APC Theory Seminar,
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OBSERVATIONAL SIGNATURES OF MULTIFIELD INFLATION WITH CURVED FIELD SPACE

BACKGROUND, LINEAR FLUCTUATIONS AND NON-GAUSSIANITIES

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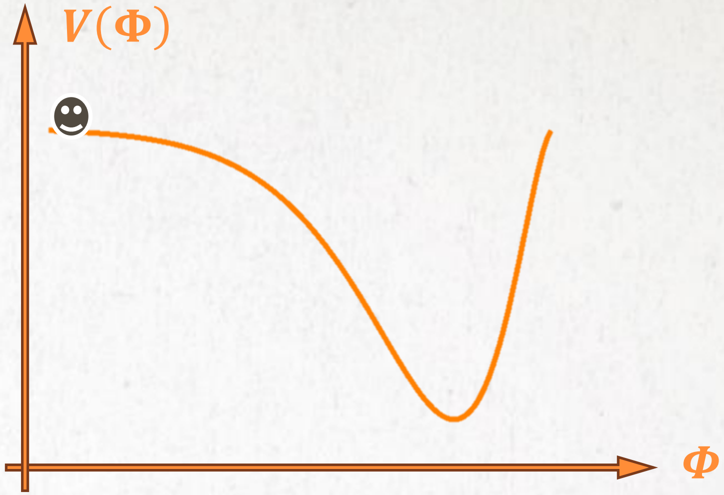
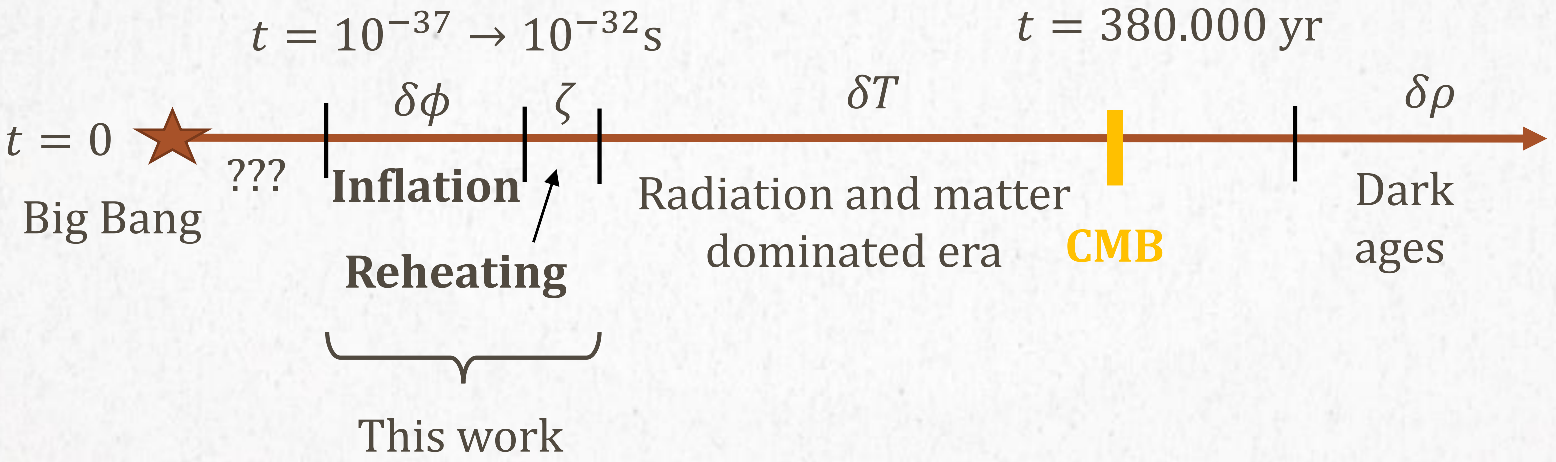
I. USUAL PICTURE OF INFLATION

A CONSISTENT COSMOLOGICAL STORY



VERY BROAD PICTURE

- Cosmology: history, content and laws of the Universe
- Early Universe: before emission of the CMB



CMB is the first observational consequence of the physics in the early universe

LINKS WITH HIGH-ENERGY THEORIES

$\log(E/GeV)$

Natural units: $\hbar = c = 1$ and the only dimension is energy (or mass)

19 — M_p: Planck mass
M_s: String theory, quantum loop gravity etc.

16 — M_{GUT} : Grand Unified Theories (GUT):
Unification of the fundamental non-grav. forces

? — H, inflation
The exact energy scale of inflation is not known
Detection of primordial GWs may tell us

3 — Standard Model, LHC

Accessible experiments on Earth

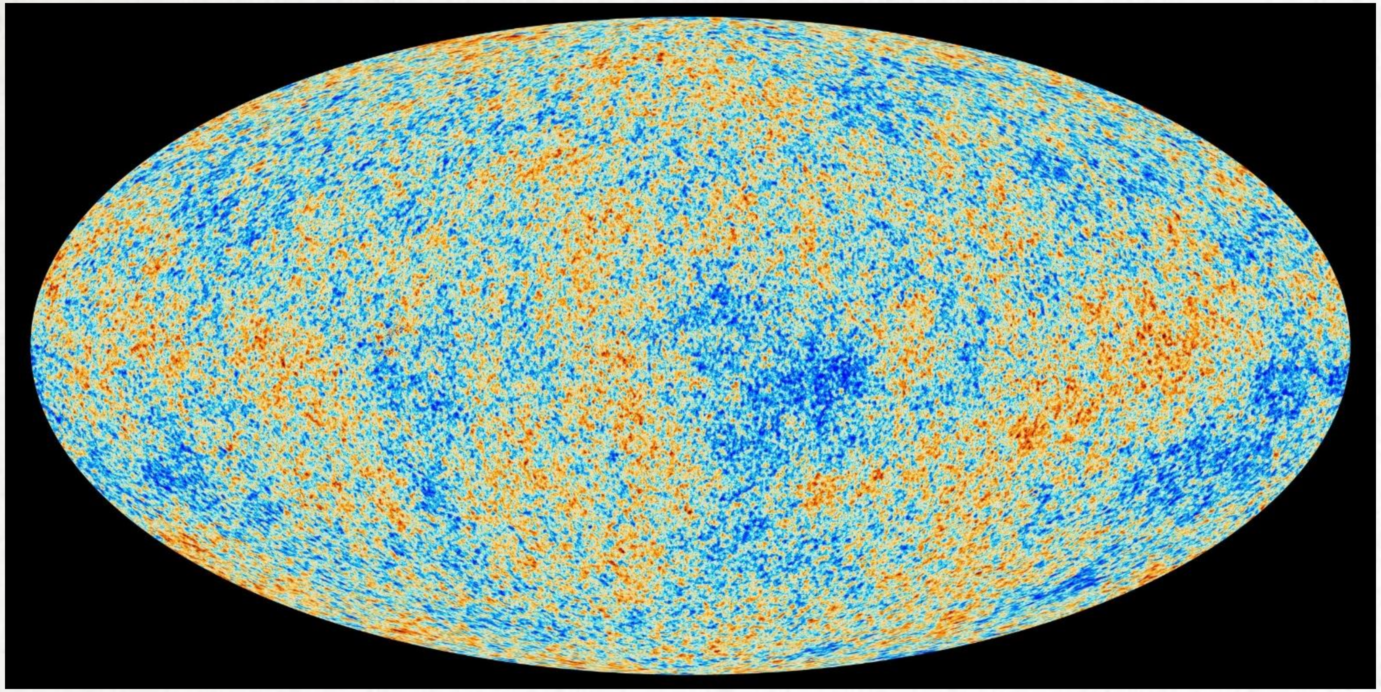
Inflation happens at high energies

It is sensitive to high-energy effects

We do not know physics at high energies
so we can not say any thing about inflation

Let’s work on inflation and hopefully
we learn about high-energy physics

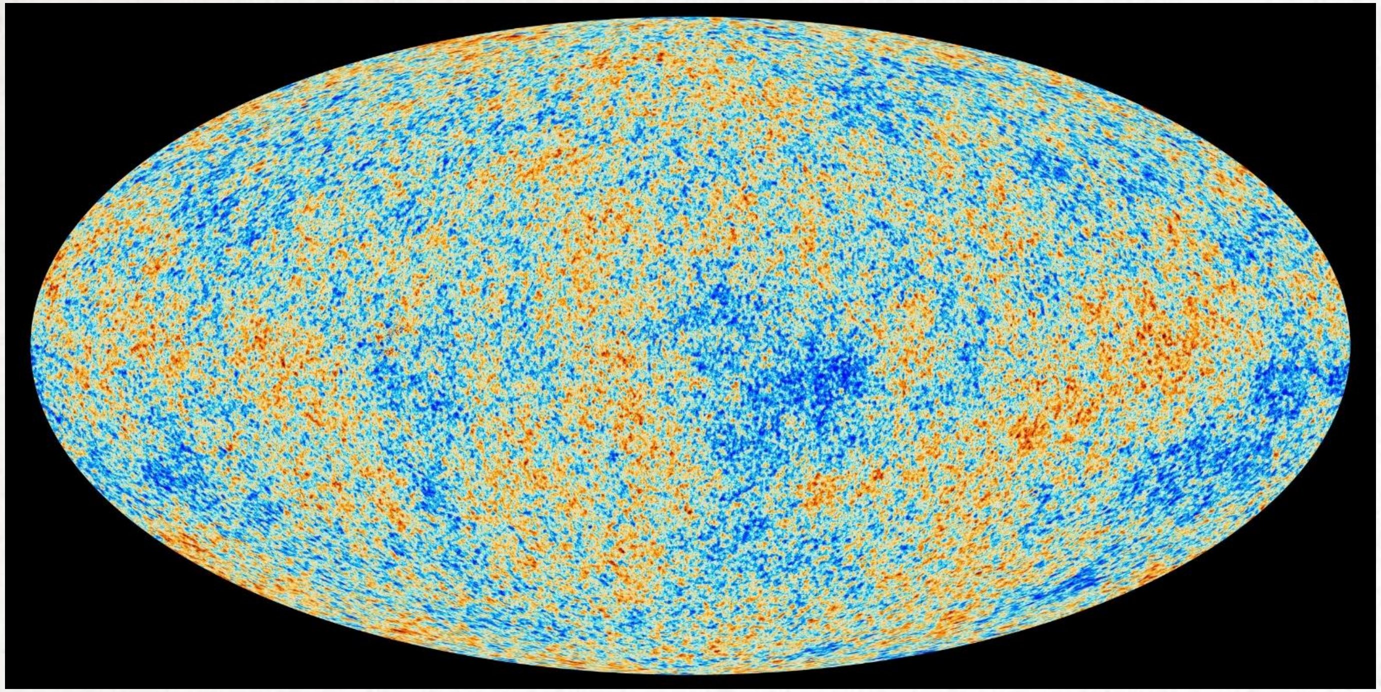
CMB OBSERVATION MOTIVATES INFLATION



$$T \sim 2.73K \; ; \; \frac{\delta T}{T} \sim 10^{-5} \; ; \; |\Omega_k| \ll 1$$

- How is the universe so homogeneous?
  Horizon problem
- Why is the universe so spatially flat?
  Flatness problem

CMB OBSERVATION MOTIVATES INFLATION



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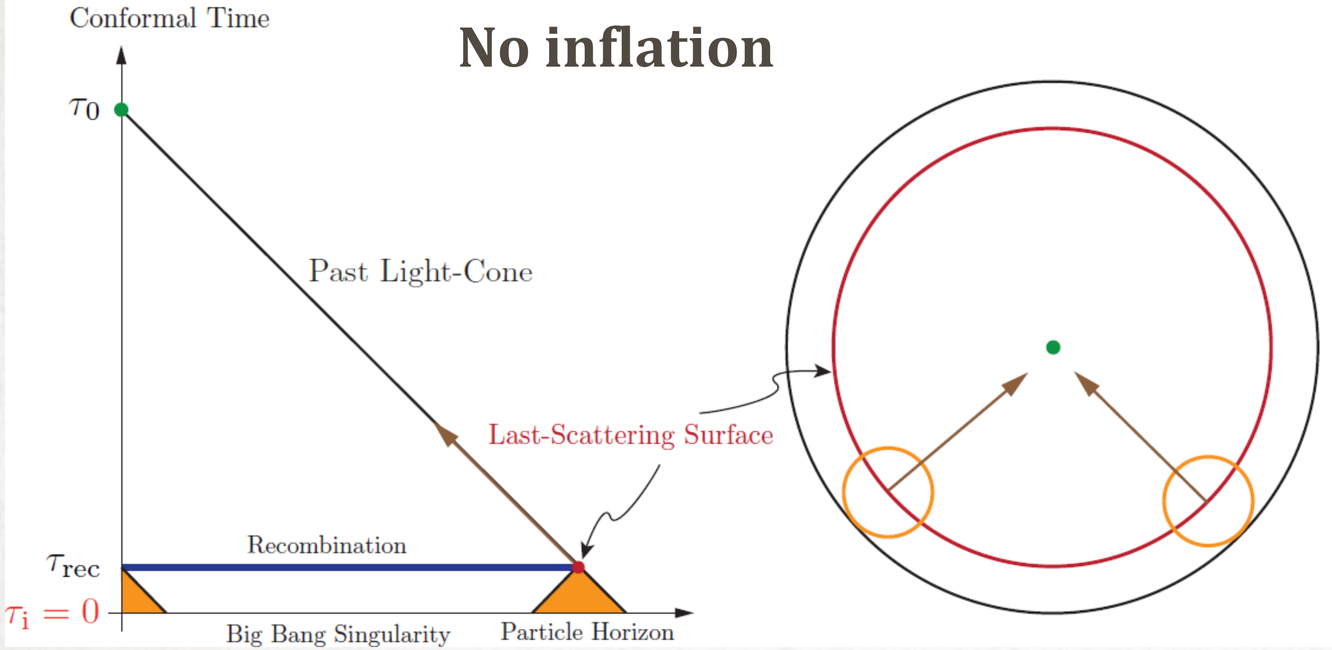
Inflation, an era of accelerated expansion of the Universe, solves both the horizon and flatness problems

SPACETIME DIAGRAM



[Figures from « Lectures on inflation », D. Baumann]

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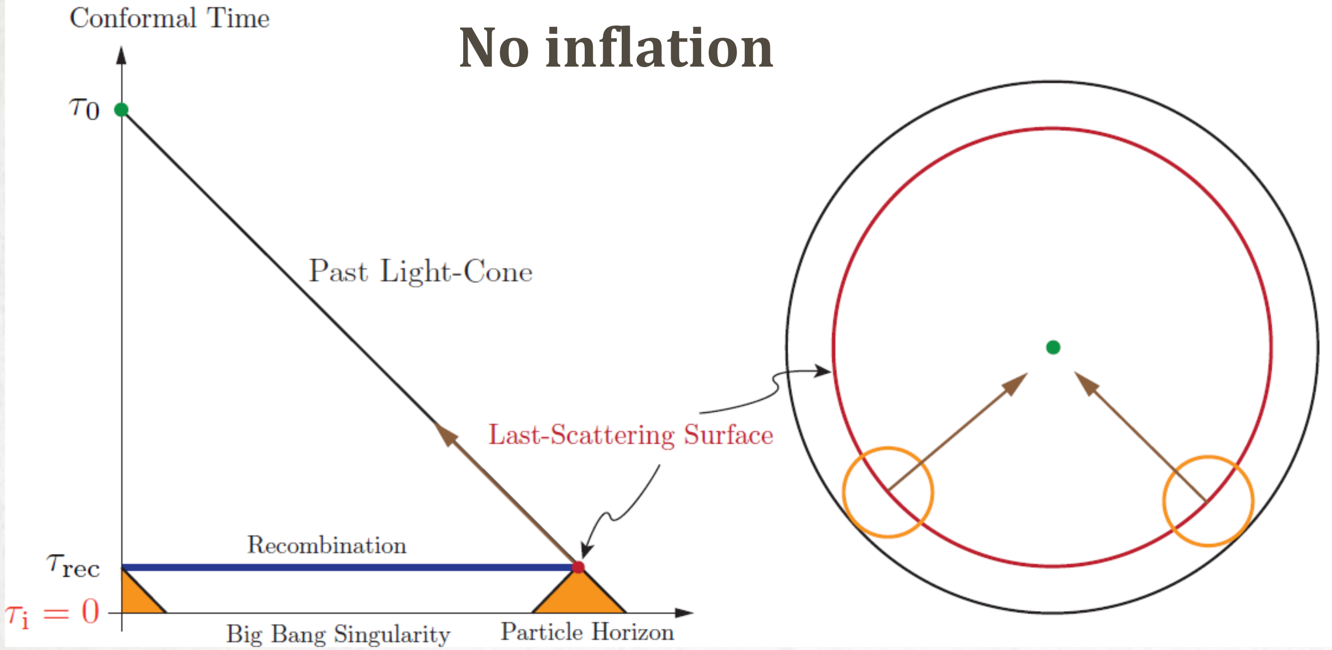
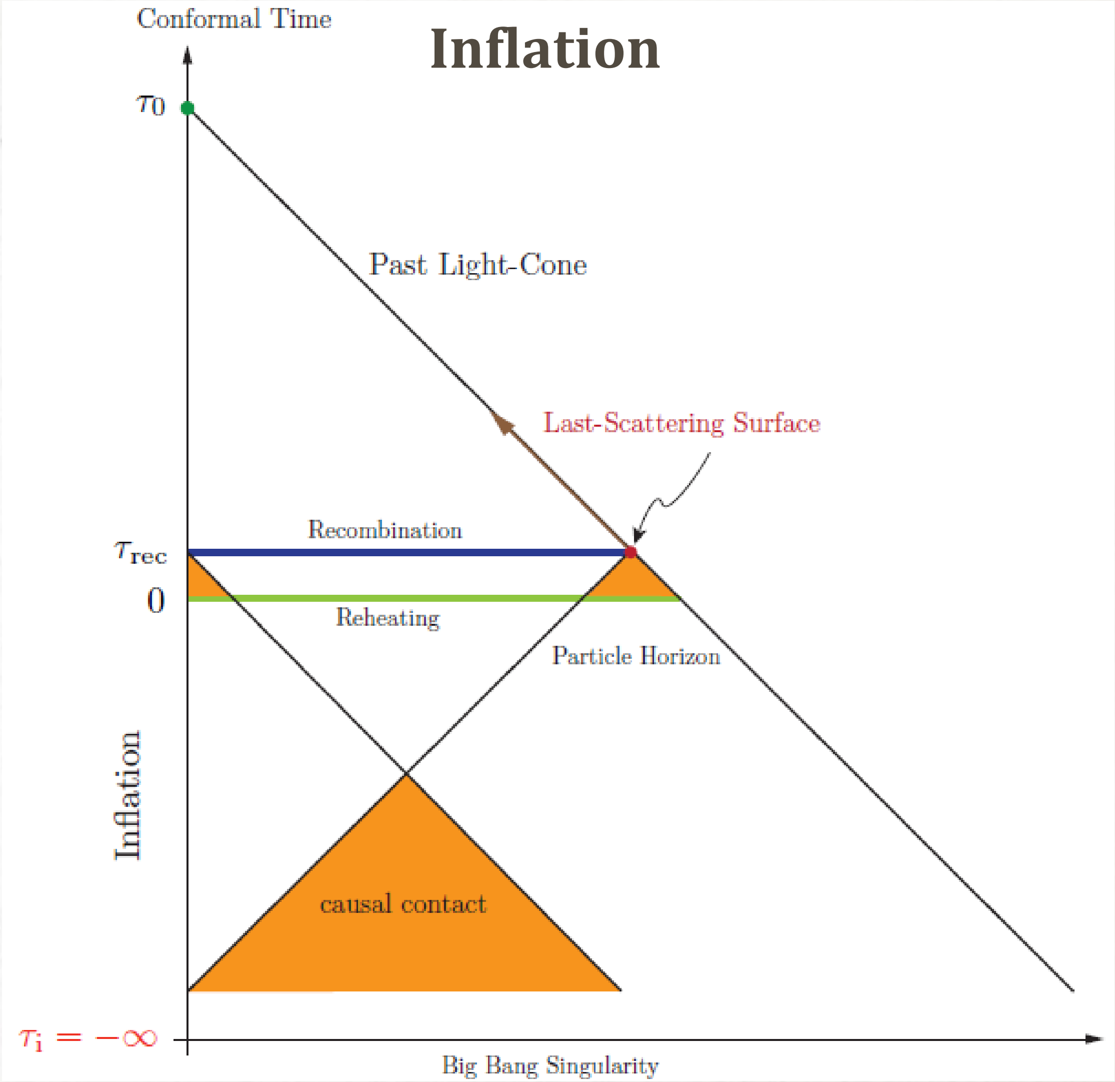




For inflation, you need matter with

$$w = \frac{P}{\rho} < -1/3$$

[Figures from « Lectures on inflation », D. Baumann]

OBSERVATIONAL CONSTRAINTS

$\zeta(\tau, \vec{x})$ the primordial curvature perturbation

$\zeta_{\vec{k}}(\tau)$ its Fourier transform

dictates the statistics of the temperature anisotropies in the CMB

$\overbrace{A_s \quad n_s \quad r}^{\text{Power spectrum}}$			$\overbrace{\vert f_{NL}\vert}^{\text{Bispectrum}}$	$\overbrace{\vert g_{NL}\vert}^{\text{Trispectrum}}$
A_s	n_s	r	$\vert f_{NL}\vert$	$\vert g_{NL}\vert$
1.6×10^{-9}	0.965 ± 0.004	< 0.10	< 50	$< 10^5$

$$\boldsymbol{r} = \frac{P_{GW}}{P_\zeta}$$

Planck constraints from the CMB

Large Scale Structure (LSS) experiments such as DESI or Euclid could constrain $|f_{NL}|<10$

- 2-point (Gaussian) statistics:

$$< \zeta_{\vec{k}} \zeta_{\vec{k}'} > = (2\pi)^3 \;\; \delta^{(3)}(\vec{k} + \vec{k'}) \times P_\zeta(k)$$

Dimensionless power spectrum is

$$\mathcal{P}_\zeta(k) = \frac{2\,\pi^2}{k^3} P_\zeta(k) = \boldsymbol{A_s} \left(\frac{k}{k_*}\right)^{\boldsymbol{n_s}-1}$$

- 3-point (Non-Gaussian) statistics:

$$< \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} >= (2\pi)^3 \;\; \delta^{(3)}(\vec{k_1} + \vec{k_2} + \vec{k_3}) \times B_\zeta(\vec{k_1}, \vec{k_2}, \vec{k_3})$$

Dimensionless bispectrum is

$$\mathcal{B}_\zeta = \frac{2\pi^2}{k^3} B_\zeta = \boldsymbol{f_{NL}} \times \left(\mathcal{P}_\zeta\right)^2$$

- 4-point (Gaussian+Non-Gaussian)

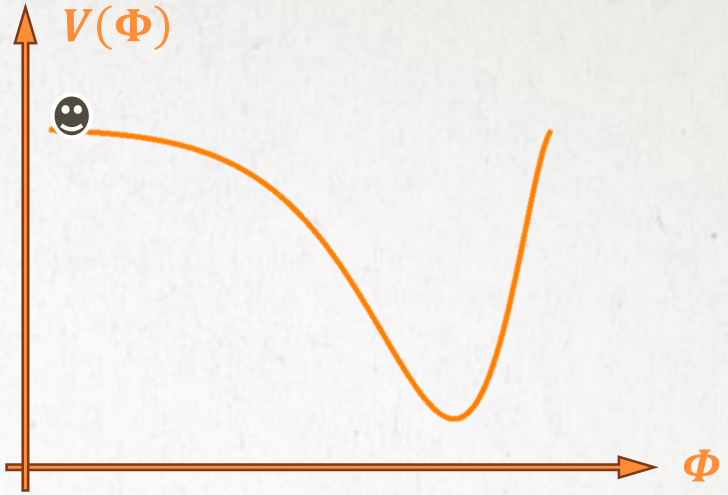
Trispectrum $\mathcal{T}_\zeta = \boldsymbol{g_{NL}} \times \left(\mathcal{P}_\zeta\right)^3$

SLOW-ROLL SINGLE FIELD INFLATION

- Quasi de Sitter space: $\epsilon = -\frac{\dot{H}}{H^2} \ll 1\,; \eta = \frac{\dot{\epsilon}}{H\epsilon} \ll 1$

$$\Rightarrow \frac{M_p\, V'}{V} \ll 1\,; \frac{M_p^2 |V''|}{V} \ll 1$$

- Single-clock: only one scalar degree of freedom
- Canonical kinetic term



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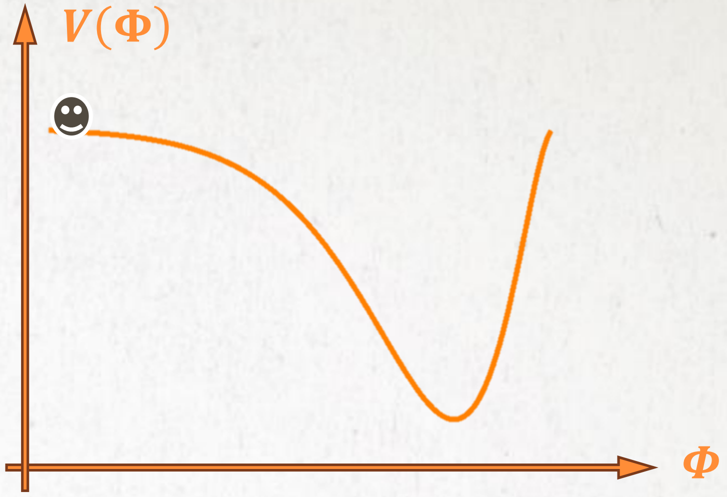


Success and failure

✓ Enough expansion to solve the horizon and flatness problems

✓ Nearly scale-invariant scalar power spectrum on large scales

✓ Small tensor-to-scalar ratio
Small non-Gaussianities

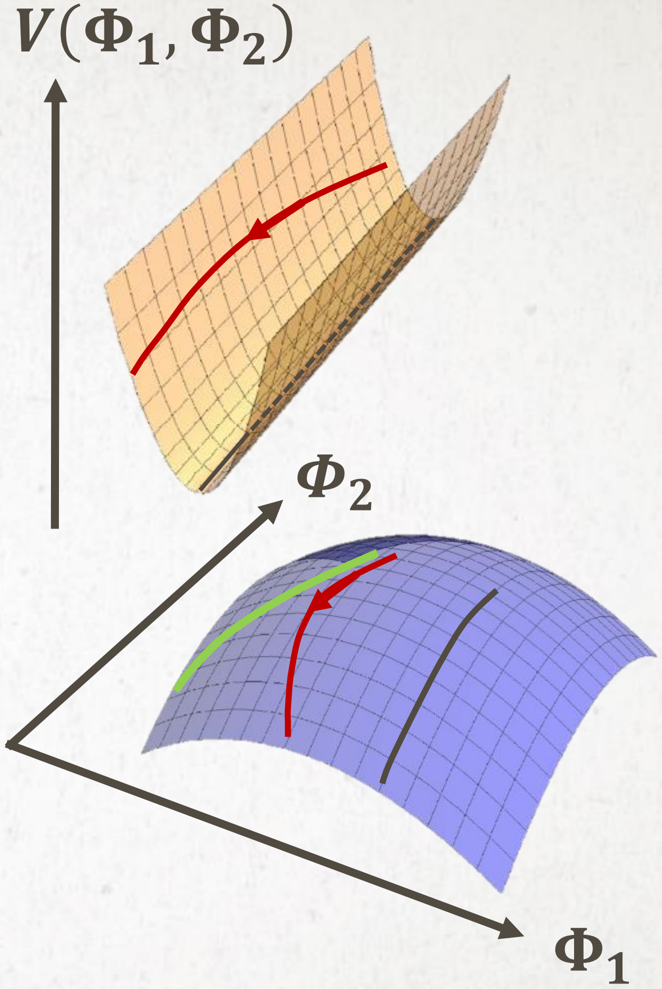
❖ Few theoretical motivation: realistic UV completions typically predict **several scalar fields with non-canonical kinetic terms**

❖ Sensitive to Planck scale suppressed operators, quantum loops renormalize the potential:

eta problem: $\boldsymbol{\frac{M_p^2|V''|}{V} > 1}$

II. MULTIFIELD INFLATION WITH CURVED FIELD SPACE

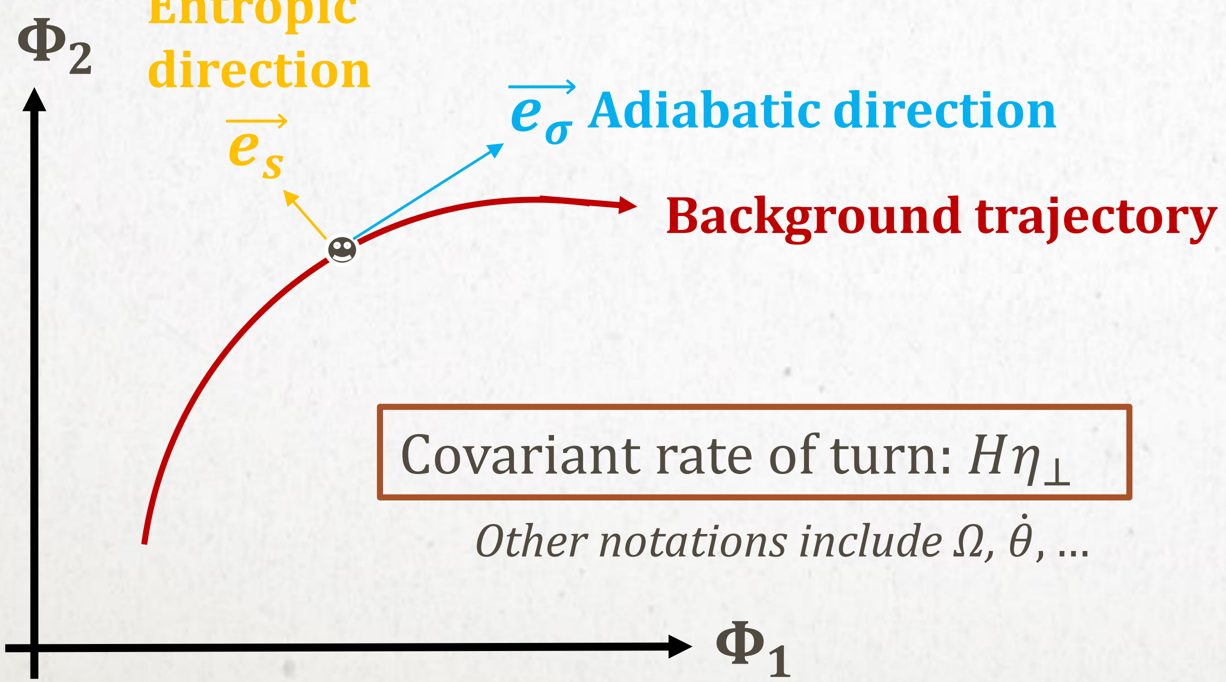
GEOMETRICAL EFFECTS UNVEILED



Curved field space

MULTIFIELD INFLATION WITH CURVED FIELD SPACE

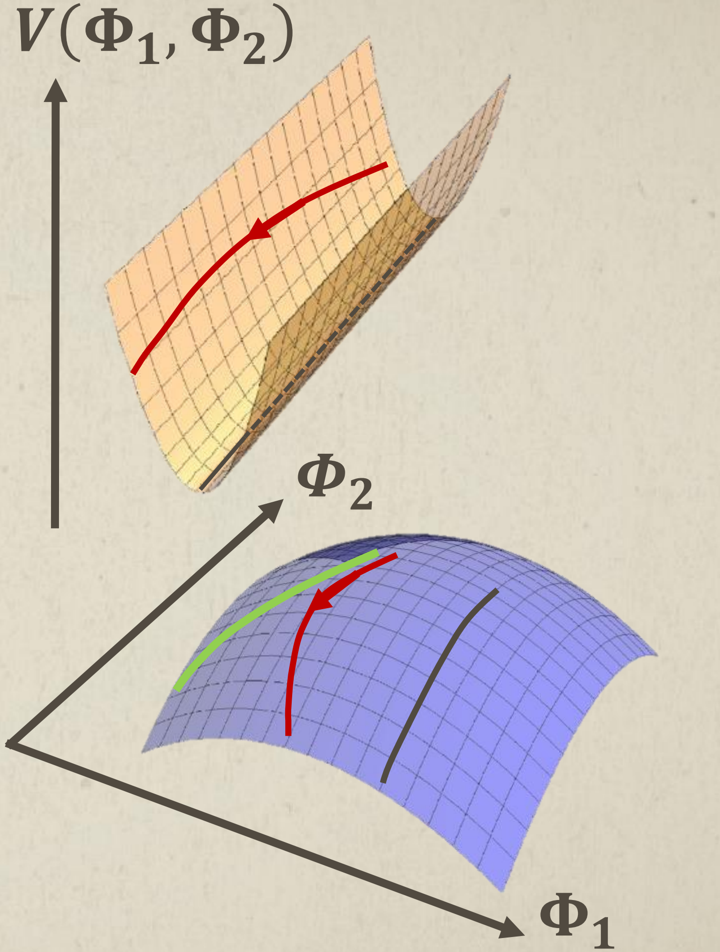
$$S = \int \sqrt{-g}\left(\frac{R}{2} - \sum_{a,b} g^{\mu\nu} \boldsymbol{G_{ab}(\phi^c)} \partial_\mu \phi^a \partial_\nu \phi^b - V(\phi^c)\right)$$



$\boxed{\text{Covariant rate of turn: } H\eta_\perp}$

Other notations include Ω, $\dot{\theta}$, ...

One geodesic
Non-geodesic motion
Minimum of the potential

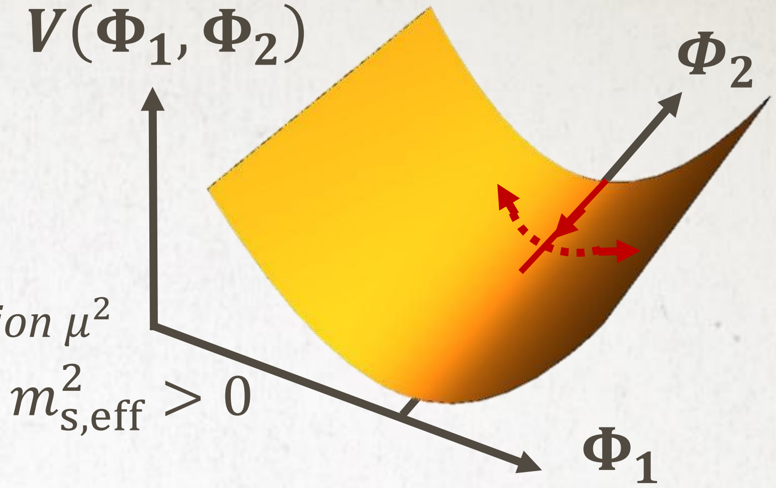


$\boxed{\text{Curved field space}}$

Local scalar curvature: R_{fs}

STABILITY OF BACKGROUND TRAJECTORIES

GEOMETRICAL DESTABILIZATION OF INFLATION



Other notation μ^2

- A stable trajectory requires $\perp$ long wavelength modes to be stable: $m^2_{\mathrm{s,eff}} > 0$

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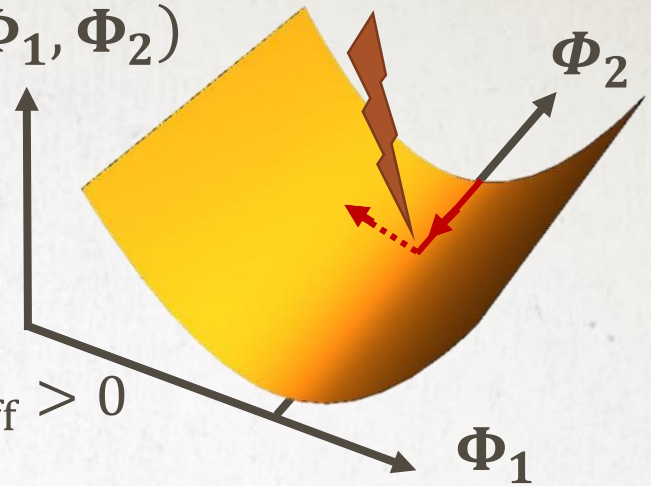

Other notation μ^2

- A stable trajectory requires ⊥ long wavelength modes to be stable: $m^2_{\mathrm{s,eff}} > 0$

Hessian of the potential   Bending   Geometry of field-space

- Geometrical destabilization of inflation: $\dfrac{m^2_{\mathrm{s,eff}}}{H^2} = \underbrace{\dfrac{V_{;ss}}{H^2} + 3\eta_\perp^2}_{>0} + \underbrace{\epsilon R_{\mathrm{fs}} M_p^2}_{<0} < 0$

< 0 for hyperbolic field spaces

[S. Renaux-Petel, K. Turzynski 2015]

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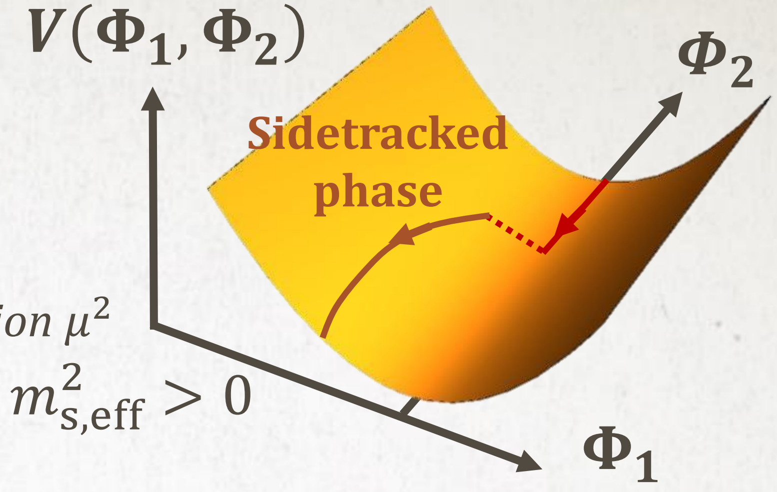

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[S. Renaux-Petel, K. Turzynski 2015]

→ Second, sidetracked phase of inflation

[O. Grocholski, M. Kalinowski, M. Kolanowski, S. Renaux-Petel, K.Turzynski, V. Vennin 2019]

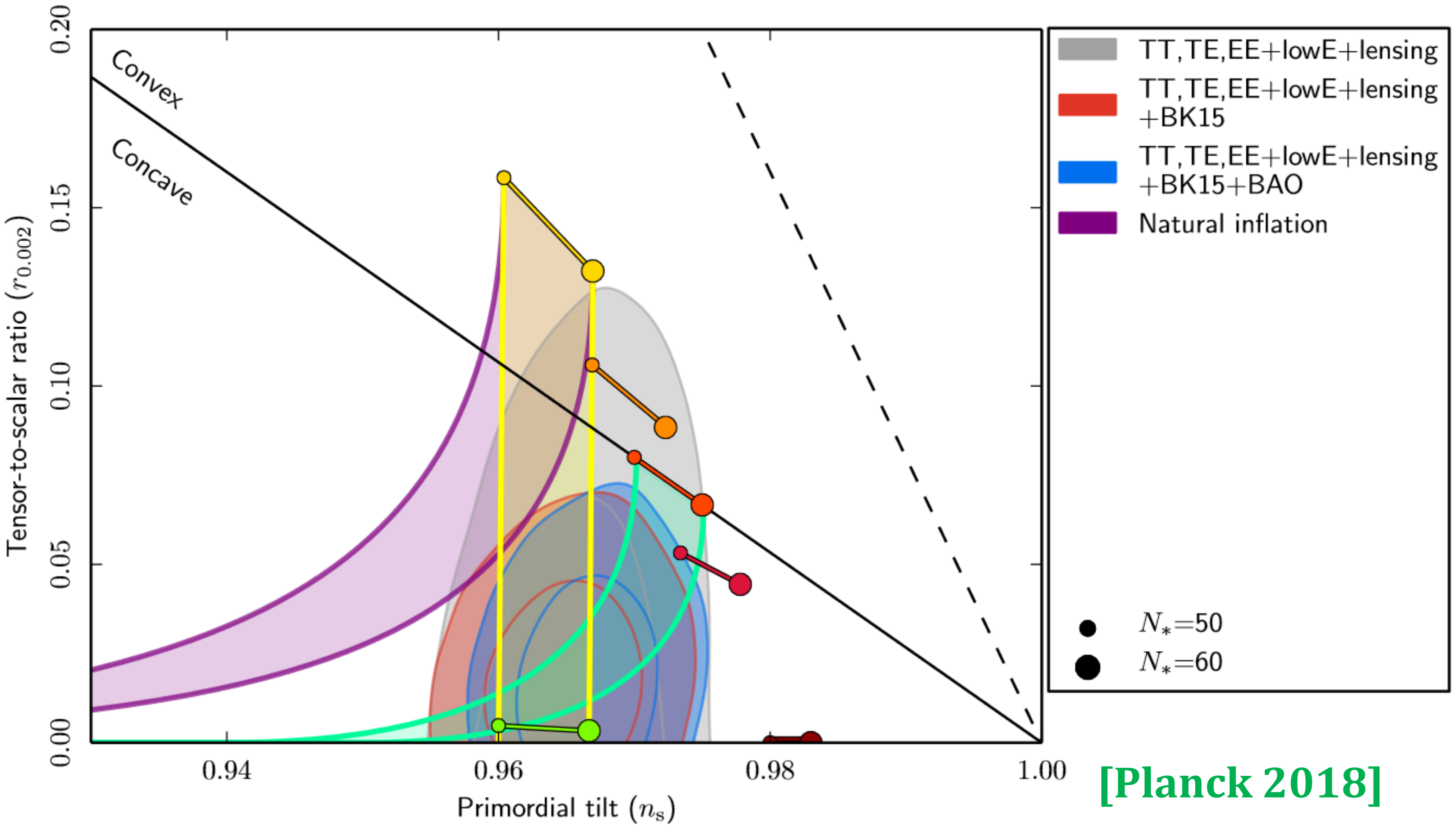
All observables ($N_{\text{inflation}}, n_s, r, f_{\text{nl}} \ldots$) affected

PHYSICS OF LINEAR FLUCTUATIONS

RESURRECTING NATURAL INFLATION?

$$V(\phi) = \Lambda^4 \left(1 + \cos\left(\frac{\phi}{f}\right)\right)$$

Discrete shift symmetry protecting potential from quantum corrections



[Planck 2018]

PHYSICS OF LINEAR FLUCTUATIONS

RESURRECTING NATURAL INFLATION?

$$V(\phi,\chi) = \Lambda^4\left(1+\cos\left(\frac{\phi}{f}\right)\right) + \frac{1}{2}m^2\chi^2$$

Negatively curved field spaces

Toy models (so far)

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Negatively curved field spaces

Toy models (so far)

[Garcia-Saenz, Renaux-Petel, Ronayne 2018]

- **Minimal metric:**

$$d\sigma^2 = \left(1+\frac{\boldsymbol{2\chi^2}}{\boldsymbol{M^2}}\right)d\phi^2 + d\chi^2$$

$$R_{\mathrm{fs}} = -\frac{4}{M^2(1+2\chi^2/M^2)^2}$$

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- **Hyperbolic metric:**

$$d\sigma^2 = \left(1+\frac{\mathbf{2\chi^2}}{\boldsymbol{M^2}}\right)d\phi^2 + \frac{\mathbf{2\sqrt{2}\,\chi}}{\boldsymbol{M}}\boldsymbol{d\phi d\chi} + d\chi^2$$

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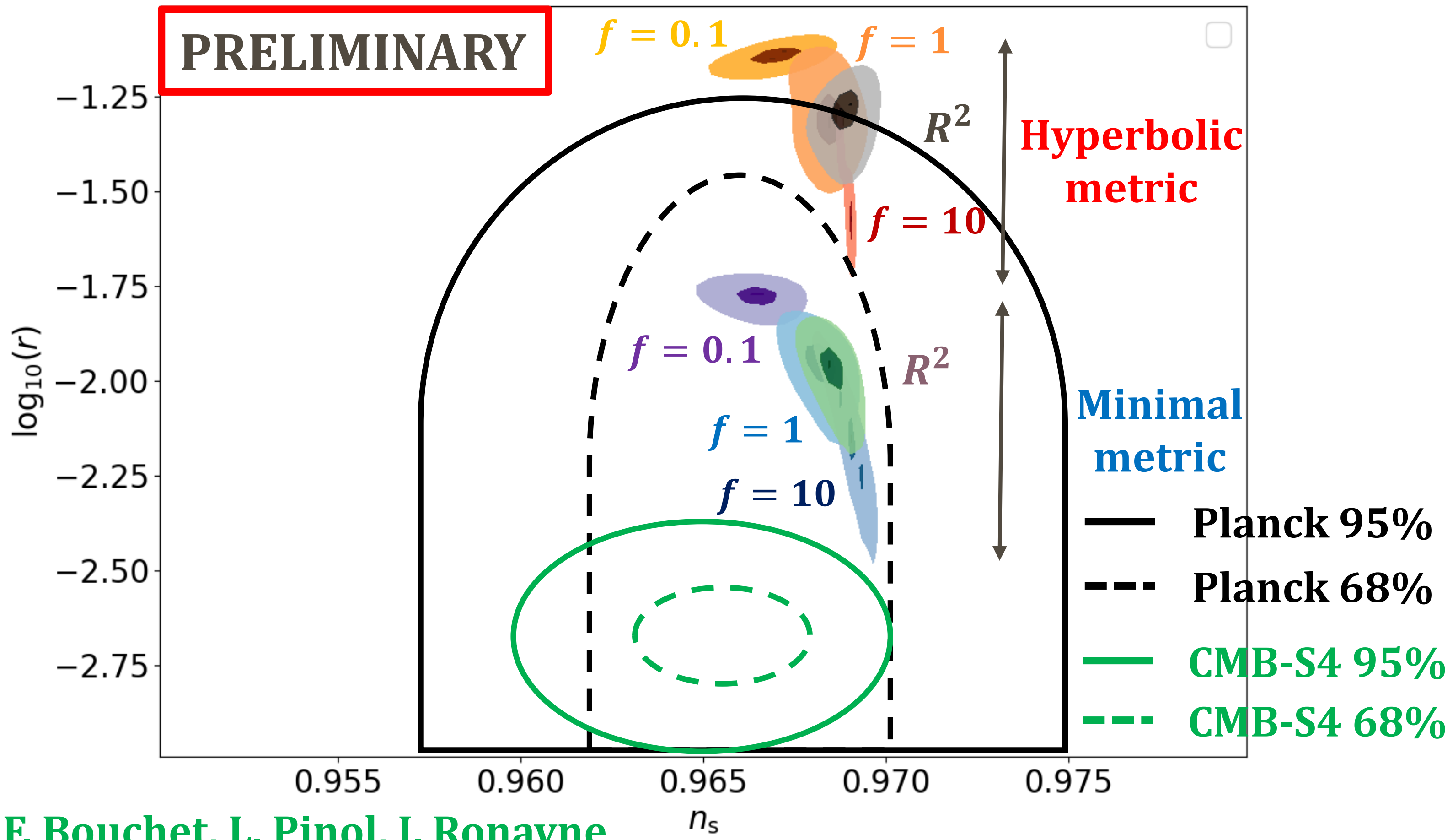

F. Bouchet, L. Pinol, J. Ronayne

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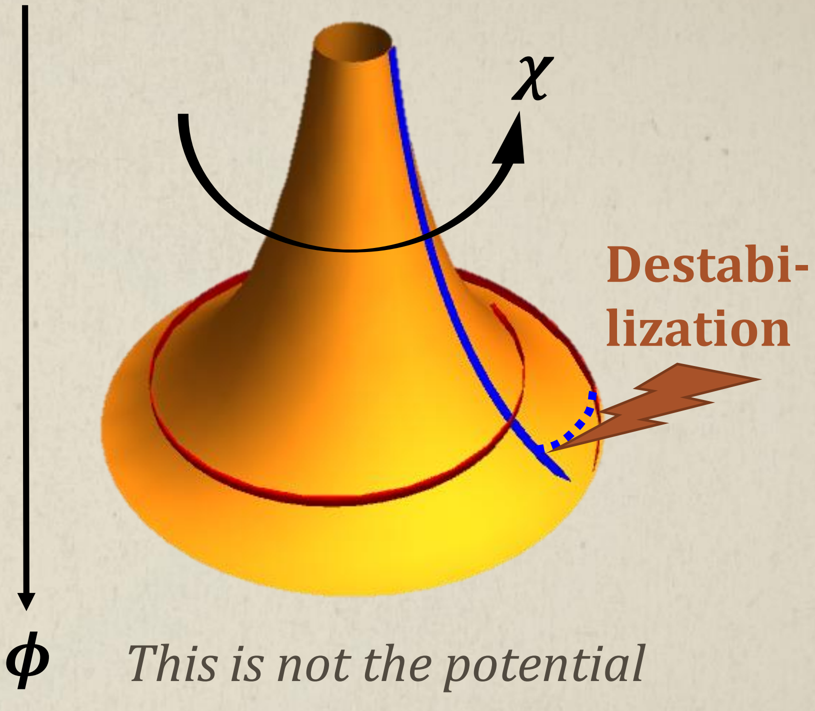
NON-GAUSSIANITIES
HYPERINFLATION

[Fumagalli, Garcia-Saenz, Pinol, Renaux-Petel, Ronayne 2019]
Phys. Rev. Lett. 123, 201302

Setup radial angular

The scalar fields ϕ , χ live on an internal hyperbolic plane

Embedding of the hyperbolic plane in 3D
Radial trajectory
Hyperinflation trajectory



This is not the potential

Hyperbolic field space

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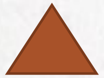
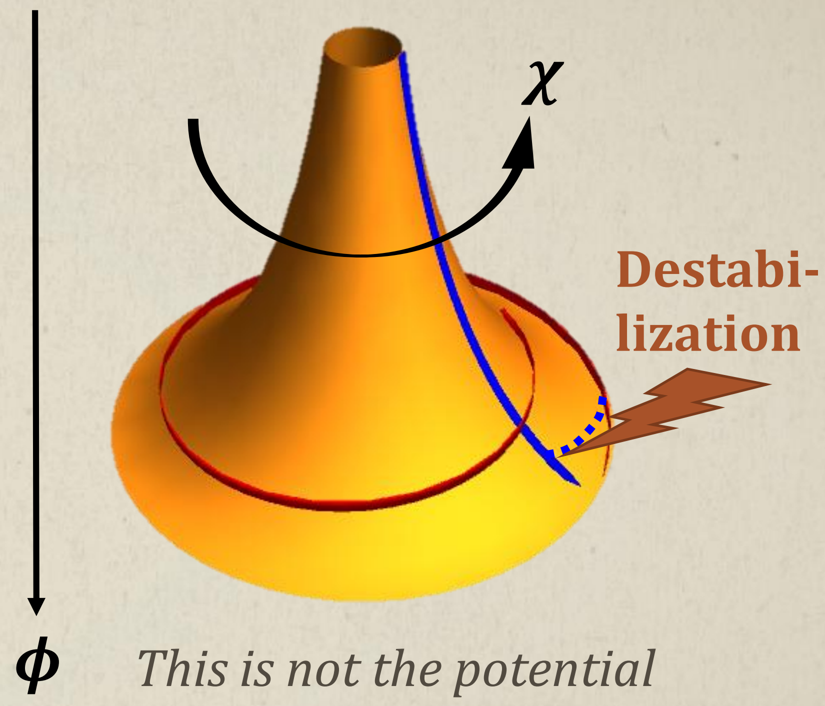
Interesting observational signatures: large non-Gaussianities in exotic flattened configurations



$$\boxed{f_{\mathrm{nl}}^{\mathrm{eq}} = \mathcal{O}(1);\ f_{\mathrm{nl}}^{\mathrm{flat}} = \mathcal{O}(50)}$$

Target for upcoming LSS experiments

Embedding of the hyperbolic plane in 3D
Radial trajectory
Hyperinflation trajectory



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III. REVISITING PRIMORDIAL NON-GAUSSIANITIES

GENERALIZING MALDACENA'S CALCULATION TO CURVED FIELD SPACE

[Garcia-Saenz, Pinol, Renaux-Petel]
J. High Energ. Phys. **2020,** 73 (2020)

$$\mathcal{L}(\zeta,\mathcal{F}) = \underbrace{\mathcal{L}^{(2)}(\zeta,\mathcal{F})}_{\substack{\textbf{Dictating the power spectrum:}\\ \textbf{2-point function}}} + \underbrace{\mathcal{L}^{(3)}_{\mathbf{Maldacena}}(\zeta) + \mathcal{L}^{(3)}_{\mathrm{new}}(\zeta,\mathcal{F}) + \mathcal{D}^{(3)}}_{\substack{\textbf{Dictating the bispectrum:}\\ \textbf{3-point function}}}$$

GAUGE FIXING

2 constrained parameters
4 dynamical scalar d.o.f.
2 can be removed

- Two scalar degrees of freedom can be fixed by a choice of gauge

- ADM formalism $ds^2 = -\boldsymbol{N}^2 dt^2 + g_{ij}\left(dx^i + \boldsymbol{N^i} dt\right)\left(dx^j + \boldsymbol{N^j} dt\right)$

  with $g_{ij}(t,\vec{x}) = a^2(t)e^{2\boldsymbol{\psi}(t,\vec{x})}\left(\delta_{ij} + \partial_i\partial_j \boldsymbol{E}(t,\vec{x})\right)$

- $\boldsymbol{Q_\sigma}(t,\vec{x}) = e_a^\sigma(t)Q^a(t,\vec{x})$ and $\boldsymbol{Q_s}(t,\vec{x}) = e_a^s(t)Q^a(t,\vec{x})$, adiabatic and entropic perturbations

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Flat gauge: $E = \psi = 0 \rightarrow Q_\sigma = Q_\sigma^{\mathrm{flat}}$ and $Q_s = Q_s^{\mathrm{flat}}$	**Comoving gauge: $E = Q_\sigma = 0 \rightarrow \psi^{\mathrm{com}} = \zeta$ and $Q_s^{\mathrm{com}} = \mathcal{F}$**
✓ $\mathcal{L}^{(3)}(Q_\sigma^{\mathrm{flat}}, Q_s^{\mathrm{flat}})$: already known **[Elliston, Seery, Tavakol 2012]** ❖ Correlation functions of observable perturbation ζ computed numerically from the ones of Q's **[Mulryne, Ronayne 2016]**	❖ $\mathcal{L}^{(3)}(\zeta, \mathcal{F})$: not known before this work: **[Garcia-Saenz, Pinol, Renaux-Petel]** *J. High Energ. Phys.* **2020,** 73 (2020) ✓ Correlation functions of observable perturbation ζ computed numerically directly ✓ Analytical studies possible

EXPANDING AND SIMPLIFYING THE ACTION

USING INTEGRATION BY PARTS AND LINEAR EQUATIONS OF MOTION

- We perform integrations by parts to make explicit the size of interactions
- Linear equations of motion $\frac{\delta S^{(2)}}{\delta \zeta} = 0$ and $\frac{\delta S^{(2)}}{\delta \mathcal{F}} = 0$ can be used at any time
- Resulting Lagrangian, after $O(40)$ integrations by parts and $O(10)$ uses of equations of motion:

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$$\mathcal{L}(\zeta, \mathcal{F}) = \mathcal{L}^{(2)}(\zeta, \mathcal{F}) + \mathcal{L}^{(3)}_{\text{Maldacena}}(\zeta, \chi) + \mathcal{L}^{(3)}_{\text{new}}(\zeta, \mathcal{F}) + \mathcal{D}$$

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$$\mathcal{L}^{(2)}(\zeta,\mathcal{F}) = \frac{a^3}{2}\left(2\epsilon M_p^2\left(\dot{\zeta}^2 - \frac{(\partial\zeta)^2}{a^2}\right) + \dot{\mathcal{F}}^2 - \frac{(\partial\mathcal{F})^2}{a^2} - \underbrace{m_s^2}\mathcal{F}^2 + 4\dot{\sigma}\eta_\perp\mathcal{F}\dot{\zeta}\right)$$

$$\boxed{m_s^2 = V_{;ss} - H^2\eta_\perp^2 + \epsilon R_{\text{fs}} H^2 M_p^2}$$

Mixing via the bending

Hessian of the potential

Bending of the trajectory

Field-space curvature

$$\frac{\partial^2 \chi}{a^2} = \epsilon\dot{\zeta} + \frac{\dot{\sigma}}{M_p^2}\eta_\perp \mathcal{F}$$

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$$\mathcal{L}^{(3)}_{\mathrm{Maldacena}}(\zeta, \chi) = a^3 M_p^2 \left[\epsilon(\epsilon - \eta)\dot{\zeta}^2\zeta + \epsilon(\epsilon + \eta)\zeta\frac{(\partial\zeta)^2}{a^2} + \left(\frac{\epsilon}{2} - 2\right)\frac{1}{a^4}(\partial\zeta)(\partial\chi)\partial^2\chi + \frac{\epsilon}{4a^4}\partial^2\zeta(\partial\chi)^2 \right]$$

[J. Maldacena 2003]

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New interactions

Boundary terms:
Total time derivatives
contribute to 3-pt functions
[C. Burrage, R. Ribeiro, D. Seery 2011]
[F. Arroja, T. Tanaka 2011]

NEW INTERACTIONS

$$\lambda_\perp = \frac{\dot{\eta_\perp}}{H\eta_\perp} \quad ; \quad \mu_s = \frac{\dot{m}_s}{Hm_s}$$

$$\frac{\partial^2\chi}{a^2} = \epsilon\dot{\zeta} + \frac{\dot{\sigma}}{M_p^2}\eta_\perp\mathcal{F}$$

$$\mathcal{L}^{(3)}_{\text{new}}(\zeta,\mathcal{F}) = \frac{1}{2}m_s^2\zeta\mathcal{F}\left((\epsilon+\mu_s)\mathcal{F} + (2\epsilon-\eta-2\lambda_\perp)\frac{2\dot{\sigma}\eta_\perp}{m_s^2}\dot{\zeta}\right) + \frac{\dot{\sigma}\eta_\perp}{a^2H}\mathcal{F}(\partial\zeta)^2$$

$$-\frac{\dot{\sigma}\eta_\perp}{H}\dot{\zeta}^2\mathcal{F} - \frac{1}{H}\left(H^2\eta_\perp^2 - \epsilon H^2M_p^2R_{fs}\right)\dot{\zeta}\mathcal{F}^2 - \frac{1}{6}\left(V_{;sss} - 2\dot{\sigma}H\eta_\perp R_{fs} + \epsilon H^2M_p^2R_{fs,s}\right)\mathcal{F}^3$$

$$+\frac{1}{2}\epsilon\zeta\left(\dot{\mathcal{F}}^2 + \frac{(\partial\mathcal{F})^2}{a^2}\right) - \frac{1}{a^2}\dot{\mathcal{F}}(\partial\mathcal{F})(\partial\chi)$$

Check: ζ is well massless at any order as it should (Weinberg adiabatic mode)

NEW INTERACTIONS

$$\lambda_\perp = \frac{\dot{\eta}_\perp}{H\eta_\perp} \quad ; \quad \mu_s = \frac{\dot{m}_s}{H m_s}$$

$$\frac{\partial^2\chi}{a^2} = \epsilon\dot{\zeta} + \frac{\dot{\sigma}}{M_p^2}\eta_\perp\mathcal{F}$$

$$\mathcal{L}^{(3)}_{\text{new}}(\zeta,\mathcal{F}) = \frac{1}{2}m_s^2\zeta\mathcal{F}\left((\epsilon+\mu_s)\mathcal{F} + (2\epsilon-\eta-2\lambda_\perp)\frac{2\dot{\sigma}\eta_\perp}{m_s^2}\dot{\zeta}\right) + \frac{\dot{\sigma}\eta_\perp}{a^2H}\mathcal{F}(\partial\zeta)^2$$
$$-\frac{\dot{\sigma}\eta_\perp}{H}\dot{\zeta}^2\mathcal{F} - \frac{1}{H}\left(H^2\eta_\perp^2 - \epsilon H^2 M_p^2 R_{fs}\right)\dot{\zeta}\mathcal{F}^2 - \frac{1}{6}\left(V_{;sss} - 2\dot{\sigma}H\eta_\perp R_{fs} + \epsilon H^2 M_p^2 R_{fs,s}\right)\mathcal{F}^3$$
$$+\frac{1}{2}\epsilon\zeta\left(\dot{\mathcal{F}}^2 + \frac{(\partial\mathcal{F})^2}{a^2}\right) - \frac{1}{a^2}\dot{\mathcal{F}}(\partial\mathcal{F})(\partial\chi)$$

$$\mathcal{D} = \frac{M_p^2}{2}\frac{d}{dt}\left\{\begin{aligned} &-\frac{1}{3aH^3}\zeta\left[\left(\partial_i\partial_j\zeta\right)^2 - (\partial^2\zeta)^2\right] + \frac{a}{H}\left[2(1-\epsilon)\zeta(\partial\zeta)^2 - \frac{1}{M_p^2}\zeta(\partial\mathcal{F})^2\right] - a^3\left[18H\,\zeta^3 + \frac{1}{M_p^2H}(m_s^2+4H^2\eta_\perp^2)\zeta\mathcal{F}^2\right] \\ &+\partial^2\chi\frac{a}{H}\left[-2\dot{\zeta}\zeta + \frac{\dot{\sigma}\eta_\perp}{M_p^2\epsilon}\zeta\mathcal{F} + \frac{1}{a^2}\left((\partial\zeta)^2 - \partial^{-2}\,\partial_i\partial_j\left(\partial_i\zeta\partial_j\zeta\right)\right) - \frac{1}{a^2}\left(\partial_i\zeta\partial_i\chi - \partial^{-2}\,\partial_i\partial_j\left(\partial_i\zeta\partial_j\chi\right)\right)\right] - \frac{a^3}{M_p^2H}\zeta\dot{\mathcal{F}}^2\end{aligned}\right\}$$

Local contribution to B_ζ that can be computed analytically with the « in-in » formalism

NEW INTERACTIONS

Applications: *quasi-single field, cosmological collider physics, single-field effective theory*

$$\mathcal{L}^{(3)}_{\mathbf{new}}(\zeta,\mathcal{F}) = \frac{1}{2}m_s^2\zeta\mathcal{F}\left((\epsilon+\mu_s)\mathcal{F}+(2\epsilon-\eta-2\lambda_\perp)\frac{2\dot\sigma\eta_\perp}{m_s^2}\dot\zeta\right)+\frac{\dot\sigma\eta_\perp}{a^2H}\mathcal{F}(\partial\zeta)^2$$

$$-\frac{\dot\sigma\eta_\perp}{H}\dot\zeta^2\mathcal{F}-\frac{1}{H}\left(H^2\eta_\perp^2-\epsilon H^2M_p^2R_{fs}\right)\dot\zeta\mathcal{F}^2-\frac{1}{6}\left(V_{;sss}-2\dot\sigma H\eta_\perp R_{fs}+\epsilon H^2M_p^2R_{fs,s}\right)\mathcal{F}^3$$

$$+\frac{1}{2}\epsilon\zeta\left(\dot{\mathcal{F}}^2+\frac{(\partial\mathcal{F})^2}{a^2}\right)-\frac{1}{a^2}\dot{\mathcal{F}}(\partial\mathcal{F})(\partial\chi)$$

$$\mathcal{D}=\frac{M_p^2}{2}\frac{\mathbf{d}}{\mathbf{dt}}\left\{\begin{aligned}&-\frac{1}{3aH^3}\zeta\left[\left(\partial_i\partial_j\zeta\right)^2-(\partial^2\zeta)^2\right]+\frac{a}{H}\left[2(1-\epsilon)\zeta(\partial\zeta)^2-\frac{1}{M_p^2}\zeta(\partial\mathcal{F})^2\right]-a^3\left[18H\,\zeta^3+\frac{1}{M_p^2H}(m_s^2+4H^2\eta_\perp^2)\zeta\mathcal{F}^2\right]\\&+\boldsymbol{\partial^2\chi}\frac{a}{H}\left[-2\dot\zeta\zeta+\frac{\dot\sigma\eta_\perp}{M_p^2\epsilon}\zeta\mathcal{F}+\frac{1}{a^2}\left((\partial\zeta)^2-\partial^{-2}\,\partial_i\partial_j\left(\partial_i\zeta\partial_j\zeta\right)\right)-\frac{1}{a^2}\left(\partial_i\zeta\partial_i\chi-\partial^{-2}\,\partial_i\partial_j\left(\partial_i\zeta\partial_j\chi\right)\right)\right]-\frac{a^3}{M_p^2H}\zeta\dot{\boldsymbol{\mathcal{F}}}^2\end{aligned}\right\}$$

NEW INTERACTIONS

Applications: *quasi-single field, cosmological collider physics,* ***single-field effective theory***

$$\mathcal{L}^{(3)}_{\mathbf{new}}(\boldsymbol{\zeta},\boldsymbol{\mathcal{F}}) = \frac{1}{2}m_s^2\zeta\mathcal{F}\left((\epsilon+\mu_s)\mathcal{F}+(2\epsilon-\eta-2\lambda_\perp)\frac{2\dot\sigma\eta_\perp}{m_s^2}\dot\zeta\right)+\frac{\dot\sigma\eta_\perp}{a^2H}\mathcal{F}(\partial\zeta)^2$$

$$-\frac{\dot\sigma\eta_\perp}{H}\dot\zeta^2\mathcal{F}-\frac{1}{H}\left(H^2\eta_\perp^2-\epsilon H^2M_p^2R_{fs}\right)\dot\zeta\mathcal{F}^2-\frac{1}{6}\left(V_{;sss}-2\dot\sigma H\eta_\perp R_{fs}+\epsilon H^2M_p^2R_{fs,s}\right)\mathcal{F}^3$$

$$+\frac{1}{2}\epsilon\zeta\left(\dot{\mathcal{F}}^2+\frac{(\partial\mathcal{F})^2}{a^2}\right)-\frac{1}{a^2}\dot{\mathcal{F}}(\partial\mathcal{F})(\partial\chi)$$

Useful form of the action for integrating out $\mathcal{F}$ when it is heavy

$$\mathcal{D}=\frac{M_p^2}{2}\frac{\boldsymbol{d}}{\boldsymbol{dt}}\left\{\begin{aligned}&-\frac{1}{3aH^3}\zeta\left[\left(\partial_i\partial_j\zeta\right)^2-(\partial^2\zeta)^2\right]+\frac{a}{H}\left[2(1-\epsilon)\zeta(\partial\zeta)^2-\frac{1}{M_p^2}\zeta(\partial\mathcal{F})^2\right]-a^3\left[18H\,\zeta^3+\frac{1}{M_p^2H}(m_s^2+4H^2\eta_\perp^2)\zeta\mathcal{F}^2\right]\\&+\boldsymbol{\partial^2\chi}\frac{a}{H}\left[-2\dot\zeta\zeta+\frac{\dot\sigma\eta_\perp}{M_p^2\epsilon}\zeta\mathcal{F}+\frac{1}{a^2}\left((\partial\zeta)^2-\partial^{-2}\,\partial_i\partial_j(\partial_i\zeta\partial_j\zeta)\right)-\frac{1}{a^2}\left(\partial_i\zeta\partial_i\chi-\partial^{-2}\,\partial_i\partial_j(\partial_i\zeta\partial_j\chi)\right)\right]-\frac{a^3}{M_p^2H}\zeta\dot{\boldsymbol{\mathcal{F}}}^2\end{aligned}\right\}$$

IV. INTEGRATING OUT HEAVY ENTROPIC FLUCTUATIONS

AN EFFECTIVE THEORY FOR THE OBSERVABLE CURVATURE PERTURBATION

$$S[\zeta,\mathcal{F}] \xrightarrow{\mathcal{F}_{\mathrm{heavy}}(\zeta)} S_{\mathrm{EFT}}[\zeta] = S[\zeta,\mathcal{F}_{\mathrm{heavy}}(\zeta)]$$

[Garcia-Saenz, Pinol, Renaux-Petel]
J. High Energ. Phys. **2020,** 73 (2020)

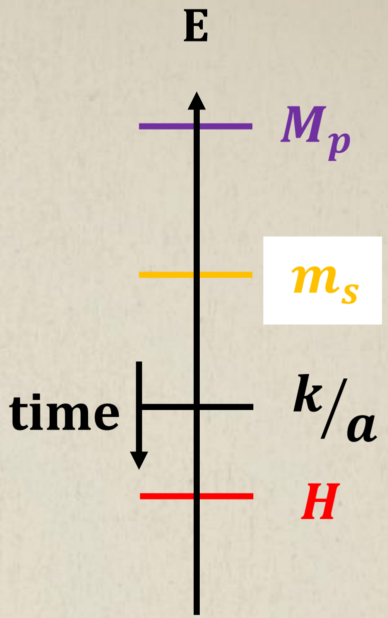
A HIERARCHY OF SCALES

WHEN ENTROPIC FLUCTUATIONS ARE HEAVY

- Equation of motion for $\mathcal{F}$:

$$\ddot{\mathcal{F}} + 3H\dot{\mathcal{F}} + \left(m_s^2 + \frac{k^2}{a^2}\right)\mathcal{F} = 2\dot{\sigma}\eta_\perp\dot{\zeta}$$

Hierarchy of scales



Energy of the "experiment"

$$H \ll m_s$$

Integrate out the heavy perturbation

Like in the Fermi theory:
Integrate out the heavy W, Z bosons and consider contact interactions for fermions

A HIERARCHY OF SCALES

WHEN ENTROPIC FLUCTUATIONS ARE HEAVY

- Equation of motion for $\mathcal{F}$:

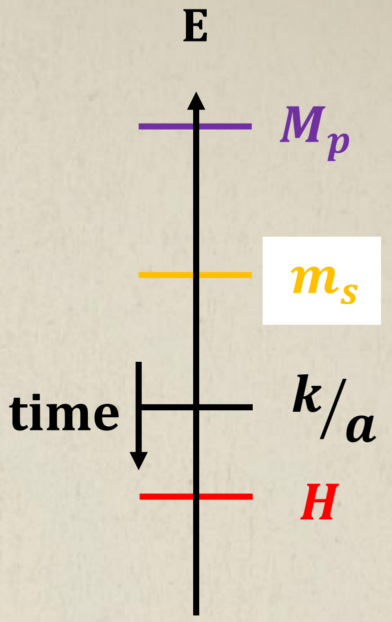
$$\cancel{\ddot{\mathcal{F}}} + \cancel{3H\dot{\mathcal{F}}} + \left(m_s^2 + \cancel{\frac{k^2}{a^2}}\right)\mathcal{F} = 2\dot{\sigma}\eta_\perp\dot{\zeta}$$

When $\mathcal{F}$ is heavy

$$\boxed{\mathcal{F}^{\mathrm{LO}}_{\mathrm{heavy}} = \frac{2\dot{\sigma}\eta_\perp}{m_s^2}\dot{\zeta}}$$

$$\boldsymbol{\omega^2, \omega H, \frac{k^2}{a^2} \ll m_s^2}$$

Hierarchy of scales



Energy of the "experiment"

$$H \ll m_s$$

Integrate out the heavy perturbation

Like in the Fermi theory:
Integrate out the heavy W, Z bosons and consider contact interactions for fermions

A HIERARCHY OF SCALES

THE QUADRATIC EFFECTIVE ACTION

➢ Equation of motion for $\mathcal{F}$:

$$\cancel{\ddot{\mathcal{F}}} + \cancel{3H\dot{\mathcal{F}}} + \left(m_s^2 + \cancel{\frac{k^2}{a^2}}\right)\mathcal{F} = 2\dot{\sigma}\eta_\perp\dot{\zeta}$$

When $\mathcal{F}$ is heavy

$$\boxed{\mathcal{F}_{\text{heavy}}^{\text{LO}} = \frac{2\dot{\sigma}\eta_\perp}{m_s^2}\dot{\zeta}}$$

$$\Downarrow$$

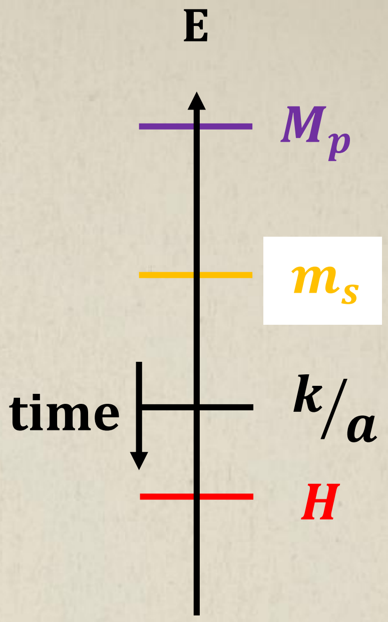
Effective single-field action for the curvature perturbation

$$S_2^{\text{EFT}}[\zeta] = \int d\tau d^3x a^2 \epsilon \left(\frac{\zeta'^2}{c_s^2} - (\partial_i\zeta)^2\right)$$

With a speed of sound c_s:

$$\boxed{\frac{1}{c_s^2} = 1 + \frac{4H^2\eta_\perp^2}{m_s^2}}$$

Hierarchy of scales



Energy of the "experiment"

$H \ll m_s$

Integrate out the heavy perturbation

Like in the Fermi theory:
Integrate out the heavy W, Z bosons and consider contact interactions for fermions

$s = \dot{c}_s / H c_s$

THE CUBIC EFFECTIVE ACTION

FULL RESULT

$$S_3^{\mathrm{EFT}}[\zeta] = \int d\tau d^3x a^2 M_p^2 \frac{\epsilon}{c_s^2} \begin{pmatrix} \frac{g_1}{\mathcal{H}}\zeta'^3 + \\ g_2 \zeta'^2 \zeta + \\ g_3 c_s^2 \zeta (\partial_i \zeta)^2 + \\ \frac{\tilde{g}_3 c_s^2}{\mathcal{H}} \zeta' (\partial_i \zeta)^2 + \\ g_4 \zeta' \partial_i \partial^{-2} \zeta' \partial_i \zeta + \\ g_5 \partial^2 \zeta (\partial_i \partial^{-2} \zeta')^2 \end{pmatrix} \text{ with } \begin{cases} g_1 = \left(\frac{1}{c_s^2} - 1\right) \boldsymbol{A} \\ g_2 = \epsilon - \eta + 2s \\ g_3 = \epsilon + \eta \\ \tilde{g}_3 = \frac{1}{c_s^2} - 1 \\ g_4 = \frac{-2\epsilon}{c_s^2}\left(1 - \frac{\epsilon}{4}\right) \\ g_5 = \frac{\epsilon^2}{4 c_s^2} \end{cases}$$

The only new parameter is **A**, and depends on the UV physics

$$X = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

THE CUBIC EFFECTIVE ACTION

RECOVERING P(X) THEORY

Redundancy of operators

$$S_3^{\mathrm{EFT}}[\zeta] = \int d\tau d^3x a^2 M_p^2 \frac{\epsilon}{c_s^2}\left(\begin{array}{c} \frac{g_1}{\mathcal{H}}\zeta'^3 + \\ g_2\zeta'^2\zeta + \\ g_3 c_s^2\zeta(\partial_i\zeta)^2 + \\ \cancel{\frac{\tilde{g}_3 c_s^2}{\mathcal{H}}\zeta'(\partial_i\zeta)^2} + \\ g_4\zeta'\partial_i\partial^{-2}\zeta'\partial_i\zeta + \\ g_5\partial^2\zeta(\partial_i\partial^{-2}\zeta')^2 \end{array}\right) \text{ with } \left\{\begin{array}{l} g_1 = \left(\frac{1}{c_s^2} - 1\right)(1 + 2\boldsymbol{A}) \\ g_2 = \frac{1}{c_s^2}\left(3(c_s^2 - 1) + \epsilon - \eta\right) \\ g_3 = \frac{1}{c_s^2}\left(-(c_s^2 - 1) + \epsilon + \eta - 2s\right) \\ g_4 = \frac{-2\epsilon}{c_s^2}\left(1 - \frac{\epsilon}{4}\right) \\ g_5 = \frac{\epsilon^2}{4c_s^2} \end{array}\right.$$

P(X) cubic lagrangian:

Direct mapping with P(X):

$$\frac{2\lambda}{\Sigma} = -\left(\frac{1}{c_s^2} - 1\right)\boldsymbol{A} \quad \text{with}$$

$$\Sigma = XP_{,X} + 2X^2P_{,XX}$$

$$\lambda = X^2P_{,XX} + \frac{2}{3}X^3P_{,XXX}$$

[X. Chen, M. Huang, S. Kachru, G. Shiu 2008]

[C. Burrage, R. Ribeiro, D. Seery 2011]

THE CUBIC EFFECTIVE ACTION

RECOVERING CANONICAL SINGLE-FIELD LIMIT

Maldacena's result:
Non-Gaussianities $\sim \boldsymbol{O(\epsilon,\eta)}$

$\boldsymbol{c_s^2 \to 1}$

$$S_3^{\mathrm{EFT}}[\zeta] = \int d\tau d^3x a^2 M_p^2 \frac{\epsilon}{c_s^2} \left(\begin{array}{c} \cancel{\frac{g_1}{\mathcal{H}}}\zeta'^3 + \\ g_2 \zeta'^2 \zeta + \\ g_3 \zeta (\partial_i \zeta)^2 + \\ \cancel{\frac{g_3}{\mathcal{H}}\zeta'}(\partial_i\zeta)^2 + \\ g_4 \zeta' \partial_i \partial^{-2} \zeta' \partial_i \zeta + \\ g_5 \partial^2 \zeta (\partial_i \partial^{-2} \zeta')^2 \end{array} \right) \text{ with } \left\{ \begin{array}{l} g_2 = \epsilon + \eta \\ g_3 = \epsilon - \eta \\ g_4 = -2\epsilon \left(1 - \frac{\epsilon}{4}\right) \\ g_5 = \frac{\epsilon^2}{4} \end{array} \right.$$

~~The only new parameter is **A**, and depends on the UV physics~~

THE CUBIC EFFECTIVE ACTION

RECOVERING THE EFT OF INFLATION

Decoupling limit result:

Non-Gaussianities $\sim \frac{1}{c_s^2} - 1$

$\epsilon, \eta, s \to 0$

$$S_3^{\text{EFT}}[\zeta] = \int d\tau d^3x a^2 M_p^2 \frac{\epsilon}{c_s^2} \left(\begin{array}{c} \frac{g_1}{\mathcal{H}} \zeta'^3 + \\ \cancel{g_2 \zeta'^2 \zeta} + \\ \cancel{g_3 c_s^2 \zeta (\partial_i \zeta)^2} + \\ \frac{\tilde{g}_3 c_s^2}{\mathcal{H}} \zeta' (\partial_i \zeta)^2 + \\ \cancel{g_4 \zeta' \partial_i \partial^{-2} \zeta' \partial_i \zeta} + \\ \cancel{g_5 \partial^2 \zeta (\partial_i \partial^{-2} \zeta')^2} \end{array} \right) \text{ with } \left\{ \begin{array}{l} g_1 = \left(\frac{1}{c_s^2} - 1 \right) \boldsymbol{A} \\ \tilde{g}_3 = \frac{1}{c_s^2} - 1 \end{array} \right.$$

The only new parameter is **A**, and depends on the UV physics

THE DECOUPLING LIMIT

REVISITED...

$$S_3^{\mathrm{EFT}}[\zeta] = \int d\tau d^3x a^2 M_p^2 \frac{\epsilon}{\mathcal{H}}\left(\frac{1}{c_s^2} - 1\right)\left(\zeta'(\partial_i\zeta)^2 + \frac{\boldsymbol{A}}{c_s^2}\zeta'^3\right)$$

$$\text{with } \boldsymbol{A} = \underbrace{-\frac{1}{2}(1 + c_s^2)}_{\textbf{Previously known}} + \cdots$$

THE DECOUPLING LIMIT

REVISITED…

Bending radius of the trajectory: $\kappa = \dfrac{\sqrt{2\epsilon}M_p}{\eta_\perp}$

$$S_3^{\mathrm{EFT}}[\zeta] = \int d\tau d^3x a^2 M_p^2 \frac{\epsilon}{\mathcal{H}}\left(\frac{1}{c_s^2} - 1\right)\left(\zeta'(\partial_i\zeta)^2 + \frac{\boldsymbol{A}}{c_s^2}\zeta'^3\right)$$

$$\text{with } \boldsymbol{A} = -\frac{1}{2}(1+c_s^2) \underbrace{- \frac{1}{6}(1-c_s^2)\frac{\kappa V_{;sss}}{m_s^2}} + \cdots$$

3rd derivative of the potential (expected)

Self-coupling of entropic fluctuations

THE DECOUPLING LIMIT
REVISITED…

Bending radius of the trajectory: $\kappa = \frac{\sqrt{2\epsilon} M_p}{\eta_\perp}$

$$S_3^{\mathrm{EFT}}[\zeta] = \int d\tau d^3x a^2 M_p^2 \frac{\epsilon}{\mathcal{H}}\left(\frac{1}{c_s^2} - 1\right)\left(\zeta'(\partial_i\zeta)^2 + \frac{\boldsymbol{A}}{c_s^2}\zeta'^3\right)$$

$$\text{with } \boldsymbol{A} = -\frac{1}{2}(1+c_s^2) - \frac{1}{6}(1-c_s^2)\frac{\kappa V_{;sss}}{m_s^2} + \underbrace{\frac{2}{3}(1+c_s^2)\frac{\epsilon R_{\mathrm{fs}} H^2 M_p^2}{m_s^2}}_{\text{Scalar curvature of the field space}} + \cdots$$

THE DECOUPLING LIMIT
REVISITED...

Bending radius of the trajectory: $\boldsymbol{\kappa} = \frac{\sqrt{2\epsilon}M_p}{\eta_\perp}$

$$S_3^{\mathrm{EFT}}[\zeta] = \int \; d\tau d^3x a^2 M_p^2 \frac{\epsilon}{\mathcal{H}}\left(\frac{1}{c_s^2} - 1\right)\left(\zeta'(\partial_i\zeta)^2 + \frac{\boldsymbol{A}}{c_s^2}\zeta'^3\right)$$

$$\text{with } \boldsymbol{A} = -\frac{1}{2}(1+c_s^2) + \frac{2}{3}(1+c_s^2)\frac{\epsilon R_{\mathrm{fs}} H^2 M_p^2}{m_s^2} - \frac{1}{6}(1-c_s^2)\left(\frac{\kappa V_{;sss}}{m_s^2} + \underbrace{\frac{\kappa\epsilon H^2 M_p^2 R_{\mathrm{fs},s}}{m_s^2}}_{\text{Derivative of the scalar curvature}}\right)$$

THE DECOUPLING LIMIT

REVISITED…

Bending radius of the trajectory: $\kappa = \dfrac{\sqrt{2\epsilon}M_p}{\eta_\perp}$

$$S_3^{\mathrm{EFT}}[\zeta] = \int d\tau d^3x a^2 M_p^2 \frac{\epsilon}{\mathcal{H}}\left(\frac{1}{c_s^2}-1\right)\left(\zeta'(\partial_i\zeta)^2 + \frac{\boldsymbol{A}}{c_s^2}\zeta'^3\right)$$

$$\text{with } \boldsymbol{A} = \underbrace{-\frac{1}{2}(1+c_s^2)}_{\text{Previously known}} + \underbrace{\frac{2}{3}(1+c_s^2)\frac{\epsilon R_{\mathrm{fs}}H^2M_p^2}{m_s^2}}_{\text{Scalar curvature of the field space}} - \frac{1}{6}(1-c_s^2)\left(\underbrace{\frac{\kappa V_{;sss}}{m_s^2}}_{3^{\text{rd}}\text{ derivative of the potential}} + \underbrace{\frac{\kappa\epsilon H^2M_p^2R_{\mathrm{fs},s}}{m_s^2}}_{\text{Derivative of the scalar curvature}}\right)$$

Previously known

3rd derivative of the potential

Scalar curvature of the field space

Derivative of the scalar curvature

$$\boxed{f_{\mathrm{nl}}^{\mathrm{eq}} \simeq \left(\frac{1}{c_s^2}-1\right)\left(-\frac{85}{324}+\frac{15}{243}\boldsymbol{A}\right)}$$

RECAP OF THIS PART

Generic 2-field inflationary model with curved field space

$$S=\int\sqrt{-g}\left(\frac{R}{2}-\sum_{a,b}g^{\mu\nu}G_{ab}(\phi^c)\partial_\mu\phi^a\partial_\nu\phi^b-V(\phi^c)\right)$$

Expanding the action to 3rd order | Choice of comoving gauge

$$\mathcal{L}(\zeta,\mathcal{F})=\mathcal{L}^{(2)}(\zeta,\mathcal{F})+\mathcal{L}^{(3)}_{\text{not simplified}}(\zeta,\mathcal{F})$$

Integrations by parts
Uses of e.o.m.

$$\boldsymbol{\mathcal{L}(\zeta,\mathcal{F})=\mathcal{L}^{(2)}(\zeta,\mathcal{F})+\mathcal{L}^{(3)}_{\text{Maldacena}}(\zeta,\chi)+\mathcal{L}^{(3)}_{\text{new}}(\zeta,\mathcal{F})+\mathcal{D}}$$

RECAP OF THIS PART

Generic 2-field inflationary model with curved field space

$$S = \int \sqrt{-g}\left(\frac{R}{2} - \sum_{a,b} g^{\mu\nu} G_{ab}(\phi^c)\partial_\mu\phi^a\partial_\nu\phi^b - V(\phi^c)\right)$$

Expanding the action to 3$^{\text{rd}}$ order | Choice of comoving gauge

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Integrations by parts
Uses of e.o.m.

$$\mathcal{L}(\zeta,\mathcal{F}) = \mathcal{L}^{(2)}(\zeta,\mathcal{F}) + \mathcal{L}^{(3)}_{\text{Maldacena}}(\zeta,\chi) + \mathcal{L}^{(3)}_{\text{new}}(\zeta,\mathcal{F}) + \mathcal{D}$$

$$\mathcal{F}_{\text{heavy}} = \frac{2\dot{\sigma}\eta_\perp}{m_s^2}\dot{\zeta}$$

Single-field effective theory

$$S_{\text{EFT}}[\zeta] = S[\zeta,\mathcal{F}_{\text{heavy}}(\zeta)]$$

RECAP OF THIS PART

Generic 2-field inflationary model with curved field space

$$S = \int \sqrt{-g}\left(\frac{R}{2} - \sum_{a,b} g^{\mu\nu} G_{ab}(\phi^c)\partial_\mu\phi^a\partial_\nu\phi^b - V(\phi^c)\right)$$

Expanding the action to 3$^{\text{rd}}$ order

Choice of comoving gauge

$$\mathcal{L}(\zeta,\mathcal{F}) = \mathcal{L}^{(2)}(\zeta,\mathcal{F}) + \mathcal{L}^{(3)}_{\text{not simplified}}(\zeta,\mathcal{F})$$

Integrations by parts

Uses of e.o.m.

$$\boldsymbol{\mathcal{L}(\zeta,\mathcal{F}) = \mathcal{L}^{(2)}(\zeta,\mathcal{F}) + \mathcal{L}^{(3)}_{\text{Maldacena}}(\zeta,\chi) + \mathcal{L}^{(3)}_{\text{new}}(\zeta,\mathcal{F}) + \mathcal{D}}$$

$$\mathcal{F}_{\text{heavy}} = \frac{2\sigma\eta_\perp}{m_s^2}\dot{\zeta}$$

Single-field effective theory

$$S_{\text{EFT}}[\zeta] = S[\zeta, \mathcal{F}_{\text{heavy}}(\zeta)]$$

P(X)

$c_s \to 1$

Canonical single-field

$\epsilon, \eta, s \to 0$

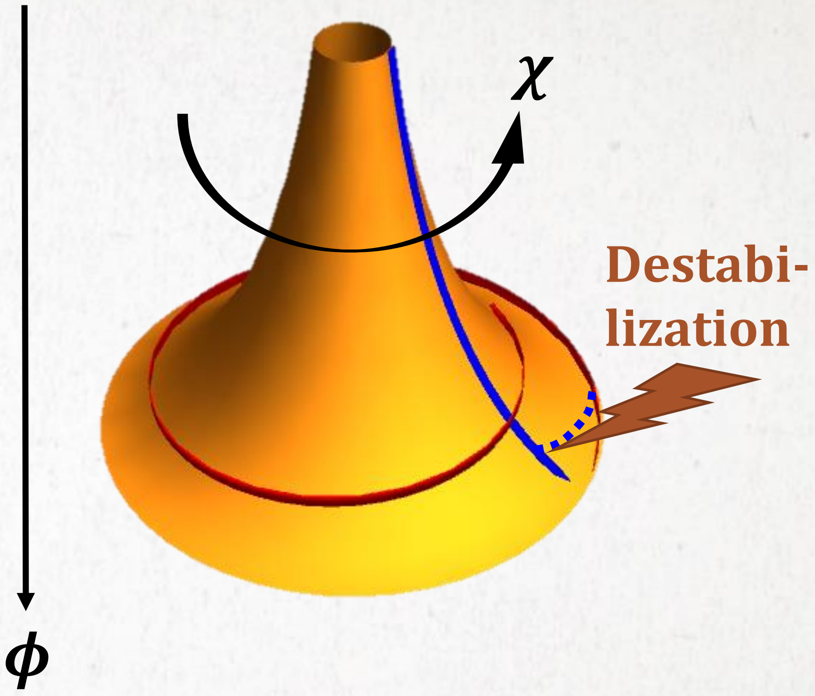
Decoupling limit in EFT of inflation: predictions for c_s^2, A and thus f_{nl}

$$f_{\text{nl}}^{\text{eq}} \simeq \left(\frac{1}{c_s^2} - 1\right)\left(-\frac{85}{324} + \frac{15}{243}A\right)$$

V. HYPERINFLATION

Embedding of the hyperbolic plane in 3D
Radial trajectory
Hyperinflation trajectory

MULTIFIELD INSTABILITY AND SINGLE-FIELD EFFECTIVE THEORY



[Fumagalli, Garcia-Saenz, Pinol, Renaux-Petel, Ronayne 2019]
Phys. Rev. Lett. 123, 201302

$\boxed{\text{Hyperbolic field space}}$

$$R_{\mathrm{fs}} = -\frac{4}{M^2}, \qquad M \ll M_p$$

HYPERINFLATION

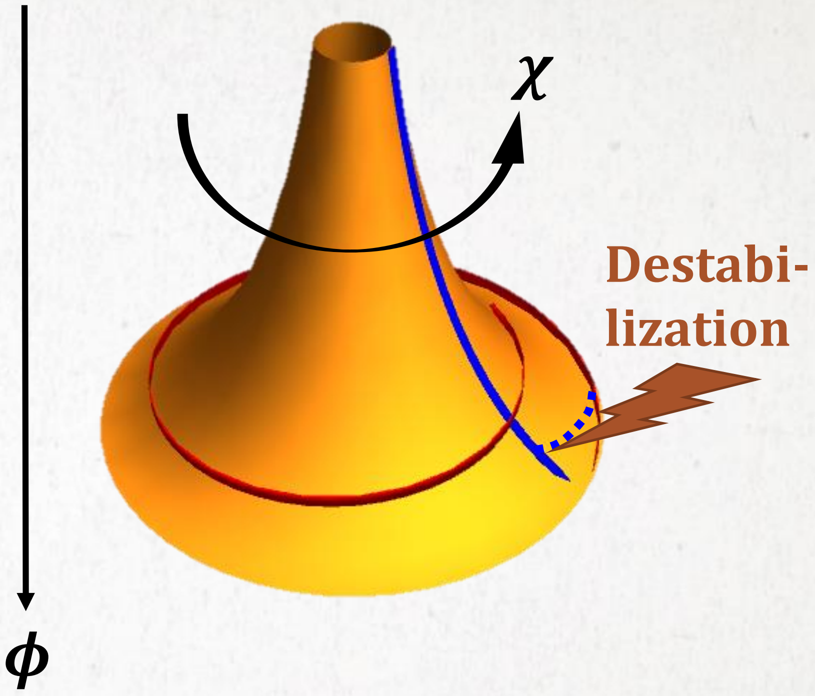
BACKGROUND ANALYSIS

Embedding of the hyperbolic plane in 3D
Radial trajectory
Hyperinflation trajectory

Angular momentum $J = a^3 M^2 \sinh^2\left(\frac{\phi}{M}\right) \dot{\chi}$

➢ $J = 0$ radial trajectory: geodesic, effectively single-field

Potentially unstable: $m_{\mathrm{s,eff}}^2 \simeq -\frac{V'}{9MH^2}\underbrace{\left(\frac{V'}{MH^2} - 9\right)}_{h^2}$



**With steep potentials,
geometrical destabilisation**

Hyperbolic field space

$$R_{\mathrm{fs}} = -\frac{4}{M^2}, \qquad M \ll M_p$$

HYPERINFLATION

BACKGROUND ANALYSIS

Angular momentum $J = a^3 M^2 \sinh^2\left(\frac{\phi}{M}\right)\dot{\chi}$

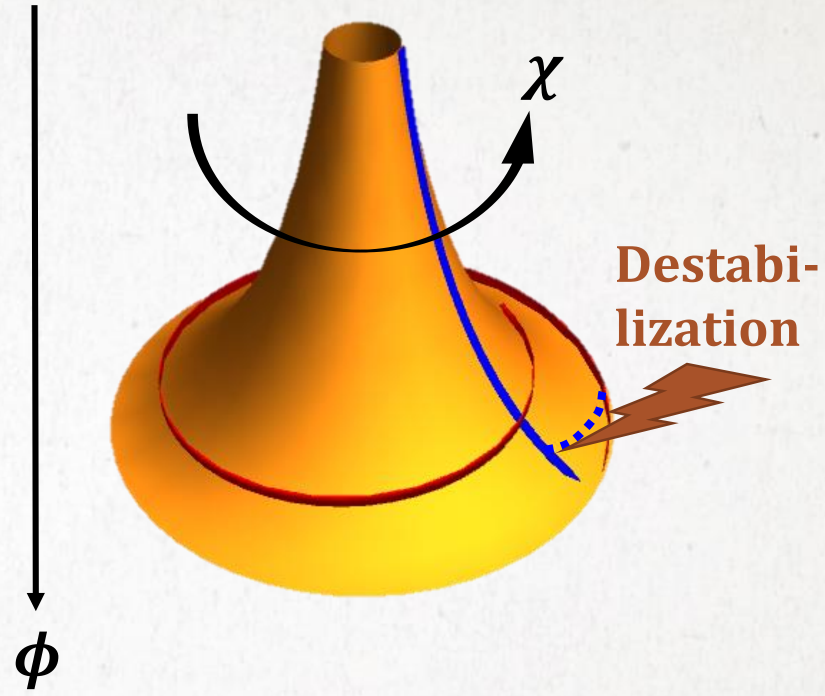
➢ $J = 0$ radial trajectory: geodesic, effectively single-field

Potentially unstable: $m_{\mathrm{s,eff}}^2 \simeq -\frac{V'}{9MH^2}\left(\frac{V'}{MH^2} - 9\right)$

➢ $J \neq 0$ spiraling (sidetracked) tajectory: hyperinflation

With steep potentials, the sidetracked phase is the attractor

Embedding of the hyperbolic plane in 3D
Radial trajectory
Hyperinflation trajectory



Hyperbolic field space

$$R_{\mathrm{fs}} = -\frac{4}{M^2}, \qquad M \ll M_p$$

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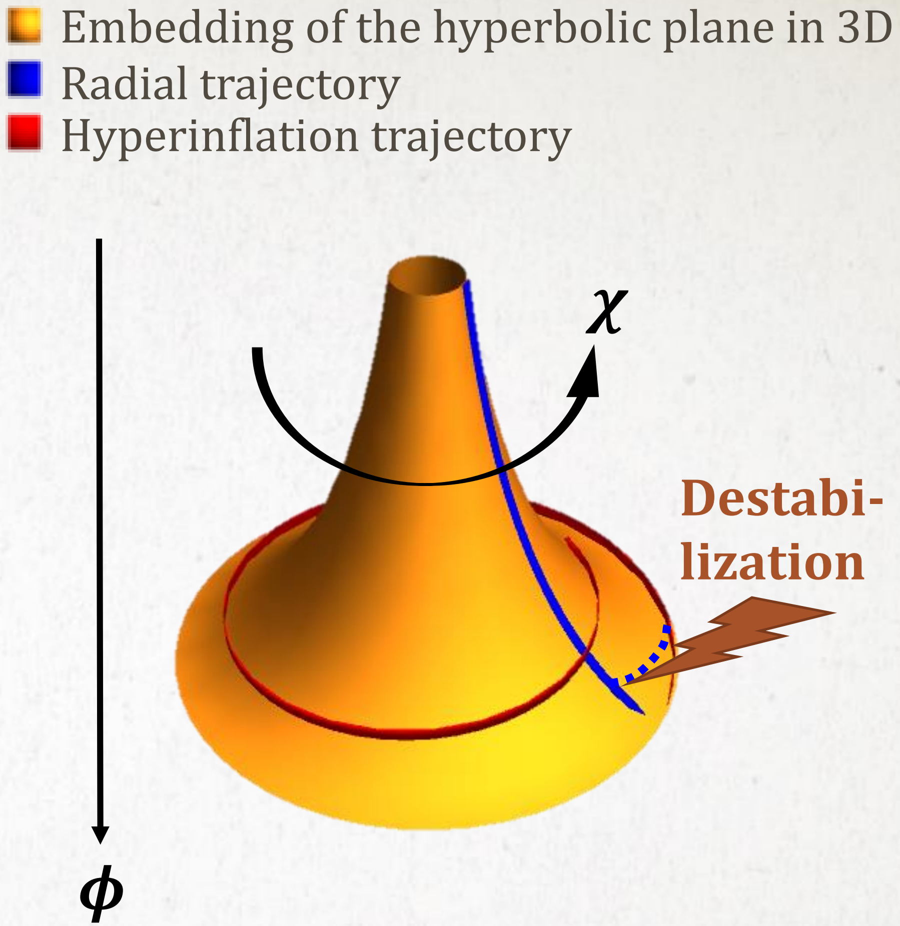
swampland conjectures, naturalness, eta problem

$\epsilon, \eta \ll 1 \Rightarrow \frac{3M^2}{M_p^2} < \frac{MV'}{V} \ll 1$ and $\frac{M|V''|}{V'} \ll 1$



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HYPERINFLATION

$$h^2 = \frac{V'}{MH^2} - 9 \gg 1$$

LINEAR PERTURBATIONS

We compute $\begin{cases} \eta_\perp^2 \approx h^2 \\ \epsilon R_{fs} M_p^2 \approx -h^2 \\ V_{;ss}/H^2 \ll 1 \end{cases} \Rightarrow \begin{cases} \dfrac{m_s^2}{H^2} \approx -2h^2 < 0 \\ \dfrac{m_{s,\mathrm{eff}}^2}{H^2} \approx 2h^2 > 0 \end{cases}$

- $\dfrac{m_s^2}{H^2} \approx -2h^2 < 0$ → Unstable, growing sub-Hubble perturbations
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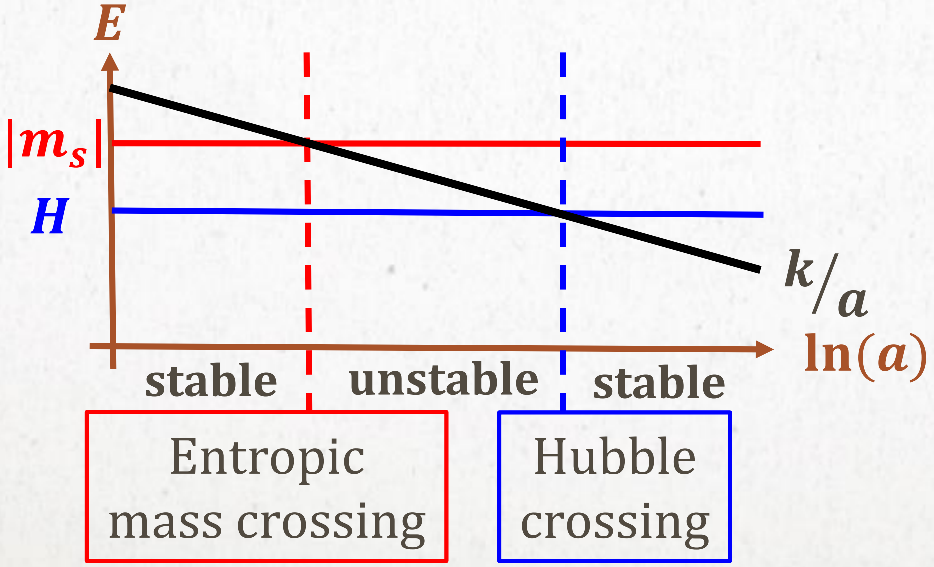
Unstable, growing sub-Hubble perturbations

Stable, decaying super-Hubble perturbations

This tachyonic instability is only transient for each k-mode

Remember in the e.o.m. for Q_s, the mass term is $\left(\frac{k^2}{a^2} + m_s^2\right) > 0$ *deep in the horizon*

$m_{s,\text{eff}}^2 > 0$ *on super-horizon scales*



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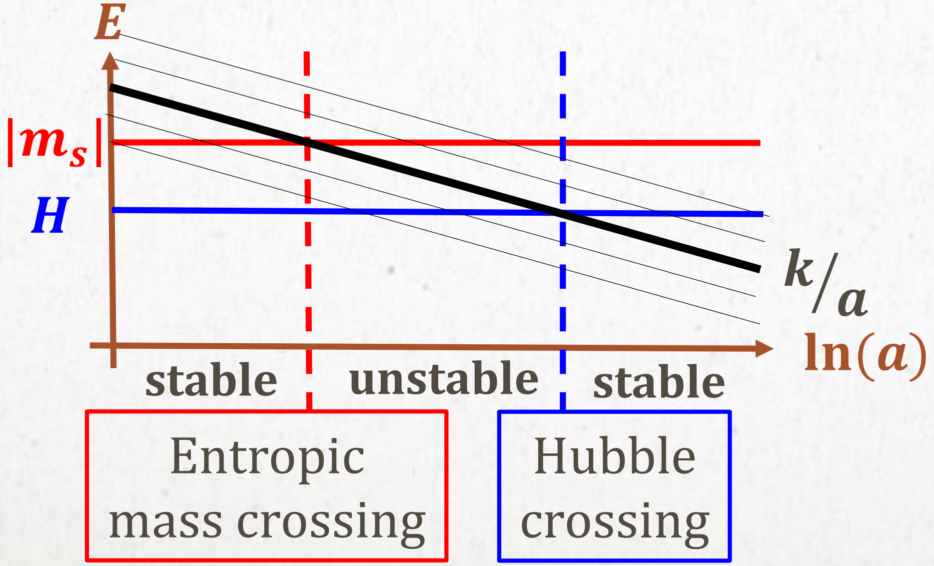

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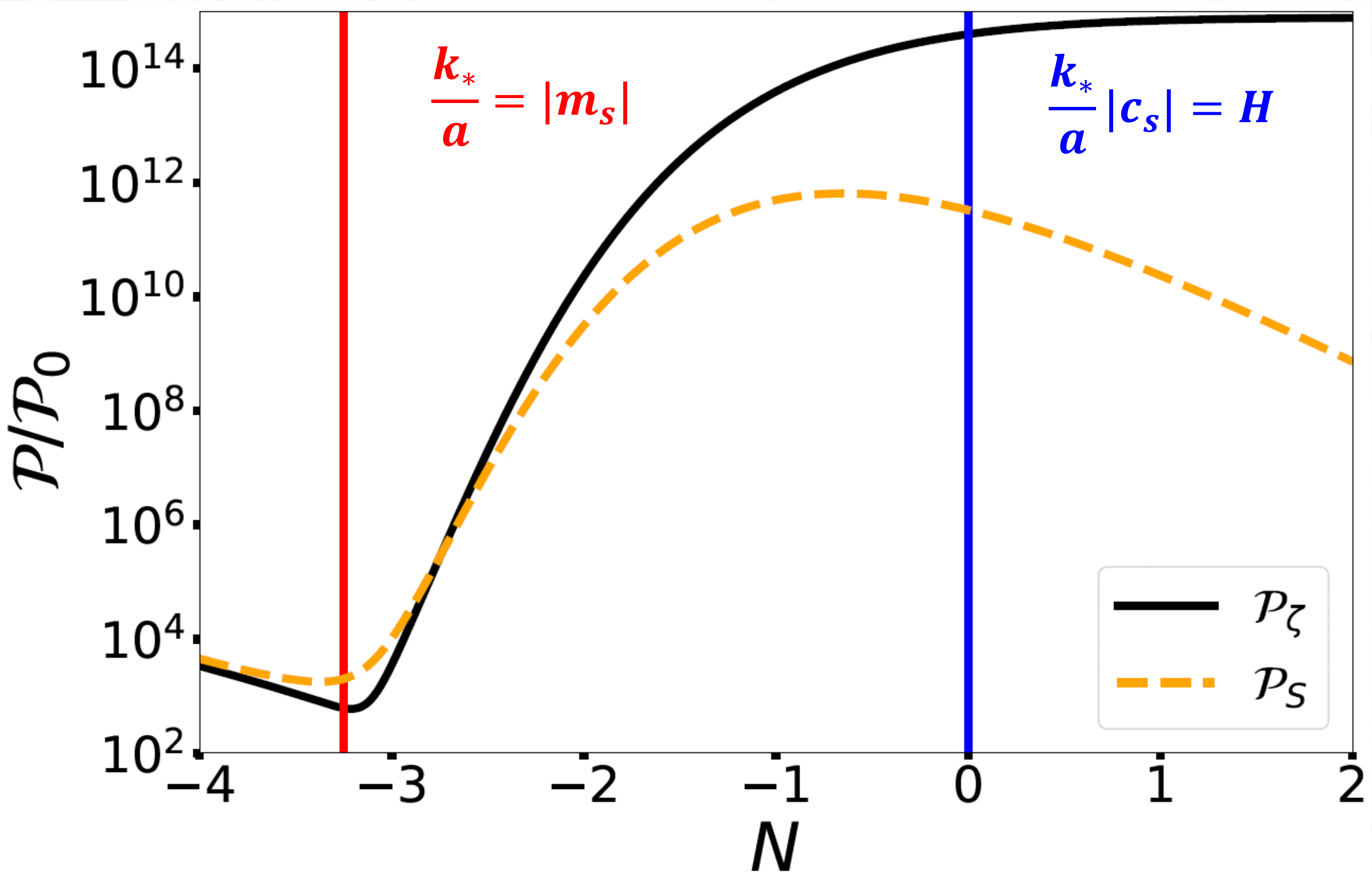


$r \ll 0.01$,
$n_s > 1$:

Excluded by CMB

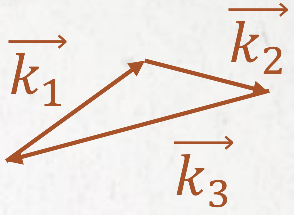
$$V = \frac{1}{2} m^2 \phi^2 \text{ with } m = M = 10^{-2} M_p$$

HYPERINFLATION

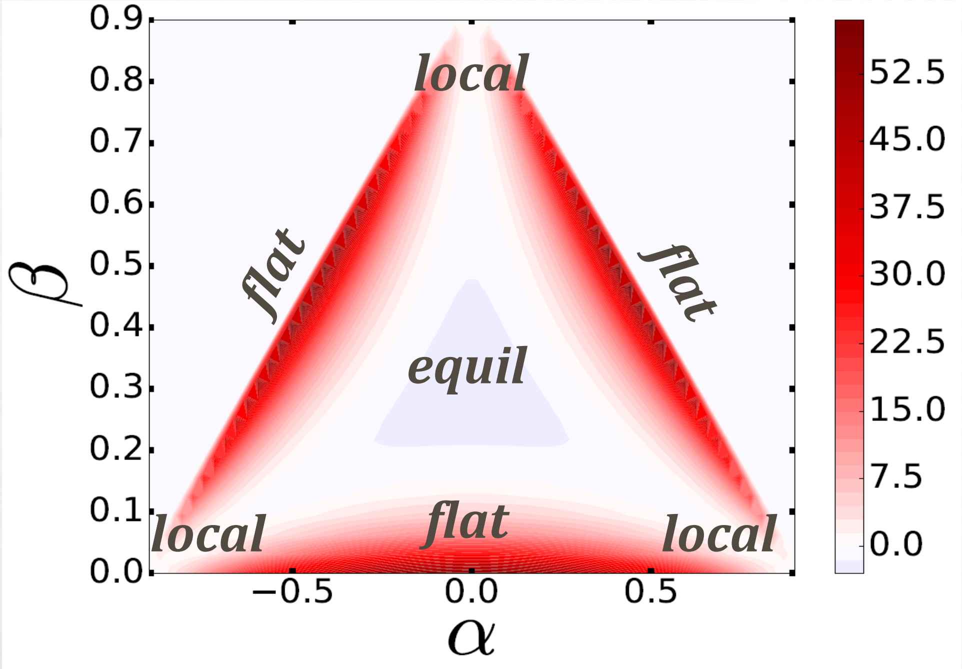
BISPECTRUM USING PyTransport 2.0

[D. Mulryne, J. Ronayne 2016]

3-point function: $< \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3} >= (2\pi)^3\ \delta^{(3)}(\overrightarrow{k_1}+\overrightarrow{k_2}+\overrightarrow{k_3}) \times B_\zeta(\overrightarrow{k_1},\overrightarrow{k_2},\overrightarrow{k_3})$



With $k_1 = \frac{3k_*}{4}(1+\alpha+\beta),\ k_2 = \frac{3k_*}{4}(1-\alpha+\beta),\ k_3 = \frac{3k_*}{2}(1-\beta)$



Characteristic large flattened bispectrum

f_{NL}^{equil}	f_{NL}^{flat}
-2.0	53.8

Single-clock inflation with Bunch-Davies initial states predicts equilateral non-Gaussianities

HYPERINFLATION

SINGLE-FIELD EFFECTIVE THEORY

- Equation of motion for $\mathcal{F}$:

$$\cancel{\ddot{\mathcal{F}}} + \cancel{3H\dot{\mathcal{F}}} + \left(m_s^2 + \cancel{\frac{k^2}{a^2}}\right)\mathcal{F} = 2\dot{\sigma}\eta_\perp\dot{\zeta}$$

$\mathcal{F}$ is 'heavy' and tachyonic

$$\boxed{\mathcal{F}_{\text{heavy}}^{\text{LO}} = -\frac{2\dot{\sigma}\eta_\perp}{|m_s^2|}\dot{\zeta}}$$

$$\boldsymbol{\omega^2, \omega H, \frac{k^2}{a^2} \ll |m_s^2|}$$

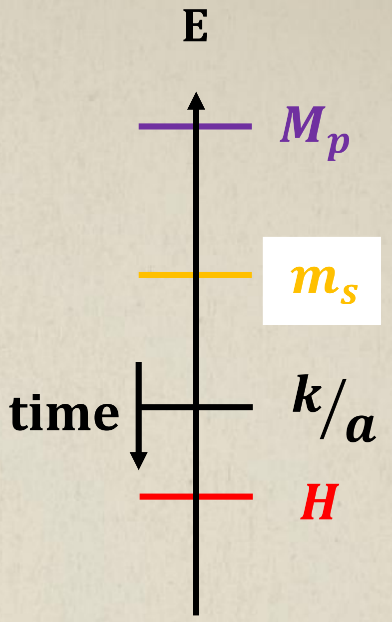
$\Downarrow$

Effective single-field action for the curvature perturbation

$$S_2^{\text{EFT}}[\zeta] = \int d\tau d^3x a^2 \epsilon \left(\frac{\zeta'^2}{c_s^2} - (\partial_i \zeta)^2\right)$$

$$\boxed{\frac{1}{c_s^2} = 1 - \frac{4H^2\eta_\perp^2}{|m_s^2|} \simeq -1}$$

Hierarchy of scales



Energy of the "experiment"

$$H \ll m_s$$

Integrate out the heavy perturbation

Like in the Fermi theory:
Integrate out the heavy W, Z bosons and consider contact interactions for fermions

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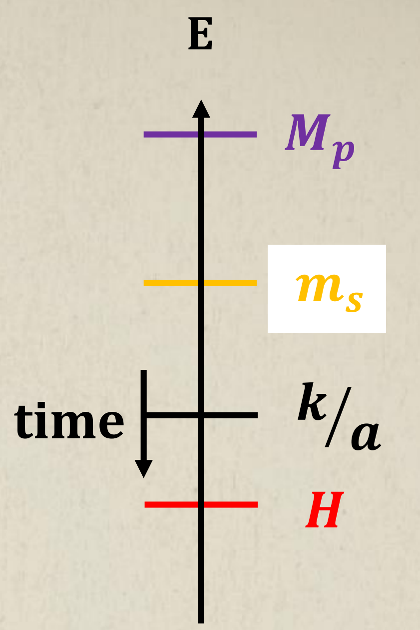
Hierarchy of scales



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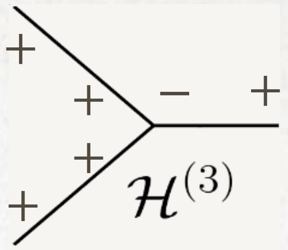
HYPERINFLATION

BISPECTRUM USING EFT

$\zeta_{\text{grown}} \sim \alpha e^{x}$



$\sim \alpha^4 e^{4x}$

Effective single-field cubic action

$$S_{(3)}^{EFT}[\zeta] = \int d\tau d^3x \;\, a\frac{\epsilon}{H} M_p^2 \left(\frac{1}{c_s^2} - 1\right)\left(\zeta'(\partial_i\zeta)^2 + \frac{\boldsymbol{A}}{c_s^2}\zeta'^3\right)$$

Need to contract one decaying mode

Growing modes are purely real

→ No exponential enhancement of $f_{NL} \sim \dfrac{<\zeta^3>}{<\zeta^2>^2}$

$$f_{\text{NL}}^{\text{flat}} = \frac{5}{576}\left(\frac{1}{|c_s^2|} + 1\right)(39(A-1) + 12x^2 + 4(A+1)x^3)$$

$f_{\text{NL}}^{\text{flat}} \sim O(50)$ $\Leftarrow$ Cubic polynomial in x with $x \sim 10$ in hyperinflation

If $A \sim 1$

[S. Garcia-Saenz, S. Renaux-Petel 2018]

HYPERINFLATION

ANALYTICAL PREDICTION VS NUMERICS

- x can be estimated from the power spectrum: $x \simeq 10$

- Our new formula enables to **compute**

$$\boldsymbol{A} = -\frac{1}{2}(1+c_s^2) + \frac{2}{3}(1+c_s^2)\frac{\epsilon R_{\mathrm{fs}} H^2 M_p^2}{m_s^2} - \frac{1}{6}(1-c_s^2)\left(\frac{\kappa V_{;sss}}{m_s^2} + \frac{\kappa \epsilon H^2 M_p^2 R_{\mathrm{fs},s}}{m_s^2}\right) \simeq -0.33$$

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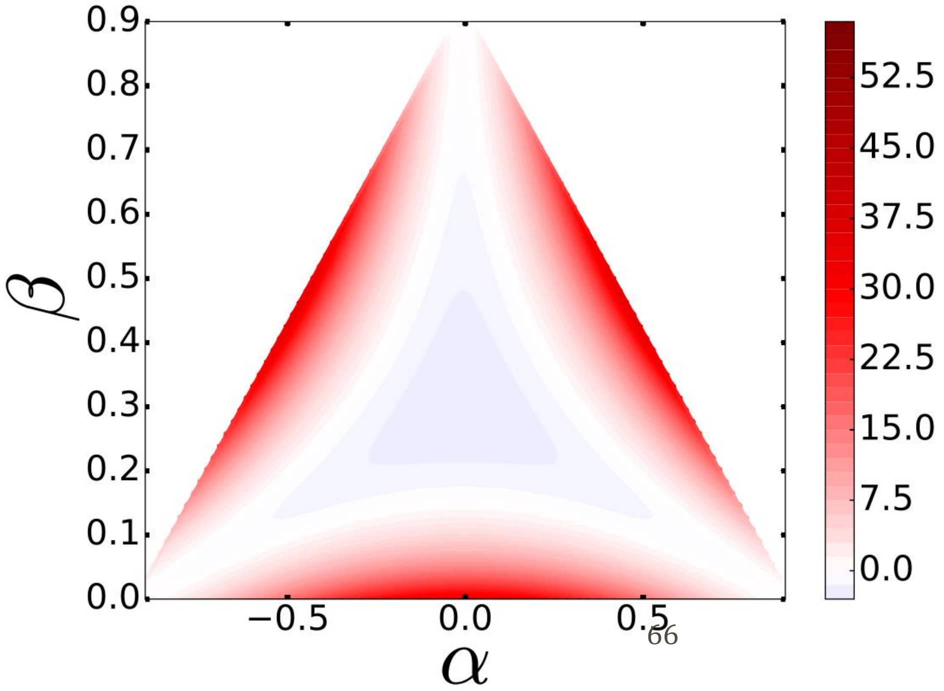

Vs. Numerics?

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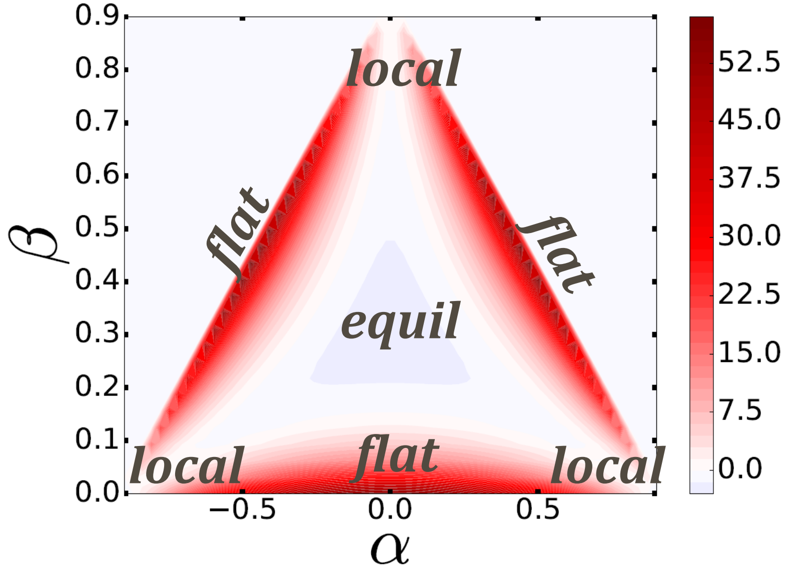

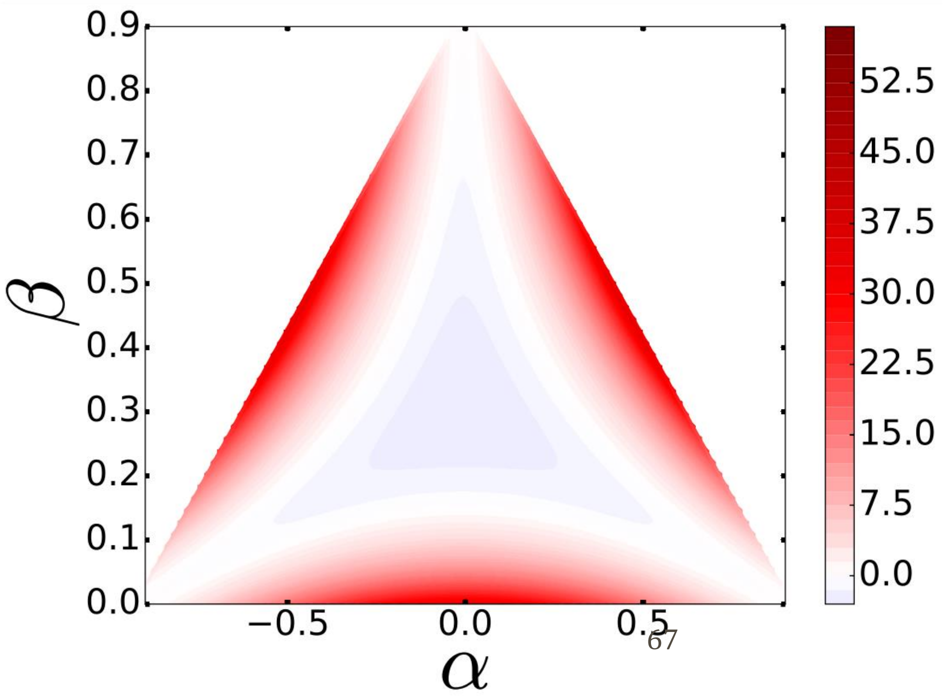

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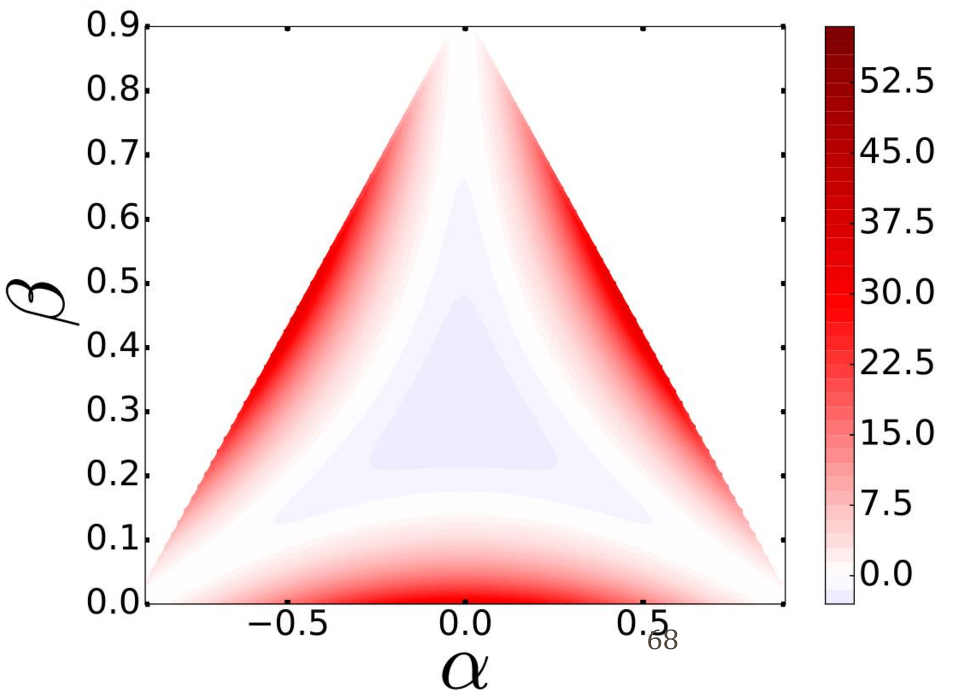

HYPERINFLATION

OTHER PHENOMENOLOGICAL ASPECTS

- Estimation of higher n-point functions with the EFT of inflation

We find enhanced flattened configurations: $$\frac{<\zeta^n>}{<\zeta^2>^{n-1}} \propto \left[\left(\frac{1}{|c_s|^2}+1\right)x^3\right]^{n-2}$$

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- If the large bending is only transient, then only scales in the instability band *at that time* are enhanced

If that happens after CMB scales have exited horizon, could produce PBHs without affecting CMB

[Fumagalli, Renaux-Petel, Ronayne, Witkowski 2020]

OTHER PROJECTS

MULTIFIELD STOCHASTIC INFLATION

- Heuristic derivation and understanding of ambiguities due to the nature of SDEs: « Inflationary stochastic anomalies »

[Pinol, Renaux-Petel, Tada 2019]
Class. Quantum Grav. **36** 07LT01

- Path integral derivation and resolution of the stochastic anomalies

[Pinol, Renaux-Petel, Tada 2020 soon?]

MULTIFIELD/MULTIFLUID REHEATING

- Generic single-field instability at small scales, can produce PBHs

[Martin, Papanikolaou, Pinol, Vennin 2020] *JCAP*

- Adiabatic and entropic perturbations in double inflation

[Martin, Pinol 2020 fall?]

CONCLUSION

- Slow-roll single-field inflation challenged: theory or model?

- Multifield inflation with curved field space is more generic and motivated by UV completions (string theory compactifications, supergravity…)

- Internal geometry plays a crucial role already at the background level: GEOmetrical DEStabilization of Inflation (ERC working group « GEODESI » led by S. Renaux-Petel at IAP)

- It crucially affects the physics of linear fluctuations and can shift (n_s, r) predictions by a lot

- Non-Gaussianities can be enhanced, thus providing exotic detectable signatures

- Step towards the general understanding of Non-Gaussianities of such models:

  Extending Maldacena's calculation

  Single-field effective theory: explicit geometry-dependent f_{nl}

- Hyperinflation: a full case study

THANKS FOR YOUR ATTENTION!

OF RELEVANT ENTROPIC MASS SCALES

- Equation of motion for $\mathcal{F}$:

$$\ddot{\mathcal{F}} + 3H\dot{\mathcal{F}} + \left(m_s^2 + \frac{k^2}{a^2}\right)\mathcal{F} = 2\dot{\sigma}\eta_\perp\dot{\zeta}$$

Dynamics dictated by:

- m_s^2
- $\eta_\perp$ and $\dot{\zeta}$

- Equation of motion for ζ on large scales:

$$\dot{\zeta} = -\frac{\dot{\sigma}\eta_\perp}{\epsilon M_p^2}\mathcal{F} + O\left(\frac{k^2}{a^2}\right)$$

- Effective equation of motion for $\mathcal{F}$ on large scales:

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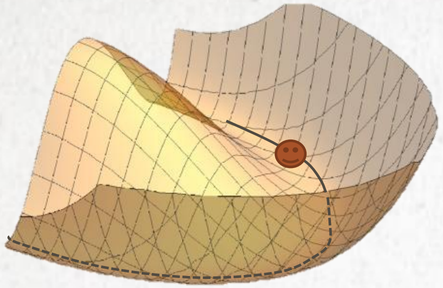
THE GELATON CHECK

The gelaton scenario



- 2 fields (ϕ, ψ), curved field-space
- ψ is very heavy and adiabatically follows the min of its effective potential
- The full field ψ can be integrated out, giving a single-field P(X) theory

Our procedure

- Keeping $\bar{\psi}$ at the level of the background
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- Get P(X)-like cubic Lagrangian

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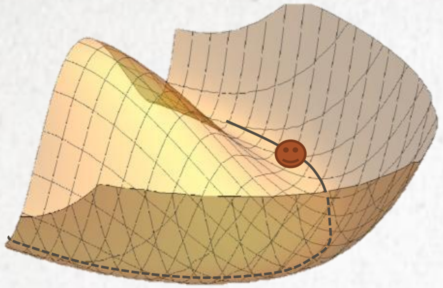

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Same P(X) theory!

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MAKING ASSUMPTIONS MORE PRECISE

$$\mathcal{F}^{\mathrm{LO}}_{\mathrm{heavy}} = \frac{2\dot{\sigma}\eta_\perp}{m_s^2}\dot{\zeta}$$

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For consistency, NLO (i=1) correction must be neglible compared to LO (i=0) in the expansion

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[D. Baumann, D. Green 2011] ω^2_{new}

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[D. Baumann, D. Green 2011]

[S. Céspedes, V. Atal, G. Palma 2012]

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- The EFT is useful only if it is well valid at sound horizon crossing: $\boxed{\frac{H^2}{m_s^2 c_s^2} \ll 1}$