Cosmological constraints on parameters of effective theory of Standard Model

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2.1   Introduction and aQGC

Effective extension of the SM consists in the parameterization of the Lagrangian with the operators of higher dimensions with some coefficients:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \sum_i \sum_n \frac{F_{i,n}}{\Lambda^n} \mathcal{O}_i^{n+4} = \mathcal{L}_{\text{SM}} + \sum_i \sum_n f_{i,n} \mathcal{O}_i^{n+4}. \tag{1}$$

In this equation $\mathcal{L}_{\text{SM}}$ is the SM Lagrangian, Λ is the new physics energy scale, $\mathcal{O}_i^{n+4}$ is the i-th operator of $n+4$ dimension, $F_{i,n}$ is the corresponding unobservable dimensionless coefficient, $f_{i,n} = F_{i,n}/\Lambda^n$ is the corresponding (Wilson's) observable coefficient which has dimension TeV^{-n}.

In this report anomalous quartic gauge couplings (aQGC) was considered. It is convenient to study this couplings with operators of eight dimensions which lead to genuine aQGC without the contribution of anomalous triple gauge couplings (aTGC) [2]. These operators are constructed from covariant derivative of the Higgs field

$$D_\mu \Phi = \left(\partial_\mu + ig\frac{\sigma_i}{2}W_\mu^i + ig'\frac{1}{2}B_\mu \right) \Phi, \tag{2}$$

$SU(2)_L$ field strength tensor

$$\hat{W}_{\mu\nu} = \frac{\sigma^i}{2}W_{\mu\nu}^i, \tag{3}$$

where

$$W_{\mu\nu}^i = \partial_\mu W_\nu^i - \partial_\nu W_\mu^i + g\varepsilon^{ijk}W_\mu^k W_\nu^k, \tag{4}$$

and $U(1)_Y$ field strength tensor

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{5}$$

and can be divided into three families. S-family operators contain just covariant derivatives of the Higgs field:

$$\begin{aligned}
\mathcal{O}_{\text{S0}} &= \left[(D_\mu\Phi)^\dagger D_\nu\Phi \right] \left[(D^\mu\Phi)^\dagger D^\nu\Phi \right], \\
\mathcal{O}_{\text{S1}} &= \left[(D_\mu\Phi)^\dagger D^\mu\Phi \right] \left[(D_\nu\Phi)^\dagger D^\nu\Phi \right].
\end{aligned} \tag{6}$$

T-family operators contain just gauge field strength tensors:

$$\mathcal{O}_{\text{T0}} = \text{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right]\text{Tr}\left[\hat{W}_{\alpha\beta}\hat{W}^{\alpha\beta}\right],$$

$$\mathcal{O}_{\text{T1}} = \text{Tr}\left[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}\right]\text{Tr}\left[\hat{W}_{\mu\beta}\hat{W}^{\alpha\nu}\right],$$

$$\mathcal{O}_{\text{T2}} = \text{Tr}\left[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}\right]\text{Tr}\left[\hat{W}_{\beta\nu}\hat{W}^{\nu\alpha}\right],$$

$$\mathcal{O}_{\text{T5}} = \text{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right]\left[B_{\alpha\beta}B^{\alpha\beta}\right],$$

$$\mathcal{O}_{\text{T6}} = \text{Tr}\left[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}\right]\left[B_{\mu\beta}B^{\alpha\nu}\right],\tag{7}$$

$$\mathcal{O}_{\text{T7}} = \text{Tr}\left[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}\right]\left[B_{\beta\nu}B^{\nu\alpha}\right],$$

$$\mathcal{O}_{\text{T8}} = \left[B_{\mu\nu}B^{\mu\nu}\right]\left[B_{\alpha\beta}B^{\alpha\beta}\right],$$

$$\mathcal{O}_{\text{T9}} = \left[B_{\alpha\mu}B^{\mu\beta}\right]\left[B_{\beta\nu}B^{\nu\alpha}\right].$$

Finally, M-family operators mix covariant derivatives of the Higgs field and gauge field strength tensors:

$$\mathcal{O}_{\text{M0}} = \text{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right]\left[(D_\beta\Phi)^\dagger D^\beta\Phi\right],$$

$$\mathcal{O}_{\text{M1}} = \text{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\nu\beta}\right]\left[(D_\beta\Phi)^\dagger D^\mu\Phi\right],$$

$$\mathcal{O}_{\text{M2}} = \left[B_{\mu\nu}B^{\mu\nu}\right]\left[(D_\beta\Phi)^\dagger D^\beta\Phi\right],$$

$$\mathcal{O}_{\text{M3}} = \left[B_{\mu\nu}B^{\nu\beta}\right]\left[(D_\beta\Phi)^\dagger D^\mu\Phi\right],\tag{8}$$

$$\mathcal{O}_{\text{M4}} = \left[(D_\mu\Phi)^\dagger\hat{W}_{\beta\nu}D^\mu\Phi\right]B^{\beta\nu},$$

$$\mathcal{O}_{\text{M5}} = \left[(D_\mu\Phi)^\dagger\hat{W}_{\beta\nu}D^\nu\Phi\right]B^{\beta\mu} + \text{h.c.},$$

$$\mathcal{O}_{\text{M7}} = \left[(D_\mu\Phi)^\dagger\hat{W}_{\beta\nu}\hat{W}^{\beta\mu}D^\nu\Phi\right].$$

From all possible quartic gauge couplings SM predicts just $WWWW$, $WWZZ$, $WWZ\gamma$, $WW\gamma\gamma$. Table 1 shows which quartic gauge couplings are affected by each operator.

Table 1: Influence of the 8-dimensional operators on quartic gauge couplings. Affected couplings are marked with a symbol $\circ$.

Operator	$WWWW$	$WWZZ$	$WWZ\gamma$	$WW\gamma\gamma$	$ZZZZ$	$ZZZ\gamma$	$ZZ\gamma\gamma$	$Z\gamma\gamma\gamma$	$\gamma\gamma\gamma\gamma$
$\mathcal{O}_{S0}, \mathcal{O}_{S1}$	$\circ$	$\circ$			$\circ$				
$\mathcal{O}_{T0}, \mathcal{O}_{T1}, \mathcal{O}_{T2}$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$
$\mathcal{O}_{T5}, \mathcal{O}_{T6}, \mathcal{O}_{T7}$		$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$
$\mathcal{O}_{T8}, \mathcal{O}_{T9}$					$\circ$	$\circ$	$\circ$	$\circ$	$\circ$
$\mathcal{O}_{M0}, \mathcal{O}_{M1}, \mathcal{O}_{M7}$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$		
$\mathcal{O}_{M2}, \mathcal{O}_{M3}, \mathcal{O}_{M4}, \mathcal{O}_{M5}$		$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$		

2.2 Amplitude decomposition

For studying processes using EFT one need to know how cross section depends on coefficient value. This dependence is considered in this section for the case when process contains not more that one new physics vertex.

In the general case, when Lagrangian is parameterized with a set of operators as

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \sum_i f_i \mathcal{O}_i, \tag{9}$$

amplitude of some process contains SM and beyond-the-SM (BSM) terms and can be written as

$$\mathcal{A} = \mathcal{A}_{\text{SM}} + \sum_i f_i \mathcal{A}_{\text{BSM},i}. \tag{10}$$

Square of this amplitude is

$$|\mathcal{A}|^2 = |\mathcal{A}_{\text{SM}}|^2 + \sum_i f_i 2\text{Re}(\mathcal{A}_{\text{SM}}^\dagger \mathcal{A}_{\text{BSM},i}) + \sum_i f_i^2 |\mathcal{A}_{\text{BSM},i}|^2 + \sum_{i,j,i>j} f_i f_j 2\text{Re}(\mathcal{A}_{\text{BSM},i}^\dagger \mathcal{A}_{\text{BSM},j}). \tag{11}$$

So, squared amplitude as well as cross section contains SM term, interference (linear) terms $\propto f_i$, quadratic terms $\propto f_i^2$ and cross terms $\propto f_i f_j$.

For setting 1D limits the Lagrangian is parameterized by a single operator as

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + f\mathcal{O}. \tag{12}$$

In this case amplitude and its square are

$$\mathcal{A} = \mathcal{A}_{\text{SM}} + f\mathcal{A}_{\text{BSM}} \tag{13}$$

and

$$|\mathcal{A}|^2 = |\mathcal{A}_{\text{SM}}|^2 + f2\text{Re}(\mathcal{A}_{\text{SM}}^\dagger \mathcal{A}_{\text{BSM}}) + f^2 |\mathcal{A}_{\text{BSM}}|^2. \tag{14}$$

Therefore, cross section contains one SM term, one interference term and one quadratic term and can be written as

$$\sigma = \sigma_{\text{SM}} + f\sigma_{\text{int}} + f^2 \sigma_{\text{quad}}. \tag{15}$$

If considered process is not predicted by the SM, then $\mathcal{A}_{\text{SM}} = 0$ and cross section is

$$\sigma = f^2 \sigma_{\text{quad}}. \tag{16}$$

3 Constraints from the CMB in the early Universe

3.1 Setting limits methodology

Such processes as $\gamma\gamma \to \nu\bar{\nu}\nu\bar{\nu}$ can affect modern relic neutrino number density $n_\nu^0 = 339.5$ cm^{-3} which can be predicted from the standard cosmological Big Bang model using observable CMB number density $n_\gamma^0 = 410.7$ cm^{-3} [3]. Of course, predicted neutrino number density from the anomalous couplings $n_\nu^{\text{pred},0}$ should be less than n_ν^0. Taking into account that predicted

neutrino number density depends on EFT coupling constant f, condition

$$n_\nu^{\text{pred},0} < n_\nu^0 \tag{17}$$

can lead to constraint on f.

Neutrino number density from the anomalous couplings can be predicted as

$$n_\nu^{\text{pred},0} = \alpha_\nu n_0 = \frac{\alpha_\nu N_0}{V_0}, \tag{18}$$

where N_0 is the number of anomalous interactions that have occurred to present-day time moment, n_0 is N_0 per unit volume, α_ν is the number of neutrinos that are produced from a single anomalous interaction and V_0 is the present-day size of the Universe.

In the following calculations the first photon is incoming and the second photon is the mobile target. Assuming that CMB photons have Planck's energy distribution and isotropic spatial distribution, one can define the flux of the incoming photons

$$\frac{\mathrm{d}^4 N_1}{\mathrm{d}E_1 \, \mathrm{d}\Omega_1 \, \mathrm{d}S \, \mathrm{d}t} = f_{\text{Pl}}(E_1|T)\frac{1}{4\pi}n_\gamma c \tag{19}$$

and the distribution of the target photons

$$\frac{\mathrm{d}^2 N_2}{\mathrm{d}E_2 \, \mathrm{d}\Omega_2} = f_{\text{Pl}}(E_2|T)\frac{1}{4\pi}n_\gamma V, \tag{20}$$

where T is the CMB temperature (Planck's distribution parameter), n_γ is CMB photons number density, c is the speed of light, V is the size of the Universe and

$$f_{\text{Pl}}(E|T) = \frac{qE^2}{e^{E/T} - 1} \tag{21}$$

is the Planck's energy distribution normalized to unity with coefficient $q = (2\zeta(3)T^3)^{-1}$.

Since interacting photons have different energies and relative angle, it is necessary to know cross section dependence on this parameters. This can be done in the following way. Choosing spatial frame so that momentum of the first photon directed along the z axis, s invariant can be represented as

$$s = (p_1 + p_2)^2 = 2(p_1 p_2) = 2(E_1 E_2 - \vec{p}_1 \vec{p}_2) = 2E_1 E_2(1 - \cos\theta_2). \tag{22}$$

Then, obtaining cross section dependence on s invariant, one can obtain its dependence on E_1, E_2 and $\cos\theta_2$. Assuming that $\sigma(s)$ is a polynomial function

$$\sigma(s) = f^2 \sum_i p_i s^i \tag{23}$$

with $i > 0$ (anomalous cross section increases with energy), then

$$\sigma(E_1, E_2, \cos\theta_2) = f^2 \sum_i 2^i p_i E_1^i E_2^i (1 - \cos\theta_2)^i. \tag{24}$$

Frequency of the considered process is

$$\dot{N} = \int \sigma(E_1, E_2, \cos\theta_2) \frac{\mathrm{d}^4 N_1}{\mathrm{d}E_1\,\mathrm{d}\Omega_1\,\mathrm{d}S\,\mathrm{d}t} \frac{\mathrm{d}^2 N_2}{\mathrm{d}E_2\,\mathrm{d}\Omega_2}\,\mathrm{d}E_1\,\mathrm{d}E_2\,\mathrm{d}\Omega_1\,\mathrm{d}\Omega_2. \tag{25}$$

Taking into account Eq. 19, 20 and 24 and integrating over all angles θ_1, ϕ_1 and ϕ_2, it can be rewritten as

$$\dot{N} = \frac{1}{2} c n_\gamma^2 V f^2 \sum_i 2^i p_i \int E_1^i E_2^i (1 - \cos\theta_2)^i f_{\mathrm{Pl}}(E_1|T) f_{\mathrm{Pl}}(E_2|T)\,\mathrm{d}E_1\,\mathrm{d}E_2\,\mathrm{d}\cos\theta_2. \tag{26}$$

Integration by energy can be performed using formula

$$\int\limits_0^\infty E^i f_{\mathrm{Pl}}(E|T)\,\mathrm{d}E = q \int\limits_0^\infty \frac{E^{i+2}}{e^{E/T} - 1}\,\mathrm{d}E = q T^{i+3} \Gamma(i+3)\zeta(i+3), \tag{27}$$

where $\Gamma(x)$ is the gamma function and $\zeta(x)$ is Riemann zeta function. So the integral from Eq. 26 is

$$\int E_1^i E_2^i (1 - \cos\theta_2)^i f_{\mathrm{Pl}}(E_1|T) f_{\mathrm{Pl}}(E_2|T)\,\mathrm{d}E_1\,\mathrm{d}E_2\,\mathrm{d}\cos\theta_2 =$$

$$= \int\limits_0^\infty E_1^i f_{\mathrm{Pl}}(E_1|T)\,\mathrm{d}E_1 \int\limits_0^\infty E_2^i f_{\mathrm{Pl}}(E_2|T)\,\mathrm{d}E_2 \int\limits_{-1}^1 (1 - \cos\theta_2)^i\,\mathrm{d}\cos\theta_2 =$$

$$= \left(\int\limits_0^\infty E^i f_{\mathrm{Pl}}(E|T)\,\mathrm{d}E\right)^2 \int\limits_0^2 y^i\,\mathrm{d}y = q^2 T^{2i+6} \left(\Gamma(i+3)\zeta(i+3)\right)^2 \frac{2^{i+1}}{i+1} =$$

$$= T^{2i} \frac{2^{i-1}}{i+1} \frac{\Gamma^2(i+3)\zeta^2(i+3)}{\zeta^2(3)}, \tag{28}$$

where $y = 1 - \cos\theta_2$. Finally, frequency of the considered process can be rewritten as

$$\dot{N} = \frac{c n_\gamma^2 V f^2}{4\zeta^2(3)} \sum_i \frac{4^i}{i+1} p_i \Gamma^2(i+3)\zeta^2(i+3)T^{2i}. \tag{29}$$

In this formula CMB number density n_γ, CMB temperature T and size of the Universe V depend on time t. In addition, anomalous interactions lead to a decrease in the CMB number density. So, n_γ depend on N. Therefore, Eq. 29 becomes a differential equation. Solving of this equation can be used for obtaining $n_\nu^{\mathrm{pred},0}$ with Eq. 18 and setting limits with Eq. 17.

3.2 Choice of the time interval and dependencies estimation

Since in EFT cross sections increases with the energy, the most interesting stage of the Universe evolution for this study is radiation dominance stage (RD). Besides, non-zero mass of the neutrino leads to impossibility of reactions of neutrino production from CMB when energies of the photons are too small. Therefore, contribution from stages after RD is negligible. So upper time point can be chosen as $t_{\max} = 10^5$ yr. In the other side, neutrino had been in the equilibrium with another particles up to moment $t \approx 1$ s ($T \approx 1$ MeV). So lower time point can be chosen as $t_{\min} = 1$ s.

Number density from the CMB photons decreases due to the Universe expansion and from the anomalous interactions:

$$n_\gamma(t) = \left(n_\gamma^0 V_0 - \alpha_\gamma N(t)\right) \frac{1}{V(t)}, \tag{30}$$

where α_γ is the reducing of the number of CMB photons per single anomalous interaction. Thus Eq. 29 can be rewritten as

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \left(n_\gamma^0 V_0 - \alpha_\gamma N\right)^2 \frac{1}{V(t)} \frac{cf^2}{4\zeta^2(3)} \sum_i \frac{4^i}{i+1} p_i \Gamma^2(i+3) \zeta^2(i+3) T^{2i}(t). \tag{31}$$

This is a differential equation, where variables N and t can be separated. After integrating by N from 0 to N_0 it can be seen that

$$\frac{1}{\alpha_\gamma} \left(\frac{1}{n_\gamma^0 V_0 - \alpha_\gamma N_0} - \frac{1}{n_\gamma^0 V_0}\right) = \int\limits_{t_{\min}}^{t_{\max}} \frac{1}{V(t)} \frac{cf^2}{4\zeta^2(3)} \sum_i \frac{4^i}{i+1} p_i \Gamma^2(i+3) \zeta^2(i+3) T^{2i}(t) \, \mathrm{d}t. \tag{32}$$

Multiplying this equation by $\alpha_\gamma V_0$ and taking into account that $V(t) \propto a^3(t)$ and $a(t) \propto T^{-1}(t)$, one can find that

$$\frac{1}{n_\gamma^0 - \alpha_\gamma n_0} - \frac{1}{n_\gamma^0} = \frac{\alpha_\gamma cf^2}{4\zeta^2(3)T_0^3} \sum_i \frac{4^i}{i+1} p_i \Gamma^2(i+3) \zeta^2(i+3) \int\limits_{t_{\min}}^{t_{\max}} T^{2i+3}(t) \, \mathrm{d}t \tag{33}$$

where $T_0 = 2.73$ K $= 2.35 \cdot 10^{-4}$ eV is observable present-day CMB temperature.

Dependence of the CMB temperature on time can be estimated from the facts that $T \propto a^{-1}$, $a \propto \sqrt{t}$ at the RD and conditions that $T(t_{\min}) = T_{\min}$ and $T(t_{\max}) = T_{\max}$, where $T_{\min} = 1$ MeV and $T_{\max} = 1$ eV. So, one can obtain

$$T(t) = g + \frac{h}{\sqrt{t}}, \tag{34}$$

where values of the parameters are $h = (T_{\min} - T_{\max})/(t_{\min}^{-1/2} - t_{\max}^{-1/2})$ and $g = T_{\min} - h t_{\min}^{-1/2}$.
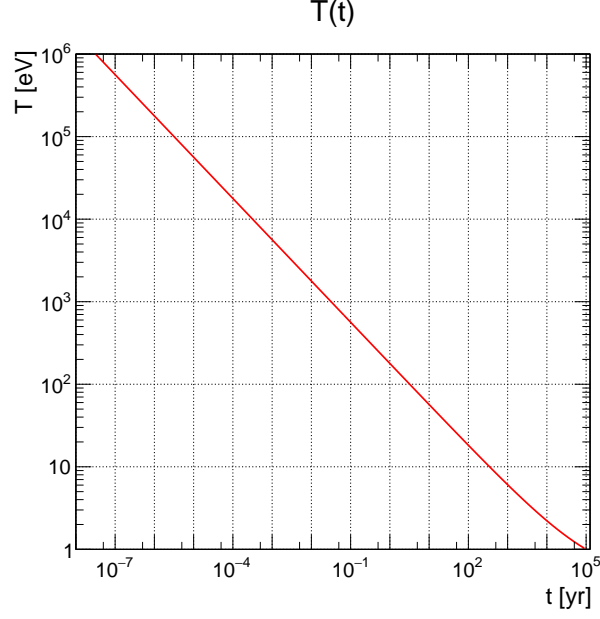
Figure 1: Temperature.

Integral from Eq. 33 denoted as I_i can be calculated using binomial theorem:

$$I_i = \int\limits_{t_{\min}}^{t_{\max}} T^{2i+3}(t)\,\mathrm{d}t = \int\limits_{t_{\min}}^{t_{\max}} \left(g + \frac{h}{\sqrt{t}}\right)^{2i+3}\mathrm{d}t = h^{2i+3}\sum_{k=0}^{2i+3} C_{2i+3}^{k}\left(\frac{g}{h}\right)^{2i+3-k}\int\limits_{t_{\min}}^{t_{\max}} t^{-k/2}\,\mathrm{d}t =$$

$$= \sum_{k=0,k\neq 2}^{2i+3} C_{2i+3}^{k} g^{2i+3-k} h^{k}\frac{t_{\max}^{1-k/2} - t_{\min}^{1-k/2}}{1-k/2} + C_{2i+3}^{2} g^{2i+1} h^{2}\ln\frac{t_{\max}}{t_{\min}}, \quad (35)$$

where $C_n^k = \frac{n!}{k!(n-k)!}$ is the binomial coefficient. Using Eq. 33, formula for n_0 can be written as

$$n_0 = \frac{n_\gamma^0}{\alpha_\gamma}\left(1 - \frac{1}{1 + f^2\frac{\alpha_\gamma n_\gamma^0 c}{4\zeta^2(3)T_0^3}\sum_i \frac{4^i}{i+1}p_i\Gamma^2(i+3)\zeta^2(i+3)I_i}\right). \quad (36)$$

Then, using Eq. 17 and 18, constraint on f can be found as

$$|f| < \sqrt{\frac{1}{\alpha_\nu n_\gamma^0 - \alpha_\gamma n_\nu^0}\frac{n_\nu^0}{n_\gamma^0}\frac{4\zeta^2(3)T_0^3}{c\sum_i \frac{4^i}{i+1}p_i\Gamma^2(i+3)\zeta^2(i+3)I_i}}. \quad (37)$$

3.3   Cross section parameterization and results

$\alpha_\gamma = 2$, $\alpha_\nu = 4$.

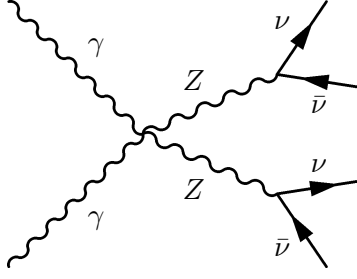For cross section parameterization Monte Carlo event generator MadGraph5 [4] was used.

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Figure 2: Diagram $\gamma\gamma \to \nu\bar{\nu}\nu\bar{\nu}$.

Table 2: f_{T0}

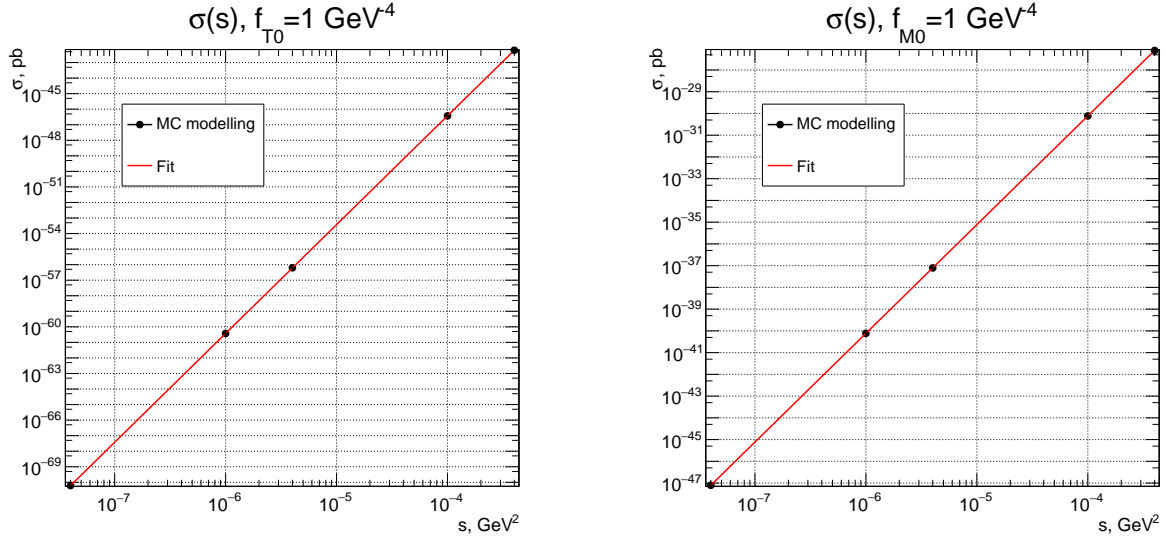| s, GeV^{-4}	$\sigma(f_{\text{T0}} = 1\ \text{GeV}^{-4})$, pb	$\sigma(f_{\text{M0}} = 1\ \text{GeV}^{-4})$, pb
$4 \cdot 10^{-8}$	$(6.13 \pm 0.08) \cdot 10^{-71}$	$(7.87 \pm 0.02) \cdot 10^{-48}$
10^{-6}	$(3.74 \pm 0.03) \cdot 10^{-61}$	$(7.70 \pm 0.02) \cdot 10^{-41}$
$4 \cdot 10^{-6}$	$(6.27 \pm 0.08) \cdot 10^{-57}$	$(7.87 \pm 0.04) \cdot 10^{-38}$
10^{-4}	$(3.73 \pm 0.01) \cdot 10^{-47}$	$(7.68 \pm 0.03) \cdot 10^{-31}$
$4 \cdot 10^{-4}$	$(6.18 \pm 0.02) \cdot 10^{-43}$	$(7.89 \pm 0.04) \cdot 10^{-28}$

Figure 3: Fit.

Table 3: Fit parameters

Coefficient	Parameters
f_{T0}	$p_7 = 3.74 \cdot 10^{-19}\ \text{pb/GeV}^6$
f_{M0}	$p_5 = 7.69 \cdot 10^{-11}\ \text{pb/GeV}^2$

Table 4: Results

Coefficient	Limit, TeV^{-4}
f_{T0}	$1.4 \cdot 10^{35}$
f_{M0}	$2.4 \cdot 10^{27}$

4 Constraints from the ultra-high-energy cosmic rays

Coming soon.

5   Conclusion

References

[1] C. Degrande, N. Greiner, W. Kilian, O. Mattelaer, H. Mebane, T. Stelzer et al., *Effective Field Theory: A Modern Approach to Anomalous Couplings*, *Annals Phys.* **335** (2013) 21 [1205.4231].

[2] O.J.P. Éboli and M.C. Gonzalez-Garcia, *Classifying the bosonic quartic couplings*, *Phys. Rev. D* **93** (2016) 093013 [1604.03555].

[3] PARTICLE DATA GROUP collaboration, *Review of Particle Physics*, *PTEP* **2020** (2020) 083C01.

[4] J. Alwall, R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, O. Mattelaer et al., *The automated computation of tree-level and next-to-leading order differential cross sections, and their matching to parton shower simulations*, *JHEP* **07** (2014) 079 [1405.0301].