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ABSTRACT
on the Cosmoparticle physics

«Experimental detection of relic neutrino with nonzero mass»

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INTRODUCTION

The concept of a "light, neutral, weakly interacting particle" was proposed in 1930 by W. Pauli [1], who was solving the problem of violating the energy–momentum conservation principle in the beta–decay reaction. The term "neutrino" was introduced in 1934 by Enrico Fermi, who had made the low–energy effective field theory of beta–decay — 4–fermionic Fermi's interaction (see, for example, [2]). In 1946, Bruno Pontecorvo put forward the idea to use the inverse beta–process:

$$\nu + (Z, A) \to e^- + (Z + 1, A) \; . \tag{1}$$

Moreover, he offered a specific chlorine–argon reaction $\nu + {}^{37}\mathrm{Cl} \to e^- + {}^{37}\mathrm{Ar}$ to detect neutrinos and quoting the Sun and the nuclear reactors as significant sources of them [3]. Finally, in 1950, Reines and Cowan registered neutrinos in the Savannah River reactor experiment [4] (later it turned out that it was not neutrinos, but the antineutrinos).

Today neutrino physics is of the great interest from the point of view of fundamental physics for the some following reasons:

- The neutrinos are the only known fundamental particles whose masses have not been determined yet [1]; moreover, the mechanism of generation of these masses is unclear:
  - The neutrinos are Dirac or Majorana particles?
  - What hierarchy of masses is realized?
- What is the fundamental theory behind neutrino oscillations? At the moment, this model is phenomenological;
- The Standard Model (SM) by its construction does not describe the presence of a neutrino mass, which is an additional argument in favor of the need for its extension;

[1] Of course, it is incorrect to say that flavor states of neutrino have masses. This issue will be discussed in more detail in 1.3.

- The problem of a possible CP–violation [5] in the lepton sector of the Standard Model — is the Dirac's (or Majorana's) phase $\delta_{\rm CP}$ equal to zero?
- The neutrinos, due to their high penetrating power, transfer information from distant objects almost without distortion, which is a powerful enough tool for astrophysics and cosmology;
- Are there sterile (or more exotic) neutrino states?
  - They may be candidates for the role of dark matter (see, for example, overview [6]);
  - They can probably explain reactor anomaliy [7];
  - They can probably explain the acceleration anomaly [8,9];
  - They can probably explain gallium anomaly [10];
- The relic neutrinos have a lot of information about processes in the early Universe, but they have not yet been detected;

In this paper we will focus on the last point — the possibilities of the detecting relic neutrino with nonzero mass. The work is structured as follows:

1) The first part presents a theoretical overview of neutrino physics in the context of the Standard Model and cosmology, discusses the mechanisms of neutrino mass generation and neutrino oscillations;
2) In the second part, a possible experiment on detecting relic neutrinos is described, and the simplest simulation of such an experiment is carried out using the material presented in the first part, an analysis of the sensitivity of such an experiment is carried out;
3) In conclusion, the results obtained are presented;

1. THEORETICAL OVERVIEW

1.1. THE MECHANISMS OF NEUTRINO MASS GENERATION

The first phenomenological theory of 2–neutrino mixing was soon proposed by Z. Maki, M. Nakagawa and S. Sakata [21] in 1962 and, regardless of them, by B. Pontecorvo and V. Gribov [22] in 1969. This section presents the modern formalism of 3–neutrino mixing [23–25].

The Dirac mass of neutrino can be introduced into the theory:

$$\mathcal{L}_m^{(\mathrm{D})} = \overline{\nu}_{\mathrm{L}}\,\mathcal{M}_{\mathrm{D}}\,\nu_{\mathrm{R}} + \mathrm{h.c.}\,, \tag{1.1}$$

where h.c. is hermitian conjugation, $\nu_{\mathrm{L}} = (\nu_e, \nu_\mu, \nu_\tau)_{\mathrm{L}}$, $\mathcal{M}_{\mathrm{D}}$ – matrix of 3×3 Dirac masses. The occurrence of such a term violates the invariance of the Lagrangian with respect to chiral transformations — with the unitary evolution of neutrinos, transitions with a change in chirality become possible, which contradicts observations

On the other hand, we can introduce the left and right **Majorana masses**:

$$\begin{aligned}
\mathcal{L}_m^{(\mathrm{MR})} &= -\frac{1}{2}\left(\overline{\nu}_{\mathrm{R}}^{\,\mathrm{C}}\,\mathcal{M}_{\mathrm{MR}}\,\nu_{\mathrm{R}}\right) + \mathrm{h.c.},\\
\mathcal{L}_m^{(\mathrm{ML})} &= -\frac{1}{2}\left(\overline{\nu}_{\mathrm{L}}^{\,\mathrm{C}}\,\mathcal{M}_{\mathrm{ML}}\,\nu_{\mathrm{L}}\right) + \mathrm{h.c.},
\end{aligned} \tag{1.2}$$

where $\nu_{\mathrm{L,R}}^{\mathrm{C}} = \mathbf{C}\,\nu_{\mathrm{L,R}}$, $\mathbf{C}$ – charge conjugation operator. Majorana neutrinos are truly neutral particles, chirality is preserved, but transitions with non-conservation of the lepton number are possible — for example, double neutrinoless β–decay [33]. It should be note that this reaction has been sought for a long time in experiments, but has not yet been observed.

In general, we introduce the neutrino mass into the theory as the sum of the Dirac and Majorana terms:

$$\begin{aligned}
\mathcal{L}_m &= -\overline{\nu}_{\mathrm{L}}\,\mathcal{M}_{\mathrm{D}}\,\nu_{\mathrm{R}} - \frac{1}{2}\left(\overline{\nu}^{\,\mathrm{C}}_{\mathrm{R}}\,\mathcal{M}_{\mathrm{MR}}\,\nu_{\mathrm{R}}\right) - \frac{1}{2}\left(\overline{\nu}^{\,\mathrm{C}}_{\mathrm{L}}\,\mathcal{M}_{\mathrm{ML}}\,\nu_{\mathrm{L}}\right) + \mathrm{h.c.} \\
&= \frac{1}{2}\begin{pmatrix}\overline{\nu}_{\mathrm{L}} & \overline{\nu}^{\,\mathrm{C}}_{\mathrm{R}}\end{pmatrix}\begin{pmatrix}\mathcal{M}_{\mathrm{LM}} & \mathcal{M}_{\mathrm{D}} \\ \mathcal{M}^{\mathrm{T}}_{\mathrm{D}} & \mathcal{M}_{\mathrm{RM}}\end{pmatrix}\begin{pmatrix}\nu^{\mathrm{C}}_{\mathrm{L}} \\ \nu_{\mathrm{R}}\end{pmatrix} + \mathrm{h.c.} \\
&= \frac{1}{2}\begin{pmatrix}\overline{\nu}_{\mathrm{L}} & \overline{\nu}^{\,\mathrm{C}}_{\mathrm{R}}\end{pmatrix}\mathcal{M}\begin{pmatrix}\nu^{\mathrm{C}}_{\mathrm{L}} \\ \nu_{\mathrm{R}}\end{pmatrix} + \mathrm{h.c.}\,, \qquad (1.3)
\end{aligned}$$

The 6×6 matrix $\mathcal{M}$ is not necessarily diagonal – states are mixed, new terms appear in the Lagrangian, for example, $-m_{e\mu}(\overline{\nu}_{e\,\mathrm{L}}\nu^{\mathrm{C}}_{\mu\,\mathrm{L}} + \overline{\nu}^{\,\mathrm{C}}_{\mu\,\mathrm{L}}\nu_{e\,\mathrm{L}})$ – neutrino transitions of one flavor into neutrinos of another are possible, that is, oscillations. For the same reason of the non-diagonality of the matrix, neutrinos with a certain flavor do not have a certain mass.

The matrix $\mathcal{M}$ can be transformed form using a bi-unitary transformation:

$$\begin{pmatrix}\mathrm{U} & \mathrm{R} \\ \mathrm{S} & \mathrm{V}\end{pmatrix}^{\dagger}\begin{pmatrix}\mathcal{M}_{\mathrm{LM}} & \mathcal{M}_{\mathrm{D}} \\ \mathcal{M}^{\mathrm{T}}_{\mathrm{D}} & \mathcal{M}_{\mathrm{RM}}\end{pmatrix}\begin{pmatrix}\mathrm{U} & \mathrm{R} \\ \mathrm{S} & \mathrm{V}\end{pmatrix}^{*} = \begin{pmatrix}\mathcal{M}_{\mathrm{L}} & 0 \\ 0 & \mathcal{M}_{\mathrm{R}}\end{pmatrix}, \qquad (1.4)$$

where $\mathcal{M}_{\mathrm{L}} = \mathrm{diag}(m_1, m_2, m_3)$, $\mathcal{M}_{\mathrm{R}} = \mathrm{diag}(M_1, M_2, M_3)$. The matrices U, R, S, V must obey the following relations:

$$\begin{aligned}
\mathrm{U}\,\mathrm{U}^{\dagger} + \mathrm{R}\,\mathrm{R}^{\dagger} = \mathrm{S}\,\mathrm{S}^{\dagger} + \mathrm{V}\,\mathrm{V}^{\dagger} &= 1\,, \\
\mathrm{U}^{\dagger}\,\mathrm{U} + \mathrm{S}^{\dagger}\,\mathrm{S} = \mathrm{R}^{\dagger}\,\mathrm{R} + \mathrm{V}^{\dagger}\,\mathrm{V} &= 1\,, \\
\mathrm{U}\,\mathrm{S}^{\dagger} + \mathrm{R}\,\mathrm{V}^{\dagger} = \mathrm{U}^{\dagger}\mathrm{R} + \mathrm{S}^{\dagger}\,\mathrm{V} &= 0\,. \qquad (1.5)
\end{aligned}$$

After diagonalization, the Lagrangian (1.3) will take the form

$$\mathcal{L}_m = -\frac{1}{2}\begin{pmatrix}\overline{\nu'}_{\mathrm{L}} & \overline{\nu'}^{\,\mathrm{C}}_{\mathrm{R}}\end{pmatrix}\begin{pmatrix}\mathcal{M}_{\mathrm{L}} & 0 \\ 0 & \mathcal{M}_{\mathrm{R}}\end{pmatrix}\begin{pmatrix}\nu'^{\mathrm{C}}_{\mathrm{L}} \\ \nu'_{\mathrm{R}}\end{pmatrix} + \mathrm{h.c.}\,, \qquad (1.6)$$

where $\nu'_{\mathrm{L}} = \mathrm{U}^{\dagger}\,\nu_{\mathrm{L}} + \mathrm{S}^{\dagger}\,\nu^{\mathrm{C}}_{\mathrm{R}}$ и $\nu'_{\mathrm{R}} = \mathrm{R}^{\mathrm{T}}\,\nu^{\mathrm{C}}_{\mathrm{L}} + \mathrm{V}^{\mathrm{T}}\,\nu_{\mathrm{R}}\,.$

Using the new field

$$\nu^{'} = \begin{pmatrix} \nu^{'}_{\mathrm{L}} \\ \nu^{'\,\mathrm{C}}_{\mathrm{R}} \end{pmatrix} + \begin{pmatrix} \nu^{'\,\mathrm{C}}_{\mathrm{L}} \\ \nu^{'}_{\mathrm{R}} \end{pmatrix} = \begin{pmatrix} \nu_{1\,\mathrm{L}} \\ \nu_{2\,\mathrm{L}} \\ \nu_{3\,\mathrm{L}} \\ \nu_{1\,\mathrm{R}} \\ \nu_{2\,\mathrm{R}} \\ \nu_{3\,\mathrm{R}} \end{pmatrix}, \tag{1.7}$$

we rewrite the Lagrangian (1.6) as follows:

$$\mathcal{L}_m = -\frac{1}{2}\overline{\nu^{'}} \begin{pmatrix} \mathcal{M}_{\mathrm{L}} & 0 \\ 0 & \mathcal{M}_{\mathrm{R}} \end{pmatrix} \nu^{'} = -\frac{1}{2}\sum_{i=1}^{3} \left(m_i\, \overline{\nu}_{i\,\mathrm{L}}\, \nu_{i\,\mathrm{L}} + M_i\, \overline{\nu}_{i\,\mathrm{R}}\, \nu_{i\,\mathrm{R}} \right) . \tag{1.8}$$

The states $\nu_{i\,\mathrm{L}}$ ($\nu_{i\,\mathrm{R}}$) are eigenstates of the mass matrix, have a certain mass m_i (M_i), do not have a certain flavor and, since the matrix $\mathcal{M}$ is Hermitian, form a complete basis of the state space. The transition from one representation to another is carried out using the matrices U, R, S, V:

$$\begin{cases} \nu^{'}_{\mathrm{L}} = \mathrm{U}^{\dagger}\, \nu_{\mathrm{L}} + \mathrm{S}^{\dagger}\, \nu^{\mathrm{C}}_{\mathrm{R}} \\ \nu^{'}_{\mathrm{R}} = \mathrm{R}^{\mathrm{T}}\, \nu^{\mathrm{C}}_{\mathrm{L}} + \mathrm{V}^{\mathrm{T}}\, \nu_{\mathrm{R}} \end{cases}, \quad \begin{cases} \nu_{\mathrm{L}} = \mathrm{U}\, \nu^{'}_{\mathrm{L}} + \mathrm{R}\, \nu^{'\mathrm{C}}_{\mathrm{R}} \\ \nu_{\mathrm{R}} = \mathrm{S}^{*}\, \nu^{'\mathrm{C}}_{\mathrm{L}} + \mathrm{V}^{*}\, \nu^{'}_{\mathrm{R}} \end{cases} . \tag{1.9}$$

Right neutrinos and left antineutrinos are not observed in experiments, which makes it possible to simplify the above model. From $\nu_{\mathrm{R}} = 0$, it immediately follows that $\nu^{\mathrm{C}}_{\mathrm{R}} = 0$. The dimension of the state space is reduced by half – from the 2-dimensional one described by the basis (ν_{L}, ν_{R}), to the 1-dimensional one with one basis state–vector ν_{L}.

In this reason, in (1.9) there will only be

$$\nu^{'}_{\mathrm{L}} = \mathrm{U}^{\dagger}\, \nu_{\mathrm{L}}\ ,\ \nu_{\mathrm{L}} = \mathrm{U}\, \nu^{'}_{\mathrm{L}}\, , \tag{1.10}$$

where $\mathrm{U}^{\dagger}\mathrm{U} = 1$, that is, the matrix U becomes unitary [1].

[1] Note that the matrix U does not have to be unitary in reality. The non-unitarity of the mixing matrix may be related to the existence of exotic neutrino states. Some experimental data of neutrino accelerator experiments indicate the non-unitarity of the mixing matrix - see, for example, [13, 14]

The matrix $\mathrm{U} \equiv \mathrm{U_{PMNS}}$ of the transition from one basis to another is called the Pontecorvo–Maki–Nakagawa–Sakata mixing matrix. It is usually parameterized in the following form:

$$\mathrm{U_{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \times \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13}\cdot e^{-i\delta} \\ 0 & 1 & 0 \\ -\sin\theta_{13}\cdot e^{-i\delta} & 0 & \cos\theta_{13} \end{pmatrix} \times$$

$$\times \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (1.11)$$

There are 6 parameters: three mixing angles $\theta_{ij} \in [0; \pi/2]$, the Dirac complex phase $\delta \in [0; 2\pi]$ (if neutrinos are Dirac particles) and complex phases of Marjorana $\rho, \sigma \in [0; 2\pi]$ (if neutrinos are Majorana particles). The inequality of complex phases to zero leads to a CP violation in the lepton sector of the Standard Model.

1.2. NEUTRINO OSCILLATIONS

In 1957 [19] B. M. Pontecorvo drew attention to the fact that if the lepton charge is not strictly conserved and if the neutrino has a small non-zero mass, then neutrino oscillations can occur. Let's describe the oscillation mechanism using the results of the previous section [2].

Let the α–flavor state of neutrino with energy E was born at the space point $x_0 = 0$, so $|\psi(x_0 = 0)\rangle = |\nu_\alpha\rangle$. Let's write the flavor state as a superposition of the masses using the mixing matrix (1.11): $|\nu_\alpha\rangle = \sum\limits_{j=1}^{3} \mathrm{U}_{\alpha j}\, |\nu_j\rangle$. In the case of unitary evolution, states ν_j are quasi-stationary and have a certain momentum at a certain mass: $p_j = \sqrt{E^2 - m_j^2}$. Then the neutrino state at point x is written as

$$|\psi(x)\rangle = \sum_{j=1}^{3} \mathrm{U}_{\alpha j}\, |\nu_j\rangle \cdot e^{-ip_j x}\,. \tag{1.12}$$

The probability that the «detector» at point x will «register» a neutrino with an α–flavor (usually it called the probability of survival) is:

$$P_{\alpha\alpha} = |\mathcal{A}_{\alpha\alpha}|^2 = |\, \langle \nu_\alpha | \psi(x) \rangle \,|^2\,. \tag{1.13}$$

Using decompositions of flavor states through mass ones, taking into account the orthogonality $\langle \nu_k | \nu_j \rangle = \delta_{kj}$, we obtain:

$$\mathcal{A}_{\alpha\alpha} = \sum_{k=1}^{3} \sum_{j=1}^{3} \mathrm{U}^*_{\alpha k} \mathrm{U}_{\alpha j}\, \langle \nu_k | \nu_j \rangle\, e^{-ip_j x} = \sum_{j=1}^{3} |\mathrm{U}_{\alpha j}|^2 e^{-ip_j x}\,. \tag{1.14}$$

Note that expression (1.14) includes the square of the modulus of the mixing matrix element – the Dirac and Majorana phases do not contribute to this amplitude. Substituting (1.14) into (1.13), using $m_j \ll E$, we get the final result (for example, for the case $\alpha = e$):

[2] It should be said that this reasoning contains a number of important assumptions: we believe that the neutrino state is described by a plane wave that spreads out at the speed of light. A more accurate reasoning can be found, for example, in [24].

$$P_{ee} = 1 - \sin^2 2\theta_{13} \left(\cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32}\right) - \\ - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21}, \tag{1.15}$$

where $\Delta_{ij} = 1.267 \cdot \Delta m^2_{ij}\frac{L}{E}$, $i, j = 1, 2, 3$; $\Delta m^2_{ij} = m^2_i - m^2_j$ — difference of squares of masses of mass states [eV^2]; L — distance from the source to the detection point [m]; E — neutrino energy [MeV].

Similarly, we can consider the probability of oscillation ν_μ into ν_e:

$$P_{\mu e} = \frac{1}{2} \sin 2\theta_{23}\sin 2\theta_{12}\cos^2\theta_{13}\sin\theta_{13}\sin\Delta_{12}\sin\Delta_{13}\cos\delta + \\ + \sin^2\theta_{23}\sin^2 2\theta_{13}\sin^2\Delta_{31} - \sin 2\theta_{23}\sin 2\theta_{12}\cos^2\theta_{13} \times \\ \times \sin\theta_{13}\sin\Delta_{12}\sin^2\Delta_{13}\sin\delta \tag{1.16}$$

The most general equation for $P_{\alpha\beta}$ can be found in [25]. Note that expressions (1.15) and (1.16) are valid if neutrinos propagate in vacuum (and, with good accuracy, in air). Otherwise, the effects of matter begin to affect the oscillations and expressions become more complicated [15, 16].

1.3. NEUTRINO IN THE STANDARD MODEL AND ITS EXTENSIONS

In the Standard Model $\mathrm{U}(1)_{\mathrm{Y}} \otimes \mathrm{SU}(2)_{\mathrm{L}} \otimes \mathrm{SU}(3)$ [26] generation of the fermion masses described by the Yukawa interaction of the fermionic fields with the Higgs scalar field:

$$\mathcal{L}_{\mathrm{Yu}} = Y^{e}_{ij} \left(\overline{L}_i \varphi\right) e_{\mathrm{R}\,j} + Y^{\nu}_{ij} \left(\overline{L}_i \tilde{\varphi}\right) \nu_{\mathrm{R}\,j} + \mathrm{h.c.}\ , \tag{1.17}$$

where $L_i = \begin{pmatrix} \nu_{\mathrm{L}\,i} \\ e_{\mathrm{L}\,i} \end{pmatrix}$ – left leptonic doublet (the fundamental representation of $\mathrm{SU}(2)_{\mathrm{L}}$), $e_{\mathrm{R}\,j}, \nu_{\mathrm{R}\,j}$ — right singletons (the trivial representation of $\mathrm{SU}(2)_{\mathrm{L}}$),

$\varphi = \begin{pmatrix} \varphi^{+} \\ \varphi^{0} \end{pmatrix}$ — isodublet of the Higgs fields, Y^{e}_{ij} and Y^{ν}_{ij} — 3×3 matrices of coupling constants which determines the masses of leptons, $i, j = e, \mu, \tau$.

In case of spontaneous violation $\mathrm{U}(1)_{\mathrm{Y}} \otimes \mathrm{SU}(2)_{\mathrm{L}} \to \mathrm{U}(1)_{\mathrm{EM}}$ vacuum mean φ:

$$\langle \varphi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ V \end{pmatrix} \neq 0, \tag{1.18}$$

where $V = \sqrt{\dfrac{1}{\sqrt{2} G_F}} \approx 250$ GeV.

Substituting (1.18) into (1.17) gives

$$\begin{aligned} \mathcal{L}_{\mathrm{Yu}} &= \frac{V}{\sqrt{2}} \left\{ Y^{e}_{ij} (\overline{e}_{\mathrm{R}} e_{\mathrm{L}} + \overline{e}_{\mathrm{L}} e_{\mathrm{R}})_{ij} + Y^{\nu}_{ij} (\overline{\nu}_{\mathrm{R}} \nu_{\mathrm{L}} + \overline{\nu}_{\mathrm{L}} \nu_{\mathrm{R}})_{ij} \right\} \\ &= m^{e}_{ij} (\overline{e}_{\mathrm{R}} e_{\mathrm{L}} + \overline{e}_{\mathrm{L}} e_{\mathrm{R}})_{ij} + m^{\nu}_{ij} (\overline{\nu}_{\mathrm{R}} \nu_{\mathrm{L}} + \overline{\nu}_{\mathrm{L}} \nu_{\mathrm{R}})_{ij}\ . \end{aligned} \tag{1.19}$$

Here $m_{ij} = \frac{V}{\sqrt{2}} Y_{ij}$ is the matrix of fermionic masses, the components of which are free parameters of the model and cannot be calculated due to infinite renormalization of the coupling constants Y_{ij}.

The structure of the Higgs sector of the Standard Model leads to a global symmetry corresponding to the conservation of the lepton number. As noted above, this fact prohibits the presence of Majorana neutrinos in the Lagrangian.

The Dirac mass terms preserve the lepton number, but they are equal to zero, since right neutrinos are not observed in nature. Thus, in the Standard Model, the zero mass of neutrinos is due to a limited set of particle fields that form the simplest representation of the group $\mathrm{U}(1)_{\mathrm{Y}} \otimes \mathrm{SU}(2)_{\mathrm{L}}$.

1.3.1. EXTENSION OF THE HIGGS SECTOR AND MAJORANA NEUTRINO GENERATION

It is possible to postulate the existence of an additional triplet of Higgs fields H with a hypercharge $Y = 2$, which can be represented as [29]

$$(\vec{\sigma}\vec{H}) = \begin{pmatrix} H^{+} & \sqrt{2}H^{++} \\ \sqrt{2}H^{0} & -H^{+} \end{pmatrix}. \tag{1.20}$$

Then additional terms of the Yukawa interaction appear in the fermionic Lagrangian:

$$\mathcal{L}^{M}_{\text{int}} = f_{ij}\,\overline{L}^{\mathrm{C}}_{i\,\mathrm{L}}\,L_{j\,\mathrm{L}}\,(\vec{\sigma}\vec{H}) + \text{h.c.}\,. \tag{1.21}$$

Substitution of a non-zero vacuum mean of the field H

$$\langle(\vec{\sigma}\vec{H})\rangle = \begin{pmatrix} 0 & 0 \\ V_H & 0 \end{pmatrix} \tag{1.22}$$

into the equation (1.21) leads to Majorana neutrino term of the form $(f_{ij}V_H)\,\overline{\nu}^{\mathrm{C}}_{i\,\mathrm{L}}\nu_{j\,\mathrm{L}}$ with mass $M_{\mathrm{M}} = \dfrac{f_{ij}V_H}{\sqrt{2}}$.

The vacuum mean V_H can't be too large for the reason that the mean of the isotriplet field gives a different contribution to the masses of $W^{\pm}$ and Z bosons:

$$\frac{\Delta M_W^2}{M_W^2} = \frac{2V_H^2}{V^2}, \quad \frac{\Delta M_Z^2}{M_Z^2} = \frac{4V_H}{V} \tag{1.23}$$

Therefore, in order for the ratio $M_W = M_Z \cos\theta_W$ to be carried out with an accuracy corresponding to experimental data (on the order of a percentage), the condition is necessary $V_H \leq 0.1V \approx 25$ GeV.

1.3.2. EXTENSIONS OF THE HIGGS AND LEPTON SECTORS, "SEE–SAW" MECHANISM

If we assume that there are right neutrinos $\nu_{\rm R}$ and a scalar Higgs field χ additional to the doublet

$$\varphi = \begin{pmatrix} \varphi^{+} \\ \varphi^{0} \end{pmatrix}$$

which is a singlet with respect to the standard group $\mathrm{U(1)_Y \otimes SU(2)_L}$, then both Dirac and Majorana neutrino terms appear in the Lagrangian of the interaction:

$$\mathcal{L}_{\rm int} = M_{\rm D}\, \overline{\nu}_{\rm L}\, \nu_{\rm R} + M_{\rm M}\, \overline{\nu}_{\rm R}^{\rm C}\, \nu_{\rm R} + \text{h.c.}\,. \tag{1.24}$$

At the same time , the Dirac mass $M_{\rm D} = \frac{\langle\varphi\rangle f_{\rm L}}{\sqrt{2}}$ in order of magnitude is equal to the mass of the charged fermion, and the Majorana mass $M_{\rm M} = \frac{\langle\chi\rangle f_{\rm R}}{\sqrt{2}}$ is close to the energy scale at which the spontaneous symmetry breaking of the fundamental group describing GUT occurs up to the SM group.

The appearance of the Majorana mass occurs due to neutrality with respect to the transformations of the group $\mathrm{U(1)_Y \otimes SU(2)_L}$. In this case, the interaction $f_R\, \overline{\nu}_{\rm R}\, (\chi^{\rm C})^{*}\, \nu_{\rm R}$ does not change the symmetry properties of the theory, and the lepton number will be preserved if a doubled lepton charge is attributed to the field χ . Spontaneous violation of the symmetry of the global group $\mathrm{U(1)_L}$ associated with the preservation of the lepton number due to the non-zero vacuum mean Higgs field χ leads to the appearance of a Majorana mass in the right neutrino and is accompanied by the appearance of a massless Majorana fermion, whose field is described by an imaginary component $\mathrm{Im}\chi$ [27].

The diagonalization of the mass matrix in the Lagrangian

$$\begin{pmatrix} \overline{\nu}_{\rm L} & \overline{\nu}_{\rm R}^{\rm C} \end{pmatrix} \begin{pmatrix} 0 & M_{\rm D} \\ M_{\rm D}^{\rm T} & M_{\rm M} \end{pmatrix} \begin{pmatrix} \nu_{\rm L} \\ \nu_{\rm R}^{\rm C} \end{pmatrix} \tag{1.25}$$

is performed similarly to subsection 1.1, and the weak states

$$\begin{aligned} \nu'_{\rm L} &= \nu_{\rm L} + \nu_{\rm L}^{\rm C}, \\ \nu'_{\rm R} &= \nu_{\rm R} + \nu_{\rm R}^{\rm C} \end{aligned} \tag{1.26}$$

are the superposition of the mass Majorana states

$$\begin{aligned}\nu_1 &\approx \nu'_{\mathrm{L}} - M_{\mathrm{M}}^{-1} M_{\mathrm{D}}\, \nu'_{\mathrm{R}}\,, \\ \nu_2 &\approx \nu'_{\mathrm{R}} + M_{\mathrm{M}}^{-1} M_{\mathrm{D}}\, \nu'_{\mathrm{L}}\,. \end{aligned} \tag{1.27}$$

In this case, the neutrino ν'_{L} is mainly a light neutrino with a mass of the order of the mass of fermions:

$$m_{\nu'_{\mathrm{L}}} \approx M_{\mathrm{M}}^{-1} M_{\mathrm{D}}^2 = \left(M_1 \sin^2\theta + M_2 \cos^2\theta\right)^{-1} (M_1 - M_2)^2 \sin 2\theta\,, \tag{1.28}$$

and the heavy neutrino with a GUT-mass:

$$m_{\nu'_{\mathrm{R}}} \approx \left(M_1 \sin^2\theta + M_2 \cos^2\theta\right) \tag{1.29}$$

Note that the mixing angle is very small in this case:

$$\tan\theta \approx \frac{m_f}{M_{\mathrm{GUT}}}, \tag{1.30}$$

where $m_f \approx 1$ MeV is the mass scale of light fermions, $M_{\mathrm{GUT}} \approx 10^{15}$ GeV — the scale of the Great Unification.

The mechanism of neutrino mass generation, in which there are two mass scales (m_f, M_{GUT}) that determine the mass matrix in the combined GUT theories, is called "see–saw" mechanism [30].

1.4. NEUTRINO IN THE BIG BANG COSMOLOGY

1.4.1. THERMODYNAMICS OF THE EARLY UNIVERSE AND ITS RELATION TO COSMOLOGY

In the early stages of the expansion of the hot Universe, neutrinos were in thermodynamic equilibrium with photons, electrons, quarks and their antiparticles [3]. This makes it possible to calculate all the parameters of the neutrino distribution in the present epoch. The distribution function of bosons and fermions can be represented as:

$$f_i = \frac{\mathrm{d}^6 N}{\mathrm{d}^3 x\ \mathrm{d}^3 p} = \frac{1}{(2\pi)^3} \frac{g_{si}}{\exp\left(E_i/T\right) \pm 1}, \tag{1.31}$$

where g_{si} – statistical weight – number of quantum states of particles ($i = f, b$, f – fermion, b – boson), T – temperature, E_i – energy , "+" corresponds to fermions, "–" corresponds to bosons. Using this distributions, we can find number densities (in the ultrarelativistic case $E_i \gg m_i$):

$$\begin{aligned} n_f &= \int f_f\,\mathrm{d}^3 p = \frac{\zeta(3)}{\pi^2} g_{sf} T^3\,, \\ n_b &= \int f_b\,\mathrm{d}^3 p = \frac{3\zeta(3)}{4\pi^2} g_{sb} T^3\,, \end{aligned} \tag{1.32}$$

and energy densities:

$$\begin{aligned} \varepsilon_f &= \int E f_f\,\mathrm{d}^3 p = \frac{7\pi^3}{240} g_{sf} T^4\,, \\ \varepsilon_b &= \int E f_b\,\mathrm{d}^3 p = \frac{\pi^3}{30} g_{sb} T^4\,, \end{aligned} \tag{1.33}$$

where $\zeta(3) = \frac{1}{2}\int\limits_0^\infty \frac{x^2\,\mathrm{d}x}{\exp(x)-1} = \frac{2}{3}\int\limits_0^\infty \frac{x^2\,\mathrm{d}x}{\exp(x)+1} \approx 1.202$ is Apery's constant.

[3] This paragraph is based on 2 courses of lectures delivered at MEPhI at the department 40 — [31,32]

In the non-relativistic case we have $E_i = m_i + E_{i\,\mathrm{kin}} \approx m_i \ll T$, so

$$n_i = \int f_i \, \mathrm{d}^3 p = g_{si} \left(\frac{T m_i}{2\pi} \right)^{3/2} \exp\left(-m_i/T\right) \tag{1.34}$$

for bosons and fermions.

For a flat universe, the energy density is uniquely related to the Hubble parameter, and this relationship follows from cosmology and general relativity. Note that cosmology "does not know" about the composition of the Universe. On the other hand, the critical density can be represented through the thermodynamics described above. Equation (1.35) reflects the fundamental connection between cosmology and thermodynamics of the early Universe:

$$\rho_{\mathrm{c}} = \frac{3H^2}{8\pi G_N} = \left[\sum_b g_{sb} \left(\frac{T_b}{T} \right)^4 + \frac{7}{8} \sum_f g_{sf} \left(\frac{T_f}{T} \right)^4 \right] \frac{\pi^2 T^4}{30} \tag{1.35}$$

Note that the expression (1.35) includes only relativistic particles — the density of non-relativistic particles is exponentially suppressed (1.34).

1.4.2. GETTING OUT OF EQUILIBRIUM — THE FREEZING

Equilibrium in a thermodynamic system exists as long as the rate of reactions is greater than the rate of change of external conditions. On the cosmological scale, it is natural to understand the Hubble parameter by the rate of change of external conditions. Thus, the condition for the termination of the reactions (*freezing*) has the form:

$$\Gamma_{ij} = n_j \langle \sigma_{ij} v_i \rangle = H \tag{1.36}$$

Here, the value Γ_{ij} — the rate of the reaction of the interaction of a particle i with j — is the number of such reactions per unit of time. Naturally, this equality $\Gamma_{ij} = H$ gives a good, but approximate estimate, since the process of getting out of equilibrium is continuous.

Neutrinos in the first seconds after the Big Bang played an important role in primary nucleosynthesis. Let's consider a moment in the Early Universe in which the reactions of the transformation of protons into neutrons and vice versa took place. There are only 5 such reactions:

$$\begin{aligned} n+\nu_e &\leftrightarrow p+e^- , \\ n+e^+ &\leftrightarrow p+\overline{\nu}_e , \\ n &\rightarrow p+e^- +\overline{\nu}_e . \end{aligned} \qquad (1.37)$$

Such transformations can also take place due to strong interaction (effectively through π mesons), however, at temperatures at which a π meson could be born, there were no protons and neutrons yet.

Termination of the first four reactions occurs, respectively, when $\Gamma_{np} = n_{\nu,e}\langle\sigma_{np}v\rangle \approx H$. The solution of this equality with respect to temperature gives the freezing temperature $T^* \approx 0.75$ MeV — when the Universe cools down to this temperature, the transformation of a neutron into a proton is possible only with beta decay (the last reaction in (1.37)). Knowing this temperature, it is possible to estimate the ratio of protons to neutrons, using the fact that they are non-relativistic particles ($T^* \approx 1$ MeV $\ll m_p \approx m_n \approx 1$ GeV) and obey the Boltzmann distribution:

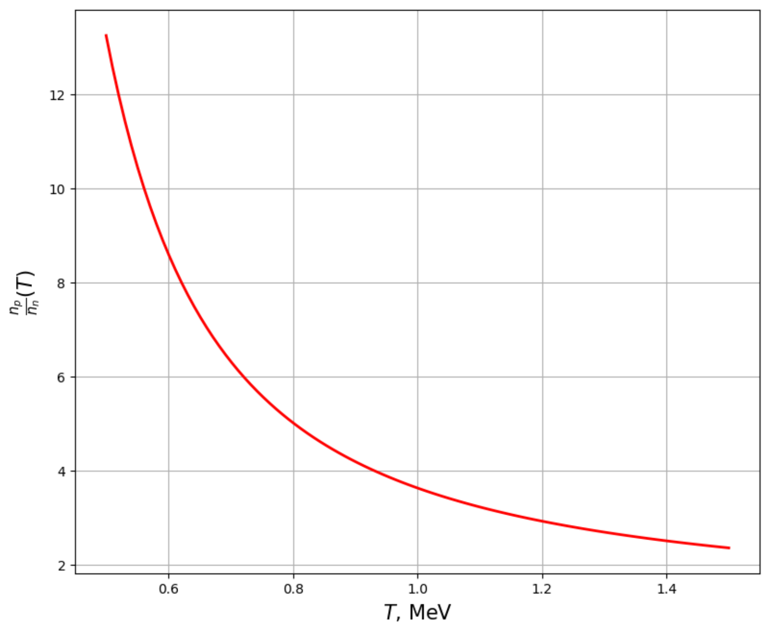

FIG. 1.1 — Dependence of the ratio of proton to neutron concentrations on temperature

At the freezing temperature $T = T^* \approx 0.75$ MeV the ratio of proton to neutron concentrations is $n_p/n_n \approx 6$. Neutrinos from (1.37) that are out of equilibrium and for which the Universe has become transparent are called **relic neutrinos** (or cosmic neutrino background, **CνB**). After quenching protons and neutrons, nucleosynthesis began.

Firstly, at temperatures 0.1 MeV $< T <$ 0.75 MeV neutrons and protons combine to form deuterium (D), but it is immediately destroyed by interactions with photons (the concentration of photons at this time exceeds the concentration of all baryons by 10^9 times):

$$\begin{aligned} p + n &\rightarrow D + \gamma \ \ (1), \\ D + \gamma &\rightarrow p + n \ \ \ (2). \end{aligned} \tag{1.38}$$

Reaction (2) in equation (1.38) is a threshold: the photon energy must be greater than $E_D \approx 2.2$ MeV. Equality of the rates of formation ($\Gamma_{pn \to D\gamma}$) and the rates of destruction ($\Gamma_{D\gamma \to pn}$) it is reached at $T = 0.1$ MeV; further, when the temperature drops, the rate of formation exceeds the rate of destruction and deuterium begins to accumulate. This corresponds to the time $t = 100$ seconds since the Big Bang.

The numerical values of temperatures presented depend on the number of neutrino varieties (above everywhere $N_\nu = 3$), while the ratio n_p/n_n radically affects the primary nucleosynthesis.

So, if neutrinos were released at temperatures greater than 0.75 MeV, that is, there would be almost the same number of protons and neutrons in the Universe. This means that later all of them would combine into the core of the main isotope of helium and there would be practically no free hydrogen left in the Universe. In principle, helium clouds could eventually undergo gravitational condensation and give rise to stars, some of which would acquire planetary systems. However, on these planets there would be no hydrogen and, consequently, water, without which we cannot imagine the origin of life.

Otherwise, if neutrinos uncoupled at temperatures lower than 0.75 MeV, the ratio n_n/n_p would be much less than one - this means that there would be no helium in the Universe and it would remain purely hydrogen until the first stars appeared.

2. NEUTRINO CAPTURE IN INVERSE β DECAY

To begin with, let's consider the reaction of quasielastic scattering of the neutrino mass state on a neutron:

$$\nu_j(p_j) + n(p_n) \to p(p_p) + e^-(p_e) \tag{2.1}$$

In a non-relativistic approximation, we can use Fermi 4–fermionic theory to effectively describe this process:

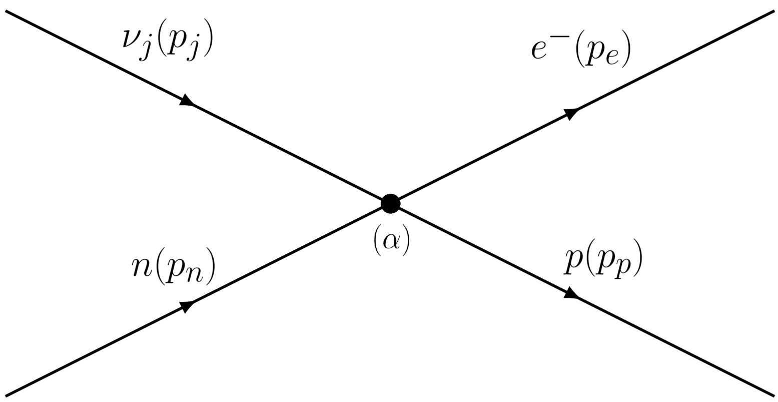

FIG. 2.1 — Feynman diagram of the neutrino capture process in Fermi theory

So, the $\mathcal{M}$-matrix has the form

$$\mathcal{M}_j = \frac{G_F}{\sqrt{2}} \mathrm{V}_{ud} \mathrm{U}^*_{ej} \Big(\overline{u}_e \gamma_\alpha (1-\gamma_5) u_j\Big) \Big(\overline{u}_p \big[\gamma^\alpha (g_V - g_A \gamma_5) + \\ + \frac{g_M}{2m_p} \sigma^{\alpha\beta} (p_p - p_n)_\beta + g_P (p_p - p_n)^\alpha \gamma_5\big] u_n\Big) \,. \tag{2.2}$$

Here G_F, V_{ud} and U_{ej} being the Fermi constant, ud–element of CKM matrix and ej–element of PMNS matrix; g_V, g_A, g_M and g_P — nuclear form factors — real functions of the square of the transmitted 4-momentum $t = (p_p - p_n)^2$. We use $\eta_{\mu\nu} = \mathrm{diag}\{1; -1; -1; -1\}$, $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$ and $\sigma_{\alpha\beta} = (\gamma_\alpha\gamma_\beta - \gamma_\beta\gamma_\alpha)/2$.

Note that due to the crossing symmetry, the $\mathcal{M}$–matrix of neutrino capture is associated with the $\mathcal{M}$–matrices of β–decay and inverse β–decay. To move from one process to another, it is necessary to redefine the Mandelstam variables s, t, u in the right way (and also discard the element of the PMNS matrix, assuming that a flavor neutrino takes part in these processes).

The nuclear form factors included in (2.2) can be represented in a "dipole" form:

$$g_V(t) = \frac{g_V(0)}{(1 - t/m_V^2)^2}, \quad g_A(t) = \frac{g_A(0)}{(1 - t/m_A^2)^2},$$
$$g_M(t) = \frac{\mu_p - \mu_n}{(1 - t/4m_p^2)(1 - t/m_V^2)^2}, \quad g_P(t) = g_A(t)\frac{m_n + m_p}{m_\pi^2 - t}, \tag{2.3}$$

where $m_V \approx m_A \approx 1$ GeV, $m_\pi \approx 140$ MeV, $\mu_p - \mu_n = 3.706$ is the difference between the proton and neutron anomalous magnetic moments in units of the nuclear magneton.

In the non-relativistic case (more precisely , if t/m_p^2, t/m_π^2 and $t/m_A^2 \approx t/m_V^2 \ll 1$), form factors can be considered constant: $g_V(t) \approx g_V(0) = 1$ and $g_A(t) \approx g_A(0) = -1.261$; $g_M(t)$ and $g_P(t)$ have the following order of smallness and can be neglected. Then the $\mathcal{M}$-matrix is written as

$$\mathcal{M}_j = \frac{G_F}{\sqrt{2}} \mathrm{V}_{ud} \mathrm{U}_{ej}^* \Big(\overline{u}_e \gamma_\alpha (1 - \gamma_5) u_j\Big) \Big(\overline{u}_p (g_V(0)\gamma^\alpha - g_A(0)\gamma_5) u_n\Big) . \tag{2.4}$$

The $\mathcal{M}$–matrix, which takes into account all the presented form factors, can be found in [39] (in [39] a cross section for the inverse β–decay is given, however, as noted above, it is not difficult to rewrite it for our case).

The cross section of the (2.1) has the following form

$$\mathrm{d}\sigma_j = \frac{1}{2E_j} \frac{1}{2E_n} \frac{1}{|\vec{v}_j - \vec{v}_n|} \left(\frac{1}{2} \sum_{\text{spin}} |\mathcal{M}_j|^2\right) \mathrm{d}\Phi , \tag{2.5}$$

where E_j, E_n and $\vec{v}_j$, $\vec{v}_n$ — neutrino and neutron energies and velocities,

$\mathrm{d}\Phi$ — lorentz–invariant phase space element:

$$\mathrm{d}\Phi = \frac{\mathrm{d}^3 p_e}{(2\pi)^3}\frac{1}{2E_e}\frac{\mathrm{d}^3 p_p}{(2\pi)^3}\frac{1}{2E_p}(2\pi)^4\delta^{(4)}(p_j + p_n - p_p - p_e) \ . \tag{2.6}$$

Here $\mathrm{d}^3 p = |\vec{p}|^2 \ \mathrm{d}|\vec{p}|\mathrm{d}\Omega$. We integrate by $\mathrm{d}^3 p_p$ and $\mathrm{d}|\vec{p}_e|$ in the laboratory system, where the neutron rests: $\vec{v_n} = 0, E_n = m_n$. We get:

$$\mathrm{d}\sigma_j = \frac{1}{4E_e E_j}\frac{1}{v_j}\left(\frac{1}{2}\sum_{\text{spin}}|\mathcal{M}_j|^2\right)\frac{|\vec{p}_e|}{4\sqrt{s}\ (2\pi)^2}\,\mathrm{d}\Omega_e \ , \tag{2.7}$$

where $\sqrt{s} = \sqrt{(p_j + p_n)^2} = \sqrt{m_n^2 + 2m_n E_j} = m_n + \frac{E_j}{m_n} + O\left(\left(\frac{E_j}{m_n}\right)^2\right) \approx m_n$.

When moving from (2.1) to real nucleons, we need to discuss three additional contributions:

1) Must be replaced $\left(\frac{1}{2}\sum\limits_{\text{spin}}|\mathcal{M}_j|^2\right) \to |\mathcal{M}_{\text{Nucl.}}|^2$, which takes into account the type of transition (allowed/forbidden, Gamow-Teller/Fermi);
2) Take into account that the β–electron is in the Coulomb field of the daughter nucleus — multiply the expression (2.7) by the Fermi function $F(Z, E_e)$ [2, 35];
3) Take into account additional corrections related to the finite size of the nucleus and the finite wavelengths of leptons [35], screening of the β–particle by atomic shell electrons [36], radiation corrections related to the possibility of emitting a virtual/real photons [37, 38] el al.

Then the most general expression for the cross section has the form [40]:

$$\mathrm{d}\sigma_j = \frac{G_F^2|\mathrm{V}_{ud}|^2|\mathrm{U}_{ej}|^2}{2(2\pi)^2}\frac{\left[\varepsilon_\mu\varepsilon_\nu^* X^{\mu\nu}\right]}{4E_e E_j}|\mathcal{M}_{\text{Nucl.}}|^2 F(Z, E_e)E_e|\vec{p}_e|\,(1+\delta)\,\mathrm{d}\Omega_e \ , \tag{2.8}$$

where ε_μ — polarization 4-vector of the parent nucleus, $X^{\mu\nu}$ — leptonic tensor the definition of which will be necessary below, $(1+\delta)$ — the multiplier that includes the various corrections mentioned above in the third item.

The Fermi function is written as follows:

$$F(Z,E_e) = 2(\gamma+1)(2p_eR)^{(2\gamma-1)}e^{(\pi\alpha ZE_e/p_e)} \cdot \frac{|\Gamma(\gamma+\mathrm{i}\alpha ZE_e/p_e)|^2}{|\Gamma(2\gamma+1)|^2}\,, \qquad (2.9)$$

where $\gamma = \sqrt{1-(\alpha Z)^2}$, $\alpha \approx 1/137$ — fine structure constant, $\Gamma(z)$ — Euler gamma–function of a complex argument $R = R(A)$ — radius of the nuclear (Elton's formulae [41]):

$$R(A) = 1.121A^{1/3} + 2.426A^{-1/3} - 6.614/A \ \ [\mathrm{fm}]. \qquad (2.10)$$

The leptonic tensor $X^{\mu\nu}$ is

$$X^{\mu\nu} = \Big(\overline{u}_e\gamma^\mu(1-\gamma_5)u_j\Big)\Big(\overline{u}_j\gamma^\nu(1-\gamma_5)u_e\Big)\,. \qquad (2.11)$$

The cross section in the form (2.8) is convenient because it allows us to consider polarized nuclei. For electron and neutrino in definite spin states we have

$$\begin{aligned}\sum_{\text{spin}} u_e\overline{u}_e &= \frac{1}{2}\left(\not{p}_e + m_e\right)\left(1+\gamma_5\not{S}_e\right)\\ \sum_{\text{spin}} u_j\overline{u}_j &= \frac{1}{2}\left(\not{p}_j + m_j\right)\left(1+\gamma_5\not{S}_j\right)\,,\end{aligned} \qquad (2.12)$$

where $\not{p} = p^\mu\gamma_\mu$ and S_e and S_j are electron and neutrino spin 4-vectors:

$$\begin{aligned}S_e &= \left(\frac{(\vec{p_e}\,\vec{s_e})}{m_e}\,,\ \vec{s_e} + \frac{(\vec{p_e}\,\vec{s_e})}{m_e(E_e+m_e)}\vec{p_e}\right)\\ S_j &= \left(\frac{(\vec{p_j}\,\vec{s_j})}{m_j}\,,\ \vec{s_j} + \frac{(\vec{p_j}\,\vec{s_j})}{m_j(E_j+m_j)}\vec{p_j}\right)\,.\end{aligned} \qquad (2.13)$$

$\vec{s_e}$ and $\vec{s_j}$ are the unit vectors in the direction of the electron and neutrino spin in their rest frames. By entering the following convenient parameters:

$$K_e = 1 - \frac{E_e}{E_e+m_e}(\vec{v_e}\vec{s_e})\,,\quad K_j = 1 - \frac{E_j}{E_j+m_j}(\vec{v_j}\vec{s_j}) \qquad (2.14)$$

and the 4-vectors:

$$A^\mu = p_e^\mu - m_e S_e^\mu, \quad B^\mu = p_j^\mu - m_j S_j^\mu\,, \tag{2.15}$$

we can rewrite $\varepsilon_\mu \varepsilon_\nu^* X^{\mu\nu}$ in the next form:

$$\begin{aligned}\varepsilon_\mu \varepsilon_\nu^* X^{\mu\nu} = 2E_e E_j \, & \left[1 - (\vec{v}_e \vec{s}_e) - K_e\,(\vec{v}_e \vec{s}_N) + \frac{m_e}{E_e}(\vec{s}_e \vec{s}_N)\right] \times \\ \times \, & \left[1 - (\vec{v}_j \vec{s}_j) - K_j\,(\vec{v}_j \vec{s}_N) + \frac{m_j}{E_j}(\vec{s}_j \vec{s}_N)\right]\end{aligned} \tag{2.16}$$

Note the following: the amplitude of the process in principle under consideration depends on five vectors: $\vec{v}_e, \vec{v}_j, \vec{s}_e, \vec{s}_j, \vec{s}_N$ from which one can form ten scalar dot-products:
$(\vec{v}_e \vec{s}_e)\,,(\vec{v}_e \vec{s}_N)\,,(\vec{s}_e \vec{s}_N)\,,(\vec{v}_j \vec{s}_j)\,,(\vec{v}_e \vec{v}_j)\,,(\vec{v}_e \vec{s}_j)\,,(\vec{v}_j \vec{s}_e)\,,(\vec{s}_e \vec{s}_j)\,,(\vec{v}_j \vec{s}_N)\,,(\vec{s}_j \vec{s}_N)$.

The first three of them do not depend on the neutrino variables, so, they are not useful for discriminating between neutrino capture and neutrino emission in β–decay The fourth — $(\vec{v}_j \vec{s}_j)$ — is not helpful because experiments on neutrino helicity measurements cannot distinguish between absorption of a left–helical neutrino and production of a right–helical (anti)neutrino. The term $(\vec{v}_j \vec{s}_j)$, however, affects the total neutrino capture rate and may also play an important role in studying Dirac vs. Majorana neutrino nature in captures of relic neutrinos.

In fact, the last two terms are included in formula (2.16), and, consequently, in the cross section (2.8).

Next , we can consider several cases:

For polarized nuclei

$$\int \frac{\mathrm{d}\Omega_e}{4\pi} \sum_{s_e} \varepsilon_\mu \varepsilon_\nu^* X^{\mu\nu} = 4E_e E_j \left\{1 - (\vec{v}_j \vec{s}_j) + \left(K_j \vec{v}_j - \frac{m_j}{E_j}\vec{s}_j\right)\vec{s}_N\right\} \tag{2.17}$$

For unpolarized nuclei

$$\int \frac{d\Omega_e}{4\pi}\frac{1}{3}\sum_{\lambda}\varepsilon_\mu(\lambda)\varepsilon^*_\nu(\lambda)X^{\mu\nu} = 2E_eE_j\left\{1-(\vec{v}_j\vec{s}_j)+\frac{1}{9}\left(1+\frac{2m_e}{E_e}\right)(\vec{v}_j\vec{s}_e)\,K_j-\frac{m_j}{E_j}(\vec{s}_e\vec{s}_j)\right\} \tag{2.18}$$

$$\int \frac{d\Omega_e}{4\pi}\frac{1}{3}\sum_{\lambda,s_e}\varepsilon_\mu(\lambda)\varepsilon^*_\nu(\lambda)X^{\mu\nu} = 4E_eE_j\left\{1-(\vec{v}_j\vec{s}_j)\right\} \tag{2.19}$$

Note that (2.18) describes anisotropies of electron emission with respect to the directions of the velocity and spin of the incoming relic neutrinos $\vec{v}_j$ and $\vec{s}_j$, which change with time in the lab frame. The electron direction anisotropy with respect to a fixed direction in this frame should therefore exhibit time variations. This in principle could be used to find out the direction of the peculiar motion of the Sun with respect to the CνB rest frame.

In order to derive the expressions for the total capture rate of the CνB neutrinos as well as for various angular correlations of interest, one has to multiply (2.8) by the neutrino velocity distribution function of the j–th neutrino mass–eigenstate $f_j(\vec{v}_j)$ and neutrino velocity v_j, integrate or sum over the relevant finite-state kinematic variables and sum the result over j.

Considering, for example, (2.19) in conjunction with (2.8), we obtain:

$$f_j(\vec{v}_j)v_j\sigma_j = \frac{G_F^2|\mathrm{V}_{ud}|^2|\mathrm{U}_{ej}|^2}{2\pi}F(Z,E_e)E_e\,p_e\,\left\{f_j(\vec{v}_j)(1-\vec{v}_j\vec{s}_j)|\mathcal{M}_{\mathrm{Nucl.}}|^2\right\}(1+\delta) \tag{2.20}$$

Recall that $\vec{s}_j$ is the *unit vector* in the direction of the neutrino spin in neutrinos rest frames. Dirac neutrinos are prepared in a definite spin state, they are left-helical, whereas both helicities are present if the neutrinos are Majorana. We will keep the calculation general for now — denote the neutrino helicity by:

$$(1-\vec{v}_j\vec{s}_j) = \begin{cases} 1-v_j, & \text{right helical} \\ 1+v_j, & \text{left helical} \end{cases}$$

Finally, we can write the final expression for capture rate:

$$\Gamma_{\mathrm{C}\nu\mathrm{B}} = \sum_{j=1}^{3}\left(\overline{\left\{f_j^{\,\mathrm{right}}(\vec{v}_j)v_j\sigma_j\right\}_{\mathrm{right}}}+\overline{\left\{f_j^{\,\mathrm{left}}(\vec{v}_j)v_j\sigma_j\right\}_{\mathrm{left}}}\right) \tag{2.21}$$

As a function of the neutrino velocity distribution, it is reasonable to take the Maxwell isotropic distribution:

$$f(\vec{v_j}) = f(v_j) = n_0 \cdot 4\pi v_j^2 \left(\frac{m_j}{2\pi T_\nu}\right)^{3/2} \exp\left(-\frac{m_j v_j^2}{2T_\nu}\right), \tag{2.22}$$

where $n_0 \approx 56\ \text{cm}^{-3}$ — the number of relic neutrinos per unit volume, $T_\nu = (4/11)^{1/3} T_\gamma \approx 0.168\ \text{meV}$ — CνB temperature.

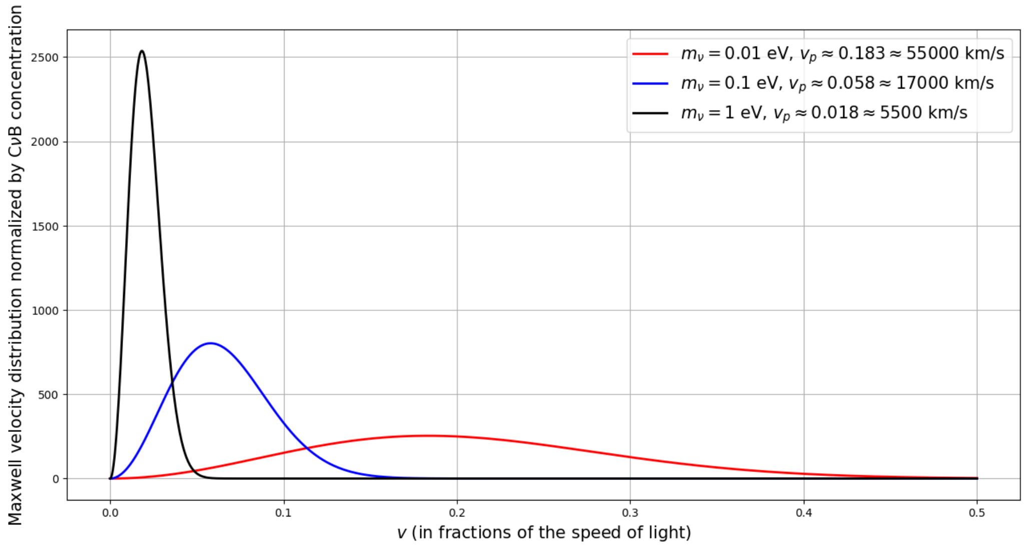

FIG. 2.2 — Maxwell velocity distribution normalized by CνB concentration, v_p — the most probable speed

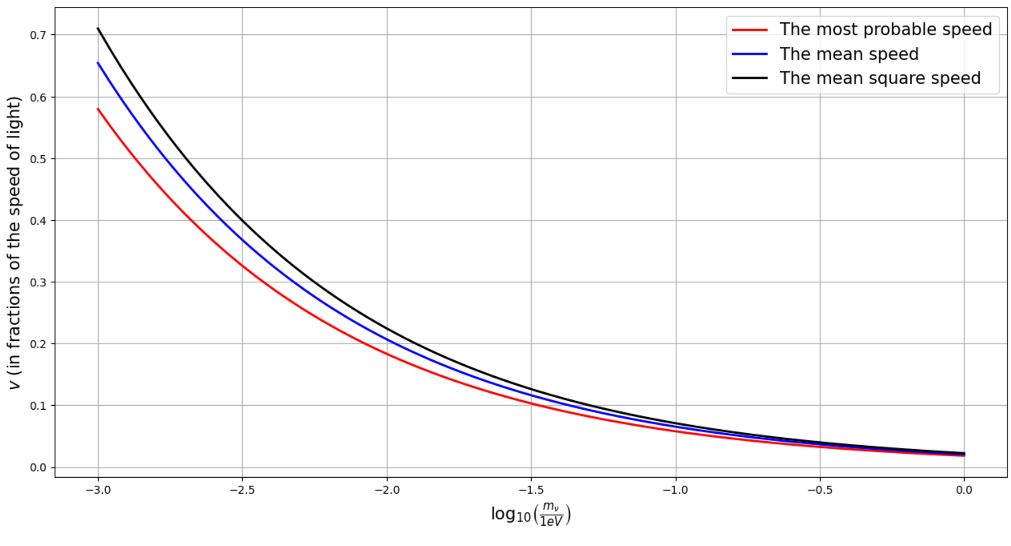

FIG. 2.3 — Characteristic neutrino velocities as a function of the neutrino mass

It follows from FIG. 2.3 that Maxwell's description of the velocity distribution function of relic neutrinos will be valid for neutrino masses of more than approximately $m_\nu \geq 3$ meV.

Dirac and Majorana neutrinos when interacting with polarized nuclei

Let us consider now CνB detection by capture on polarized targets in the lab frame. For the squared amplitude of the allowed transition one finds

$$\left\langle \frac{\varepsilon_\mu \varepsilon_\nu^* X^{\mu\nu}}{4E_e E_j} \right\rangle = \frac{1}{2E_e}\left(A^0 - (\vec{A}\cdot\vec{s}_N)\right)\left\{1 - \lambda_j v_j + \left(1 - \frac{2}{3}\lambda_j v_j - \frac{v_j^2}{3}\right)(\vec{u}\cdot\vec{s}_N)\right\} \tag{2.23}$$

where $\lambda_j = \pm 1$ with the upper (lower) sign corresponding to right-helical (left-helical) neutrinos, $\vec{u}$ — the velocity of the reference frame associated with the Earth moving relative to the CνB reference frame, A^μ is defined in the equation (2.15), $\vec{s}_N$ — spin of the nuclear.

The part of this expression relevant to our discussion is the last factor on the right hand side. As the direction of $\vec{u}$ in the lab frame changes during the day because of the rotation of the Earth, this factor gives rise to periodic variations of the signal, provided that the nuclear polarization vector $\vec{s}_N$ is not oriented along the Earth's rotation axis. The amplitude of the time variations is maximal when $\vec{s}_N$ is orthogonal to this axis. Let us now examine the effects of these time-dependent angular correlations.

Let $\mathcal{F}_j(\lambda_j)$ denote the expression in the curly brackets (2.23), so

$$\mathcal{F}_j(\lambda_j = \pm 1) = 1 \mp v_j + \left(1 \mp \frac{2}{3}v_j - \frac{1}{3}v_j^2\right)(\vec{u}\cdot\vec{s}_N) \tag{2.24}$$

For right–helical neutrino states we have to distinguish between Dirac and Majorana neutrinos. In the Dirac case, right–helical states are antineutrinos which cannot be detected in inverse β–decay processes, that is, one has to set $\mathcal{F}_j^D(\lambda_j = 1) = 0$ in that case. Assuming that CνB contains equal numbers of left–helical and right–helical neutrinos and summing over the helicities, we find

$$\mathcal{F}_j = \sum_{\lambda_j = \pm 1} \mathcal{F}_j(\lambda_j) = \begin{cases} 1 + v_j + (1 + \frac{2}{3}v_j - \frac{1}{3}v_j^2)(\vec{u}\cdot\vec{s}_N), \text{ Dirac neutrinos} \\ 2\left[1 + \left(1 - \frac{1}{3}v_j^2\right)(\vec{u}\cdot\vec{s}_N)\right], \text{ Majorana neutrinos} \end{cases} \tag{2.25}$$

In the limiting case of non-relativistic neutrinos this gives:

$$\mathcal{F}_j = \sum_{\lambda_j=\pm1} \mathcal{F}_j(\lambda_j) = \begin{cases} 1 + v_j + (1 + \frac{2}{3}\,v_j)(\vec{u}\cdot\vec{s}_N), \text{ Dirac neutrinos} \\ 2\,[1 + (\vec{u}\cdot\vec{s}_N)]\,, \text{ Majorana neutrinos} \end{cases} \tag{2.26}$$

For highly relativistic neutrinos we obtain

$$\mathcal{F}_j^D \approx \mathcal{F}_j^M \approx 2\left(1 + \frac{2}{3}(\vec{u}\cdot\vec{s}_N)\right) \tag{2.27}$$

Let us consider more closely the regime of non-relativistic neutrinos, in which the results for Dirac and Majorana neutrinos differ. Dropping in (2.26) the term proportional to $(\vec{u}\cdot\vec{s}_N)$, we recover that the detection cross section for Majorana neutrinos is about twice as large as that for Dirac neutrinos, which means that detection of relic neutrinos could in principle shed light on neutrino nature. This is, however, complicated by the fact that the local CνB density at the Earth may differ from n_0 due to gravitational clustering effects and so is not precisely known. Because the detection rate of relic neutrinos is proportional to their local density, by measuring only the absolute CνB detection rate one may not be able to determine neutrino nature unambiguously.

3. MODELING OF POSSIBLE EXPERIMENTS

3.1. NEUTRINO ABSORPTION ON THE UNPOLARIZED TRITIUM

Using the results outlined above, consider the process

$$\nu_j + {}^3\text{H} \to {}^3\text{He} + e^- \,. \tag{3.1}$$

We obtain the velocity-multiplied capture cross section for mass eigenstate j:

$$\begin{aligned} f_j(\vec{v_j})\sigma_j(s_\nu)v_j = {} & \frac{G_F^2 |\text{V}_{ud}|^2 |\text{U}_{ej}|^2}{2\pi} F(Z, E_e) E_e \, p_e \, \frac{m_{^3\text{He}}}{m_{^3\text{H}}} \times \\ & \times \left(\langle f_\text{F} \rangle^2 + \frac{g_A^2}{g_V^2} \langle \, g_\text{GT} \rangle^2 \right) (1 - \vec{v_j} \vec{s_j}) f_j(\vec{v_j}) \end{aligned} \tag{3.2}$$

here $\langle f_\text{F} \rangle^2 \approx 0.9987$, $\langle \, g_\text{GT} \rangle^2 \approx 2.788$, $g_A \approx 1.27$, $g_V \approx 1$.

Consider the kinematics of the reaction. The calculation simplifies greatly if we neglect the momentum of the incident neutrino. For typical CνB neutrinos, which have a momentum $p_j \approx 6 \cdot 10^{-4}$ eV and a mass $m_j \approx 0.1$ eV, A simple calculation gives the kinetic energy of the emitted electron to be

$$K_e^{\text{C}\nu\text{B}} = \frac{(m_{^3\text{H}} - m_e + m_j)^2 - m_{^3\text{He}}^2}{2(m_{^3\text{H}} + m_j)} \tag{3.3}$$

The background reaction is β–decay of tritium with electron endpoint energy

$$K_\text{end} = \frac{(m_{^3\text{H}} - m_e)^2 - (m_{^3\text{He}} + m_j)^2}{2m_{^3\text{H}}} \tag{3.4}$$

Displacement above this β-decay endpoint is given by

$$\Delta K = K_e^{\mathrm{C}\nu\mathrm{B}} - K_{\mathrm{end}} = \frac{(m_{^3\mathrm{H}} + m_{^3\mathrm{He}} + m_j)^2 - m_e^2}{2m_{^3\mathrm{H}}(m_{^3\mathrm{H}} + m_j)} m_j \tag{3.5}$$

It is convenient to consider separately the part of the full cross-section, which depends only on the neutrino mass, but not on chirality:

$$\bar{\sigma}(m_\nu) = \frac{G_F^2 |\mathrm{V}_{ud}|^2}{2\pi} F(Z, E_e) E_e \, p_e \, \frac{m_{^3\mathrm{He}}}{m_{^3\mathrm{H}}} \left(\langle f_{\mathrm{F}} \rangle^2 + \frac{g_A^2}{g_V^2} \langle \, g_{\mathrm{GT}} \rangle^2 \right) , \tag{3.6}$$

where $E_e = K_e^{\mathrm{C}\nu\mathrm{B}} + m_e$, $p_e = \sqrt{E_e^2 - m_e^2}$ — the total energy and momentum of the electron, which depend on the mass of the neutrino.

Figure 3.1 illustrates the dependence of this cross-section, attributed to the calculated in the work [42], on the neutrino mass.

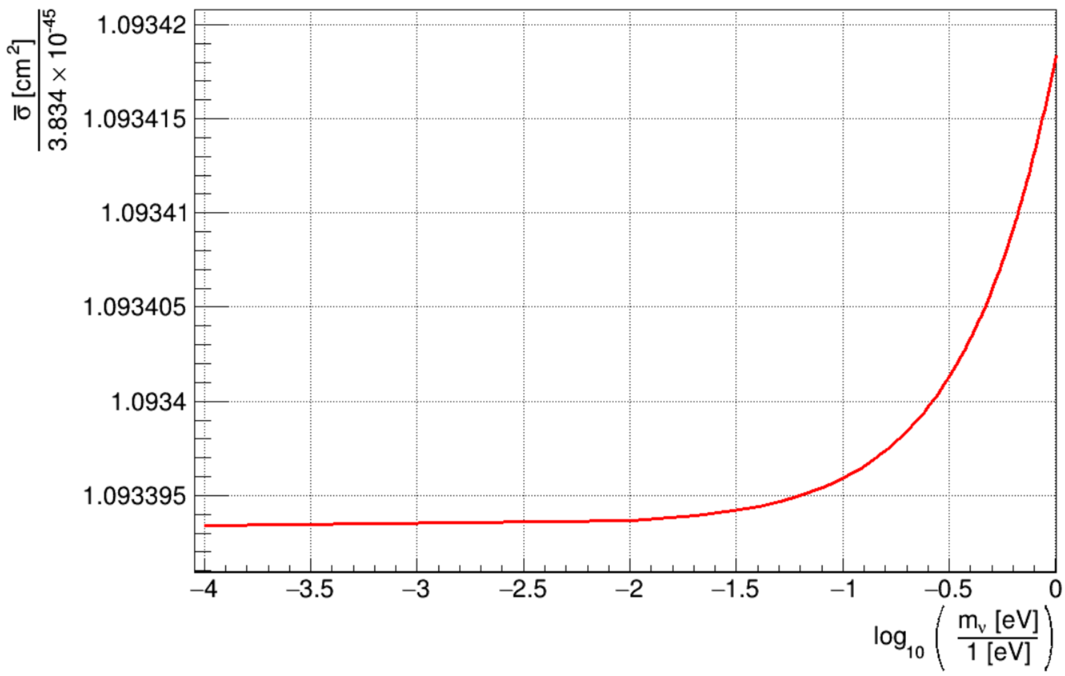

FIG. 3.1 — Dependence of the capture cross section on the neutrino mass

It is expected that this value is very weakly dependent on the neutrino mass, so

$$\overline{\sigma} \approx 1.0934 \cdot 3.834 \cdot 10^{-45} \approx 4.2 \cdot 10^{-45} \text{ cm}^2. \tag{3.7}$$

The difference between our calculation and the calculation in [42] may be due to different parameter values during the calculation. So, equation (3.2) can now be rewritten in a more compact form

$$f(v_j)\sigma_j(s_\nu)v_j = \overline{\sigma} \cdot |\mathrm{U}_{ej}|^2 \cdot (1 - \vec{v_j}\vec{s_j}) \cdot f(v_j)\,, \tag{3.8}$$

where $f(v_j)$ is described (2.22). Finally,

$$\begin{aligned}
\Gamma^{\text{left}}_{\text{C}\nu\text{B}} &= \int_0^1 \sum_{j=1,2,3} \overline{\sigma} \cdot |\mathrm{U}_{ej}|^2 \cdot (1 + v_j) \cdot f(v_j)\,\mathrm{d}v_j = \\
&= \overline{\sigma} \sum_{j=1,2,3} |\mathrm{U}_{ej}|^2 \int_0^1 (f(v_j) + v_j f(v_j))\,\mathrm{d}v_j = \\
&= \overline{\sigma} \sum_{j=1,2,3} |\mathrm{U}_{ej}|^2 (n_0 + n_0\langle v_j\rangle) = \\
&= \overline{\sigma} \cdot n_0 \sum_{j=1,2,3} |\mathrm{U}_{ej}|^2 (1 + \langle v_j\rangle)\,,
\end{aligned} \tag{3.9}$$

$$\Gamma^{\text{right}}_{\text{C}\nu\text{B}} = \overline{\sigma} \cdot n_0 \sum_{j=1,2,3} |\mathrm{U}_{ej}|^2 (1 - \langle v_j\rangle)\,. \tag{3.10}$$

Note that in this model we assume the neutrino concentration $n_0 = 53$ cm^{-3}, although, as discussed above (p. 26), this is not entirely correct.

Let's move from left and right chiral neutrinos to Dirac and Majorana.

$$\begin{aligned}
\Gamma^{\text{D}}_{\text{C}\nu\text{B}} &= \Gamma^{\text{left}}_{\text{C}\nu\text{B}} \\
\Gamma^{\text{M}}_{\text{C}\nu\text{B}} &= \Gamma^{\text{left}}_{\text{C}\nu\text{B}} + \Gamma^{\text{right}}_{\text{C}\nu\text{B}} = 2\overline{\sigma} n_0 = \text{const}
\end{aligned} \tag{3.11}$$

The calculation for $m_\nu = \{0.003, 0.01, 0.1\}$ eV gives

$$\Gamma^{\text{D}}_{\text{C}\nu\text{B}} \approx 9.29 \cdot 10^{-33}\ \text{s}^{-1} \tag{3.12}$$

$$\Gamma^{\text{M}}_{\text{C}\nu\text{B}} \approx 14.1 \cdot 10^{-33}\ \text{s}^{-1} \tag{3.13}$$

From an experimental point of view, we are interested in the number of events in 1 year per 1 mass of tritium (for example, per 100 grams), so

$$\Gamma^{\mathrm{D}}_{\mathrm{C}\nu\mathrm{B}} \approx 9.29 \cdot 10^{-33}\ \mathrm{s}^{-1} \cdot \left(\frac{M_{^3\mathrm{H}}}{\mu_{^3\mathrm{H}}}\right)[\mathrm{mole}] \cdot N_A[\mathrm{mole}^{-1}] \cdot T[\mathrm{s\ year}^{-1}] \tag{3.14}$$

$$\Gamma^{\mathrm{M}}_{\mathrm{C}\nu\mathrm{B}} \approx 14.1 \cdot 10^{-33}\ \mathrm{s}^{-1} \cdot \left(\frac{M_{^3\mathrm{H}}}{\mu_{^3\mathrm{H}}}\right)[\mathrm{mole}] \cdot N_A[\mathrm{mole}^{-1}] \cdot T[\mathrm{s\ year}^{-1}], \tag{3.15}$$

where $M_{^3\mathrm{H}}$ — tritium mass [g], $N_A \approx 6.02 \cdot 10^{23}\,[\,\mathrm{mole}^{-1}]$ — Avogadro's number, $\mu_{^3\mathrm{H}} \approx 3.01605\,[\mathrm{g/mole}]$ — molar mass of tritium, $T = 3.1536 \cdot 10^7$ s.

Figure (3.2) shows a graph of the number of events of relic neutrinos as a function of the the neutrino mass.

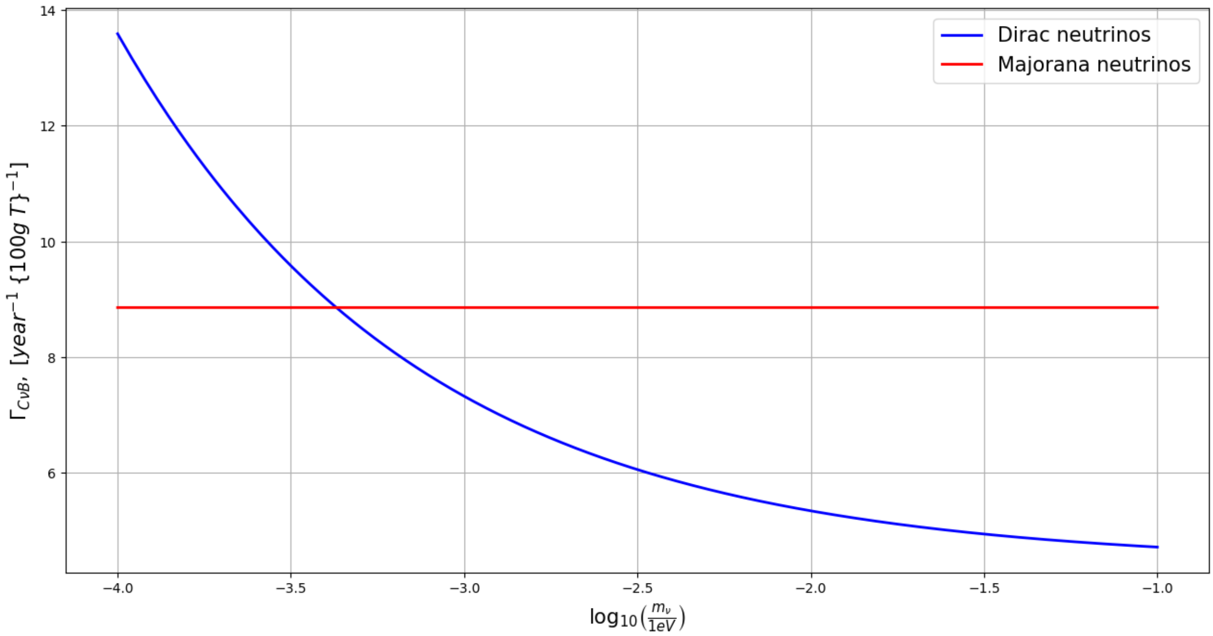

FIG. 3.2 — The number of events CνB as a function of the neutrino mass

Suppose that the detector has a Gaussian response function:

$$\rho(E, \varepsilon, \sigma) = \frac{1}{\sqrt{2\pi}\sigma(E)} \exp\left(-\frac{(E-\varepsilon)^2}{2\sigma^2(E)}\right), \tag{3.16}$$

where $\sigma(E)$ — energy resolution, E — true energy, ε — the energy registered in the detector.

So, the numbers of events is given by:

$$\begin{aligned}
\frac{\mathrm{d}N_\beta}{\mathrm{d}\varepsilon_e} &= \int\limits_{-\infty}^{\infty} \frac{\mathrm{d}\Gamma_\beta}{\mathrm{d}E_e}\rho(E_e,\varepsilon_e,\sigma)\,\mathrm{d}E_e \\
\frac{\mathrm{d}N_{\mathrm{C}\nu\mathrm{B}}}{\mathrm{d}\varepsilon_e} &= \int\limits_{-\infty}^{\infty} \Gamma_{\mathrm{C}\nu\mathrm{B}}(E_e)\,\delta(E_e - K_e^{\mathrm{C}\nu\mathrm{B}} - m_e)\,\rho(E_e,\varepsilon_e,\sigma)\,\mathrm{d}E_e
\end{aligned} \tag{3.17}$$

Let's discuss the normalization of spectra. Let at the beginning of the experiment the detector contains $M_{^3\mathrm{H}}$ [g] tritium, i.e. $N_{^3\mathrm{H}} = N_A(M_{^3\mathrm{H}}/\mu_{^3\mathrm{H}})$ particles. During the measurement of T [year] , according to the law of radioactive decay, it will decay:

$$N_{^3\mathrm{H}}(T, M_{^3\mathrm{H}}) = N_A\frac{M_{^3\mathrm{H}}}{\mu_{^3\mathrm{H}}}\left(1 - 2^{T/T_{1/2}}\right), \tag{3.18}$$

where $T_{1/2} \approx 12.32\,[\text{year}]$ — tritium half-life. Accordingly, exactly as many events will be registered if the detector covers a solid angle 4π and registers all events, so, $N_\beta = N_{^3\mathrm{H}}(T, M_{^3\mathrm{H}})$.

Thus, the equations (3.17) can be presented in the following convenient form:

$$\begin{aligned}
\frac{\mathrm{d}N_\beta}{\mathrm{d}\varepsilon_e} &= N_{^3\mathrm{H}}(T, M_{^3\mathrm{H}}) \int\limits_{-\infty}^{\infty} \frac{\mathrm{d}\omega_\beta}{\mathrm{d}E_e}\rho(E_e,\varepsilon_e,\sigma)\,\mathrm{d}E_e\,, \\
\frac{\mathrm{d}N_{\mathrm{C}\nu\mathrm{B}}}{\mathrm{d}\varepsilon_e} &= \frac{N_{\mathrm{C}\nu\mathrm{B}}(T, M_{^3\mathrm{H}})}{\sqrt{2\pi}\sigma(E)}\exp\left(-\frac{(K_e^{\mathrm{C}\nu\mathrm{B}} - \varepsilon)^2}{2\sigma^2(E)}\right),
\end{aligned} \tag{3.19}$$

where $\frac{\mathrm{d}\omega_\beta}{\mathrm{d}E_e}$ — the probability-normalized beta spectrum.

In fact, we are not interested in the entire spectral range of electron energies $[0; K_e^{\mathrm{C}\nu\mathrm{B}}]$, but only in a small neighborhood near the endpoint energy $[K_e^{\mathrm{C}\nu\mathrm{B}} - 5\sigma\,; K_e^{\mathrm{C}\nu\mathrm{B}} + 5\sigma]$. For the Gaussian distribution of CνB events, we can assume that all events fall into this segment. The number of events from tritium beta-decay can be estimated as:

$$\widetilde{N}_\beta(\sigma) = \int\limits_{K_e^{\mathrm{C}\nu\mathrm{B}}-5\sigma}^{K_e^{\mathrm{C}\nu\mathrm{B}}+5\sigma} \frac{\mathrm{d}N_\beta}{\mathrm{d}\varepsilon_e}\mathrm{d}\varepsilon_e\,, \quad \widetilde{N}_{\mathrm{C}\nu\mathrm{B}}(\sigma) = \int\limits_{K_e^{\mathrm{C}\nu\mathrm{B}}-5\sigma}^{K_e^{\mathrm{C}\nu\mathrm{B}}+5\sigma} \frac{\mathrm{d}N_{\mathrm{C}\nu\mathrm{B}}}{\mathrm{d}\varepsilon_e}\mathrm{d}\varepsilon_e\,. \tag{3.20}$$

In this model we will always assume $\sigma(E) > m_\nu$, so it will not be possible to distinguish the mass states of neutrinos among themselves. The resolution of the detector will be identified with the width of the energy bin of Monte–Carlo histograms.

The most important value is the ratio of the number of events:

$$r_{\mathrm{SN}}(\sigma) = \frac{\widetilde{N}_{\mathrm{C}\nu\mathrm{B}}(\sigma)}{\widetilde{N}_{\beta}(\sigma)}. \tag{3.21}$$

If $r_{\mathrm{SN}}(\sigma) \gg 1$, we reliably register relic neutrinos, if $r_{\mathrm{SN}}(\sigma) \ll 1$, C$\nu$B events are indistinguishable from background tritium beta decay events. The dependence of this value on the neutrino mass was obtained in the graph is shown in the figure 3.3.

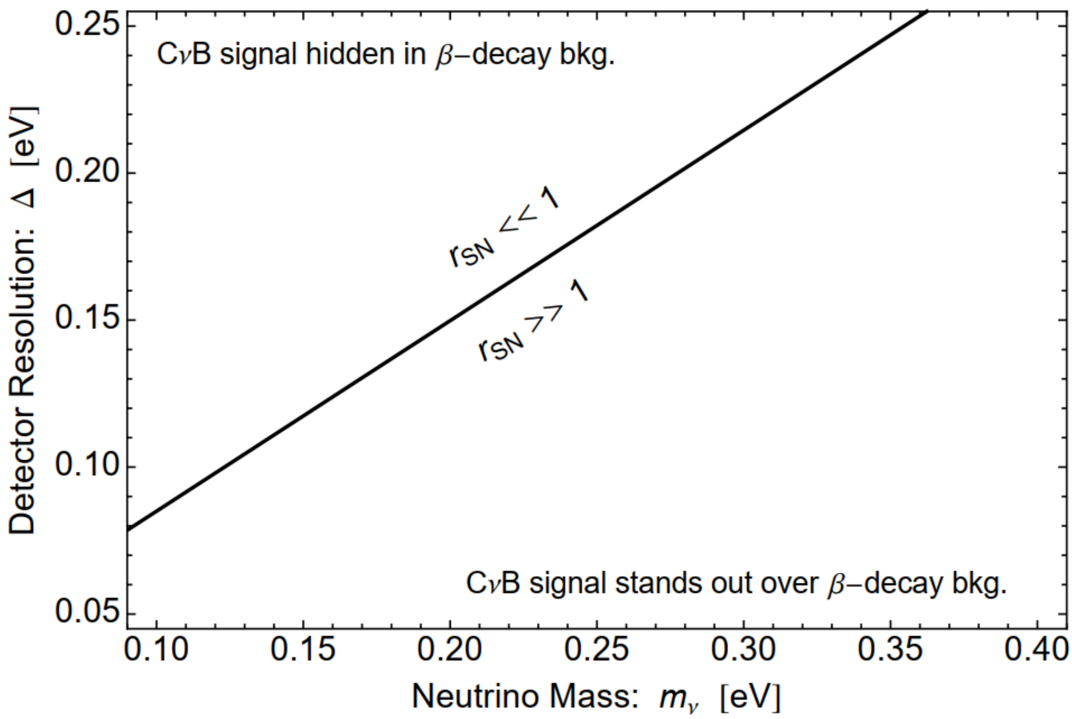

FIG. 3.3 — Dependence of the signal ratio on the neutrino mass [42]

This figure shows a contour plot of signal to noise ratio, r_{SN}, for a range of detector resolutions, $\Delta \approx 2.35 \cdot \sigma$, and neutrino masses, m_ν, for Majorana neutrinos with a degenerate neutrino mass spectrum. In the region below the $r_{\mathrm{SN}} = 1$ line, the CνB signal stands out over the beta decay background, and in the region above this line, the background events dominate. For Majorana neutrinos, $r_{\mathrm{SN}} \geq 1$ corresponds to $\Delta \leq 0.7\, m_\nu$.

If $r_{\rm SN}$ is large then the systematic error arising from the beta decay endpoint is negligible, and (in the absence of other systematic errors) the limiting factor is statistics. If N events are detected, then the counting error is expected to go like $\sqrt{N}$, and the statistical significance can be estimated as $N/\sqrt{N} = \sqrt{N}$. A 3σ detection requires $N \approx 9$ events in the signal region, and a 5σ detection requires $N \approx 25$ events. With an event rate of $\Gamma^{\rm M}_{{\rm C}\nu{\rm B}} \approx 8 - 9\,[{\rm year}^{-1}]$ these significances would require approximately 1 and 3 years of data taking, respectively [42].

CONCLUSION

The purpose of this paper was to show the potential for the registration of relic neutrinos in the neutrino capture reaction on beta-decaying nuclei.

In the first half, a theoretical description of neutrino oscillations and mass mixing is given; an essay is written on various mechanisms of neutrino mass generation; the question of the role of neutrinos in primary nucleosynthesis in the first minutes of the early Universe is touched upon. . In the second, substantial part, the calculation of neutrino capture cross sections by beta-decay nuclei is given, an example of a possible tritium experiment is considered, restrictions on the energy resolution of the detector in the simplest model are presented without taking into account background events (solar neutrinos, atmospheric neutrinos, etc.) and the approximation that the density of neutrinos of the same grade corresponds to $n_0 \approx 56$ cm^{-3} on the Earth.

In the future, it is planned to study the issue related to the registration of relic neutrinos due to modulations of the number of events over time due to the effects of the motion of the laboratory reference frame associated with the Earth relative to the CνB system, which was already mentioned in section 2.

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