Fake Scattering, Finite String Field Theory like Formulation

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Novel String Field Theory and Unitarity, although in non-relativistic case (Fake Scattering, Hope for finiteness). Part 1

Fake Scattering Concept

I am fashinated by the idea of making a quantum field theory like theory so that in a **fundamental** sense there is **No Timedevelopment**, but when looking at it appropriately, then you can "see" it as e.g. string field theory (a theory of second quantized strings).

This fake-scattering concept is implemented in the "Novel string field theory" long put forward by Masao Ninomiya and me (HBN).

Hamiltonian =0 gives no time-development

So quantum mechanically the **no time-development theory** is just a Hilbert space of the states, and they never develop - there is basically no time needed -.

In the **"Novel string field theory"** of ours the states in this Hilbert space are described formally by a second quantized theory of particles that can occur in different numbers just like in usual second quantized theory. We call these particles "objects" and they are crudely to be considered small pieces of strings like in the Charles Thorn string bit theory. But very importantly **we** first split the string into bits, **after we hav gone to the light cone varibles on the string**: $\tau - \sigma$ and $\tau + \sigma$.

Introduction of Fake Degrees of Freedom

In the philosophy that the true **fundamental thory** has **no time** (or say no time development) means that all development with time has to be fake. That is to say it has to be in some degrees of freedom, that do not really exist in nature, but which we the physicists introduce formally so as to make a theory more in agreement with our usual picture of how physics is.
Abstractly we replace each basis vector in a basis for the second quantized Hilbert space by a series of bases-vectors.
Then we allow the "fake-development" - the fake Hamiltonian - to only move arround the basis-vectors into each other which belong to the same fundamental basisvector.

A String Field Theory Inspired Example

To a good enough approximation the listener can imagine that our "objects" (after some techniqual details of only using the "even" ones among them) are (scalar) **particles** with position and momenta in a 25+1 dimensional world (or if we choose an infinite momentum frame in 24 transverse dimensions), and that there in any single particle state for such a particle can be a number of particles, just as in second quantization.
To avoid the problems with relativity, Dirac sea etc., we like to for pedagogical reasons effectively consider a non-relativistic theory, or almost equivalent an infinite momentum frame formulation.

The Pedagogical Non-relativistic model with Zero Hamiltonian

We consider a model with say non-relativistic bosons - so that they can occur in any number in any sigle particle state -. To make not develop in time we want to simplify to make the Hamiltonian zero

$$H = 0, \tag{1}$$

which in addition to having no interactions mean that we let the non-relativistic mass

$$m \rightarrow \infty, \tag{2}$$

so that even the kinetic term $\frac{\vec{p}^2}{2m}$ goes to zero.
We can choose a basis for the single particle states to be e.g. either the momentum eigenstates or the position eigenstates (a priori as we wish).

Second Quantizing our $H = 0$ Particle Model:

As basis to use in single particle Hilbert space we shall here choose the position eigenstates because we like to talk about a "nearness" concept (we want to say if two particles described by such basis vectors chosen are close or far apart.)
Then the corresponding basis in the second quantization state space is enumerated by a function, that to every position $\vec{x}$ assigns a number $n(\vec{x})$ giving the number of particles with exactly the position $\vec{x}$.
In other words

$$n : \mathbf{R}^{24} \rightarrow \{0, 1, 2, ...\} \tag{3}$$

and we cannot require it continuos unless we take it to be only constant, but we at this stage do not talk about continuity.

Second Quantized Basis

A basis - and this is the one we now have chosen to use - in the second quantized state space consists of vectors like

$$|n> \;\; = \;\; \prod_{\vec{x}} \frac{a^{\dagger}(\vec{x})^{n(\vec{x})}}{\sqrt{n(\vec{x})}} |n = 0> \qquad (4)$$

where $a^{\dagger}(\vec{x})$ is the creation operator for a particle at the position $\vec{x}$. The symbol **R** stands for the set of real numbers.
Remember

$$n : \mathbf{R}^{24} \rightarrow \{0, 1, 2, ...\}. \qquad (5)$$

Introduction of the Fake Degree of Freedom "The Successor Function" f

Our extremely simple $H = 0$ theory just introduced has a priori nothing to do with strings (nor much other sensible physics for that matter), but now we want by just talking make it into a string field theory!
For each single one $|n>$ of our basis states in the second quantized space we want to introduce a "sucessor function " f, which is a permutation of the particles present in that state.
In the state $|n>$ there are

$$N(n) = \sum_{\vec{x}} n(\vec{x}) \quad (6)$$

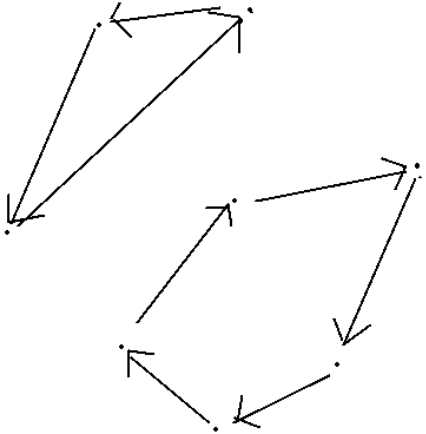
particles present. Here we cheated and assumed that there were not infinitely many particles present.

The "Successor function" f is a Permutation of the Particles present in the state $|n>$.

Assuming that there are only finitely many particles in a second quantized state vector $|n>$ we can think of these $N(n)$ particles as true particles, and you could define $N(n)!$ permutations f of the $N(n)$ particles present.

The dots are points with a $n(\vec{x}) = 1$.

The arrows denote action of f.



We Think of a Phantasy-space with $|n>$ replaced by $N(n)!$ new phatasy basis vectors representing the same true physics.

So the new basis-vectors in the second quantized space should be denoted

$$|n, f> \;=\; (|n>, f) \text{ where } f \in P_{N(n)} \tag{7}$$

where again

$$N(n) \;=\; \sum_{\vec{x}} n(\vec{x}) \tag{8}$$

is the number of particles in the state $|n>$.

Working with Phantasy Makes Life Easier

Of course it is f which is the phantasy degree of freedom. It was just introduced by us.
So we can decide - just we like so for some reason to explained possibly later - to say that we throw away all the choices of the permutation f, for which the position of a particle $\vec{x}_{first}$ and the particle into which f maps it $\vec{x}_{f(first)}$ are not close. I.e. we require only to include in our phantasy the f's satisfying

$$f(first) \quad \text{close to} \quad first \tag{9}$$

I.e.

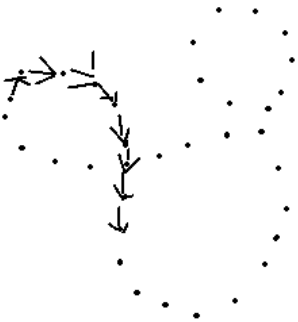
$$\left|\vec{x}_{first} - \vec{x}_{f(first)}\right| \quad \text{small.} \tag{10}$$

If f does not obey this restriction, we simply take it out and let there be fewer state vectors in the phantasy Hilbert space.

We can phantasize that f describes successors in long almost connected chains

We can choose the f permutations, we allow, to be such that they describe connected closed loop chains of the particles in the state, so well it now is possible.

From our purpose of making theory to be part of a speculated theory for everything we could be allowed to postulate something - if beautifull enough - also **about the state of the universe**, such as that the most liklely type of state is one in which the particles sit in long circular chains with rather small distance between the neighbors and even further assumtions involving the momenta.



If the points of the state |n>
happens to be in chains one can
easily find a successor function f
so that successors are close.

About Fundamental Physics We can further only make assumptions about the Initial and /or Final states

After we settled on no time-development ($\sim$ Hamiltonian being zero) we can **not** as physicists looking for the right theory of nature anymore **speculate about the Hamiltonian**, because that we already took to zero (as operator).
But we may **want to** have a bit of chanse to **assume** a little bit to adjust to fit our hoped for model to experimental information etc.
Then we have the chanse of speculating about the **initial state** (which is also the final state though, when no development).

Assumptions about Initial and final state

For our purpose with the string theory towards which we are driving in mind we like to make assumptions about initial condition like this:

- An approximate constraint on the relative state of a couple of particles A and $f(A)$, namely

$$k(\vec{x}_{f(a]} - \vec{x}_A) \approx \vec{p}_{f(A)} + \vec{p}_A, \qquad (11)$$

  where k is a constant, actually related (as to be seen) to the Regge slope α' so important in string theory.
- The particles shall approximately form cyclic chains.
- And they shall even especially locally along the chains have a certain wave function like they would have in string theory if they were identified with the "objects" of ours (which I have not yet described in detail.)

Assumptions about (Initial) State Formulated by Density Matrix ρ

Whatever assumption about a quantum system one might want to make it can in principle be written by means of a **density matrix** ρ.
ρ is a positve operator on the Hilbert space of state vectors for the system normalized to $Tr(\rho) = 1$.
We have one $\rho_{fundamental}$ for the “fundamental degrees of freedom, and we can partly choose one ρ_{full} for the combined system of the fundamantal and the phantasy degrees of fredom systaem. Then you can act

$$\rho_{fundamental}|n>$$

or

$$\rho_{full}(|n>, f)$$

Density Matrix Relation

We shall naturally require for consistency

$$< p|\rho_{fundamental}|n > \quad = \quad \sum_f (< p|, f)\rho_{full}(|n >, f) \quad (12)$$

or formulated differently:

$$\rho_{fundamental} \quad = \quad Tr_{w.r.t.\ phantasy}\rho_{full} \quad (13)$$

So far we talk about timeless density matrices.
But could we make a purely phantasy time development of only the phantsy or f-degrees of freedom without distrubing the fundamental ($|n >$, $|p >$,...) degrees of freedom ?

Stringy Initial State Assumptions, and Phantasy Notation give String Field Theory

The point is we put a fair amount of **string theory into assumptions about the initial state**, partly because we cannot do it in the proper Hamiltonian.

The assumption,

"An approximate constraint on the relative state of a couple of particles A and $f(A)$, namely

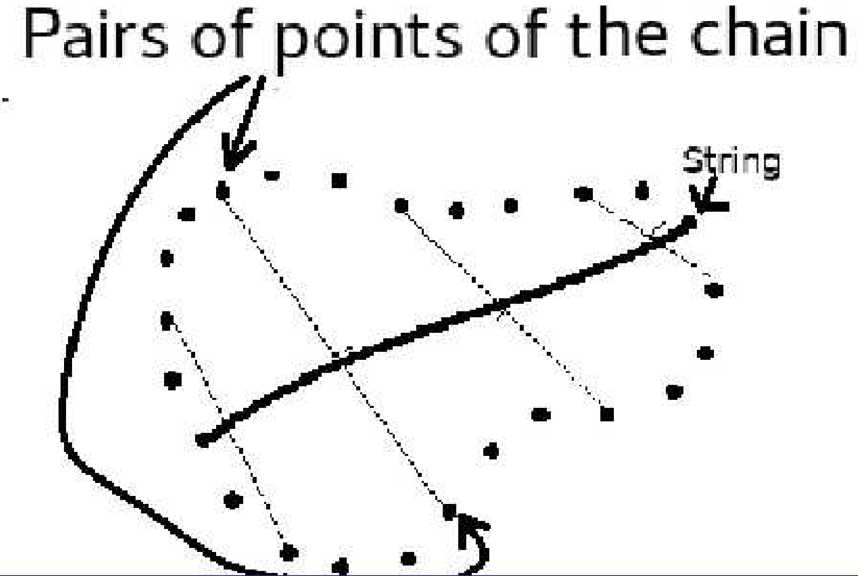
$$k(\vec{x}_{f(a]} - \vec{x}_A) \quad \approx \quad \vec{p}_{f(A)} + \vec{p}_A, \tag{14}$$

where k is a constant, actually related (as to be seen) to the Regge slope α' so important in string theory."

would if the particles did not have infinite masses mean that the cyclic chain would move along itself.

Yet a complication in relating the trivial static theory to string theory

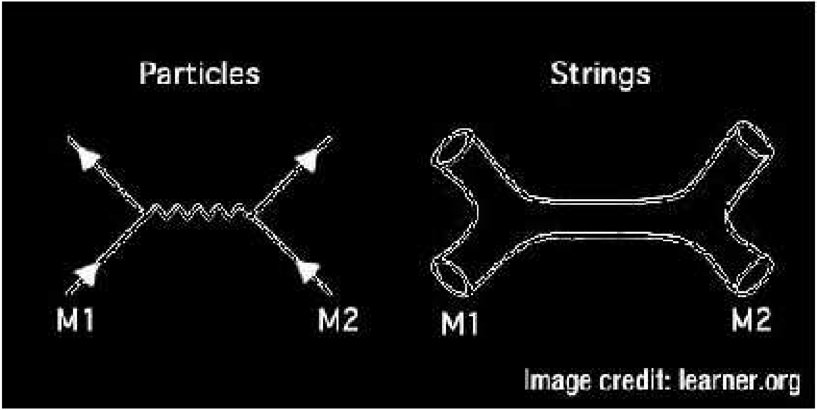
The cyclic chains of particles are **not** simply the strings when we identify with string theory - as it would be in Charles Thorns theory -, No,
We have to choose a starting point and go along the cyclical chain from that with two marks in opposite directions along the chain, and then construct for each step an average of the two "poeple" that satrted at the start. It is the series of average under this trip of the two "people" that makes up the string.
In this way you see we get an open string.


Pairs of points of the chain
String

Main Point: Brought although a bit complicated a correspondance to String Theory

To a set of strings in a known state - e.g. the ground state of their oscillations - one can calculate the state of the correpending particles (which we usually call "objects") sitting - ordered by the faked f description - in a cyclic chain for each open string (we postpone the closed strings for the moment).
I.e. **We can pretend to see string theory in our game with infinitely heavy particles.**
Most remarkably: When we calculated the overlap between two different sets of string representing second quantized states, we got - apart from a wrong sign (a missing i) - the form of the **Veneziano model**.

Correspondance with Veneziano Model rather short via thinking on surfaces of string development



Important step in Showing Veneziano Model from Our Novel String field theory

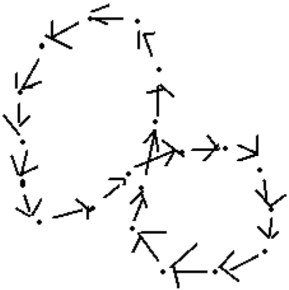
You think of external ground state strings. They can be produced as in general ground states - by a long imaginary time development with the appropriate Hamiltonian. This development is then written as in complex time development of the string, very reminiscent of what it always used in string theory to compute say Veneziano model.
Very crudely we just give a motivation for this kind of functional integral description of the strings.
Really we do it with a doubled string; i.e. we have a closed string digram describe the open string. So there are some complications but we did mannage to one of the three terms.

Changing Phantasy Degrees of Fredom can Change Number of Cyclic Chains and thus of (Open) Strings

For different "successor functions" f_1 and f_2 you can find different numbers of cyclic chains even for completely the same configuration of the infinitely heavy particles (="objects") and thus in fundamental physics-wise the same situation $|n>$.

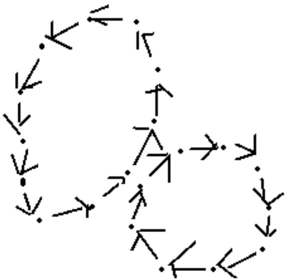
Take a fundamental physics situation $|n>$ like this:

In choice f_1 of the phantasy you have one open string



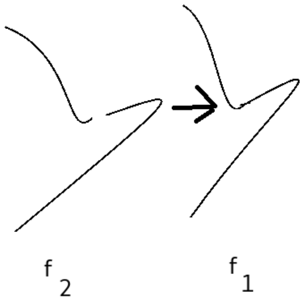
Successor function f_1 chosen so that only one cyclic chain.

In another choise f_2 of the Phantasy gives Two chains, thus Two open strings



Successor function f_2 chosen so that there are two cyclic chains.

Unification of strings can be change of f, thus phantasy



Two strings can become one just in the phantasy of $f_2 \dashrightarrow f_1$.

How to make a purely Phantasy Hamiltonian ?

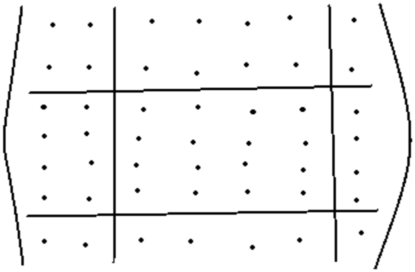
The excercise we want to do now is to see what Hamiltonian is allowed working on the extended Hilbert space containing also the phantasy degrees of freedom, so that the basis states are

$$\text{(Extended) Basis states} \quad \text{of the form} \quad (|n>, f) \tag{15}$$

while

$$\text{Fundamental basis states} \quad \text{of the form} \quad |n> \tag{16}$$

Thinking of matrices the extended operator (matrix) consists of a lot of blocks, one block for each matrix element in the (original) fundamental Hamiltonian (which is actually zero).



7 * 7 = 49 matrix elements

3 * 3 = 9 blocks

Attempting to find a Hamiltonian only moving the Phantsy Degrees of Freedom

For any operator depending only on the physical degrees of freedom O we want the "only phantasy" Hamiltonoian $H_{phantasy}$ candidate to commute with it:

$$[O, H_{phantasy}] = 0. \tag{17}$$

This condition is too strong, since it would not allow the hamiltonian to depend at all on the "fundamental" degrees of freedom, because if so, the conjugate variable to the one it depended on would be made to vary (and that we wanted to exclude).
We must be satisfied with only having this requirement **approximately.**

Not so good Argument that we can have a wanted $H_{phantasy}$ approximately.

We want the development in the f or phantasy degrees of freedom only to depend on that some cyclic chains come very close / touch; and that is dependent on only very few particles/ "objects", so at least it does only involve at first few among an extremely big number of "objects" in the interesting situation.

Purpose of this Faked Scattering String Theory Formulation

Hope you got the idea of **considering a completely trivial** $H = 0$ **quantum field theory** and built up a story of e.g. strings **just by defining some extra "phantasy degrees of freedom"**.
What is the purpose ?:

- It is a method to make a second quantized string theory (competing with works by Kaku and Kikkawa and by Witten, ...). You can describe states with several strings.
- You may use the idea to look for further models sharing the great property of string theory of **not having the usual divergencies**. Likely this is the only hope for making theories, that make sense, in high dimensions.

Problem of Ultraviolet Divergences Worse the Higher Dimension of Space-time

Each momentum-formulated loop intergal in a Feynman diagram bring a $\int ... d^d q$ integration and unless there are very many propagators in the loop we cannot avoid divergence for large loop momenta q.

The higher dimension the more different loop integrals lead to divergencies.

To absorb the divergencies into bare coupling constants you need in high dimensions so many that the theory ends up with infinitely many parameters, and is in principle useless.

Direction of Hope for High Dimensional Theories: Formfactors

One needs some factor that can make converge the loops in the high dimensional theory, otherwise you have ultraviolet divergensies and in high dimensions it gets too many different divergencies.

Best hope:
some exponentially falling off factor

$$\text{Factor extra in loop} \quad \propto \quad \exp(-k * q_E^2) \tag{18}$$

much like what one gets from formfactors when one has effective theories for hadrons.

Suggestion:
Replace the particles in the high dimensional theory by **composite (bound) states**, like the hadrons are composite in QCD.

Just Bound States Not Good Enough: Partons

If as we now believe hadrons are bound states but of quarks and gluons called in this connection partons the effective vertices will NOT go down exponentially for very big momentum transfers but will be dominated by the coupling to a single parton and behave at the end more like in the theory of just particles. Thus it will only help a part of the way, but finally at high momenta the divergencies reappear.

Only if there are infinitely many consituents(=partons) in the bound states and they have Bjorken $x = 0$, you can postpone parton dominance from popping up, and thus only then we can use the replacement of the original particles in high dimensional theory by bound states.

Hadrons Scatter Crudely by Exchange of Bunches of Constituents

Hadron scattering at energies below where partons collissions become important was described by exchange of other hadrons, pions, vector bosons like ω, again hadrons which again consists of many partons. So it was mainly exchange of lots of partons between one hadron and another one, while the single partons hardly were seen.

Moderate energy Hadron scattering in terms of partons is much like the fake-scattering of just exchanging bunches from one bound state to another one.
(we here ignored the relativity and effects of vacuum)

A Major Achievement of Phantasy Hamiltonian Formulation is Unitarity of Time-deveopment Operator.

If the theory has a formal /phantasy time development given by a Hamiltonian $H_{phantasy}$ then we have automatically that developping during some time interval wil result in a unitary operator development.
Essentially unitary S-matrix.

Perturbation Expansion in Coefficient on the "Phantasy Hamiltonian" $H_{phantasy}$

Really the overall scale of the $H_{phantasy}$ is a matter of the time unit. In fact there is no time in the theory before we introduce the phantasy degrees of freedom and make them move.

Natural to make perturbation theory in the coefficient on $H_{phantasy}$.

Then we get one shift in the topology or way of connection of the cyclic chains for each order in the perturbation. That corresponds to different topologies of string surface diagrams as describing unitarity corrections to the Veneziano model.

Coclusion

- We have put forward a very trivial second quantized theory (of infinitely heavy non-relativsic particles identified as our earlier "objects") and assumed for it a Hamiltonian that is zero as operator. So no timedevelopment in this "fundamental" theory.
- We can only make it more interesting or adjustable by assuming something about the state of it. Say by a density matrix $\rho_{fundamental}$. We use this option to assume that the particles (="objects") sit in (long) closed chains (cyclic chains).
- We interpret each cyclic chain to describe an open string in a string theory.

Conclusion Continued

- We introduced a phantasy system of degrees of freedom by introducing a "successor function" f, which puts all the "objects" ($\sim$ paricles) into a series of closed chains, thereby making explicite the such chains assumed to be present by the assumption about the likely state of the trivial second quantized system.
- Mostly we imagine the cyclic ordering is given by the "fundamental" state of the trival theory, but in some cases it will be ambiguous which chains there are. Then it is we introduce the fake/phantasy/f-variable to distinguish possibilities.

Conclusion yet continued

- Then the idea was to make a Hamiltonian supposed to mainly make this fake degree of freedom move, but approximately to avoid varying the "fundamantal" degrees of freedom.
  With this we then get a quite phantasy time. We only get timedevelopment due to the phantasy degrees of freedom.

Conclusion on Hopes and Applications

- Really the formulation of ours is a **solution** of second quantized string theory, in the sense that we could say we solved the time development by identifying string theory with several strings with a theory without time development.
- Hope to generalize our "object" picture to different models which have the same great property as string theory of **not having usual divergencies**! This would be absolutely needed in high dimensions, because with point particles high dimensions cause rather hopeless divergencies.
- As a speciall case we may generalize to p-adic Veneziano model.