Ур-ие состояния однородного массивного скалярного поля

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$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)(\partial^\mu \varphi) - \frac{1}{2}m^2\varphi^2, \quad \varphi = \varphi(x,t) \in \mathbb{R}^4, \quad \eta^{\mu\nu} = diag(1,-1,-1,-1)$$

$$(\partial_\mu \partial^\mu + m^2)\varphi(x,t) = 0$$

Поле φ однородно $\Rightarrow \varphi = \varphi(t)$; Тогда

$$\ddot{\varphi} + m^2\varphi = 0 \Rightarrow \varphi(t) = A\cos(mt+\alpha)$$

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial \partial_\mu \varphi}\partial^\nu \varphi - \eta^{\mu\nu}\mathcal{L} = (\partial^\mu\varphi)(\partial^\nu\varphi) - \frac{1}{2}\eta^{\mu\nu}(\partial_\rho\varphi)(\partial^\rho\varphi) + \frac{1}{2}\eta^{\mu\nu}m^2\varphi^2$$

$$\mathcal{E} = T^{00} = (\partial^0\varphi)^2 - 1/2\left((\partial_0\varphi)^2 + (\underbrace{\cancel{\vec{\partial}\varphi}}_{0})^2\right) + 1/2\, m^2\varphi^2 = A^2m^2\sin^2(mt+\alpha) - 1/2\, A^2m^2\sin^2(mt+\alpha) + 1/2\, A^2m^2\cos^2(mt+\alpha)$$

$$T^{00} = 1/2\, A^2m^2$$

$$p^i = T^{0i} = T^{i0} = (\partial^0\varphi)(\overset{0}{\cancel{\partial^i\varphi}}) - 1/2\,\overset{0}{\cancel{\eta^{0i}}}(\partial_\rho\varphi)(\partial^\rho\varphi) + 1/2\,\overset{0}{\cancel{\eta^{0i}}}m^2\varphi^2 = 0$$

$$T^{0i} = T^{i0} = 0$$

$$T^{ii} = (\overset{0}{\cancel{\partial^i\varphi)(\partial^i\varphi}}) - 1/2\,\eta^{ii}\left((\partial_0\varphi)^2 + (\underbrace{\cancel{\vec{\partial}\varphi}}_{0})^2\right) + 1/2\,\eta^{ii}m^2\varphi^2 = 1/2\, A^2m^2\sin^2(mt+\alpha) - 1/2\, m^2A^2\cos^2(mt+\alpha) =$$

$$= -1/2\, A^2m^2\cos(2mt+2\alpha)$$

$$T^{ii} = -1/2\, A^2m^2\cos(2mt+2\alpha)$$

$$T^{ij} = (\overset{0}{\cancel{\partial^i\varphi)(\partial^j\varphi}}) - 1/2\,\underbrace{\cancel{\eta^{ij}}}_{0}\left((\partial_0\varphi)^2 + (\vec{\partial}\varphi)^2\right) + 1/2\,\underbrace{\cancel{\eta^{ij}}}_{0}m^2\varphi^2 = 0$$

$$T^{ij} = 0, \quad i \neq j$$

$$T^{ij} = -1/2\, A^2m^2\cos(2mt+2\alpha)\,\delta_{ij}$$

$$T^{\mu\nu} = \begin{pmatrix} 1/2\, A^2m^2 & \vec{0} \\ \downarrow 0 & -1/2\, A^2m^2\cos(2mt+2\alpha)\,\delta_{ij} \end{pmatrix}$$

$$P = -1/2\, A^2m^2\cos(2mt+2\alpha) \text{ — давление}$$

$mt \ll 1$: $P = -1/2\, A^2m^2\left(\cos(2\alpha) - \sin(2\alpha)\cdot 2mt - 1/2\cos(2\alpha)\cdot(mt)^2 + O((mt)^3)\right)$

Пренебрегая членами $O(mt)$, получим $P = -1/2\, A^2m^2\cos(2\alpha)$

Если $\alpha = 0$ (почему так?), $\begin{matrix} P = -1/2\, A^2m^2 \\ \mathcal{E} = 1/2\, A^2m^2 \end{matrix} \Rightarrow \boxed{P = -\mathcal{E}}$

Если $\alpha \neq 0$, то $\boxed{P = -\mathcal{E}\cdot\cos(2\alpha)}$

$mt \gg 1$:
$$\bar{P} = \lim_{T\to+\infty}\int_0^T \left(-1/2\, A^2m^2\cos(2mt+\alpha)\right)dt\cdot\frac{1}{T} = -\frac{1}{4}A^2m\cdot\lim_{T\to\infty}\frac{1}{T}\int_0^T \cos(2mt+2\alpha)\,d(2mt+2\alpha) =$$

$$= -\frac{1}{4}A^2m\lim_{T\to\infty}\frac{\sin(2mT+2\alpha)}{T} = 0$$

$$\boxed{\bar{P} = 0, \quad mt \gg 1}$$