

Topological Structures in Unified Theories

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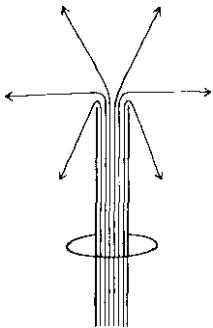
G. Lazarides, G. Dvali, T. Kibble, A. Pal, N. Okada, D. Raut, M. Rehman, V.N. Senoguz,
C.S. Un, R. Maji, J. Chakraborty, T. Vachaspati

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beyond the standard models?” [Virtual Workshop]-Bled 2021
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Grand Unified Theories (GUTs)

- Unification of SM/MSSM gauge couplings
- Unification of matter/quark-lepton multiplets
- Electric charge quantization, Magnetic monopoles predicted (as Dirac wanted)
- Proton Decay
- $b - \tau$ Yukawa unification in realistic models.
- Seesaw physics, neutrino oscillations
- Baryogenesis/leptogenesis
- Inflation/gravity waves, $\delta\rho/\rho$ and cosmic strings

Dirac Monopole (1931)



Annu. Rev. Nucl. Part. Sci. 1984.34:461-530



$$eg = \frac{n}{2}$$

t'Hooft-Polyakov Monopole (Toy Model)

- Scalar triplet ϕ^a in the adjoint representation of $SU(2)$ breaks $SU(2) \rightarrow U(1)_{em}$.
- We can choose the identity map or "hedgehog" configuration such that $\lim_{r \rightarrow \infty} \phi^a(\vec{x}) = v \hat{r}^a$.
- To ensure a finite energy solution, we require $D_\mu \phi^a(x) = 0$ at the boundary.
- Ansatz for the Higgs and gauge fields,

$$\begin{aligned}\phi^a(\vec{x}) &= v f(r) \hat{r}^a, \\ A_i^a(\vec{x}) &= a(r) \frac{\varepsilon_{aij} \hat{r}^j}{er}.\end{aligned}$$

- Monopole mass $M \sim \frac{M_w}{\alpha}$, core size $\sim M_w^{-1}$.

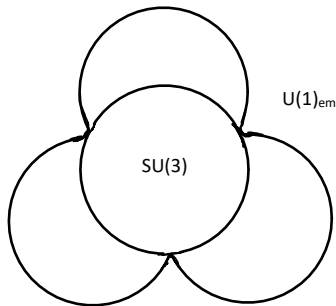
Magnetic Monopoles in Unified Theories

Any unified theory with electric charge quantization predicts the existence of topologically stable ('tHooft-Polyakov) magnetic monopoles. Their mass is about an order of magnitude larger than the associated symmetry breaking scale.

Example :

$SU(5) \rightarrow SM (3-2-1)$

Lightest monopole carries one unit of Dirac magnetic charge even though there exist fractionally charged quarks;



$$\text{monopole mass} \sim \frac{M_G}{\alpha_G}$$

$SU(5)$ Monopole

$$SU(5) \xrightarrow{\text{24-plet Higgs}} SU(3)_c \times SU(2)_L \times U(1)_Y$$

$$SU(3)_c \times SU(2)_L \times U(1)_Y \xrightarrow{\text{5-plet Higgs}} SU(3)_c \times U(1)$$

$$Q_{em} = \begin{pmatrix} 1/3 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

$SU(5)$ Monopole

- A 2π rotation with Q_{em} yields:

$$\text{diag} \left(\frac{2\pi}{3}, \frac{2\pi}{3}, \frac{2\pi}{3}, 1, 1 \right)$$

- Next, we perform a $\frac{2\pi}{3}$ rotation with

$$Q_{color} = \text{diag} \left(-\frac{1}{3}, -\frac{1}{3}, \frac{2}{3}, 0, 0 \right)$$

→ return to identity element.

- The monopole carries one unit of Dirac magnetic charge and color magnetic charge.

○ $SU(4)_c \times SU(2)_L \times SU(2)_R$ (Pati-Salam)

Electric charge is quantized with the smallest permissible charge being $\pm(e/6)$; Lightest monopole carries two units of Dirac magnetic charge;

○ $SO(10) \rightarrow 4-2-2 \rightarrow 3-2-1$

Two sets of monopoles: First breaking produces monopoles with a single unit of Dirac charge.

Second breaking yields monopoles with two Dirac units.

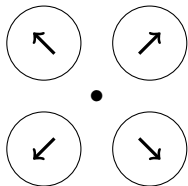
○ E_6 breaking to the SM can yield intermediate mass monopoles carrying three units of Dirac charge.

○ $E_6 \rightarrow SU(3)_c \otimes SU(3)_L \otimes SU(3)_R \rightarrow$
 $SU(3)_c \otimes SU(2)_L \otimes U(1)_{em}$

The discovery of primordial magnetic monopoles would have far-reaching implications for high energy physics & cosmology.

Primordial Monopoles

They are produced via the Kibble Mechanism as $G \rightarrow H$:



Center of monopole has G
symmetry $\langle \phi \rangle = 0$

Initial no. density $\propto T_c^{-3}$. With big bang cosmology such numbers are unacceptable.

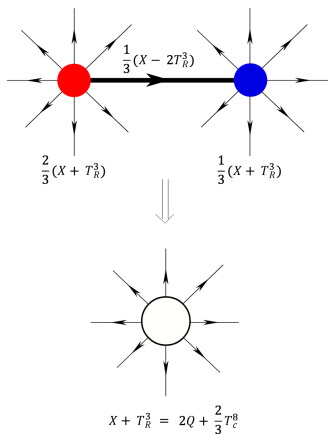
$$r_{in} = \frac{N_m}{N_\gamma} \sim 10^{-2}.$$

\Rightarrow Primordial Monopole Problem (Zeldovich & Khlopov, Preskill)

(Need Inflation)

'Schwinger' Monopole

$$\begin{aligned}
 SU(4)_c \times SU(2)_L \times SU(2)_R &\rightarrow SU(3)_c \times U(1)_{B-L} \times SU(2)_L \times U(1)_R \\
 &\rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \quad (1)
 \end{aligned}$$



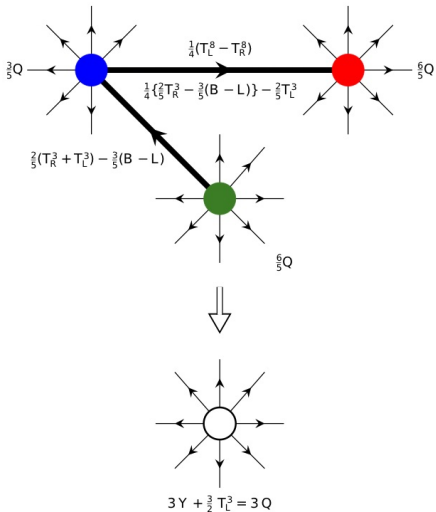


Figure 1: Emergence of the topologically stable triply charged monopole from the symmetry breaking $G \rightarrow SU(3)_c \times SU(2)_L \times U(1)_{Y_L} \times U(1)_{Y_R} \times U(1)_R \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_c \times U(1)_{em}$. An $SU(2)_R$ (green) monopole is connected by a flux tube to an $SU(3)_L$ (blue) monopole which, in turn, is connected to an $SU(3)_R$ (red) monopole by a superconducting flux tube. The constituent monopoles are pulled together to form the triply charged monopole. The fluxes along the tubes and around the monopoles are indicated.

Cosmic Necklaces

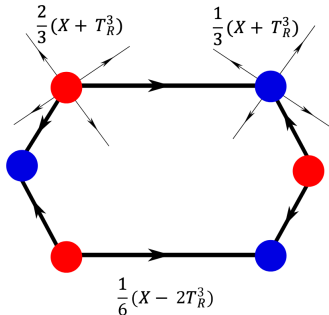


FIG. 2: Necklace with $SU(4)_c$ and $SU(2)_R$ monopoles from the symmetry breaking $SU(4)_c \times SU(2)_L \times SU(2)_R \rightarrow SU(3)_c \times U(1)_{B-L} \times SU(2)_L \times U(1)_R \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \times Z_2$, where the last step is achieved by a **126**-plet of $SO(10)$. Notation as in Fig. 1. We display explicitly only the Coulomb magnetic flux of two of the monopoles and the magnetic flux along one of the tubes. This necklace may survive inflation.

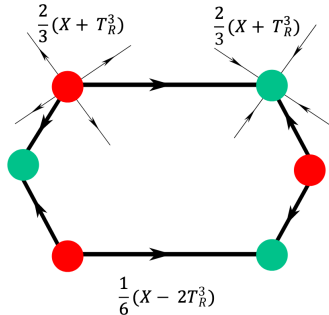


FIG. 3: Necklace with $SU(4)_c$ monopoles (red) and anti-monopoles (green) from the symmetry breaking $SO(10) \rightarrow SU(4)_c \times SU(2)_L \times U(1)_R \rightarrow SU(3)_c \times U(1)_{B-L} \times SU(2)_L \times U(1)_R \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \times Z_2$, where the last step is achieved by a **126**-plet of $SO(10)$. We assume that the monopoles from the first step of symmetry breaking are inflated away. We display explicitly only the Coulomb magnetic flux of one monopole and one antimonopole and the magnetic flux along one of the tubes. This necklace may survive inflation.

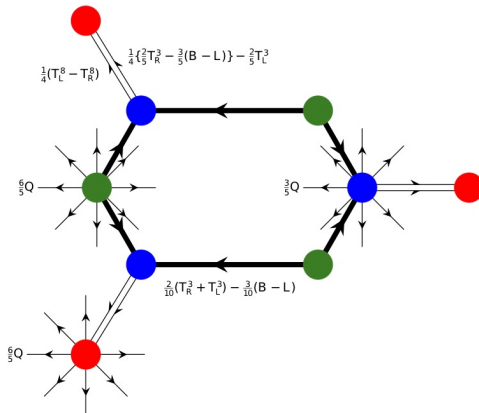


Figure 2: Necklace configuration with alternating $SU(3)_L$ (blue) and $SU(2)_R$ (green) monopoles from the symmetry breaking $G \rightarrow SU(3)_c \times SU(2)_L \times U(1)_{Y_L} \times U(1)_{Y_R} \times U(1)_R \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \times Z_2 \rightarrow SU(3)_c \times U(1)_{em} \times Z_2$. These are connected by half flux tubes along the necklace as indicated. Each $SU(3)_L$ (blue) monopole in the necklace is also connected by a flux tube with an $SU(3)_R$ (red) monopole hanging outside the necklace. We display explicitly only the Coulomb magnetic flux of three of the constituent monopoles and the flux along two of the tubes.

Monopole Searches in Colliders

- Gauge symmetries such as $SU(4)_c \times SU(2)_L \times SU(2)_R$ and $SU(3)_c \times SU(3)_L \times SU(3)_R$ are not truly unified without additional assumptions.

However, electric charge is quantized in these models, and it's plausible that their symmetry breaking scale lies well below the GUT scale.

- If the scale is \sim few TeV or so, the corresponding monopoles may be accessible in HE colliders.
- Monopoles carry two and three quanta of Dirac magnetic charges (respectively).
- In addition, we may find exotic states that are color singlets but carry fractional electric charges, $\pm e/2$ ($\pm e/3$).

Electroweak Monopole and Magnetic (Nambu) Dumbbell

- Consider the $SU(5)$ couplings $5^\dagger \times 24 \times 5$ and $5^\dagger \times 24^2 \times 5$
- After electroweak breaking the heavy $SU(2)_L$ triplet scalar ($Y = 0$) in 24_{\sim} acquires an induced $VEV \propto \frac{\langle H \rangle}{M_T} \langle H \rangle$
- Ignoring EW breaking by H ,
 $SU(2)_L \xrightarrow{\langle \phi \rangle} U(1)_L$ yields a monopole, with magnetic flux corresponding to a 2π rotation around T_L^3
- Reintroducing $\langle H \rangle$, such that $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$, this electroweak monopole ceases to be topologically stable

$$Q = \frac{T_L^3}{2} + Y \text{ is unbroken;}$$

$$\mathcal{B} = \frac{T_L^3}{2} - \frac{3Y}{5} \text{ is broken.}$$

Electroweak Monopole and Magnetic (Nambu) Dumbbell

- The monopole with one unit of charge along T_L^3 carries a Coulomb magnetic charge of $\frac{3}{4} \left(\frac{2\pi}{e} \right)$, and is attached to a Z-flux tube.
- A monopole and an antimonopole are expected to pair up and form a magnetic dumbbell (Nambu) connected by this flux tube (1977/1978).

- Consider (Nambu)

$$H = \frac{v_D}{\sqrt{2}} \begin{pmatrix} \cos(\theta/2) \\ \sin(\theta/2)e^{i\phi} \end{pmatrix},$$

such that $H^\dagger \sigma_i H = \frac{v_D^2}{2} \frac{x_i}{r}$

- Monopole is accompanied by a string (Z-flux) along the negative x_3 -axis where $\theta = \pi$.
- If the neutrino is covariantly transported around the string its wavefunction acquires an Aharonov-Bohm phase $\exp(iQ_Z^\nu \Phi_Z)$
 - Q_Z^ν is $SU(2)_L$ Z-charge
 - $= \frac{e}{\sin(\theta_W)\cos(\theta_W)} (T_L^3/2 - Q\sin^2(\theta_W))$
 - Φ_Z is the Z-flux
 - $\Rightarrow \Phi_Z = (4\pi/e)\sin(\theta_W)\cos(\theta_W)$
- Similarly $\Phi_{em} = (4\pi/e)\sin^2(\theta_W)$
- In GUTs, $\sin^2(\theta_W) = \frac{3}{8}$

Electroweak Monopole and Magnetic (Nambu) Dumbbell

- Ignoring the flux tube, following Nambu, the monopole mass is $\sim 700 \text{ GeV}$.

$$\rho_{str} \sim 2 \times 10^{-2} \text{ GeV}^{-1} \quad , \quad \mu_{str} \sim 3 \times 10^5 \text{ GeV}^2$$

Dumbbell mass $\sim 5 - 6 \text{ TeV}$

- These topological structures also appear in more elaborate GUTs such as $SO(10)$ and SUSY extensions.

Combining Nambu and Dirac Monopoles in the SM

- For a $U(1)_Y$ Dirac monopole in the Standard Model, Dirac quantization condition gives,

$$m_Y = \frac{2\pi}{y}n = \frac{12\pi}{g'}n, \quad n \in \mathbb{Z}, \quad (1)$$

- Consider Nambu monopole in the symmetry breaking $SU(2)_L \rightarrow U(1)_L$.

$$m_L = \frac{2\pi}{g/2}n' = \frac{4\pi}{g}n', \quad n' \in \mathbb{Z}, \quad (2)$$

- The net Z and A magnetic charges on a conglomerate of n $U(1)_Y$ and n' $SU(2)_L$ monopoles are

$$m_{Y+L,Z} = \frac{4\pi n'}{g} \cos \theta_w - \frac{12\pi n}{g'} \sin \theta_w, \quad (3)$$

$$m_{Y+L,A} = \frac{4\pi n'}{g} \sin \theta_w + \frac{12\pi n}{g'} \cos \theta_w \quad (4)$$

Combining Nambu and Dirac Monopoles in the SM

- This configuration should not have any net Z magnetic charge because Z magnetic fields are confined once the electroweak symmetry is broken. Any net Z flux would form a string that would confine the monopole conglomerate to an anti-conglomerate. Thus, we require

$$\frac{4\pi n'}{g} \cos \theta_w - \frac{12\pi n}{g'} \sin \theta_w = 0.$$

the above constraint gives

$$n' = 3n,$$

and so the conglomerate should contain three times as many Nambu monopoles as the Dirac Y-monopoles.

- The electromagnetic magnetic charge on the conglomerate is

$$m_{Y+L,A} = \frac{12\pi}{e} n$$

Combining Nambu and Dirac Monopoles in the SM

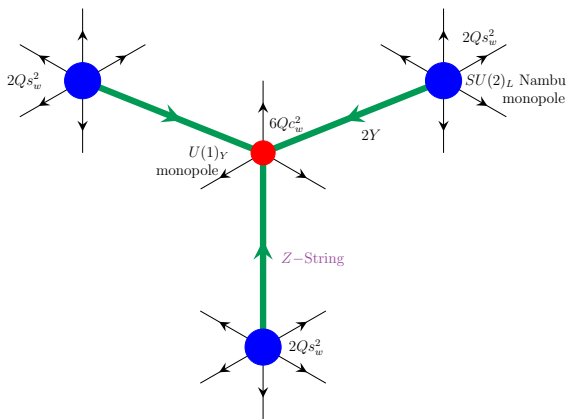


Figure: A purely $U(1)_Y$ monopole (red color) with winding number six from the breaking $SU(5) \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$ has a core of size M_{GUT}^{-1} and mass $\sim 10 M_{GUT}$. It merges following electroweak breaking with three $SU(2)_L$ (Nambu) monopoles to yield a purely electromagnetic monopole that carries six quanta ($12\pi/e$) of Dirac magnetic charge.

Colored $U(1)_Y$ Dirac Monopole

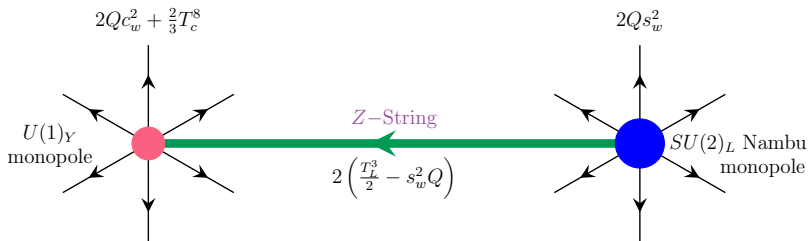


Figure: The Y-monopole emits EM and Z flux as well as color magnetic flux. The combined system made up of this monopole and the $SU(2)_L$ Nambu monopole carries two units ($2 \times 2\pi/e$) of EM charge in addition to the color charge, compatible with the Dirac quantization condition.

Colored $U(1)_Y$ Dirac Monopole

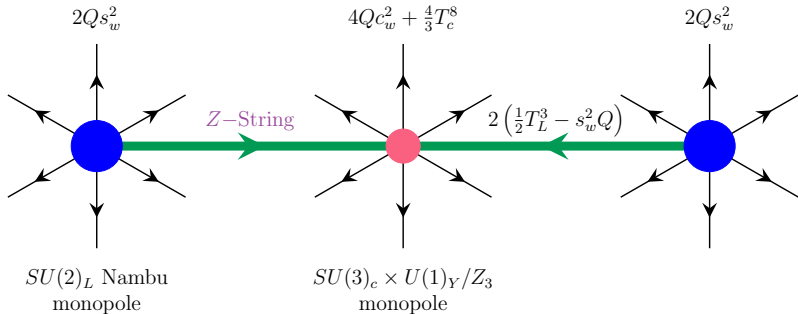
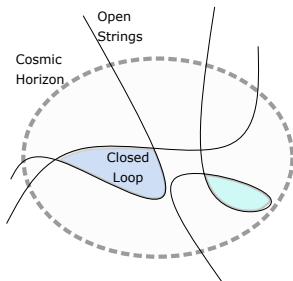


Figure: This conglomerate consists of two Nambu monopoles and a colored $U(1)_Y$ monopole.

Cosmic Strings from $SO(10)$

Cosmic Strings arise during symmetry breaking of $G \rightarrow H$ if $\pi_1(G/H)$ is non-trivial. Consider

$SO(10) \xrightarrow{M_{GUT}} SU(4) \times SU(2)_L \times SU(2)_R \xrightarrow{M_I} SM \times Z_2$ Mass per unit length of string is $\mu \sim M_I^2$, with $M_I \ll M_P$. The strength of string gravity is determined by the dimensionless parameter $G\mu \ll 1$.



Non-SUSY SO(10)

Usually broken via one or more intermediate steps to the SM

- $G = SO(10)/Spin(10)$
- $H = SU(3)_c \times U(1)_{e.m.}$
- $\Pi_2(G/H) \cong \Pi_1(H) \Rightarrow$ Monopoles
- $\Pi_1(G/H) \cong \Pi_0(H) = \mathbb{Z}_2 \Rightarrow$ Cosmic Strings (provided $G \rightarrow H$ breaking uses only tensor representations)
- $\mathbb{Z}_2 \subset \mathbb{Z}_4$ (center of $SO(10)$)
[T. Kibble, G. Lazarides, Q.S., PLB, 1982]
- Intermediate scale monopoles and cosmic strings may survive inflation.
- Recent work suggests that this Z_2 symmetry can yield plausible cold dark matter candidates.

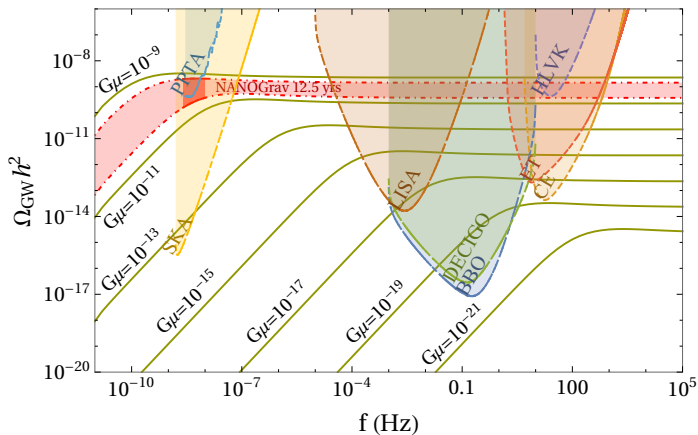
[Mario Kadastik, Kristjan Kannike, and Martti Raidal Phys. Rev. D 81 (2010), 015002; Yann Mambrini, Natsumi Nagata, Keith A. Olive, Jeremi Quevillon, and Jiaming Zheng Phys.Rev. D91 (2015) no.9, 095010 ; Sofiane M. Boucenna, Martin B. Krauss, Enrico Nardi Phys.Lett. B755 (2016) 168-17]

Generation of Stochastic Gravity Waves: Analytic Approximation

Three different regimes for decaying cosmic string loops:

- Loops that are produced and decay during radiation dominance. They produce the plateau in the spectrum.
- Loops that are produced during radiation dominance but decay during matter dominance. They generate a sharply peaked spectrum at lower frequencies, which is responsible for the overall peak of the spectrum.
- Loops that are produced and decay during matter domination. They also generate a sharply peaked spectrum which is though overshadowed by the previous case.

GWs spectrum and observational prospects



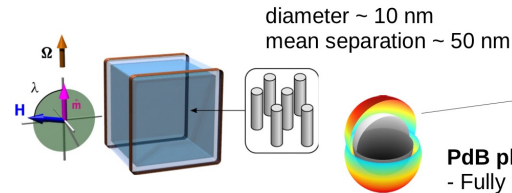
Walls Bounded by Strings

- Consider the breaking chain

$$\begin{array}{ccc} SO(10) & \xrightarrow{54} & SU(4)_c \times SU(2)_L \times SU(2)_R \\ & & \downarrow \widetilde{126} \\ & & SU(3)_c \times SU(2)_L \times U(1)_Y. \end{array}$$

- The first step leaves unbroken the discrete symmetry ‘C’ (also known as ‘D’) that interchanges left and right, and conjugates the representations.
- The $\widetilde{126}$ vev breaks ‘C’ which produces domain walls
- Thus we end up with walls bounded by strings.
Similar structures also arise in axion models.

Experimental System



We can control:

- Magnetic field
- RF excitation
- Angular velocity
- Temperature
- Pressure

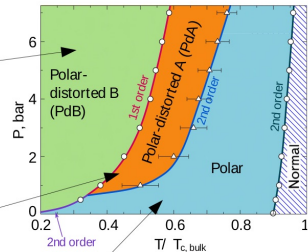
^3He – p-wave superfluid
→ multiple s.f. phases

PdB phase:
- Fully gapped

PdA phase:
- Two node points

Polar phase:
- Nodal line

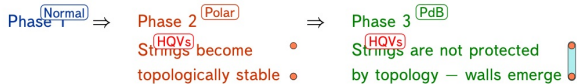
Phase diagram (on cooling):



HQVs in the PdB phase

KIBBLE-LAZARIDES-SHAFI (KLS) WALL or WALL BOUNDED BY STRINGS

Composite defect suggested in the context of phase transitions in the early Universe:

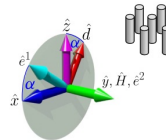
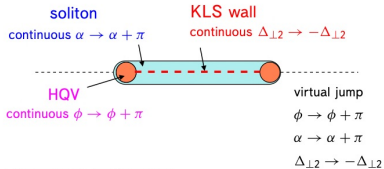


Kibble *et al*,
PRD **26**, 435 (1982)

Polar-distorted B phase:

$$A_{\mu i} = e^{i\phi} (\Delta_{\parallel} \hat{d}_{\mu} \hat{z}_i + \Delta_{\perp 1} \hat{e}_{\mu}^1 \hat{x}_i + \Delta_{\perp 2} \hat{e}_{\mu}^2 \hat{y}_i)$$

$$|\Delta_{\perp 1}| = |\Delta_{\perp 2}| = q |\Delta_{\parallel}|, \quad q < 1$$



$$\hat{\mathbf{d}} = \hat{\mathbf{x}} \cos \alpha - \hat{\mathbf{z}} \sin \alpha$$

$$\hat{\mathbf{e}}^1 = \hat{\mathbf{z}} \cos \alpha + \hat{\mathbf{x}} \sin \alpha$$

$$\hat{\mathbf{e}}^2 = \hat{\mathbf{y}} \parallel \mathbf{H}$$

Walls Bounded by Strings [J. Makinen, V. Dmitriev, et al. (Nature, 2019)]

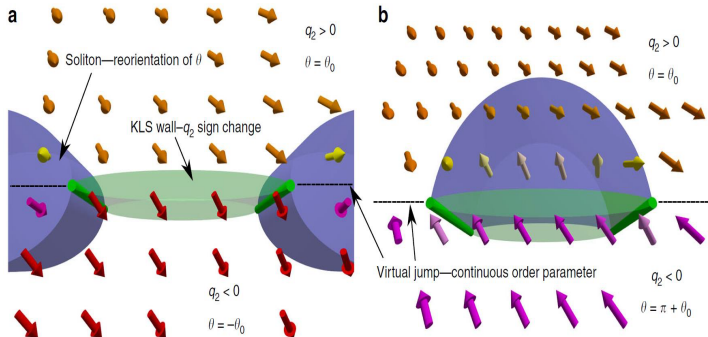


Fig. 4 Kibble-Lazarides-Shafi (KLS) wall configurations in the PdB phase. Each HQV core terminates one soliton—reorientation of the spin part of the order parameter denoted by the angle θ —and one KLS wall. The orientation of the $\hat{\mathbf{d}}$ -vector is shown as arrows where their color indicates the angle θ , based on numerical calculations (Supplementary Figure 2). **a** The KLS wall is bound between a different pair of HQV cores as the soliton. Ignoring the virtual jumps, the angle θ winds by $\pi - 2\theta_0$ across the soliton and by $2\theta_0$ across the KLS wall. The order parameter is continuous across the virtual jumps, where $\phi \rightarrow \phi + \pi$, $\theta \rightarrow \theta + \pi$, and $q_2 \rightarrow -q_2$. **b** The soliton and the KLS wall are bound between the same pair of HQV cores. The total winding of the $\hat{\mathbf{d}}$ -vector is π across the structure. In principle, the KLS wall may lie inside or outside the soliton. Here the KLS wall and the soliton are spatially separated for clarity

SUSY Higgs (Hybrid) Inflation

[Dvali, Shafi, Schaefer; Copeland, Liddle, Lyth, Stewart, Wands, '94]
[Lazarides, Schaefer, Shafi, '97] [Senoguz, Shafi, '04; Linde, Riotto '97]
[Rehman, Shafi, Wickman '10] [Buchmuller, Domcke, Kamada, Schmitz '13]

- Attractive scenario in which inflation can be associated with symmetry breaking $G \rightarrow H$
- Simplest inflation model is based on

$$W = \kappa S (\Phi \bar{\Phi} - M^2)$$

S = gauge singlet superfield, $(\Phi, \bar{\Phi})$ belong to suitable representation of G

- Need $\Phi, \bar{\Phi}$ pair in order to preserve SUSY while breaking $G \rightarrow H$ at scale $M \gg \text{TeV}$, SUSY breaking scale.
- R-symmetry

$$\Phi \bar{\Phi} \rightarrow \Phi \bar{\Phi}, \quad S \rightarrow e^{i\alpha} S, \quad W \rightarrow e^{i\alpha} W$$

Note: If $G = U(1)$ cosmic strings appear at the end of inflation in the simplest scenario.

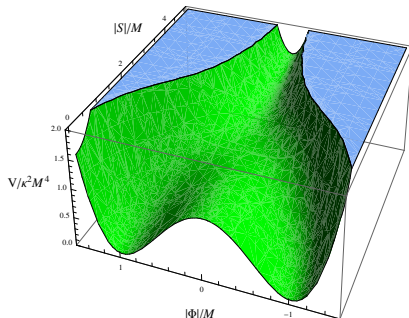
SUSY Higgs (Hybrid) Inflation

- Tree Level Potential

$$V_F = \kappa^2 (M^2 - |\Phi|^2)^2 + 2\kappa^2 |S|^2 |\Phi|^2$$

- SUSY vacua

$$|\langle \bar{\Phi} \rangle| = |\langle \Phi \rangle| = M, \quad \langle S \rangle = 0$$



SUSY Higgs (Hybrid) Inflation

Take into account radiative corrections (because during inflation $V \neq 0$ and SUSY is broken by $F_S = -\kappa M^2$)

- Mass splitting in $\Phi - \bar{\Phi}$

$$m_{\pm}^2 = \kappa^2 S^2 \pm \kappa^2 M^2, \quad m_F^2 = \kappa^2 S^2$$

- One-loop radiative corrections

$$\Delta V_{\text{1loop}} = \frac{1}{64\pi^2} \text{Str}[\mathcal{M}^4(S)(\ln \frac{\mathcal{M}^2(S)}{Q^2} - \frac{3}{2})]$$

- In the inflationary valley ($\Phi = 0$)

$$V \simeq \kappa^2 M^4 \left(1 + \frac{\kappa^2 \mathcal{N}}{8\pi^2} F(x) \right)$$

where $x = |S|/M$ and

$$F(x) = \frac{1}{4} \left((x^4 + 1) \ln \frac{(x^4 - 1)}{x^4} + 2x^2 \ln \frac{x^2 + 1}{x^2 - 1} + 2 \ln \frac{\kappa^2 M^2 x^2}{Q^2} - 3 \right)$$

SUSY Higgs (Hybrid) Inflation

Tree level + radiative corrections + minimal Kähler potential yield:

$$n_s = 1 - \frac{1}{N} \approx 0.98.$$

$\delta T/T$ proportional to M^2/M_p^2 , where M denotes the gauge symmetry breaking scale. Thus we expect $M \sim M_{GUT}$ for this simple model. In practice, $M \approx (1 - 5) \times 10^{15}$ GeV

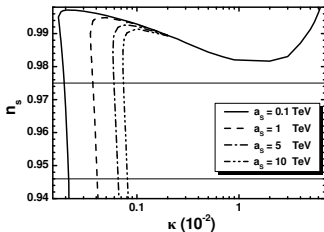
Since observations suggest that n_s lie close to 0.97, there are at least two ways to realize this slightly lower value:

- include soft SUSY breaking terms, especially a linear term in S ;
- employ non-minimal Kähler potential.

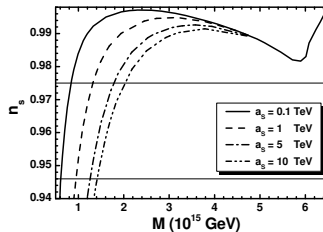
Note: Using non-minimal K one can realize $M \sim M_{GUT}$ and $r \sim 0.01$ with field values staying below M_P .

SUSY Higgs (Hybrid) Inflation

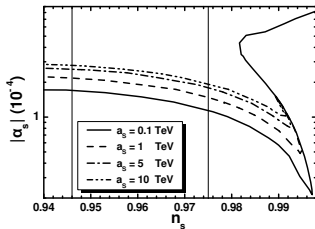
[Pallis, Shafi, 2013; Rehman, Shafi, Wickman, 2010]



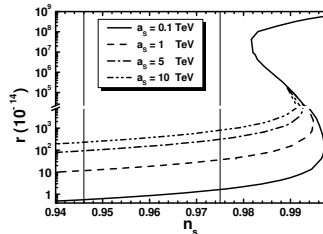
(a)



(b)



(a)

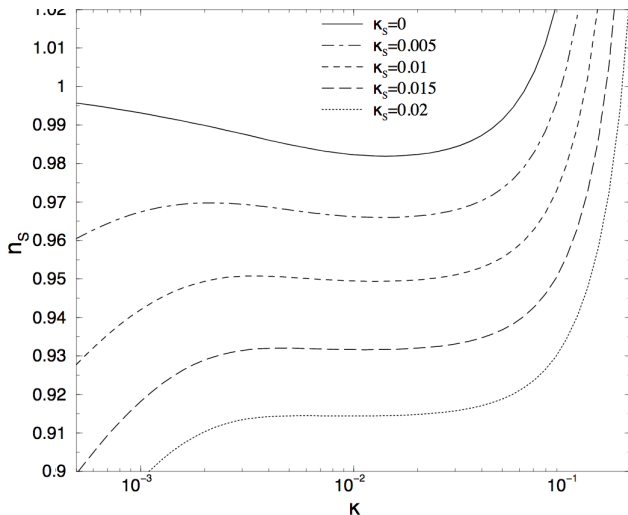


(b)

SUSY Higgs (Hybrid) Inflation

- $K \supset \kappa_s (S^\dagger S)^2$

[M. Bastero-Gil, S. F. King and Q. Shafi, 2006]



$$SU(5) \times U(1)_\chi (\chi SU(5))$$

Group	Representations					
	Matter					
$SU(5)$	$F_i(\bar{5})$	$T_i(10)$	$\nu_i^c(1)$			
$2\sqrt{10}U(1)_\chi$	3	-1	-5			
	Scalars					
$SU(5)$	$\Phi(24)$	$H(5)$	$\bar{H}(\bar{5})$	$\chi(1)$	$\bar{\chi}(1)$	$S(1)$
$2\sqrt{10}U(1)_\chi$	0	2	-2	10	-10	0

Table: Matter and Higgs content in minimal $SU(5) \times U(1)_\chi$. $\chi, \bar{\chi}$ fields implement $U(1)_\chi$ breaking and $\bar{\chi}$ provides masses to the right handed neutrinos, ν_i^c . The singlet S plays an important role during inflation.

$SU(5) \times U(1)_\chi (\chi SU(5))$: Salient Features

- $U(1)_\chi$ prevents rapid proton decay
- $U(1)_\chi \rightarrow Z_2$ ('matter parity'); Stable LSP.
- Observable Proton Decay
- Stable Cosmic Strings
- Yukawa Unification

Current Experimental Efforts

- Dark Matter (WIMP, axion, WIMPzilla...)
- Supersymmetry (LHC Run-3, HL-LHC...)
- Gravity Waves (Primordial, phase transition, cosmic string...)
- Neutrino Physics (Majorana or Dirac, absolute mass?)
- Proton Decay (Hyper-K, DUNE...)
- Dark Energy

Summary

- Unification of all forces remains a compelling idea.
- Grand unification explains charge quantization, predicts monopoles and proton decay.
- Also explains tiny neutrino masses via seesaw mechanism.
- Intermediate scale monopoles and cosmic strings may survive inflation.
- In non-SUSY inflation with Higgs potential, $r \gtrsim 0.01$ (minimal coupling to gravity).
- SUSY and Non-SUSY models offer plausible dark matter candidates such as TeV mass higgsino, axions....
- SUSY offers compelling inflation models

Thank You