Scalar Induced Gravitational Waves from a Universe filled with Primordial Black Holes

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  [Published in JCAP, ArXiV:1907.04236, Authors: J. Martin, T. P. and V. Vennin]

- Metric Preheating and Radiative Decay in Single-Field Inflation
  [Published to JCAP, ArXiV: 2002.01820, Authors: L. Pinol, J. Martin, T. P. and V. Vennin]

- **Scalar Induced GWs from a Universe filled with PBHs**
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Introduction

- Primordial Black Holes (PBHs) are astrophysical objects which are created in the early universe out of the collapse of enhanced curvature perturbations.

- They are different from astrophysical black holes since they are not created out of the collapse of a star.

- PBHs can be used to probe different physical phenomena depending on their mass.
- Low Mass PBHs ($M<10^{15}$g) => Probe the Physics of the Early Universe: Phase Transitions, BBN, CMB Distortions
- Intermediate Mass PBHs ($M\approx 10^{15}$g) => Probe High Energy Astrophysics: Cosmic Rays, γ-ray Background
- Large Mass PBHs ($M> 10^{15}$g) => Probe Gravity and Dark Sector: Large Scale Stucture, Dark Matter, Dark Energy

- In our work, we focus on **low mass PBHs** which could **dominate the universe energy content** at the early stages of the cosmic history.
  [1903.05053, 1907.04236, 2003.10455, 2005.05693]

- In order to constrain them, we probe the **scalar induced GWs** produced due to 2nd order gravitational interactions during a phase of PBH domination.

2. The Matter Power Spectrum

- As regards the PBH matter field we assume an initially randomly distributed gas of PBHs (Poisson Statistics) [1806.10414, 1805.05912] with the same mass [1906.08978] .

$$\left\langle \frac{\delta\rho_{\rm PBH}}{\bar{\rho}}(\vec{r})\frac{\delta\rho_{\rm PBH}}{\bar{\rho}}(\vec{r}\,')\right\rangle = \xi\delta(\vec{r}-\vec{r}\,'), \text{where } \xi \text{ is r independent}$$

- By assuming a universe filled with PBHs and a fluid with equation-of-state parameter w, we coarse-grain the PBH matter field and we obtain that

$$\langle\,|\,\delta_{k,\rm f}\,|^2\,\rangle = \frac{\xi}{a_{\rm f}^3} = \frac{4\pi}{3}\left(\frac{\bar{r}_{\rm f}}{a_{\rm f}}\right)^3\Omega^2_{\rm PBH,f}$$

- The PBH matter power spectrum is $\mathscr{P}_{\delta,\rm PBH}(k) \equiv \dfrac{k^3}{2\pi^2}\langle\,|\,\delta_k\,|^2\,\rangle$.

- The total matter power spectrum is: $\mathscr{P}_{\delta}(k) = \mathscr{P}_{\delta,\rm prim}(k) + \mathscr{P}_{\delta,\rm PBH}(k)$, where $\mathscr{P}_{\delta,\rm prim}(k)$ is the almost scale invariant primordial power spectrum at the "long" CMB scales.

- In the "small" PBH scales, $\mathscr{P}_{\delta,\rm PBH}(k) \gg \mathscr{P}_{\delta,\rm prim}(k)$.

2. The Matter Power Spectrum

The UV cut-off

- The scales considered here should be larger than the mean separation of PBHs. Otherwise, we probe the granularity of the matter density field which at very small scales is described by different physics. This the same as demanding being in the linear regime, $\mathscr{P}_\delta(k) < 0.1$.

$$r(t) > \bar{r}_{\rm PBH}(t) \Leftrightarrow k < k_{\rm UV} \equiv \frac{a_{\rm pbhd}}{\bar{r}_{\rm PBH,pbhd}} \Leftrightarrow k < a_{\rm f} H_{\rm f} \Omega_{\rm PBH,f}^{1/3}$$

The PBH formation energy scales

- a) PBHs dominate before they evaporate. Thus,

$$t_{\rm pbhd} - t_{\rm f} < t_{\rm evap} - t_{\rm f} = \Delta t_{\rm evap} \Leftrightarrow \Omega_{\rm PBH,f} > 10^{-3}\sqrt{\frac{\rho_{\rm f}}{M_{\rm Pl}^4}}$$

- b) PBHs form after the end of inflation. From Planck data, in the case of single-field slow-roll inflation, $H_{\rm end} < 8 \times 10^{13}{\rm GeV} \Leftrightarrow \rho_{\rm end} < 10^{-10} M_{\rm Pl}^4$. Thus,

$$\rho_{\rm f} < \rho_{\rm end} < 10^{-10} M_{\rm Pl}^4$$

- c) PBHs should evaporate before BBN. Thus,

$$\rho_{\rm evap} > \rho_{\rm BBN} \Leftrightarrow \frac{4\rho_{\rm f}^3 \Omega_{\rm PBH,f}^4}{9} > \rho_{\rm BBN}\left[\left(\frac{30720}{g}(4\pi)^3\right)^2 \Omega_{\rm PBH,f}^4 M_{\rm Pl}^8 + \frac{\rho_{\rm f}^2}{36} + \left(\frac{30720}{g}(4\pi)^3\right)^2 \Omega_{\rm PBH,f}^2 M_{\rm Pl}^4 \rho_{\rm f}\right]$$

$$\Rightarrow \rho_{\rm f} > 10^{-24} M_{\rm Pl}^4$$

- The relevant mass range therefore is: $10^{-24} M_{\rm Pl}^4 < \rho_{\rm f} < 10^{-10} M_{\rm Pl}^4 \Rightarrow M \in [10^{-32} M_\odot, 10^{-25} M_\odot]$

3. The Gauge Issue

- We stress out here the notion of the **PBH formation frame**. It is the coordinate system which is related to the **spatial threading of spacetime at PBH formation time.**

- The PBH formation frame is related to the **PBH production mechanism** which as we assume produces randomly distributed PBHs.

- Since we do not have in mind a production mechanism which gives rise to randomnly distributed PBHs, we model this **freedom of choice** by choosing two classes of gauges when we write the matter power spectrum at small scales.

- a) The class of Newtonian gauges

$$\delta_k \propto \left[\left(\frac{k}{aH}\right)^2 + c\right]\zeta_k, \text{ where c} = \text{constant,}$$

- b) The class of comoving gauges

$$\delta_k \propto \left(\frac{k}{aH}\right)^2 \zeta_k$$

3. The Incompatibility of the Class of the Comoving Gauges

- As an example of the class of the comoving gauges we choose the comoving-orthogonal gauge where,
$$\delta^{\rm cm}_k = -\frac{2(1+w)}{5+3w}\left(\frac{k}{aH}\right)^2\zeta_k \, .$$
- The curvature power spectrum is then written as
$$\mathcal{P}_{\zeta,\rm cm}(k) = \frac{k^3}{2\pi^2}\left\langle |\zeta_k|^2 \right\rangle = \ldots = \frac{25}{6\pi}\left(\frac{k}{a_{\rm f}H_{\rm f}}\right)^3\left(\frac{k}{a_{\rm pbhd}H_{\rm pbhd}}\right)^{-4}\Omega_{\rm PBH,f} \propto k^{-1}$$
- Two issues with the class of the comoving gauges.
- a) Due to the k^{-1} scaling $\mathcal{P}_\zeta(k)$, at small k values $\mathcal{P}_\zeta(k)$ “exploses”. Thus, one should imposes an IR cutoff such as $\mathcal{P}_\zeta(k) < 0.1 \text{ for } \mathrm{k} \in [\mathrm{k_{IR}}, \mathrm{k_{UV}}]$. However, there is a non a-priori a physical IR scale.
- b) In addition, due to the “explosion” of $\mathcal{P}_\zeta(k)$ at the “large” CMB scales one can not recover the 10^{-9} value of the primordial $\mathcal{P}_\zeta(k)$.
- Consequently, the class of the comoving gauges is not compatible with Poissonian-like distributed PBHs at formation!

4. Scalar Induced Gravitational Waves

- Choosing as the gauge for the GW frame the Newtonian gauge, the metric is written as

$$\mathrm{d}s^2 = a^2(\eta)\left[-(1+2\Phi)\mathrm{d}\eta^2 + \left((1-2\Phi)\delta_{ij} + \frac{h_{ij}}{2}\right)\mathrm{d}x^i\mathrm{d}x^j\right]$$

- The second order tensor modes, h_{ij} , can be decomposed in Fourier modes as follows

$$h_{ij}(\eta, \vec{x}) = \int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^{3/2}}\left[h^{(+)}_{\vec{k}}(\eta)e^{(+)}_{ij}(\vec{k}) + h^{(\times)}_{\vec{k}}(\eta)e^{(\times)}_{ij}(\vec{k})\right]e^{i\vec{k}\cdot\vec{x}}.$$

- The equation of motion for the Fourier modes, $h_{\vec{k}}$, read as

$$h^{s,''}_{\vec{k}} + 2\mathscr{H}h^{s,'}_{\vec{k}} + k^2 h^s_{\vec{k}} = 4S^s_{\vec{k}}$$

- The source term, $S_{\vec{k}}$, reads as

$$S^s_{\vec{k}} = \int \frac{\mathrm{d}^3\vec{q}}{(2\pi)^{3/2}}e^s_{ij}(\mathbf{k})q_iq_j\left(2\Phi_{\vec{q}}\Phi_{\vec{k}-\vec{q}} + \frac{4}{3(1+w)}(\mathscr{H}^{-1}\Phi'_{\vec{q}} + \Phi_{\vec{q}})(\mathscr{H}^{-1}\Phi'_{\vec{k}-\vec{q}} + \Phi_{\vec{k}-\vec{q}})\right)$$

4. Scalar Induced Gravitational Waves

- The solution of the equation of motion of tensor modes can be written analytically by the use of the Green function, which satisfies the homogeneous equation,

$$h^{s}_{\vec{k}}(\eta) = \frac{4}{a(\eta)} \int_{\eta_{\rm pbhd}}^{\eta} \mathrm{d}\bar{\eta} G_{\vec{k}}(\eta, \bar{\eta}) a(\bar{\eta}) S^{s}_{\vec{k}}(\bar{\eta})$$

- The Bardeen potential, $\Phi_{\vec{k}}$, present in the source term, $S^{s}_{\vec{k}}$, in the absence of the entropic modes, satisfies the following equation,

$$\Phi''_{\vec{k}} + \frac{6(1+w)}{1+3w}\frac{1}{\eta}\Phi'_{\vec{k}} + wk^2\Phi_{\vec{k}} = 0.$$

- The solution of the above equation can be written analytically as.

$$\Phi_{\vec{k}}(\eta) = \frac{1}{y^{\lambda}}\left(C_1(k)J_{\lambda}(y) + C_2(k)Y_{\lambda}(y)\right) \text{ where } \lambda = \frac{1}{2}\frac{5+3\mathrm{w}}{1+3\mathrm{w}} \text{ and } \mathrm{y} = \sqrt{\mathrm{w}}\mathrm{k}\eta$$

- If there is a dominant mode, which is the case when $w = 0$ or $1/3$ then $\Phi_{\vec{k}} = \phi_{\vec{k}}\Phi(k\eta)$.

- The definition of the tensor power spectrum, $\mathscr{P}_h(\eta, k)$ reads as

$$\langle h^{r}_{\vec{k}_1}(\eta) h^{s,*}_{\vec{k}_2}(\eta)\rangle \equiv \delta^{(3)}(\vec{k}_1 - \vec{k}_2)\delta^{rs}\frac{2\pi^2}{k_1^3}\mathscr{P}_h(\eta, k_1)\,.$$

4. Scalar Induced Gravitational Waves

- Then, following a long but straightforward calculation, $\mathscr{P}_h(\eta,k)$ reads as

$$\mathscr{P}_h(\eta,k)=4\int_0^{\Lambda}\mathrm{d}v\int_{|1-v|}^{\min(\Lambda,1+v)}\mathrm{d}u\left(\frac{4v^2-(1+v^2-u^2)^2}{4uv}\right)^2 I^2(u,v,x)\mathscr{P}_\zeta(kv)\mathscr{P}_\zeta(ku)$$

where $\Lambda = k_{\mathrm{UV}}/k$.

- The function $I(v,u,x)$ reads as

$$I(u,v,x)=\int_{x_{\mathrm{pbhd}}}^{x}\mathrm{d}\bar{x}\,\frac{a(\bar{x})}{a(x)}kG_k(x,\bar{x})f_k(u,v,\bar{x}),\ \text{where}$$

$$f_k(u,v,\bar{x})=\frac{6(w+1)}{3w+5}\Phi(u\bar{x})\Phi(v\bar{x})+\frac{6(1+3w)(w+1)}{(3w+5)^2}(\bar{x}\partial_{\bar{x}}\Phi(v\bar{x})\Phi(u\bar{x})+\bar{x}\partial_{\bar{x}}\Phi(u\bar{x})\Phi(v\bar{x}))$$
$$+\frac{3(1+3w)^2(1+w)}{(3w+5)^2}\bar{x}^2\partial_{\bar{x}}\Phi(v\bar{x})\partial_{\bar{x}}\Phi(u\bar{x})$$

- Here, we have a convolution between the curvature power spectra at two different points. Therefore, from the energy-momentum conservation $k\leq 2k_{\mathrm{UV}} \Leftrightarrow \Lambda > 1/2$.

4. Scalar Induced Gravitational Waves

The Energy Density of GWs

- In the case of sub-horizon scales where one can consider the flat spacetime approximation since on small scales one does not feel the curvature of spacetime the energy density of GWs reads as [M. Maggiore, GWs - Theory and Experiments: Volume 1]

$$\rho_{\rm GW}(\eta, \vec{x}) = \frac{M_{\rm Pl}^2}{8}\langle \partial_t h_{\alpha\beta}\partial_t h^{\alpha\beta} + \partial_i h_{\alpha\beta}\partial_i h^{\alpha\beta}\rangle\,.$$

  It's the sum of a kinetic and a gradient term- equipartition of energies.

- In our case, we are not in the regime of a free wave equation. We have an equation of motion for the tensor modes $h^r_{\vec{k}}$ with a constant source term, $S^r_{\vec{k}}$. Thus, in the sub-horizon scales one has that $h^r_{\vec{k}} \simeq \dfrac{4S^r_{\vec{k}}}{k^2} = \mathrm{constant}$ in time. Therefore the contribution from the kinetic term is negligible.

- Consequently, the spectral abundance, $\Omega_{\rm GW}(\eta, k)$, of GWs can be written as:

$$\Omega_{\rm GW}(\eta, k) \simeq \Omega^{\rm grad}_{\rm GW}(\eta, k) = \frac{1}{\rho_{\rm tot}}\frac{\mathrm{d}\rho^{\rm grad}_{\rm GW}}{\mathrm{d}\ln k} = \ldots = \frac{1}{48}\left(\frac{k}{a(\eta)H(\eta)}\right)^2 \mathcal{P}_h(\eta, k)\,.$$

5. The Gravitational Wave Spectrum

- Having discarded the class of the comoving gauges, we work with the Newtonian gauges. For the purpose of our work we make use of the Newtonian gauge for the PBH formation frame. In this gauge,

$$\delta_k^{\mathrm{Nt}} = -\frac{2}{5}\left[\left(\frac{k}{aH}\right)^2 + 3\right]\zeta_k$$

- The curvature power spectrum reads as

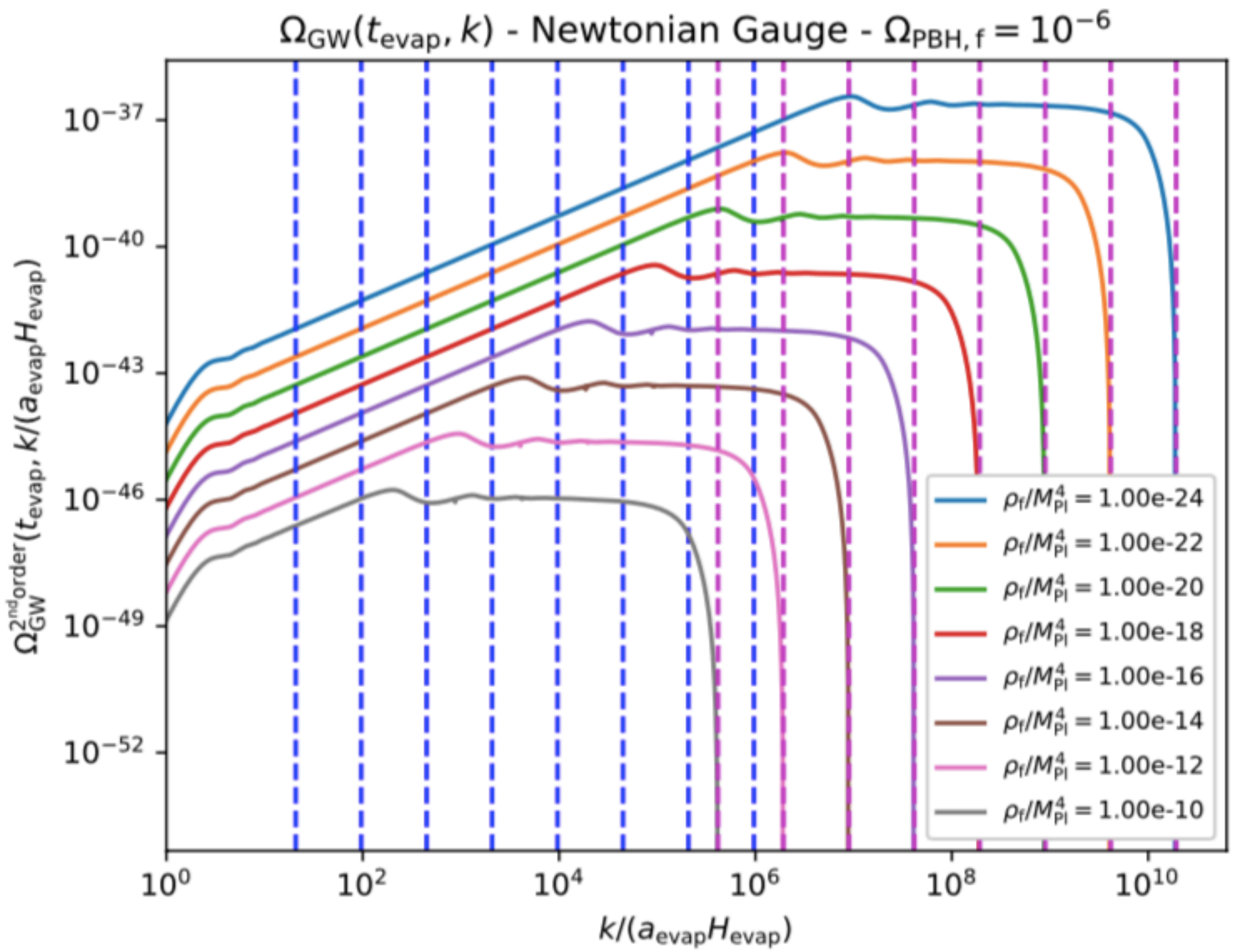
$$\mathcal{P}_{\zeta,\mathrm{Nt}}(k) = \frac{k^3}{2\pi^2}|\zeta_k|^2 = \frac{25}{8\pi^2}|\delta_k^{\mathrm{Nt}}|^2\left[\left(\frac{k}{a_{\mathrm{pbhd}}H_{\mathrm{pbhd}}}\right)^2 + 3\right]^{-2} = \frac{25}{6\pi}\left(\frac{k}{a_{\mathrm{f}}H_{\mathrm{f}}}\right)^3\Omega_{\mathrm{PBH,f}}\left[\left(\frac{k}{a_{\mathrm{pbhd}}H_{\mathrm{pbhd}}}\right)^2 + 3\right]^{-2}$$

- The abundance then of GWs , $\Omega_{\mathrm{GW}}(\eta, k)$ after a straightforward calculation reads as:

$$\Omega_{\mathrm{GW}}^{\mathrm{Nt}}(\eta,k) = \frac{4}{48}\left(\frac{25}{6\pi}\right)^2\left(\frac{k}{aH}\right)^2\left(\frac{k}{a_{\mathrm{f}}H_{\mathrm{f}}}\right)^6\Omega^2_{\mathrm{PBH,f}}\overline{I^2_{\mathrm{MD}}(x\gg 1)}$$
$$\times\int_0^{\Lambda=a_{\mathrm{f}}H_{\mathrm{f}}/k}\mathrm{d}v\int_{|1-v|}^{\min(\Lambda,1+v)}\mathrm{d}u\left(\frac{4v^2-(1+v^2-u^2)^2}{4uv}\right)^2 u^3v^3$$
$$\times\left[\left(\frac{kv}{a_{\mathrm{pbhd}}H_{\mathrm{pbhd}}}\right)^2+3\right]^{-2}\left[\left(\frac{ku}{a_{\mathrm{pbhd}}H_{\mathrm{pbhd}}}\right)^2+3\right]^{-2}\quad \text{where } x = k\eta = \frac{2k}{aH} \text{ for } w = 0.$$

- The source function $I_{\mathrm{MD}}(x \gg 1)$ is calculated on the sub-horizon regime, where $x \gg 1$ and it is written as $I_{\mathrm{MD}}(x\gg 1) \simeq \frac{6}{5}$.

5. The Gravitational Wave Spectrum

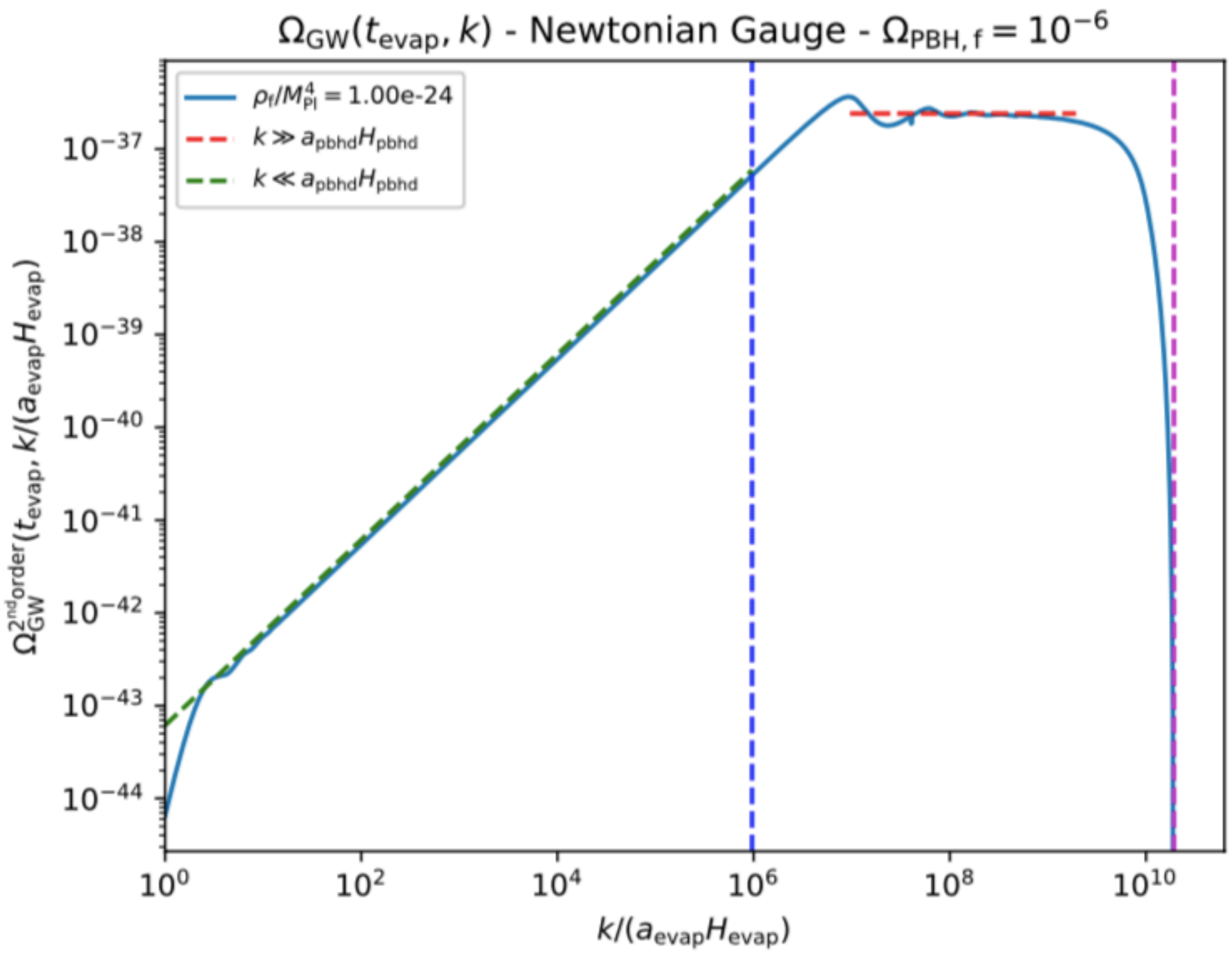


- One identifies a broken power law for the GW spectrum. Two scales enter in the problem, $k_{\rm d} = a_{\rm d}H_{\rm d}$ and $k_{\rm UV} = a_{\rm f}H_{\rm f}\Omega^{1/3}_{\rm PBH,f}$.

- As $\rho_{\rm f}$ decreases the amplitude of GWs increases. This is expected since as $\rho_{\rm f}$ decreases the PBH mass increases=> $\Rightarrow \Delta t_{\rm evap} \propto M^3/M^4_{\rm Pl} \uparrow$. The period of GW production is increased.

5. The Gravitational Wave Spectrum

- The abundance of GWs, $\Omega_{\rm GW}^{\rm Nt}(\eta, k)$, can be approximated analytically as

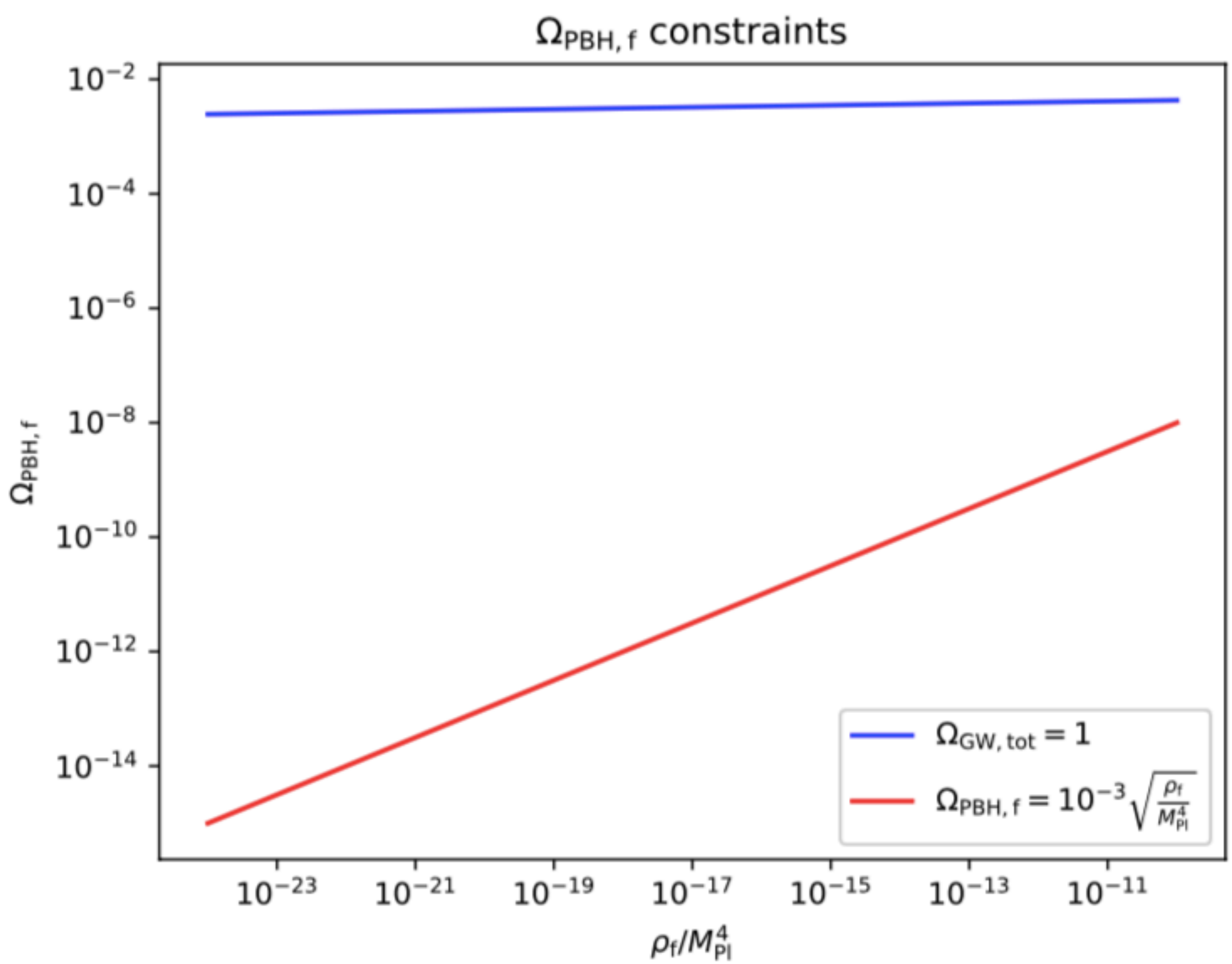


$$\Omega_{\rm GW}^{\rm Nt}(\eta, k) = \frac{4}{48}\overline{I_{\rm MD}^2(x \gg 1)}\left(\frac{25}{6\pi}\right)^2\left(\frac{k}{aH}\right)^2\left(\frac{k}{a_{\rm f}H_{\rm f}}\right)^6\Omega_{\rm PBH,f}^2\left(\frac{k}{a_{\rm d}H_{\rm d}}\right)^{-7}$$

$$\times\begin{cases}\frac{1}{45}\left[5\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}\Omega_{\rm f}^{2/3}}\right) - 3\frac{11\Omega_{\rm f}^{2/3} + 40\Omega_{\rm f}^2 + 45\Omega_{\rm f}^{10/3}}{\left(1+3\Omega_{\rm f}^{4/3}\right)^3}\right] \propto k \ \text{ for } k \ll a_{\rm pbhd}H_{\rm pbhd} \\ \frac{\pi^2}{8}\left(\frac{k}{a_{\rm pbhd}H_{\rm pbhd}}\right)^{-1} \propto k^0 \ \text{ for } k \gg a_{\rm pbhd}H_{\rm pbhd}\end{cases}$$

5. The Gravitational Wave Spectrum

- One also can constrain from above the initial abundance of PBHs by demanding that GWs are not overproduced during the transient PBH domination era, i.e. $\Omega_{\mathrm{GW,tot}}(\Omega_{\mathrm{PBH,f}}, \rho_{\mathrm{f}}) = \int \mathrm{d}\ln k\, \Omega_{\mathrm{GW}}(\eta, k) < 1 \Leftrightarrow \Omega_{\mathrm{PBH,f}} < \Omega^{\mathrm{max}}_{\mathrm{PBH,f}} \simeq 3 \times 10^{-3}$.

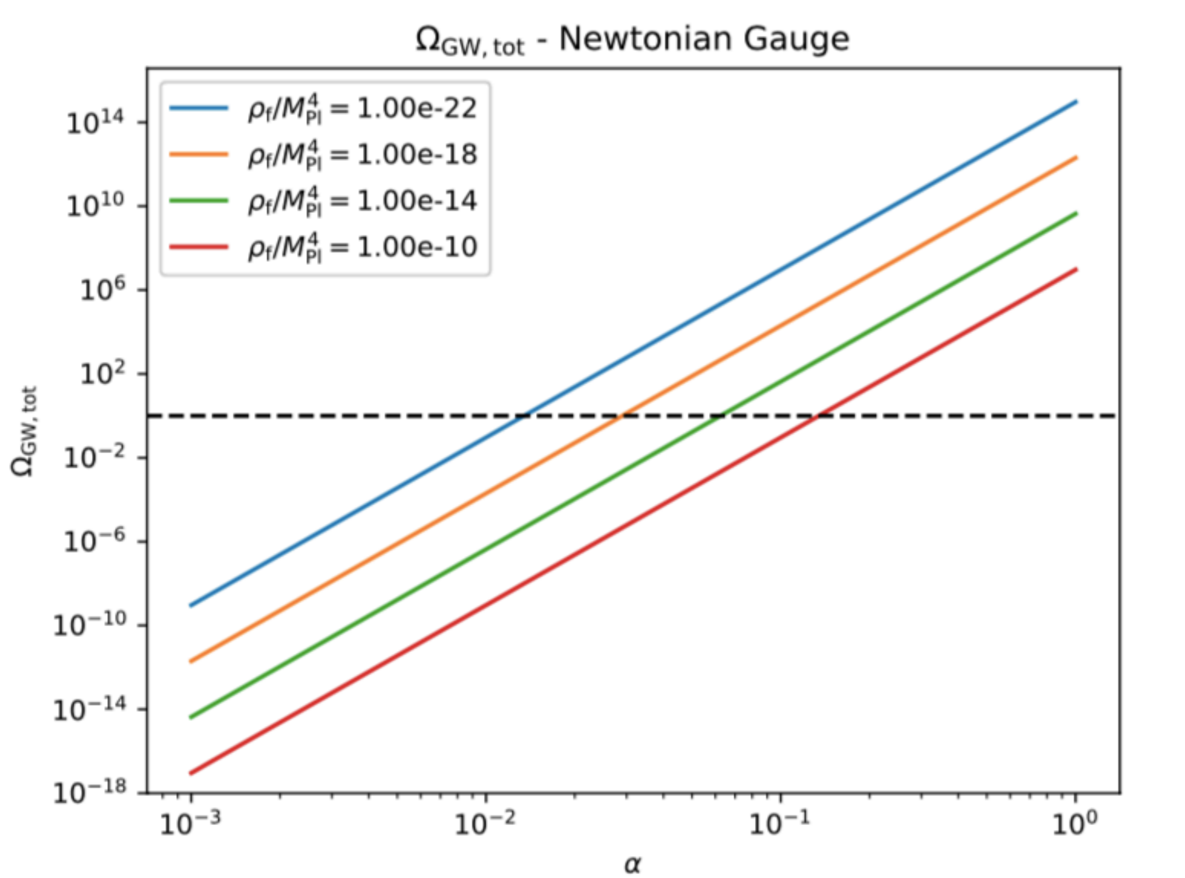


5. The Gravitational Wave Spectrum

- In the case where $\Omega_{\rm PBH,f} = 1$, i.e. PBHs dominate upon formation, we enter the non-linear regime when $k = k_{\rm max} = 2k_{\rm UV}$. In particular,

$$\mathscr{P}_\zeta(k_{\rm max} = 2k_{\rm UV}) = \frac{25}{54\pi}\left(\frac{2k_{\rm UV}}{a_{\rm f}H_{\rm f}}\right)^3 = \frac{25}{54\pi}\left(\frac{2a_{\rm f}H_{\rm f}}{a_{\rm f}H_{\rm f}}\right)^3 \simeq 1.18$$

- Then one, can parametrise the upper bound on k by a parameter α such as that $k_{\rm max} = 2\alpha k_{\rm UV}$. Thus, $\mathscr{P}_\zeta(k_{\rm max} = 2\alpha k_{\rm UV}) = 0.1 \Leftrightarrow a_{\rm linear} = \left(\frac{0.1}{1.18}\right)^{1/3} = 0.44$.
- For $k < k_{\rm max}$, one is always in the linear regime since in the Newtonian gauge, $\mathscr{P}_\zeta(k) \propto k^3$.



- In the case when PBHs dominate upon formation, GWs are overproduced!

6. Conclusions

- We studied the scalar induced GWs during a transient period of PBH domination in the early universe.
- Assuming Poissonian-like distributed PBHs at formation, we found that the class of comoving gauges where $\delta_k \propto \left(\frac{k}{aH}\right)^2 \zeta_k$ is not compatible to describe the spatial threading of spacetime at PBH formation time.

- The abundance of the GWs produced crucially depends on the PBH formation energy scale, ρ_f as well as on the abundance of PBHs at formation time, $\Omega_\mathrm{PBH,f}$.

- By demanding that PBHs dominate the universe energy content before their evaporation one derives a lower bound on the initial abundance of PBHs, $\Omega_\mathrm{PBH,f} > 10^{-3}\sqrt{\frac{\rho_\mathrm{f}}{M_\mathrm{Pl}^4}}$.
- In addition, by requiring that GWs do not lead to a backreaction problem, i.e. $\Omega_\mathrm{GW,tot} < 1$, we derive an upper bound on the initial abundance of PBHs, $\Omega_\mathrm{PBH,f} < 3 \times 10^{-3}$.
- In particular, we show that PBHs cannot have dominated the universe content from their formation time on.

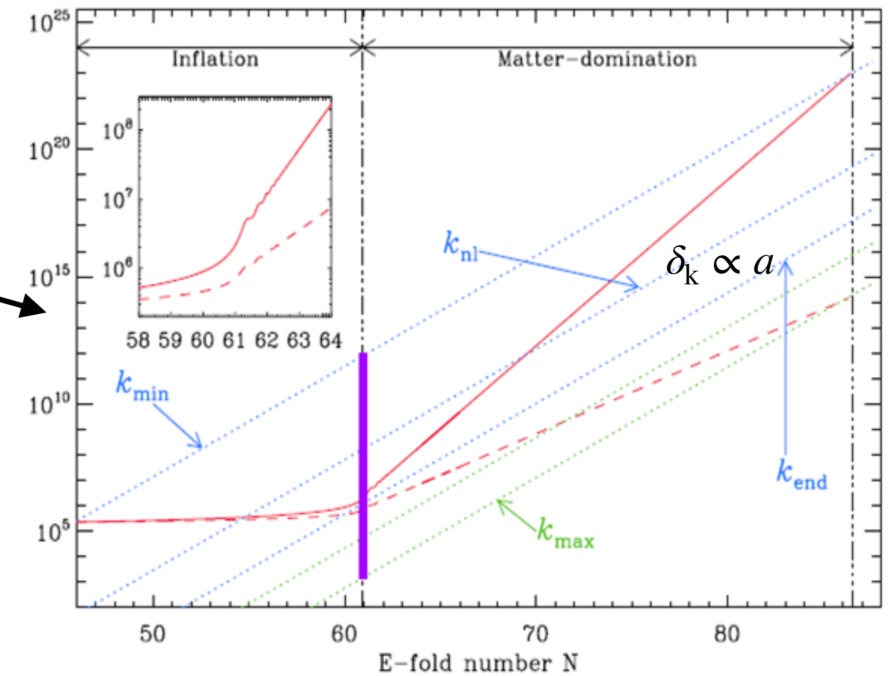
Thank you for your attention!

Appendix

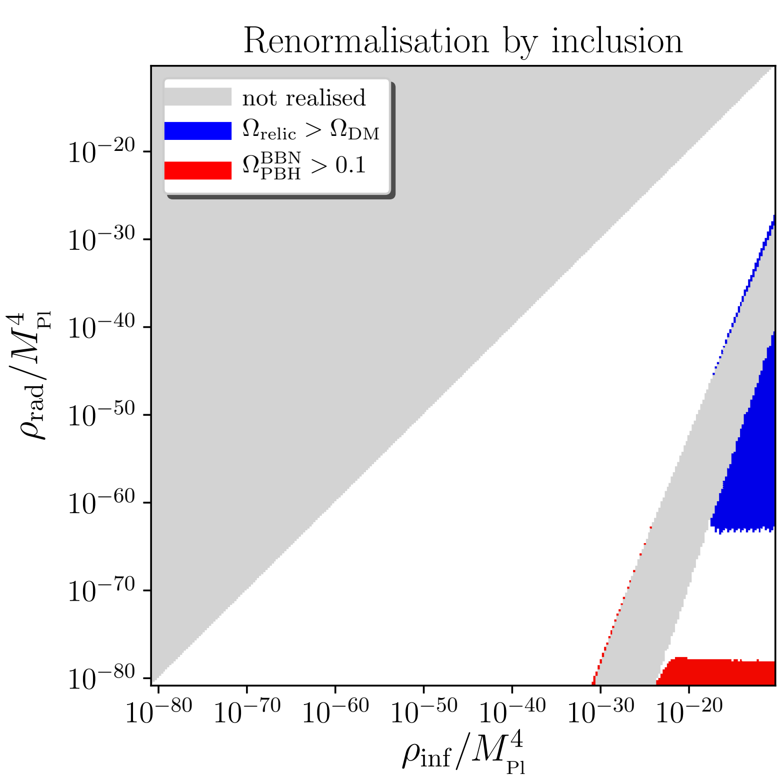
Primordial Black Holes from the Preheating Instability

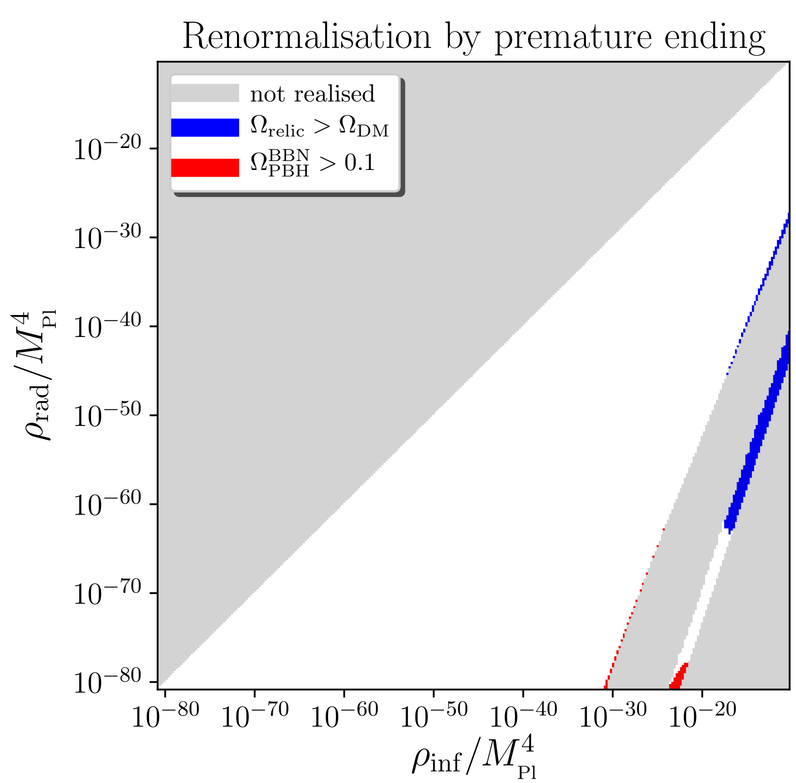
[Published in JCAP, ArXiV:1907.04236, J. Martin, TP, V. Vennin]

- After the end of inflation, **the inflaton oscillates** around a local minimum of its potential. These oscillations trigger a **resonant instability** for some of its fluctuation modes which exit the Hubble radius close to the end of inflation.
- At these **enhanced scales** we study the production of PBHs. The production mechanism can be so efficient that **PBHs** subsequently **dominate the content of the universe** and **reheating proceeds from their evaporation**.
- In order for our model to be theoretically consistent and not to give rise to overproduction of PBHs, we consider **two complementary renormalisation schemes of the PBH mass fraction, β(M)**
- Observational constraints on the PBH abundance also restrict the duration of the resonant instability phase, leading to tight **constraints on the reheating temperature** that we derive as well as on the **energy scale at the end of inflation.**



[J. Martin, M. Lemoine, K. Jedamzik – 2010]



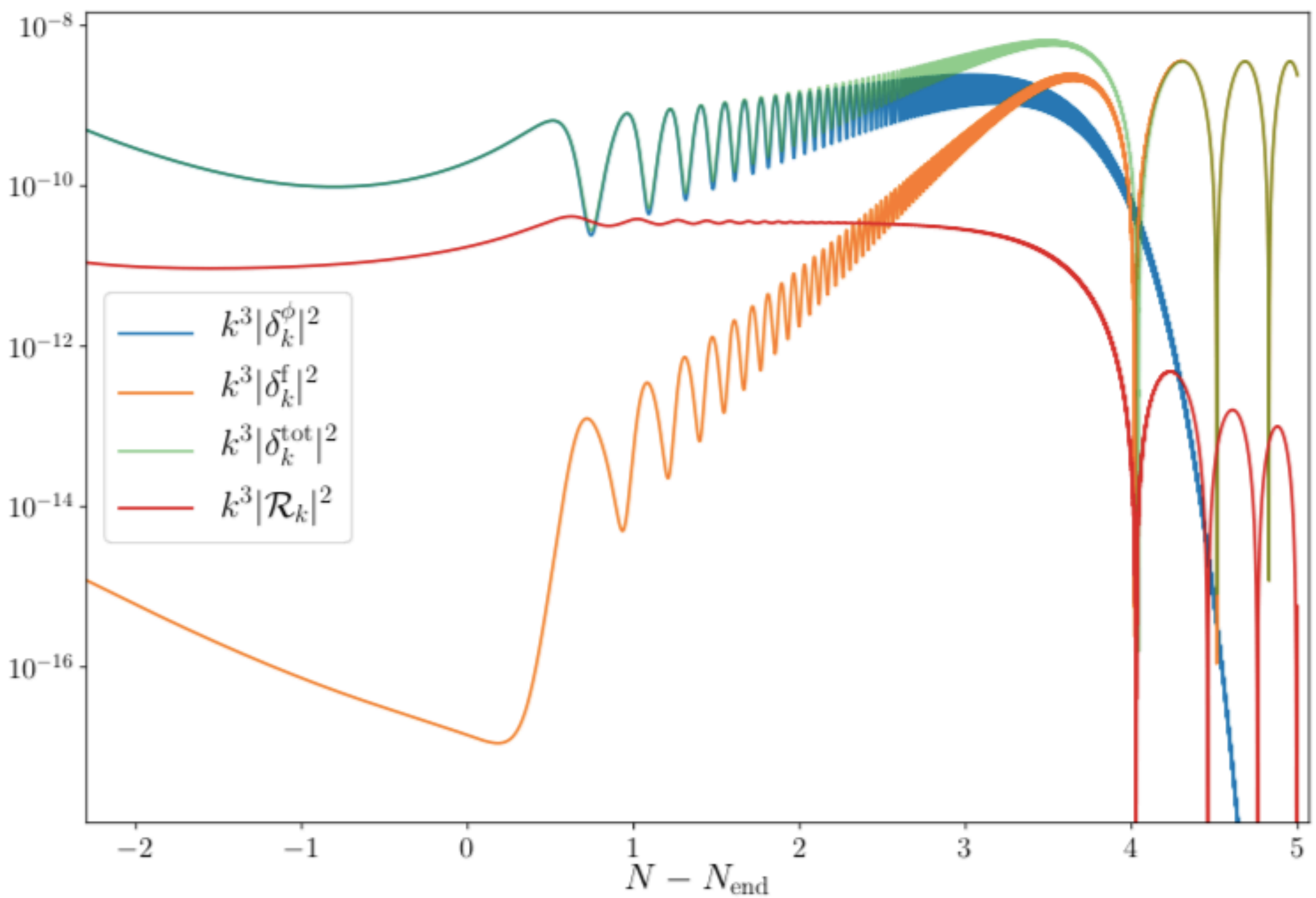


Metric Preheating and Radiative Decay in Single-Field Inflation

With L.Pinol, J. Martin, TP and V. Vennin

[Published to JCAP, ArXiV: 2002.01820]

- The inflation is coupled to a radiation fluid to ensure the transition to the radiation-dominated era.
- The inflation decay does not alter the resonant instability structure until the radiation fluid dominates the energy content of the universe.



Anisotropic Collapse of PBHs

I. Musco, Theodoros Papanikolaou

- We consider a general spherical symmetric metric which is of the form

$$\mathrm{d}s^2 = -\,\mathrm{d}t^2 + a^2(t)\left[\frac{\mathrm{d}r^2}{1-K^2(r)} + r^2\mathrm{d}\Omega^2\right]$$

- We account for the anisotropy of the collapse by introducing an anisotropic pressure in a covariant way:

$$p_\mathrm{t} = p_\mathrm{r} + \lambda f(r,t)k^\mu \nabla_\mu p_\mathrm{r} \quad \text{, pressure gradients}$$

$$p_\mathrm{t} = p_\mathrm{r} + \lambda f(r,t)k^\mu \nabla_\mu \rho \quad \text{, density gradients}$$

- Choosing $f(r,t) = R(r,t)$ the parameter λ becomes a dimensionless anisotropy parameter.

- By doing a gradient expansion approximation, energy and velocity perturbations are expressed in terms of the curvature profile $K(r)$, the dimensionless parameter λ and the pressure/density gradients $\{\frac{\partial p_\mathrm{r}}{\partial r} \text{ or } \frac{\partial \rho}{\partial \mathrm{r}}\}$.
- Then using as initial conditions the density and velocity perturbations profiles we solve numerically the Misner-Sharp-Hernadez hydrodynamical equations and we extract the PBH formation threshold δ_c.
- Intuitively, for configurations where $p_\mathrm{r} > p_\mathrm{t}$ the collapse to PBHs is favoured.