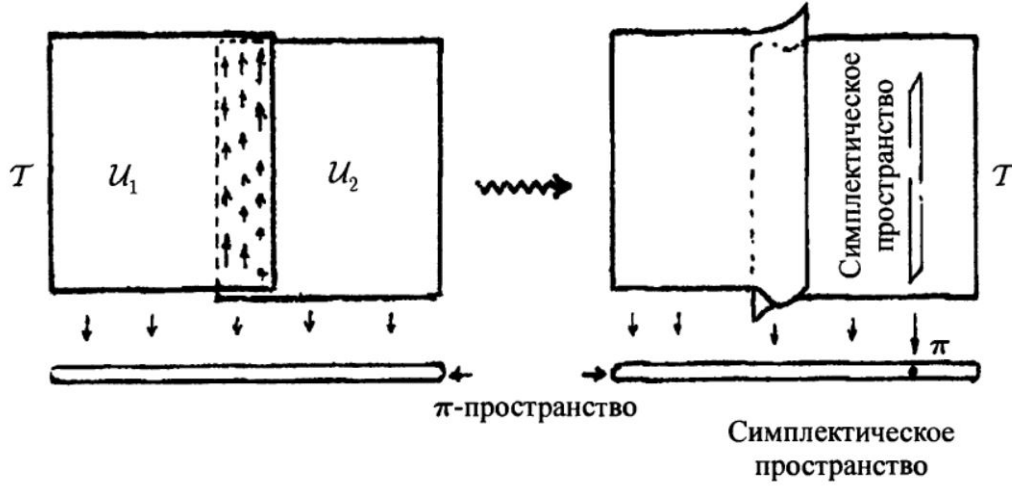

Рисунок 2.4 — The curved twistor space τ is projected onto the space of π-spinors

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From the deformed space τ, it is now possible to construct a curved complex Minkowski space. It is called a nonlinear graviton. The entire scheme for constructing a nonlinear graviton is shown in Figure 2.5.

We describe the construction of a nonlinear graviton. a) With the standard twistor correspondence for a flat space, the points P and Q of the space $\mathbb{CM}$ are separated by a zero interval whenever the lines P and Q intersect in the space $\mathbb{PT}$. b) We want to somehow deform $\mathbb{PT}$ into a curved twistor space τ, however mathematical theorems state that this cannot be done globally. Accordingly, as our initial spacetime, we will take only a small neighborhood U of the point R in $\mathbb{CM}$. c) This neighborhood corresponds to the tubular neighborhood Q of the line R in $\mathbb{PT}$. d) Now we deform the region Q, which is divided into two sets U_1 and U_2. e) The original line R is now broken and cannot be used to unambiguously define a "space-time point". f) Kodaira's theorem comes to the rescue, from which it follows that there is a 4-parametric family of "lines" R^* that can serve for this purpose. g) The points of the desired space of the "nonlinear graviton" M (complex 4-space) are determined by the curves of Kodaira R^*. The complex conformal metric of the space M is determined (as
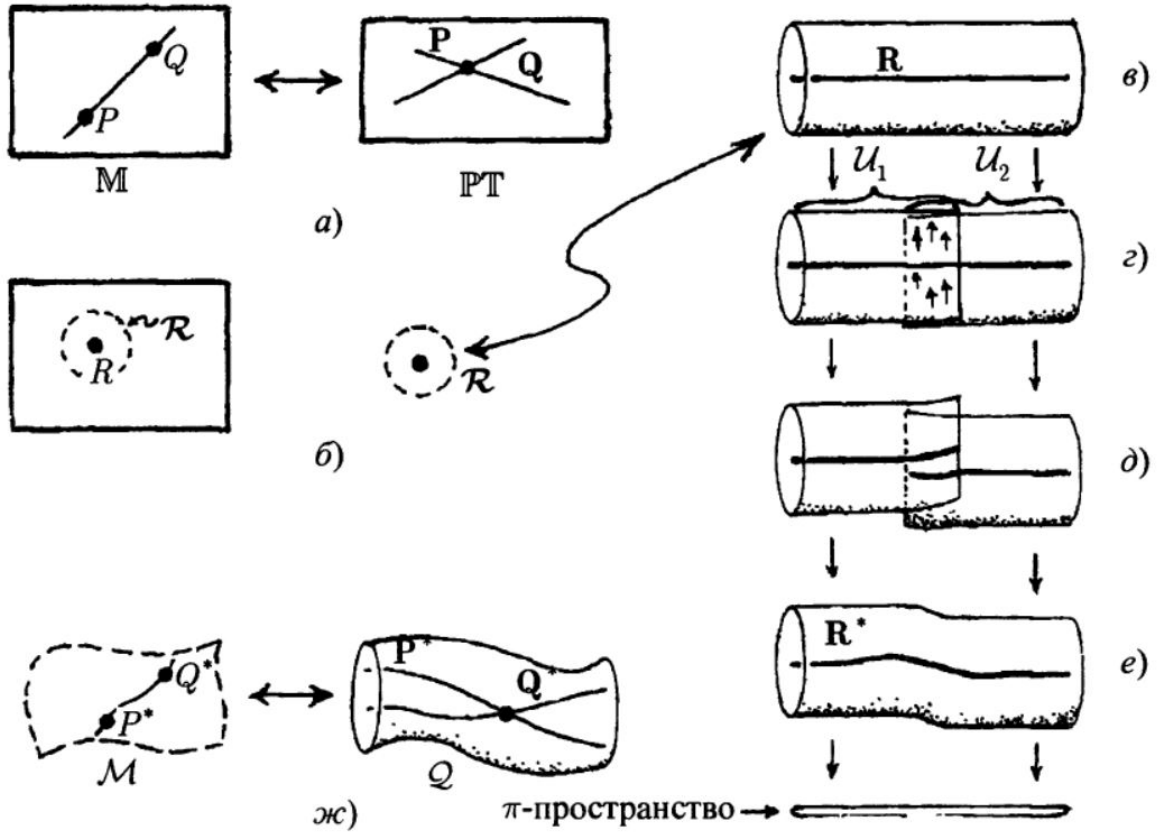
Рисунок 2.5 — A scheme for constructing a nonlinear graviton
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in the case of a) by the condition that the points P^* and Q^* are separated by a zero interval at the intersection of the corresponding lines P^* and Q^* in τ.

Thus, a deformed Minkowski space M is obtained, which is the space of sections of the bundle τ. Two points in this space are isotropically located if the corresponding sections intersect. The complex conformal given by this definition the structure on M can be extended to the complete complex Riemannian metric g_{ab} on M. It turns out that this metric satisfies Einstein's equations in the void $R_{ab} = 0$.

3. CONCLUSION

Twistor theory is a set of non-local constructions with roots in nineteenth-century projective geometry. To date, the ideas of twistors have been expanded and generalized in many different directions and applied to many completely different problems of mathematics and physics. A feature of this theory is the correspondence between points in spacetime and holomorphic curves in twistor space. Einstein's equations and Yang-Mills equations in spacetime are replaced by algebra-geometric problems in twistor space.

Recently, the problem of describing a graviton with helicity 2, which is described by a twistor function of the degree of homogeneity -6. The degree of homogeneity -6.) was solved. This made it possible to include both the left-polarized and right-polarized part of the graviton in the theory.

At the moment, the theory of twistors is not complete. There are a number of problems with the description of massive particles.

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