The Cosmological Cheshire Cat

Recent progress to explain dark matter halos

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Dark matter: only a mischievous grin in our universe

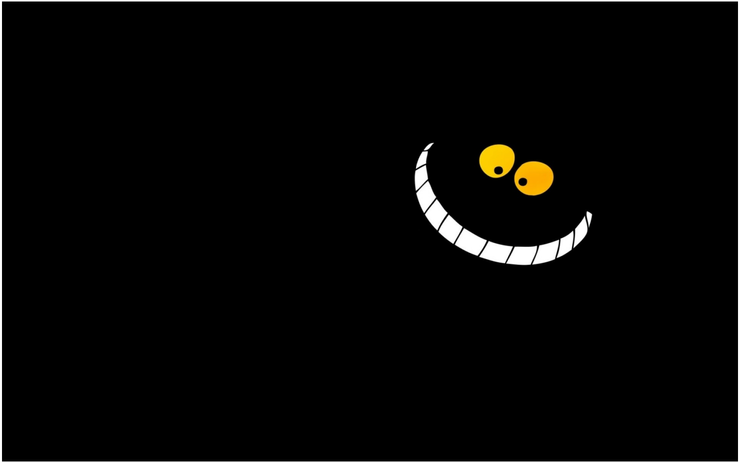
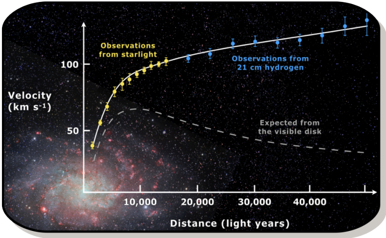


Image credits: http://getwallpapers.com/collection/cheshire-cat-wallpapers, Disney

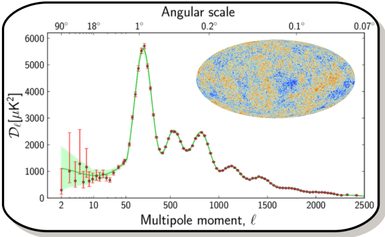
Observational evidence for dark matter

strong gravitational lensing

galaxy rotation curves



cosmic microwave background



multi-band observations

Image credits: R. Durrer (2008), ESA (Planck), Mario De Leo, ESA/Hubble, NASA/CXC/M. Weiss - Chandra X-Ray Observatory: 1E 0657-56

Dark matter structures in cosmological simulations

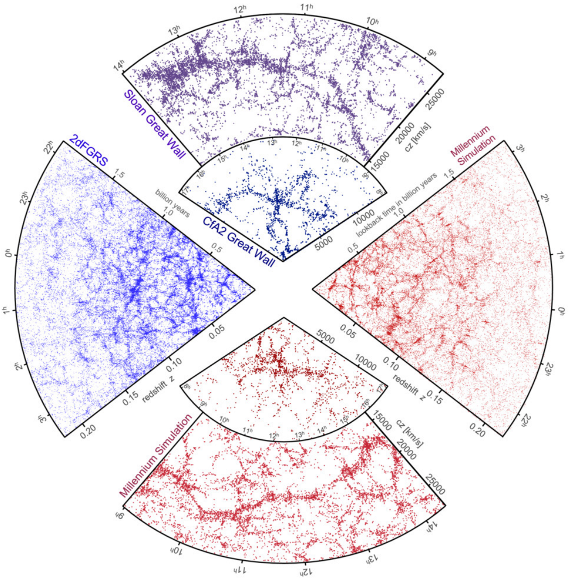
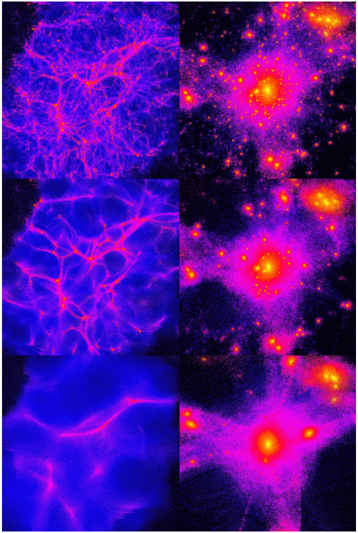


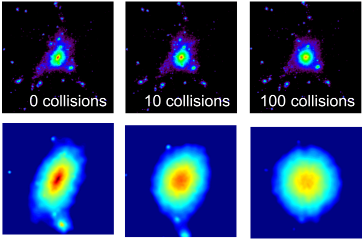
Image credits: Frenk & White (2012), Navarro, Frenk & White (1996)

Constraints on dark matter properties from simulations

velocity of dark matter



self-interactions of dark matter



luminous matter feedback

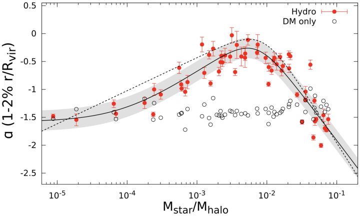
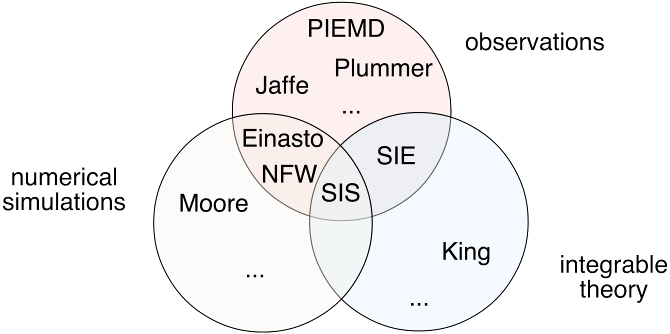


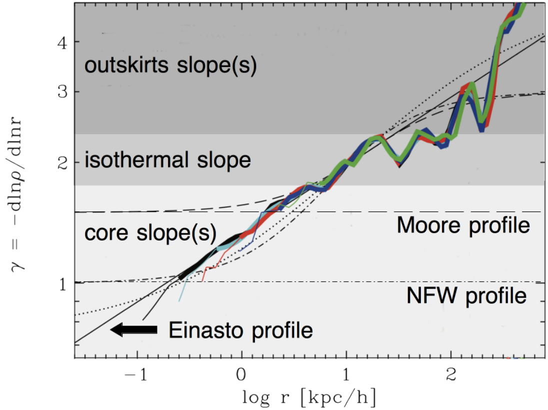
Image credits: Ben Moore, Yoshida et al. (2000), Tollet et al. (2016)

Goal: one model for individual dark matter halo mass densities $\rho(r)$



Prerequisites for a testable, evidence-based model

almost universal mass density profiles



reproduce γ

predict γ

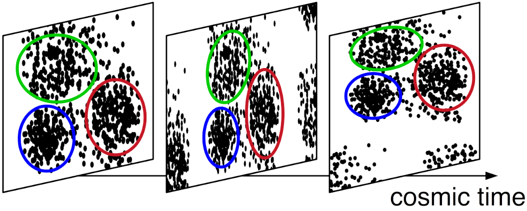
Image credits: Navarro et al. (2010)

Prerequisites for an effective, macroscopic model

scales of interest:

- simulations: $M \geq \mathcal{O}(10^4\, M_\odot),\ L \geq \mathcal{O}(10\,\mathrm{kpc})$
- observations: $M \geq \mathcal{O}(10^8\, M_\odot),\ L \geq \mathcal{O}(10\,\mathrm{kpc})$

decoupling of structure and dynamics



$\rightarrow$ **individual particle-based halo model at each time**

Prerequisites for a simplified, abstract model

physical picture

- n_p dark matter particles
- halo $=$ sphere of radius r_max
- Newtonian gravity as the only interaction
- collisionless particles

mathematical picture

- identical, identically distributed
- finite ensemble props $(n_\mathrm{p}, r_\mathrm{max})$
- linear superposition, spatial power-law PDF
- independent particles

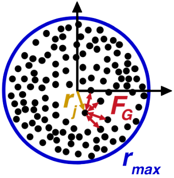
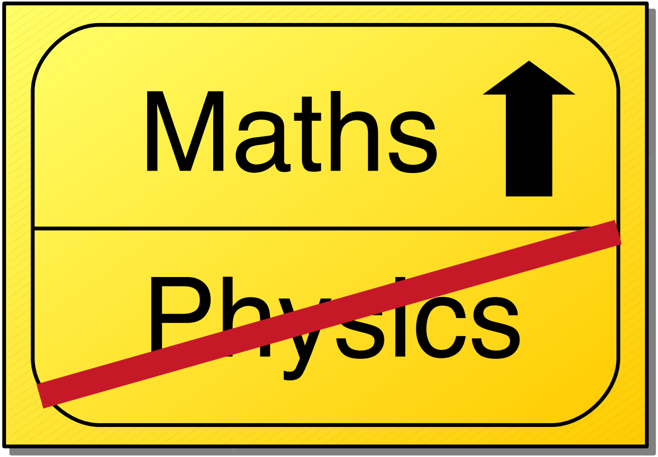


Image credits: Aquarius A1 simulation MPA Garching


Maths
~~Physics~~

Individual power-law probability densities

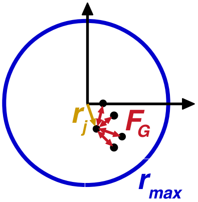
For each of the n_p particles holds:

- probability for a particle to be at r_j:

$$p(r_j) = N \left(\frac{r_j}{r_\sigma} \right)^{-\gamma} \,, \; \gamma > 0$$

- normalisation N given by finite volume:

$$\int_{V_\mathrm{max}} \mathrm{d}V \, p(r_j) \stackrel{!}{=} 1$$



$$4\pi N \int\limits_{r_\mathrm{min}}^{r_\mathrm{max}} \mathrm{d}r \, r^2 \left(\frac{r}{r_\sigma} \right)^{-\gamma} \stackrel{!}{=} 1 \qquad \rightarrow \qquad N = N(\gamma, r_\sigma, r_\mathrm{max}, r_\mathrm{min})$$

$$\boxed{\boldsymbol{p(r_j) = N(\gamma, r_\sigma, r_\mathrm{max}, r_\mathrm{min}) \left(\frac{r_j}{r_\sigma} \right)^{-\gamma}}}$$

Joint ensemble

Joining the independent, identically distributed particles:

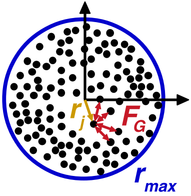
- identically distributed = same PDF for each particle

$$p(r) = N(\gamma, r_\sigma, r_{\max}, r_{\min}) \left(\frac{r}{r_\sigma}\right)^{-\gamma}$$

- independent = joint PDF is product of individual PDFs

$$p_{\mathrm{E}}(r_1, ..., r_{n_\mathrm{p}}) = \prod_{j=1}^{n_\mathrm{p}} p(r_j)$$

- (in)distinguishable particles ?



$$\boldsymbol{p_{\mathrm{E}}(r_1, ..., r_{n_\mathrm{p}}) = \left(\frac{1}{n_\mathrm{p}!}\right) \prod_{j=1}^{n_\mathrm{p}} p(r_j)}$$

Most likely configuration of the joint ensemble

- joint PDF

$$p_{\mathrm{E}}(r_1, ..., r_{n_\mathrm{p}}) = \left(\frac{1}{n_\mathrm{p}!} \right) \prod_{j=1}^{n_\mathrm{p}} p(r_j) = \left(\frac{1}{n_\mathrm{p}!} \right) \prod_{j=1}^{n_\mathrm{p}} N(\gamma) \left(\frac{r_j}{r_\sigma} \right)^{-\gamma}$$

- log-likelihood

$$\mathcal{L}(r_1, ..., r_{n_\mathrm{p}}) = \log \left(p_{\mathrm{E}}(r_1, ..., r_{n_\mathrm{p}}) \right) = -\log(n_\mathrm{p}!) + \sum_{j=1}^{n_\mathrm{p}} \log \left(p(r_j) \right)$$

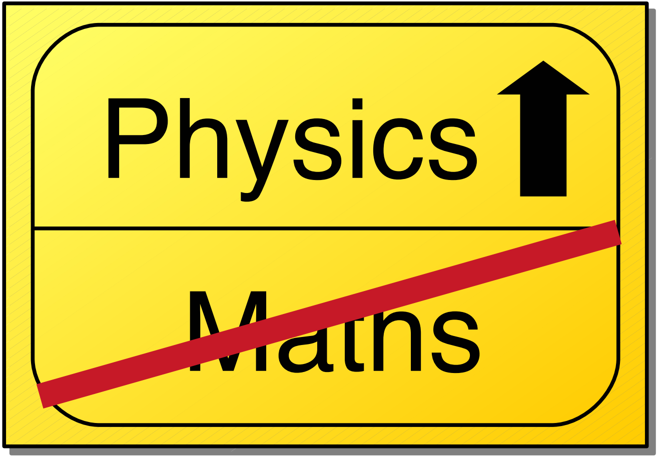
$$= -\log(n_\mathrm{p}!) + n_\mathrm{p} \log \left(N(\gamma) \right) - \gamma \sum_{j=1}^{n_\mathrm{p}} \log \left(\frac{r_j}{r_\sigma} \right)$$

- most likely configuration to determine γ

$$\partial_\gamma \mathcal{L}(r_1, ..., r_{n_\mathrm{p}}) \stackrel{!}{=} 0$$

$$\boxed{\frac{\partial_\gamma N(\gamma, r_\sigma, r_{\max}, r_{\min})}{N(\gamma, r_\sigma, r_{\max}, r_{\min})} - \frac{1}{n_\mathrm{p}} \sum_{j=1}^{n_\mathrm{p}} \ln \left(\frac{r_j}{r_\sigma} \right) \stackrel{!}{=} 0}$$

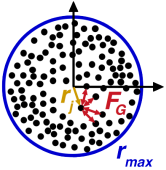
Physical interpretation of mathematical results

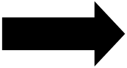


Transfer from particle ensemble to mass density

- Density of a dark matter halo

$$\rho(r) = m_\mathrm{p}\, n(r) = m_\mathrm{p}\, n_\mathrm{p}\, p(r) = m_\mathrm{p}\, n_\mathrm{p}\, N(\gamma, r_\sigma, r_\mathrm{max}, r_\mathrm{min}) \left(\frac{r}{r_\sigma}\right)^{-\gamma}$$





$\rightarrow$ $\rho(r)$ from a finite number of spatially bounded particles

$\rightarrow$ most likely spatial configuration determines γ

Physically relevant cases

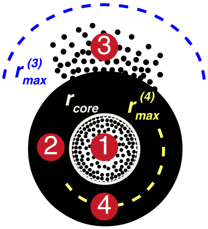
Choose $r_\sigma \ll r_j$ in

$$\frac{\partial_\gamma N(\gamma, r_\sigma, r_\mathrm{max}, r_\mathrm{min})}{N(\gamma, r_\sigma, r_\mathrm{max}, r_\mathrm{min})} - \frac{1}{n_\mathrm{p}} \sum_{j=1}^{n_\mathrm{p}} \ln\left(\frac{r_j}{r_\sigma}\right) \overset{!}{=} 0$$

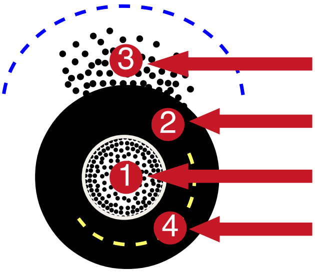
to obtain

$$\gamma = 2 + \frac{n_\mathrm{p}}{\sum\limits_{j=1}^{n_\mathrm{p}} \ln\left(\frac{r_j}{r_\mathrm{max}}\right)}$$

1. core case: $0 < r_\mathrm{min} \leq r_j \leq r_\mathrm{core}$
2. isothermal fluid case: $n_\mathrm{p} \gg 1$
3. thinning outskirts: $r_j \ll r_\mathrm{max}$
4. overflowing outskirts: $r_\mathrm{max} \leq r_j \approx 2 r_\mathrm{max}$



Limiting approximations

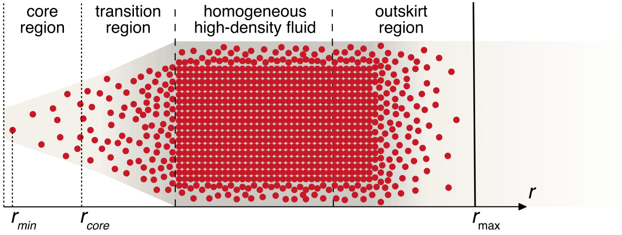


thinning outskirts: $\rho(r) \propto r^{-3}$ $(r_{\max} \to \infty)$

isothermal fluid case: $\rho(r) \propto r^{-2}$ $(n_{\mathrm{p}} \to \infty)$

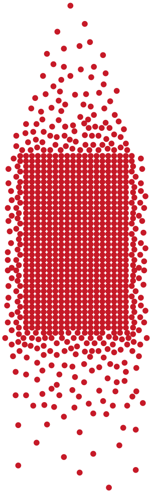
core case: $\rho(r) \propto r^{-1}$ possible (uniform distr.)

overflowing outskirts: $\rho(r) \propto r^{-4}$ $(\langle \delta r \rangle \to r_{\max})$



Summary: dark matter halo profiles from a fundamental theory

- **Our approach so far**
  - purely probabilistic (mathematical) approach
    $\longrightarrow$ **separation of structure & dynamics**
    $\longrightarrow$ **independence of cosmic assembly history**
  - simple, analytic approximations as solutions
    $\longrightarrow$ **clear understanding of halo shapes**
    $\longrightarrow$ **optimisation of simulations & observations**
  - translations of mathematical terms to physical ones exist
    $\longrightarrow$ **connection to thermodynamics**
    $\longrightarrow$ **reduction to necessary prerequisites**



- **Further reading**
  - Essay (hon. mention of the Gravity Research Foundation)
    $\longrightarrow$ **arXiv:2005.08975**
  - Full paper (in press in General Relativity & Gravitation)
    $\longrightarrow$ **arXiv:2002.00960**

Thank you for your attention

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