Formation and Abundance of Primordial Black Holes

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APC - Paris

12 November 2020

- ***C.Germani, IM*** **- PRL 122, 141302 (2019)**
- ***IM*** **- PRD 100, 123524 (2019) - <u>editor suggestion</u>**
- **S. Young, IM, C. Byrnes - JCAP 11, 012 (2019)**
- **A. Kehagias, IM, A. Riotto - JCAP 12, 029 (2019)**
- **A. Kalaja et al - JCAP 10, 031 (2019)**

Past collaborators:
John Miller (Oxford)
Alexander Polnarev (London)
Luciano Rezzolla (Frankfurt)

PBHs: Introduction

- **Primordial Black Holes (PBHs)** [Hawking (1971)] could form from the collapse of cosmological perturbation during the radiation dominated era.
- **PBHs** can in principle span a large wide range of masses and if not evaporated ($M > 10^{15}\,g$) are interesting candidates for dark matter, intermediate mass black holes and the seeds of supermassive black holes.
- I study **PBH formation in spherical symmetry** solving the combined set of Einstein + hydro equations putting initial conditions in the cosmic time slicing.

$$ds^2 = -A^2(r,t)dt^2 + B^2(r,t)dr^2 + R^2(r,t)d\Omega^2$$

$$U(r,t) := \frac{1}{A}\partial_t R \qquad \Gamma(r,t) = \frac{1}{B}\partial_r R$$

$$M(r,t) = \int_0^R 4\pi R^2 \rho dR \qquad \Gamma^2 = 1 + U^2 - \frac{2M}{R}$$

In radiation: $p(r,t) = \frac{1}{3}\rho(r,t)$

Trapping Horizons

Expansion of **ingoing/outgoing** null-rays :

$$k^a/l^a = \left(\frac{1}{A}, \pm\frac{1}{B}, 0, 0\right) \implies \theta_\pm = h^{cd}\nabla_c k_d = \frac{2}{R}(U \pm \Gamma)$$

$$h_{ab} = g_{ab} + \frac{1}{2}(k_a l_b + l_a k_b) \qquad k^a l_a = -2$$

Black Hole / Cosmological horizon : $\theta_\pm = 0 \Rightarrow \left.\frac{1}{A}\frac{dR}{dt}\right|_\pm = 0 \Rightarrow \Gamma^2 = U^2$

$$\boxed{R(r,t) = 2M(r,t)}$$

The horizon condition is independent of the slicing and holds also within a non-vacuum moving medium

The so-called **apparent horizon** of a black hole (which is a future trapping horizon) is the **outermost trapped surface for outgoing radial null rays** while the **trapping horizon for an expanding universe** (which is a past trapping horizon) **is foliated by the innermost anti-trapped surfaces for ingoing radial null rays**.

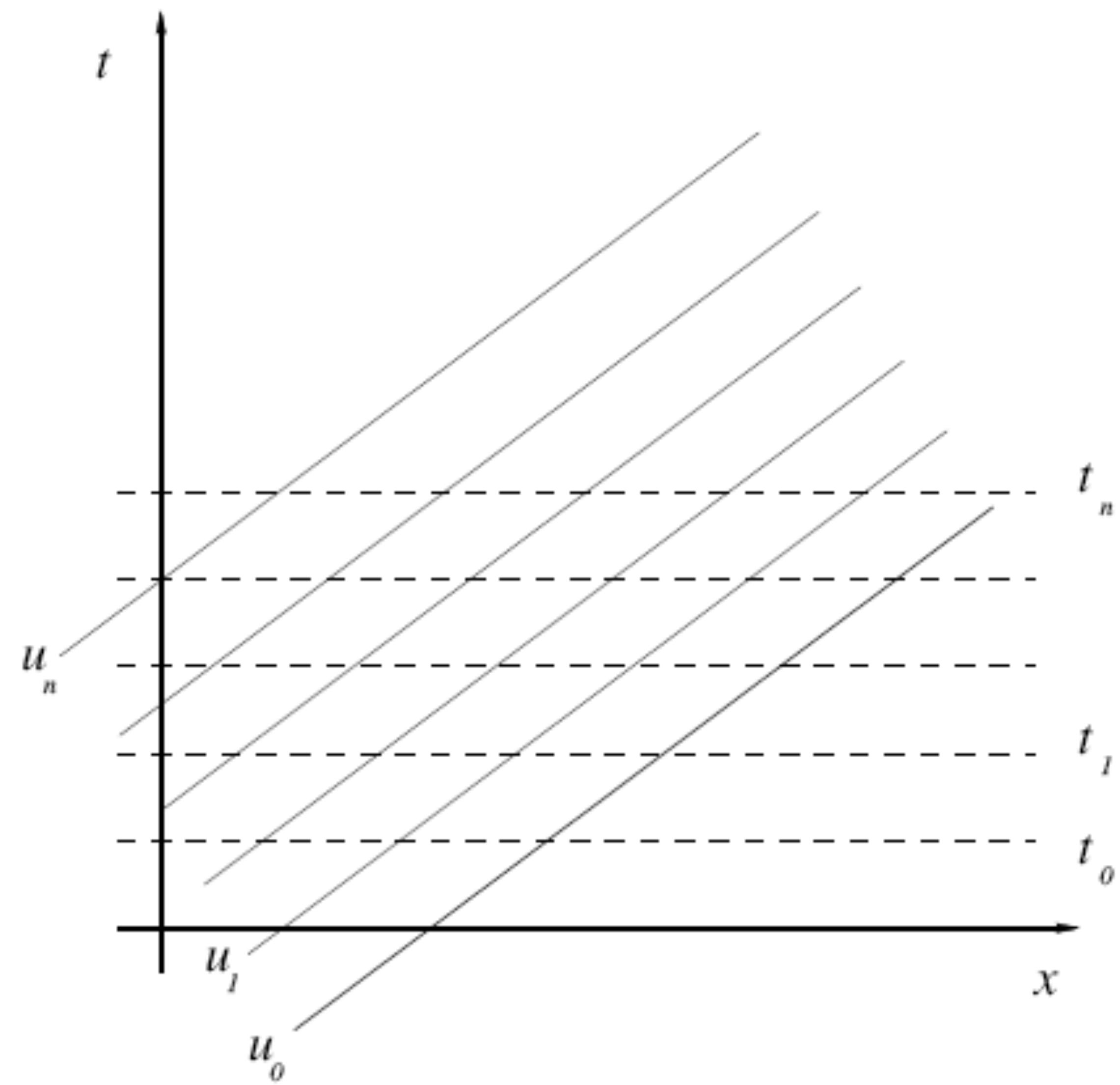
Numerical Results: the method

- My simulations are performed using a **Lagrangian spherically symmetric GR hydro code with an adaptive grid** (AMR).

- We set initial conditions using a **cosmic time coordinate** t.

- We transfer those onto a **null foliation** of the space time, then evolved using an **observer time coordinate** u.

- The **formation of a PBH is seen by a distant external observer** (the singularity is hidden by the asymptotic formation of the apparent horizon).



$$ds^2 = -A^2 dt^2 + B^2 dr^2 + R^2 d\Omega^2 \qquad ds^2 = -G^2 du^2 - 2GB\,dr\,du + R^2 d\Omega^2$$

COSMIC TIME $\quad G\,du = A\,dt - B\,dr \quad$ **NULL TIME**

COSMIC TIME

$$D_t \equiv \frac{1}{A}\left(\frac{\partial}{\partial t}\right)_r \qquad D_r \equiv \frac{1}{B}\left(\frac{\partial}{\partial r}\right)_t$$

$$D_t U = -\left[\frac{\Gamma}{\rho + p} D_r p + \frac{M}{R^2} + 4\pi R p\right]$$

$$D_t \rho_0 = -\frac{\rho_0}{\Gamma R^2} D_r (R^2 U)$$

$$D_t \rho = \frac{\rho + p}{\rho_0} D_t \rho_0$$

$$D_t M = -4\pi R^2 U p$$

$$D_r A = -\frac{A}{\rho + p} D_r$$

$$D_r M = 4\pi R^2 \Gamma \rho$$

$$\Gamma^2 = 1 + U^2 - \frac{2M}{R}$$

NULL TIME

$$D_t \equiv \frac{1}{G}\left(\frac{\partial}{\partial u}\right)_r \qquad D_k = D_r + D_t$$

$$D_t U = -\frac{1}{1 - c_s^2}\left[\frac{\Gamma}{\rho + p} D_k p + \frac{M}{R^2} + 4\pi R p + \right.$$
$$\left. + c_s^2 \left(D_k U + \frac{2U\Gamma}{R}\right)\right]$$

$$D_t \rho_0 = -\frac{\rho_0}{\Gamma}\left[D_t U - D_k U - \frac{2U\Gamma}{R}\right]$$

$$D_t \rho = \frac{\rho + p}{\rho_0} D_t \rho_0$$

$$D_t M = -4\pi R^2 U p$$

$$D_k \left[\frac{\Gamma + U}{G}\right] = -\frac{4\pi R}{G}(\rho + p)$$

$$D_k M = 4\pi R^2 (\rho\Gamma - pU)$$

$$\Gamma^2 = D_k R - U = 1 + U^2 - \frac{2M}{R}$$

Equation of State

energy density: $\rho = \rho_0(1+e)$

pressure: $p = (\gamma - 1)\rho_0 e$

rest mass density

specific internal energy (velocity dispersion)

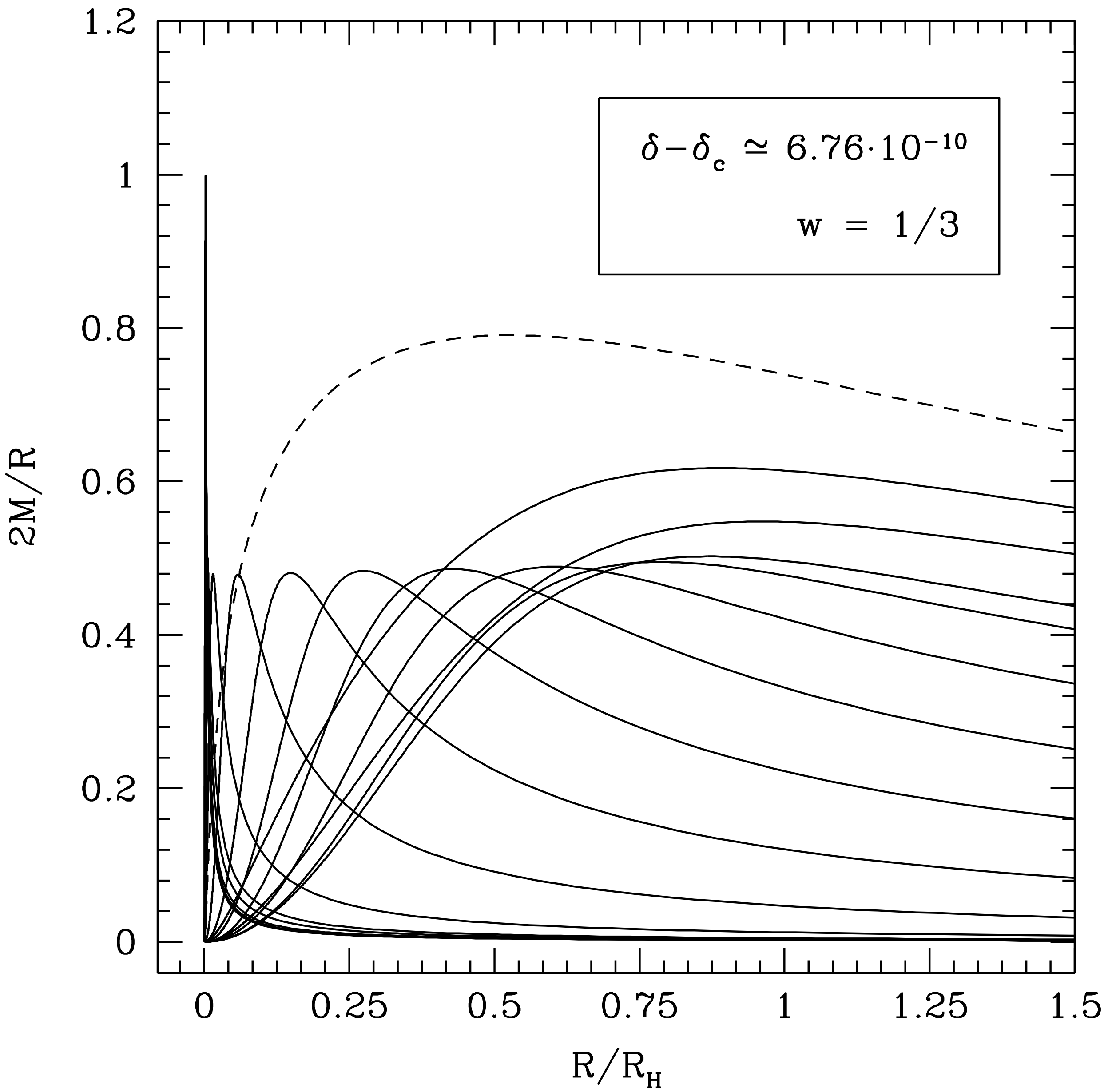
adiabatic index - particle degree of freedom

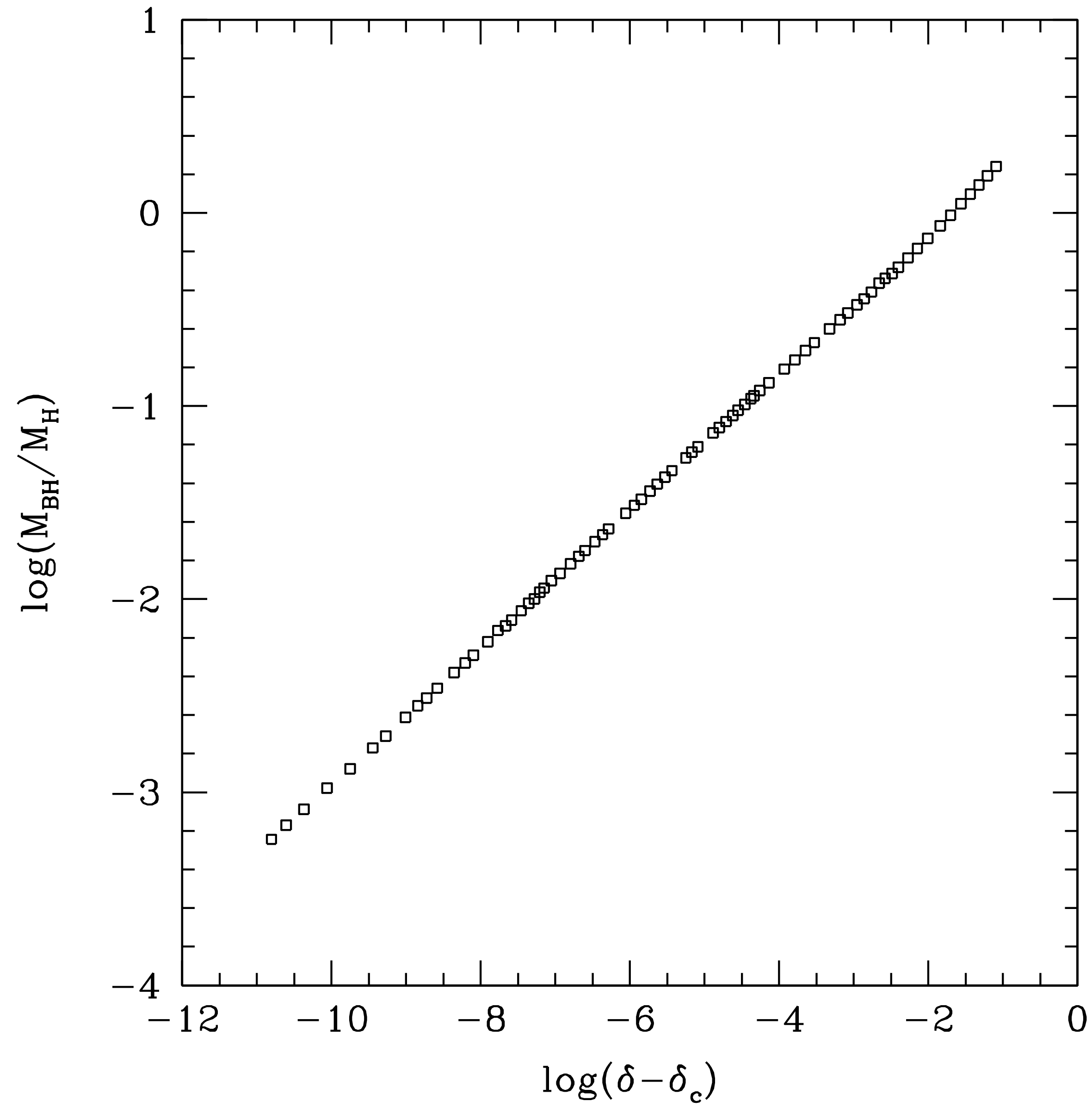
- Barotropic fluid (no rest mass density): $p = w\rho$ with $w \in [0,1]$
  - radiation dominated era: $w = 1/3$ RADIATION $(\gamma = 4/3)$
  - matter dominated era: $w = 0$ DUST $(\gamma = 1)$
- Polytropic fluid: $p = K(s)\rho_0^\gamma$ $(\gamma = 5/3,\ 4/3,\ 2)$
  - If the fluid is adiabatic (no entropy change): $K(s) = K$ (constant)

Numerical Results: PBH formation/bounce

$$\boxed{R(r,t) = 2M(r,t)}$$

$$\boxed{M_{PBH} = \mathcal{K}(\delta - \delta_c)^{\gamma} M_H}$$





IM, Miller, Rezzolla Polnarev - (2005-2013)

Curvature profile & quasi homogenous solution

- The asymptotic metric ($t \longrightarrow 0$), describing super-horizon cosmological perturbations in the comoving synchronous gauge can be written as:

$$ds^2 \simeq -dt^2 + a^2(t)\left[\frac{1}{1-K(\tilde{r})\tilde{r}^2}d\tilde{r}^2 + \tilde{r}^2 d\Omega^2\right]$$

$$ds^2 \simeq -dt^2 + a^2(t)e^{2\zeta(r)}\left[dr^2 + r^2 d\Omega^2\right]$$

- In the "linear regime" of cosmological perturbations, adiabatic perturbations on super horizon scales can be described by a time independent curvature profile using the **quasi-homogeneous / gradient expansion approach**.

$$\boxed{K(\tilde{r})\tilde{r}^2 = -r\zeta'(r)\left[2 + r\zeta'(r)\right]} \qquad \tilde{r} = re^{\zeta(r)}$$

- The zero-order perturbation in the curvature is related to the first non zero order perturbation in the metric/hydro variable.

$$\Gamma^2 = 1 + U^2 - \frac{2M}{R} = 1 - K(\tilde{r})\tilde{r}^2 = \left[1 + r\zeta'(r)\right]^2$$

PBH Threshold

$$\frac{\delta\rho}{\rho_b} = \left(\frac{1}{aH}\right)^2 \frac{3(1+w)}{5+3w}\left[K(\tilde{r}) + \frac{\tilde{r}}{3}K'(\tilde{r})\right]$$

$$\frac{\delta\rho}{\rho_b} = -\left(\frac{1}{aH}\right)^2 \frac{2(1+w)}{5+3w}\left[\nabla^2\zeta(r) + \frac{1}{2}\left(\nabla\zeta(r)\right)^2\right] e^{-2\zeta(r)}$$

$$\mathcal{C} := \frac{2[M(r,t) - M_b(r,t)]}{R(r,t)} = -\frac{3(1+w)}{5+3w} r\zeta'(r)\left[2 + r\zeta'(r)\right] = \frac{3(1+w)}{5+3w} K(\tilde{r})\tilde{r}^2$$

- The typical scale of the perturbation is identified as the location where the perturbation reaches its maximum compactness.

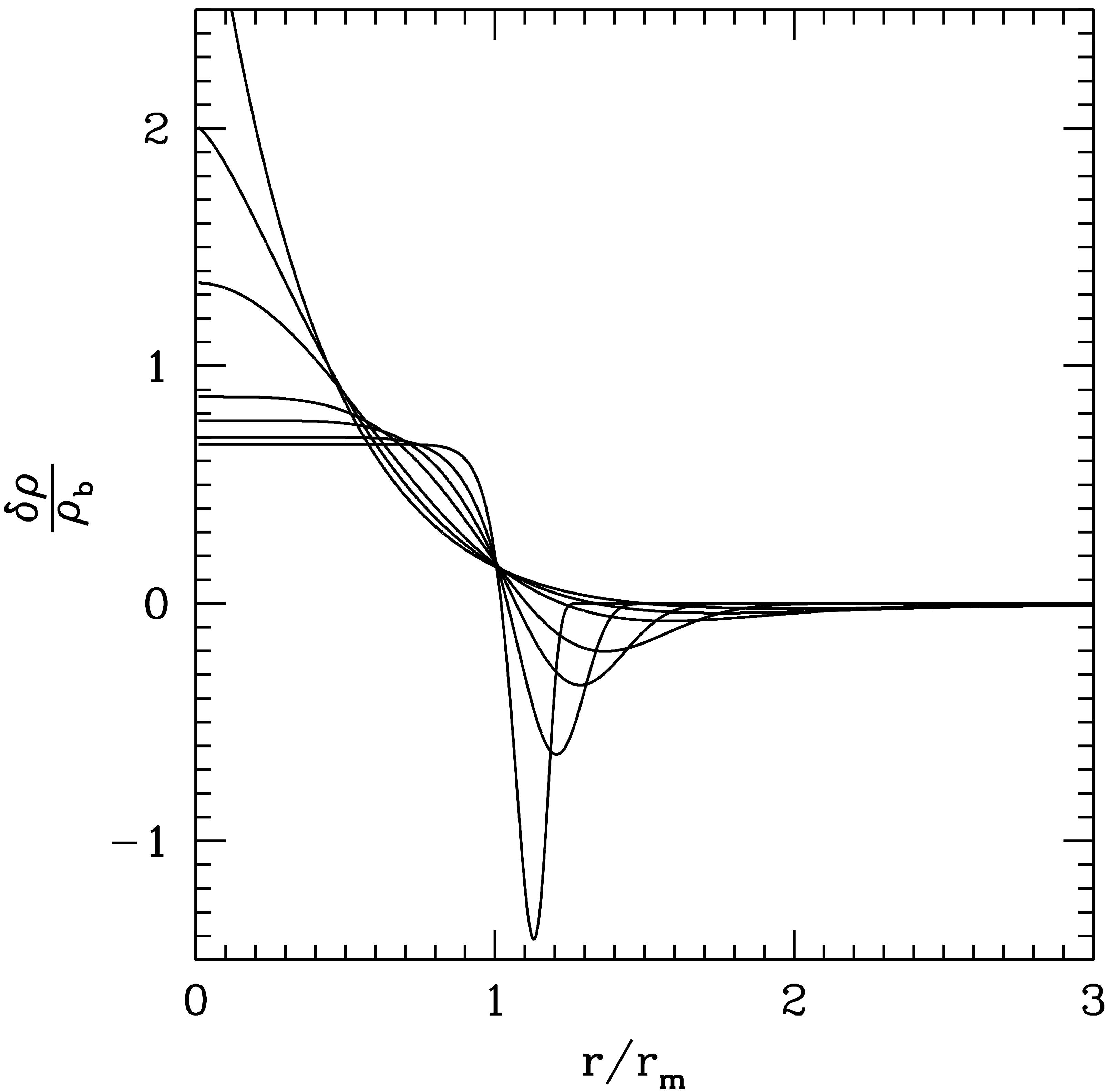
$$r_m \ : \ C'(r_m) = 0 \qquad\qquad \delta(r_m, t_H) := \frac{1}{V_b}\int_0^{r_m} 4\pi \frac{\delta\rho}{\rho_b} r^2\, dr = \mathcal{C}(r_m)$$

- The perturbation amplitude can measured as the mass excess within a characteristic scale at "horizon crossing time", independent from the curvature profile.

$$\boxed{\delta(r_m, t_H) = 3\frac{\delta\rho}{\rho_b}(r_m, t_H)}$$

Musco - PRD (2019)

$$K(r) = \mathcal{A} \exp\left[-\frac{1}{\alpha}\left(\frac{\tilde{r}}{\tilde{r}_m}\right)^{2\alpha}\right] \quad \Rightarrow \quad \frac{\delta\rho}{\rho_b} = \frac{\delta\rho_0}{\rho_b}\left[1 - \frac{2}{3}\left(\frac{\tilde{r}}{\tilde{r}_m}\right)^{2\alpha}\right]\left[-\frac{1}{\alpha}\left(\frac{\tilde{r}}{\tilde{r}_m}\right)^{2\alpha}\right]$$



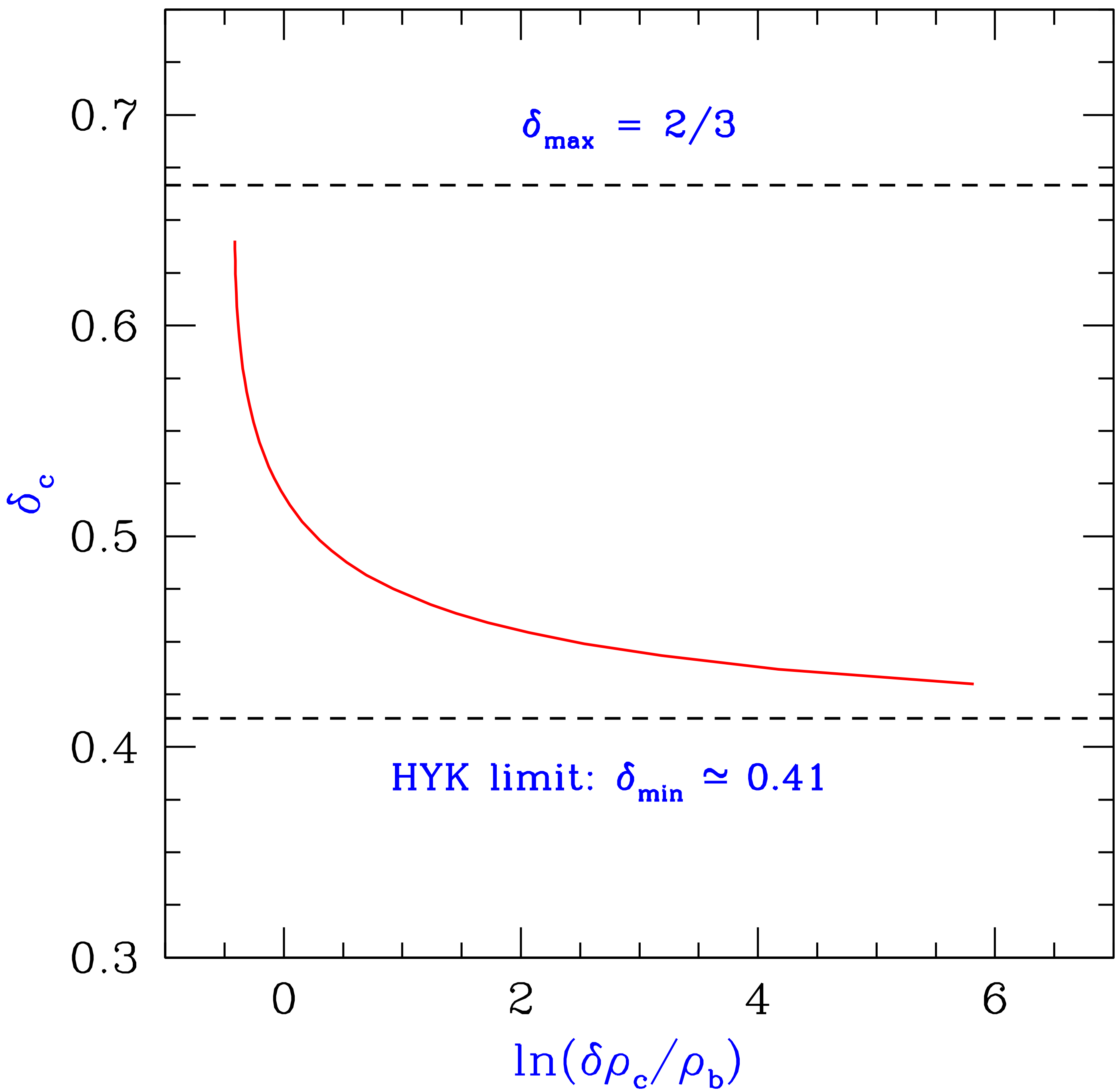
Profile Steepness

$$\alpha = -\frac{\mathcal{C}''(\tilde{r}_m)\tilde{r}_m^2}{4\delta_m}$$

$$\frac{\tilde{r}_0}{\tilde{r}_m} = \left(\frac{3}{2}\right)^{1/2\alpha} \geq 1$$

Musco - PRD (2019)

PBH threshold and peak amplitude



$$\frac{\delta\rho_0}{\rho_b} = e^{1/\alpha}\delta_m$$

$$\boxed{0.41 \lesssim \delta_c \leq \frac{2}{3} \qquad \frac{\delta\rho}{\rho_b} \geq \frac{2}{3}}$$

Musco - PRD (2019)

$$\delta_c \simeq \frac{4}{15} e^{-\frac{1}{\alpha}} \frac{\alpha^{1-\frac{5}{2\alpha}}}{\Gamma\left(\frac{5}{2\alpha}\right) - \Gamma\left(\frac{5}{2\alpha}, \frac{1}{\alpha}\right)}$$

Escrivá, Germani, Sheth - arXiv (2019)

PBH abundance (using peak theory)

- Mean initial energy density profile: $$\frac{\delta\rho}{\rho_b}(r,t) = \frac{\delta\rho}{\rho_b}(0,t)\frac{\xi^{(2)}(r,t)}{\xi^{(2)}(r,0)}$$

- Two point correlation function: $$\xi^{(2)}(r,t) = \frac{1}{2\pi^2 \times (2\pi)^3}\int dk k^2 \frac{\sin(kr)}{kr} P_\Delta(k,t)$$

- **PBH Abundance:** $$\beta_f \simeq \sqrt{\frac{2}{\pi}}\mathcal{K}\left(\frac{k_*}{a_m H_m}\right)^3 \sigma_0^\gamma \nu_c^{1-\gamma}\gamma^{\gamma+1/2} e^{-\frac{\nu_c^2}{2}}$$

- **Critical parameter:** $$\nu_c \equiv \frac{1}{\sigma_0}\frac{\delta\rho_{0c}}{\rho_b} \gg 1\,, \qquad \frac{\delta\rho_{0c}}{\rho_b}\,, \sigma_0 \propto \frac{1}{a^2 H^2}$$

- Moments of the power spectrum: $$\sigma_j^2(t) \equiv \int \frac{k^2 dk}{2\pi^2} P_\Delta(k,t) k^{2j}\,, \qquad k_* \equiv \frac{\sigma_1}{\sqrt{3}\sigma_0}$$

C.Germani, IM - PRL (2019)

Primordial power spectrum / PBHs as dark matter

- Power spectrum of cosmological perturbations: $\mathcal{P}(k) = \mathcal{P}_{CMB} + \mathcal{P}_0 e^{-\frac{(k-k_p)^2}{2\sigma_{\mathcal{P}}^2}}$

- In the limit of a **broad/narrow peak of the power spectrum**:

$$\delta_c \simeq 0.51\,, \qquad \frac{\delta\rho_{0c}}{\rho_b} \simeq 1.25$$

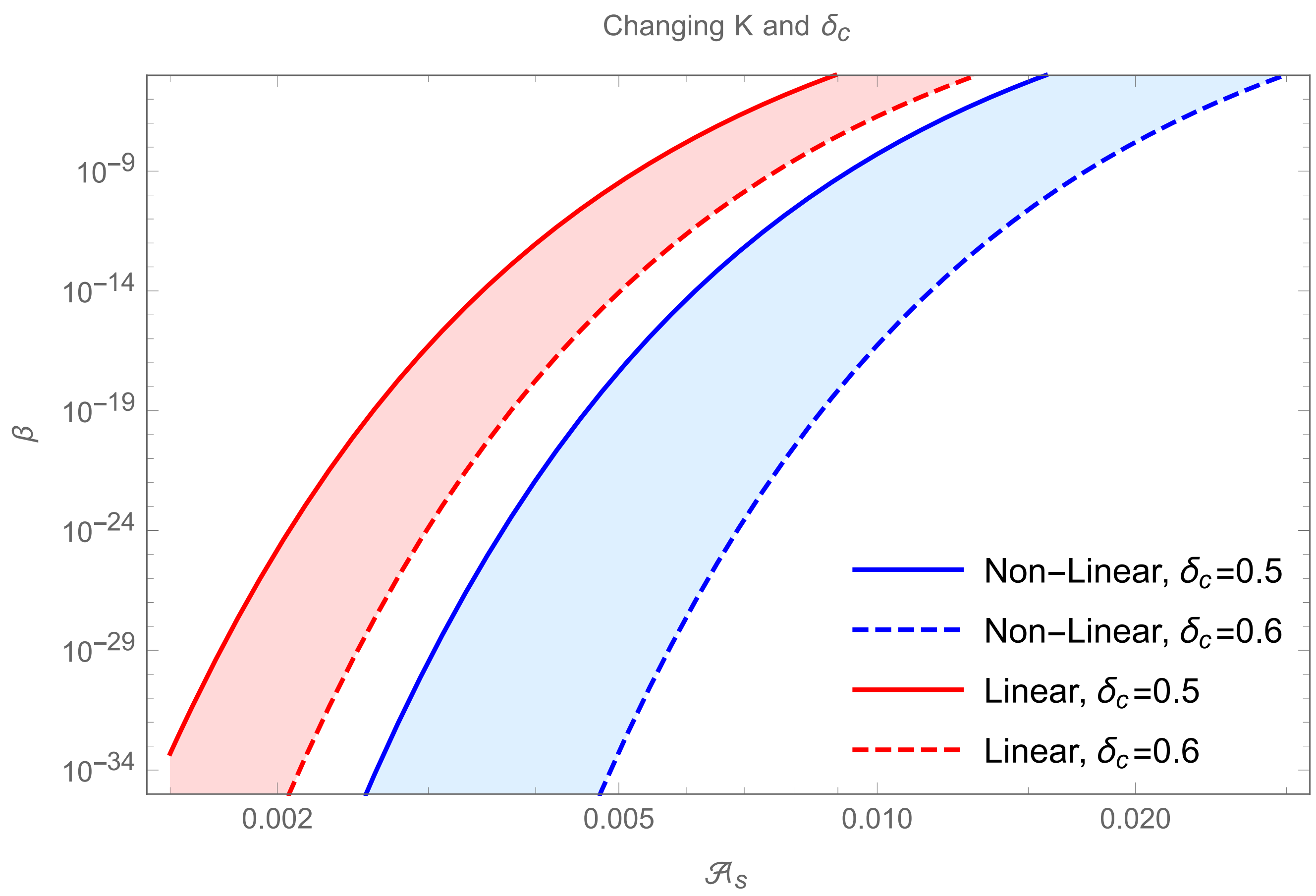
- If $M_{PBH} \sim 10^{16} g$ are Dark Matter $\Rightarrow \beta_f \simeq 10^{-8}\sqrt{\frac{M_{PBH}}{M_\odot}} \simeq 10^{-16}$

- **Narrow peak:** $\frac{k_p}{\sigma_{\mathcal{P}}} \gg 1 \;\Rightarrow\; \nu_c \simeq 0.22\sqrt{\frac{k_p}{\sigma_{\mathcal{P}}\mathcal{P}_0}} \;\Rightarrow\; \mathcal{P}_0 \sim 7\times 10^{-4}\frac{k_p}{\sigma_{\mathcal{P}}} \gg 10^{-3}$

- **Broad peak:** $\frac{k_p}{\sigma_{\mathcal{P}}} \ll 1 \;\Rightarrow\; \nu_c \simeq 0.46(\mathcal{P}_0)^{-1/2} \;\Rightarrow\; \mathcal{P}_0 \sim 3\times 10^{-3}$

- **Press-Schecther approach** (neglecting the shape): $\nu_c \simeq \frac{9}{4}\frac{\delta_c}{\sqrt{\mathcal{P}_0}} \;\Rightarrow\; \mathcal{P}_0 \sim 1$

Power spectrum constraints are weakened by a factor ~ 2



Delta function power spectrum

S.*Young, IM, C.Byrnes* JCAP (2019)

- In order to generate the same number of PBHs when taking the non-linear (NL) relation into account, compared to the linear relation, the power spectrum amplitude needs to increase

$$\delta = \delta_l \left(1 - \frac{3}{8}\delta_l\right)$$

$$\delta_l = -\frac{4}{3} r_m \zeta'(r_m)$$

$$1.5 \lesssim \frac{\mathcal{P}_{0_{NL}}}{\mathcal{P}_{0_L}} = \frac{16\left(1 - \sqrt{1 - \frac{3}{2}\delta_c}\right)^2}{9\delta_c^2} \lesssim 4$$

- For the typical value of $\delta_c \sim 0.55$, power spectrum constraints are weakened by a factor of 2

Non-Linearities & Non-Gaussianities

In general for a peaked power spectrum considering the 3-point correlation function of the power spectrum, one gets:

$$\frac{\delta\rho(r)}{\rho_b} = \frac{\nu}{\sigma}\left[\xi^{(2)}(r) + \frac{\nu}{2\sigma}\xi^{(3)}(\vec{x}_1,\vec{x}_2,\vec{x}_2) + \frac{\nu^2}{6\sigma^2}\xi^{(4)}(\vec{x}_1,\vec{x}_2,\vec{x}_2,\vec{x}_2) + \cdots\right]\exp\left(-\sum_{n=3}^{\infty}(\nu\sigma)^n\xi^{(n)}(0)/n!\right)$$

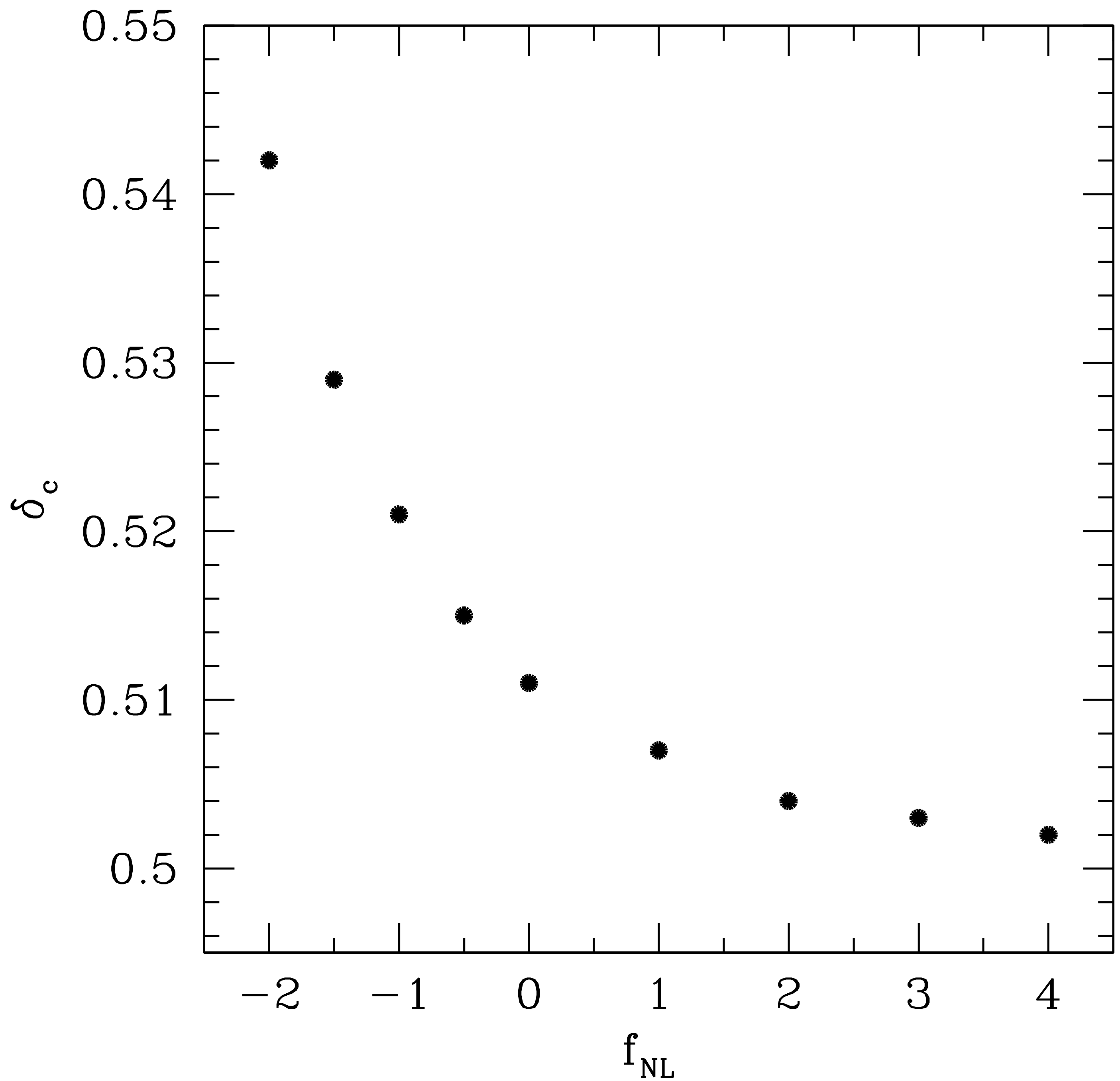
$$\frac{\delta\rho}{\rho_b}(x) = \frac{\delta\rho_{0_G}}{\rho_b}\left[\frac{\sin x}{x} + \frac{1}{x_{m_G}^2}\frac{\delta\rho_{0_G}}{\rho_b}\mathcal{F}(x)\right] \qquad x \equiv k_p r$$

$$\mathcal{F}(x) \equiv \frac{27}{2}\left[\mathcal{F}_1(x) + \frac{3}{5}f_{NL}\mathcal{F}_2(x)\right] \quad \mathcal{F}(0) = \frac{27}{2}\left(1 + \frac{3}{5}f_{NL}\right) \qquad \mathcal{P}_0 = \frac{A}{k_p^2}$$

$$\nu = \frac{9}{8}\frac{1}{\sqrt{\pi A}x_{m_G}^2}\frac{\delta\rho_{0_G}}{\rho_b} \qquad \frac{\delta\rho_0}{\rho_b} = \frac{\delta\rho_{0_G}}{\rho_b}\left[1 + \frac{27}{2x_{m_G}^2}\frac{\delta\rho_{0_G}}{\rho_b}\right]\exp\left[-\frac{1}{2\pi A}\left(\frac{9}{4x_{m_G}^2}\frac{\delta\rho_{0_G}}{\rho_b}\right)^3\right]$$

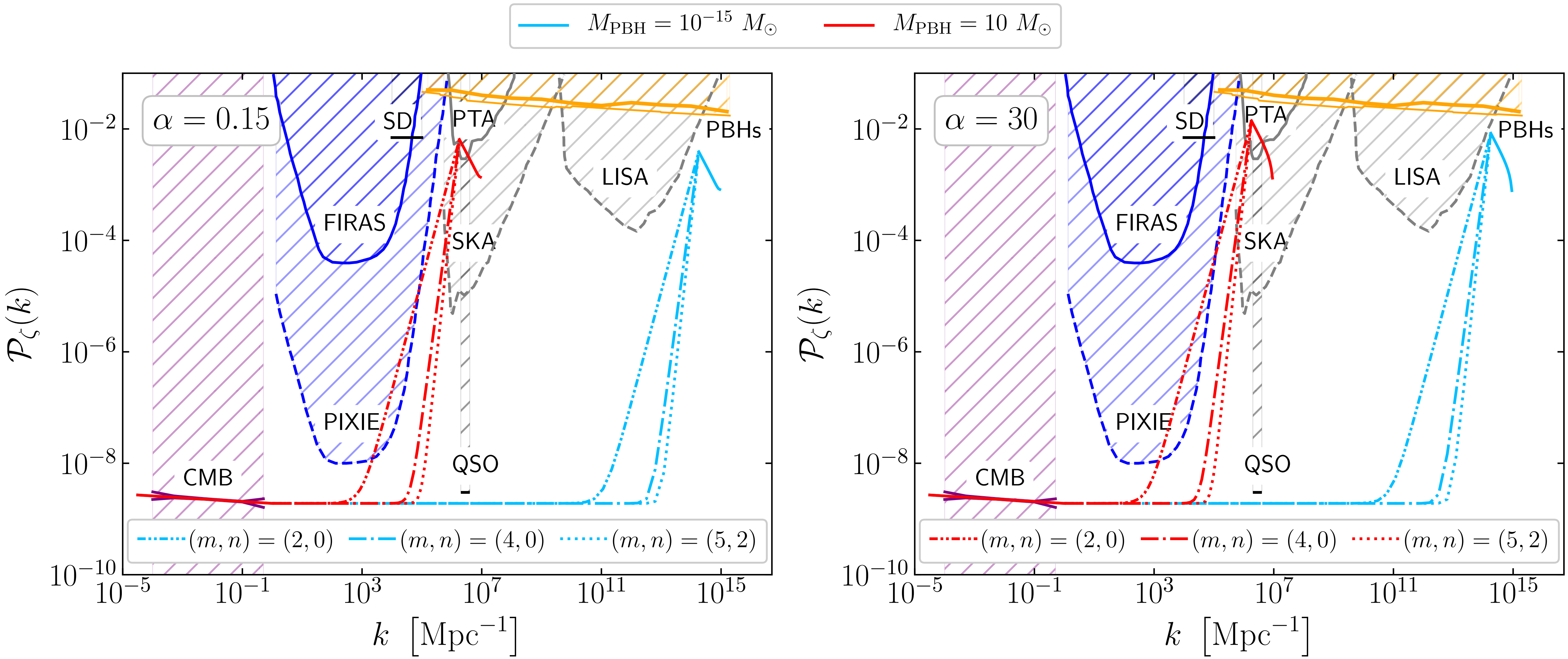
A. Kehagias, IM, A. Riotto JCAP (2019)

Non Gaussian Threshold



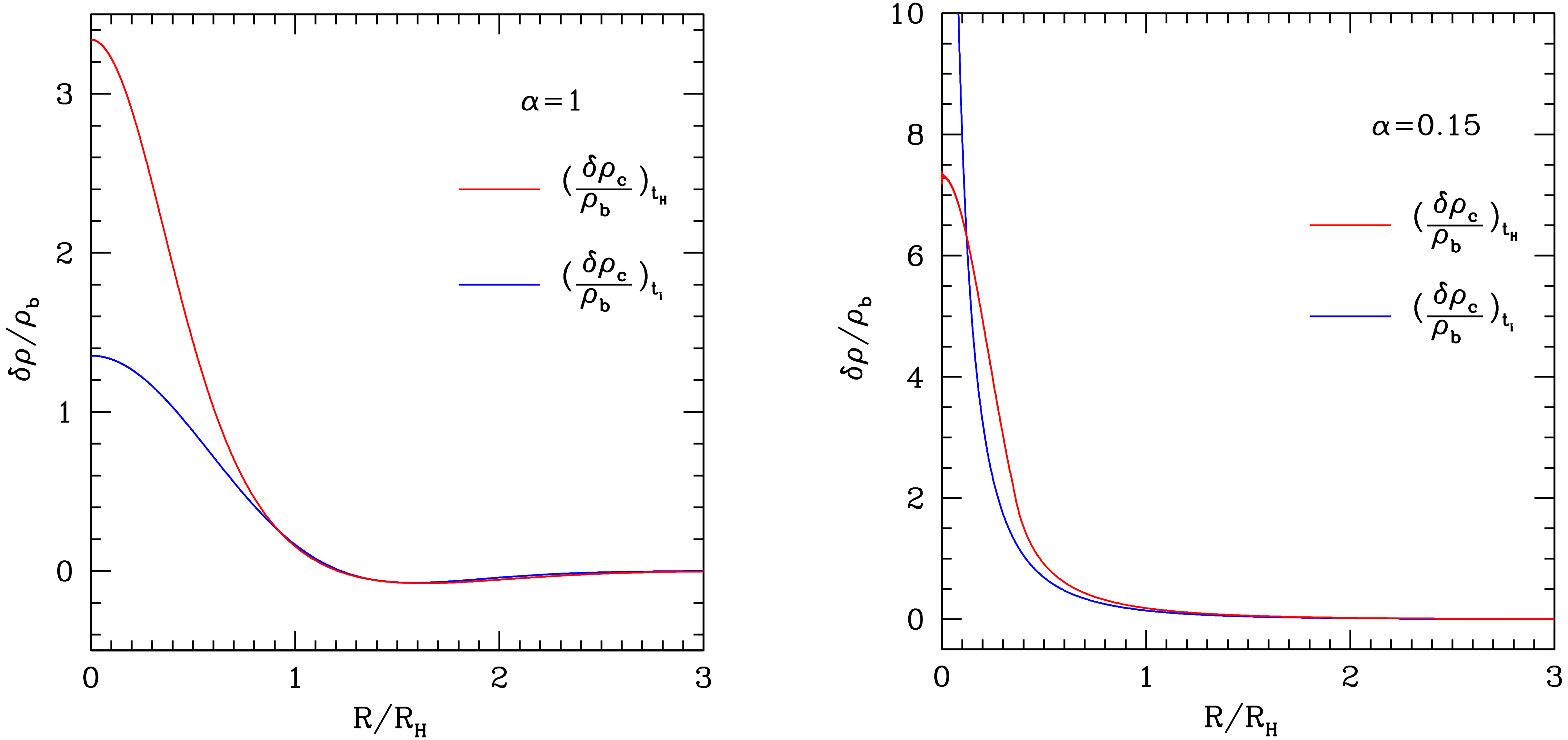
A. Kehagias, IM, A. Riotto JCAP (2019)

PBHs as Dark Matter (reconstructing the power spectrum)

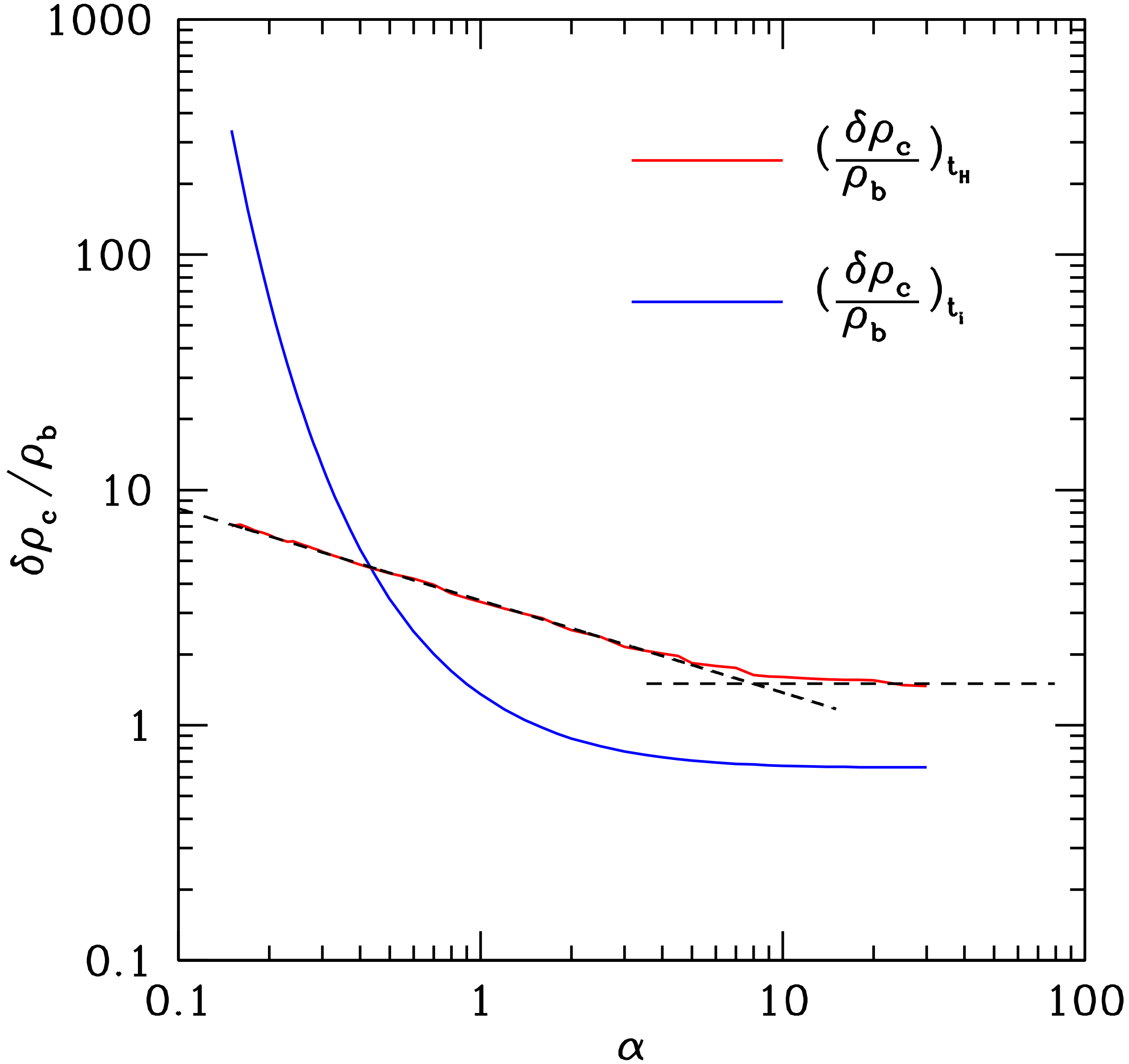


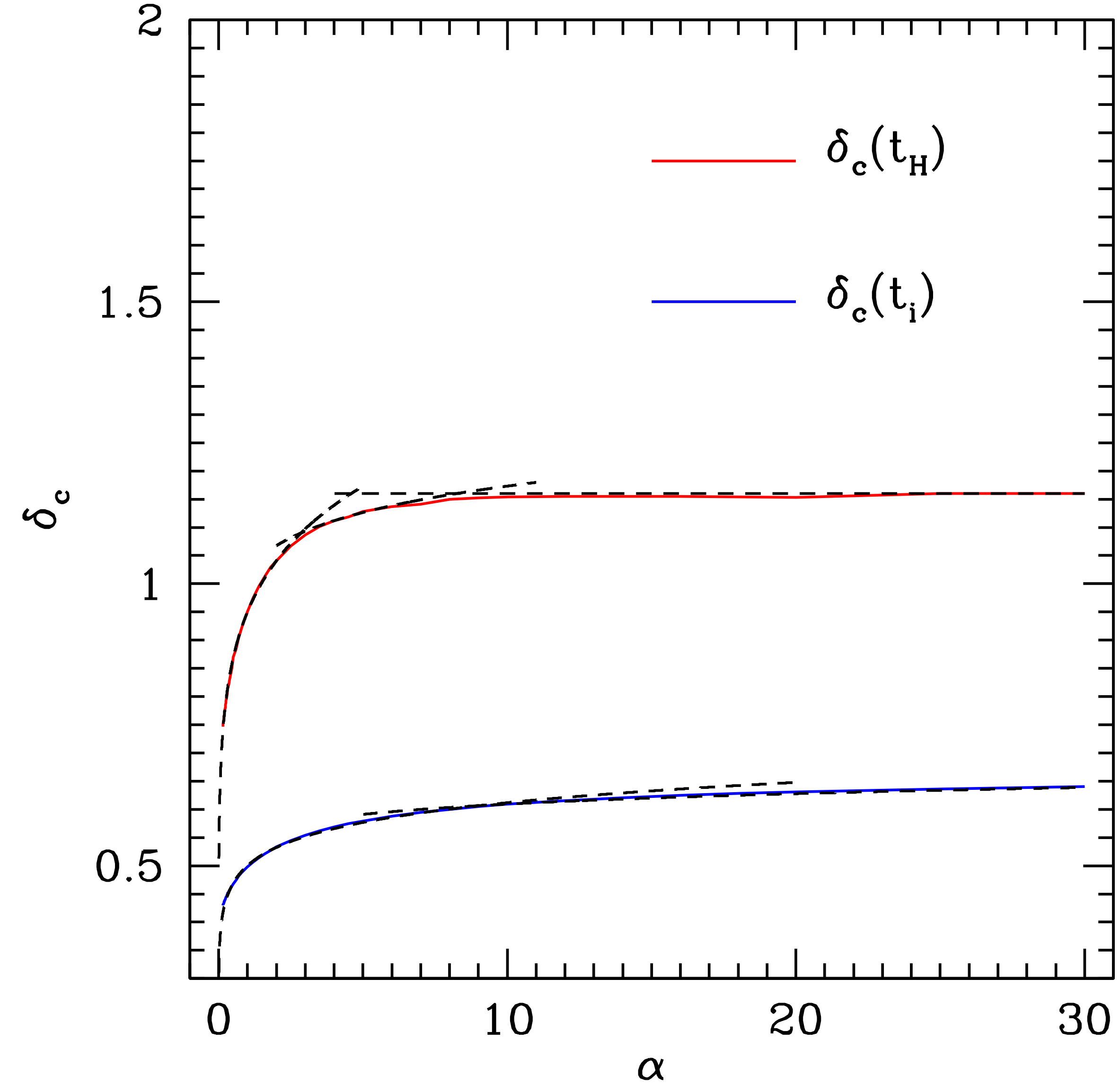
Alba Kalaja, Nicola Bellomo, Nicola Bartolo, Daniele Bertacca, Sabino Matarrese, IM, Alvise Racanelli, Licia Verde - JCAP (2019)

Non linear Horizon crossing (work in progress)



IM & A. Riotto - **in progress**





$$\left.\frac{\delta\rho_c}{\rho_b}\right|_{t_i} = e^{1/\alpha}\delta_c(t_i)$$

$$\delta_c(t_i) \simeq \begin{cases} \alpha^{0.046} - 0.5 & \alpha \lesssim 9 \\ \alpha^{0.025} - 0.45 & \alpha \gtrsim 9 \end{cases}$$

$$\left.\frac{\delta\rho_c}{\rho_b}\right|_{t_H} \simeq \begin{cases} 10^{\,0.53-0.17\ln(\alpha)} & \alpha \lesssim 8 \\ 1.5 & \alpha \gtrsim 8 \end{cases}$$

$$\delta_c(t_H) \simeq \begin{cases} \alpha^{0.125} - 0.5 & \alpha \lesssim 2.85 \\ \alpha^{0.06} + 0.025 & 2.85 \lesssim \alpha \lesssim 8.25 \\ 1.16 & \alpha \gtrsim 8.25 \end{cases}$$

Conclusions

- **There is no universal threshold for PBH formation.** The curvature profile (pressure gradients) play a key role determining the particular value of the threshold. **This can be related to the morphology of the power spectrum of cosmological perturbations.**

- **PBH formation is characterised by non liner curvature profile**, the linear approximation does not gives accurate results.

- **The threshold,** and the correspondent smooth peak amplitude, **computed at horizon crossing, are important quantities to compute PBH abundance.** These are affected by **non Gaussian statistics** (*Germani & Sheth 2020*).

- **The abundance of PBHs is exponentially sensitive to the threshold of the energy density. The shape of the peak of the power spectrum is very important.**

- **The value of averaged threshold seems to be quite solid with respect to non-linearities and non-Gaussianities(?)** (non linear effects to be investigated more).

- **A large enough feature of the power spectrum on small scales (large k) could account for an important component of dark matter in PBHs.**