Perturbations of black holes in modified gravity

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Why do we modify gravity ?

- GR is very powerful but cannot explain where dark energy comes from
- Cosmological constant: discrepancy of $\sim 10^{60}$ between measured and predicted
- Important to have a parametrization of deviations from GR

Outline

1. Modifying gravity : the DHOST theories
    - Freedom available for modification
    - The scalar-tensor theories
2. Black hole perturbations
    - Black holes in DHOST
    - Perturbations of the Schwarzschild solution
    - New results for stealth Schwarzschild

Modifying gravity : the DHOST theories

Unicity of GR

Lovelock's unicity theorem

Consider a Lagrangian $\mathcal{L}$ for a metric $g_{\mu\nu}$ and require

- Second-order field equations,
- 4-dimensional spacetime.

$\Rightarrow$ only possibility is GR!

Modified gravity $\rightarrow$ *break at least one of these hypotheses*:

- Higher-dimensional spacetimes,
- New fields,
- Higher-order derivatives.

Leads to DHOST theories

Unicity of GR solutions

No-hair theorem

Consider GR, look for black hole solutions with

- stationarity
- asymptotical flatness

$\Rightarrow$ only solution is Kerr!

Still valid in some modified gravity theories. To get new solutions:

- Add higher derivatives
- Break stationarity

Leads to specific ansatz in DHOST solutions

DHOST theories

Ostrogradsky ghost

Higher order derivatives in the Lagrangian lead in general to **instabilities**.

Most general theory:

Degenerate Higher-Order Scalar Tensor

Evade the Ostrogradski instability

Obtain new physical behaviour

One new dynamical field (“fifth force”)

Usual metric part of GR

DHOST action

$$
\begin{aligned}
S = \frac{1}{16\pi G}\int \mathrm{d}^4x\,(F(\phi, X)R + P(\phi, X) + Q(\phi, X)\Box\phi \\
+A_1(\phi, X)\phi_{\mu\nu}\phi^{\mu\nu} + A_2(\phi, X)\left(\Box\phi\right)^2 \\
+A_3(\phi, X)\Box\phi\phi^{\mu}\phi_{\mu\nu}\phi^{\nu} + A_4(\phi, X)\phi^{\mu}\phi_{\mu\rho}\phi^{\rho\nu}\phi_{\nu} \\
+A_5(\phi, X)\left(\phi^{\mu}\phi_{\mu\nu}\phi^{\nu}\right)^2\Big)
\end{aligned}
\tag{1}
$$

Degeneracy conditions

Relations between A_i functions: **3 classes**. Only stable class:

- 5 free functions A_1, A_3, F, P, Q,
- $F \neq XA_1$.

$\Rightarrow$ most general stable (GR + scalar field) theory

Constraints on modified theories

Observation of electromagnetic counterpart

Observation of gravitational waves and counterparts:

- Speed of gravitational waves is c up to 10^{-15}: $A_1 = 0$.
- No gravitational wave decay: $A_3 = 0$.

$\Rightarrow$ many constraints, but still interesting to study DHOST as an effective theory for higher energies.

Black hole perturbations

Motivation: quasinormal modes and GW ringdown

Ringdown of binary black hole merger:

- Specific frequencies called quasinormal modes,
- Information about the background solution ⇒ provides test of GR
- Obtained via first-order perturbation theory

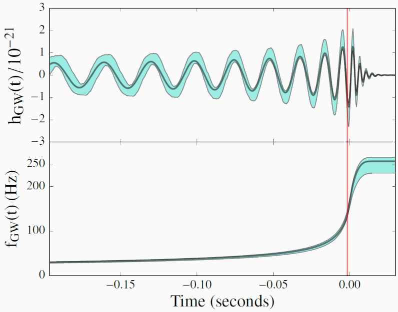


Figure 1: Signal from a binary black hole merger, and fit used to obtain the ringdown frequency (The LIGO Scientific Collaboration et al., 2016)

Finding DHOST solutions: “stealth” black holes

- Many free functions for DHOST $\longrightarrow$ many different solutions
- Look for solutions that **match the GR solution** in the metric sector: “stealth” black holes

Simplified subtheory

- $X = \text{cst}$,
- F and A_i depend only on X,
- **Horndeski theory**: $A_3 = 0$ and $A_1 = 2F'(X)$.

Full solution equations

- Need a **time-dependant scalar field** to evade no-hair theorems (Babichev & Charmousis, 2014).
- Full solution proposed by Babichev and Esposito-Farese, 2017

$$\mathrm{d}s^2 = -\left(1-2M/r\right)\mathrm{d}t^2 + \left(1-2M/r\right)^{-1}\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega^2$$

$$\phi = qt + \psi(r) \quad \text{with} \quad \psi'(r) = q\frac{\sqrt{r/2M}}{1-2M/r}$$

$$X = X_0 = -q^2$$

$$F(X_0) = 1, F'(X_0) = \alpha, F''(X_0) = \beta$$

$$P(X_0) = 0, P'(X_0) = 0, Q''(X_0) = \gamma$$

$$Q(X_0) = 0, P'(X_0) = 0, Q''(X_0) = \delta$$

The perturbation framework

1. Metric perturbations $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$
2. Decompose on parities wrt φ: **even** and **odd** perturbations
3. Decompose each parity onto spherical harmonics $Y_{\ell,m}$
4. Fix the gauge

Even perturbations

$$h_{\mu\nu} = \begin{pmatrix} \left(1-\frac{2M}{r}\right) H_0 & H_1 & 0 & 0 \\ \text{sym} & \frac{H_2}{1-\frac{2M}{r}} & 0 & 0 \\ 0 & 0 & K & 0 \\ 0 & 0 & 0 & K \end{pmatrix} Y_{\ell,m}$$

Odd perturbations

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & \frac{h_0}{\sin\theta}\frac{\partial}{\partial\varphi} & -h_0 \sin\theta \frac{\partial}{\partial\theta} \\ 0 & 0 & \frac{h_1}{\sin\theta}\frac{\partial}{\partial\varphi} & -h_1 \sin\theta \frac{\partial}{\partial\theta} \\ \text{sym} & \text{sym} & 0 & 0 \\ \text{sym} & \text{sym} & 0 & 0 \end{pmatrix} Y_{\ell,m}$$

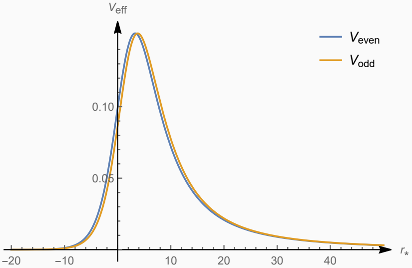
Schrödinger-like equation

Manipulation of equations

- Change spatial coordinate $\mathrm{d}r = (1 - 2M/r)\,\mathrm{d}r_*$
- Change variables: $\psi_{\text{even}} = f(H_0, H_1, H_2, K)$, $\psi_{\text{odd}} = f(h_0, h_1)$
- Fourier transform

Obtain two propagation equations:

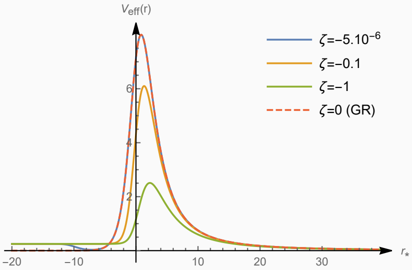
$$\begin{cases} \dfrac{\partial^2 \psi_{\text{even}}}{\partial r_*^2} + \left(\omega^2 - V_{\text{even}}(r)\right) = 0 \\ \dfrac{\partial^2 \psi_{\text{odd}}}{\partial r_*^2} + \left(\omega^2 - V_{\text{odd}}(r)\right) = 0 \end{cases}$$



Propagation equations for stealth Schwarzschild

What changes in modified gravity

- Same metric perturbation + scalar perturbation
- New even degree of freedom propagates (scalar waves)
- Time-dependant scalar field: must change time variable $t_* = t + \nu(r)$
- Odd perturbations: only $\alpha = \zeta/q^2$ plays a role



Conclusion

- Modified gravity is important to better understand the uniqueness of GR
- A good test is the computation of the resonance modes of black holes: the quasinormal modes
- To compute them, perturb Einstein's equations and find Schrödinger-like equations
- Next steps: compute new frequencies in stealth Schwarzschild, extend to Kerr

Thanks for your attention!