In a homogeneous & isotropic universe the interval b/w^n 2 events in a co moving co-ordinates has the form:

$$ds^2 = c^2 dt^2 - a^2(t)\, dl^2 \longrightarrow (1)$$

dl = Co-ordinate distance.

$$d\eta = \frac{dt}{a(t)} \Big\} \text{ Cosmological red shift.}$$

(1) can also be written as.

$$d^2s^2 = a^2(\eta)\left(d\eta^2 - dl^2\right) \longrightarrow (2)$$

& dl^2 - metric of homogeneous & isotropic space of constant curvature 1, 0, -1.

Scalar wave eqn:

$$\left(\nabla_i \nabla^i + m^2 + \frac{R}{6}\right)\phi(x) = 0 \longrightarrow \boxed{3}$$

∇_i = Covariant differentiation.
R = Scalar curvature.

Taking $\phi = a^{-1} u_\lambda(\eta)\, \psi_J(x)$ ⟶ (4)

where $J = (\lambda, l, m)$, m = integer such that $-l \le m < l$.

for $K = 1$, $0 < \lambda < \infty$, ~~$l = 0, 1, 2 \ldots$~~

for $K = 0, -1$, $l = 0, 1, 2 \ldots$

Substituting (4) in (3) and seperating variables, we obtain an oscillator type eqn with variable frequency.

$$\ddot{u}_\lambda + \omega^2(\eta)\, u_\lambda = 0, \quad \omega^2(\eta) = \lambda^2 + m^2 a^2(\eta) \longrightarrow (5)$$

ψ_J = eigenfunctions of Laplace operator in 3-space with eigenvalues $\lambda^2 - K$.

Energy momentum tensor:

$$T_{\mu\nu} = 2\,\phi^*_{(;\mu}\phi_{;\nu)} - g_{\mu\nu}\left[\phi^*_{;l}\,\phi^{;l} - \left(m^2 + \frac{R}{6}\right)\phi^*\phi\right]$$
$$- \frac{1}{3}\left[R_{\mu\nu} + \nabla_\mu\nabla_\nu - g_{\mu\nu}\nabla_l\nabla^l\right]\phi^*\phi \longrightarrow (6)$$

Applying Normalizing operation on (6)

$$N_\eta(T_{\mu\nu}) = T_{\mu\nu} - \langle 0_\eta | T_{\mu\nu} | 0_\eta \rangle$$

we get energy.

$$\varepsilon(\eta) = \langle 0 | N_\eta(T_0^0) | 0 \rangle = \frac{1}{\pi^2 a^4}\int_0^\infty d\lambda\, \lambda^2 \omega\, |\beta_\lambda|^2, \longrightarrow 7(i)$$

and pressure

$$P(\eta) = -\langle 0 | N_\eta (T_i^i) | 0 \rangle = \frac{1}{3\pi^2 a^4} \int_0^\infty d\lambda\, \lambda^2 \omega \Big[|\beta_\lambda|^2 - \frac{m^2 a^2}{2\omega} \times \left(|U_\lambda|^2 - \frac{1}{\omega} \right) \Big] \longrightarrow \varepsilon \rightarrow (ii)$$

$$\text{where } |\beta_\lambda|^2 = \frac{\left(|U_\lambda|^2 + \omega^2 |U_\lambda|^2 - 2\omega \right)}{4\omega}$$ } no density of pairs created in ~~the~~ mode λ

~~As~~ @ $t << m^{-1}$,

$$\beta_\lambda = \frac{m^2}{4\lambda^2} \int_0^\eta \frac{da^2(\eta_1)}{d\eta_1} e^{-2i\lambda\eta_1} d\eta_1$$

& for $\lambda << \eta^{-1}$,

$$|\beta_\lambda|^2 = \frac{(\omega(\eta) - \lambda)^2}{4\lambda\, \omega(\eta)}.$$

ε = Integration ? i & ii in the whole range of λ.

$$\varepsilon = \frac{m^4}{16\pi^2} \left[\ln \frac{1}{mt} + \varphi\left(\frac{1+q}{1-q} \right) + \ln(1-q) + \frac{1}{4q} - \frac{1}{2} \right].$$

$$p = -\frac{m^4}{16\pi^2} \left[\ln \frac{1}{mt} + \varphi\left(\frac{1+q}{1-q} \right) + \ln(1-q) - \frac{1}{12q} - \frac{1}{2} \right)$$

ie $p_z - \varepsilon$. $\longrightarrow$ vaccum state.