

About Fractons: further investigations O.M. Lecian

I. FRACTIONAL-ELECTRIC-CHARGED-PARTICLES: AB INITIO DERIVATION, CALCULATIONS, EXPERIMENTAL VERIFICATIONS

Fractional-electric-charged-particles can be due either to a broken color symmetry, or to an 'enlarged' GUT theory to which color singlets can be included; according to these two mechanisms, fractons are expected to acquire $1/3$ electric charge and/or other values of fractional charges.

In an **unbroken color symmetry**, an $SU(5)$ model which allows for fractons in $S(7)$ non-trivial change of embedding can be considered. Such a non-trivial change of embedding consists of

- 2 normal $SU(5)$ families, and
- 2 charge-shifted conjugated families. The latter must be light ($\lesssim 100 GeV$) for them to acquire mass only after the breaking of $SU(2) \otimes U(1)$.

Charge shifted families contain both fractionally-charged leptons and fractionally-charged quarks, with charges q not in sequence

$$q \equiv \left(\frac{2}{3} + n\right),$$

, with n integer. As from [1], **a fracton can be either a lepton or a hadron**.

From [2], a **broken** $SU(7)$

$$SU(7) \rightarrow SU(3) \otimes SU(2) \times U(1)$$

at super-heavy mass scales, is obtained by scalars in the irreps of high dimensionality, and cannot couple directly with fermions.

Given the Lagrangian model L_0 [2] invariant under supergauge transformations in the one-loop approximation.

$$L_0 = -\frac{1}{2}(\partial_\mu A)^2 - \frac{1}{2}(\partial_\mu B)^2 - \frac{1}{2}i\bar{\psi}\gamma^\mu\partial_\mu\Psi + \frac{1}{2}F^2 + \frac{1}{2}G^2$$

, with A scalar, B pseudoscalar, ψ Majorana particle, F and G auxiliary fields. Conformal transformations and γ^5 transformations in the algebra imply that only theories with massless particles can be invariant under supergauge transformations: it is possible to eliminate the pertinent commutators and anticommutators [3]; differently, it is necessary to restrict to the consequent subalgebra to remove massless particles, and obtain supergauges with constant parameters.

Given the invariant Λ

$$\Lambda \equiv \lambda F$$

, the auxiliary fields can be eliminated from the Lagrangean.

It is therefore possible to study of higher order corrections, construct more complex and more realistic models, invariant under a combination of supergauge and internal symmetries.

As from [4], a **broken color** $SU(3)$ implies the presence of quarks and 'diquarks'. The observed charges are the color singlet 1_c -particles with fractional charges.

This implies a kind of GUT of electroweak interactions and strong interactions, which equals to minimal extensions of $SU(5)$ to include fractionally-charged color singlets.

The model $SU(6)$ does not fit the requirements for such constructions.

The mixing parameter $\sin^2\theta_w$ implies the presence of many light doublets to be added on purpose.

A. Constraints for the obtention of fractons

Representation constraints (R):

in the simplest case

- R₁: fermionic representations $\underline{1}_c, \underline{3}_c, \underline{1}_c^*$;
- R₂: reducible fermionic representations:
 - complex;
 - flavor-chiral;
 - free of Adler-Bell-Jackiw anomalies;
- R₃: the found representations of the symmetry group must contain at least $\underline{3}_c$ or $\underline{1}_c^*$;
- R₄: the sum of the charges of the found representations should equal zero;
- R₅: $\Delta Q = \pm 1$ for weak currents $\Rightarrow \exists$ two $\underline{1}_c$ -fields which differ by ± 1 .
- R₇: for pure vector couplings of the unbroken $SU_c(3) \otimes U_{em}(1)$, Weyl fields must be pairs of fermionic representations $\{\tilde{F}\}$ with $q \neq 0$.

Dynamical constraints (D)

- D₁: symmetries must break at a mass scale $\mu, \mu \simeq 100 GeV$ as $SU_c(3) \otimes U_{em}(1)$;
- D₂: the spectrum must be such that the GUT is at a mass $M, M \gtrsim 10^{14} GeV$ to avoid proton decay;
 - $\alpha_s(100 GeV)$: $0.1 < \alpha_s < 0.3$;
 - $\alpha_M \lesssim 0.3$;
 - the mixing angle θ_w : $\sin^2 \theta_w \sim 0.20$.

By combining the constrains, the conditions are obtained:

- R₁, R₂ iff anomaly-free red. repr.'s of $SU(N)$ constructed from irrepr.'s corresponding to single-color Young tables (with all indices desymmetrized);
 - R₃, R₅ + request of fractionally-charged color singlets:
 - $SU(6)$ is excluded;
 - $SU(7)$ must contain a vector coupling $SU_c(3) \otimes U_{em}(1)$;
- from spinorial representations of $O(14)$.

$Q = \frac{2}{3}, Q = -\frac{1}{3}, Q = 0$ do not violate the R constraints.

$Q = -\frac{1}{3}, Q = 0$ contain only one generation of ordinary quarks and leptons.

As a result, (the values of the charge(s) of) the possible eigenvalues of the charge operator Q can be assigned. For this purpose,

- three copies of the fermionic representations $\{\tilde{F}\}$ are required for the theory to contain the known quarks and leptons;
- all fermions must be light with respect to M ;
- Higgs scalars contributing to the fermionic-mass matrix must breaks the weak $SU(2)$ group.

- D₁, D₂: two different predicted values of $\sin^2 \theta_w$:

- non-exotic Dirac fields;
- exotic Dirac fields.

Both:

- Q_5 's, i.e. $SU(5)$ charge operators, are defined;
- exotic-charge operators Q_e are conserved;

separately by *any* symmetry-breaking Higgs scalar (which can be decomposed of two fermions) for the conservation of the total charge.

B. Further tentative models

In [5], the implications of a $SU(5)$ GUT's containing

$$SU(3)_c \times SU(2)_L \times U(1),$$

the mixing angle and the proton lifetime have been analyzed

for a single super-heavy mass scale \tilde{m}_s (unification mass scale).

The lifetime of a proton becomes smaller when superheavy-Higgs-boson-mediated amplitudes become significant (higher-order amplitudes of effective dimension 5 give rise to proton life-times proportional to m_s^2 rather than m_s^4).

The experimental value [6]

$$\sin^2(\hat{\theta}_W(M_W)) = 0.215 \pm 0.014$$

is here confirmed; the proton results nevertheless unstable. In order to increase the lifetime of the proton, supersymmetric constraints have to be imposed. Such constraints require N_H relatively light weak isodoublets and n_g generations of quarks and leptons for the pertinent low-energy limit. The supersymmetric hypotheses requested about the spectrum of particles are:

- gauge bosons to have spin $\frac{1}{2}$ fermion partners;
- the ordinary spin $\frac{1}{2}$ quarks and leptons to have scalar partners; and
- the Higgs scalars to have spin $\frac{1}{2}$ fermionic partners.

The following assumptions can be done to simplify the analysis:

- varying $\tilde{\mu}$;
- all the added supersymmetric partners have a mass $\tilde{\mu} \leq \tilde{m}_W$.

For $\tilde{\mu} \leq \tilde{m}_W$, the β function can be investigated.

2-loops β function is used to let the coupling from $\tilde{\mu}$ to \tilde{m}_s to evolve.

Theories involving

$SU(2)_L \times U(1)$ involve two parameters, the mixing parameter $\sin^2(\hat{\theta}_W)$ and the parameter ρ (obtained from ν_μ deep inelastic scattering).

Such theories predict proton decay; it can be avoided by:

- improving and better determining the mixing parameter $\sin^2(\hat{\theta}_W)$; and
- finding the pertinent agreement of ρ .

It is possible to use a satisfactory Weinberg angle instead of using the VEV's techniques at $\sim 10^9 GeV$:

- from irrep.'s for fixing the charge shifts of the two conjugated families[7].

Differently, the choice of $SU(7)$ allows one to avoid

$\alpha m_{ss} \lesssim m_W$, i.e. the mixing Weinberg parameter (too high in minimal $SU(5)$ supersymmetric GUT's).

The choice of $SU(7)$ with fractons allows one to avoid

$$\alpha m_{ss} \lesssim m_W,$$

i.e. it allows one to avoid a value of $\sin^2(\theta(m_W))$ too high in minimal $SU(5)$ supersymmetric GUT('s)

In particular, in a minimal $SU(7)$ supersymmetric GUT, which includes fractons, with the minimum two light Higgs doublets and no intermediate mass scales (not gauge-breaking mass scales and not supersymmetry-breaking mass ones) can give a value for the mixing parameter close to the experimental one.

From [8], by assuming the two charges-shifted singlets as light, the low-energy thresholds from the charges-shifted families and their scalar partners, the errors in the one-loop calculations for the values of m_W and $\sin^2 \theta$ are large enough allow for a consistent complete agreement with the experimental standard deviations.

II. FRACTIONALLY-CHARGED-QCD-SINGLET PARTICLES

An $SO(18)$ GUT can contain fractionally-charged-color-singlet fermions and exotic quarks [9]. The interest in this model is that it can be broken up to $SU(3)_c \times U(1)_{em}$.

Given T the generators of the $SO(18)$ subgroups, three different ways to define the electric-charge operator Q are possible.

The differences in the ways of the assignments of the charge operator are due to the presence of 'auxiliary' symmetries within the steps of the breach.

The generalized charge operator Q' is defined as

$$Q' = Q + \sqrt{2}aT_3^{L'} + \sqrt{2}bT_3^{R'} + \sqrt{2}cT_3^{L''} + \sqrt{2}dT_3^{R''},$$

with a, b, c, d constants:

- $a = c = 0, b = 1/3, d = 2/3$;
- $a = c = 0, b = d = 1/3$;
- $a = b = c = d = 1/3$.
- (- $a = b = c = d = 0$).

The breaking of the $SO(18)$ symmetry is achieved as

$$\begin{aligned} SO(18) &\xrightarrow{M} SO(10)_S \times SO(8)_W \\ &\xrightarrow{M'} SO(8)_S \times SO(8)_W \times U'(1) \\ &\xrightarrow{M''} SU(3)_c \times SU(2)_W \times U(1) \times \tilde{G}' \\ &\xrightarrow{M_W} SU(3)_c \times U(1)_{em} \end{aligned}$$

with

M : mass scale at Grand Unified Point,

M', M'' : intermediate mass scales,

M_W : mass scale of the intermediate bosons of weak interaction; and

where

- $\tilde{G}' \equiv -U''(1)$;
- discrete symmetry;
 - chiral symmetry;
 - possibly not necessary (by adjusting masses after the choice of M'').

III. FURTHER APPLICATIONS

A. Proton stability and neutrino masses

The problem of the proton stability and that of the neutrino masses have been proposed to be discussed unficatedly [10] by hypothesizing the existing of a further single field.

A $U(1)_{B-L}$ broken by the presence of a single field of charge 2; this way,

- the remaining Z_2 symmetry avoids the proton to decay;
- the charge-2 field
 - can couple with rh ν 's and lh ν 's;
 - endows the lh ν 's with a large mass;
 - allows fractionally-charged particles to acquire a large mass.

In [11], the model $SO(10)$ gauge symmetry is considered for the breach at the string level to

- $SO(6) \times SO(4) \times SU(5) \times U(1)$, and
- $SU(3) \times SU(2) \times U(1)^2$.

The investigation is conducted about the *thermal history* of the Universe.

The resulting exotic-matter states can be stable, and are demonstrated to be classified according to the properties of the $SO(10)$ symmetry breaking.

For $SO(10)$, the states have nonstandard charges under the $U(1)_{Z'}$ symmetry, as embedded in $SO(10)$, and are orthogonal to $U(1)_Y$:

- these states are stable if the $U(1)_{Z'}$ gauge symmetry

- is unbroken down to low energies, or;
- if some residual local discrete symmetry is still unbroken after the $U(1)_{Z'}$ symmetry breaking.

There can exist a fractional-charge particle stable under the $U(1)_{Z'}$ symmetry.

1) The densities of fractionally-charged hadronic bound states at low temperature are severely constrained and cannot avoid decaying without the hypothesis of an inflationary phase of the Universe. The constraints are aimed at allowing the model with current Astrophysical evidences [12].

2) Differently, under a non-Abelian gauge group the fractionally-charged states are confined, s.t. they would form integrally-charged bound states:

this way, the conditions established in [12] should not be violated.

3) At lowering temperatures connected with the expansion of the universe, the remaining fractionally-charged hadrons are predicted to give rise to bound states of integer charge with fractionally-charged heavy leptons.

4) The static properties of hadrons, i.e. electromagnetic mass splitting of mesons containing heavy-light quarks are model-dependent;

transition and elastic form factors are calculated in the heavy-quark effective theory:

this way, it is possible to calculate the mass-splitting relation between the two heavy-light mesons \tilde{U}_0 and \tilde{U}_1 .

Within the framework of the thermal history of the Universe, high-energy quark from the decay from a heavy particle (i.e. inflatons, modulus or gravitino) can be demonstrated to undergo flavor oscillation and is thermalized after scatterings with the ambient thermal plasma, the scattering being due to the presence of a dimension-nine-baryon-number-violating operator because of the presence of a baryon number symmetry operator, which conserves the baryon parity [13].

The evolution of barionic matter and of fractionally-charged particles within the thermal history of the Universe, as well as a description of the pertinent experimental probes and investigation, has been exposed also in [14].

IV. FURTHER MODELS INVOLVING MIRROR SYMMETRY

In [15], the experimental verifications of the parameter space of supersymmetric GUT theories including also a Yukawa coupling is proposed.

In [16], fractionally-charged branes on orbifold are predicted by including also mirror symmetry.

In [17], anyon quasiparticles with fractional quantum numbers are investigated on orbifolds.

In [18], within the framework of mirror symmetry in $N = 2$ superconformal field theories, charged particles acquire fractional exchange statistics parametrized by the phases; the model exhibits charge non-conservation for the $U(1)$ particles, which applies to quantum Hall effect.

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