Dynamical evolution of the cluster of primordial black holes

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Introduction

Quantum fluctuations of scalar fields during inflation can lead to formation of cluster of PBHs.

Some astrophysical effects can be attributed to PBHs:

- Supermassive black holes at large redshift[1] $z > 6$.
- PBHs form a part of the Dark Matter[2].
- The data from mergers of black holes from LIGO/Virgo[3].
- PBHs can contribute to the reionization of the Universe[4].

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[1] ApJ 833 (2016) 222
[2] Phys. Rev. D 94 (2016) 083504
[3] Phys. Rev. Lett. 118 (2017) 221101
[4] Int. J. Mod. Phys. D 26 (2017) 1750102

Main equations (1)

The Boltzmann equation:

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \nabla_{\boldsymbol{r}} f - \nabla_{\boldsymbol{r}} \phi \cdot \nabla_{\boldsymbol{v}} f = I[f, \boldsymbol{v}, \boldsymbol{r}], \quad (1)$$

The Poisson equation:

$$\nabla^2 \phi = 4\pi G \rho(\mathbf{r}), \quad (2)$$

our assumptions:

- Two-body relaxation $\rightarrow$ the Fokker-Planck equation.
- The quasi-stationary case $\rightarrow$ Jeans's theorem: $f(E, L, ...)$.
- $T_{relax} \gg T_{orbital}$ $\rightarrow$ an orbital integration.
- The isotropy in the space of angular momentum $\rightarrow$ $f(E, t)$.
- The cluster evolves in empty space.

Main equations (2)

The Fokker-Planck equation[5]:

$$p(E)\frac{\partial f_a}{\partial t} = \frac{\partial}{\partial E}\left[m_a f_a B(E,f) + \frac{\partial f_a}{\partial E}C(E,f)\right] - \nu(E,f)f_a, \qquad (3)$$

νf — the loss cone term.
The Poisson equation:

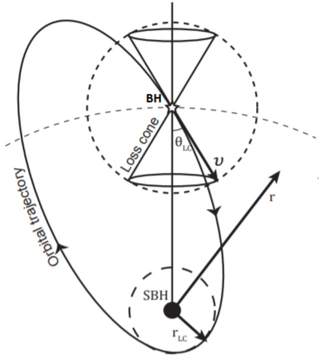
$$\phi(r) = -4\pi G\left[\frac{1}{r}\int_0^r dr' r'^2 \rho(r') + \int_r^\infty dr' r' \rho(r')\right]. \qquad (4)$$

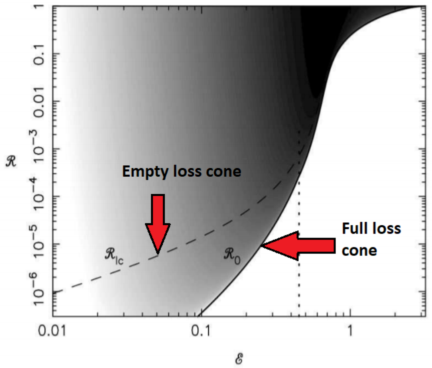
The density profile:

$$\rho(r) = 4\pi \sum_a m_a \int_{\phi(r)}^0 dE \sqrt{2(E-\phi(r))}\, f_a(E). \qquad (5)$$

[5] ApJ 848 (2017) 10

Loss Cone





$\mathcal{R} = \frac{L^2}{L_c^2(E)}$, $\mathcal{R}_{lc} = \frac{L_{lc}^2}{L_c^2}$, $L_{lc} = 2mcr_g$, $r_{lc} = 4r_g$.
In one orbital period, a BH can acquire an angular momentum greater than L_{lc}, as a result, the critical region «decreases»[6].

[6] Class. Quantum Grav. 30 (2013) 244005

Initial data

The density profile:

$$\rho_a(r) = \rho_0 \left(\frac{r}{r_0}\right)^{-\gamma} \left[1 + \left(\frac{r}{r_0}\right)^{\alpha}\right]^{(\gamma-\beta)/\alpha}, \qquad (6)$$

$\gamma = 1$, $\alpha = 2$, $\beta = 5$, $r_0 = 1$ pc.
The distribution function:

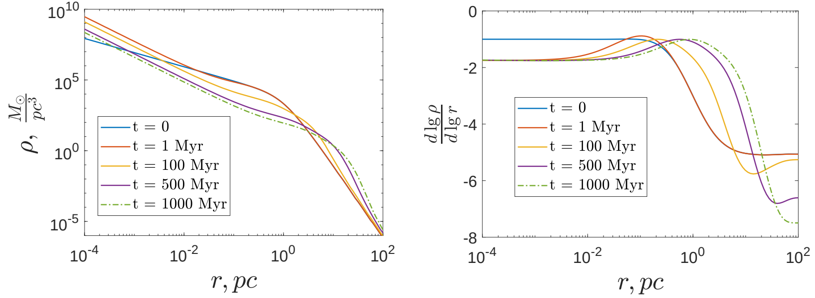
$$f_a(E) = \frac{\sqrt{2}}{4\pi^2 m_a} \frac{d}{dE} \int_E^0 \frac{d\phi}{\sqrt{\phi - E}} \frac{d\rho_a}{d\phi}. \qquad (7)$$

The mass spectrum:

$$\frac{dN}{dM} \propto \frac{1}{M_\odot} \left(\frac{M}{M_\odot}\right)^s \Bigg|_{10^{-4} M_\odot}^{10 M_\odot}, \qquad (8)$$

$s = -2,\ N_{tot} = 10^8,\ M_{tot} = 5.4 \cdot 10^4 M_\odot,\ M_{cbh} = 10^3 M_\odot.$

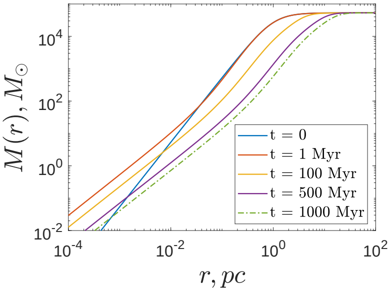
Density profile and its slope

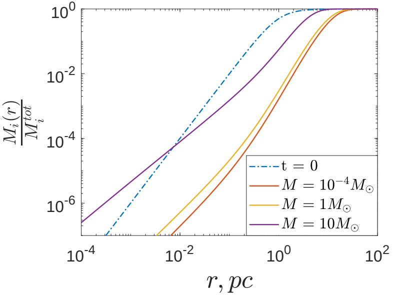


- $\rho \propto r^{-7/4}$ — the Bahcall–Wolf cusp[7] in the central region after 1 Myr.
- After the cluster relaxation, the evolution goes into a slow mode.
- The B-W law of the density profile extends over large r.
- The magnitude of the cusp decreases.

[7] ApJ 209 (1976) 214-232

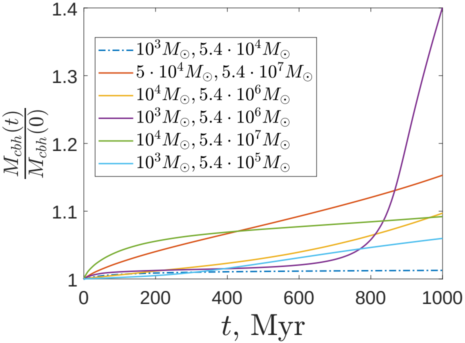
Dependence of mass of radius





- The initial radius of the cluster is $R \sim 3$ pc, final is $R \sim 23$ pc.
- The mass segregation.

Rate of growth of the central black hole



The first value in the legend is the mass of the central black hole, the second is the total mass of the cluster without the central black hole. The dotted-dashed line is the cluster in question.

As a result of an accretion, the mass of the central black hole will also increase that can affect on the system evolution.

Conclusion

The dynamic evolution of a cluster of primordial black holes is studied.

- In the central region, the cusp: $\rho \propto r^{-7/4}$.
- The size of the cluster increases 10 times.
- The mass segregation occurs.
- The mass of the central black hole grows insignificant.

Open question: how much will external conditions affect the evolution of the cluster?

Expression for coefficients

$$B(E) = \Gamma \sum_b m_b \int_{E_{crit}}^{E} p(E')f_b(E')\, dE',$$

$$C(E) = \Gamma \sum_b m_b^2 \left(\int_{E_{crit}}^{E} q(E')f_b(E')\, dE' + q(E) \int_{E}^{0} f_b(E')\, dE' \right),$$

$$q(E) = \frac{4}{3} \int_0^{\phi^{-1}(E)} dr\, r^2 [2(E - \phi(r))]^{3/2},$$

$$p(E) = 4 \int_0^{\phi^{-1}(E)} dr\, r^2 \sqrt{2(E - \phi(r))},$$

$\Gamma = 16\pi^2 G^2 \ln \Lambda$, $\Lambda \sim N$.

E_{crit} is $\sim$ energy of the last stable circular orbit around the central black hole.

Loss cone term

$$\nu(E) = \frac{p(E)\mathcal{D}(E)}{\alpha + \ln(1/\mathcal{R}_{lc})}, \quad \alpha = (\xi^2 + \xi^4)^{1/4},$$

$$\xi(E) = \frac{\mathcal{D}(E)\, P(E)}{\mathcal{R}_{lc}},$$

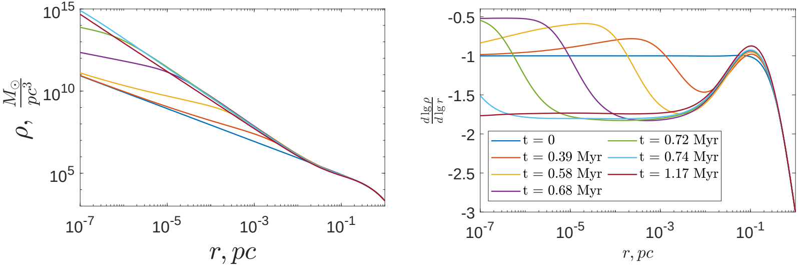
$$\mathcal{D}(E) = \frac{2}{p(E)} \int_0^{\phi^{-1}(E)} \frac{\langle \Delta v_\perp^2 \rangle\, r^2\, dr}{\sqrt{2(E - \phi(r))}},$$

$$\langle \Delta v_\perp^2 \rangle = \Gamma \sum_i m_i^2 \left(\frac{4}{3} I_{0,i} + 2J_{1/2,i} - \frac{2}{3} J_{3/2,i} \right),$$

$$I_{0,i}(E) = \int_E^0 f_i(E')\, dE',$$

$$J_{n,i}(E, r) = \int_{\phi(r)}^{E} dE'\, f_i(E') \left(\frac{E' - \phi(r)}{E - \phi(r)} \right)^n.$$

Establishment of Bahcall-Wolf cusp



Some expressions

Orbital integration:

$$dN = f(\mathbf{v},\mathbf{r})\sqrt{g}\,d\mathbf{v}d\mathbf{r} \to N(E,L) = f(E,L)\int_{orb}\sqrt{g}\,d\mathbf{r},$$
$$\sqrt{g} = \frac{L}{r^2 v_r} \to N(E,L)dEdL = 16\pi^2 L f(E,L)\left(\int_{r_-}^{r_+}\frac{dr}{v_r}\right)dEdL. \quad (9)$$

Estimation of relaxation and orbital time:

$$\langle(\Delta v)^2\rangle \cdot T_r = v^2 \to T_r = \frac{v^3}{8\pi G^2 m\rho \ln\Lambda}, \quad T_{orb} \sim \frac{2\pi R}{v}. \quad (10)$$

$v \sim 10$ km$\cdot$s^{-1}, $\rho \sim 10^5 M_\odot\cdot$pc^{-3}, $m \sim 10^{-2} M_\odot$, $\Lambda \sim N$, $N = 10^8$, $R = 1$ pc $\to T_r \sim 100$ Myr, $T_{orb} \sim 1$ Myr.

Strong scattering:

$$\sigma_s = \frac{4\pi G^2 m^2}{v^4}\,\mathrm{ctg}^2\,\psi \to T_s = \frac{1}{n\sigma_s v} \sim \frac{v^3}{\pi G^2 m\rho} \sim 20 Gyr. \quad (11)$$