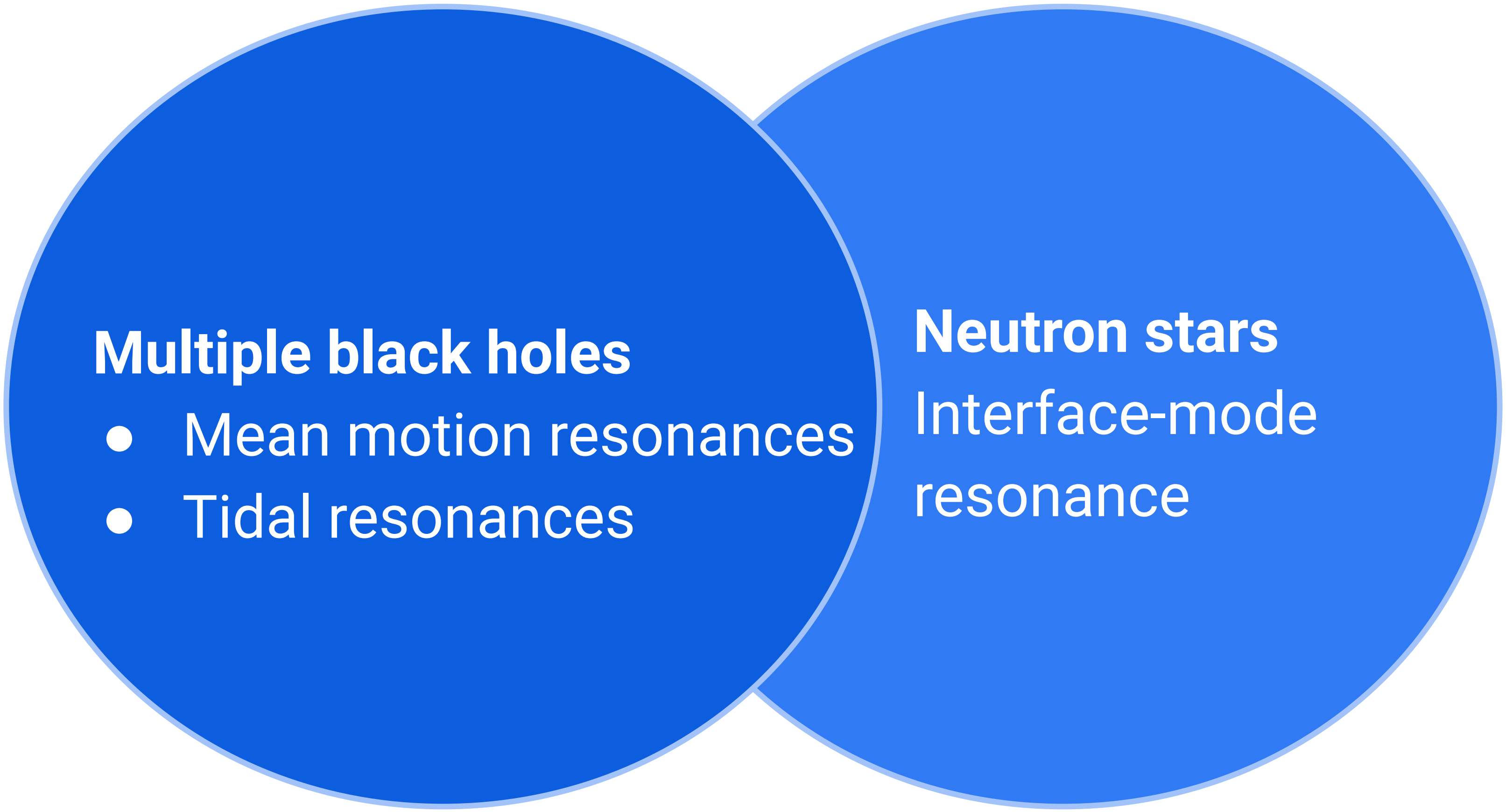
Resonances in black hole spacetimes and neutron stars

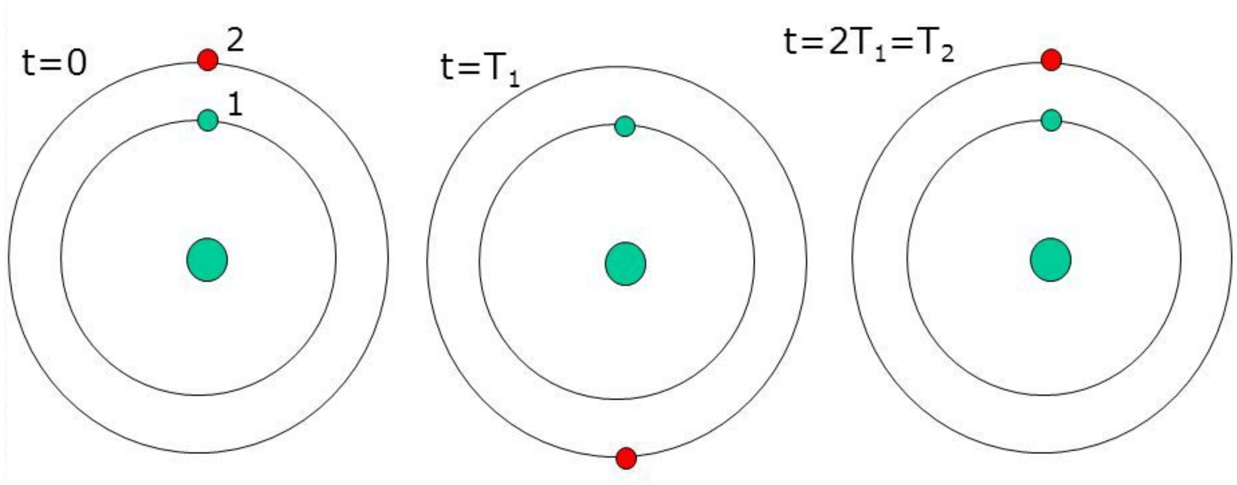
Béatrice Bonga - 2 June 2020
Virtual Seminar APC (University of Paris) & Service de Physique Théorique (ULB, Brussels)

Based on BB, Yang, Hughes [PRL, 1905.00030], Yang, BB, Peng, Li [PR, 1910.07337] and Pan, Lyu, BB, Ortiz and Yang [arXiv: 2003.03330]

Tidal forces & resonances

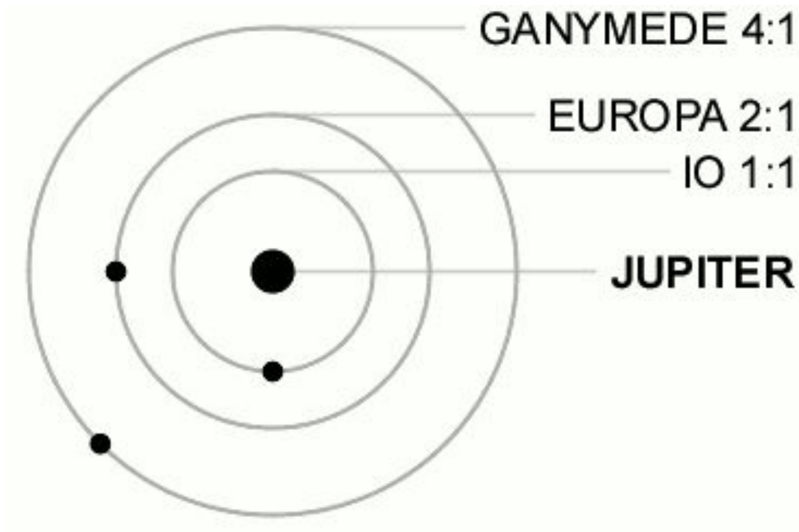


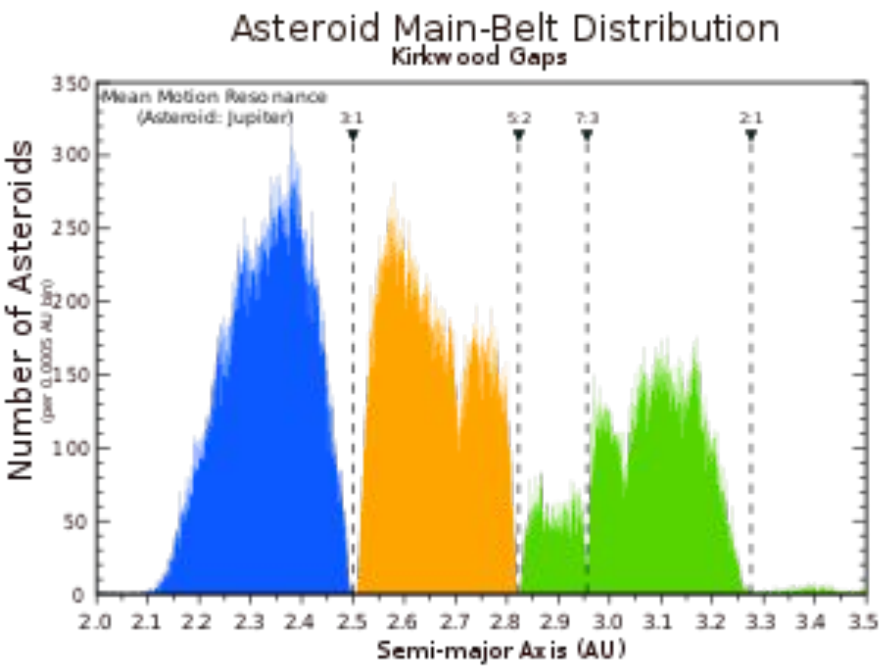
Mean motion resonance

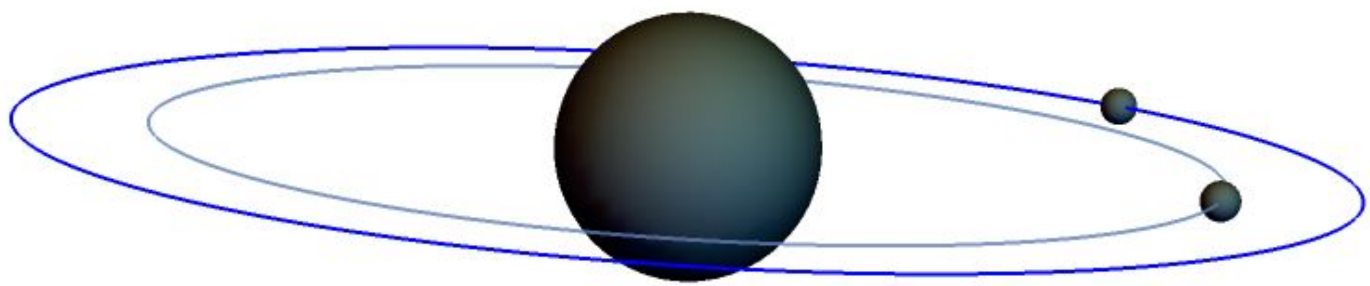


Orbital resonance due to the gravitational interaction between two small bodies orbiting a central massive object

Stable and unstable configurations







Focus on mean motion resonance around massive black holes in the *post-Newtonian regime*

Post-Newtonian Hamiltonian

Oscillatory terms, secular terms and resonant terms → only include first order resonance

$$H = H_1 + H_2 + H_{\mathrm{int}}$$

$$H_1 = \frac{1}{2m_1}\left(p_r^2 + \frac{p_\phi^2}{r^2}\right) - \frac{m_1 M}{r} + \frac{1}{c^2}\left[-\frac{1}{8m_1^3}\left(p_r^2 + \frac{p_\phi^2}{r^2}\right)^2 + \frac{m_1 M^2}{2r} - \frac{3M}{2m_1 r}\left(p_r^2 + \frac{p_\phi^2}{r^2}\right)\right] + \mathcal{O}\left(\frac{1}{c^4}\right)$$

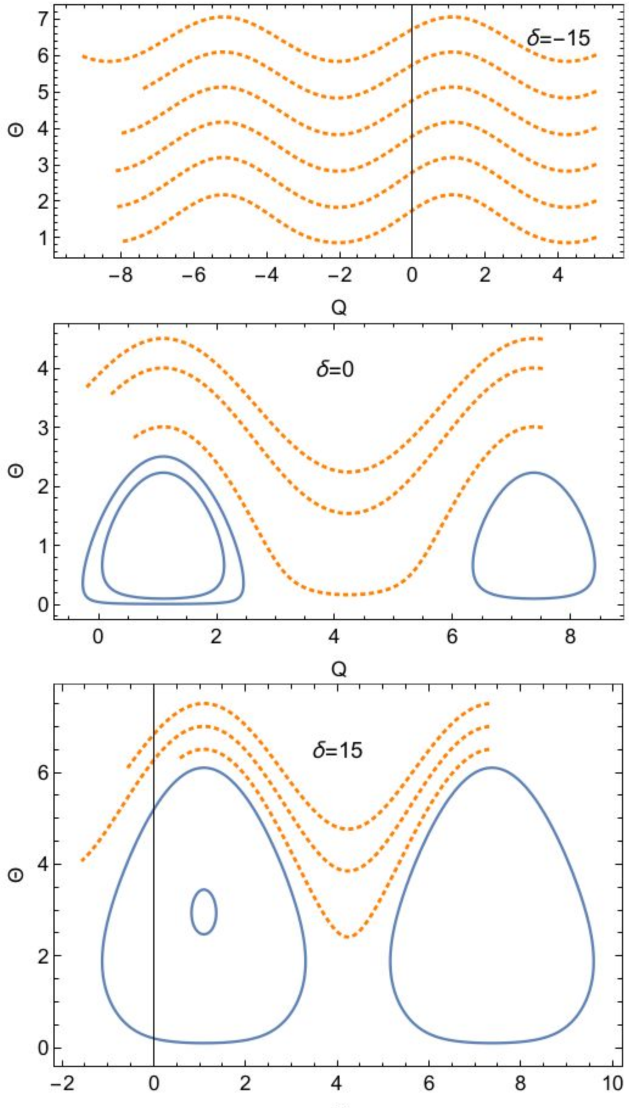
Canonical coordinates & momenta → Action-angle variables → Poincaré variables → Small eccentricity expansion

$$H = \alpha\underline{\Gamma} + \beta\underline{\Gamma}^2 + \kappa\sqrt{2\underline{\Gamma}}\cos\theta_2 + \mathcal{O}\left(\frac{1}{c^4}, f_d^2, e^3\right)$$

Measure of proximity of resonance

Resonant/libration angle

Phase space: guiding trajectories



No resonance possible

Resonance possible

Parametric evolution of α

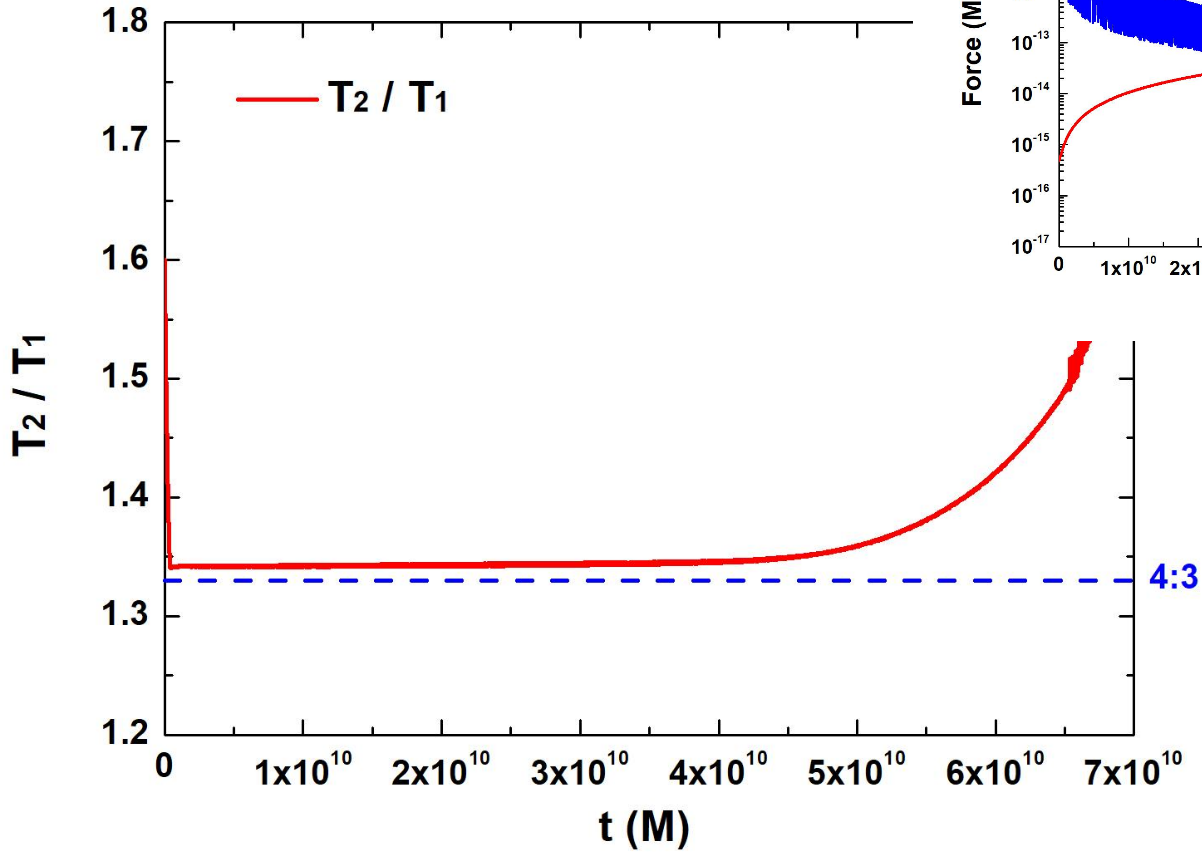
$$H = \alpha\underline{\Gamma} + \beta\underline{\Gamma}^2 + \kappa\sqrt{2\underline{\Gamma}}\cos\theta_2 + \mathcal{O}\left(\frac{1}{c^4}, f_d^2, e^3\right)$$

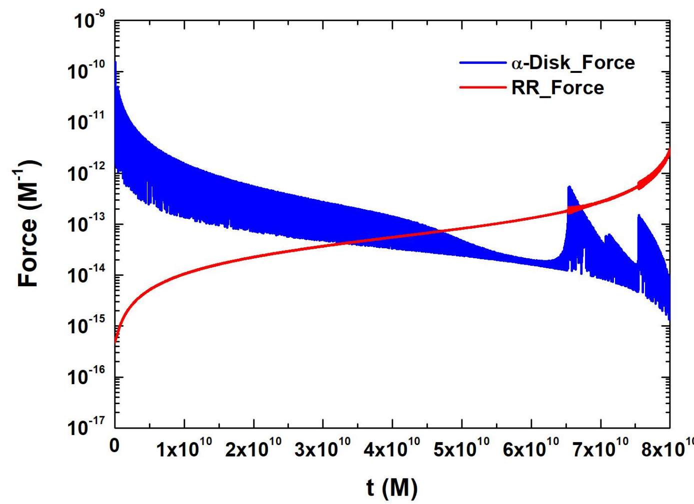
Capture and escape

Sustained resonance in planetary systems: $\eta > C\,(\tau_e / \tau_a)^{3/2}$

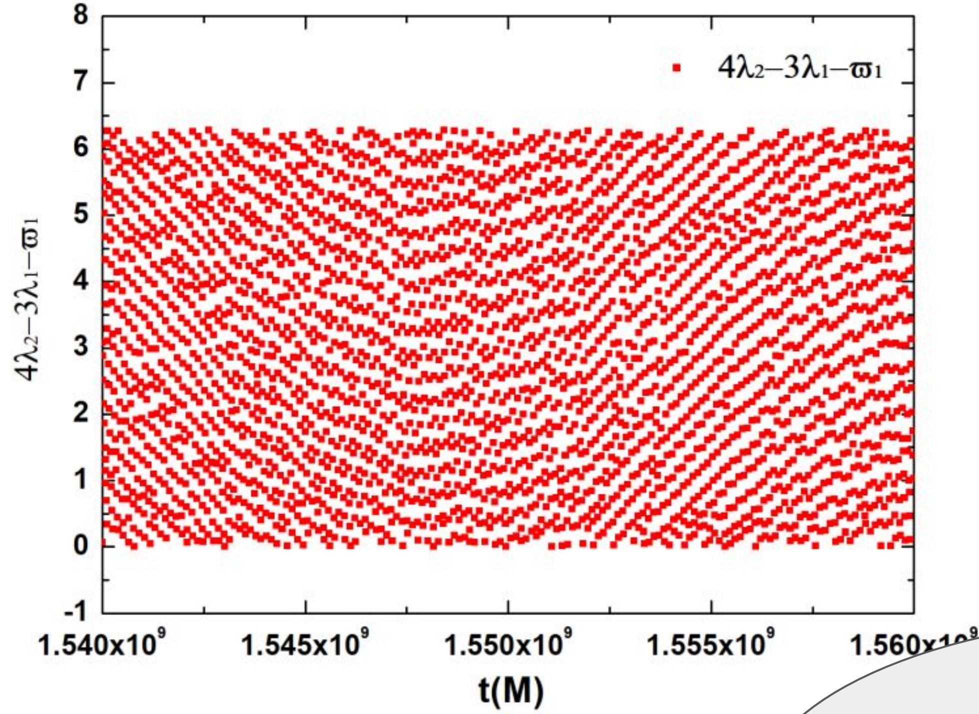
⇨ Gravitational radiation alone cannot lead to sustained resonance

Numerical evolution (disk + post-Newtonian effects)



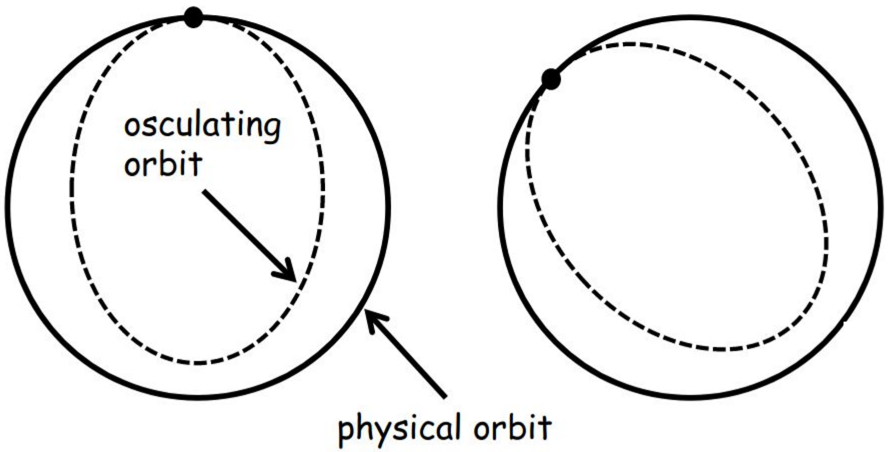


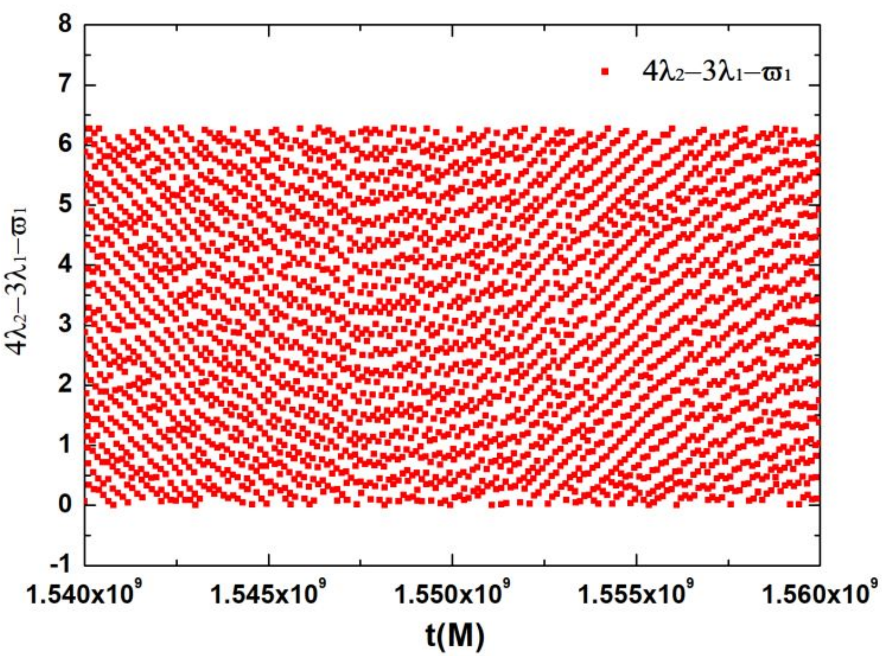
Cautionary tale



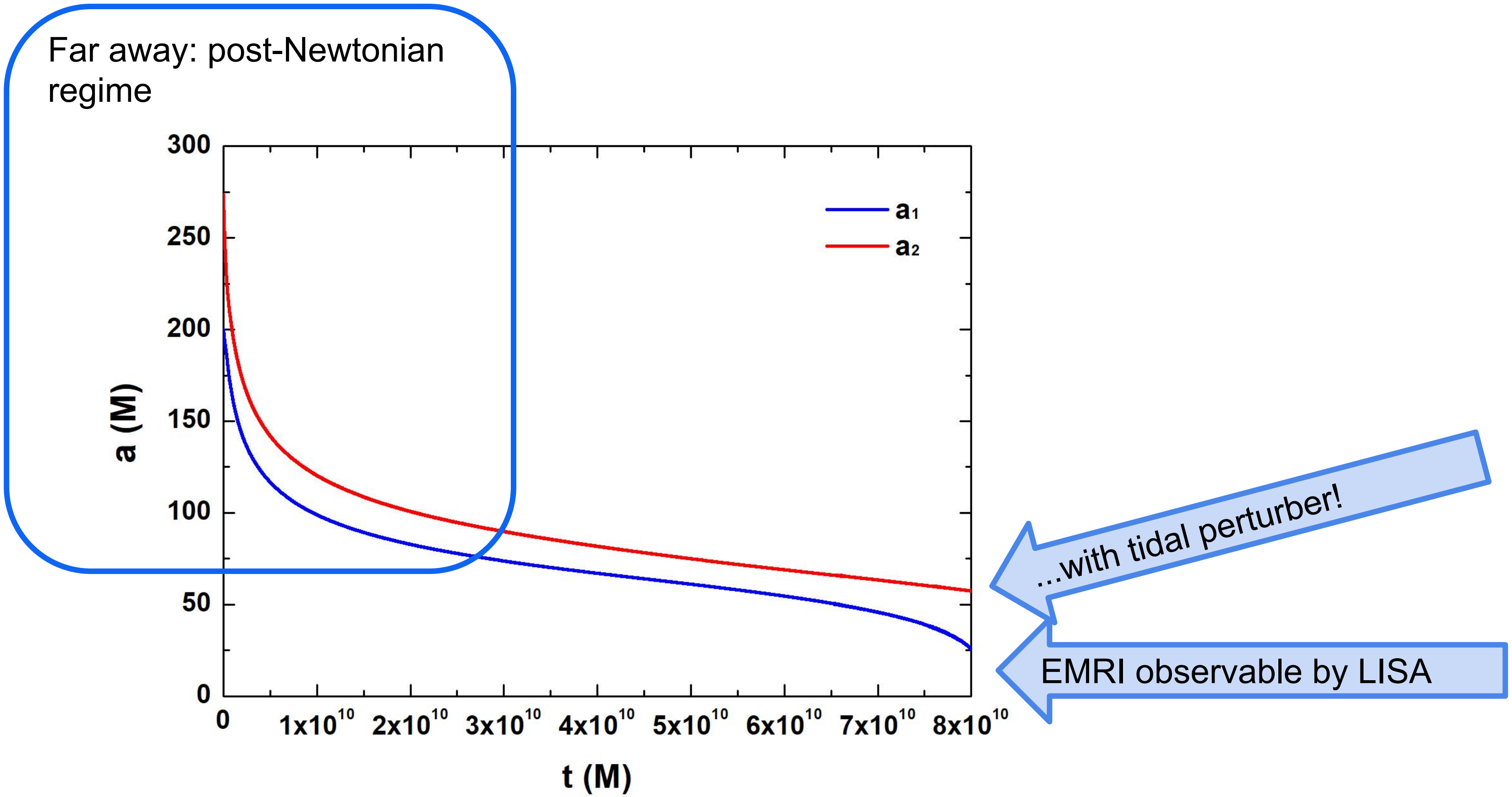
Hold on…
Resonant angle should be locked if true resonance!

“Nature is perfectly happy with a circle!”





Multiple black holes close to central black hole



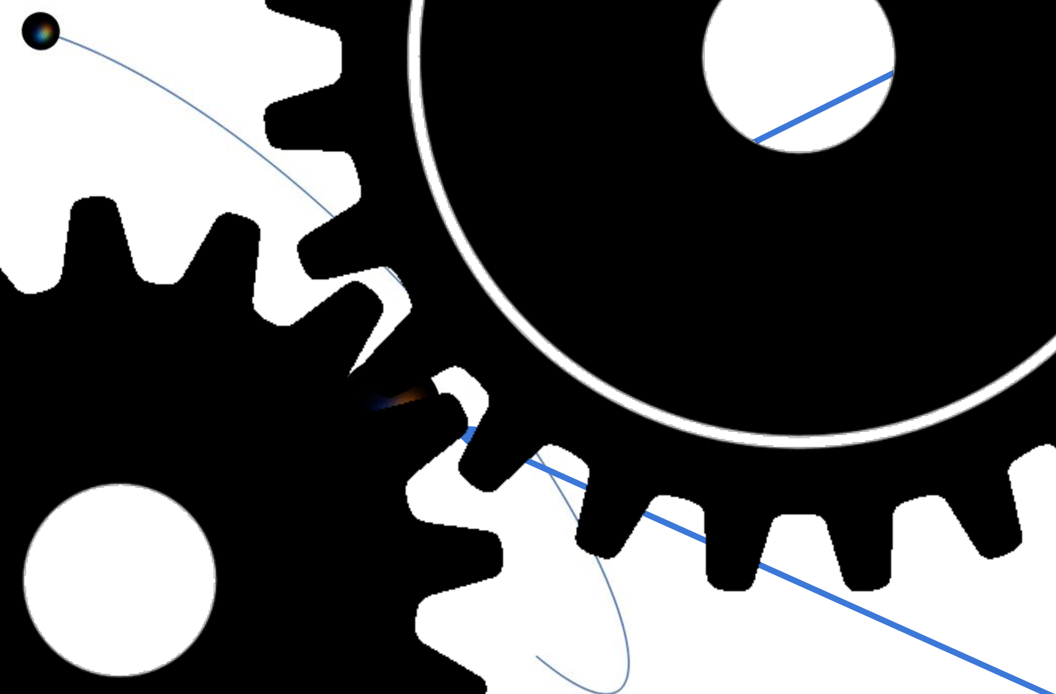
Mean motion resonance summary

- Mean motion resonances are due to the interaction of two small bodies in the gravitational field of a massive body

- Observed in many planetary systems

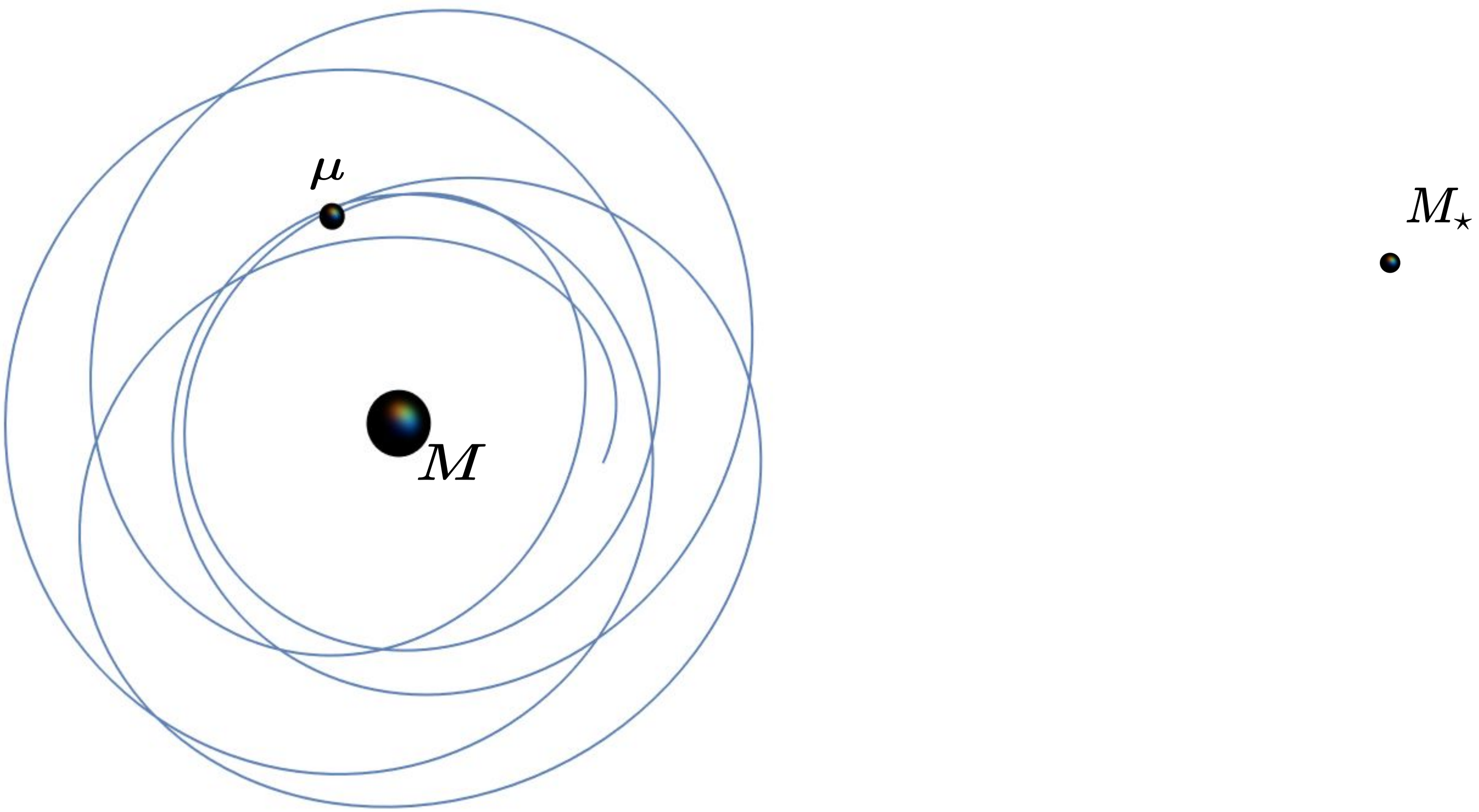
- During mean motion resonances the two small bodies evolve in sync and their motion is locked together

Extreme Mass Ratio Insp[redacted]

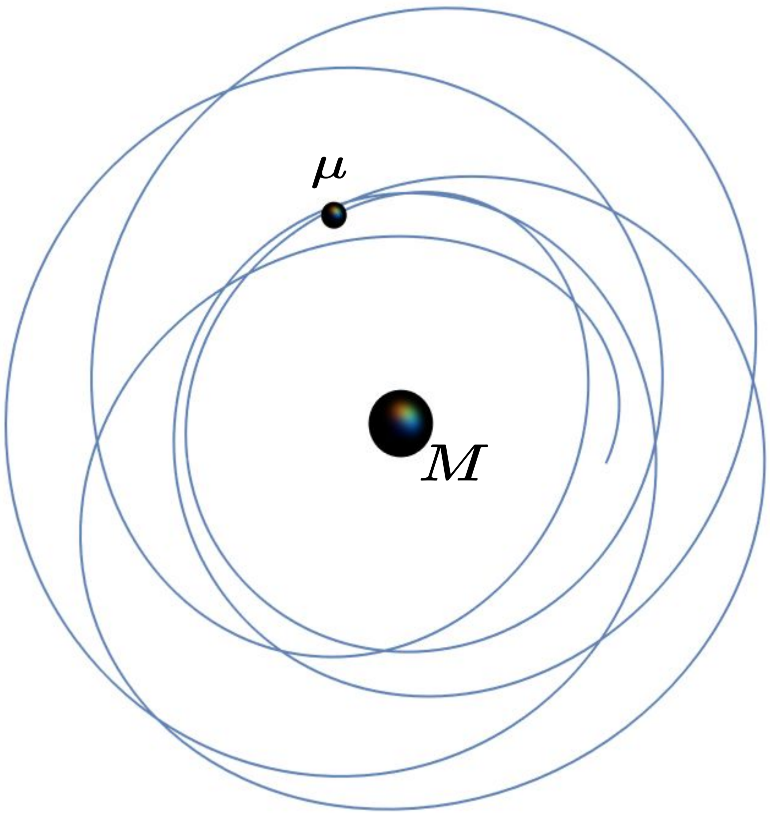


[redacted]

EMRIs as isolated systems ?



Zeroth order description



Four constants of motion:
$\{\mu$, E, L_z, Q$\}$

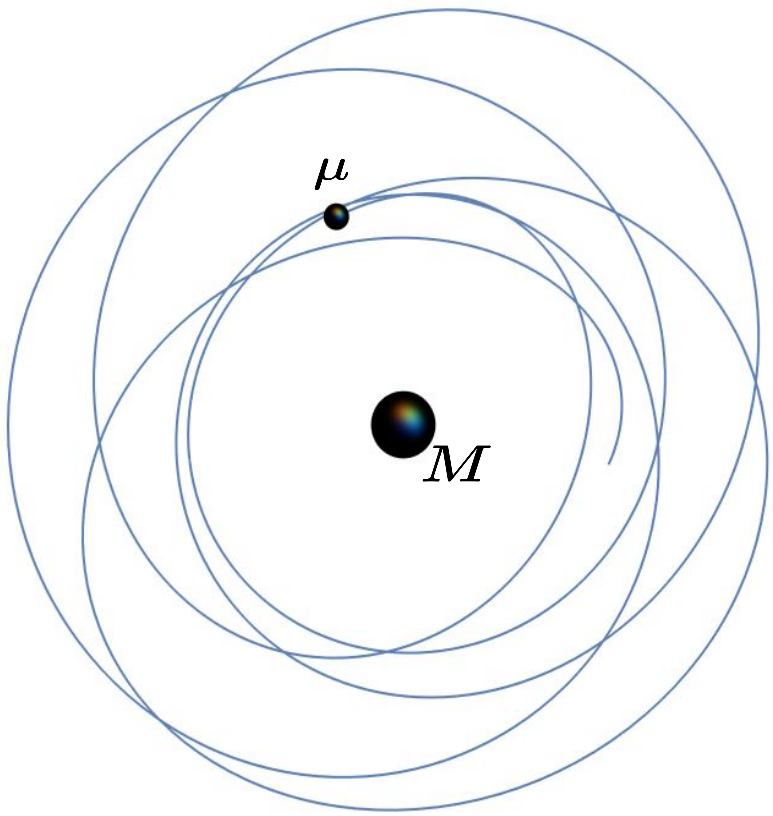
→ Geodesic equation is integrable

→ Action-angle variables are useful

$$\frac{dq_i}{d\tau} = \omega_i(\mathbf{J})$$
$$\frac{dJ_i}{d\tau} = 0$$

$$\mathbf{J} := \{J_r, J_\theta, J_\phi\}$$

Before introducing a perturber….



… motion is not geodesic!

Gravitational radiation changes description

$$\frac{dq_i}{d\tau} = \omega_i(\mathbf{J}) + \eta g^{(1)}_{i,\mathrm{sf}}(q_\theta, q_r, \mathbf{J}) + \mathcal{O}(\eta^2)$$

$$\frac{dJ_i}{d\tau} = \eta G^{(1)}_{i,\mathrm{sf}}(q_\theta, q_r, \mathbf{J}) + \mathcal{O}(\eta^2)$$

Mass ratio: μ/M

Self force 101: adiabatic approximation

$$\frac{dq_i}{d\tau} \approx \omega_i(\mathbf{J})$$
$$\frac{dJ_i}{d\tau} \approx \eta \left\langle G^{(1)}_{i,\mathrm{sf}}(q_\theta, q_r, \mathbf{J}) \right\rangle$$

[Flanagan & Hinderer, PRD 2008]

Averaging

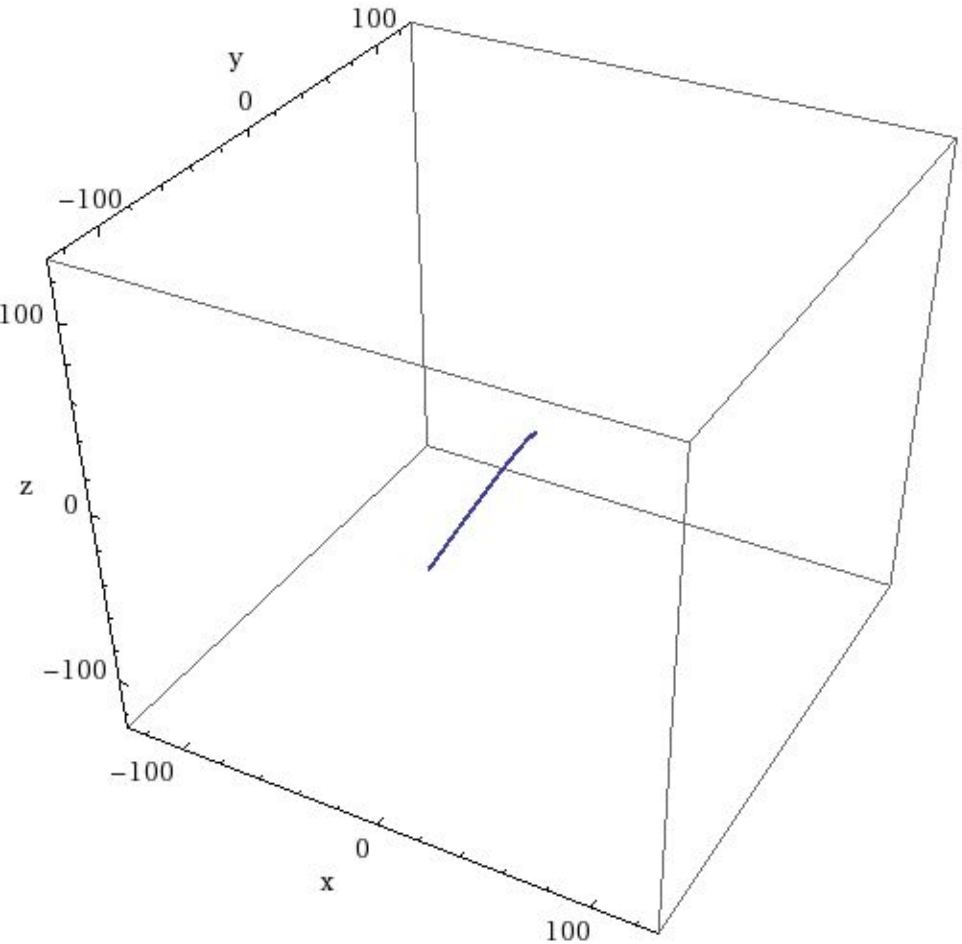
$$G_{\rm sf}^{(1)}(q_\theta, q_r, \mathbf{J}) = \sum_{k,n} G_{\rm sf,kn}^{(1)}(\mathbf{J}) e^{i(kq_\theta + nq_r)}$$

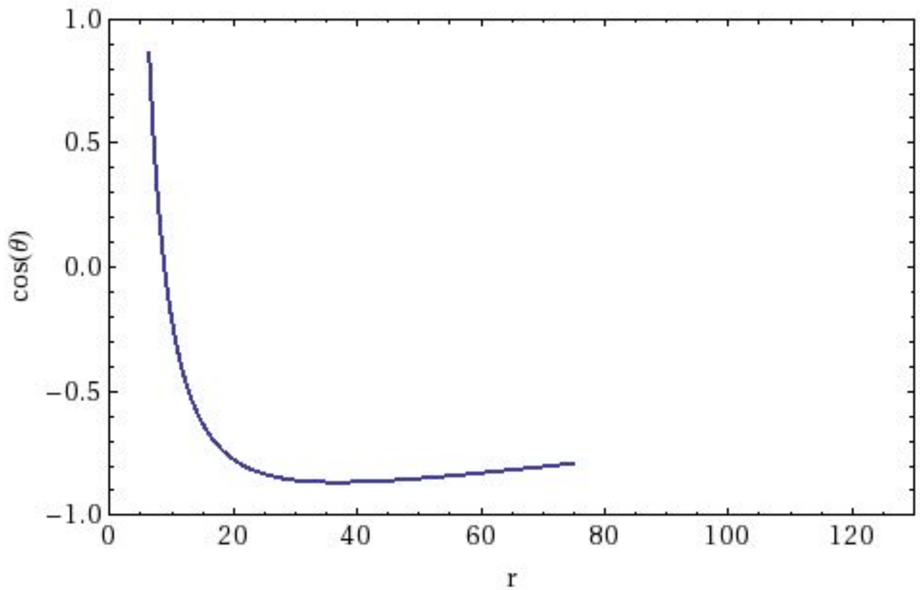
$$\Rightarrow \quad \left\langle G_{\rm sf}^{(1)}(q_\theta, q_r, \mathbf{J}) \right\rangle = G_{\rm sf,00}^{(1)}(\mathbf{J})$$

Not true when $k\omega_\theta + n\omega_r \approx 0$

[Flanagan & Hinderer, PRL 2012]

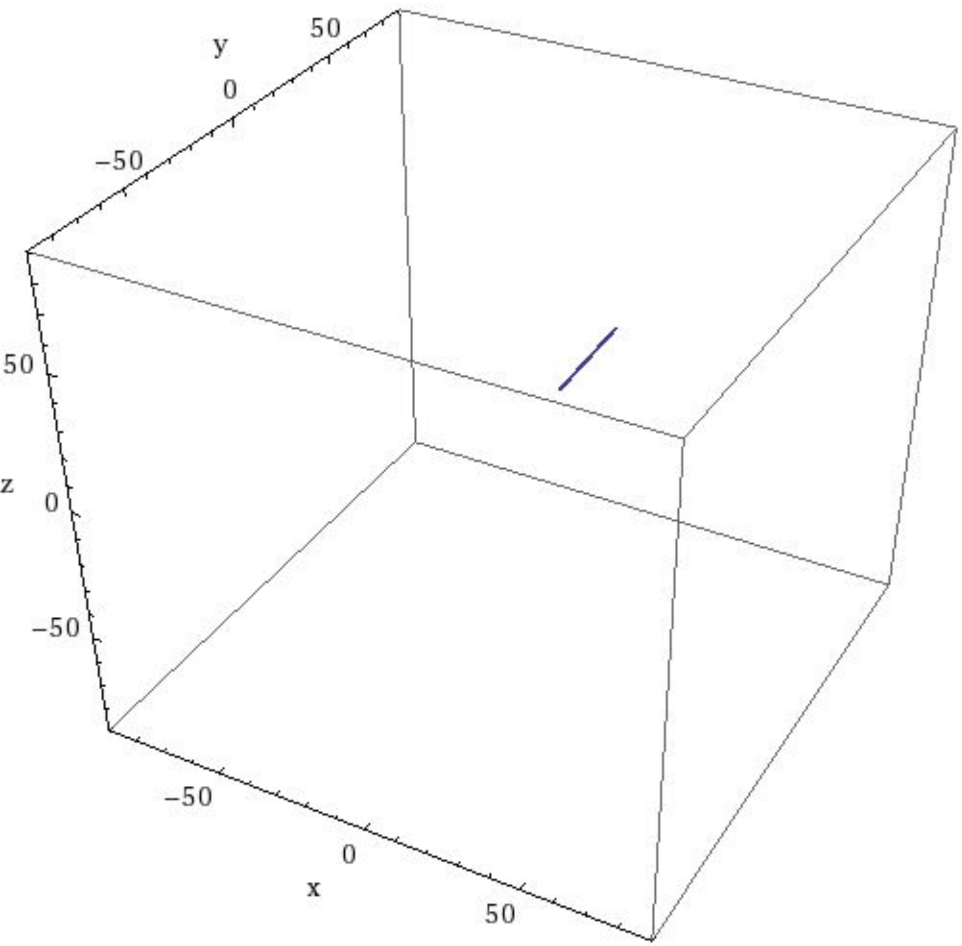
Generic evolution

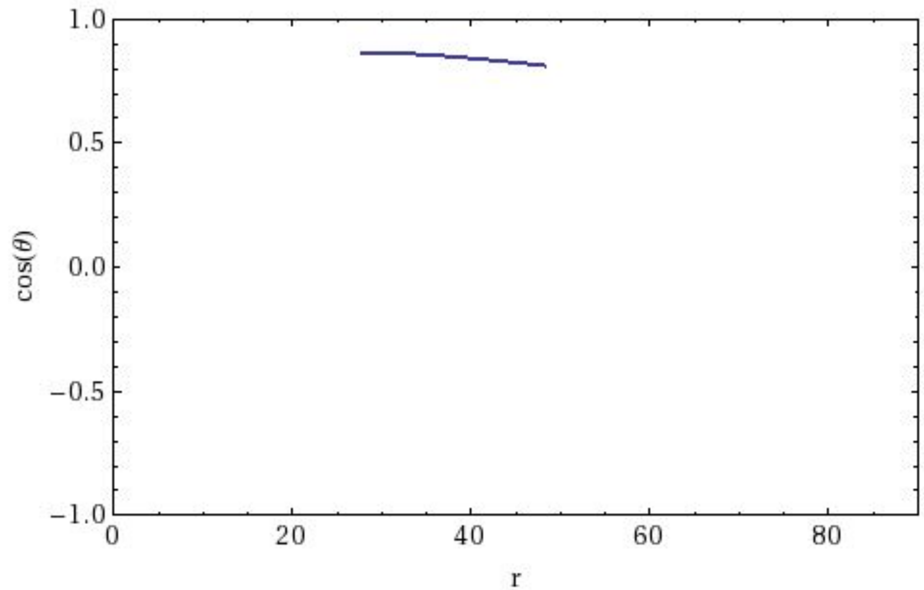




Credit: Rob Cole

Resonant orbit

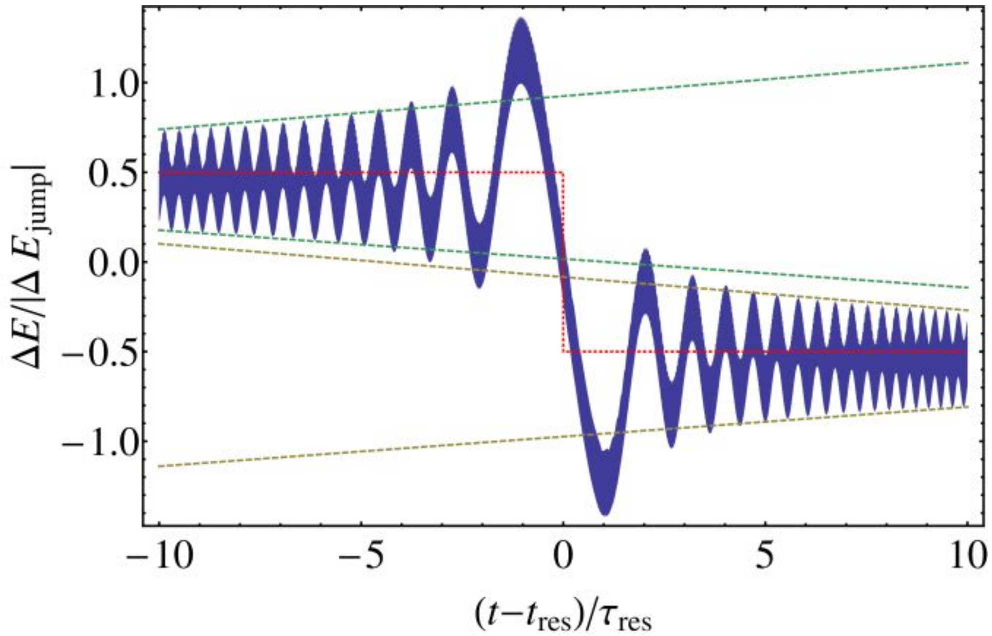




Credit: Rob Cole

Self-force resonance

[Flanagan & Hinderer, PRL 2012]



Are self-force resonances astrophysically important?

- They happen generically for EMRIs in the LISA band.
- Kick size is typically small but if early in the inspiral → significant dephasing of the waveform.
- Bigger kicks when
  (1) Lower order resonances (=small *n* and *k*)
  (2) More eccentric orbits

Only lose a few percent for detection purposes, but important for parameter estimation!
[Berry, Cole, Canizares, Gair, PRD 2016]

How about the tidal perturber?

Action-angle variables ***with*** tidal perturber

$$\frac{dq_i}{d\tau} = \omega_i(\mathbf{J}) + \epsilon\, g^{(1)}_{i,\mathrm{td}}(q_\phi, q_\theta, q_r, \mathbf{J}) + \eta\, g^{(1)}_{k,\mathrm{sf}}(q_\theta, q_r, \mathbf{J}) + \mathcal{O}(\eta^2, \epsilon^2, \eta\epsilon)$$

$$\epsilon = M_\star M^2/a^3$$

$$\frac{dJ_i}{d\tau} = \boxed{\epsilon\, G^{(1)}_{i,\mathrm{td}}(q_\phi, q_\theta, q_r, \mathbf{J})} + \eta\, G^{(1)}_{i,\mathrm{sf}}(q_\theta, q_r, \mathbf{J}) + \mathcal{O}(\eta^2, \epsilon^2, \eta\epsilon)$$

Tidal resonance

$$G_i^{(1)}(q_\phi, q_\theta, q_r, \mathbf{J}) = \sum_{m,k,n} G_{i,mkn}^{(1)}(\mathbf{J}) e^{i(mq_\phi + kq_\theta + nq_r)}$$

$$\omega_{mkn} := m\omega_\phi + k\omega_\theta + n\omega_r = 0$$

- More general condition than for self-force resonance
- *Also* occurs for low eccentricity orbits

Size of the effect?

$$a^\alpha = -\frac{1}{2}(g^{\alpha\beta}_{\mathrm{Kerr}} + u^\alpha u^\beta)(2h_{\beta\lambda;\rho} - h_{\lambda\rho;\beta})u^\lambda u^\rho$$

Metric of tidally perturbed Kerr $h_{\mu\nu}$ from [Gonzales + Yunes, 2005]

> Teukolsky equation + metric reconstruction

> Takes as input $\mathcal{E}_{ij}$

> Assumes tidal field is stationary

> Caveat: only takes into account m=±1 and m=±2

Mathematica   Fri 08:58

metricperturbation-withEtidal-foranyspin-hx.m - Wolfram Mathematica 12.0

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Functions Sections Update   Debug Run All Code

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(* Created with the Wolfram Language : www.wolfram.com *)
{{(m2*Sin[ϑ]^2*((-30*a^8 + 4*a^4*(2*ml - 9*r)*r^3 +
      96*r^6*(-2*ml + r)^2 + a^6*(44*ml^2 + 84*ml*r - 78*r^2) +
      16*a^2*r^4*(14*ml^2 - 16*ml*r + 3*r^2) +
      a^2*(-45*a^6 + 32*a^2*ml*(2*ml - r)*r^2 +
        a^4*(62*ml^2 + 70*ml*r - 99*r^2) + 16*r^4*(10*ml^2 - 24*ml*r +
          9*r^2))*Cos[2*ϑ] - 2*a^4*(9*a^4 - 2*(8*ml - 9*r)*(2*ml - r)*
         r^2 + a^2*(-10*ml^2 + 10*ml*r + 9*r^2))*Cos[4*ϑ] +
      a^6*(-3*a^2 + 2*ml^2 - 6*ml*r + 3*r^2)*Cos[6*ϑ])*
     Cos[2*ϕ] - 4*a*(a^2 + 2*r^2 + a^2*Cos[2*ϑ])*
     (4*a^2*r^2*(-11*ml + 6*r) + a^4*(-13*ml + 9*r) +
      8*r^3*(4*ml^2 - 8*ml*r + 3*r^2) + 4*a^2*(a^2*(-4*ml + 3*r) +
        r^2*(-13*ml + 6*r))*Cos[2*ϑ] + 3*a^4*(-ml + r)*
       Cos[4*ϑ])*Sin[2*ϕ]))/
  (128*b^3*(r^2 + a^2*Cos[ϑ]^2)^3),
 (m2*Sin[ϑ]^2*((3*a^6 + 16*a^2*(ml - r)*r^3 + 24*(2*ml - r)*r^5 -
      a^4*r*(10*ml + r) + 4*a^2*(a^4 + 2*(2*ml - 3*r)*r^3 -
        a^2*r*(2*ml + r))*Cos[2*ϑ] + a^4*(a^2 + (2*ml - 3*r)*r)*
       Cos[4*ϑ])*Cos[2*ϕ] +
    8*a*r*(a^2 + 2*r^2 + a^2*Cos[2*ϑ])*(a^2 + 2*r*(-ml + r) +
      a^2*Cos[2*ϑ])*Sin[2*ϕ]))/
  (16*b^3*(r^2 + a^2*Cos[ϑ]^2)^2),
 (m2*Sin[2*ϑ]*((3*a^6*(4*ml - r) + 8*(2*ml - r)*r^6 +
      a^4*r*(-2*ml^2 + 32*ml*r - 11*r^2) + 8*a^2*r^3*(ml^2 + 6*ml*r -
        2*r^2) + 4*a^2*(a^4*(4*ml - r) + 3*a^2*(4*ml - r)*r^2 -
        2*(ml - r)^2*r^3)*Cos[2*ϑ] + a^4*(a^2*(4*ml - r) + 2*ml^2*r -
        r^3)*Cos[4*ϑ])*Cos[2*ϕ] +
    2*a*(a^2 + 2*r^2 + a^2*Cos[2*ϑ])*(a^4 - 4*ml^2*r^2 + 2*r^4 +
      a^2*(-2*ml^2 - 4*ml*r + 3*r^2) + a^2*(a^2 - 2*ml^2 + 4*ml*r + r^2)*
       Cos[2*ϑ])*Sin[2*ϕ]))/
  (32*b^3*(r^2 + a^2*Cos[ϑ]^2)^2),
 (m2*Sin[ϑ]^2*
   (a*(2*(5*a^8 + 8*r^6*(-9*ml^2 + 2*ml*r + 2*r^2) +
        2*a^2*r^4*(-26*ml^2 - 5*ml*r + 20*r^2) +
        a^6*(-11*ml^2 - 9*ml*r + 23*r^2) + a^4*r^2*(-25*ml^2 - 15*ml*r +
          42*r^2)) + (15*a^8 + 16*ml*(5*ml - 2*r)*r^6 +
        16*a^2*r^4*(-5*ml^2 + ml*r + 3*r^2) + a^6*(-31*ml^2 - 3*ml*r +
          63*r^2) + a^4*r^2*(-97*ml^2 - 15*ml*r + 96*r^2))*Cos[2*ϑ] +
      2*a^2*(3*a^6 + 2*ml*r^4*(-2*ml + r) + a^2*r^2*(-23*ml^2 + 23*ml*r +
```

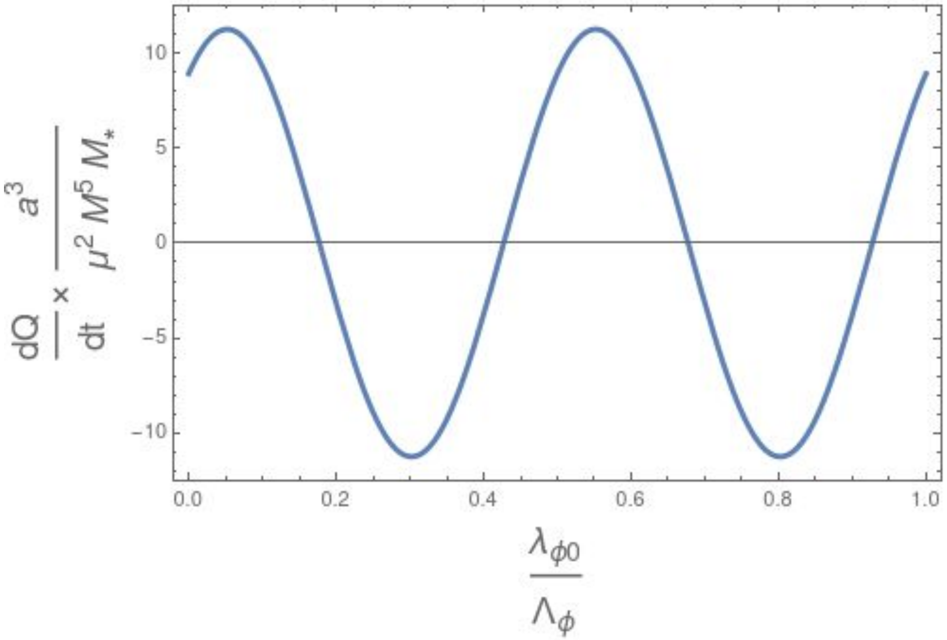
100%

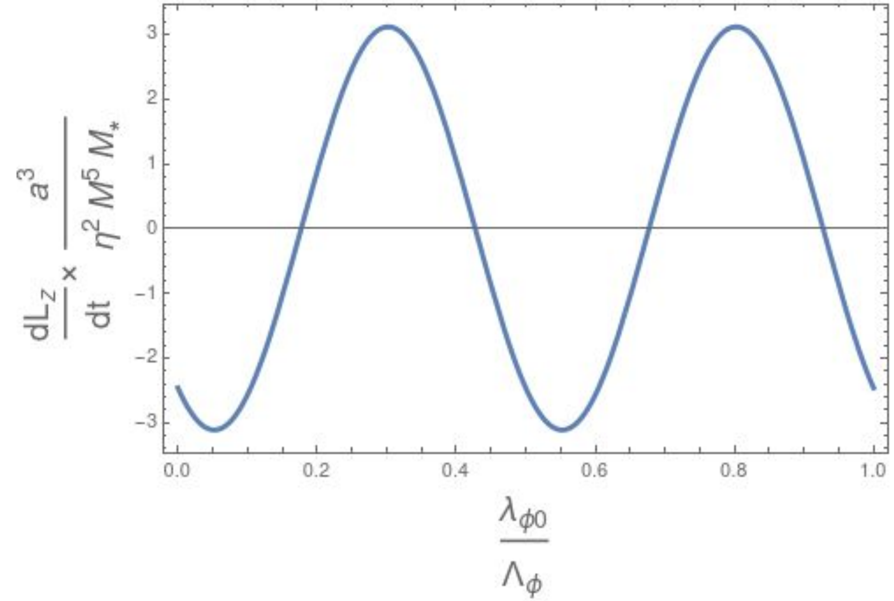
Proof of principle with m=-2,k=2,n=1

$$\left\langle G_i^{(1)}(q_\phi, q_\theta, q_r, \mathbf{J}) \right\rangle \approx G_{i,-2,2,1}^{(1)}(\mathbf{J}) e^{-2iq_{\phi 0}} + \mathrm{cc}\,.$$

χ	$r_{\min}$	$r_{\max}$	$\theta_{\min}$	$\dot{Q}_{-2,2,1}$	$\dot{L}_{z\,-2,2,1}$
0.7	3.5	5.1628033	$\pi/3$	$1.66 + 2.27i$	$-0.35 - 0.47i$
0.9	3	6.6159726	$\pi/4$	$6.60 + 7.70i$	$-1.72 - 2.01i$
0.99	3	5.3718120	$\pi/4$	$4.46 + 3.43i$	$-1.23 - 0.95i$

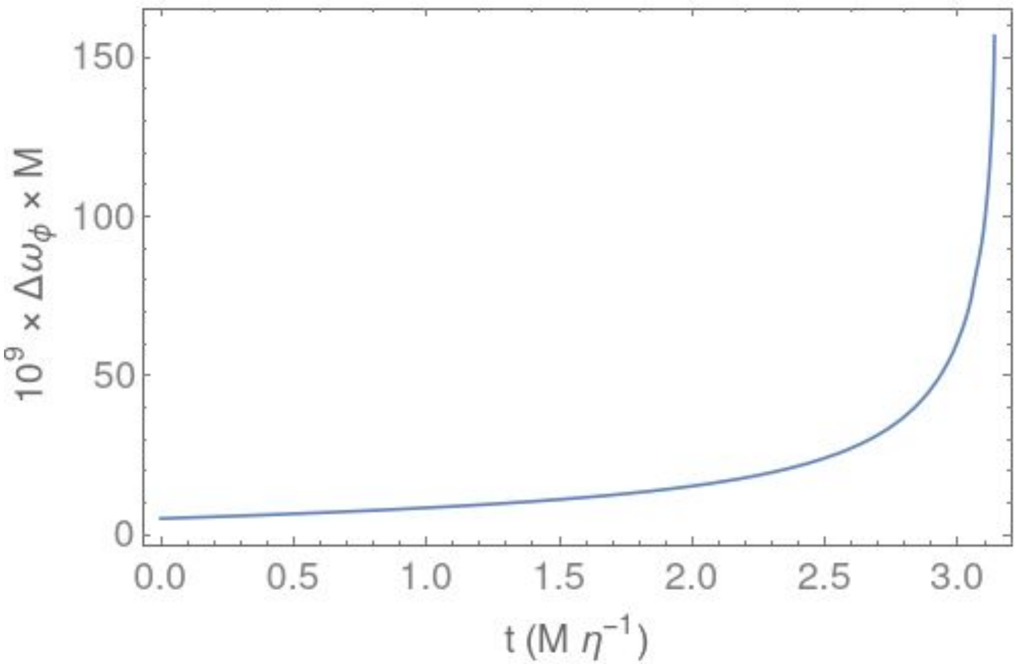
Rate of change depends on the phase





Compare two orbits

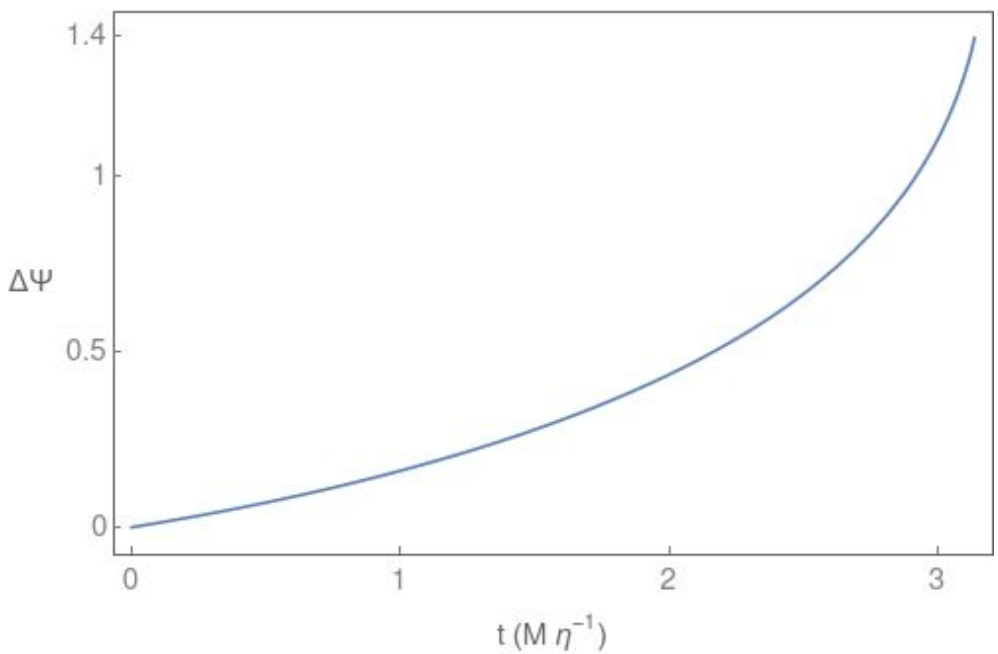
$$\{E, Q, L_z\} \to \omega_\phi^{(1)} \quad \text{versus} \quad \{E, Q + \Delta Q, L_z + \Delta L_z\} \to \omega_\phi^{(2)}$$



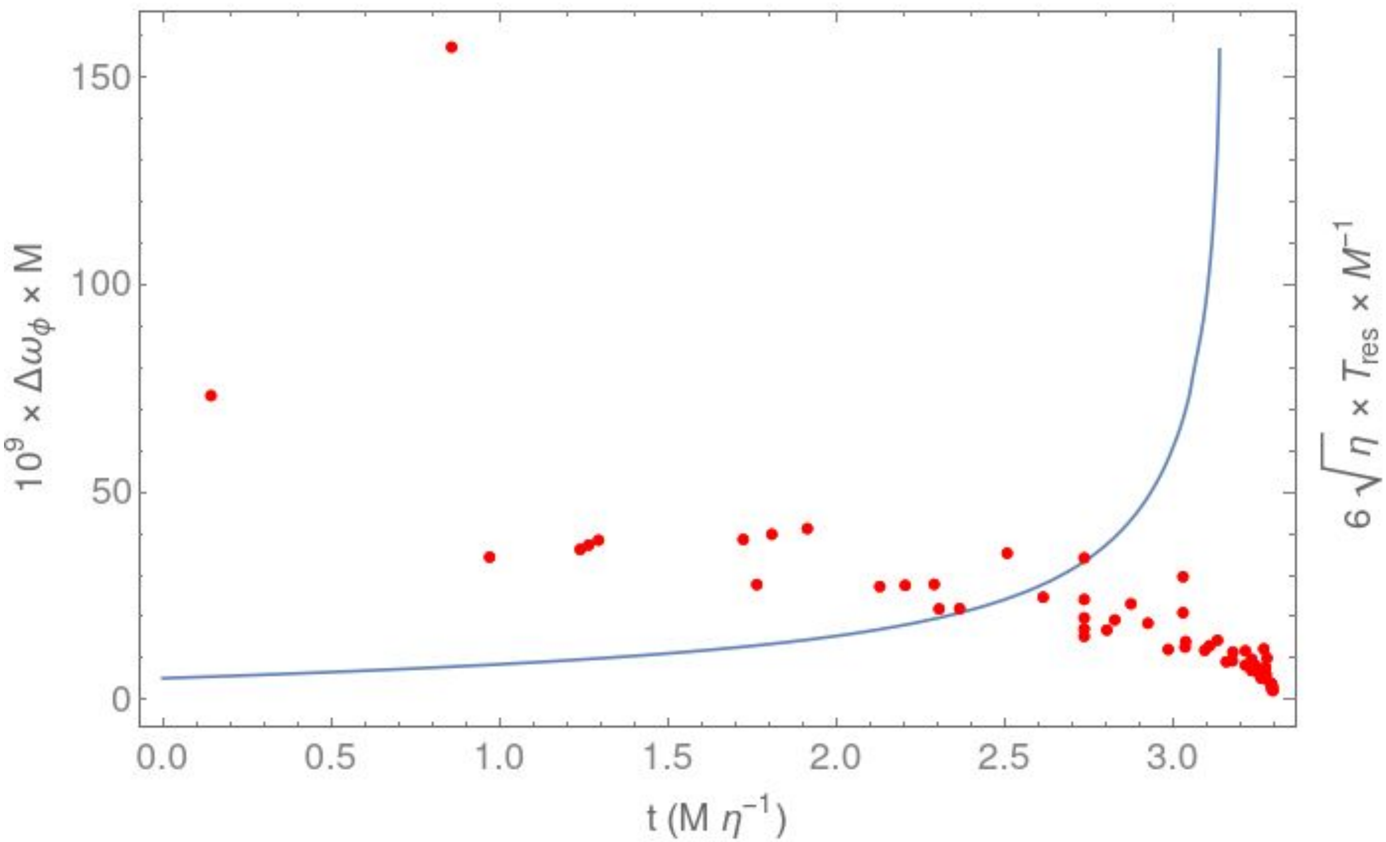
Influence on phase gravitational waveform

$$\Delta\Psi := \int_0^{T_{\rm plunge}} 2\Delta\omega_\phi dt$$

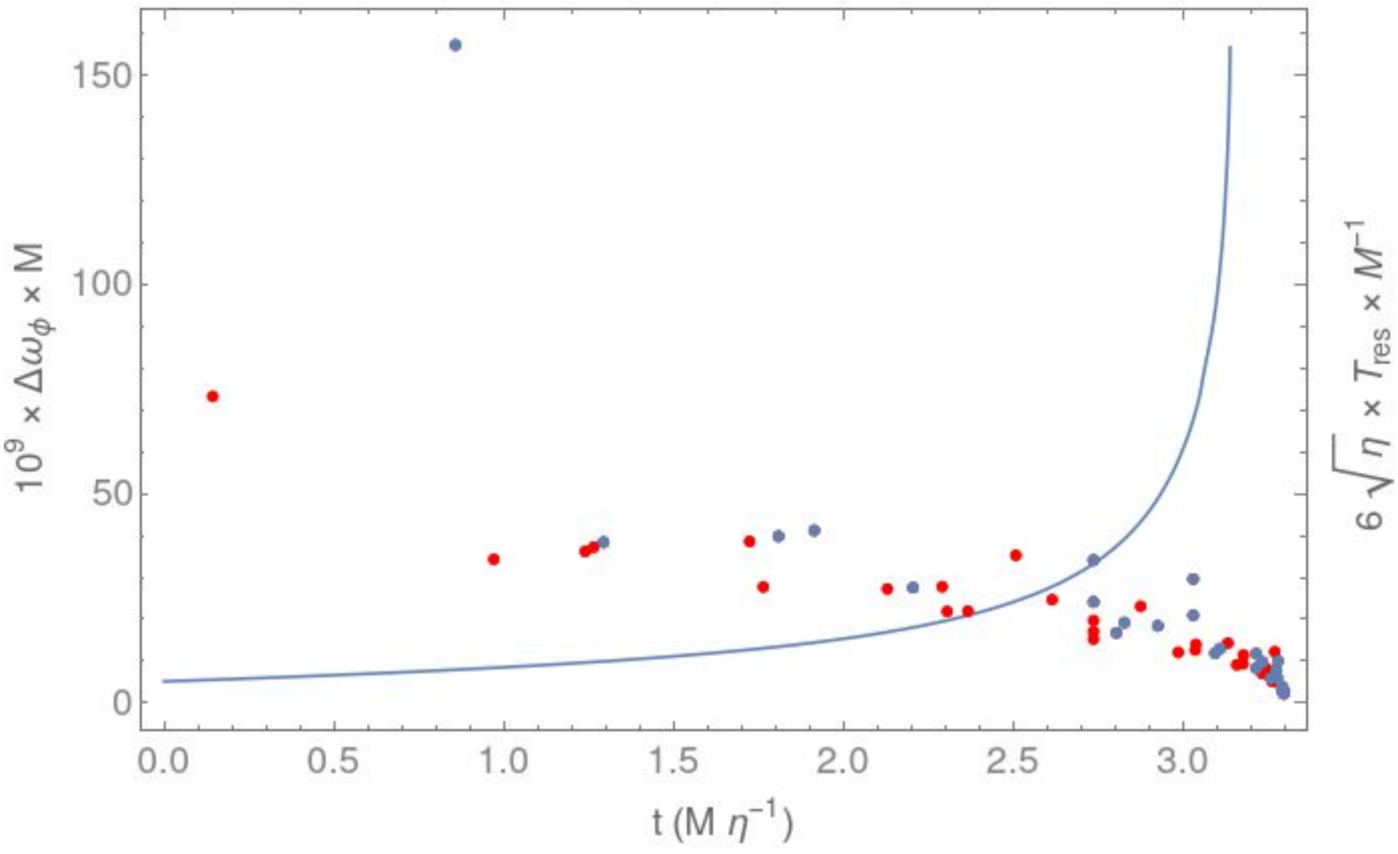
$$= 1.4\left(\frac{\mu}{10M_\odot}\right)^{-\frac{1}{2}}\left(\frac{M}{M_{\rm SgrA^*}}\right)^{\frac{7}{2}}\left(\frac{M_*}{10\,M_\odot}\right)\left(\frac{a}{4.3\,{\rm AU}}\right)^{-3}$$



Many resonances



Many resonances

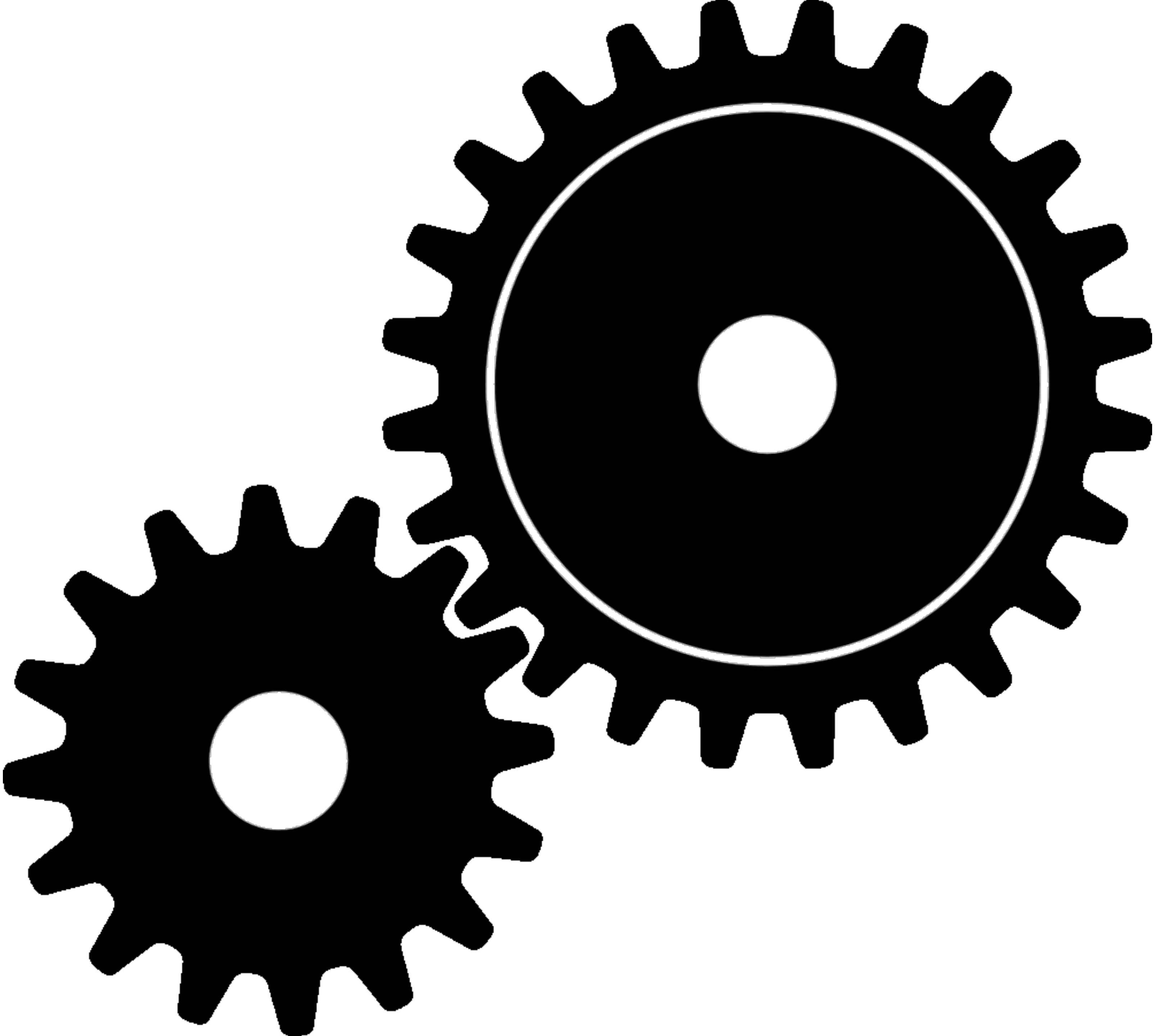


Tidal resonances summary

- Tidal field can change EMRI waveforms appreciably depending on the distance and mass of the tidal perturbers

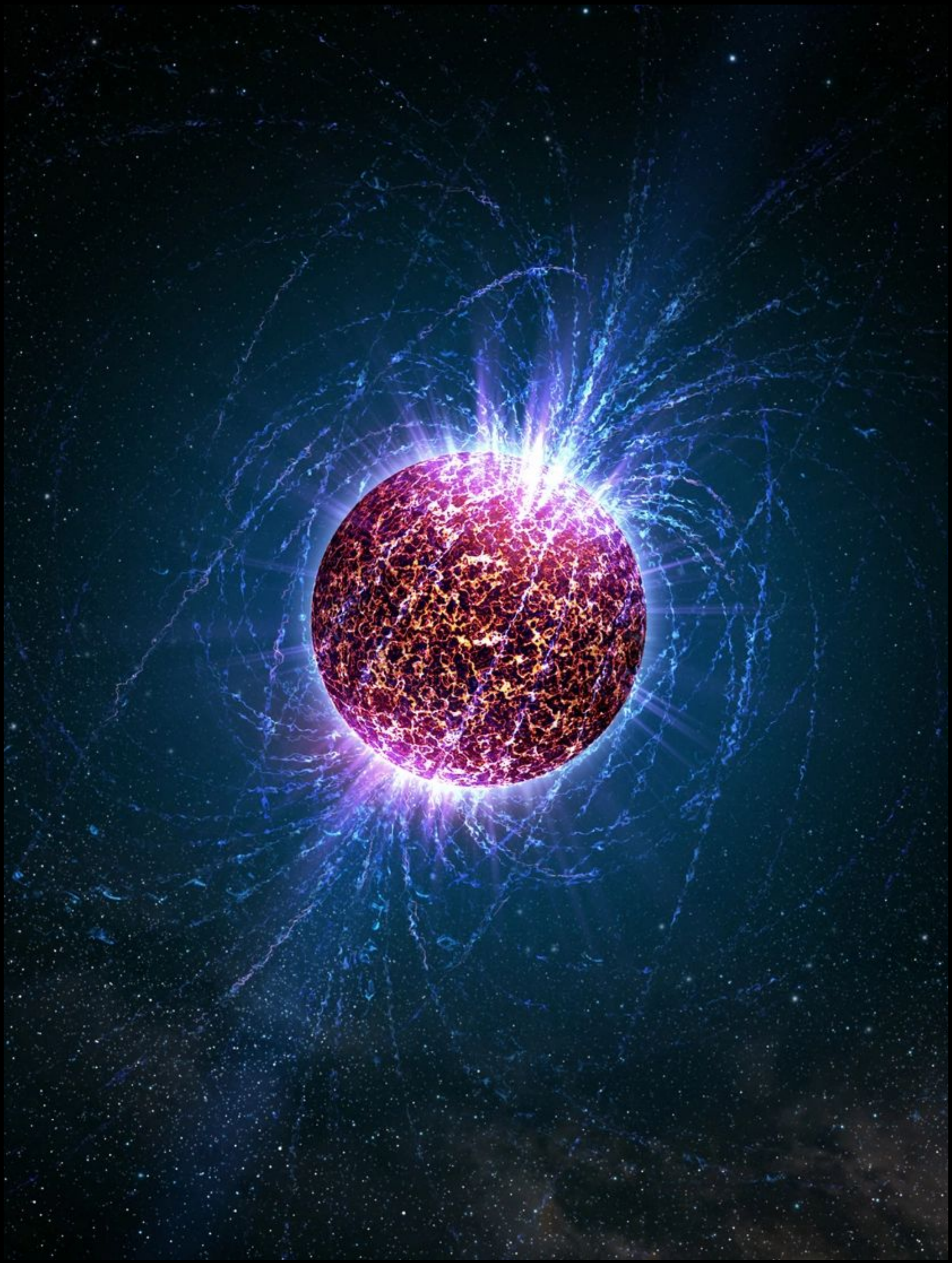
- Opportunity to learn about “light” black holes or stars “close” to massive black holes

- Critical to understand when constraining deviations from General Relativity



How do we constrain Equation of State (EOS) of neutron stars?

1. M_{max} Maximimum observed mass (Radio astronomy)
2. M vs R (X-ray astronomy)
3. Deformability (Gravitational wave astronomy)
4. NS seismology (Light curves)
5. NS seismology (Gravitational wave astronomy)



NS seismology in binaries with gravitational waves

Principle

Mode resonance is excited when $f_0 \approx f_{GW}$

→

Binary loses orbital energy to the mode

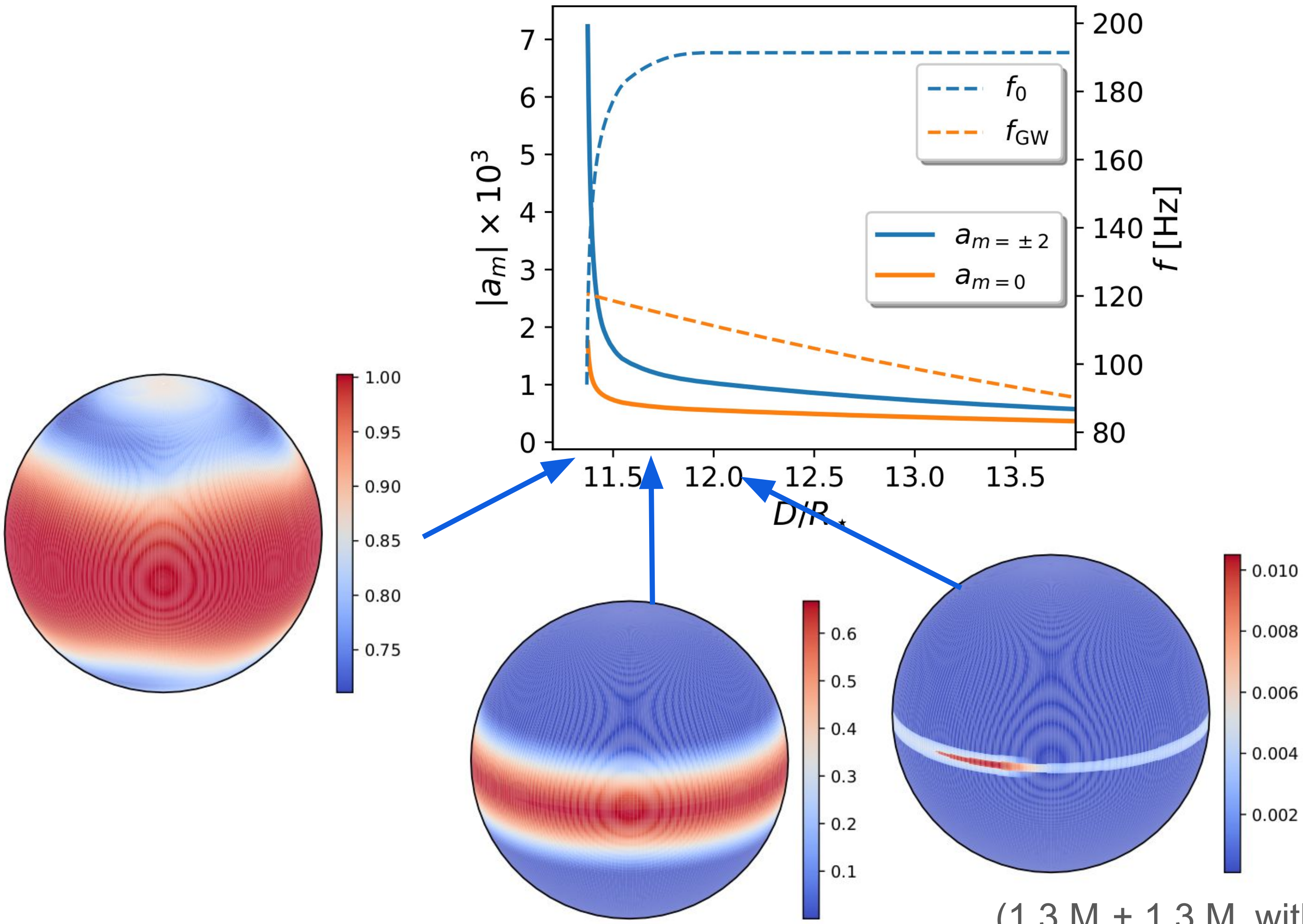
→

Phase shift $\delta\varphi$ in GW

$$\delta\varphi \simeq 5 \cdot 10^{-3} \left(\frac{100\,\mathrm{Hz}}{f_0}\right)^2 \left(\frac{Q}{3 \cdot 10^{-4}}\right)^2 \left(\frac{1.4\,M_\odot}{M}\right)^4 \left(\frac{R}{10\,\mathrm{km}}\right)^2$$

f	$f_0 \simeq 2$ kHz $\quad Q \simeq 0.5$	Too high frequency for LIGO
p	$f_0 \geq 2$ kHz	Too high frequency for LIGO
g	$f_0 \simeq 100$Hz $\quad Q \simeq 10^{-4}$	Small $\delta\varphi$
r	$f_0 \simeq f_{star}$	Requires highly spinning NS (≥100Hz)
i	$f_0 \simeq (30,200)$Hz $\quad Q \simeq 10^{-2}$	$\delta\varphi = (1,50)$

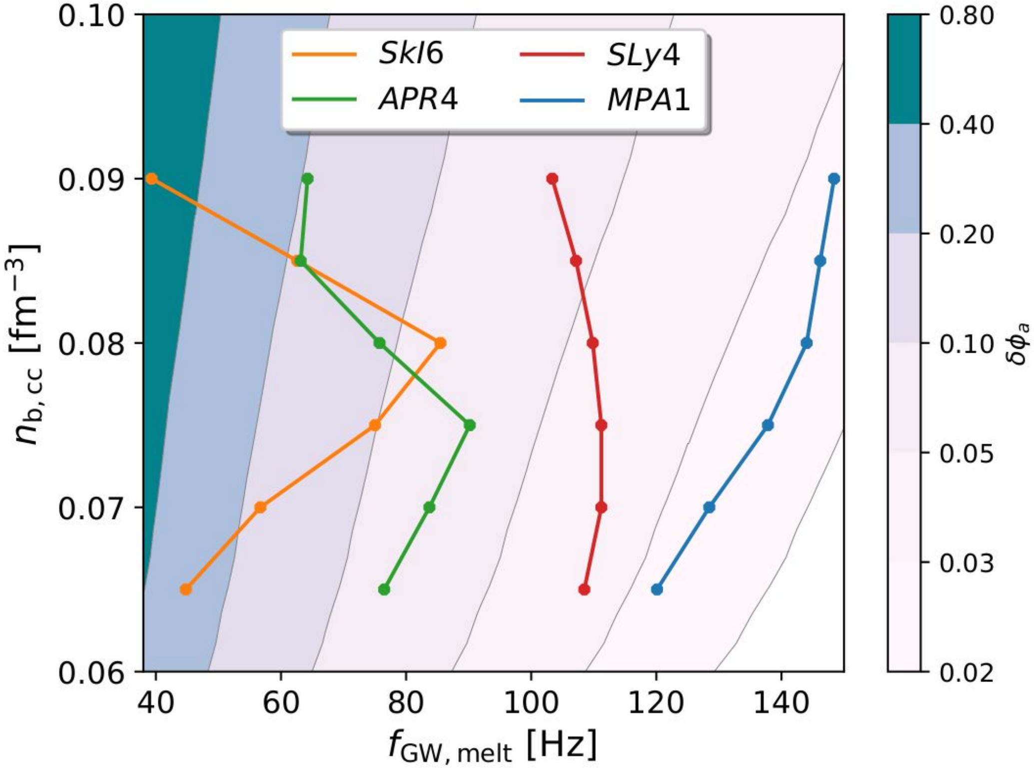
More interesting: the crust melts!



(1.3 $M_\oplus$ + 1.3 $M_\oplus$ with R=11.7 km SLy4)

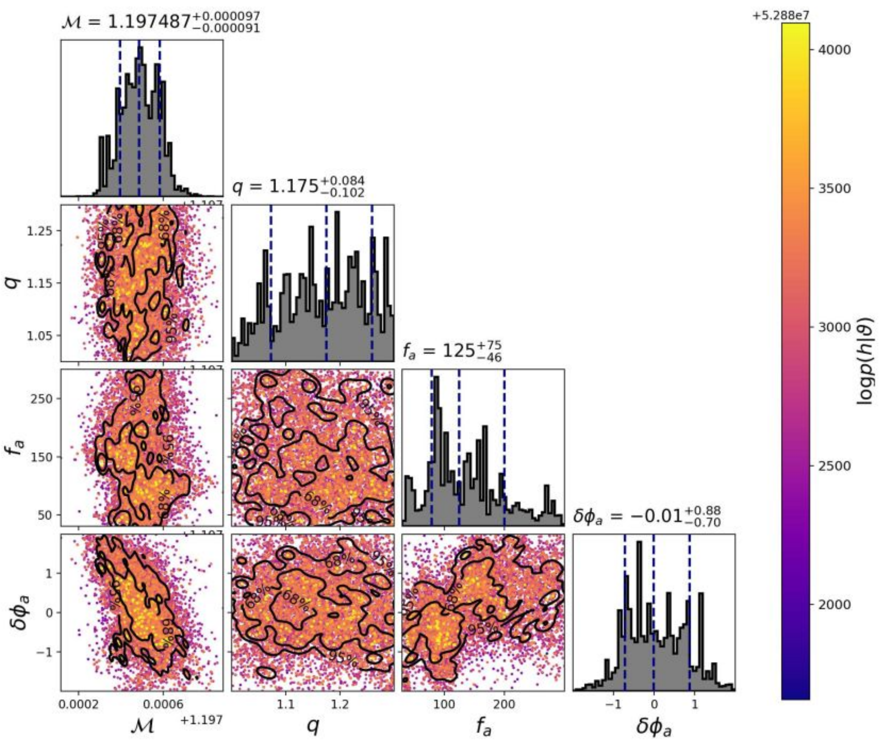
Phase shift depends on EOS

$$\delta\varphi \simeq \underbrace{\frac{\delta\varphi}{\delta t}}_{=2\omega_{\mathrm{orbit}}} \times \underbrace{\frac{\delta t}{\delta E}}_{=P_{\mathrm{GW}}^{-1}} \times \underbrace{\delta E}_{=E_{\mathrm{melt}}}$$

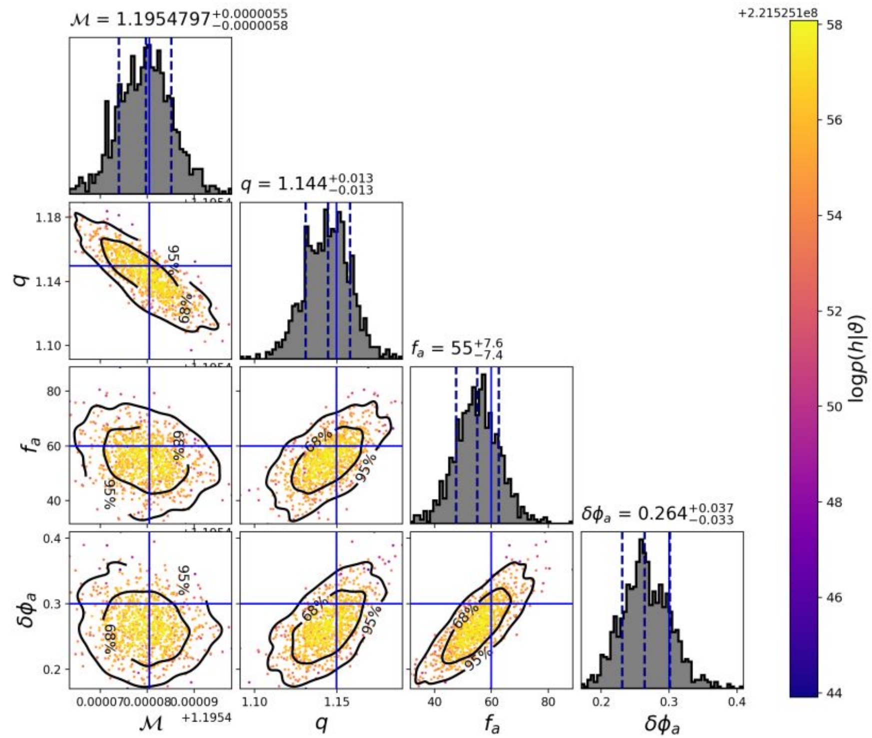


Observable by LIGO A+

GW170817



Example with LIGO A+



Summary i-mode

- Tidal interactions in binary neutron stars can excite the core-crust interface mode

- This can melt the crust and change the waveform

- Mode analysis is based on Newtonian perturbations on a relativistic background

  → work in progress: fully relativistic calculation i-mode


Merci pour
votre attention

Amplitude interface mode

$$\ddot{a}_m(t) + \gamma(t)\,\dot{a}_m + \omega_0^2(t)\,a_m = \frac{GMQ}{D(t)^3} e^{-im\Phi(t)}$$