Expanding Universe

Lecture from the course
« Introduction to cosmoparticle physics »

Structure of visible Universe

1 pc = 3.26 l.y. = 3.086×10^{18} cm

$M_{Gal} \sim 10^{11} M_{Sun}$

Length scales:

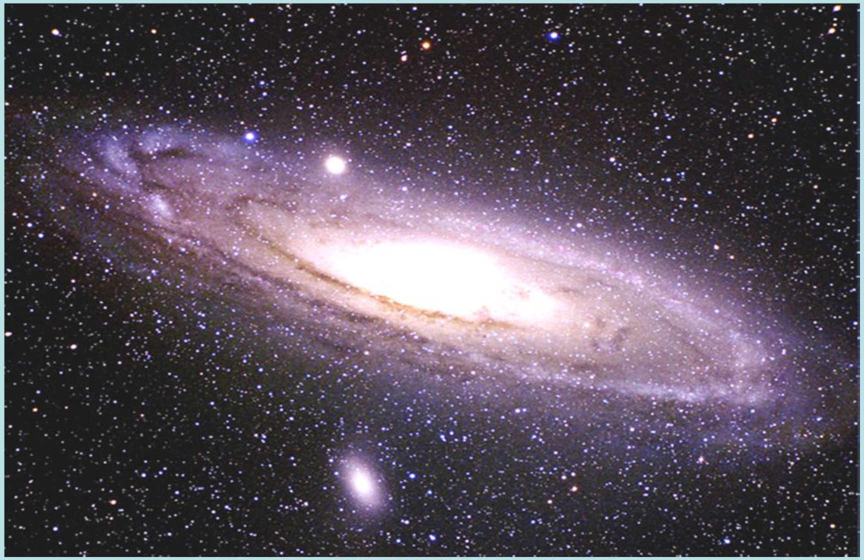
Galaxy --------------- ~30 kpc

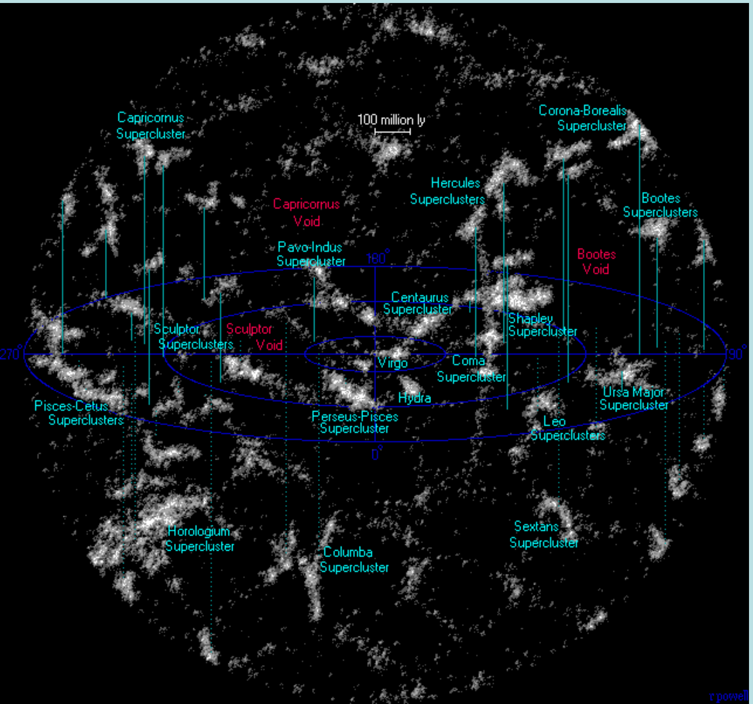
Local group ---------- ~1-10 Mpc

Cluster ---------------- ~100 Mpc

Universe* ------------- ~ 5000 Mpc

*) the size of visible part of Universe





At the scales >> 100 Mpc, the Universe looks homogenous and isotropic.

The Hubble law

Galaxies have systematic red shifts of spectral lines, what can be interpreted due to Doppler effect as a recessional velocity (Vesto Slipher, 1912-1917):

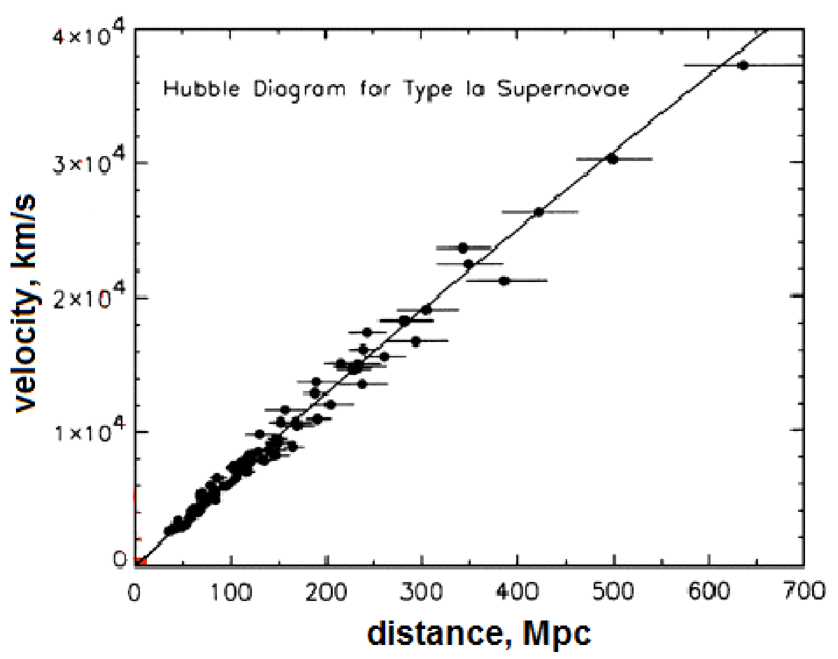
$$z=\Delta\lambda/\lambda \cong v/c \quad \text{(for } v \ll c\text{)}$$

In 1929, **Edwin Hubble**, having determined the distances to the galaxies ("extra-Galactic nebulas"), revealed the law of galaxies recession:

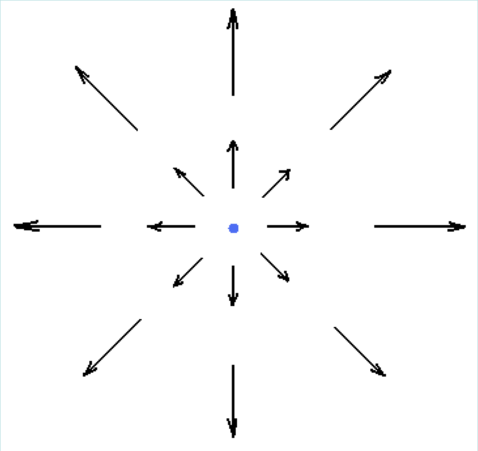
$$v = H^*R$$

Modern measurements give
H = 74.2 ± 3.6 (km/s)/Mpc

Hubble's result was H=576±6 (km/s)/Mpc



The Universal Expansion

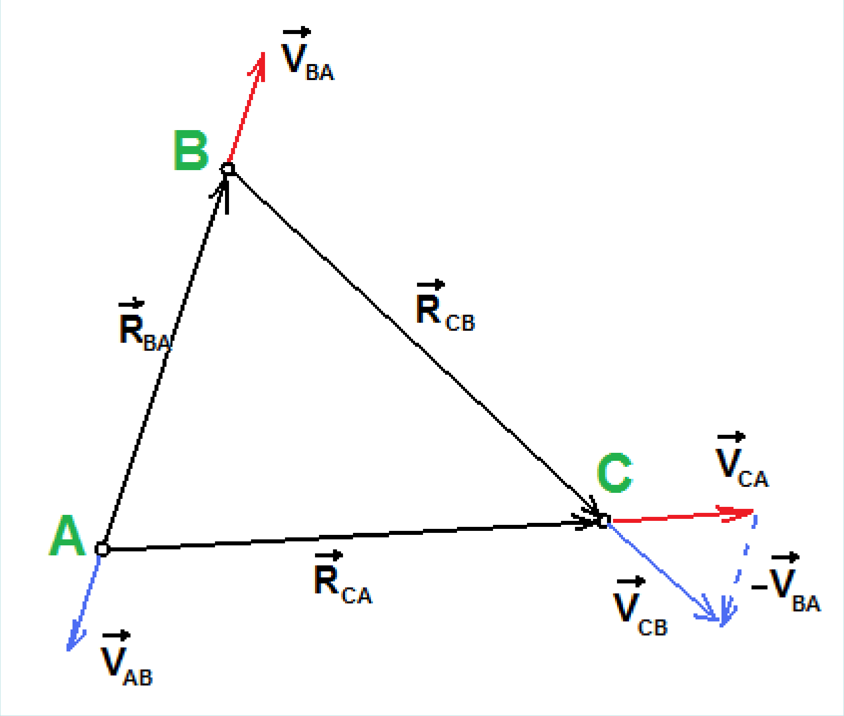


Such a law of recession does not contradict to homogeneity and isotropy of Universe (cosmological principle).

$$\vec{V}_{AB} = -\vec{V}_{BA} = -H\vec{R}_{BA} = H\vec{R}_{AB}$$

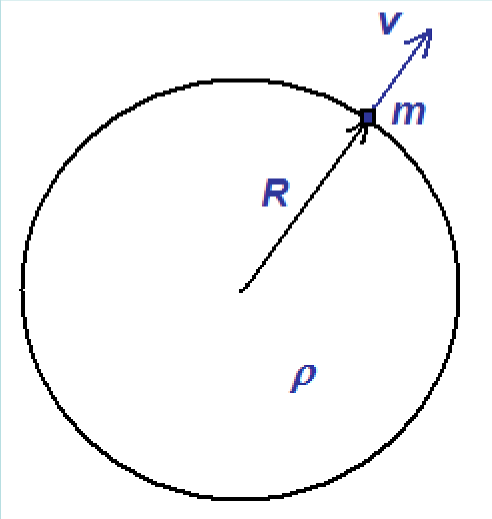
$$\vec{V}_{CB} = \vec{V}_{CA} - \vec{V}_{BA} = H\vec{R}_{CA} - H\vec{R}_{BA} =$$

$$= H(\vec{R}_{CA} - \vec{R}_{BA}) = H\vec{R}_{CB}$$



The Critical Density

To understand an ultimate fate of Universe (whether or not expansion is infinite), let us determine the density, at which recession tends to zero at infinity



$$\begin{cases} \dfrac{mv^2}{2} = \dfrac{GMm}{R} \\ v = HR \\ M = \dfrac{4}{3}\pi R^3 \cdot \rho \end{cases} \Rightarrow \boxed{\rho_{cr} = \dfrac{3H^2}{8\pi G}}$$

$\rho > \rho_{cr}$ – expansion is finite, it must change by contraction (closed world)
$\rho < \rho_{cr}$ – expansion is infinite (open world)
$\rho = \rho_{cr}$ – expansion is infinite, but its rate tends asymptotically to zero (flat world)

Expansion in General Relativity

GR (Einstein) equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad - \Lambda g_{\mu\nu}$$

Alexander Friedmann obtained the equations of evolution of Universe from GR under suppositions of homogeneity and isotropy of Universe.

$$\begin{cases} \dfrac{\ddot{a}}{a} = -\dfrac{4\pi G}{3}\left(\varepsilon + \dfrac{3p}{c^2}\right) \quad + \dfrac{\Lambda c^2}{3} \\ \left(\dfrac{\dot{a}}{a}\right)^2 - \dfrac{8\pi G \varepsilon}{3} = -\dfrac{K c^2}{a^2} \quad + \dfrac{2\Lambda c^2}{3} \end{cases} \qquad H \equiv \frac{\dot{a}}{a}$$

Here a is the scale factor, ε and p are the energy density and pressure of matter respectively, K is the parameter of curvature.

Equation of state

$$p = 0$$ **- non-relativistic ("dust"-like) matter**

Compare $p = nkT \sim \varepsilon_{kin} << mc^2 \cdot n = \varepsilon$

$$p = \frac{\varepsilon}{3}$$ **- (ultra)relativistic (radiation-like) matter**

$$p = -\varepsilon$$ **- vacuum-like matter (vacuum energy)**

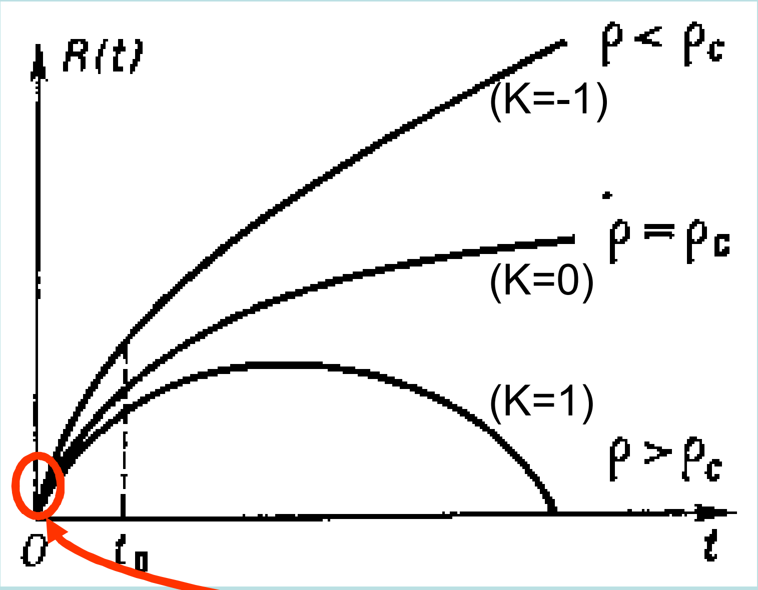
Here and further we use the units

$$\hbar = c = k = 1$$

The Beginning of Expansion

t **~1/*H* defines the age of the Universe – time from the beginning of expansion**

Λ=0



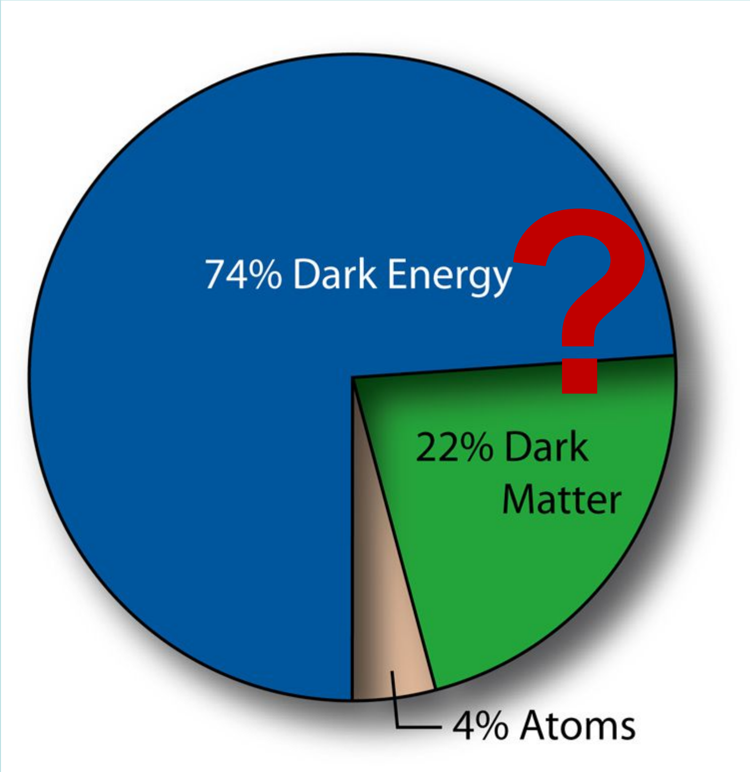
$$m_{Pl} = \sqrt{\frac{\hbar c}{G}} \approx 2\cdot 10^{-5}\,\mathrm{g} \approx 1.2\cdot 10^{19}\,\mathrm{GeV}$$

$$l_{Pl} = \sqrt{\frac{G\hbar}{c^3}} \approx 1.6\cdot 10^{-33}\,\mathrm{cm}$$

$$t_{Pl} = \sqrt{\frac{G\hbar}{c^5}} \approx 0.5\cdot 10^{-43}\,\mathrm{s}$$

K=-1 corresponds to open world, K=0 to flat world and K=1 to closed world. In the early Universe effects of curvature were negligeble and the expansion was very close to flat model. Formally, the model is singular at *t*=0, but the classical description of space time is limited by Planck scales, at which quantum effects of gravity should be taken into account.

The Modern Composition



$$\Omega \equiv \frac{\rho}{\rho_{cr}}$$

$$\Omega_b \approx 0.044 \qquad \Omega_{CMB} \approx 0.5 \cdot 10^{-4}$$

$$\Omega_{DM} \approx 0.20$$

$$\Omega_{\Lambda} \approx 0.7$$

$$\Omega_{tot} \approx 1.0$$

In the modern Universe dominate dark energy and dark matter – their nature will be the subject of our analysis, which strongly involves the important role of a tiny component of CMB.

Relic Radiation

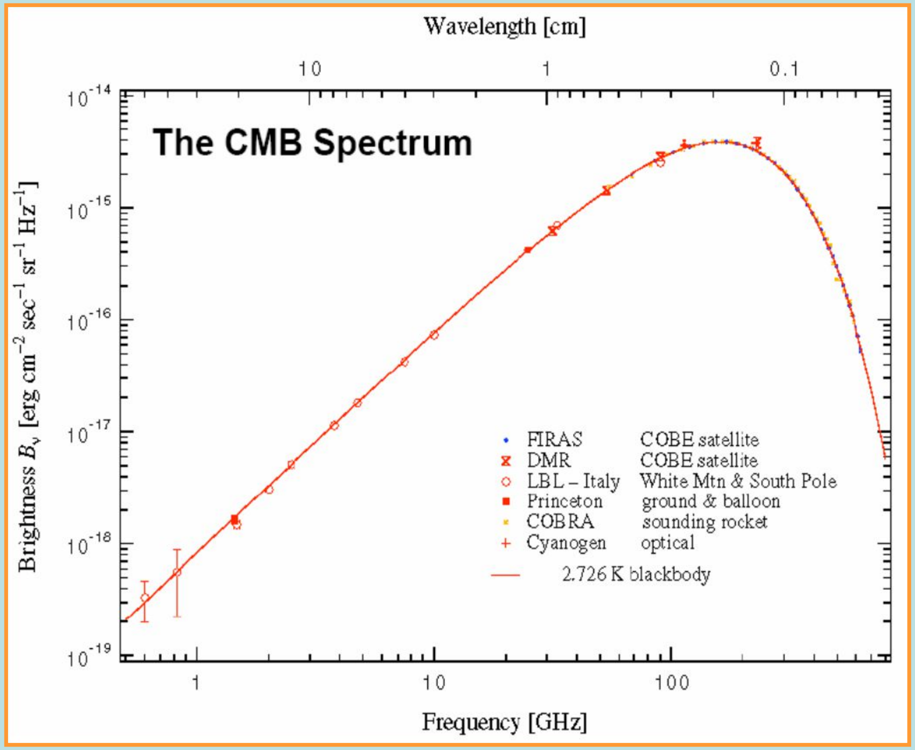
Cosmic microwave background (CMB):

Planckian black body spectrum with

T=2.73 K

n_{γ}=411 cm^{-3}

$\delta T/T$~ 10^{-4}



T is proportional to $1/a$.

Energy density of this radiation ~$3Tn_{\gamma}$ was much larger in the early Universe.

For the fixed ratio of number densities of baryons (nucleons) and photons :

n_b/n_{γ} ~ 0.6x10^{-9} it means inevitable dominance of radiation over the baryonic matter in the early Universe

Adiabatic Law of Expansion

From Friedmann's equations one can get

$$\dot{\varepsilon} = -3H(\varepsilon + p)$$

$d(\varepsilon \cdot V) + p \cdot dV = 0$ where $dV = d(a^3)/a^3 \cdot V = 3Hdt \cdot V$

This means that expansion can be interpreted like adiabatic process (*dQ=dU+dA=0*), what justifies application of respective laws of thermodynamics.

Going back in time, we come to denser and hotter matter. This trend can be followed up to Planck scales.

It was just this trend that made people to think about the early Universe as on a natural laboratory for super high energy physics.

However, it turned out that the history of the Universe was much more complicated and inevitably involved super high energy physics as the basic elements of this history.

Conclusions

- **Cosmological expansion leads to conclusion that the early Universe was very dense.**
- **The discovery of Cosmic Microwave background radiation in the expanding Universe proved that these early stages were very hot.**
- **Combination of these two trends leads to the idea of a hot expanding Universe – to the Big Bang model.**
- **It makes early Universe a natural laboratory of super high energy physics.**
- **However, as we'll see in the future lectures this physics in its turn causes strong influence on the properties of the early Universe.**