Vainshtein screening for slowly rotating stars

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work to appear, with E. Babichev

Introduction

- Modified gravity: a way to test GR, may explain other problems linked to dark energy, interesting in its own right...
- Many ways to modify GR (break assumptions of Lovelock's theorem), a simple way is to add an extra scalar field mediating the gravitational force
- The "fifth force" changes the gravitational interaction, but gravity in the solar system is very well constrained !

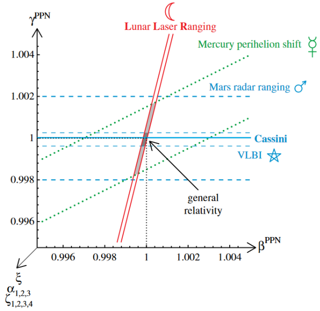

Figure 1: Solar-system constraints on the PPN parameters.

Esposito-Farèse, 2005

- How to screen the fifth force ?

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How to screen the fifth force ?

- Examples:
  - k-Mouflage gravity (**Babichev+, 2009**) similar to the Vainshtein mechanism in massive gravity, non-linearities quench the fifth force
  - Chameleon gravity, the mass of the scalar increases in regions of high density (**Khoury,Weltman, 2003**)
  - Symmetron fields, vev of the scalar (coupling to matter) depends on mass density (**Hinterbichler,Khoury, 2010**)
- Idea of Vainshtein mechanism: introduce derivative self-interactions of the scalar field.

$$S = \frac{M_P^2}{2} \int \mathrm{d}^4x \sqrt{-g}\, [(1+\alpha\phi)\, R + K_{\mathrm{NL}}[\phi]] + S_m[g_{\mu\nu}, \psi_m] \qquad (1)$$

- Deviations from GR are predicted far from the source, but it is recovered inside some radius r_V (Vainshtein radius) because of non-linearities

1. Vainshtein screening in spherical symmetry

1. Example: cubic galileon

Simple example in spherical symmetry: cubic Galileon

(Luty+, 2003)

$$S = M_P^2 \int \mathrm{d}^4x\sqrt{-g}\left[\left(\frac{1}{2} + \alpha\phi\right)R - \frac{\eta}{2}(\partial\phi)^2 - \frac{\beta}{2}(\partial\phi)^2\Box\phi\right] + S_m[g_{\mu\nu}, \psi_m]$$

$$T^{\mu\nu} = (\rho + P)\,u^\mu u^\nu + Pg^{\mu\nu}$$

- $\mathrm{d}s^2 = -e^{\nu(r)}\mathrm{d}t^2 + e^{\lambda(r)}\mathrm{d}r^2 + r^2\left[\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right]$
- Weak gravity $\{\lambda, r\lambda', \nu, r\nu'\} \ll 1$, and also assume $\{\phi, r\phi', r^2\phi''\} \ll 1$
- The metric equations with $P \ll \rho$ read:

$$\lambda + r\lambda' = \rho r^2 + 2\alpha r\left(2\phi' + r\phi''\right)$$

$$r\nu' - \lambda = -4\alpha r\phi'$$

- Combining these with the scalar equation yields, with $M = 4\pi\int\rho r^2\mathrm{d}r$:

$$2\alpha GM + \left(6\alpha^2 + \eta\right)r^2\phi' - 2\beta r\phi'^2 = 0$$

Simple example in spherical symmetry: cubic Galileon

- We keep the branch that decays at ∞

$$\phi' = \frac{2\alpha r r_S}{k_2 r_V^3}\left[1 - \sqrt{1 + \frac{2GMr_V^3}{r_S r^3}}\right]$$

where we defined $k_2 = \eta + 6\alpha^2$ and $r_V^3 = 8\alpha\beta r_S/k_2^2$:

- Linear regime ($r \gg r_V$):

$$\phi' = -\frac{2\alpha r_S}{k_2 r^2} \Rightarrow \lambda = \frac{2GM}{r}\left[1 + \mathcal{O}(1)\right]$$

- Non-linear regime outside the star ($R < r \ll r_V$):

$$\phi' \sim \frac{r_S}{\sqrt{r_V^3\ r}} \ll \{\lambda', \nu'\} \Rightarrow \lambda = \frac{2GM}{r}\left[1 + \mathcal{O}\left(\frac{r^{3/2}}{r_V^{3/2}}\right)\right]$$

1. Vainshtein screening in spherical symmetry

2. DHOST Ia theories

Degenerate Higher Order Scalar-Tensor (DHOST) theories

$$S = M_P^2 \int \mathrm{d}^4x\sqrt{-g}\left(f(\phi,X)R + K(\phi,X) - G_3(\phi,X)\Box\phi + \sum_{i=1}^{5} A_i(\phi,X)\mathcal{L}_i\right) + S_\mathrm{m}\left[g_{\mu\nu},\psi_\mathrm{m}\right]$$

$$\mathcal{L}_1 = \phi_{\mu\nu}\phi^{\mu\nu}, \quad \mathcal{L}_2 = (\Box\phi)^2, \quad \mathcal{L}_3 = \phi_{\mu\nu}\phi^\mu\phi^\nu\Box\phi,$$
$$\mathcal{L}_4 = \phi_\mu\phi^\nu\phi^{\mu\alpha}\phi_{\nu\alpha}, \quad \mathcal{L}_5 = (\phi_{\mu\nu}\phi^\mu\phi^\nu)^2$$

- Different classes of DHOST theories can be obtained (**Langlois, Noui; Crisostomi+, 2016**), but only one (subclass Ia) is viable for phenomenology. In this class 3 functions are constrained:

$$A_2 = -A_1$$
$$A_4 = \frac{8XA_1^3 + A_1^2(3f + 16Xf_X) - X^2fA_3^2 + A_3A_1(8X^2f_X - 6Xf) + 2f_XA_1(3f + 4Xf_X) + 2fA_3(Xf_X - f) + 3ff_X^2}{2(f + 2XA_1)^2}$$
$$A_5 = \frac{(f_X + A_1 + XA_3)(A_1^2 - 3XA_1A_3 + f_XA_1 - 2fA^3)}{2(f + 2XA_1)^2}$$

Screening in DHOST Ia theories

- The Vainshtein screening works well for Horndeski theories (**Babichev+, 2009; Kimura+, 2012; Kase+, 2015...**)
- It was shown to be broken inside the matter source for a time-dependent scalar in Beyond-Horndeski theories (**Kobayashi+, 2015; Dima+, 2018; Langlois+,2018**), where the Newtonian forces were shown to be of the form:

$$\frac{\nu'}{2} = \frac{\bar{G}M}{r^2} + \alpha_1 \bar{G} M''$$
$$\frac{\lambda}{2r} = \frac{\bar{G}M}{r^2} + \alpha_2 \bar{G} \frac{M'}{r}$$

- More recently, it was shown the the the mechanism can be broken outside the source also (**Hirano+; Crisostomi+, 2019**)
- What happens for more realistic axi-symmetric spacetimes ?

2. Slow rotation

2. Slow rotation

1. Hartle formalism

Hartle formalism

- Introduce a new function accounting for slow rotation (Hartle, 1967), and add a scalar field:

$$\mathrm{d}s^2 = -e^{\nu(t,r)}\mathrm{d}t^2 + e^{\lambda(t,r)}\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\,[\mathrm{d}\varphi - \varepsilon\omega(t,r)\mathrm{d}t]^2$$
$$\phi = qt + \phi(r)$$

- Assume matter is a perfect fluid with uniform angular velocity Ω:

$$T^{\mu\nu} = (\rho + P)\,u^\mu u^\nu + Pg^{\mu\nu}$$

$$u^\mu = \left(e^{-\nu/2}, 0, 0, \varepsilon\Omega e^{-\nu/2}\right)$$

- We now have a $(t\varphi)$ metric equation

$$\mathcal{E}^t{}_\varphi = \frac{1}{2M_P^2}T^t{}_\varphi$$

Frame-dragging equation

- The differential equation for ω is

$$\omega'' + K_1\omega' + K_2\,(P+\rho)\,(\omega-\Omega) = 0$$

- The coefficients are different from GR

$$K_1 = \frac{4}{r} - \frac{\lambda' + \nu'}{2} + \frac{\mathrm{d}}{\mathrm{d}r}\ln\left(f + 2XA_1\right),$$
$$K_2 = -\frac{e^{\lambda}}{f + 2XA_1},$$

- How does modifying gravity change the solution for ω ? We use the solutions for $\{\lambda, \nu, \phi\}$ (in spherical symmetry) to find the expressions of K_1 and K_2

2. Slow rotation

2. Relativistic stars in shift-symmetric theories

Vacuum equation in shift-symmetric theories

- In GR, frame-dragging function in vacuum is related to J, the angular momentum of the star (**Papapetrou, 1948**)

$$\omega'' + \frac{4}{r}\omega' = 0 \Rightarrow \omega = \frac{2GJ}{r^3}$$

- This was also shown to be the case in some shift-symmetric ($\phi \to \phi + \text{const.}$) quadratic GLPV theories

$$\mathcal{L} = K(X) + f(X)R + f_X\left[(\Box\phi)^2 - \phi_{\mu\nu}\phi^{\mu\nu}\right] + \frac{A_3(X)}{2}\varepsilon^{\mu\nu\alpha\sigma}\varepsilon^{\lambda\eta\kappa}{}_{\sigma}\phi_{\mu\lambda}\phi_{\nu\eta}\phi_{\alpha}\phi_{\kappa}$$

- $K = \alpha X$, $f = \kappa + \eta X$, $A_3 = 0$ (**Cisterna+, 2016**), coupling $\sim G_{\mu\nu}\phi^\mu\phi^\nu$
- $f = \frac{1}{2}, K = \alpha_0 + \alpha_1 X + \alpha_2 X^2, A_3 = \text{const.}$ (**Sakstein+, 2017**)

Example: quadratic Horndeski theory

- This can be generalized to all quadratic GLPV theories. As an example, take $A_3 = 0$ (Horndeski theory) and assume $f_{XX} \neq 0$

$$\mathcal{L} = K(X) + f(X)R + f_X\left[(\Box\phi)^2 - \phi_{\mu\nu}\phi^{\mu\nu}\right]$$

$$\phi'^2 = \frac{e^\lambda\left[2f_X\left(1 + r\nu' - e^\lambda\right) + r\left(2q^2 f_{XX}\nu' e^{-\nu} - rK_X e^{-\lambda}\right)\right]}{2f_{XX}\left(1 + r\nu'\right)}$$

$$e^\lambda = \frac{2\left(1 + r\nu'\right)\left(f_X^2 + ff_{XX}\right)}{2f_X^2 + r^2 f_X K_X + f_{XX}\left(2f + r^2K + r^2P/M_P^2\right)}$$

- We can get λ' in terms of ϕ'^2 and e^λ, and write K_1 in the form

$$K_1 = \frac{4}{r} + F(r, \nu, \nu', P)\left(P + \rho\right)$$

3. Weak-field approximation

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1. Solution for ω

Weak-field approximation

- We now work in the weak-field approximation

$$\{r^n \frac{\mathrm{d}^n\lambda}{\mathrm{d}r^n}, r^n \frac{\mathrm{d}^n\nu}{\mathrm{d}r^n}, r^n \frac{\mathrm{d}^n\phi}{\mathrm{d}r^n}\} \ll 1$$

- The corresponding condition for ω is

$$\omega \ll \Omega$$

- The $(t\varphi)$ equation becomes

$$\omega'' + \frac{4}{r}\left[1 + \frac{r\delta K_1}{4}\right]\omega' = \frac{K_2(r)\Omega}{M_P^2}\rho(r)$$

Weak-field approximation

- We set $\omega'(0) = 0$ and $\lim_{r\to\infty} \omega = 0$, and $\Xi_1(r) = e^{-\int \delta K_1 \mathrm{d}r}$

$$\omega(r) = \frac{\Omega}{M_P^2} \int_\infty^r \frac{\Xi_1(v)}{v^4} \left(\int_0^v \frac{K_2(u)\rho(u)}{\Xi_1(u)} u^4 \mathrm{d}u \right) \mathrm{d}v$$

- Assume the coefficients are

$$\begin{aligned} \Xi_1 &= 1 + \varepsilon \delta \Xi_1 \ , \\ K_2 &= \kappa_2 \left(1 + \varepsilon \delta K_2\right) \end{aligned}$$

- The leading term is

$$\omega(r) = \frac{\kappa_2 \Omega}{M_P^2} \int_\infty^r \frac{1}{v^4} \left(\int_0^v \rho(u) u^4 \mathrm{d}u \right) \mathrm{d}v + \mathcal{O}(\varepsilon)$$

- A priori $\kappa_2 \neq -2$ like in GR, but this constant can be absorbed in the definition of J as seen from an exterior observer (unless one knows ρ precisely)

Power-law corrections

- The leading term is unchanged, but what about leading corrections ? Assume power-law corrections to coefficients, $(r/r_i)^{s_i} \ll 1$

$$\frac{r\delta K_1}{4} = \left(\frac{r}{r_1}\right)^{s_1} H_{r\leq R} + \left(\frac{r}{r_2}\right)^{s_2} H_{R<r\leq r_V} + \left(\frac{r}{r_3}\right)^{s_3} H_{r>r_V} \ ,$$
$$\delta K_2 = \left(\frac{r}{r_0}\right)^{s_0}$$

- The leading corrections outside the source are

$$\omega = \frac{2\Omega G \tilde{J}}{r^3}\left[1 + \mathcal{O}\left(\frac{r}{r_2}\right)^{s_2} H_{R<r\ll r_V} + \mathcal{O}\left(\frac{r}{r_3}\right)^{s_3} H_{r\gg r_V}\right]$$

- Inside the source, assuming a constant density star

$$\omega(r) - \omega(0) = \frac{3\kappa_2 G J_0 r^2}{2R^5}\left[1 + \frac{10\varepsilon}{(s_0+5)(s_0+2)}\left(\frac{r}{r_0}\right)^{s_0} - \frac{40\varepsilon}{(s_1+5)(s_1+2)}\left(\frac{r}{r_1}\right)^{s_1}\right]$$

3. Weak-field approximation

2. Examples with $q \neq 0$

Time-dependent scalar

- We consider $\phi = qt + \phi(r)$ with $q \neq 0$, and the assumption

$$\phi'^2 \ll q^2$$

- Weak-field expression for coefficients

$$K_1 = \frac{4}{r} - \frac{\lambda' + \nu'}{2} + \frac{2(f_\phi + q^2 A_{1\phi})\phi' - q^2 \left(f_X + 2A_1 + q^2 A_{1X}\right)\left(\nu' + \frac{2\phi'\phi''}{q^2}\right)}{2(f + q^2 A_1)} ,$$
$$K_2 = -\frac{1}{f + q^2 A_1} + \mathcal{O}(\lambda, \frac{\phi'^2}{q^2})$$

- The corrections to $4/r$ are generally small, at least if $r\phi''/\phi' \sim \mathcal{O}(1)$, which means the leading term for ω is the same as GR. What about leading corrections to ω ?

Metric potentials in spherical symmetry

$$x = \frac{\phi'}{r}, \quad y = \frac{\nu'}{2r}, \quad z = \frac{\lambda}{2r^2}, \quad M(r) = 4\pi \int_0^r \rho(\bar{r})\bar{r}^2 \mathrm{d}\bar{r}, \quad \mathcal{A}(r) = \frac{GM(r)}{q^2 r^3}$$

- We define the Vainshtein radius as $\mathcal{A}(r_V) \sim 1$

$$r_V^3 \equiv \frac{r_S}{q^2}$$

- Assuming $\dot{z} \sim qz$, $\dot{y} \sim qy$ and dimensionless coeffs of $\mathcal{O}(1)$, we obtain for $qr \ll 1$

$$y = \alpha_1 \mathcal{A} + \beta_1 x + \gamma_1 x^2 + \delta_1 r x x' + \eta_1 \ ,$$
$$z = \alpha_2 \mathcal{A} + \beta_2 x + \gamma_2 x^2 + \delta_2 r x x' + \eta_2$$

- We use these expressions in the scalar and $(t\varphi)$ equations

Scalar equation and K_1

- We get a cubic equation for x (**Dima+, 2018; Langlois+,2018**)

$$C_3x^3 + C_2x^2 + \left(C_1 + \Gamma_1\mathcal{A} + \Gamma_2\frac{(r^3\mathcal{A})'}{r^2}\right)x + \Gamma_0\mathcal{A} + \eta_3 = 0$$

- The coefficient K_1 is

$$K_1 = \frac{4}{r}\left[1 + \alpha_0 r^2\mathcal{A} + \zeta_0\left(r^3\mathcal{A}\right)' + \beta_0 r^2 x + \kappa_0 r^3 x' + \gamma_0 r^2 x^2 + \delta_0 r^3 x x' + \sigma_0 r^4\left(x x'' + x'^2\right) + \eta_0 r^2\right]$$

- We find the solution for x, and deduce the correction to K_1. This will give us the leading corrections to ω in the weak-field approximation

Outside the Vainshtein radius ($\mathcal{A} \ll 1$) and $qr \ll 1$

- For $r_V \ll r \ll 1/q$, linear terms in x dominate. Assuming $C_1 \neq 0$, the scalar equation is

$$C_1 x + \Gamma_0 \mathcal{A} + \eta_3 = 0$$

- Two cases depending on η_3
  - If $\eta_3 = 0$ (sufficient condition $K = G_{3\phi} = 0$)

$$x = -\frac{\Gamma_0 \mathcal{A}}{C_1} \Rightarrow K_1 = \frac{4}{r}\left[1 + \mathcal{O}\left(\frac{r_S}{r}\right)\right]$$

  - Otherwise $\Gamma_0 \mathcal{A} \ll \eta_3$

$$x = \text{const.} \Rightarrow K_1 = \frac{4}{r}\left[1 + \mathcal{O}\left(q^2 r^2\right)\right]$$

- The corrections to ω are

$$\omega = -\frac{\kappa_2 G J}{r^3}\left[1 + \mathcal{O}\left(\frac{r_S}{r}, q^2 r^2\right)\right]$$

- The corrections are not suppressed by r_V (less effective screening)

Inside the Vainshtein radius ($\mathcal{A} \gg 1$), assuming $C_3\Gamma_1 < 0$

$$C_3 x^3 + C_2 x^2 + \left(\Gamma_1 \mathcal{A} + \Gamma_2 \frac{(r^3\mathcal{A})'}{r^2}\right) x + \Gamma_0 \mathcal{A} = 0$$

- We first consider the case $C_3 \neq 0$, and obtain (assume $\Gamma_2 < 0$)

$$x = \pm\sqrt{\frac{-\Gamma_1 \mathcal{A} - \Gamma_2 \frac{(r^3\mathcal{A})'}{r^2}}{C_3}}$$

- Inside the star, we have

$$K_1 = \frac{4}{r}\left[1 + \frac{\mathrm{d}}{\mathrm{d}r}\left(\iota_0 r^3 \mathcal{A} + \iota_1 r^4 \mathcal{A}' + \iota_2 r^5 \mathcal{A}''\right) + \mathcal{O}\left(q^2 r^2 \sqrt{\mathcal{A}}\right)\right]$$

meaning leading corrections to ω_{GR} won't generically be suppressed by r_V

- Outside the star

$$K_1 = \frac{4}{r}\left[1 + \mathcal{O}\left(\frac{r_S\sqrt{r}}{r_V^{3/2}}\right)\right] \Rightarrow \omega = \frac{2G\tilde{J}}{r^3}\left[1 + \mathcal{O}\left(\frac{r_S\sqrt{r}}{r_V^{3/2}}\right)\right]$$

so corrections are suppressed by r_V (analogous to spherical symmetry)

Example: Shift-symmetric quadratic GLPV theories

$$\mathcal{L} = K(X) + f(X)R + f_X\left[(\Box\phi)^2 - \phi_{\mu\nu}\phi^{\mu\nu}\right] + \frac{A_3(X)}{2}\varepsilon^{\mu\nu\alpha\sigma}\varepsilon^{\lambda\eta\kappa}{}_{\sigma}\phi_{\mu\lambda}\phi_{\nu\eta}\phi_{\alpha}\phi_{\kappa}$$

- We already that the GR expression for K_1 is recovered in vacuum for these theories. The metric potentials are (**Kobayashi+, 2015**)

$$y = \tilde{G}\left(\frac{M}{r^3} - \frac{q^4 A_3^2}{2[f(q^2 A_{3X} + 4A_3 + 2f_{XX}) + q^2 A_3 f_X + 2f_X^2]}\cdot\frac{M''}{r}\right),$$
$$z = \tilde{G}\left(\frac{M}{r^3} + \frac{q^2 A_3(q^4 A_{3X} + 2f_X + 5q^2 A_3 + 2q^2 f_{XX})}{2[f(q^2 A_{3X} + 4A_3 + 2f_{XX}) + q^2 A_3 f_X + 2f_X^2]}\cdot\frac{M'}{r^2}\right)$$

- We redefined Newton's constant as

$$\tilde{G} = \frac{G}{2f - 4q^2 f_X - 2q^4 f_{XX} - 5q^4 A_3 - q^6 A_{3X}}$$

Example: Shift-symmetric quadratic GLPV theories

- Inside the star, ω verifies

$$\omega'' + \frac{4}{r}\left[1 - \frac{GM'}{4(2f - 2q^2 f_X - q^4 A_3)}\right]\omega' - \frac{4GM'}{r^2(2f - 2q^2 f_X - q^4 A_3)}(\omega - \Omega) = 0$$

- One can redefine Newton's constant to get the same equation as GR

$$G^* = \frac{G}{2f - 2q^2 f_X - q^4 A_3} \neq \tilde{G}$$

- The 2 redefinitions do not coincide, even when $A_3 = 0$. In this case the screening works in spherical symmetry but is less effective for ω (corrections not suppressed by r_V)
- Even if the Vainshtein mechanism works in spherical symmetry, corrections to ω_{GR} are not necessarily suppressed by powers of r_V

Inside the Vainshtein radius ($\mathcal{A} \gg 1$), $C_3 = \Gamma_1 = 0$ and $C_2 \neq 0$

$$C_2 x^2 + \left(C_1 + \Gamma_2 \frac{(r^3 \mathcal{A})'}{r^2} \right) x + \Gamma_0 \mathcal{A} + \eta_3 = 0$$

$$\boxed{f A_{1X} + A_1 f_X - f A_3 = 0}$$

- The scalar field verifies

$$\boxed{x = -\frac{r^2 C_1 + \Gamma_2 (r^3 \mathcal{A})' \pm \sqrt{[r^2 C_1 + \Gamma_2 (r^3 \mathcal{A})']^2 - 4 r^4 \mathcal{A} C_2 \Gamma_0}}{2 r^2 C_2}}$$

- In the region $R < r \ll r_V$, this simplifies to (assuming $\Gamma_0 C_2 < 0$)

$$\boxed{x_{\mathrm{out}} = \pm \sqrt{\frac{-\mathcal{A} \Gamma_0}{C_2}}}$$

and we get

$$\boxed{K_1 = \frac{4}{r} \left[1 + \xi \frac{r_S}{r} + \mathcal{O}\left(\frac{r_S \sqrt{r}}{r_V^{3/2}} \right) \right]}$$

Example: Survivor theory

- In the context of the EFT of dark energy, $c_T = 1$ and no decay of graviton into dark energy kill most of DHOST Ia (**Ezquiaga+;Creminelli+, 2017; Creminelli+,2018**)

$$\mathcal{L} = f(\phi, X)R + K(\phi, X) - G_3(\phi, X)\Box\phi + \frac{3f_X^2}{2f}\phi_\mu\phi^\nu\phi^{\mu\alpha}\phi_{\nu\alpha}$$

- For these theories, we have (assume $f_X \neq 0$)

$$\xi = \frac{f_X[2f_\phi(f + 5q^2 f_X) - 3fq^2 G_{3X} - 2fq^2 f_{X\phi}]}{8(q^2 f_X - 2f)^2(fG_{3X} - 3f_X f_\phi)}$$

- The condition for the Vainshtein mechanism to work in spherical symmetry (**Hirano+;Crisostomi+, 2019**) implies that the screening is also effective for ω

Example: Survivor theory

- Inside the star, assume $\Gamma_2 \neq 0$ and choose the branch (**Hirano+, 2019**)

$$x \simeq \frac{\Gamma_0}{\Gamma_2} \frac{r^2 \mathcal{A}}{(r^3 \mathcal{A})'} \sim \mathcal{O}(1)$$

- In this case we have

$$\omega'' + \frac{4}{r}\left[1 - \frac{q^2 f_X}{(2f - q^2 f_X)^2} \frac{GM}{r} - \frac{(f - q^2 f_X) GM'}{2(2f - q^2 f_X)^2}\right] \omega' - \frac{2GM'}{fr^2}(\omega - \Omega) = 0$$

- Deviations from GR will appear at first order in the weak-field expansion (unless $f = 1/2$, example cubic galileon)

Example of "large" corrections for ω outside the source

$$K_1 = \frac{4}{r}\left[1 + \xi\frac{r_S}{r} + \mathcal{O}\left(\frac{r_S\sqrt{r}}{r_V^{3/2}}\right)\right]$$

- In the previous example, Vainshtein in spherical symmetry $\Rightarrow r_V$ suppressed corrections for ω outside the star ($\xi = 0$). Can we have $\xi \neq 0$?

$$\xi = \left[f + q^2 A_1\right]\left[f\left(q^2 A_{1X} + 2f_X\right) + A_1\left(2f + q^2 f_X\right)\right]\xi_0$$

- A necessary condition for non-rotating Vainshtein is $y \simeq z$. In our case

$$r^2(y - z) = -4\xi_0 f_X\left(f + q^2 A_1\right)^2 \cdot \frac{r_S}{r} + \mathcal{O}\left(\frac{r_S\sqrt{r}}{r_V^{3/2}}\right)$$

- If $f_X \neq 0$, then $\xi_0 = 0 \Rightarrow \xi = 0$
- Look for theories with $f_X = 0$, but $\xi_0 \neq 0$, for simplicity we set $f = 1/2$

Example of "large" corrections for ω outside the source

$$fA_{1X} + A_1 f_X - fA_3 = 0$$

- $f = 1/2 \Rightarrow A_{1X} = A_3$

$$\xi = -\frac{\left(1+2q^2A_1\right)\left(2q^2A_1+q^4A_{1X}\right)\left[2A_{1\phi}\left(4+6q^2A_1-q^4A_{1X}\right)+\left(1+2q^2A_1\right)\left(3G_{3X}+2q^2A_{1\phi X}\right)\right]}{2\left(2+6q^2A_1+q^4A_{1X}\right)^2\left[3A_{1\phi}\left(2+2q^2A_1-q^4A_{1X}\right)+2\left(1+2q^2A_1\right)\left(G_{3X}+q^2A_{1\phi X}\right)\right]} \neq 0$$

- On the other hand

$$r^2 y = \iota_3 \frac{r_S}{r} + \mathcal{O}\left(\frac{r_S\sqrt{r}}{r_V^{3/2}}\right),$$
$$r^2 z = \iota_3 \frac{r_S}{r} + \mathcal{O}\left(\frac{r_S\sqrt{r}}{r_V^{3/2}}\right)$$

- This means that the corrections to ω are larger than the corrections to the metric potentials (which are suppressed by r_V as usual)

3. Weak-field approximation

3. Examples with $q = 0$

Almost shift-symmetric theories with q=0

- We consider theories of the form

$$\mathcal{L} = [f(X) + \alpha\phi]\, R + K(X) - G_3(X)\Box\phi + \sum_{i=1}^{5} A_i(X)\mathcal{L}_i$$

- In this way, we evade the no-hair theorem of (**Lehébel+, 2017**). We introduce the scalar current which is conserved when $\alpha = 0$

$$\nabla_\mu J^\mu = -\alpha R$$

- In the weak-field regime, the scalar equation reads

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left[r^2 J^r + \alpha\left(2r\lambda - r^2\nu'\right)\right] = 0$$

- We will set the integration constant to 0 for regularity of J^2 at the origin

Example 1: K-essence

- Consider the following theory with an integer $p \geq 2$

$$\mathcal{L} = \left[\frac{1}{2} + \alpha\phi\right] R + \mu X^p$$

- We neglect non-linear corrections to the (tt) and (rr) eqs

$$\lambda = \frac{2GM(r)}{r} + 2\alpha r\phi' \, ,$$
$$\nu' = \frac{2GM(r)}{r^2} - 2\alpha\phi'$$

- The scalar verifies

$$\frac{\alpha GM(r)}{r^2} + 3\alpha^2\phi' - p\left(-\frac{1}{2}\right)^p \mu\phi'^{2p-1} = 0$$

Example 1: K-essence

- For large radii, the linear term dominates

$$\phi' = -\frac{r_S}{6\alpha r^2} \Rightarrow \omega = \frac{2GJ}{r^3}\left[1 + \mathcal{O}\left(\frac{r_S}{r}\right)\right]$$

- Define r_V by equating linear and non-linear terms

$$r_V^2 \sim \frac{r_S}{6}\left(\frac{|\mu| p}{3 \cdot 2^p \alpha^{2p}}\right)^{\frac{1}{2p-2}}$$

- For $r \ll r_V$, the non-linear term dominates

$$\phi' = \mathrm{sgn}\left[(-1)^p \alpha\mu\right]\left(\frac{2^p|\alpha| G M(r)}{p|\mu| r^2}\right)^{\frac{1}{2p-1}}$$

Example 1: K-essence

- Compare the fifth force to the GR potentials

$$\left|\frac{\phi'}{\{\lambda'_{\mathrm{GR}}, \nu'_{\mathrm{GR}}\}}\right| \sim \frac{1}{6\alpha}\left(\frac{r_S r^2}{2GM(r) r_V^2}\right)^{\frac{2p-2}{2p-1}}$$

- Screening in spherical symmetry breaks down close to the center of the source, for $r \leq R^3/r_V^2 \equiv r_*$
- The corrections to K_1 and K_2 (assuming $\rho = \mathrm{const.}$) are

$$K_1^{\mathrm{out}} = \frac{4}{r}\left[1 + \mathcal{O}\left(\frac{r_S}{r_V}\left(\frac{r}{r_V}\right)^{\frac{2p-3}{2p-1}}\right)\right],$$

$$K_1^{\mathrm{in}} = \frac{4}{r}\left[1 + \frac{r r_S}{8 r_V^2}\left(\frac{3-4p}{3(1-2p)}\left(\frac{r}{r_*}\right)^{\frac{1}{2p-1}} - \frac{3r}{r_*}\right)\right],$$

$$K_2 = -2\left[1 + \frac{r_S r^2}{R^3}\left(1 + \mathcal{O}\left(\frac{r_*}{r}\right)^{\frac{2p-2}{2p-1}}\right)\right]$$

- Leading corrections also change for ω when $r \leq r_*$, but $r_* \sim 10$ m so not physically relevant

Example 2: Cubic galileon

$$\mathcal{L} = \left[\frac{1}{2} + \alpha\phi\right] R + \eta X - \beta X \Box \phi$$

- The linear regime is the same as for K-essence. We can define r_V, and in the region $r \ll r_V$ we obtain for $\rho = \text{const}$.

$$K_1^{\text{out}} = \frac{4}{r}\left[1 + \mathcal{O}\left(\frac{r_S\sqrt{r}}{r_V^{3/2}}\right)\right] ,$$

$$K_1^{\text{in}} = \frac{4}{r}\left[1 - \frac{3 r_S r^2}{8R^3}\left(1 + \mathcal{O}\left(\frac{R^{3/2}}{r_V^{3/2}}\right)\right)\right] ,$$

$$K_2 = -2\left[1 + \frac{r_S r^2}{R^3}\left(1 + \mathcal{O}\left(\frac{R^{3/2}}{r_V^{3/2}}\right)\right)\right]$$

and the corrections to ω_{GR} are also suppressed by powers of r_V

Example 3: Quadratic sector of Horndeski theory

$$\mathcal{L} = [f(X) + \alpha\phi]\, R + f_X \left[(\Box\phi)^2 - \phi_{\mu\nu}\phi^{\mu\nu}\right]$$

- We consider theories of the form

$$f(X) = \frac{1}{2} + \kappa X^p$$

  with an integer $p \geq 1$
- For $p > 2$, case similar to k-essence, screening breaks very close to the center of the star
- For $p = 1, 2$, similar to the cubic galileon, the screening is effective everywhere for $r \ll r_V$.

4. Conclusion

Conclusion

- We have shown that in the weak-field approximation, the Vainshtein mechanism can be extended to axi-symmetric spacetimes outside the star (and also inside when $\kappa_2 = -2$)
- In many cases, if the screening operates in spherical symmetry, then the corrections to ω_{GR} are also suppressed by powers of r_V, but there are counter-examples
- A notable difference to the spherically symmetric case happens in the region $r \gg r_V$. While the metric potentials receive leading order corrections, the frame-dragging function still takes the GR form to leading order. However, the screening is less effective, because deviations from GR are not suppressed by powers of r_V like in the region $r \ll r_V$
- We have shown in a particular class of shift-symmetric theories that the vacuum equation for the frame-dragging function is exactly the same as in GR (even for relativistic stars). This means that the screening for ω is perfect in these cases.

Thank you !

Example 3: Quadratic sector of Horndeski theory

$$\mathcal{L} = [f(X) + \alpha\phi]\, R + f_X \left[(\Box\phi)^2 - \phi_{\mu\nu}\phi^{\mu\nu}\right]$$

- The scalar equation is

$$\alpha G M(r) + 3\alpha^2 r^2 \phi' + \phi'^3 f_{XX} = 0$$

- Assume $f(X) \neq \sqrt{X}$ (otherwise the non-linear term becomes constant). We will consider theories of the form

$$f(X) = \frac{1}{2} + \kappa X^p$$

with an integer $p \geq 1$

Example 3: Quadratic sector of Horndeski theory $p > 1$

- We first assume $p \geq 2$, so that $f_{XX} \neq 0$. In this case

$$K_1^{\text{out}} = \frac{4}{r}\left[1 + \mathcal{O}\left(\frac{r_S r}{r_V^2}\right)\right] ,$$

$$K_1^{\text{in}} = \frac{4}{r}\left[1 - \frac{3 r_S r^2}{8R^3}\left(1 + \frac{R^2}{r_V^2} \cdot \mathcal{O}\left(\frac{R}{r}\right)^{\frac{2p-4}{2p-1}}\right)\right] ,$$

$$K_2 = -2\left[1 + \frac{r_S r^2}{R^3}\left(1 + \frac{R^2}{r_V^2} \cdot \mathcal{O}\left(\frac{R}{r}\right)^{\frac{2p-4}{2p-1}}\right)\right]$$

- For $p > 2$, the screening in spherical symmetry stops working close to the center of the star (like in K-essence). The leading corrections to ω_{GR} are also modified in this region. However, once again, the size of this region is not physically relevant.

Example 3: Quadratic sector of Horndeski theory $p = 1$

- For p=1, the leading term in the scalar current disappears, since $f_{XX} = 0$

$$\alpha GM(r) + 3\alpha^2 r^2 \phi' + \cancel{\phi'^3 f_{XX}} = 0$$

- One must keep next-to leading terms in both the metric and scalar equations

$$\lambda = \frac{2GM(r)}{r} + 2\alpha r \phi' + 2\kappa \phi'^2 \,,$$
$$\nu' = \frac{2GM(r)}{r^2} - 2\alpha \phi'$$

$$\alpha GM(r) + 3\alpha^2 r^2 \phi' \left[1 + 2\frac{\kappa\phi'}{\alpha r} + \frac{2}{3}\left(\frac{\kappa\phi'}{\alpha r}\right)^2\right] = 0$$

Example 3: Quadratic sector of Horndeski theory $p = 1$

- For the Vainshtein screening to be successful in spherical symmetry, one must assume that the cubic term dominates

$$\phi' \simeq -\frac{r_S}{\alpha r_V^2}\left(\frac{GM(r)}{2r_S}\right)^{1/3}$$

- In this case, we have for $\rho = \text{const.}$

$$K_1^{\text{out}} = \frac{4}{r}\left[1 + \mathcal{O}\left(\frac{r_S r}{r_V^2}\right)\right] ,$$

$$K_1^{\text{in}} = \frac{4}{r}\left[1 - \frac{3r_S r^2}{8R^3}\left(1 + \mathcal{O}\left(\frac{R^2}{r_V^2}\right) + \mathcal{O}\left(\frac{r_S r^2}{R^2 r_V}\right)\right)\right] ,$$

$$K_2 = -2\left[1 + \frac{r_S r^2}{R^3}\left(1 + \mathcal{O}\left(\frac{R}{r_V}\right)\right)\right]$$

- The corrections to ω_{GR} are screened by powers of r_V