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Inflation, primordial black holes, and induced gravitational waves from modified supergravity

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A review based on joint work with
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My personal encounter with A.D. Sakharov

As a graduate student, I was in Moscow at Lebedev Physical Institute (FIAN) during the years 1980-1986, in its Theoretical Department (Ginzburg Lab).

Academician A.D.Sakharov also belonged to the same Department, as well as some other great cosmologists (e.g., A. Linde and V. Mukhanov).

Over 1980-1986 A.D. Sakharov was put in exile by Soviet authorities. I was able to see him in person during the International Seminar on Quantum Gravity in Moscow, in May 1987.

Besides his great contributions to physics and politics, A.D. Sakharov was strongly advocating academic freedom.

PLAN of TALK

- General motivation: why inflation, why PBHs, why supergravity
- Starobinsky inflation (review and main ideas to be elevated to supergravity)
- The standard approach to cosmology in supergravity, and its problems
- The alternative minimal description of Starobinsky inflation in supergravity
- Generating seeds of PBHs after inflation (scenarios and their realizations) in gravity and supergravity
- Specific modified supergravity models for a more fundamental description of Starobinsky inflation and PBHs seeds produced after inflation
- Conclusion and Outlook: gravitational waves

WHY INFLATION in THE EARLY UNIVERSE

- Cosmological inflation (a phase of quasi-de-Sitter accelerated expansion with an exit) was proposed to explain homogeneity and spatial flatness of our Universe at large scales, its large size and entropy; inflation can explain the almost scale-invariant spectrum of CMB radiation; cosmological perturbations from quantum fluctuations during inflation can seed the CMB anisotropy and the LSS.
- Inflation is a paradigm, not a theory! Theoretical mechanisms of inflation use a driver (called inflaton field) with proper scalar potential.
- The physical nature and origin of inflaton and scalar potential, as well as its interactions with other fields are the big mysteries.
- There is a more fundamental (vs. phenomenological) way of thinking about inflation, and it is given by *supergravity* and *string theory*.

WHY PRIMORDIAL BLACK HOLES

- Primordial density fluctuations may also be responsible for seeding PBHs, when their amplitude is larger by a factor of $\sim 10^7$ compared to the amplitude observed at CMB scales;
- PBHs as possible non-particle DM, and seeds of supermassive BHs;
- Induced GW may be observed (advanced LIGO/VIRGO/KAGRA, LISA).
- Theoretical description of the PBH formation from inflation is possible either in the context of *single-field* inflation or in the context of *multi-field* inflation.
- There is a more fundamental (vs. phenomenological) way of thinking about PBH seeds, given by supergravity and string theory. A good theory is needed. "*There is nothing more practical than a good theory*" (E. Witten).
- PBHs study constrains physics, "*even when PBHs never formed*" (B. Carr).
- IPMU is the right place for PBHs studies: M.Sasaki, T. Suyama, T. Tanaka, S. Yokoyama, arXiv:1801.05235 for a review.

WHY SUPERSYMMETRY (N=1 in 4D)

1. $\{Q_\alpha, \bar{Q}_\beta\} = 2\sigma^\mu_{\alpha\beta} P_\mu$

2. $\{Q_\alpha, Q_\beta\} = 0$

3. $[P_\mu, Q_\alpha] = 0$

4. $[M_{\mu\nu}, Q_\alpha] = i(\sigma_{\mu\nu})_\alpha^{\ \beta} Q_\beta$

WHY SUPERGRAVITY

- Supergravity is a field theory with local SUSY that automatically implies GR.
- Supergravity is the only way to consistently describe a spin-3/2 field in GR;
- Supergravity remains the primary candidate for new physics beyond the SM; it connects gravity to particle physics, unifies bosons and fermions, and severely restricts their couplings;
- SUSY leads to a cancellation of quadratic divergences in quantum loops;
- Some supergravity theories arise as the low-energy effective actions in (compactified) superstring theory (quantum gravity) in String Landscape; a viable description of inflation and PBHs in those supergravities thus leads to their UV-completion in string theory.
- Supergravity as a more fundamental theoretical framework to the phenomenological model building (though not an ultimate one).

Supergravity with spontaneously broken SUSY has a natural candidate (LSP) for DM.

Starobinsky inflationary model (not from the historical perspective!)

$\boxed{\text{but from scratch}}$

The Starobinsky model of inflation is defined by the action (Starobinsky,1980)

$$S_{\text{Star.}} = \frac{M_{\text{Pl}}^2}{2} \int \mathrm{d}^4x \sqrt{-g} \left(R + \frac{1}{6m^2} R^2 \right) , \tag{1}$$

where we have introduced the reduced Planck mass $M_{\text{Pl}} = 1/\sqrt{8\pi G_{\text{N}}} \approx 2.4 \times 10^{18}$ GeV, and the scalaron (inflaton) mass m as the only parameter. We use the spacetime signature $(-,+,+,+,)$.

The $(R+R^2)$ gravity model (1) can be considered as the simplest extension of the standard Einstein-Hilbert action in the context of modified $F(R)$ gravity theories with an action

$$S_F = \frac{M_{\text{Pl}}^2}{2} \int \mathrm{d}^4x \sqrt{-g}\, F(R) , \tag{2}$$

in terms of the function $F(R)$ of the scalar curvature R.

Equivalence between $f(R)$ gravity and scalar-tensor gravity I

The $F(R)$ gravity action (2) is classically equivalent to

$$S[g_{\mu\nu},\chi] = \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\,\left[F'(\chi)(R-\chi)+F(\chi)\right] \tag{3}$$

with the real scalar field χ, provided that $F'' \neq 0$ that we always assume. The primes denote the derivatives with respect to the argument.

The equivalence is easy to verify because the χ-field equation implies $\chi = R$. In turn, the factor F' in front of the R in (3) can be (generically) eliminated by a Weyl transformation of metric $g_{\mu\nu}$, which transforms the action (3) into the action of the scalar field χ minimally coupled to Einstein gravity and having the scalar potential

$$V = \left(\frac{M_{\rm Pl}^2}{2}\right)\frac{\chi F'(\chi)-F(\chi)}{F'(\chi)^2}\ . \tag{4}$$

Equivalence between $f(R)$ gravity and scalar-tensor gravity II

The kinetic term of χ becomes canonically normalized after the field redefinition $\chi(\varphi)$ as

$$F'(\chi) = \exp\left(\sqrt{\tfrac{2}{3}}\varphi/M_{\mathrm{Pl}}\right) \ , \quad \varphi = \frac{\sqrt{3}M_{\mathrm{Pl}}}{\sqrt{2}} \ln F'(\chi) \ , \tag{5}$$

in terms of the canonical inflaton field φ, with the total acton

$$S_{\mathrm{quintessence}}[g_{\mu\nu},\varphi] = \frac{M_{\mathrm{Pl}}^2}{2}\int \mathrm{d}^4x\sqrt{-g}R - \int \mathrm{d}^4x\sqrt{-g}\left[\tfrac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + V(\varphi)\right] . \tag{6}$$

The classical and quantum stability conditions of $F(R)$ gravity theory are given by

$$F'(R) > 0 \quad \text{and} \quad F''(R) > 0 \ , \tag{7}$$

and they are obviously satisfied for Starobinsky model (1) for $R > 0$.

The inverse transformation

The inverse transformation reads

$$R = \left[\frac{\sqrt{6}}{M_{\mathrm{Pl}}}\frac{dV}{d\varphi} + \frac{4V}{M_{\mathrm{Pl}}^2}\right] \exp\left(\sqrt{\tfrac{2}{3}}\varphi/M_{\mathrm{Pl}}\right), \tag{8}$$

$$F = \left[\frac{\sqrt{6}}{M_{\mathrm{Pl}}}\frac{dV}{d\varphi} + \frac{2V}{M_{\mathrm{Pl}}^2}\right] \exp\left(2\sqrt{\tfrac{2}{3}}\varphi/M_{\mathrm{Pl}}\right). \tag{9}$$

In the case of Starobinsky model (1), one finds the famous potential

$$V(\varphi) = \frac{3}{4}M_{\mathrm{Pl}}^2 m^2 \left[1 - \exp\left(-\sqrt{\tfrac{2}{3}}\varphi/M_{\mathrm{Pl}}\right)\right]^2 . \tag{10}$$

This scalar potential is bounded from below (non-negative and stable), and it has the absolute minimum at $\varphi = 0$ corresponding to a Minkowski vacuum. The scalar potential (10) also has a plateau of positive height (related to the inflationary energy density), that gives rise to slow roll of inflaton during the inflationary era.

The inflationary features

A duration of inflation is measured in the slow roll approximation by the e-foldings number

$$N_e \approx \frac{1}{M_{\rm Pl}^2}\int_{\varphi_{\rm end}}^{\varphi_*}\frac{V}{V'}d\varphi \ , \tag{11}$$

where φ_* is the inflaton value at the reference scale (horizon crossing), and $\varphi_{\rm end}$ is the inflaton value at the end of inflation when one of the slow roll parameters

$$\varepsilon_V(\varphi) = \frac{M_{\rm Pl}^2}{2}\left(\frac{V'}{V}\right)^2 \quad \text{and} \quad \eta_V(\varphi) = M_{\rm Pl}^2\left(\frac{V''}{V}\right) \ , \tag{12}$$

is no longer small (close to 1).

The amplitude of scalar perturbations at horizon crossing is given by

$$A_s = \frac{V_*^3}{12\pi^2 M_{\rm Pl}^6 (V_*{}')^2} = \frac{3m^2}{8\pi^2 M_{\rm Pl}^2}\sinh^4\left(\frac{\varphi_*}{\sqrt{6}M_{\rm Pl}}\right) \ . \tag{13}$$

Spectral predictions of the one-field inflationary scenario in GR (according to A.A. Starobinsky)

Scalar (adiabatic) perturbations:

$$P_\zeta(k) = \frac{H_k^4}{4\pi^2\dot\phi^2} = \frac{GH_k^4}{\pi|\dot H|_k} = \frac{128\pi G^3 V_k^3}{3V_k'^2}$$

where the index k means that the quantity is taken at the moment $t = t_k$ of the Hubble radius crossing during inflation for each spatial Fourier mode $k = a(t_k)H(t_k)$. Through this relation, the number of e-folds from the end of inflation back in time $N(t)$ transforms to $N(k) = \ln \frac{k_f}{k}$ where $k_f = a(t_f)H(t_f)$, t_f denotes the end of inflation.

The spectral slope

$$n_s(k) - 1 \equiv \frac{d \ln P_\zeta(k)}{d \ln k} = \frac{1}{\kappa^2}\left(2\frac{V_k''}{V_k} - 3\left(\frac{V_k'}{V_k}\right)^2\right)$$

Tensor perturbations (A. A. Starobinsky, JETP Lett. 50, 844 (1979)):

$$P_g(k) = \frac{16GH_k^2}{\pi}; \quad n_g(k) \equiv \frac{d\ln P_g(k)}{d\ln k} = -\frac{1}{\kappa^2}\left(\frac{V_k'}{V_k}\right)^2$$

The consistency relation:

$$r(k) \equiv \frac{P_g}{P_\zeta} = \frac{16|\dot{H}_k|}{H_k^2} = 8|n_g(k)|$$

Tensor perturbations are always suppressed by at least the factor $\sim 8/N(k)$ compared to scalar ones. For the present Hubble scale, $N(k_H) = (50-60)$.

Starobinsky inflation and CMB (Planck)

The Starobinsky model (1) is in very good agreement with the Planck data. The Planck (2018) satellite mission measurements of the Cosmic Microwave Background (CMB) radiation give the scalar perturbations tilt as $n_s \approx 1+2\eta_V - 6\varepsilon_V \approx 0.9649 \pm 0.0042$ (68%CL) and restrict the tensor-to-scalar ratio as $r \approx 16\varepsilon_V < 0.064$ (95%CL). The Starobinsky inflation yields $r \approx 12/N_e^2 \approx 0.004$ and $n_s \approx 1 - 2/N_e$, where N_e is the e-foldings number between 50 and 60, with the best fit at $N_e \approx 55$.

The Starobinsky model (1) is geometrical (based on gravity only), while its (mass) parameter m is fixed by the observed CMB amplitude (COBE, WMAP) given by $\log(10^{10}A_s) = 2.975 \pm 0.056$ (68%CL) (or $A_s \approx 1.96 \cdot 10^{-9}$) as

$$m \approx 3 \cdot 10^{13}\ \text{GeV} \quad \text{or} \quad \frac{m}{M_{\text{Pl}}} \approx 1.3 \cdot 10^{-5}\,. \tag{14}$$

A numerical analysis of (11) with the potential (10) yields (with $N_e \approx 55$)

$$\sqrt{\frac{2}{3}}\varphi_*/M_{\text{Pl}} \approx \ln\left(\frac{4}{3}N_e\right) \approx 5.5\,, \quad \sqrt{\frac{2}{3}}\varphi_{\text{end}}/M_{\text{Pl}} \approx \ln\left[\frac{2}{11}(4+3\sqrt{3})\right] \approx 0.5 \tag{15}$$

More comments about Starobinsky inflation

- Universality for slow roll: see Eqs. (8) and (9);
- No free parameters (high predictive power);
- Einstein criterium ("*simple but not too simple*"):

Starobinsky potential (10) won against a power potential (Planck mission, 2018);

- Attractor solution with an exit: $H(t) \approx \left(\frac{M}{6}\right)^2 (t_{\text{end}} - t) + \ldots$ that is driven by the $+R^2$ term (scale invariance, no ghost; uniqueness in quadratically modified gravity); scalaron as the Nambu-Goldstone boson of spontaneously broken scale invariance.
- The UV-cutoff of $(R + R^2)$ gravity is $M_{\text{Pl}} \gg H_{\text{inf.}}$, after expanding the Starobinsky potential (10) in powers of ϕ;
- Starobinsky potential as the mass term: $\frac{3}{2} g(1 - e^{-\sqrt{2/3}\phi}) = \varphi$ yields the non-canonical kinetic term with a singularity at $\varphi_{\text{cr.}} = 3g/(2m)$ and the critical exponent $\alpha = \sqrt{2/3}$ (the universality again);
- Any viable inflationary model should be close to the Starobinsky model!

(among single-field models of slow-roll inflation)

Example: "Higgs inflation"

Basic ideas (Bezrukov, Shaposhnikov, 2007):

(i) identify inflaton field with Higgs field of the SM,

(ii) assume no new physics beyond the SM up to Planck scale,

(iii) add non-minimal coupling of Higgs field to gravity.

The Lagrangian (in Jordan frame) reads $(M_{\mathrm{Pl}} = 1)$

$$\mathcal{L}_J = \sqrt{-g}\left[\frac{1}{2}(1+\xi\phi^2)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V_H(\phi)\right] \tag{16}$$

where

$$V_H(\phi) = \frac{\lambda}{4}\left(\phi^2 - v^2\right)^2 \tag{17}$$

Details of Higgs inflation

- go from the original (Jordan) frame to Einstein frame via

$$g_J^{\mu\nu} = g_E^{\mu\nu}(1+\xi\phi^2) \tag{18}$$

- get a canonical scalar kinetic term for $\varphi = \varphi(\phi)$ by

$$\frac{d\varphi}{d\phi} = \frac{\sqrt{1+\xi(1+6\xi)\phi^2}}{1+\xi\phi^2} \tag{19}$$

This yields the standard (quintessence) Lagrangian

$$\mathcal{L}_E = \sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\varphi)\right] \tag{20}$$

with the (nonrenormalizable) potential

$$V(\varphi) = \frac{V_H(\phi(\varphi))}{\left[1+\xi\phi^2(\varphi)\right]^2} \tag{21}$$

The large field approximation

- In the large field approximation, $\varphi \gg \xi^{-1}$, a solution to (19) is

$$\varphi \approx \sqrt{\frac{3}{2}} \ln\left(1 + \xi\phi^2\right) \tag{22}$$

so that one arrives at the same Starobinsky potential (10):

$$V(\varphi) = \frac{\lambda}{4\xi^2}\left(1 - e^{-\sqrt{2/3}\varphi}\right)^2 \tag{23}$$

Thus, Starobinsky and Higgs inflation are in the same universality class.

- The (CMB) observations require $\xi/\sqrt{\lambda} \approx 5 \cdot 10^4$, or ξ of the order 10^4.
- The UV-cutoff of Higgs inflation is $M_{\text{Pl}}/\xi \sim H_{\text{inf.}}$, after expanding Eq. (21).
- Due to the nonrenormalizability of GR, the scalar potential of Higgs field at the inflationary scale cannot be predicted. The SM renormalization of λ becomes invalid below the inflationary scale.

Comments about Higgs inflation

- Actually, the SM Higgs field H is a doublet, though one can choose the unitary gauge in which $H = \phi/\sqrt{2}$ in the Higgs Lagrangian

$$\mathcal{L}_H = \sqrt{-g}\left[\frac{1}{2}R + \xi H^\dagger H R - g^{\mu\nu}\partial_\mu H^\dagger \partial_\nu H - \lambda\left(H^\dagger H - \frac{1}{2}v^2\right)^2\right] \quad (24)$$

- iI the large field approximation and during slow roll (inflation) we can ignore the scalar kinetic term and simplify the potential as

$$\mathcal{L}_H \approx \sqrt{-g}\left[\frac{1}{2}(1+\xi\phi^2)R - \frac{\lambda}{4}\phi^4\right] \quad (25)$$

Then varying with respect to the auxiliary field ϕ yields $\xi\phi R = \lambda\phi^3$ or

$$\phi^2 = \frac{\xi}{3}R \quad (26)$$

Substituting it into $\mathcal{L}_H$ gives the Starobinsky model again:

$$\mathcal{L}_H \approx \sqrt{-g}\left(\frac{1}{2}R + \frac{\xi^2}{4\lambda}R^2\right) \quad (27)$$

The standard approach to inflation in supergravity

- assumes inflaton in a chiral supermultiplet (max. spin 1/2), and it requires complexification of inflaton. Another chiral supermultiplet with spin-1/2 goldstino, required by spontaneous SUSY breaking caused by inflation, is also needed (thus leads to multi-field inflation); $\Phi(x,\Theta) = \phi(x) + \Theta\psi(x) + \Theta^2 F(x)$.
- Slow-roll inflation is obtained by engineering the scalar potential V in terms of a Kähler potential K and a superpotential W as $(M_{\rm Pl} = 1)$

$$V_F = e^K \left(|DW|^2 - 3\,|W|^2 \right) \quad \text{with} \quad DW = W' + K'W \ .$$

- Problems: (i) η-problem, (ii) need of stabilization of non-inflaton scalars that may easily spoil inflation, (iii) no fundamental input for a choice of (K, W) and inflationary (single-field) trajectory (low predictive power), (iv) no UV-completion, and (v) no control over quantum (gravity) corrections.

The alternative $N = 1$ supergravity frameworks

- Basic Proposition: minimize the theoretical input (# d.o.f. and interactions) for higher predictive power; employ (modified) supergravity only ;

- Basic Ideas:

(i) assign inflaton to a massive vector multiplet (max. spin 1) to get rid of sinflaton; it appears to be suitable for single-field inflation and PBHs;
(ii) use modified supergravity interactions (no ”supermatter”) similarly to GR in the $(R + R^2)$ modified gravity; it appears be suitable for two-field inflation and PBHs;

- Methods (technology):

(a) supergravity in curved superspace with manifest local SUSY;
(b) the $N = 1$ superconformal tensor calculus.

arXiv: 1011,0240, 1203.0805, 1309.7494, 1510.03524, 1607.05366, 1911.01008 [hep-th], with A.A. Starobinsky, T. Terada, Y. Aldabergenov, S. Tsujikawa, et al.

1. Inflaton in a massive vector supermultiplet

The inflaton (scalaron) can belong to a massive vector multiplet V that has a single physical scalar. The scalar potential of a vector multiplet is given by the D-term instead of the F-term. Any desired values of the CMB observables (n_s and r) and a nearly inflection point are possible. The only restriction by SUSY reads: *the inflaton scalar potential is a real function squared*, governed by arbitrary real potential $J(gV)$. The Lagrangian is

$$\mathcal{L} = \int d^2\theta 2\mathcal{E} \left\{ \tfrac{3}{8}(\overline{\mathcal{D}}\overline{\mathcal{D}} - 8\mathcal{R})e^{-\frac{2}{3}J} + \tfrac{1}{4}W^\alpha W_\alpha \right\} + \text{h.c.} \,, \tag{2}$$

and its bosonic part in Einstein frame reads

$$e^{-1}\mathcal{L} = \tfrac{1}{2}R - \tfrac{1}{4}F_{mn}F^{mn} - \tfrac{1}{2}J''\partial_m C\partial^m C - \tfrac{g^2}{2}J''B_m B^m - \tfrac{g^2}{2}J'^2 \,, \tag{3}$$

where $C = V|$ is the real scalar inflaton field and $J = J(C)$.

The D-type scalar potential of the Starobinsky inflationary model is obtained by choosing ($M_{\rm Pl} = 1$)

$$J(C) = \tfrac{3}{2}(C - \ln C) \quad \text{and} \quad C = \exp\left(\sqrt{2/3}\phi\right) \,. \tag{4}$$

$\boxed{\text{Super-Higgs mechanism}}$

Consider the master function $J(V)$ as a function $\tilde{J}(He^{2V}\overline{H})$ where we have introduced the Higgs chiral superfield H. The $\tilde{J}$ is invariant under the gauge transformations

$$H \to e^{-iZ}H \ , \quad \overline{H} \to e^{i\overline{Z}}\overline{H} \ , \quad V \to V + \tfrac{i}{2}(Z - \bar{Z}) \ , \qquad (3)$$

whose gauge parameter Z itself is a chiral superfield. The original theory of the massive vector multiplet governed by the master function J is recovered in the supersymmetric gauge $H = 1$.

We can now choose the different (Wess-Zumino) supersymmetric gauge in which $V = V_1$, where V_1 describes the irreducible massless vector multiplet minimally coupled to the dynamical Higgs chiral multiplet H (Aldabergenov, SVK, 2017). The *standard* Higgs mechanism appears when choosing the *canonical* function $J = \frac{1}{2}He^{2V}\bar{H}$ that corresponds to a *linear* function $\tilde{J}$.

2. The modified supergravity approach

The most straightforward way of extending the $(R + R^2)$ gravity to supergravity (Starobinsky and SVK 2011, Terada and SVK 2013) is described by a generic action

$$S = \int d^4x d^4\theta E^{-1} N(\mathcal{R}, \bar{\mathcal{R}}) + \left[\int d^4x d^2\Theta 2\mathcal{E} F(\mathcal{R}) + h.c. \right] \qquad (1)$$

in terms of the $\mathcal{N} = 1$ chiral superfield $\mathcal{R}$ having a complex scalar X as its first field component and the scalar curvature R as its last field component at Θ^2.

In particular, the action above is merely quadratic in R (no higher powers of R).

The action (1) can be transformed into the standard matter-coupled Einstein supergravity with two chiral matter superfields (Cecotti 1987, Gates and SVK 2009).

Modified supergravity models

Let us expand the functions N and $\mathcal{F}$ in Taylor series and keep only a few *leading* terms, as our first probe of modified supergravity ($M_{\rm Pl} = 1$),

$$N = \frac{12}{M^2}\mathcal{R}\overline{\mathcal{R}} - \frac{\xi}{2}(\mathcal{R}\overline{\mathcal{R}})^2 \, , \quad \mathcal{F} = \alpha + 3\beta\mathcal{R} \, , \tag{1}$$

with real parameters M and ξ, and complex parameters α and β.

- The chiral superfields $\mathcal{R}$ and $\mathcal{E}$ read

$$\begin{aligned} \mathcal{R} &= X + \Theta\left(-\tfrac{1}{6}\sigma^m\bar{\sigma}^n\psi_{mn} - i\sigma^m\overline{\psi}_m X - \tfrac{i}{6}\psi_m b^m\right) + \\ &\quad +\Theta^2\left(-\tfrac{1}{12}R - \tfrac{i}{6}\overline{\psi}^m\bar{\sigma}^n\psi_{mn} - 4X\overline{X} - \tfrac{1}{18}b_m b^m + \tfrac{i}{6}\nabla_m b^m + \right. \\ &\quad \left. + \tfrac{1}{2}\overline{\psi}_m\overline{\psi}^m X + \tfrac{1}{12}\psi_m\sigma^m\overline{\psi}_n b^n - \tfrac{1}{48}\varepsilon^{abcd}(\overline{\psi}_a\bar{\sigma}_b\psi_{cd} + \psi_a\sigma_b\overline{\psi}_{cd})\right) , \qquad (2) \\ 2\mathcal{E} &= e\left[1 + i\Theta\sigma^m\overline{\psi}_m + \Theta^2(6\overline{X} - \overline{\psi}_m\bar{\sigma}^{mn}\overline{\psi}_n)\right] \, , \qquad (3) \end{aligned}$$

- The standard supergravity is reproduced when $N = 0$ and $\mathcal{F} = -3\mathcal{R}$.
- Starobinsky inflation is realized when $\alpha = 0$, $\beta = -3$, and M equals to the scalaron mass, and dynamics of X is suppressed (Addazi and SVK, 2017).

Effective two-scalar field Lagrangian

In the notation

$$\frac{M^4\xi}{144} \equiv \zeta \quad \text{and} \quad |X| \equiv \frac{M}{2\sqrt{6}}\sigma \ , \tag{4}$$

where σ is the radial part of the complex scalar X, after ignoring Its angular part that does *not* appear in the scalar potential, together with $b_m = 0$, the bosonic part of the Lagrangian in our model takes the familiar form

$$e^{-1}\mathcal{L} = \frac{1}{2}f(R,\sigma) - \tfrac{1}{2}(1-\zeta\sigma^2)(\partial\sigma)^2 - U \ , \tag{5}$$

where we have the specific functions dictated by modified supergravity,

$$f(R,\sigma) = \left(1+\tfrac{1}{6}\sigma^2 - \tfrac{11}{24}\zeta\sigma^4\right)R + \frac{1}{6M^2}(1-\zeta\sigma^2)R^2 \ , \tag{6}$$

$$U = \frac{1}{2}M^2\sigma^2\left(1-\tfrac{1}{6}\sigma^2+\tfrac{3}{8}\zeta\sigma^4\right) \ . \tag{7}$$

(Standard) transfer to Einstein frame in field components

After introducing the auxiliary field χ and rewriting the Lagrangian as

$$e^{-1}\mathcal{L} = \tfrac{1}{2}\left[f_\chi(R-\chi)+f\right] - \tfrac{1}{2}(1-\zeta\sigma^2)(\partial\sigma)^2 - U\ , \qquad (8)$$

where $f_\chi \equiv \frac{\partial f}{\partial\chi}$ and in $f \equiv f(\chi,\sigma)$, R was replaced by χ, varying w.r.t. χ gives back the initial Lagrangian. On the other hand, after Weyl rescaling,

$$g_{mn} \to f_\chi^{-1} g_{mn}\ , \quad e \to f_\chi^{-2} e\ , \quad e f_\chi R \to eR - \tfrac{3}{2} e f_\chi^{-2}(\partial f_\chi)^2\ , \qquad (9)$$

with

$$f_\chi = A + B\chi \quad A \equiv 1 + \tfrac{1}{6}\sigma^2 - \tfrac{11}{24}\zeta\sigma^4\ , \quad B \equiv \frac{1}{3M^2}(1-\zeta\sigma^2)\ , \qquad (10)$$

in terms of the canonically normalized scalaron φ defined by

$$f_\chi = \exp\left[\sqrt{\tfrac{2}{3}}\varphi\right], \quad \chi = \tfrac{1}{B}\left(e^{\sqrt{\frac{2}{3}}\varphi} - A\right)\ , \quad f = \frac{1}{2B}\left(e^{2\sqrt{\frac{2}{3}}\varphi} - A^2\right)\ , \qquad (11)$$

the Lagrangian in Einstein frame takes the form

$$e^{-1}\mathcal{L} = \tfrac{1}{2}R - \tfrac{1}{2}(\partial\varphi)^2 - \tfrac{1}{2}(1-\zeta\sigma^2)e^{-\sqrt{\frac{2}{3}}\varphi}(\partial\sigma)^2 - V\ , \qquad (12)$$

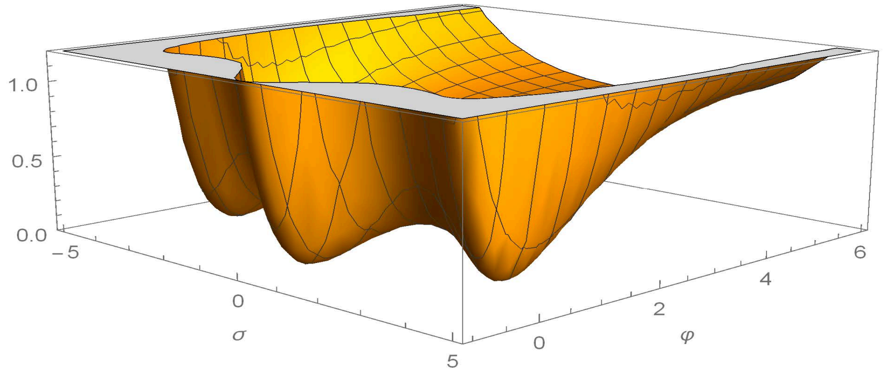
whose two-field scalar potential reads

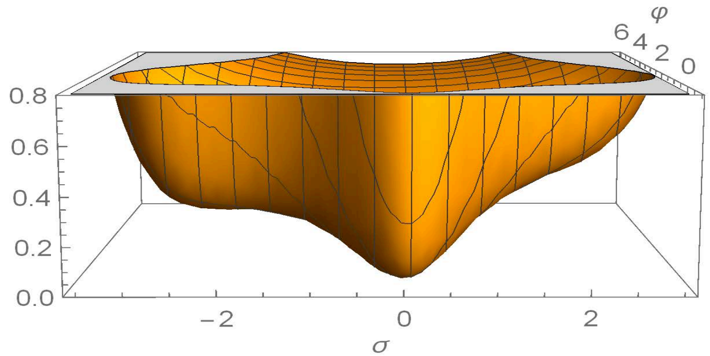
$$
\begin{aligned}
V = \frac{1}{4B}\left(1 - Ae^{-\sqrt{\frac{2}{3}}\varphi}\right)^2 + e^{-2\sqrt{\frac{2}{3}}\varphi}U = \\
= \frac{3M^2}{4(1-\zeta\sigma^2)}\left[1 - e^{-\sqrt{\frac{2}{3}}\varphi} - \frac{\sigma^2}{6}\left(1 - \frac{11}{4}\zeta\sigma^2\right)e^{-\sqrt{\frac{2}{3}}\varphi}\right]^2 \\
+ \frac{M^2}{2}e^{-2\sqrt{\frac{2}{3}}\varphi}\sigma^2\left(1 - \frac{1}{6}\sigma^2 + \frac{3}{8}\zeta\sigma^4\right) \ . \qquad (13)
\end{aligned}
$$

When $\sigma^2 > 1/\zeta$, the scalar σ becomes a ghost. However, when approaching $\sigma^2 = 1/\zeta$, the scalar potential becomes singular, so that it would take the infinite amount of energy to turn σ into a ghost (assuming its starting value in the region $\sigma^2 < 1/\zeta$).

Scalar potential in Einstein frame

$$V = \frac{1}{4B}(1 - Ax)^2 + x^2 U\,, \quad e^{-\sqrt{\frac{2}{3}}\varphi} \equiv x\,, \begin{cases} A = 1 + \frac{1}{6}\sigma^2 - \frac{11}{24}\zeta\sigma^4\,, \\ B = \frac{1}{3M^2}(1 - \zeta\sigma^2)\,, \\ U = \frac{M^2}{2}\sigma^2\left(1 - \frac{1}{6}\sigma^2 + \frac{3}{8}\zeta\sigma^4\right)\,. \end{cases}$$





The scalar potential on the left with $\zeta = 1/54 \approx 0.019$ and three Minkowski minima; on the right with $\zeta = 0.027$, a single Minkowski minimum at $\sigma = 0$ and two inflection points. In both cases $M = 1$.

Superfield transfer to Einstein matter-coupled supergravity

After introducing the Lagrange multiplier superfield $\mathbf{T}$ as (Terada and SVK, 2013)

$$\mathcal{L} = \int d^2\Theta 2\mathcal{E} \left\{ -\tfrac{1}{8}(\overline{\mathcal{D}}^2 - 8\mathcal{R})N(\mathbf{S},\bar{\mathbf{S}}) + \mathcal{F}(\mathbf{S}) + 6\mathbf{T}(\mathbf{S}-\mathcal{R}) \right\} + \text{h.c.} \,, \quad (14)$$

varying the Lagrangian w.r.t. the $\mathbf{T}$ gives back the original Lagrangian. On the other hand, the Lagrangian can above be rewritten to the form

$$\mathcal{L} = \int d^2\Theta 2\mathcal{E} \left\{ \tfrac{3}{8}(\overline{\mathcal{D}}^2 - 8\mathcal{R}) \left[\mathbf{T} + \bar{\mathbf{T}} - \tfrac{1}{3}N(\mathbf{S},\bar{\mathbf{S}}) \right] + \mathcal{F}(\mathbf{S}) + 6\mathbf{T}\mathbf{S} \right\} + \text{h.c.} \quad (15)$$

that can be put into the standard form in supergravity,

$$\mathcal{L} = \int d^2\Theta 2\mathcal{E} \left[\tfrac{3}{8}(\overline{\mathcal{D}}^2 - 8\mathcal{R})e^{-K/3} + W \right] + \text{h.c.} \,, \quad (16)$$

where the Kähler potential K and the superpotential W in our basic model are

$$K = -3\log(\mathbf{T} + \bar{\mathbf{T}} - \tilde{N}) \,, \quad \tilde{N} \equiv \mathbf{S}\bar{\mathbf{S}} - \tfrac{3}{2}\zeta(\mathbf{S}\bar{\mathbf{S}})^2 \,, \quad (17)$$

$$W = 3M\mathbf{S}\left(\mathbf{T} - \tfrac{1}{2}\right) \,. \quad (18)$$

Two-field scalar Lagrangian

takes the form of a non-linear sigma-model (NLSM) minimally coupled to gravity,

$$e^{-1}\mathcal{L} = \tfrac{1}{2}R - \tfrac{1}{2}G_{AB}\partial\phi^A\partial\phi^B - V\ , \tag{1}$$

where $\phi^A = \{\varphi, \sigma\}$, $A = 1, 2$, and the NLSM target space metric is given by

$$G_{AB} = \begin{pmatrix} 1 & 0 \\ 0 & (1-\zeta\sigma^2)e^{-\sqrt{\frac{2}{3}}\varphi} \end{pmatrix} \tag{2}$$

With the FLRW spacetime metric $g_{mn} = \mathrm{diag}(-1, a^2, a^2, a^2)$ the EoM read

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{1}{\sqrt{6}}(1-\zeta\sigma^2)e^{-\sqrt{\frac{2}{3}}\varphi}\dot{\sigma}^2 + \partial_\varphi V = 0\ , \tag{3}$$

$$\ddot{\sigma} + 3H\dot{\sigma} - \frac{\zeta\sigma\dot{\sigma}^2}{1-\zeta\sigma^2} - \sqrt{\frac{2}{3}}\dot{\varphi}\dot{\sigma} + \frac{e^{\sqrt{\frac{2}{3}}\varphi}}{1-\zeta\sigma^2}\partial_\sigma V = 0\ , \tag{4}$$

Friedmann equation and inflationary parameters I

In addition, Friedmann equation reads (in terms of Hubble function $H \equiv \dot{a}/a$)

$$3H^2 = \frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}(1-\zeta\sigma^2)e^{-\sqrt{\frac{2}{3}}\varphi}\dot{\sigma}^2 + V \ , \tag{5}$$

The standard slow roll parameter (in terms of $\tilde{t} \equiv Mt$ and $\tilde{H} = H/M$) is

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = -\frac{\dot{\tilde{H}}}{\tilde{H}^2} \quad . \tag{6}$$

Following *Gundhi and Steinwachs* (2018), we introduce the field-space velocity and acceleration unit vectors as

$$\Sigma^A \equiv \frac{\dot{\phi}^A}{|\dot{\phi}|} \ , \quad \Omega^A \equiv \frac{\omega^A}{|\omega|} \ , \tag{7}$$

respectively, where the absolute value of a field-space vector a^A is defined by $|a| \equiv \sqrt{G_{AB}a^Aa^B}$, and the acceleration vector ω^A is defined by

Inflationary parameters II

$$\omega^A \equiv \dot{\Sigma}^A + \Gamma^A_{BC}\Sigma^B\dot{\phi}^C \quad \begin{cases} \omega^\varphi = \dot{\Sigma}^\varphi + \frac{1}{\sqrt{6}}(1-\zeta\sigma^2)e^{-\sqrt{\frac{2}{3}}\varphi}\Sigma^\sigma\dot{\sigma} \ , \\ \omega^\sigma = \dot{\Sigma}^\sigma - \frac{1}{\sqrt{6}}(\Sigma^\varphi\dot{\sigma} + \Sigma^\sigma\dot{\varphi}) - \frac{\zeta\sigma}{1-\zeta\sigma^2}\Sigma^\sigma\dot{\sigma} \ . \end{cases} \tag{8}$$

Then the effective mass matrix is given by

$$\mathcal{M}^A_B \equiv G^{AC}\nabla_B\partial_C V - R^A_{CDB}\dot{\phi}^C\dot{\phi}^D \ , \tag{9}$$

where R^A_{CDB} is the Riemann tensor of the NLSM scalar manifold. The adiabatic and isocurvature parameters are defined by

$$\eta_{\Sigma\Sigma} \equiv \frac{\mathcal{M}^A_B\Sigma_A\Sigma^B}{V} \ , \quad \eta_{\Omega\Omega} \equiv \frac{\mathcal{M}^A_B\Omega_A\Omega^B}{V} \ , \tag{10}$$

respectively, where $\eta_{\Sigma\Sigma}$ plays the role of the second slow-roll parameter.

Transfer functions and inflationary observables

The transfer functions are defined by

$$T_{SS}(t_1, t_2) \equiv \exp\left[\int_{t_1}^{t_2} dt' \beta(t') H(t')\right] , \tag{11}$$

$$T_{RS}(t_1, t_2) \equiv 2\int_{t_1}^{t_2} dt' |\omega(t')| T_{SS}(t_1, t_2) , \tag{12}$$

where

$$\beta(t) \equiv -2\epsilon + \eta_{\Sigma\Sigma} - \eta_{\Omega\Omega} - \frac{4|\omega|^2}{3H^2} . \tag{13}$$

The transfer functions describe the evolution of perturbations on superhorizon scales, i.e. from the moment of horizon exit t_1 (of the k-mode of interest) until some later time t_2. According to the PLANCK data (arXiv:1807.06211[astro-ph.CO]), the observed values of n_s and r are

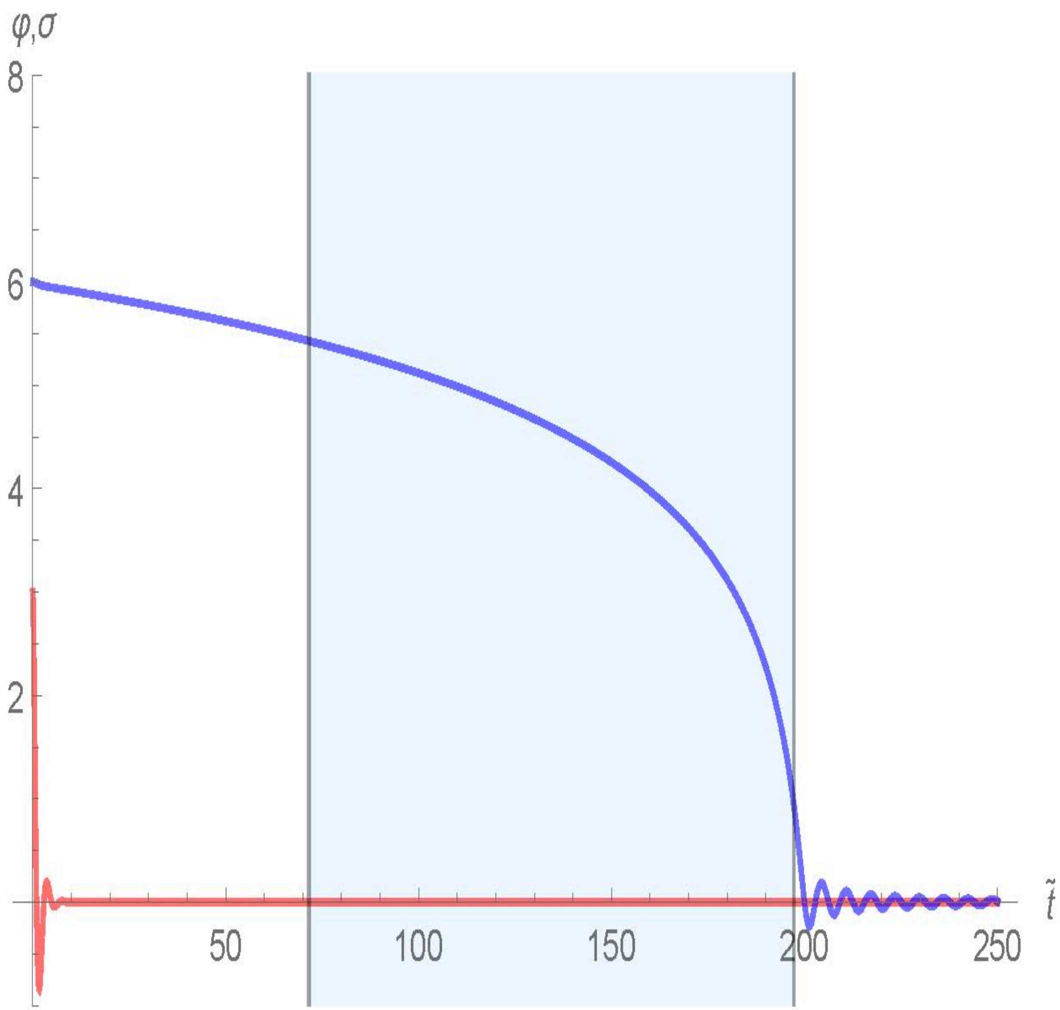
$$n_s = 0.9649 \pm 0.0042 \; (1\sigma \text{ CL}) \quad \text{and} \quad r < 0.064 \; (2\sigma \text{ CL}) . \tag{14}$$

$\boxed{\text{Inflationary solution I}}$ (generic)

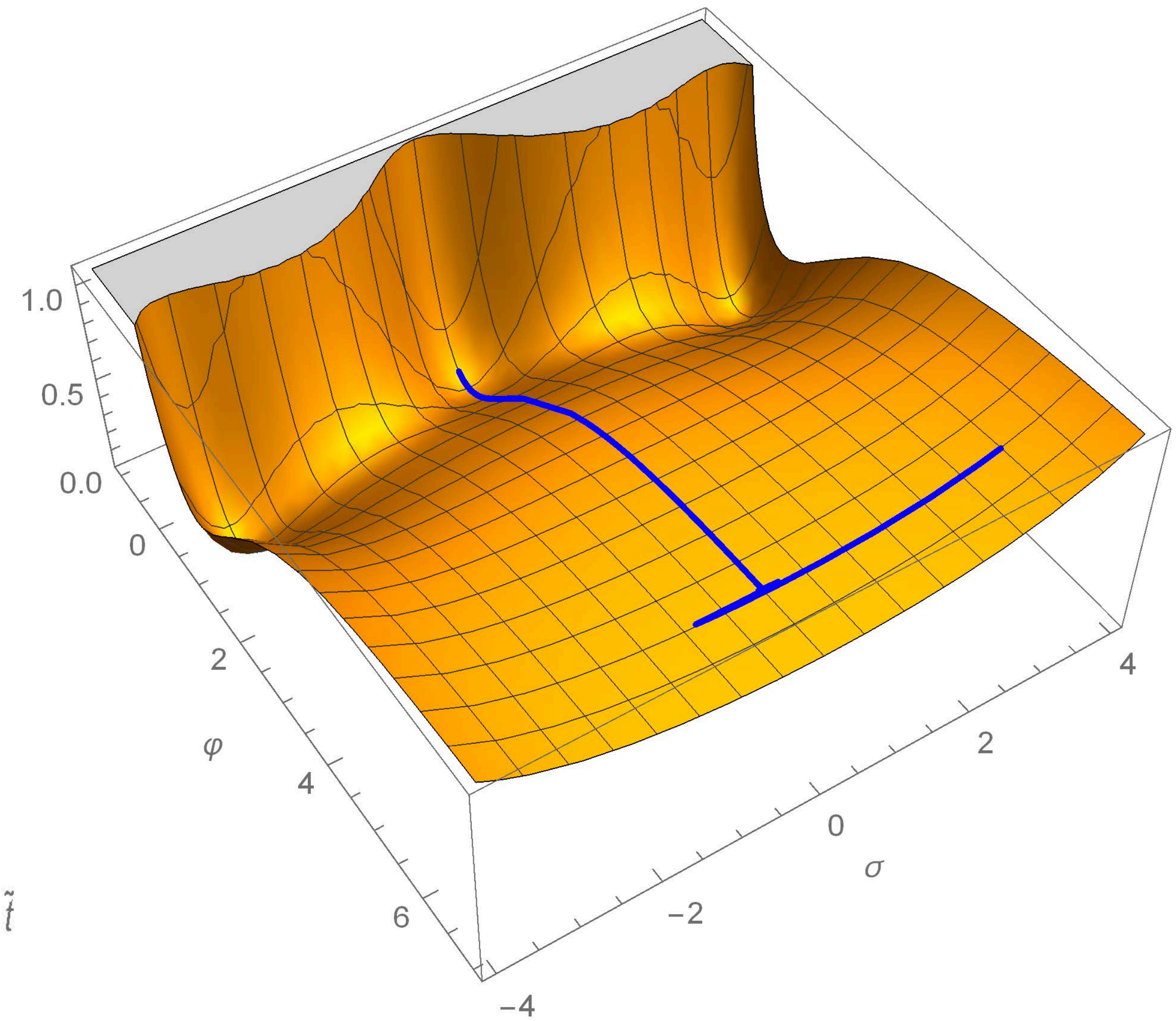
Consider the case (I) of $\zeta = 1/54 \approx 0.019$ with three Minkowski minima. We *numerically* solved the field equations with the initial conditions $\varphi(0) = 6, \sigma(0) = 3$, and the vanishing initial velocities. The field σ quickly drops to its minimum $\sigma = 0$, so that the trajectory becomes similar to that in the single-field Starobinsky inflation. It is a generic feature when the initial velocities are zero (or almost zero), $\varphi(0) \gtrsim 6$ and $|\sigma(0)| \lesssim \sigma_{\rm max}$, where $\sigma_{\rm max} = 1/\sqrt{\zeta}$ is the upper bound on σ where the potential is infinite. When $\zeta = 1/54$ we find $\sigma_{\rm max} \approx 7.35$.

The solution (I) leads to the spectral tilt and the tensor-to-scalar ratio as $n_s \approx 0.9662$ and $r_{\rm max} \approx 0.003$, respectively, which are consistent with the observed values and the theoretical (Starobinsky) predictions of chaotic single-field inflation.

Field-space trajectories I. The blue shaded region is last 60 e-folds.



The time dependence of scalaron (blue) and another scalar (red).



The scalar potential and the trajectory I.

$\boxed{\text{Inflationary solution II}}$ (special)

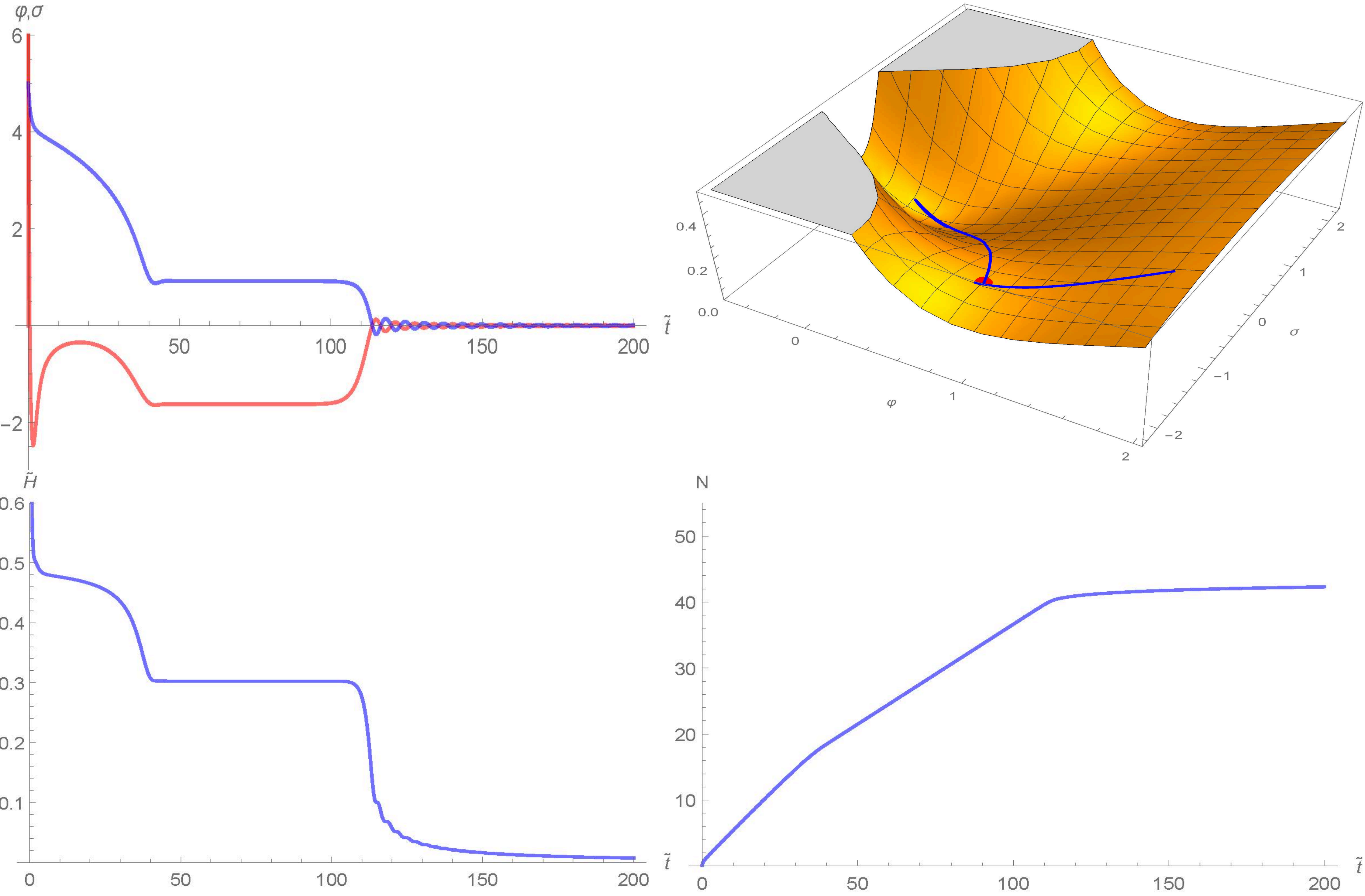
As regards PBH production after inflation, let us consider the field-space trajectory going through the saddle point of the potential, that is a maximum in the σ-direction and a (local) minimum in the φ-direction. Then the saddle point divides inflation into two stages. We found a set of initial conditions that leads to such trajectory (solution II) with

$$\varphi(0) = 5 \,, \quad \dot{\sigma}(0) = 79.784527415607 \,, \quad \sigma(0) = \dot{\varphi}(0) = 0 \,. \tag{1}$$

The total number of e-foldings is around 40, though it can be larger for larger values of $\varphi(0)$ with more fine-tuning of the initial velocities.

Thus, in order to achieve two-stage inflation, where the field-space trajectory passes through the saddle point, we have to significantly fine-tune initial conditions. When φ is large, the potential takes the shape of a valley with the minima at $\sigma = 0$, so a generic behavior of σ is to quickly relax at $\sigma = 0$, and let φ drive the entire inflationary period. Therefore, our basic model needs to be generalized for the sake of PBHs production.

Field-space trajectories II, Hubble function and e-folds.



More general models in modified supergravity

Adding the next-order terms to the modified supergravity potentials yields

$$N = \frac{12}{M^2}|\mathcal{R}|^2 - \frac{72}{M^4}\zeta|\mathcal{R}|^4 - \frac{768}{M^6}\gamma|\mathcal{R}|^6 \ , \tag{1}$$

$$F = -3\mathcal{R} + \frac{3\sqrt{6}}{M}\delta\mathcal{R}^2 \ . \tag{2}$$

The corresponding Lagrangian in Einstein frame reads

$$e^{-1}\mathcal{L} = \tfrac{1}{2}R - \tfrac{1}{2}(\partial\varphi)^2 - \frac{3M^2}{2}Be^{-\sqrt{\frac{2}{3}}\varphi}(\partial\sigma)^2 - \frac{1}{4B}\left(1 - Ae^{-\sqrt{\frac{2}{3}}\varphi}\right)^2 - e^{-2\sqrt{\frac{2}{3}}\varphi}U\,,$$

where the functions A, B, U are given by

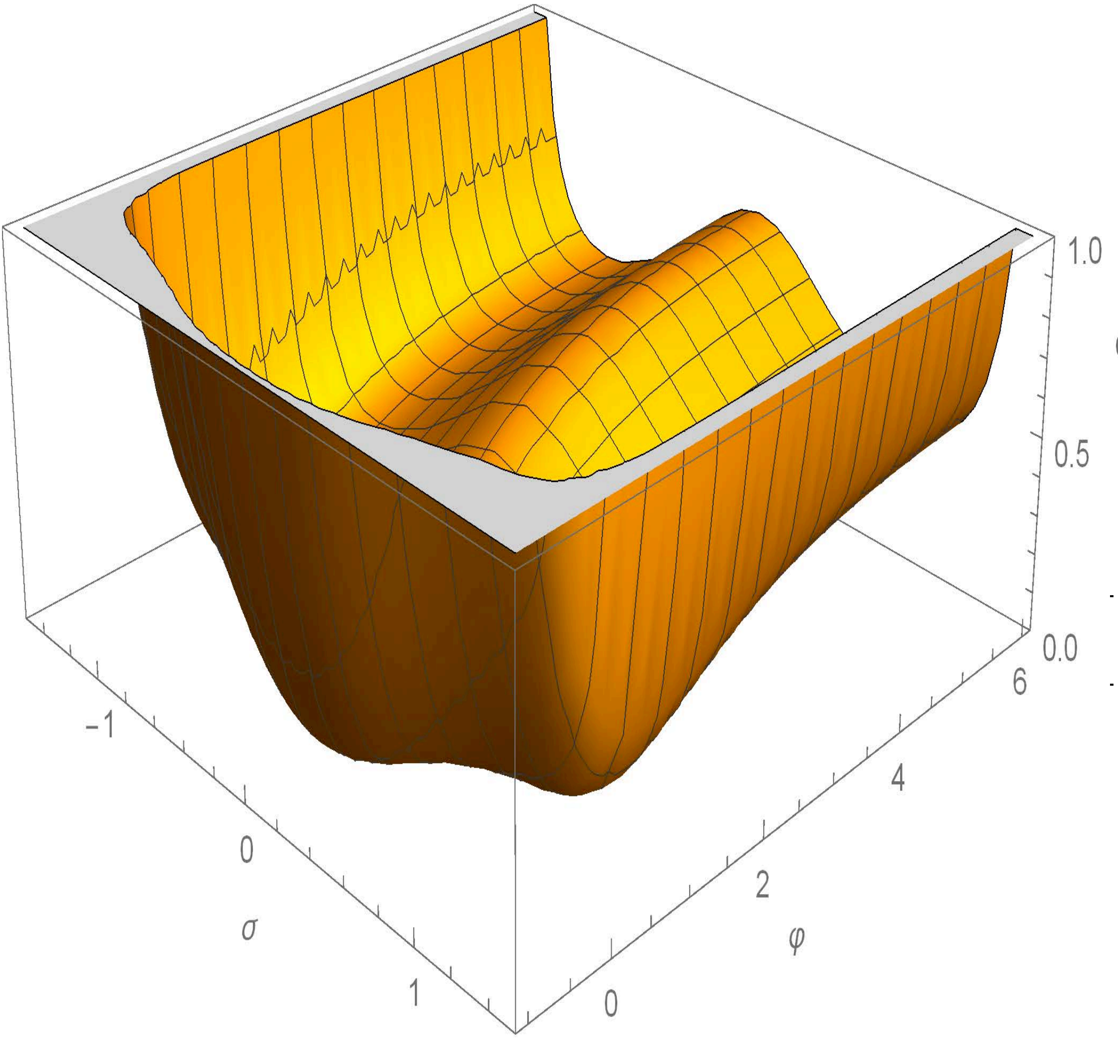
$$\begin{aligned} A &= 1 - \delta\sigma + \tfrac{1}{6}\sigma^2 - \tfrac{11}{24}\zeta\sigma^4 - \tfrac{29}{54}\gamma\sigma^6\ , \\ B &= \frac{1}{3M^2}(1 - \zeta\sigma^2 - \gamma\sigma^4)\ , \\ U &= \frac{M^2}{2}\sigma^2\left(1 + \tfrac{1}{2}\delta\sigma - \tfrac{1}{6}\sigma^2 + \tfrac{3}{8}\zeta\sigma^4 + \tfrac{25}{54}\gamma\sigma^6\right)\ . \end{aligned} \tag{3}$$

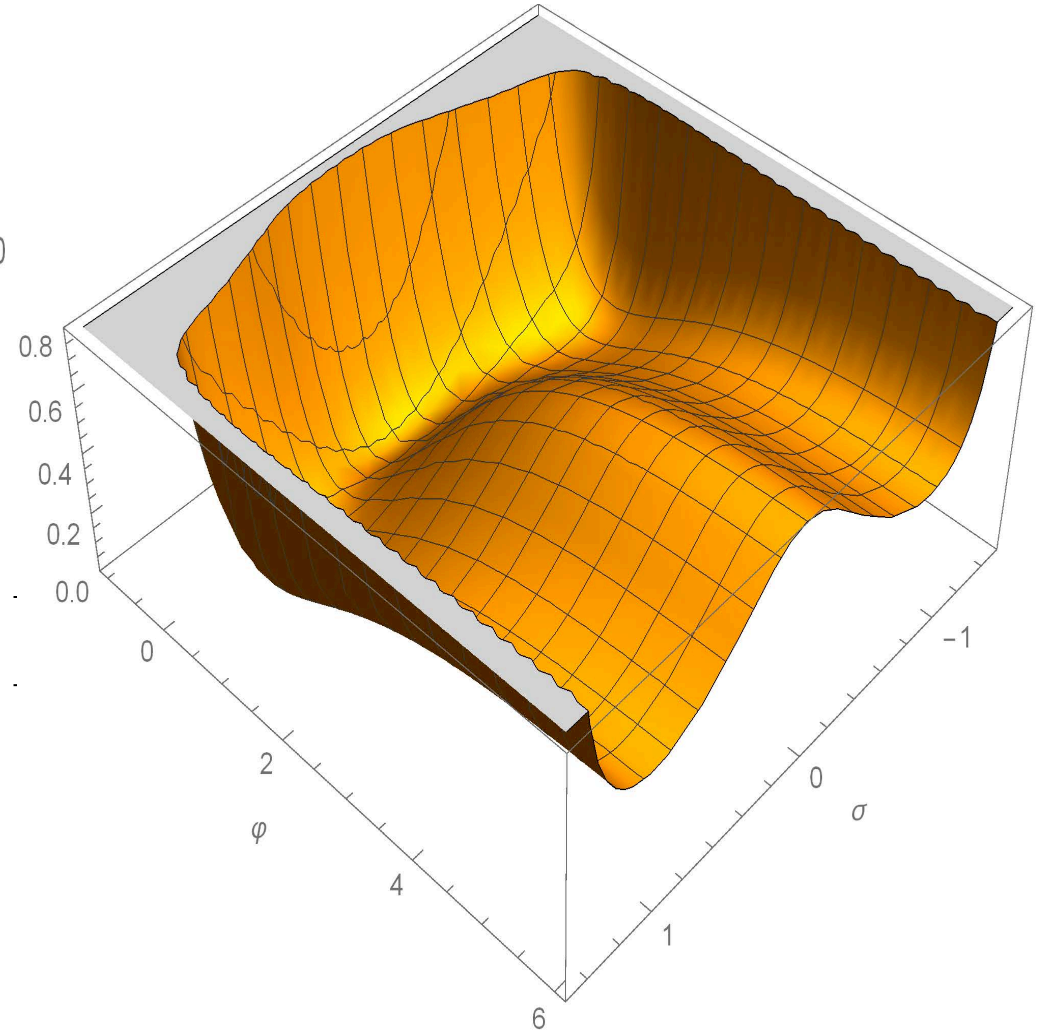
PBHs in the γ-model with $\delta = 0$

Let us choose $\gamma = 1$ and $\zeta = -1.7774$ for a numerical analysis. The scalar potential has two valleys and a single Minkowski minimum at $\sigma = \varphi = 0$. The first slow-roll (SR) inflation is possible along either of the valleys. The valleys merge into the Minkowski minimum by passing through inflection points (or near-inflection points) followed by the second, ultra-slow-roll (USR) inflationary stage.

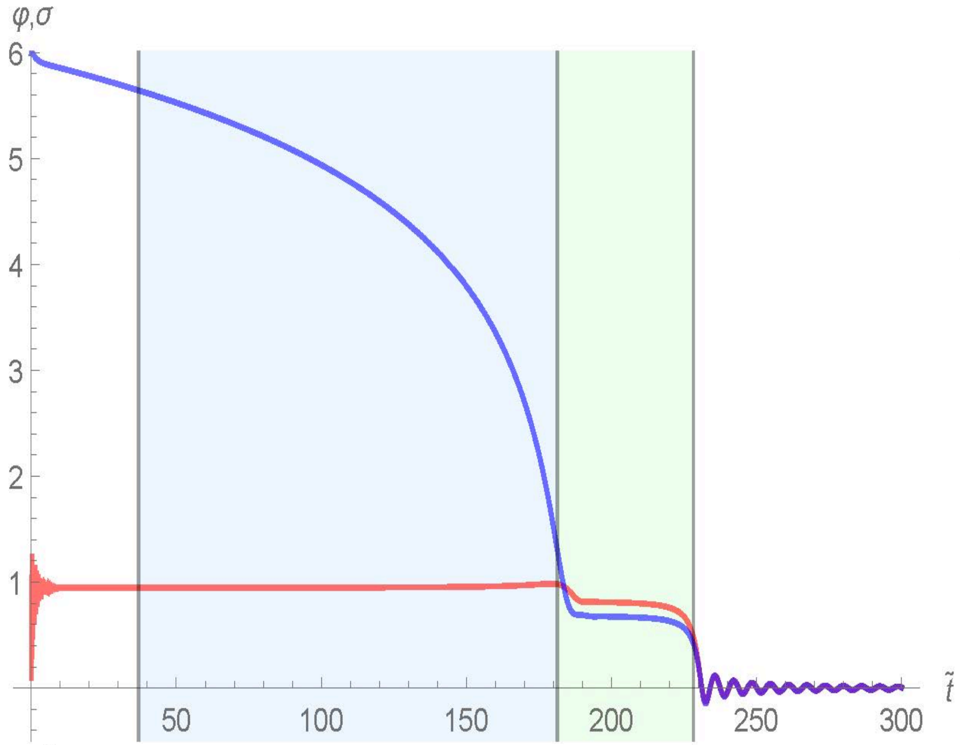
After solving the equations of motion numerically, we plot the solutions. The total number of e-foldings is set to $\Delta N = 60$, and the end of the first stage of inflation is defined by the time when $\eta_{\Sigma\Sigma}$ first crosses unity. It leads to an enhancement in the scalar power spectrum. Inflation ends when $\epsilon = 1$. With the chosen parameters, the first stage lasts $\Delta N_1 \approx 50$ e-foldings, whereas the second stage lasts for $\Delta N_2 \approx 10$: the first stage of inflation is represented by the blue shaded region, whereas the second stage is marked by the green shaded region. The length of the second stage is controlled by the parameter ζ for a given γ.

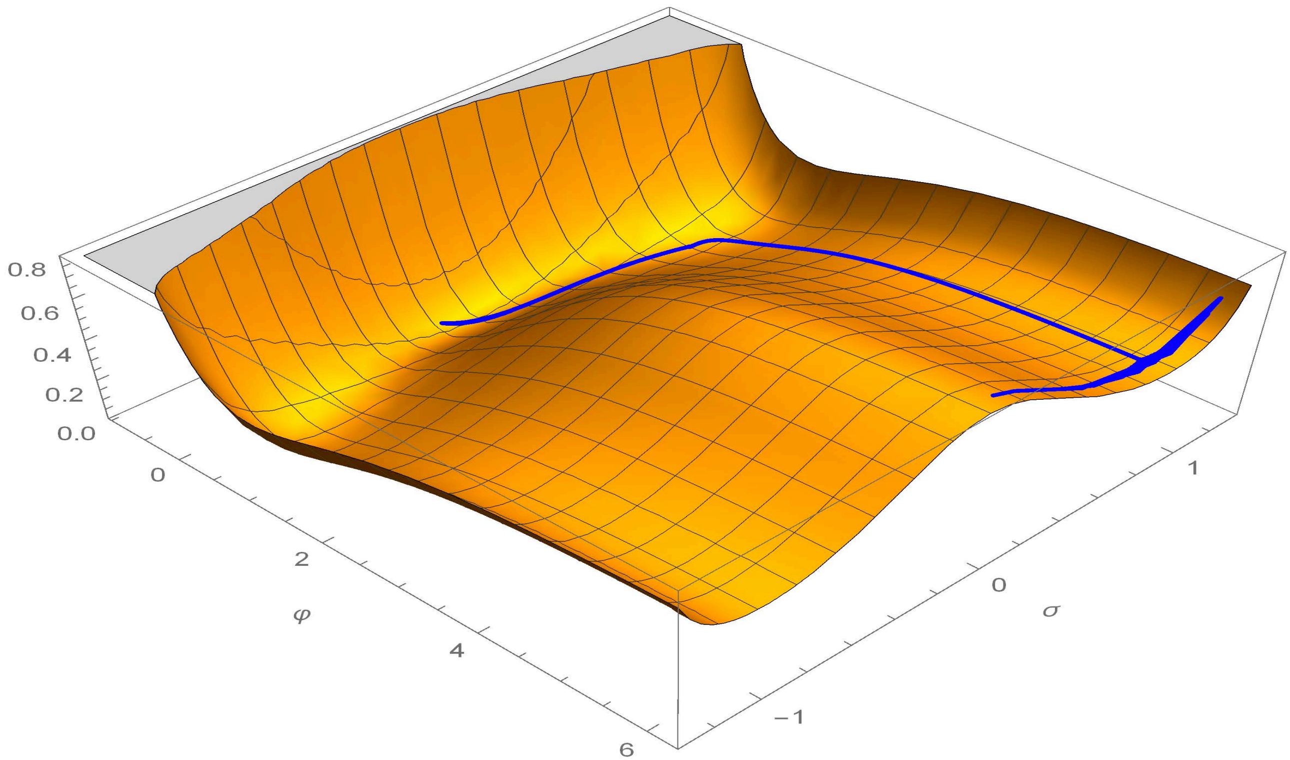
The scalar potential of the gamma-model, delta =0

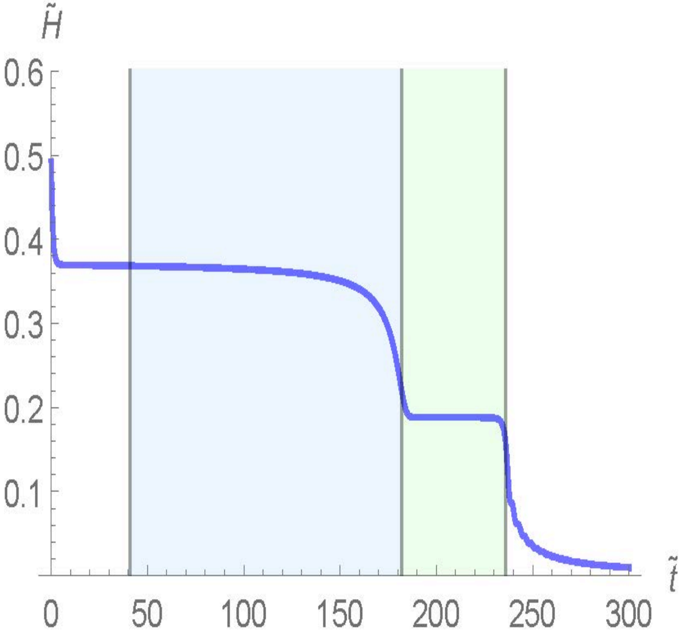


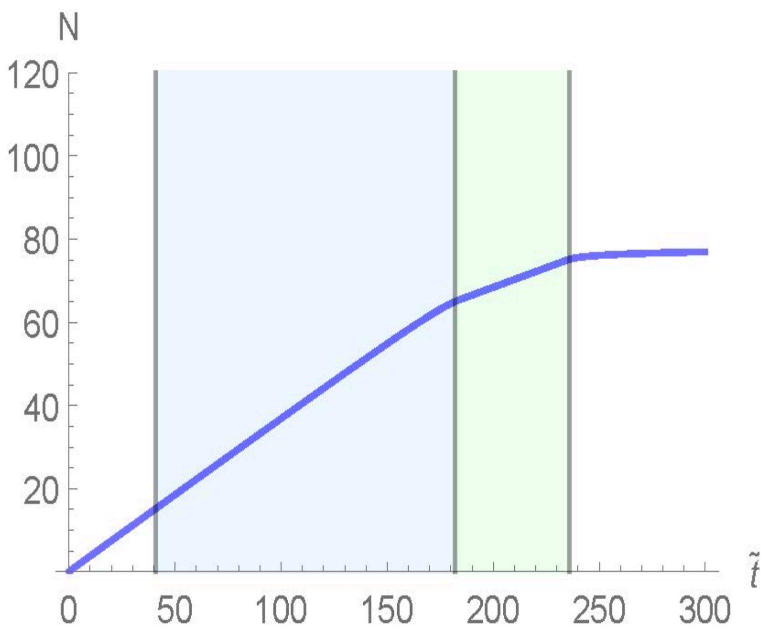


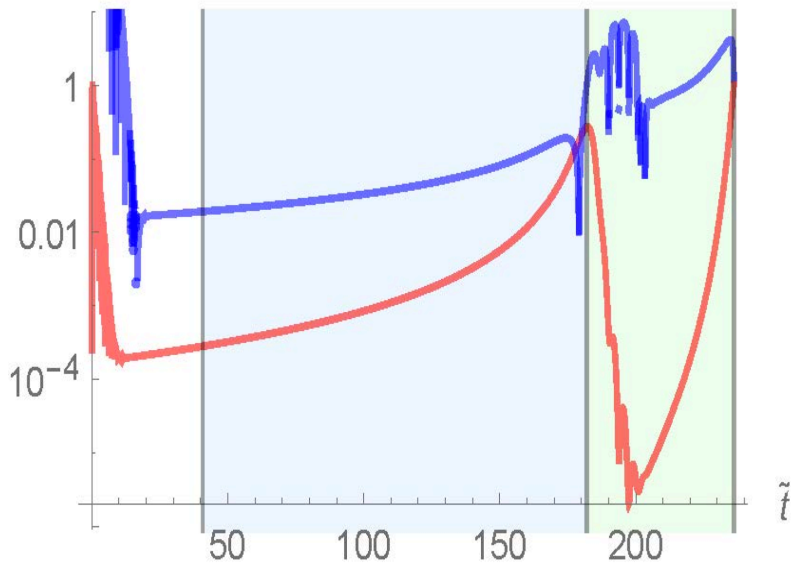
The solution, trajectory, Hubble function, e-foldings, and slow roll parameters











The observables and the parameter space

We computed the inflationary (CMB) observables as

$$n_s \approx 0.9600 \quad \text{and} \quad r_{\text{max}} \approx 0.004 \,, \tag{1}$$

corresponding to the pivot scale k_{60} (wavenumber) that leaves the horizon 60 e-folds before the end of inflation.
The parameters leading to a scalar potential with the desired properties are not unique, and for any γ greater than ~ 0.004 there is a value of ζ that leads to a similar shape of the potential (with two inflection points, unique Minkowski minimum, etc.). For a given γ, one can solve the system of equations

$$\partial_\varphi V = \partial_\sigma V = \mathbf{H} = 0 \,, \tag{2}$$

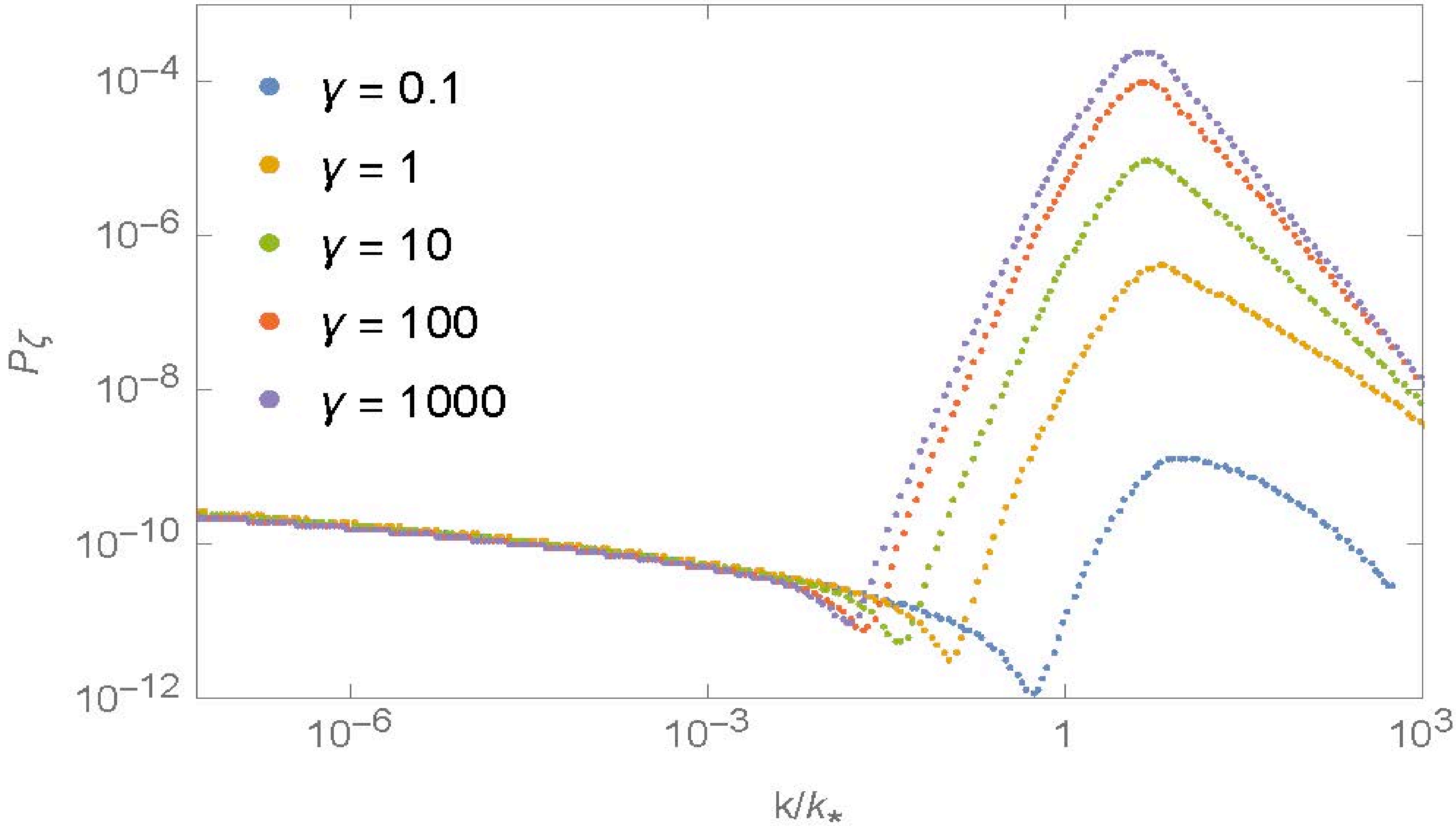
where $\mathbf{H}$ is the Hessian determinant of the potential, in order to obtain the value of ζ leading to the desired inflection points. Then, by fine-tuning ζ around that value, one can change a duration of the USR stage ΔN_2.

Our computational methods and strategy

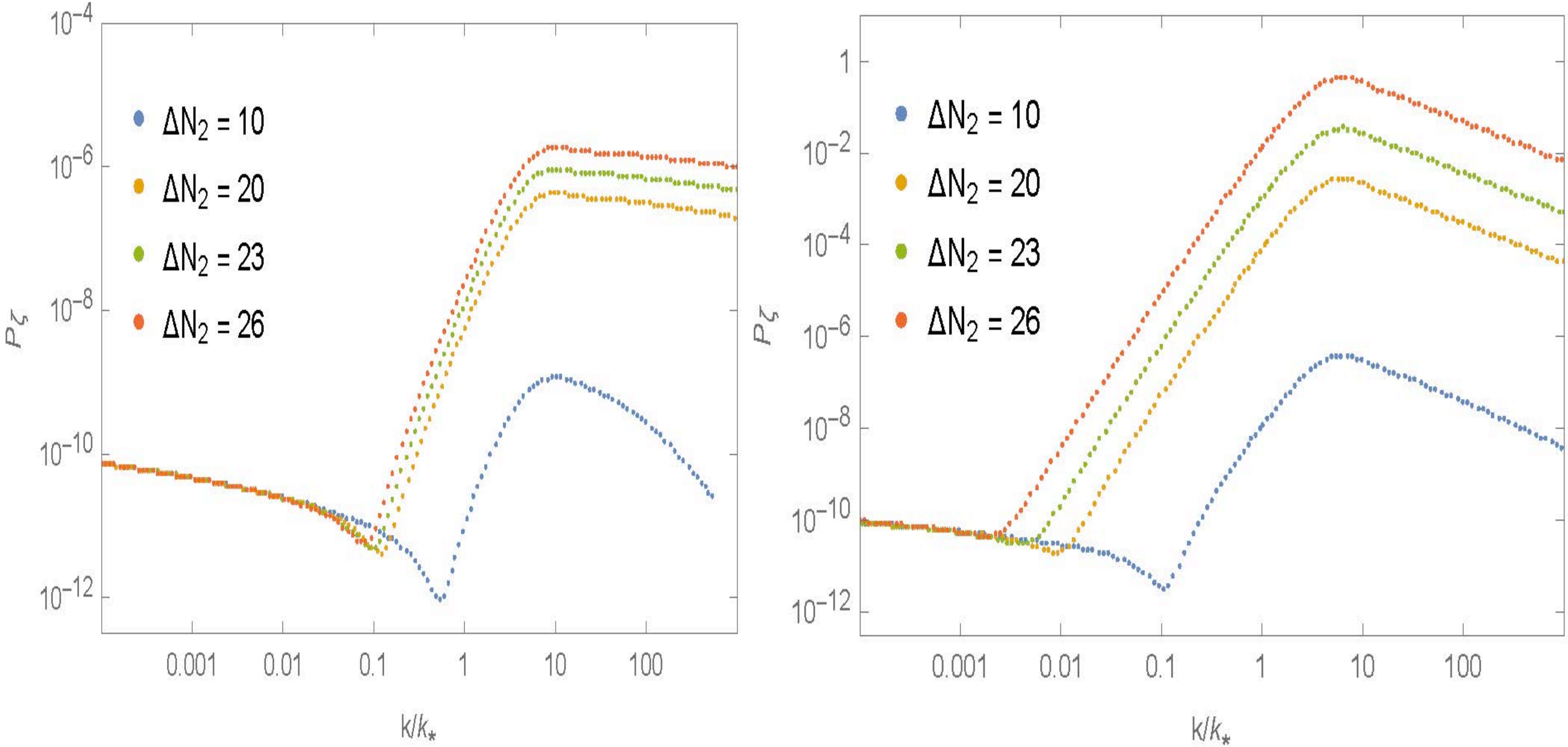
We numerically computed the power spectrum of curvature perturbations by using the *transport method* (Mulryne, 2009-2010) with the *Mathematica* package of Dias (2015), around the pivot scale k_* that leaves the horizon at the end of the first stage, i.e. ΔN_2 e-folds before the end of inflation (let us call this scale $k_{\Delta N_2}$). The inflaton mass was adjusted in each case around $\sim 10^{-5} M_{\mathrm{Pl}}$ by requiring $P_\zeta \approx 2 \times 10^{-9}$ for the mode k_{60}, first studying various values of γ (at fixed ΔN_2), and then various values of ΔN_2 for some values of γ.

ΔN_2	10	20	23	26
n_s	0.96	0.95	0.945	0.94
r_{max}	0.004	0.007	0.008	0.009

Power spectrum at $\Delta N_2 = 10$ for various values of γ



Power spectrum at $\gamma = 0.1$ (left) and $\gamma = 1$ (right) with changing ΔN_2



PBHs masses in the γ-model

The mass of a PBH created by late-inflationary overdensities was estimated by Pi, Zhang, Huang and Sasaki in arXiv:1712.09896:

$$M_{\mathrm{PBH}} \simeq \frac{M_{\mathrm{Pl}}^2}{H(t_{\mathrm{peak}})} \exp\left[2(N_{\mathrm{end}} - N_{\mathrm{peak}}) + \int_{t_{\mathrm{peak}}}^{t_{60}} \epsilon(t)H(t)dt\right] , \qquad (1)$$

where t_{peak} is the time when the wavenumber corresponding to the power spectrum peak (k_{peak}) exits the horizon, whereas t_{60} is the time when k_{60} exits the horizon (the beginning of observable inflation). By using this equation, we estimated the values of M_{PBH} for various values of ΔN_2 in our model:

ΔN_2	10	20	23	26
M_{PBH}, g	10^8	10^{16}	10^{19}	10^{21}
n_s	0.96	0.95	0.945	0.94

Comments about the PBHs masses

Our estimates are universal across the values of $\gamma = 0.1, 1, 10, 100$. PBHs with masses smaller than $\sim 10^{16} g$ would have already evaporated by now via Hawking radiation. Thus, we require $\Delta N_2 > 20$. On the other hand, the lower 3σ limit on the spectral index, $n_s \approx 0.946$, requires $\Delta N_2 < 23$, so that viable PBH masses are restricted by $\mathcal{O}(10^{16} g) < M_{\rm PBH} < \mathcal{O}(10^{19} g)$ even before considering observational constraints on PBHs masses.

As regards the constraints on γ, the obtained power spectrum tells us for $\Delta N_2 > 20$ that it is sufficient to have $\gamma \gtrsim \mathcal{O}(1)$ in order to produce the required enhancement in the spectrum.

PBHs density fraction

We numerically estimated the PBHs density fraction by using *Press-Schechter* (1973) formalism. The useful formulae include the PBH mass $\tilde{M}_{\rm PBH}(k)$, the production rate $\beta_f(k)$, and the density contrast $\sigma(k)$ coarse-grained over k:

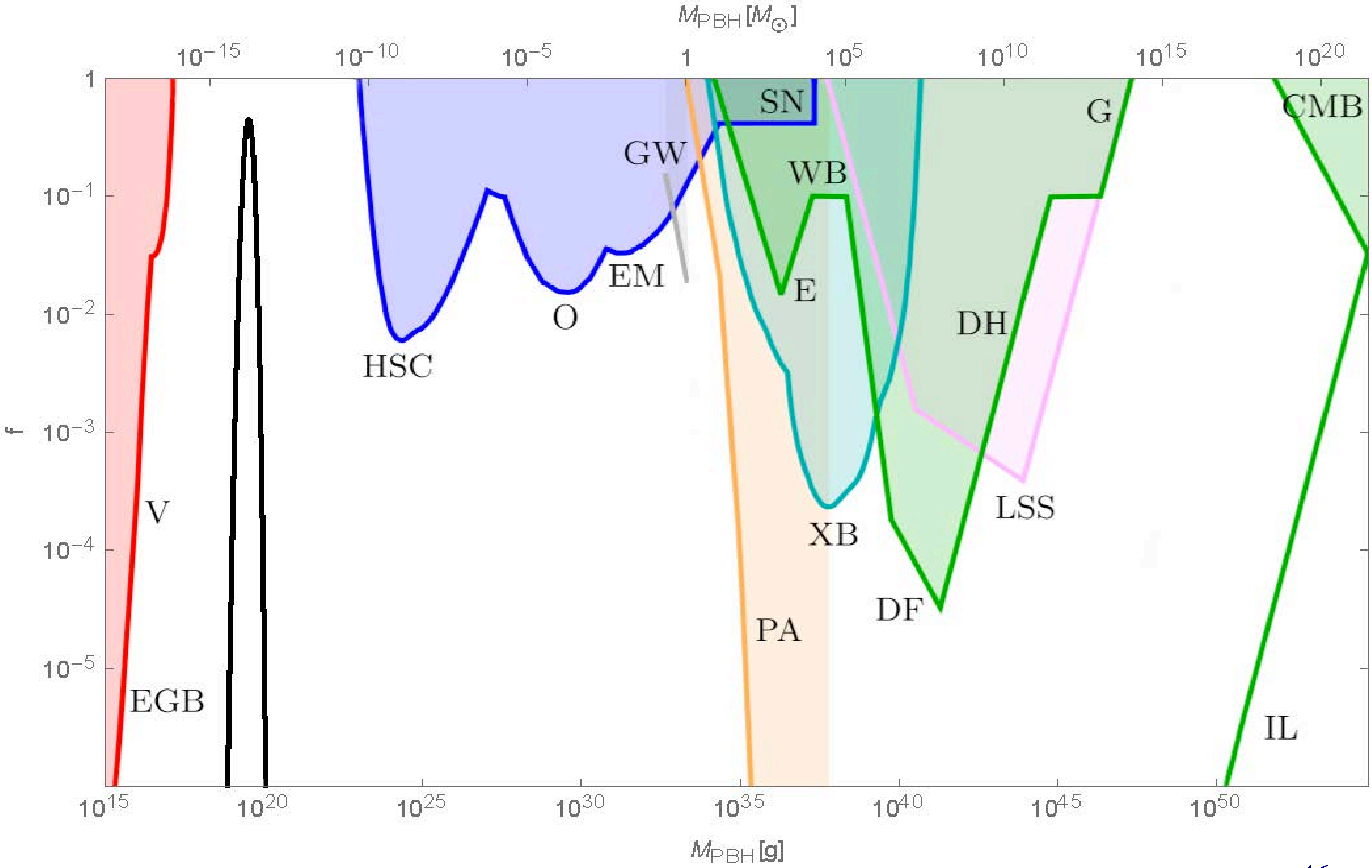
$$\tilde{M}_{\rm PBH} \simeq 10^{20}\left(\frac{7\times 10^{12}}{k\ {\rm Mpc}}\right)^2 {\rm g}\ , \quad \beta_f(k) \simeq \frac{\sigma(k)}{\sqrt{2\pi}\delta_c} e^{-\frac{\delta_c^2}{2\sigma^2(k)}}\ , \tag{2}$$

$$\sigma^2(k) = \frac{16}{81}\int \frac{dq}{q}\left(\frac{q}{k}\right)^4 e^{-q^2/k^2} P_\zeta(q)\ .$$

We have chosen the Gaussian window function for the density contrast, and have introduced δ_c is a constant representing the density threshold for PBH formation. According to Carr (1975), the naive estimate is $\delta_c \approx 1/3$, while its more precise value depends upon details of the power spectrum. Then the PBHs-to-DM density fraction is

$$\frac{\Omega_{\rm PBH}(k)}{\Omega_{\rm DM}} \equiv f(k) \simeq \frac{1.4\times 10^{24}\beta_f(k)}{\sqrt{\tilde{M}_{\rm PBH}(k){\rm g}^{-1}}}\ . \tag{3}$$

Comparison with observations based on Carr et al. (2020), in gamma-model



Comments on the comparison

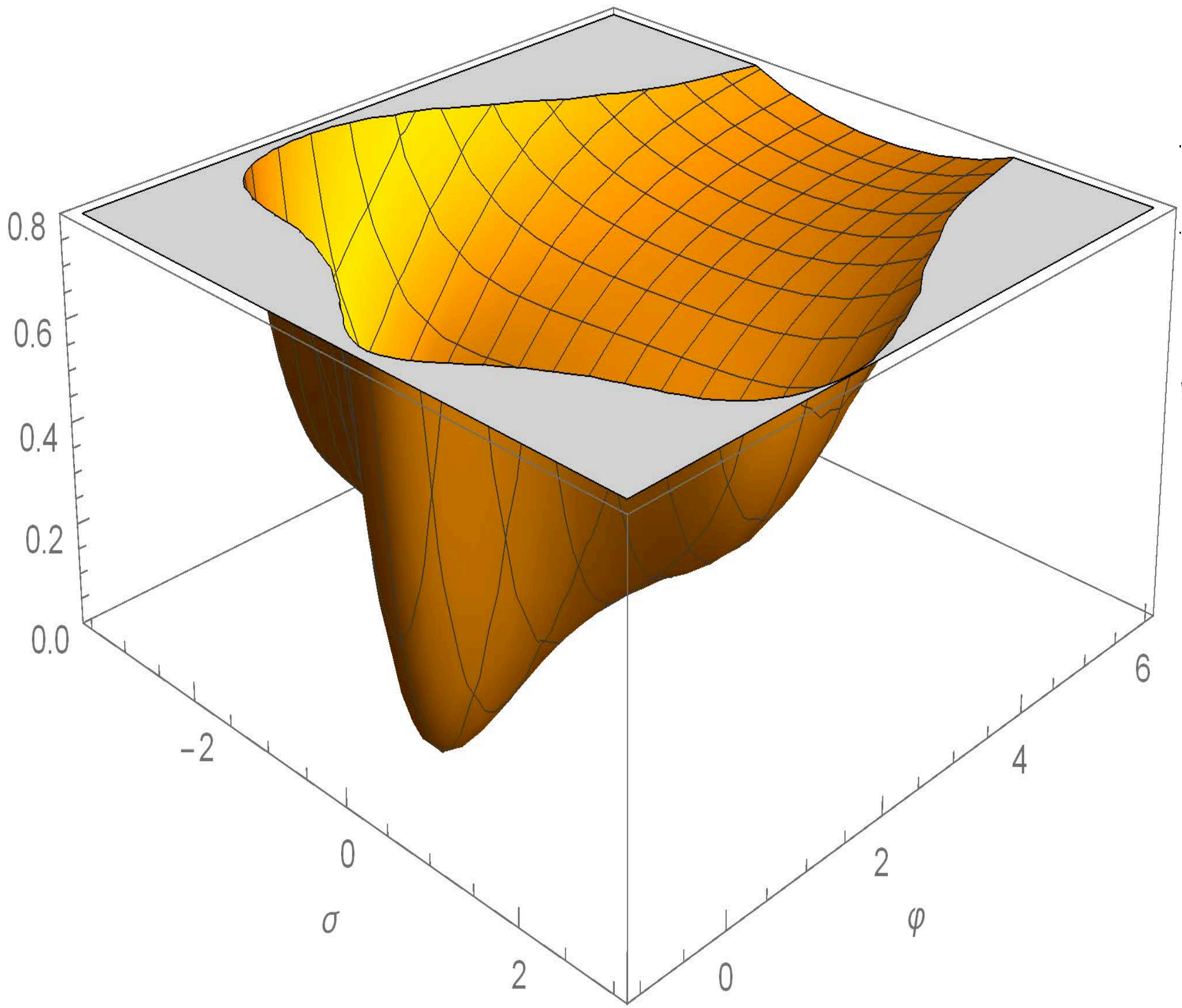
The PBHs fraction was obtained with the parameters $\gamma = 1$, $\Delta N_2 = 22$, and $\delta_c = 0.275$ (black curve). The shaded regions represent the observational constraints: from evaporation (red), lensing (purple), various dynamical effects (green), accretion (light blue), large-scale structure (dark blue), CMB distortions (orange), and background effects (grey). In the relevant regions, the notation F, WD, and NS is used to refer to femtolensing, white dwarfs, and neutron stars, respectively.

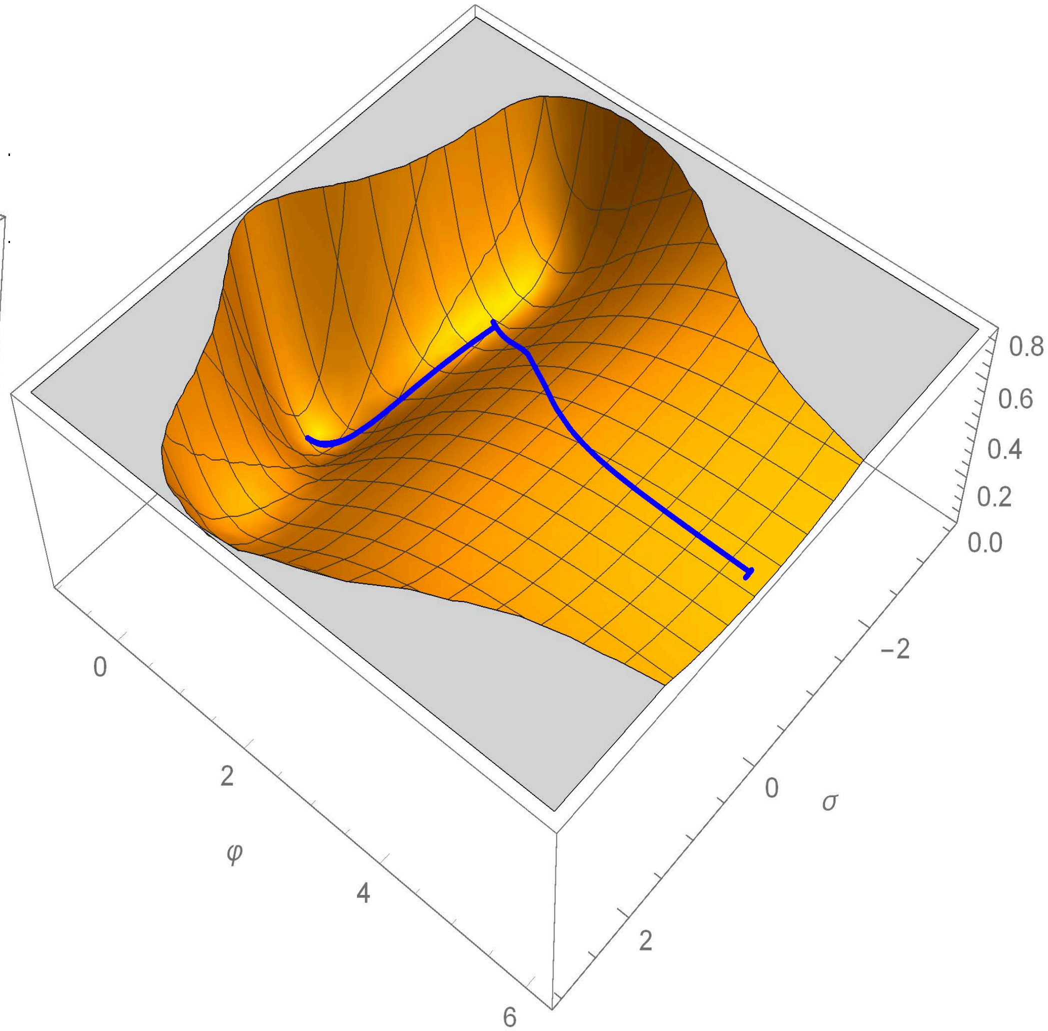
We choose the scale k_{60} to represent the largest observable scale today, which is around $10^{-4}\ \mathrm{Mpc}^{-1}$. Our numerical evaluation shows, in order to obtain a substantial density fraction, we need a relatively small δ_c.

Our peak overlaps with the constraints coming from observations of white dwarfs and neutron stars.

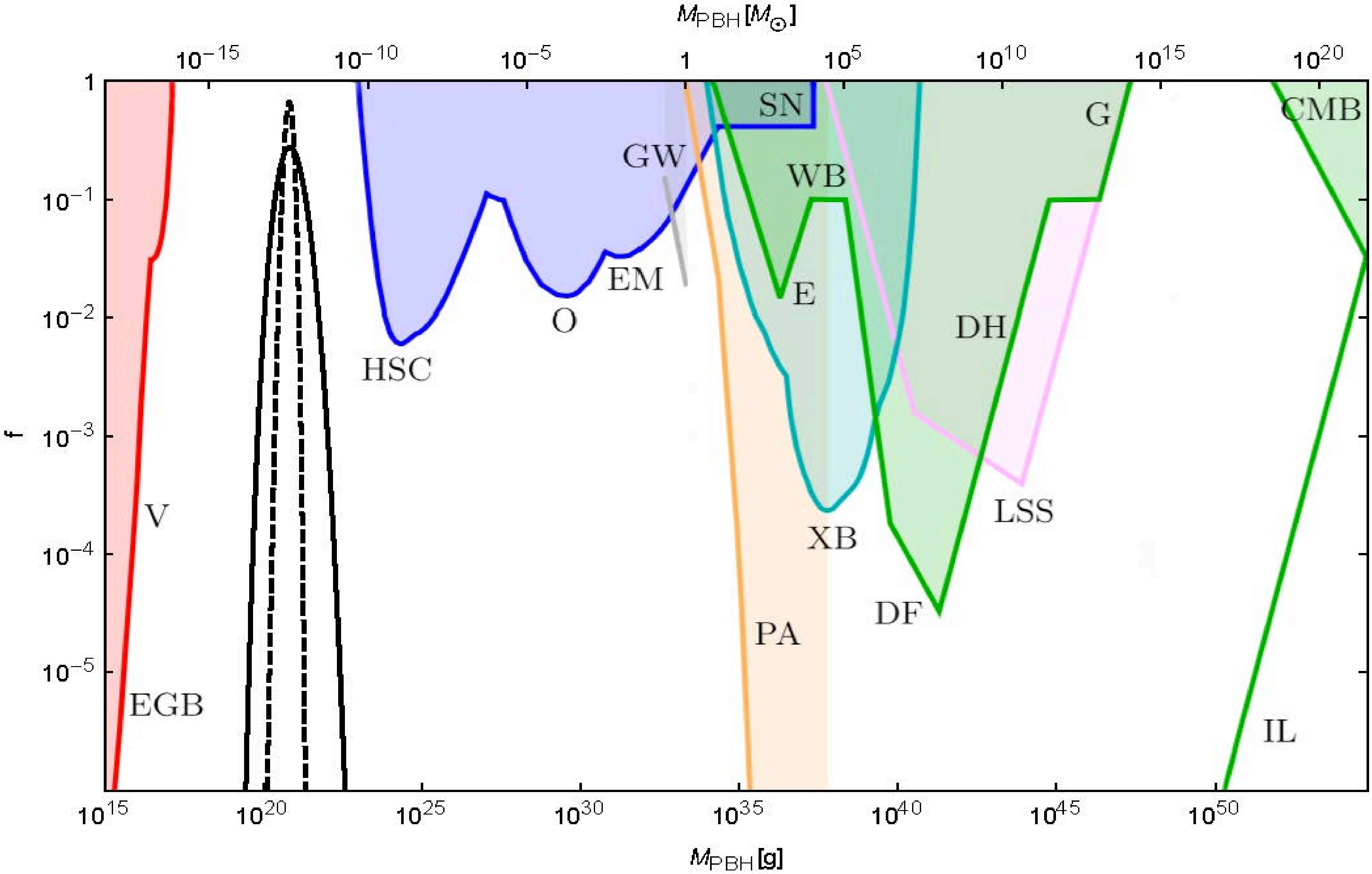
Comments about the δ-model vs. the γ-model

The scalar potential has only a single valley. The trajectories of solutions, Hubble functions, e-folding numbers and the slow-roll parameters are similar, as well as the power spectra, albeit with larger $\delta_c > 1/3$, and larger PBHs masses (up to 10^{23} g).

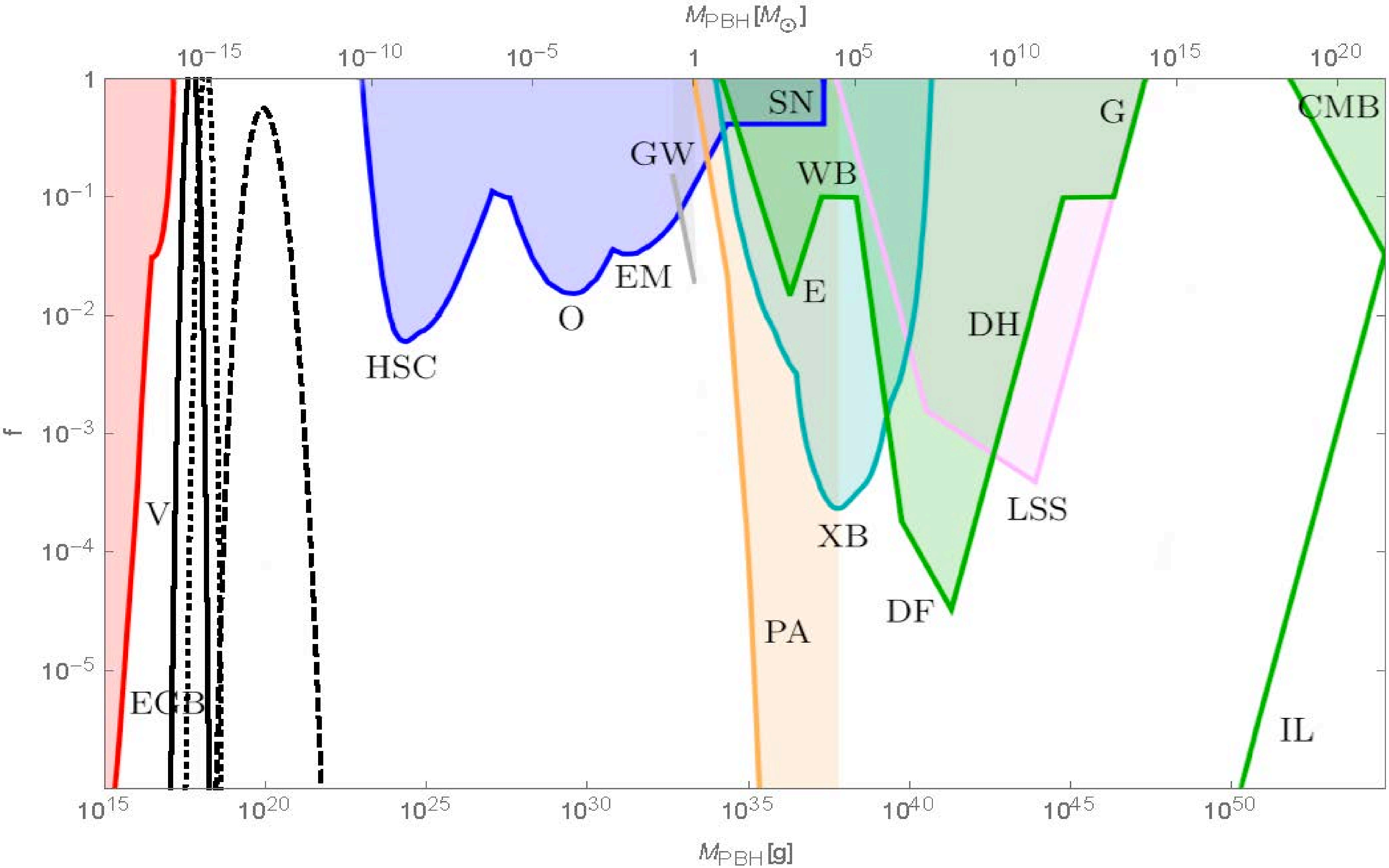




Comparison with observations (Carr et al. 2020) in the δ-model



The PBH density fraction in the models with $\gamma=1$, $\delta=0$, $\Delta N\_2=22.45$ (solid line), and $\delta=0.58$, $\Delta N\_2=23.36$ (dotted line). In both cases f_total=1.



Conclusion

- Starobinsky inflationary model is *not* only the best phenomenological fit to CMB observations but also is an important theoretical insight into physics of the early Universe and quantum gravity, namely, via the importance of the R^2 term. This claim is supported by supergravity theory.

- In particular, Higgs inflation is equivalent to Starobinsky inflation. In (standard) Einstein supergravity, both can appear in two different supersymmetric gauges of a single (*new-minimal*) supergravity model.

- The (*old-minimal*) supergravity extension of $(R + R^2)$ gravity is capable to describe both Starobinsky inflation and PBHs production (as a two-field double inflation) without adding matter d.o.f., i.e. by using a supergravity multiplet of fields and its (supergravitational) interactions only.

- The *modified* supergravity approach to inflation and PBHs production can be reformulated to the *standard* (Einstein) supergravity form with the specific (*no-scale*) Kähler potential K and superpotential W.

Outlook towards string theory

There are several *non-trivial* indications towards the existence of UV-completion of the proposed modified supergravities in superstring theory because of

- the appearance of the no-scale Kähler potential in modified supergravity, which generically arises in superstring compactifications (John Ellis et al. since 1985; S.J. Gates Jr. and SVK 2009);

- the existence of the Dirac-Born-Infeld-type extensions of our *single-field*-inflationary models in supergravity, which do not significantly alter observational constraints on inflation and PBHs production (H. Abe, S. Aoki, Y. Aldabergenov and SVK, arXiv:1808.00669 and 1812.01297);

- a possibility of interpreting our modified supergravity actions as parts of a D3-brane world-volume actions in type II strings (Binetruy, Dvali, Kallosh and Van Proeyen 2004; Aoki, Aldabergenov and SVK, arXiv:2001.09574).

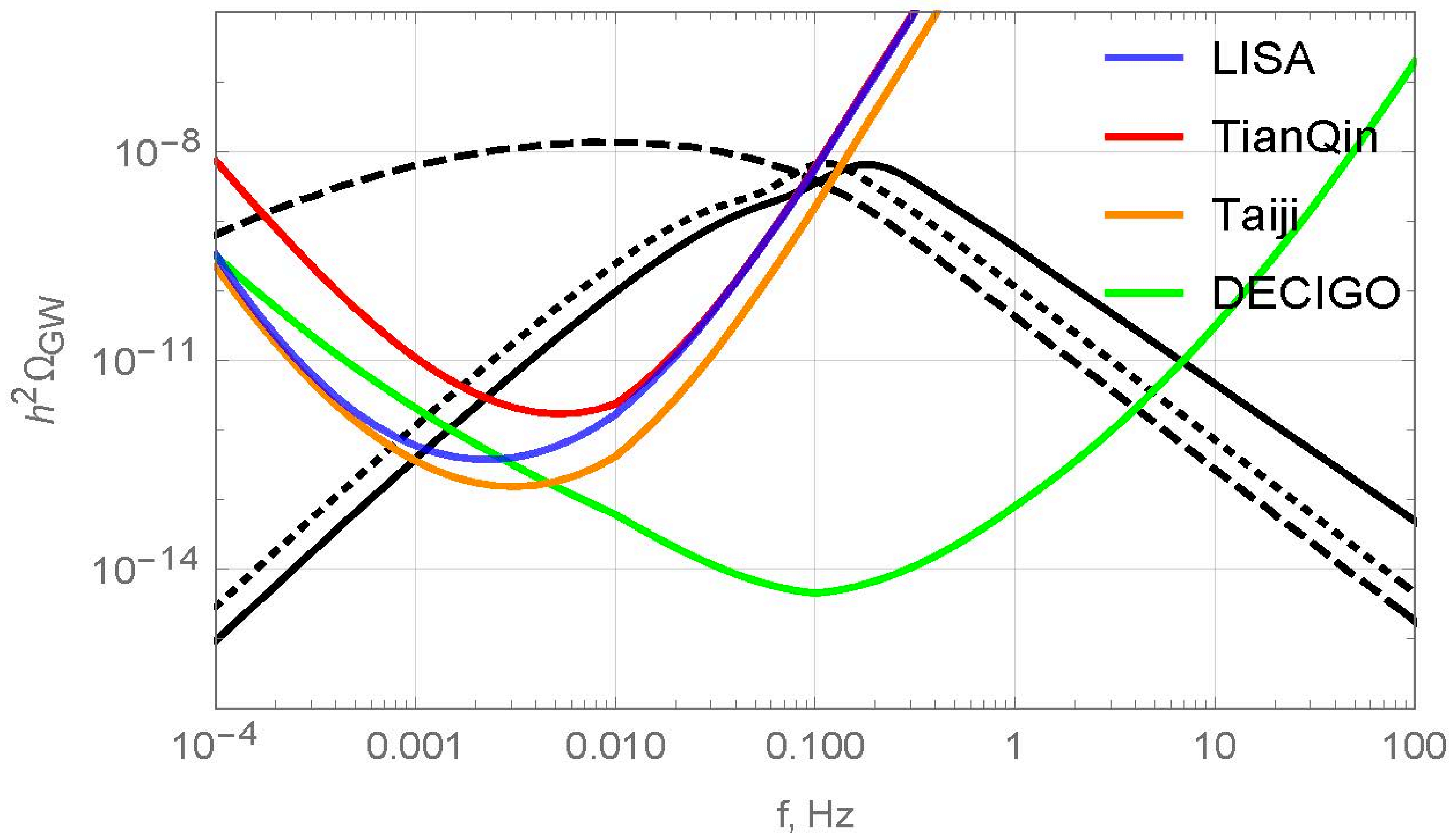
Outlook towards observations

The exploration of cosmological predictions from modified supergravity provides a remarkable bridge between quantum gravity on one side and phenomenology of inflation and PBHs on the other side.

- PBHs formation necessarily leads to Gravitational Waves (GWs) because large scalar overdensities act as a source for GWs background. Frequencies of those GWs can be directly related to expected PBHs masses and duration of the second stage of inflation.

- Those GWs may be detected in the future ground-based experiments, such as the Einstein telescope and the *global* network of GWs interferometers including advanced LIGO, Virgo and KAGRA, as well as in the space-based GWs interferometers such as LISA (or eLISA), TAIJI (old ALIA), and DECIGO.

The density of stochastic gravitational waves induced by the power spectrum enhancement in the our supergravity models (solid+dashed+dotted black curves) against the expected sensitivity curves of the space-based GW interferometers.



Thank You for Your Attention!