Constraints on particles from Big Bang model

Lecture from the course

« Introduction to cosmoparticle physics »

Stable particles from Big Bang

- Stable particles should be created in very early Universe and remain at successive stages
- If theory predicts new stable particles, they should obey this rule.
- Their presence in the Universe should not contradict its observed properties

Cosmochronology

Very early Universe

T~1 MeV, t~1 s

n/p freezing out,
ν-decoupling

T~0.1 MeV, t~100 s

Nucleosynthesis

T~1 eV, t~30000 yr

RD->MD

T~3000 K,
t~400000 yr

Recombination

t~1 Gyr

Galaxy
formation

t~3 Gyr

Λ-dominance

t~10 Gyr

Modern
epoch

Constraint on number of neutrino species

Constraint on the mass of neutrino

Relic neutrinos

Primordial neutrinos are inevitable relic of hot stages with T>1MeV. Their modern concentration is related with the concentration of relic photons

$$n_{\nu\bar{\nu}}^{(\mathrm{mod})} = \frac{4}{11}\frac{T_{\mathrm{mod}}^3}{T_*^3} n_{\nu\bar{\nu}}^{(*)} = \frac{4}{11}\frac{T_{\mathrm{mod}}^3}{T_*^3}\frac{3}{4} n_{\gamma}^{(*)} = \frac{3}{11} n_{\gamma}^{(\mathrm{mod})}$$

for one neutrino species

$$\boxed{n_{\nu\bar{\nu}}^{(\mathrm{mod})} = \frac{3}{11} n_{\gamma}^{(\mathrm{mod})} \approx 110\,\mathrm{cm}^{-3}}$$

If these neutrinos have nonzero mass one can take multiply their mass by their concentration and compare the predicted value with the total density.

Density of massive neutrinos

Contribution of massive neutrinos into the total density should in no case exceed this density. For massive (non-relativistic) neutrinos it leads to constraint on neutrino mass

$$\Omega_\nu = \frac{\sum\limits_{\text{types of }\nu} m_\nu n_{\nu\bar{\nu}}^{(\text{mod})}}{\varepsilon_{\text{cr}}} < \Omega_{\text{tot}}$$

$$\boxed{\sum_{\text{types of }\nu} m_\nu < 50\,\text{eV}\cdot\Omega_{\text{tot}}}$$

Constraints on neutrino mass

In 1970th, an upper limit on the total density was set from the estimation of age of Universe by the method of Nuclear Cosmochronology. It is based on analysis of nuclear isotope compositions. It gave *t*>5 Gyr, whence

$$\Omega_{\text{tot}} < 2 \qquad \sum_{\text{types of }\nu} m_\nu < 100\,\text{eV}$$

The modern estimation of total density give

$$\Omega_{\text{tot}} \approx 1 \qquad \sum_{\text{types of }\nu} m_\nu < 50\,\text{eV}$$

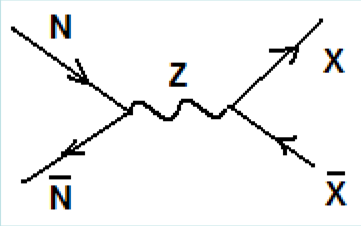
Theory of formation of Large Scale Structure (galaxies) of Universe allows to restrict contribution into density of neutrinos along with all the Hot Dark Matter (HDM), to be discussed in future, as

$$\Omega_\nu \le \Omega_{\text{HDM}} < 0.015 \qquad \sum_{\text{types of }\nu} m_\nu < 0.8\,\text{eV}$$

Heavy neutrinos

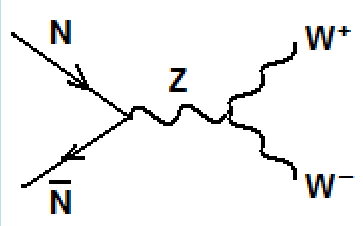
If heavy neutrinos (with mass *m*) existed, they might be in equilibrium in early Universe. At *T<m* their equilibrium number density would go down due to annihilation process.

m must be >~1 MeV in order Heavy neutrinos had time to become non-relativistic before they decoupled



$$\sigma_{N\bar{N}}\upsilon \approx \frac{\sum_{channels} Z_X}{4\pi} \frac{G_F^2 m_Z^4 m^2}{(4m^2 - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \sim \begin{cases} \frac{m^2}{m_Z^4}, & m << m_Z/2 \\ \frac{1}{m^2}, & m >> m_Z/2 \end{cases}$$

In non-relativistic limit



$$\sigma_{N\bar{N}}\upsilon \approx \frac{G_F^2 m^2}{8\pi} \sim \frac{m^2}{m_Z^4}, \quad m >> m_W$$

At *m*~200 GeV perturbative approach becomes invalid (since Yukawa couplings to the Higgs filed, defining masses of both *N* and *W* participating in the process of question, become >1).

At *m*~2 TeV **unitarity limit** is reached, which gives

$$\sigma_{N\bar{N}}\upsilon \approx \frac{4\sqrt{\pi m / T_*}}{m^2} \sim \frac{1}{m^2}$$

Freezing out of Heavy neutrinos

Neutrinos are frozen when: $n_N \sigma_{N\bar{N}} \upsilon = H$ $\Rightarrow$ ***T_*~10^{-1}m*.**

$$\Downarrow$$

$$n_N^* = \frac{H}{\sigma_{N\bar{N}} \upsilon}$$

Note, frozen out density is **inverse** proportional to annihilation cross section!

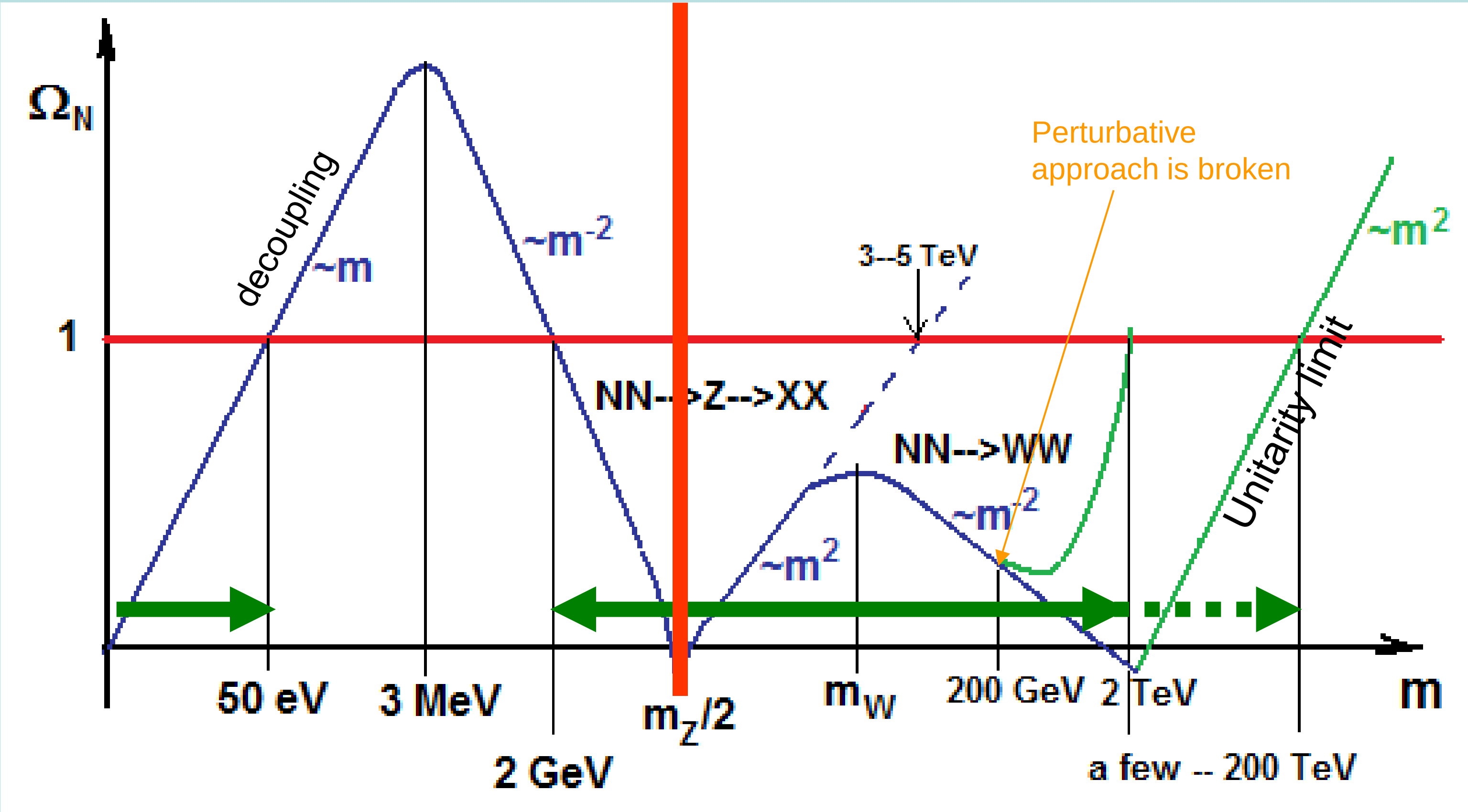
$$n_N^{(\mathrm{mod})} = n_N^* \frac{a_*^3}{a_{\mathrm{mod}}^3} \sim \frac{H}{\sigma_{N\bar{N}} \upsilon} \frac{T_{\mathrm{mod}}^3}{T_*^3} \sim \frac{1}{\sigma_{N\bar{N}} \upsilon} \frac{T_*^2}{m_{\mathrm{Pl}}} \frac{T_{\mathrm{mod}}^3}{T_*^3} \sim \frac{T_{\mathrm{mod}}^3}{m_{\mathrm{Pl}} \cdot \sigma_{N\bar{N}} \upsilon \cdot T_*} \sim \frac{n_\gamma^{(\mathrm{mod})}}{m_{\mathrm{Pl}} \cdot \sigma_{N\bar{N}} \upsilon \cdot m}$$

$$\boxed{r_N^{(\mathrm{mod})} \equiv \frac{n_N^{(\mathrm{mod})}}{n_\gamma^{(\mathrm{mod})}} \sim \frac{1}{m_{\mathrm{Pl}} \cdot \sigma_{N\bar{N}} \upsilon \cdot m} \sim \begin{cases} \dfrac{m_Z^4}{m_{\mathrm{Pl}} m^3}, & m << \dfrac{m_Z}{2} \\ \dfrac{m}{m_{\mathrm{Pl}}}, & \dfrac{m_Z}{2} << m < m_W \\ \dfrac{m_Z^4}{m_{\mathrm{Pl}} m^3}, & m >> \dfrac{m_Z}{2} \end{cases}}$$

$$\Omega_N \propto m r_\gamma^{(\mathrm{mod})} \sim \frac{1}{m_{\mathrm{Pl}} \cdot \sigma_{N\bar{N}} \upsilon}$$

Constraint on the mass of neutrino

$$\Omega_N \sim \begin{cases} \dfrac{m_Z^4}{m_{Pl} m^2}, & m << \dfrac{m_Z}{2} \\ \dfrac{m^2}{m_{Pl}}, & \dfrac{m_Z}{2} << m < m_W \\ \dfrac{m_Z^4}{m_{Pl} m^2}, & m >> m_W \end{cases}$$



Thus **2 GeV < *m* < a few÷200 TeV** is cosmologically allowable.

However ***m* < 45 GeV** is forbidden by accelerator data.

Freezing out of n/p ratio

The ratio between the numbers of neutrons and protons in early Universe was regulated by reactions

$$\tilde{\nu}_e + p \leftrightarrow e^+ + n$$

$$\nu_e + n \leftrightarrow e^- + p$$

which were frozen out together with other weak interaction processes at

$$T_* \sim \frac{\kappa_\varepsilon^{1/6}}{(G_F^2 m_{Pl})^{1/3}} \approx 1\,\mathrm{MeV}$$

This corresponds to

$t_* \sim 1$ s

If more relativistic species were present, freezing out of n/p took place at smaller t and larger T.

Primordial helium abundance

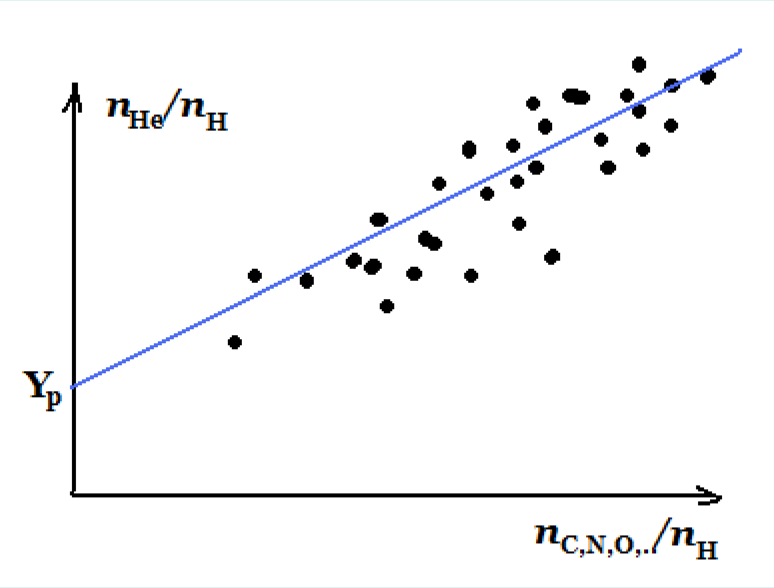
All reactions of Big Bang Nucleosynthesis took place at t~100 s, in the result of which a primordial chemical composition has been established, in which virtually all frozen out neutrons entered primordial helium-4:

$$\frac{\rho_{\mathrm{He}}}{\rho_{\mathrm{B}}} \equiv Y_p \approx \frac{2(n/p)}{1+n/p} \approx 0.25$$

This abundance should be compared with observed abundance of He-4. However, we don't observe primordial helium. The modern helium abundance also includes products of nuclear burning in stars. Special analysis of observations is needed to deduce the primordial component of helium.

Y_p from observation

The observed Helium is of both primordial and secondary (star) origin. To define primordial amount (Y_p), to be compared with prediction of BBN theory, an observed dependence of helium amount from amount of secondary elements (O, C, N,… - called *metalicity*) is extrapolated to the zero.



$$\boxed{Y_p^{(obs)} = 0.249 \pm 0.004 \pm 0.018}$$

Number of neutrino species

Number of ultrarelativistic species affects the number of neutrons and, correspondingly, predicted primordial Helium abundance (*Y_p*):

$$\frac{n_n}{n_p} = \exp\left(-\frac{\Delta m (G_F^2 m_{Pl})^{1/3}}{\kappa_\varepsilon^{1/6}} \right)$$

The larger number of neutrinos N_ν, the larger κ_ε is, the larger n/p is, the larger Y_p.

From comparison with the Y_p deduced from observation one gets

$$\boxed{2.0 < N_\nu < 4.5}$$

If to take into account only statistical errors, then $2.8 < N_\nu < 3.5$.

Constraint on mirror world

Data on primordial Helium-4 restricts number of any possible unknown species of particles. Hypothesis on mirror world existence implies doubling all known particle species. As a consequence for primordial Helium amount (*Y_p*) we should have:

$$Y_{\mathrm{p}}^{(\mathrm{mirror})} \approx 0.28$$

However, it is forbidden by observational data.

Hypothesis on mirror world to be viable should be modified **=> Shadow word**

Problems of very Hot Universe

Gravitino

In models mSUGRA, gravitino has typically the following properties:

is Majorana fermion with spin 3/2

$$m_{\tilde{G}} \sim 100 \div 1000\,\mathrm{GeV}$$

Interaction amplitude $\sim G = m_{\mathrm{Pl}}^{-1}$.

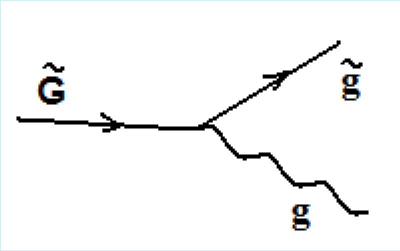
As a consequence, annihilation cross section is

$$\sigma_{ann} v \sim \frac{1}{m_{Pl}^2}$$

and, if gravitino is unstable, its lifetime

$$\tau_{\tilde{G}} \sim \frac{m_{\mathrm{Pl}}^2}{m_{\tilde{G}}^3} \sim \mathrm{yr}\left(\frac{100\ \mathrm{GeV}}{m_{\tilde{G}}}\right)^3$$

That is gravitino is long-lived particle.



Possible decay mode

Constraint on relic gravitinos

Processes of gravitational interactions should be frozen out soon after Plankian temperature $T=T_*<m_{Pl}$. Given so, (relativistic) gravitinos are decoupled from plasma at $T=T_*$. Standard estimation of modern relic density of gravitinos (see the part about light neutrinos) formally gives:

$$\Omega_{\tilde{G}}^{(\mathrm{mod})} = \frac{m_{\tilde{G}} n_{\tilde{G}}^{(\mathrm{mod})}}{\varepsilon_{\mathrm{cr}}^{(\mathrm{mod})}} = r_{\tilde{G}*} \cdot \frac{\kappa_s^{(\mathrm{mod})}}{\kappa_s^*} \cdot \frac{m_{\tilde{G}} n_\gamma^{(\mathrm{mod})}}{\varepsilon_{\mathrm{cr}}^{(\mathrm{mod})}} \sim 10^8 \frac{m_{\tilde{G}}}{100\,\mathrm{GeV}}$$

Comparison with observational data put strong constraint

$$\boxed{m_{\tilde{G}} < 1\,\mathrm{keV}}$$

which strongly disfavours minimal framework of SUGRA model.

Nonthermal relic gravitinos

Another way to reach agreement between mSUGRA and cosmological data is to assume that no period of $T \sim m_{Pl}$ took place in our Universe.

Let us suppose that initial temperature of primordial plasma had been equal to

$$T_R \ll m_{Pl}$$

In this case, thermal production of gravitinos in plasma (in collisions of particles of view $i + j \to \tilde{G} + X$) become suppressed (due to very small interaction constant), but not vanishing.

Let us estimate production rate. For it we have in the comoving volume V

$$\dot{N}_{\tilde{G}} = n_i n_j \left\langle \sigma_{ij \to \tilde{G}X} v_{ij} \right\rangle V$$

The volume changes as $V = N_\gamma / n_\gamma$

For other we have $n_{i,j,\gamma} \sim T^3 \qquad dt \sim m_{\mathrm{Pl}} dT / T^3$

Constraint on Very Hot Universe

For modern moment we obtain

$$\frac{n_{\tilde{G}}}{n_\gamma} = \int\limits_{t(T=T_\mathrm{R})}^{t(T\sim m_G)} \frac{\dot{N}_{\tilde{G}}}{N_\gamma} dt \sim \int\limits_{T=m_{\tilde{G}}}^{T_\mathrm{R}} T^6 m_\mathrm{Pl}^{-2} T^{-3} \frac{m_\mathrm{Pl}\, dT}{T^3} = \frac{T_\mathrm{R}}{m_\mathrm{Pl}}$$

$$\boxed{\Omega_{\tilde{G}} = \frac{m_{\tilde{G}} n_{\tilde{G}}}{\varepsilon_\mathrm{cr}} \sim \frac{m_{\tilde{G}} n_\gamma}{\varepsilon_\mathrm{cr}} \frac{T_\mathrm{R}}{m_\mathrm{Pl}}}$$

If gravitino is stable, then to satisfy data on modern density we get constraint on parameter of cosmological model T_R (for m_G~100 GeV)

$$\boxed{T_\mathrm{R} < 10^{9\div 10}\ \mathrm{GeV}}$$

Magnetic monopoles in GUTs

Dirac suggested an existence of magnetic monopole with magnetic charge

$$g = (2e)^{-1}$$

as condition of quantization of electric charge.

T'Hooft and Polyakov have shown, that in GUT models, where $U(1)_{e/m}$ symmetry is included to SU(3) or wider symmetry, magnetic monopole must appear in the result of spontaneous breaking of GUT symmetry as a topological defect of respective Higgs' field. The mass of monopole are predicted to be

$$m_M \sim \Lambda_{\mathrm{GUT}} / \sqrt{\alpha} \qquad (\Lambda_{\mathrm{GUT}} \sim 10^{15}\ \mathrm{GeV})$$

Magnetic monopoles in SO(3)

Let $U(1)_{e/m}$ is included in SO(3) in GUT model. The Higgs' field, violating $SO(3) \rightarrow U(1)_{e/m}$, can have SO(3) triplet form

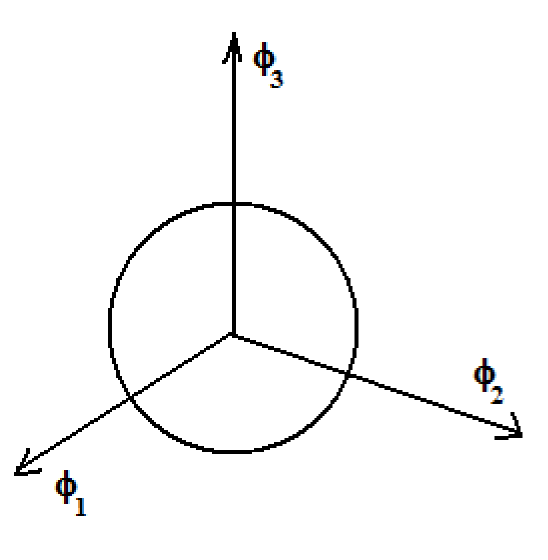
$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$$

Let Higgs' field potential have a view $f(|\phi|^2)$ with a minimum at

$$\phi^2 = \upsilon^2 = \phi_1^2 + \phi_2^2 + \phi_3^2$$

Formation of magnetic monopoles

In the isotopic space of Higgs' field, the minimum of potential corresponds to sphere. At the sphere, Higgs' field can be defined by angles θ and φ.



After phase transition (violation of SO(3) symmetry), at $T<T_{cr}\sim\upsilon$, Higgs' field acquires vacuum expectation value. In all (coordinate) space ϕ gets the value υ and different magnitudes of θ and φ, which vary within the length scale

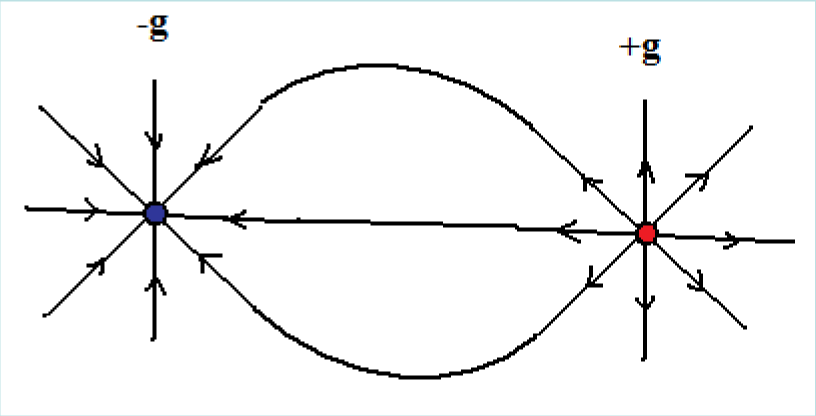
$$\Delta l \sim \frac{1}{e\upsilon}$$

However, it is not possible for θ and φ to vary continuously over 2π and not to get a singularity – the point where θ and φ are indefinite (like a pole on globe, "one cannot brush a hedgehog").

Magnetic monopole pairs

Such a singularity is topological defect – monopole. Its size is defined by **Δl**. In its center **ϕ=0,** it corresponds to non-zero energy density of ϕ (outside of minimum) – mass, pointed previously.

Gradients of **$\theta(x)$** and **$\varphi(x)$**, issuing from singularity, define intensity of magnetic filed. This is accounted for by the fact that the field ϕ is connected with gauging of electromagnetic field. Singularity, where gradients of **$\theta(x)$** and **$\varphi(x)$** come to, corresponds to an antimonopole.



Catalysis of proton decay

Inside monopole, GUT symmetry is reproduced (ϕ has not v, violating symmetry).

As a consequence, baryon and lepton numbers are not conserved, as soon as GUT does violate them.

Effect of Callan-Rubakov is predicted:

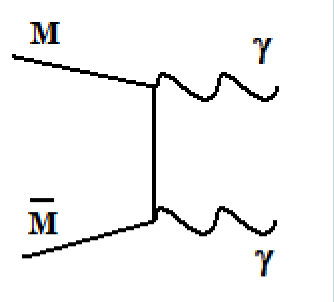
Monopole can induce (catalyze) proton decay

$$p + M \rightarrow e^{+} + e^{-} + e^{+} + M$$

Density of relic magnetic monopoles

If one formally supposes usual freezing out of magnetic monopoles, then, taking cross section of direct annihilation by analogy with e^+e^- annihilation

$$\left\langle \sigma_{ann} \upsilon \right\rangle \sim (gg/m_M)^2 \sim \frac{\alpha^{-2}}{\Lambda_{GUT}^2}$$



one would obtain (see the part about Heavy neutrino)

$$\Omega_M = \frac{m_M r_M^{(\mathrm{mod})} n_\gamma^{(\mathrm{mod})}}{\varepsilon_{cr}} \sim \frac{n_\gamma^{(\mathrm{mod})}}{\varepsilon_{cr}} \frac{1}{m_{\mathrm{Pl}} \cdot \sigma_{ann} \upsilon} \sim 10^{14} \left(\frac{\Lambda_{GUT}}{10^{15}\, GeV} \right)^2$$

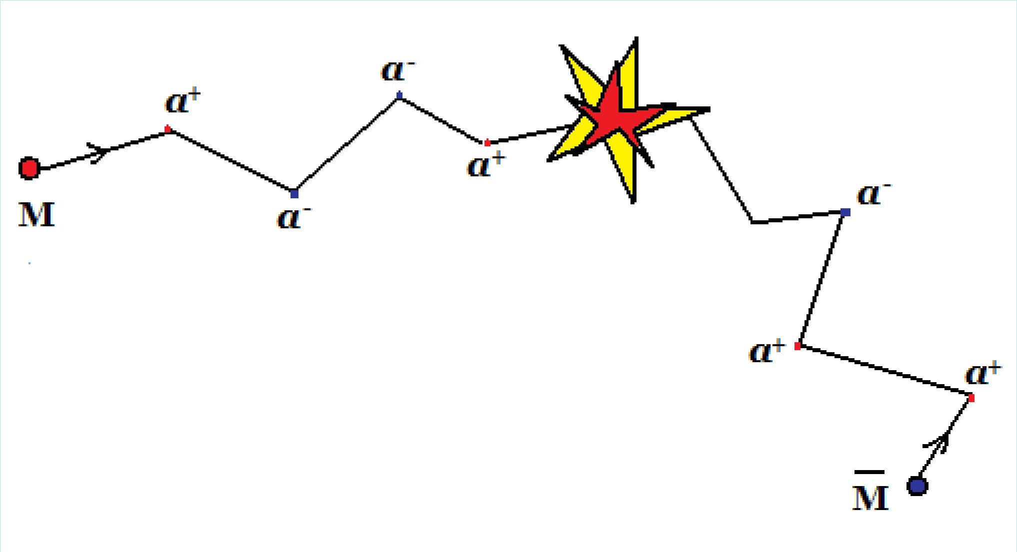
It strongly contradicts to modern data.

Magnetic monopoles in hot plasma

However, for standard picture of particles freezing out in early Universe to be applicable to magnetic monopoles, one needs to have 1) thermal production (corresponding to thermodynamic equilibrium) of monopoles preceding to freezing out, 2) approximation of free monopole annihilation justified.

The first condition does not realize, since monopoles are porduced as result of phase transition (not thermally). But their successive annihilation quickly lead to saturation of thermodynamic number density.

The second condition is not satisfied at all. In fact, magnetic monopole should experience multiple scattering in plasma. Their motion has diffusive character.



Diffusion of magnetic monopoles

Two charge particles (with magnetic charge *g*) feel each other in plasma at distance when

$$\frac{g^2}{r} \sim E \sim T \quad \Rightarrow \quad r \sim r_0 \equiv \frac{g^2}{T}$$

Diffusive approximation is proved by condition

$$\lambda \ll r_0$$

where $\lambda \sim \frac{1}{n_a \sigma} \sim \frac{1}{T^3} \frac{Tm}{(ge)^2} \sim \frac{m}{T^2 (ge)^2}$ is the scattering length.

In this case annihilation rate is defined as

$$\frac{dn_M}{dt} = -n_M^2 \, 4\pi D r_0$$

where $D = \frac{1}{3}\lambda v$ is diffusion coefficient.

Magnetic monopole overproduction

Finally, for monopole relic density one finds

$$\Omega_M \sim m \frac{m}{g^5 (eg) m_{Pl}} \frac{n_\gamma^{(\mathrm{mod})}}{\varepsilon_{cr}} \sim 10^{15} \left(\frac{\Lambda_{GUT}}{10^{15} GeV} \right)^2$$

That is conclusion does not change in principle, the problem of overproduction of magnetic monopoles remains. In fact, diffusion slows down annihilation rate with respect to direct annihilation (in approximation of free monopole motion) and more monopoles should survive.

This problem either excludes magnetic monopole with given properties, or implies completely different conditions in very early Universe.

Conclusions

- Upper limit on the modern cosmological density puts constraints on neutrino mass both for known light neutrinos and hypothetical heavy stable leptons, if they survive to the present time.
- Primordial helium abundance puts restrictions on number of neutrino species and everything present in the Universe in the 1 s (mirror world, in particular)
- Problem of relic gravitino causes doubts in existence of planckean temperatures in early Universe.
- Magnetic Monopole overproduction became a real problem for very Hot Early Universe. GUT physics became incompatible with GUT temperatures.