Gravitational radiation from a binary black hole coalescence in Einstein-scalar-Gauss-Bonnet gravity

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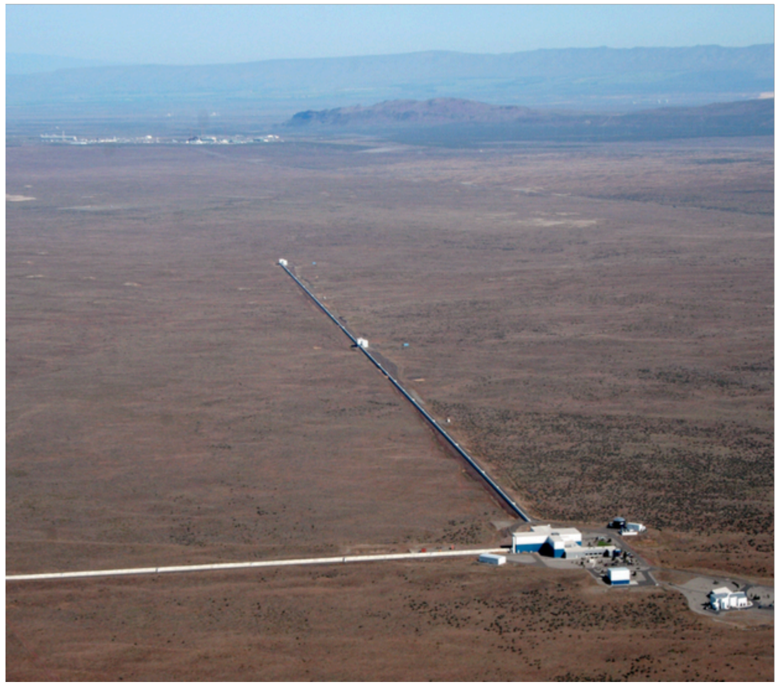
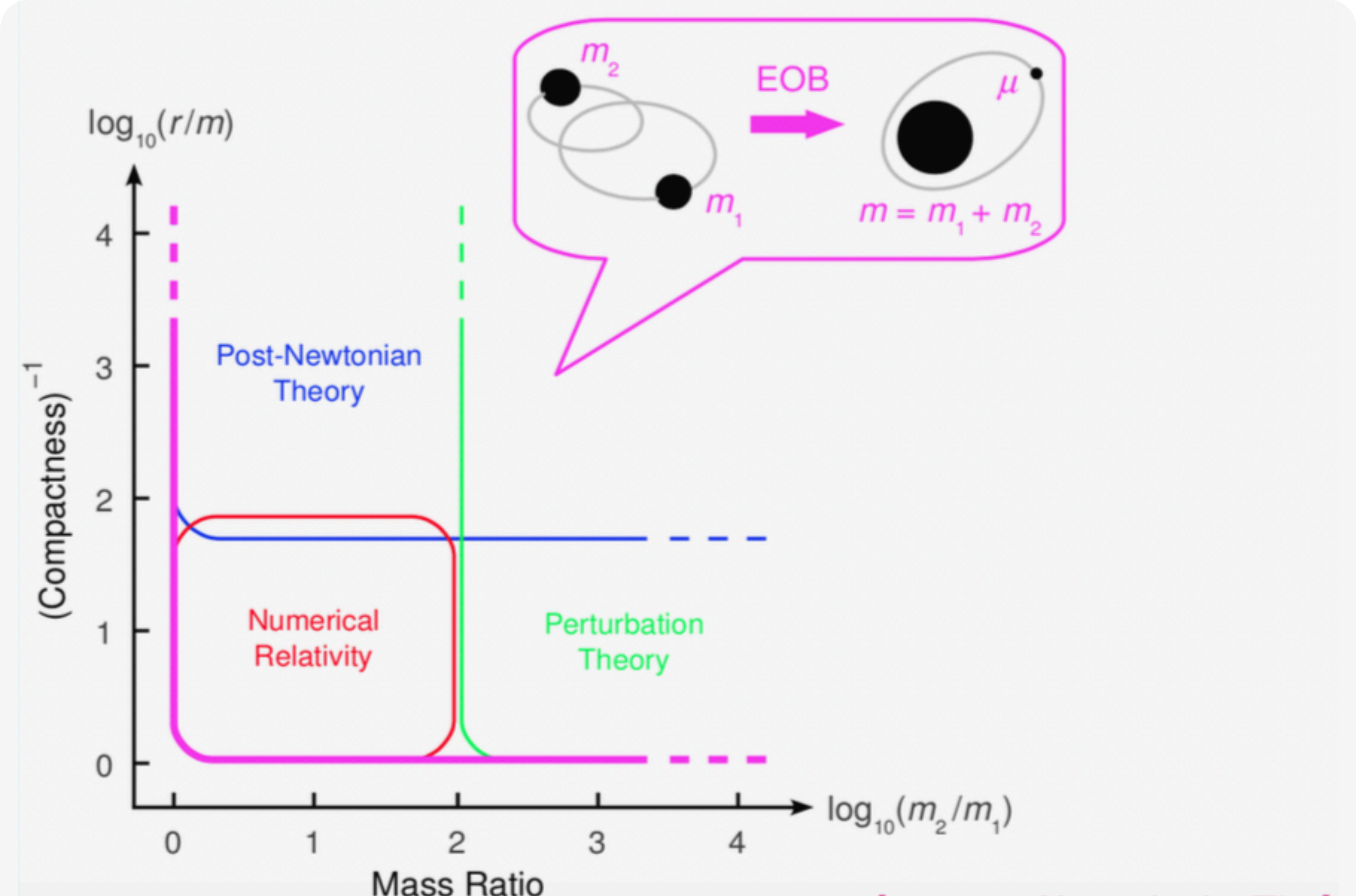
The era of gravitational wave astronomy

- GW150914: first observation of a BBH coalescence by LIGO-Virgo
- GW170817: first BNS with EM counterparts (multimessenger astronomy)
- Since April 2019: third observation run (O3) ongoing...



Opportunity of **new tests of general relativity and modified gravities**, in the strong-field regime of a compact binary merger.

"Knowing the chirp to hear it"...



[courtesy Alexandre Le Tiec]

In general relativity: PN theory, self-force calculations, EOB framework, numerical relativity...

How to adapt these tools to derive analytical waveforms in modified gravities ?

Consider the example of **Einstein-scalar-Gauss-Bonnet** (ESGB) theories.

- **Félix-Louis Julié, Emanuele Berti**, “Post-Newtonian dynamics and black hole thermodynamics in Einstein-scalar-Gauss-Bonnet gravity,” Phys.Rev. D100 (2019) no.10, 104061

- **Marcela Cardenas, Félix-Louis Julié, Nathalie Deruelle**, ”Thermodynamics sheds light on black hole dynamics,” Phys. Rev. D97, 12, 124021, 2018.

- **Félix-Louis Julié**, ”Gravitational radiation from compact binary systems in Einstein-Maxwell-dilaton theories,” JCAP 1810, 10, 033, 2018.

- **Félix-Louis Julié**, ”Reducing the two-body problem in scalar-tensor theories to the motion of a test particle: a scalar-tensor effective-one-body approach,” Phys. Rev. D97, 2, 024047, 2018.

- **Félix-Louis Julié, Nathalie Deruelle**, ”Two-body problem in scalar-tensor theories as a deformation of general relativity: an effective-one-body approach,” Phys. Rev. D95, 12, 124054, 2017.

Einstein-Scalar-Gauss-Bonnet gravity

ESGB vacuum action ($G = c = 1$)

$$I_{\mathrm{ESGB}} = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \alpha f(\varphi)\mathscr{R}^2_{\mathrm{GB}}\right)$$

- Massless scalar field φ
- Gauss-Bonnet scalar $\mathscr{R}^2_{\mathrm{GB}} = R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 4R^{\mu\nu}R_{\mu\nu} + R^2$
- Fundamental coupling α with dimensions L^2 and $f(\varphi)$ defines the ESGB theory
- $\int d^Dx\sqrt{-g}\mathscr{R}^2_{\mathrm{GB}}$ is a boundary term in $D \leqslant 4$ [Myers 87]

Second order field equations

$$R_{\mu\nu} = 2\partial_\mu\varphi\partial_\nu\varphi - 4\alpha\left(P_{\mu\alpha\nu\beta} - \frac{g_{\mu\nu}}{2}P_{\alpha\beta}\right)\nabla^\alpha\nabla^\beta f(\varphi)$$

$$\Box\varphi = -\frac{1}{4}\alpha f'(\varphi)\mathscr{R}^2_{\mathrm{GB}}$$

with $P_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} - 2g_{\mu[\rho}R_{\sigma]\nu} + 2g_{\nu[\rho}R_{\sigma]\mu} + g_{\mu[\rho}g_{\sigma]\nu}R$

Hairy black holes in ESGB gravity

Analytical solutions in the small Gauss-Bonnet coupling α limit

- **Einstein-dilaton-Gauss-Bonnet,** $f(\varphi) = e^{\varphi}$

  Mignemi-Stewart 93 at $\mathcal{O}(\alpha^2)$, *Maeda at al. 97* at $\mathcal{O}(\alpha)$, *Yunes-Stein 11* at $\mathcal{O}(\alpha)$

  Ayzenberg-Yunes 14 at $\mathcal{O}(\alpha^2, S^2)$, *Pani et al. 11* at $\mathcal{O}(\alpha^2, S^2)$, *Maselli et al. 15* at $\mathcal{O}(\alpha^7, S^5)$

- **Shift-symmetric theories,** $f(\varphi) = \varphi$

  Sotiriou-Zhou 14 at $\mathcal{O}(\alpha^2)$

- **Generic ESGB theories**

  Julié-Berti 19 at $\mathcal{O}(\alpha^4)$

Numerical solutions

- **Einstein-dilaton-Gauss-Bonnet,** $f(\varphi) = e^{\varphi}$

  Kanti et al. 95, *Pani-Cardoso 09*, *Kleihaus 15* (includes spins)

- **Shift-symmetric theories,** $f(\varphi) = \varphi$

  Delgado et al. 20 (includes spin)

- **Generic ESGB theories**

  Antoniou et al. 18

- **Quadratic couplings,** $f(\varphi) = \varphi^2(1 + \lambda\varphi^2)$ and $f(\varphi) = -e^{-\lambda\varphi^2}$

  Doneva-Yazadjiev 17, *Silva et al. 17*, *Minamitsuji-Ikeda 18*, *Macedo et al. 19*, etc...

How to address (analytically) the motion and gravitational radiation of two coalescing ESGB black holes?

See also *Witek et al. 19*, and *Okounkova 20* for numerical waveforms in the small α limit.

1. **ESGB black holes and their thermodynamics**

2. The post-newtonian (PN) dynamics of an ESGB black hole binary

3. Beyond the PN approximation: “EOBization” of an ESGB black hole binary

4. Gravitational radiation from an ESGB black hole binary

Static, spherically symmetric ESGB black holes

Just coordinate system

$$ds^2 = -A(r)\,dt^2 + \frac{dr^2}{A(r)} + B(r)\,r^2(d\theta^2 + \sin^2\theta\,d\phi^2)$$

Solve iteratively the field equations around a Schwarzschild spacetime with $\boxed{\epsilon = \frac{\alpha f'(\varphi_\infty)}{m^2} \ll 1}$

$$A(r) = 1 - \frac{2m}{r} + \sum_i \epsilon^i A_i(r)\,, \qquad B(r) = 1 + \sum_i \epsilon^i B_i(r)\,, \qquad \varphi(r) = \varphi_\infty + \sum_i \epsilon^i \varphi_i(r)$$

$$\boxed{\begin{aligned} &R_{\mu\nu} = 2\partial_\mu\varphi\partial_\nu\varphi - 4\alpha\left(P_{\mu\alpha\nu\beta} - \frac{g_{\mu\nu}}{2}P_{\alpha\beta}\right)\nabla^\alpha\nabla^\beta f(\varphi) \\ &\Box\varphi = -\frac{1}{4}\alpha f'(\varphi)\mathscr{R}^2_{\mathrm{GB}} \end{aligned}}$$

with $\mathscr{R}^2_{\mathrm{GB}} = R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 4R^{\mu\nu}R_{\mu\nu} + R^2$

ESGB black hole, at leading order for simplicity:

$$A = 1 - \frac{2m}{r} + \mathcal{O}\left(\frac{\alpha f'_\infty}{m^2}\right)^2, \qquad B = 1 + \mathcal{O}\left(\frac{\alpha f'_\infty}{m^2}\right)^2, \qquad \varphi = \varphi_\infty + \frac{\alpha f'(\varphi_\infty)}{m^2}\left(\frac{m}{2r} + \frac{m^2}{2r^2} + \frac{2m^3}{r^3}\right) + \mathcal{O}\left(\frac{\alpha f'_\infty}{m^2}\right)^2$$

Two integration constants: m and φ_∞, at all orders in the Gauss-Bonnet coupling.

ESGB black hole thermodynamics

- **Temperature:**

$$\boxed{T = \frac{\kappa}{4\pi}} \quad \text{where} \quad \kappa^2 = -\frac{1}{2}(\nabla_\mu \xi_\nu \nabla^\mu \xi^\nu)_{r_\mathrm{H}} \quad \text{is the surface gravity}$$

- **Wald entropy:**

$$S_\mathrm{w} = -8\pi \int_{r_\mathrm{H}} d\theta d\phi \sqrt{\sigma} \frac{\partial \mathscr{L}}{\partial R_{\mu\nu\rho\sigma}} \epsilon_{\mu\nu}\epsilon_{\rho\sigma} \qquad \text{with } \epsilon_{\mu\nu} = n_{[\mu} l_{\nu]}$$

$$\boxed{S_\mathrm{w} = \frac{\mathscr{A}_\mathrm{H}}{4} + 4\alpha\pi f(\varphi_\mathrm{H})} \qquad \text{in ESGB gravity.}$$

- **Mass as a global charge:**

$$\boxed{M = m + \int D\, d\varphi_\infty} \qquad D \text{ is the scalar ``charge'' defined as } \varphi = \varphi_\infty + \frac{D}{r} + \mathcal{O}\left(\frac{1}{r^2}\right)$$

[Henneaux et al. 02, Cardenas et al. 16, Anabalon-Deruelle-FLJ 16,...]

The quantities above are calculated in terms of m and φ_∞. At leading order for simplicity:

$$T = 8\pi m \left[1 + \mathcal{O}\left(\frac{\alpha f'_\infty}{m^2}\right)^2\right], \quad S_\mathrm{w} = 4\pi m^2 \left[1 + \frac{\alpha f(\varphi_\infty)}{m^2} + \mathcal{O}\left(\frac{\alpha f'_\infty}{m^2}\right)^2\right], \quad D = \frac{\alpha f'(\varphi_\infty)}{2m}\left[1 + \mathcal{O}\left(\frac{\alpha f'_\infty}{m^2}\right)\right]$$

The variations of S_w and M with respect to m and φ_∞ are such that:

$$T\delta S_\mathrm{w} = \delta M$$

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“Skeletonizing” an ESGB black hole

[in GR: Mathisson 1931, Infeld 1950,...]

$$I_{\mathrm{ESGB}} = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \alpha f(\varphi)\mathscr{R}^2_{\mathrm{GB}}\right) + I^A_{\mathrm{pp}}$$

Generic ansatz for compact bodies

$$I^A_{\mathrm{pp}}[g_{\mu\nu}, \varphi, x^\mu_A] = -\int m_A(\varphi)\, ds_A$$

with $ds_A = \sqrt{-g_{\mu\nu}dx^\mu_A dx^\nu_A}$.

- $m_A(\varphi)$ is a function of the local value of φ to encompass the effect of the background scalar field on the equilibrium of a body [Eardley 75, Damour-Esposito-Farèse 92].

- Strong equivalence principle violation

Question: How to derive $m_A(\varphi)$ for an ESGB black hole?

Answer: by identifying the BH's fields to those sourced by the particle.

Comparing the asymptotic expansions of the fields

$$R_{\mu\nu} = 2\partial_\mu\varphi\partial_\nu\varphi - 4\alpha\left(P_{\mu\alpha\nu\beta} - \frac{1}{2}g_{\mu\nu}P_{\alpha\beta}\right)\nabla^\alpha\nabla^\beta f(\varphi) + 8\pi\left(T^A_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T^A\right)$$

$$\Box\varphi = -\frac{1}{4}\alpha f'(\varphi)\mathscr{R}^2_{\mathrm{GB}} + 4\pi\frac{ds_A}{dt}\frac{dm_A}{d\varphi}\frac{\delta^{(3)}(\mathbf{x}-\mathbf{x}_A(t))}{\sqrt{-g}}$$

with $T_A^{\mu\nu} = m_A(\varphi)\dfrac{\delta^{(3)}(\mathbf{x}-\mathbf{x}_A(t))}{\sqrt{g g_{\alpha\beta}\frac{dx_A^\alpha}{dt}\frac{dx_A^\beta}{dt}}}\dfrac{dx_A^\mu}{dt}\dfrac{dx_A^\nu}{dt}$

Fields of particle A in its rest frame, $x_A^i = 0$

$$g_{\mu\nu} = \eta_{\mu\nu} + \delta_{\mu\nu}\left(\frac{2m_A(\varphi_\infty)}{\tilde{r}}\right) + \mathcal{O}\left(\frac{1}{\tilde{r}^2}\right)$$

$$\varphi = \varphi_\infty - \frac{1}{\tilde{r}}\frac{dm_A}{d\varphi}(\varphi_\infty) + \mathcal{O}\left(\frac{1}{\tilde{r}^2}\right)$$

Fields of the ESGB black hole

$$g_{\mu\nu} = \eta_{\mu\nu} + \delta_{\mu\nu}\left(\frac{2m}{\tilde{r}}\right) + \mathcal{O}\left(\frac{1}{\tilde{r}^2}\right)$$

$$\varphi = \varphi_\infty + \frac{D}{\tilde{r}} + \mathcal{O}\left(\frac{1}{\tilde{r}^2}\right)$$

Matching

- the **identification** yields

Matching conditions

$$m_A(\varphi_\infty) = m$$
$$m'_A(\varphi_\infty) = -D$$

- **For an ESGB black hole** with **"secondary hair"**, $D = D(m, \varphi_\infty)$ yields a first order differential equation.

At leading order, for simplicity:

$$\frac{dm_A}{d\varphi} + \frac{\alpha f'(\varphi)}{2m_A(\varphi)}\left[1 + \mathcal{O}\left(\frac{\alpha f'}{m_A^2}\right)\right] = 0$$

- Its resolution involves a **unique integration constant** μ_A.

The sensitivity of a hairy ESGB black hole

$$I^A_{\rm pp}[g_{\mu\nu}, \varphi, x^\mu_A] = -\int m_A(\varphi)\, ds_A$$

- In an **arbitrary ESGB theory**, BHs are described by a **unique constant parameter:**

$$\boxed{m_A(\varphi) = \mu_A\left(1 - \frac{\alpha f(\varphi)}{2\mu_A^2} + \cdots\right)}$$

where $\mu_A = M_{\rm irr} = \sqrt{\frac{S_{\rm w}}{4\pi}}$

- Recall: ESGB first law of thermodynamics: $\boxed{T\delta S_{\rm w} = \delta M}$

where $\delta M = \delta m + D\delta\varphi_\infty$.

Matching conditions

(a) $m_A(\varphi_\infty) = m$

(b) $m'_A(\varphi_\infty) = -D$

(a) and (b) $\Rightarrow$ $\delta M = 0$

As a consequence, $\delta S_{\rm w} = 0$

When φ_∞ varies slowly, the black hole readjusts its equilibrium configuration, i.e. m, in keeping its Wald entropy fixed.

$$I_{\rm ESGB} = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \alpha f(\varphi)\mathscr{R}^2_{\rm GB}\right) - \sum_A \int m_A(\varphi)ds_A$$

ESGB two-body Lagrangian at 1PN order

- Harmonic gauge $\partial_\mu(\sqrt{-g}g^{\mu\nu}) = 0$
- Conservative 1PN dynamics: $\mathcal{O}\left(\frac{v}{c}\right)^2 \sim \mathcal{O}\left(\frac{GM}{r}\right)$ corrections to Newtonian dynamics
- Solve iteratively the field equations with point particle sources around

$$g_{00} = -e^{-2U} + \mathcal{O}(v^6) \qquad \varphi = \varphi_0 + \delta\varphi$$
$$g_{0i} = -4g_i + \mathcal{O}(v^5)$$
$$g_{ij} = \delta_{ij}e^{2U} + \mathcal{O}(v^4)$$

$$\boxed{\begin{aligned} R_{\mu\nu} &= 2\partial_\mu\varphi\partial_\nu\varphi - 4\alpha\left(P_{\mu\alpha\nu\beta} - \frac{1}{2}g_{\mu\nu}P_{\alpha\beta}\right)\nabla^\alpha\nabla^\beta f(\varphi) + 8\pi\sum_A\left(T^A_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T^A\right) \\ \Box\varphi &= -\frac{1}{4}\alpha f'(\varphi)\mathscr{R}^2_{\rm GB} + 4\pi\sum_A \frac{ds_A}{dt}\frac{dm_A}{d\varphi}\frac{\delta^{(3)}(\mathbf{x} - \mathbf{x}_A(t))}{\sqrt{-g}} \end{aligned}}$$

- The sensitivities $m_A(\varphi)$ and $m_B(\varphi)$ are expanded around φ_0

$$\ln m_A(\varphi) = \ln m_A^0 + \alpha_A^0(\varphi - \varphi_0) + \frac{1}{2}\beta_A^0(\varphi - \varphi_0)^2 + \cdots$$

$$\ln m_B(\varphi) = \ln m_B^0 + \alpha_B^0(\varphi - \varphi_0) + \frac{1}{2}\beta_B^0(\varphi - \varphi_0)^2 + \cdots$$

Gauss-Bonnet contributions

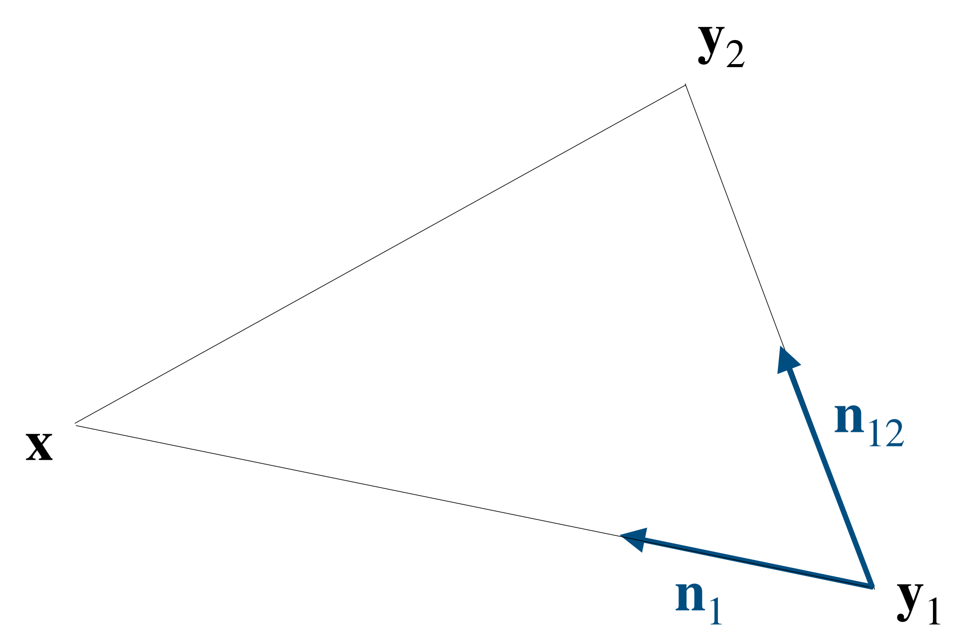
$$\Delta h(\mathbf{x}) = \Delta\frac{1}{|\mathbf{x}-\mathbf{y}|}\Delta\frac{1}{|\mathbf{x}-\mathbf{y}|} - \partial_{ij}\frac{1}{|\mathbf{x}-\mathbf{y}|}\partial_{ij}\frac{1}{|\mathbf{x}-\mathbf{y}|}$$

(i) Introduce $\mathbf{y}_1 \neq \mathbf{y}_2$

$$\begin{aligned}\Delta h_{12} &= \Delta\frac{1}{|\mathbf{x}-\mathbf{y}_1|}\Delta\frac{1}{|\mathbf{x}-\mathbf{y}_2|} - \partial_{ij}\frac{1}{|\mathbf{x}-\mathbf{y}_1|}\partial_{ij}\frac{1}{|\mathbf{x}-\mathbf{y}_2|}\\ &= \left(\frac{\partial^2}{\partial y_1^i \partial y_1^i}\frac{\partial^2}{\partial y_2^j \partial y_2^j} - \frac{\partial^2}{\partial y_1^i \partial y_2^i}\frac{\partial^2}{\partial y_1^j \partial y_2^j}\right)\frac{1}{|\mathbf{x}-\mathbf{y}_1||\mathbf{x}-\mathbf{y}_2|}\end{aligned}$$

(ii) Use Fock's "perimeter formula" (1939)

$$\Delta^{-1}\left(\frac{1}{|\mathbf{x}-\mathbf{y}_1||\mathbf{x}-\mathbf{y}_2|}\right) = \ln(|\mathbf{x}-\mathbf{y}_1| + |\mathbf{x}-\mathbf{y}_2| + |\mathbf{y}_1-\mathbf{y}_2|)$$



(iii) Take the limit $\epsilon = |\mathbf{y}_2 - \mathbf{y}_1| \to 0$

$$h_{12}(\mathbf{x}) = \frac{1-3(\mathbf{n}_{12}\cdot\mathbf{n}_1)^2}{2|\mathbf{x}-\mathbf{y}_1|^3\epsilon} + \frac{2-9(\mathbf{n}_{12}\cdot\mathbf{n}_1)+15(\mathbf{n}_{12}\cdot\mathbf{n}_1)^3}{4|\mathbf{x}-\mathbf{y}_1|^4} + \mathcal{O}(\epsilon)$$

(iv) Average out $\mathbf{n}_{12}$

$$\langle n_{12}^i\rangle = 0\,, \quad \langle n_{12}^i n_{12}^j\rangle = \delta_{ij}/3\,, \quad \langle n_{12}^i n_{12}^j n_{12}^k\rangle = 0$$

Finite Gauss-Bonnet contribution

$$h(\mathbf{x}) = \frac{1}{2|\mathbf{x}-\mathbf{y}|^4}$$

ESGB two-body Lagrangian at 1PN order [FLJ-Berti 2019)]

$$L_{AB} = -m_A^0 - m_B^0 + \frac{1}{2} m_A^0 \mathbf{v}_A^2 + \frac{1}{2} m_B^0 \mathbf{v}_B^2 + \frac{G_{AB} m_A^0 m_B^0}{r}$$
$$+ \frac{1}{8} m_A^0 \mathbf{v}_A^4 + \frac{1}{8} m_B^0 \mathbf{v}_B^4 + \frac{G_{AB} m_A^0 m_B^0}{r} \left[\frac{3}{2}(\mathbf{v}_A^2 + \mathbf{v}_B^2) - \frac{7}{2}(\mathbf{v}_A \cdot \mathbf{v}_B) - \frac{1}{2}(\mathbf{n} \cdot \mathbf{v}_A)(\mathbf{n} \cdot \mathbf{v}_B) + \bar{\gamma}_{AB}(\mathbf{v}_A - \mathbf{v}_B)^2 \right]$$
$$- \frac{G_{AB}^2 m_A^0 m_B^0}{2r^2} \left[m_A^0 (1 + 2\bar{\beta}_B) + m_B^0 (1 + 2\bar{\beta}_A) \right] + \Delta L_{AB}^{\mathrm{GB}} + \mathcal{O}(v^6)$$

- L_{AB} has the **same structure** as the scalar-tensor Lagrangian at 1PN...

$$\boxed{\begin{aligned} G_{AB} &= G(1 + \alpha_A^0 \alpha_B^0) \\ \bar{\gamma}_{AB} &= -2 \frac{\alpha_A^0 \alpha_B^0}{1 + \alpha_A^0 \alpha_B^0} \\ \bar{\beta}_A &= \frac{1}{2} \frac{\beta_A^0 \alpha_B^{0\,2}}{(1 + \alpha_A^0 \alpha_B^0)^2} \quad \text{and} \quad (A \leftrightarrow B). \end{aligned}}$$

where $\alpha_A^0 = (d \ln m_A / d\varphi)(\varphi_0)$, $\beta_A^0 = (d\alpha_A^0 / d\varphi)(\varphi_0)$

- ... except for one **new and finite Gauss-Bonnet contribution**:

$$\boxed{\Delta L_{AB}^{\mathrm{GB}} = \frac{\alpha f'(\varphi_0)}{(GM)^2} \left(\frac{GM}{r} \right)^2 \frac{G^2 m_A^0 m_B^0}{r^2} \left[m_A^0 (\alpha_B^0 + 2\alpha_A^0) + m_B^0 (\alpha_A^0 + 2\alpha_B^0) \right]}$$

- can be regarded as a 3PN correction whenever $\alpha f'(\varphi_0) \lesssim M^2$.
- In scalar-tensor theories, L_{AB} is known at 2PN [Mirshekari-Will 13] and 3PN [Bernard 19]
- In the regime above, the **conservative dynamics in ESGB** gravity is hence **known at 3PN**.

Example: Einstein-dilaton-Gauss-Bonnet black holes

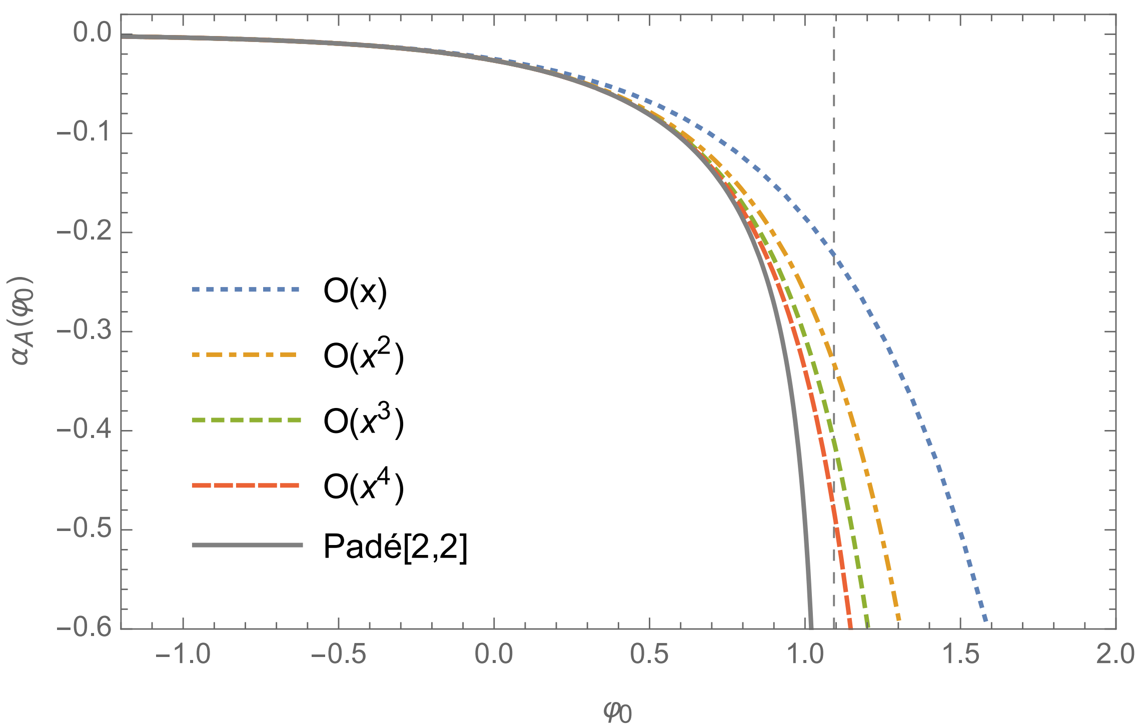
$$\boxed{f(\varphi) = \frac{e^{2\varphi}}{4}}$$

$$\alpha_A^0 = (d \ln m_A / d\varphi)(\varphi_0)\ , \quad \beta_A^0 = (d\alpha_A^0/d\varphi)(\varphi_0)$$

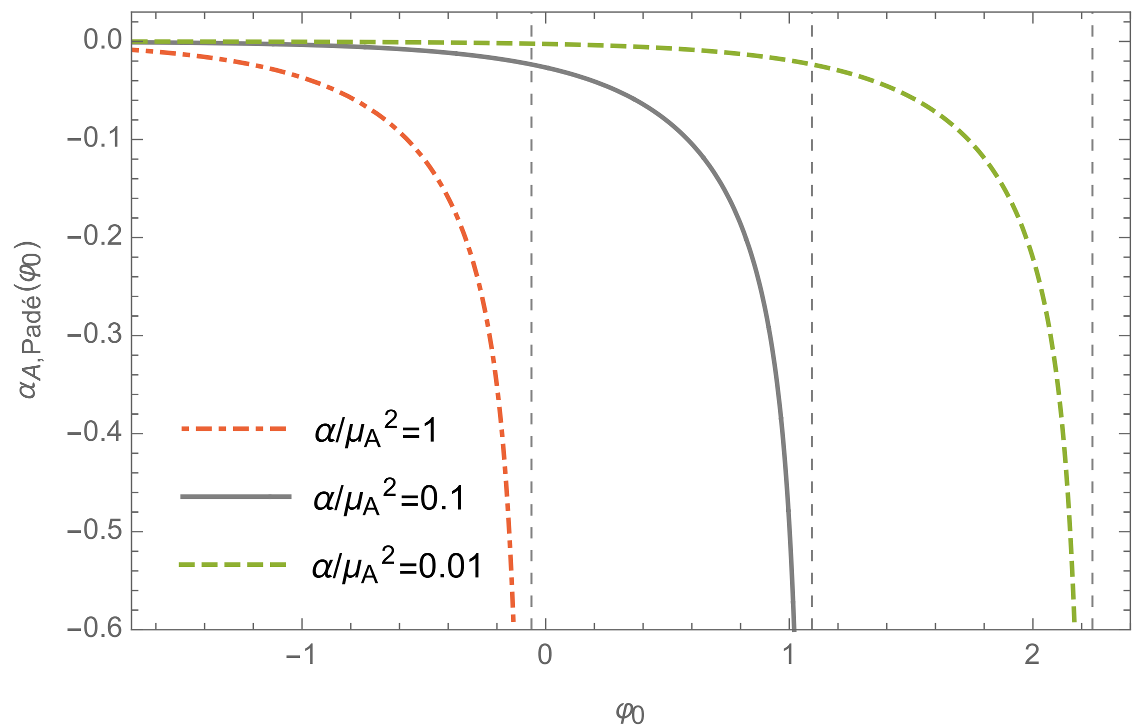
At fourth order in the Gauss-Bonnet coupling α:

$$\alpha_A^0 = -\frac{x}{2} - \frac{133}{240}x^2 - \frac{35947}{40320}x^3 - \frac{474404471}{266112000}x^4 + \mathcal{O}\left(x^5\right) \quad \text{with} \quad x = \frac{\alpha e^{2\varphi_0}}{2\mu_A^2}$$

$\alpha/\mu_A^2 = 0.1$



$\varphi_0^{\rm pole} = (1/2)\ln(2x_{\rm pole}\,\mu_A^2/\alpha)$



- α_A^0 diverges at large φ_0, with a slope which increases with the truncation order in α.
- The (2,2) Padé approximant $\mathscr{P}_2^2[\alpha_A^0]$ predicts a **pole** at $x_{\rm pole} = \dfrac{\alpha e^{2\varphi_0^{\rm pole}}}{2\mu_A^2} = 0.445$
- This pole could be the sign of **naked singularities** [Kanti at al. 95, Doneva-Yazadjiev 17]

$$\boxed{24\alpha^2 f'(\varphi_H)^2 < \left(\frac{\mathscr{A}_H}{4\pi}\right)^2} \quad \Rightarrow \quad \frac{\alpha e^{2\varphi_H}}{2\mu_A^2} < \frac{2}{1+\sqrt{6}}$$ for a skeletonized EdGB BH.

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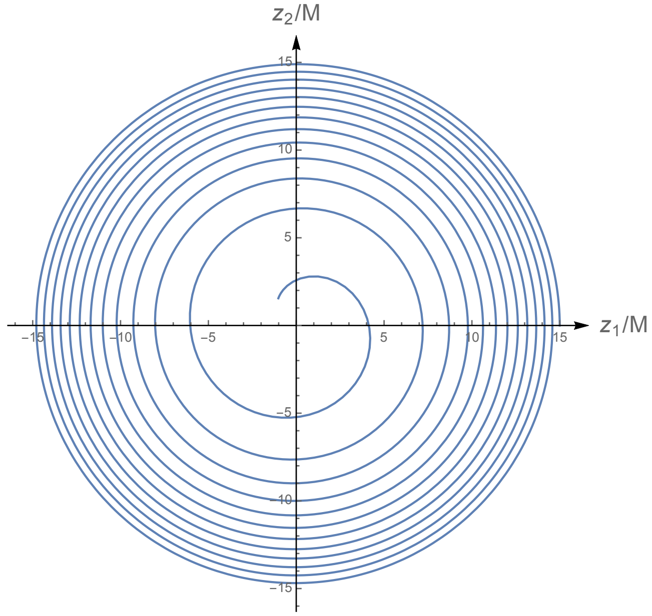
In general relativity, "effective-one-body" (EOB) :

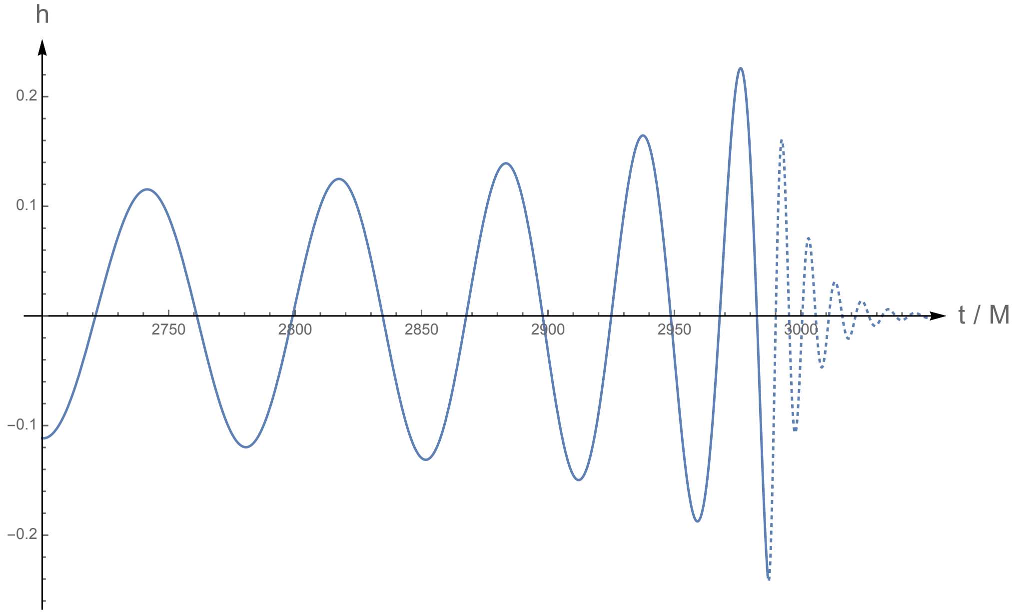
- Map the two-body PN dynamics to the motion of a **test particle** in an **effective static, spherically symmetric metric** [Buonanno-Damour 98]

$$H(Q,P)\ , \quad \epsilon = \left(\frac{v}{c}\right)^2 \qquad \longrightarrow \qquad H_e(q,p)\ , \quad ds_e^2 = g^e_{\mu\nu} dx^\mu dx^\nu$$

$$\boxed{H_e = f_{\text{EOB}}(H)}$$

- Defines a resummation of the PN dynamics, hence describes **analytically** the coalescence of 2 compact objects in **general relativity**, from inspiral **to merger**.





- Instrumental to build libraries of waveform templates for LIGO-Virgo

In practice, on the simple example of ESGB at 1PN:

Compute the two-body hamiltonian $H(Q,P) = P_R\dot{R} + P_\phi\dot{\phi} - L_{AB}$.

In the center-of-mass frame :

$$\boxed{\overrightarrow{P}_A + \overrightarrow{P}_B = \overrightarrow{0}}$$

7 coefficients (polar coordinates)

$$H = M + \left(\frac{P^2}{2\mu} - \mu\frac{G_{AB}M}{R}\right) + H^{1\mathrm{PN}} + \cdots$$

with $$\frac{H^{1\mathrm{PN}}}{\mu} = \left(h_1^{1\mathrm{PN}}\hat{P}^4 + h_2^{1\mathrm{PN}}\hat{P}^2\hat{P}_R^2 + h_3^{1\mathrm{PN}}\hat{P}_R^4\right) + \frac{1}{\hat{R}}\left(h_4^{1\mathrm{PN}}\hat{P}^2 + h_5^{1\mathrm{PN}}\hat{P}_R^2\right) + \frac{h_6^{1\mathrm{PN}}}{\hat{R}^2}$$

The 7 $h_i^{N\mathrm{PN}}$ coefficients are computed explicitly and depend on the 6 parameters $(m_A^0, \alpha_A^0, \beta_A^0)$ and $(m_B^0, \alpha_B^0, \beta_B^0)$ built from $m_A(\varphi)$ and $m_B(\varphi)$

The effective Hamiltonian H_e

Geodesic motion in a static, spherically symmetric metric

In Schwarzschild-Droste coordinates (equatorial plane $\theta = \pi/2$) :

$$\boxed{ds_e^2 = -A(r)dt^2 + B(r)dr^2 + r^2 d\phi^2}$$

$A(r)$ and $B(r)$ are arbitrary

Effective Hamiltonian $H_e(q,p)$:

$$H_e(q,p) = \sqrt{A\left(\mu^2 + \frac{p_r^2}{B} + \frac{p_\phi^2}{\hat{r}^2}\right)} \quad \text{with} \quad p_r = \frac{\partial L_e}{\partial \dot{r}} \, , \quad p_\phi = \frac{\partial L_e}{\partial \dot{\phi}}$$

Can be expanded :

$$\boxed{\begin{aligned} A(r) &= 1 + \frac{a_1}{r} + \frac{a_2}{r^2} + \cdots \\ B(r) &= 1 + \frac{b_1}{r} + \cdots \end{aligned}}$$

and depends on 3 effective parameters at 1PN order, to be determined.

EOB mapping [Buonanno-Damour 98]

(i) Canonically transform H :

$$H(Q,P) \to H(q,p)$$

Generic ansatz $G(Q,p)$ that depends on **3 parameters** at 1PN order :

$$\boxed{G(Q,p) = R\,p_r\left(\alpha_1 \mathscr{P}^2 + \beta_1 \hat{p}_r^2 + \frac{\gamma_1}{\hat{R}} + \cdots\right)}$$

(ii) Relate H to H_e through the quadratic relation [Damour 2016]

$$\boxed{\frac{H_e(q,p)}{\mu} - 1 = \left(\frac{H(q,p)-M}{\mu}\right)\left[1 + \frac{\nu}{2}\left(\frac{H(q,p)-M}{\mu}\right)\right]}$$

where $\qquad \nu = \frac{m_A^0 m_B^0}{(m_A^0 + m_B^0)^2}\,, \qquad M = m_A^0 + m_B^0\,, \qquad \mu = \frac{m_A^0 m_B^0}{M}$

3. Beyond the PN approximation: "EOBization" of an ESGB black hole binary

$$ds_e^2 = -A(r)dt + B(r)dr^2 + r^2 d\phi^2$$

It works, i.e., it yields a **unique** solution in ESGB theories:

FLJ, N. Deruelle [PRD 95, 12, 124054, 2017]

$$A(r) = 1 - 2\left(\frac{G_{AB}M}{r}\right) + 2\left[\langle\bar{\beta}\rangle - \bar{\gamma}_{AB}\right]\left(\frac{G_{AB}M}{r}\right)^2 + \cdots$$

$$B(r) = 1 + 2\left[1 + \bar{\gamma}_{AB}\right]\left(\frac{G_{AB}M}{r}\right) + \cdots$$

we recognize the PPN Eddington metric written in Droste coordinates, with :

$$\boxed{\beta^{\rm Edd} = 1 + \langle\bar{\beta}\rangle \ , \quad \gamma^{\rm Edd} = 1 + \bar{\gamma}_{AB}}$$

where

$$\langle\bar{\beta}\rangle \equiv \frac{m_A^0\bar{\beta}_B + m_B^0\bar{\beta}_A}{m_A^0 + m_B^0} \qquad \bar{\gamma}_{AB} \equiv -\frac{2\alpha_A^0\alpha_B^0}{1+\alpha_A^0\alpha_B^0} \qquad \bar{\beta}_A \equiv \frac{1}{2}\frac{\beta_A^0(\alpha_B^0)^2}{(1+\alpha_A^0\alpha_B^0)^2}$$

(See also [Damour, Jaranowski, Schaefer 15] at 4PN in GR; and [FLJ, N.Deruelle 17] at 2PN in scalar-tensor theories.)

A resummed dynamics

- The inversion of $$\frac{H_e(q,p)}{\mu} - 1 = \left(\frac{H(q,p)-M}{\mu}\right)\left[1 + \frac{\nu}{2}\left(\frac{H(q,p)-M}{\mu}\right)\right]$$

defines a "resummed" EOB Hamiltonian :

$$H_{\mathrm{EOB}} = M\sqrt{1 + 2\nu\left(\frac{H_e}{\mu} - 1\right)}\ , \quad \text{where} \quad H_e = \sqrt{A\left(\mu^2 + \frac{p_r^2}{B} + \frac{p_\phi^2}{r^2}\right)}$$

- HEOB hence defines a resummed dynamics, e.g., up to the innermost stable circular orbit (ISCO) or light-ring (LR).

$$\dot{r} = \frac{\partial H_{\mathrm{EOB}}}{\partial p_r}\ , \qquad \dot{p}_r = -\frac{\partial H_{\mathrm{EOB}}}{\partial r}\ , \qquad \dot{\phi} = \frac{\partial H_{\mathrm{EOB}}}{\partial p_\phi}\ , \qquad \dot{p}_\phi = -\frac{\partial H_{\mathrm{EOB}}}{\partial \phi}$$

1. ESGB black holes and their thermodynamics

2. The post-newtonian (PN) dynamics of an ESGB black hole binary

3. Beyond the PN approximation: “EOBization” of an ESGB black hole binary

4. **Gravitational radiation from an ESGB black hole binary**

Radiated energy fluxes at infinity

$$-\frac{d\mathscr{E}}{dt} = \mathscr{F}_g + \mathscr{F}_\varphi \qquad \text{with} \qquad \mathscr{E} = \int d^3x\,|g|\left(t^{00} + T^{00}_{(\varphi)} + T^{00}_{(\mathrm{m})}\right)$$

$$\mathscr{F}_g = \int_{x\to\infty} t^{0i}\, n_i x^2 d\Omega^2\,, \qquad \mathscr{F}_\varphi = \int_{x\to\infty} T^{0i}_{(\varphi)}\, n_i x^2 d\Omega^2\,,$$

- $\mathscr{F}_g$ is given by Einstein's 2nd quadrupole formula at leading order, $t^{\mu\nu}$ reduces to the Landau-Lifshitz pseudo-tensor
- $\mathscr{F}_\varphi$ is the extra scalar flux.

In the center-of-mass frame ($P^i_A + P^i_B = 0$) and for circular orbits: [Yagi et al. 2012, FLJ 2018]

- **Metric flux** ("dressed up" quadrupole formula)

Kepler: $(G_{AB}M\dot{\phi})^{2/3} = \mathscr{O}(v^2)$

$$\mathscr{F}_g = \frac{32}{5}\frac{\nu^2\left(G_{AB}M\dot{\phi}\right)^{10/3}}{G\left(1+\alpha^0_A\alpha^0_B\right)^2} + \cdots$$

- **Scalar flux** (with dipolar contribution if $\alpha^0_A \neq \alpha^0_B$)

$$\mathscr{F}_\varphi = \frac{\nu^2\left(G_{AB}M\dot{\phi}\right)^{8/3}}{G_*\left(1+\alpha^0_A\alpha^0_B\right)^2}\Bigg\{\frac{1}{3}(\alpha^0_A-\alpha^0_B)^2 + \left(G_{AB}M\dot{\phi}\right)^{2/3}\Bigg[\frac{16}{15}\left(\frac{m^0_A\alpha^0_B + m^0_B\alpha^0_A}{M}\right)^2 + \frac{2}{9}(\alpha^0_A-\alpha^0_B)^2\left(\nu - 3 - \bar{\gamma}_{AB} - 2\langle\bar{\beta}\rangle\right)$$
$$+2(\alpha^0_A-\alpha^0_B)\left(\frac{(m^0_A)^2\alpha^0_B - (m^0_B)^2\alpha^0_A}{5M^2} + \frac{m^0_A\left[\alpha^0_B + \alpha^0_A(\alpha^0_B)^2 + \beta^0_B\alpha^0_A\right] - (A\leftrightarrow B)}{3M(1+\alpha^0_A\alpha^0_B)}\right)\Bigg] + \cdots\Bigg\}$$

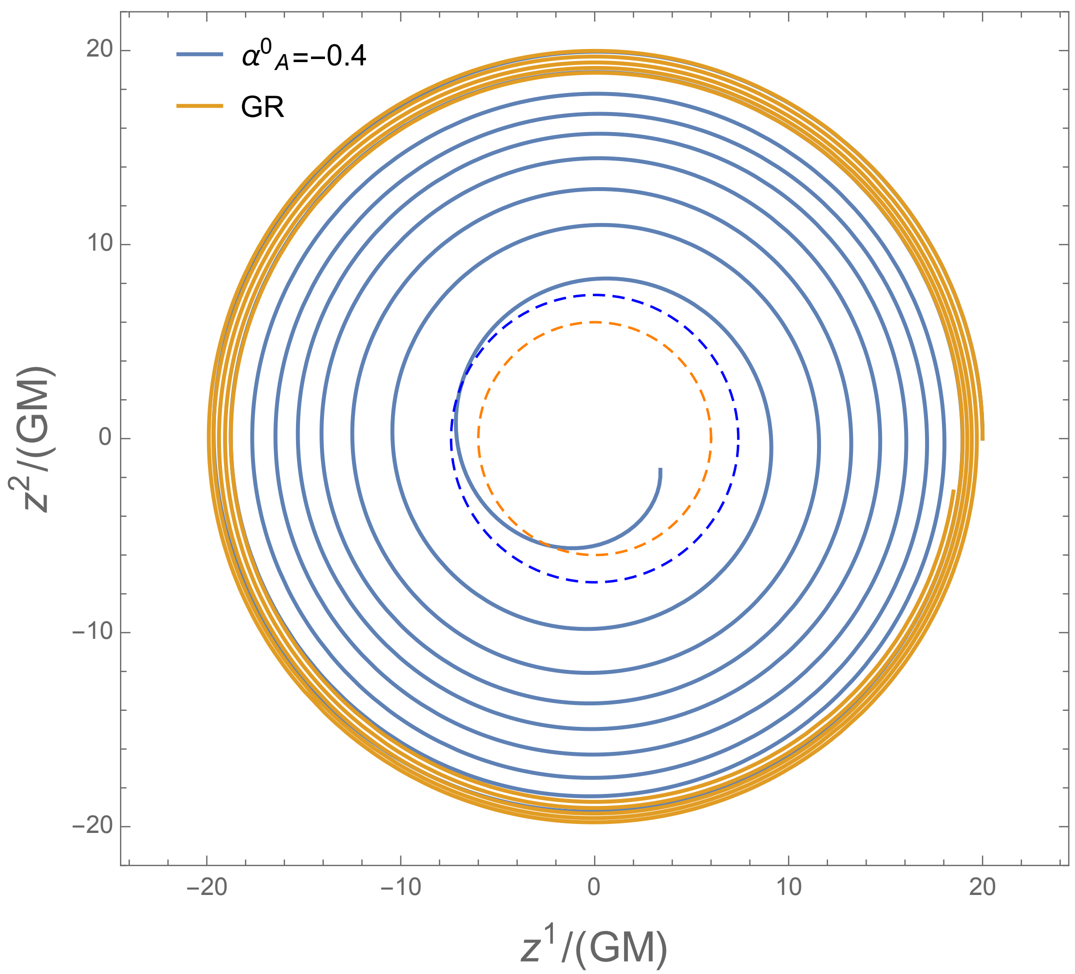
EOB dynamics including the radiation reaction force

- On quasi-circular orbits : tangential force $\boxed{F_\phi = -(\mathscr{F}_g + \mathscr{F}_\varphi)/\dot{\phi}}$

$$\dot{r} = \frac{\partial H_{\rm EOB}}{\partial p_r}\,, \qquad \dot{p}_r = -\frac{\partial H_{\rm EOB}}{\partial r}\,, \qquad \dot{\phi} = \frac{\partial H_{\rm EOB}}{\partial p_\phi}\,, \qquad \dot{p}_\phi = -\frac{\partial H_{\rm EOB}}{\partial \phi} + F_\phi$$

where $H_{\rm EOB} = M\sqrt{1 + 2\nu\left(\frac{H_e}{\mu} - 1\right)}$ and $H_e = \mu\sqrt{A\left(1 + \frac{p_r^2}{\mu^2 B} + \frac{p_\phi^2}{\mu^2 r^2}\right)}$

Example: effective trajectory for two EdGB black holes ($f(\varphi) = e^{2\varphi}/4$):



- $z^1 = r\cos(\phi)\,, \quad z^2 = r\sin(\phi)$
- Asymmetric binary: $\frac{m_A^0}{m_B^0} = 2 \quad (\nu \simeq 0.22)$
- BHs with scalar hair ($\alpha_A^0 = -0.4\,,\ \alpha_B^0 = -1.6$)
- GR limit in yellow
- Note : $(\dot{r}/r\dot{\phi})^2_{\rm ISCO} = 0.01$

Last step : compute the ESGB-EOB waveforms up to the ISCO

- Mirrors follow the geodesics of the **Jordan metric** (in the solar system)

$$\tilde{g}_{\mu\nu} = \mathscr{A}^2(\varphi) g_{\mu\nu} = \mathscr{A}^2_\odot \left[\eta_{\mu\nu}(1 + 2\alpha_\odot \delta\varphi) + h^{TT}_{\mu\nu}\right] + \mathcal{O}\left(\frac{1}{x^2}\right)$$

- New "**breathing**" mode

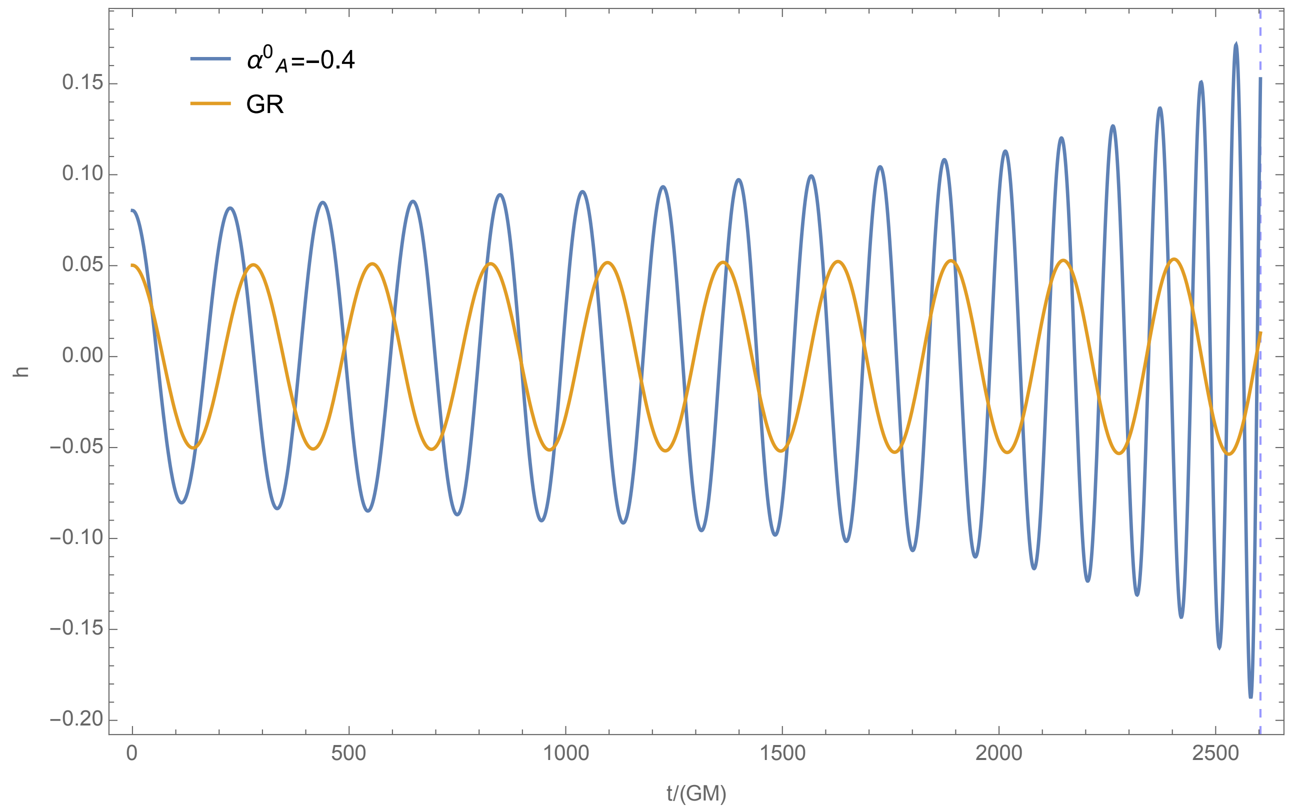
$$\frac{d^2\xi^i}{dt^2} \simeq -\tilde{R}^i{}_{0j0}\,\xi^j \quad \text{with} \quad \tilde{R}^i{}_{0j0} = -\mathscr{A}^2_\odot \left[\frac{1}{2}\ddot{h}^{TT}_{ij} + \alpha_\odot \delta\ddot{\varphi}\left(\delta_{ij} - n_i n_j\right)\right] + \mathcal{O}\left(\frac{1}{x^2}\right)$$

where $\alpha_\odot = \dfrac{d \ln \mathscr{A}}{d\varphi}(\varphi_\odot)$ and

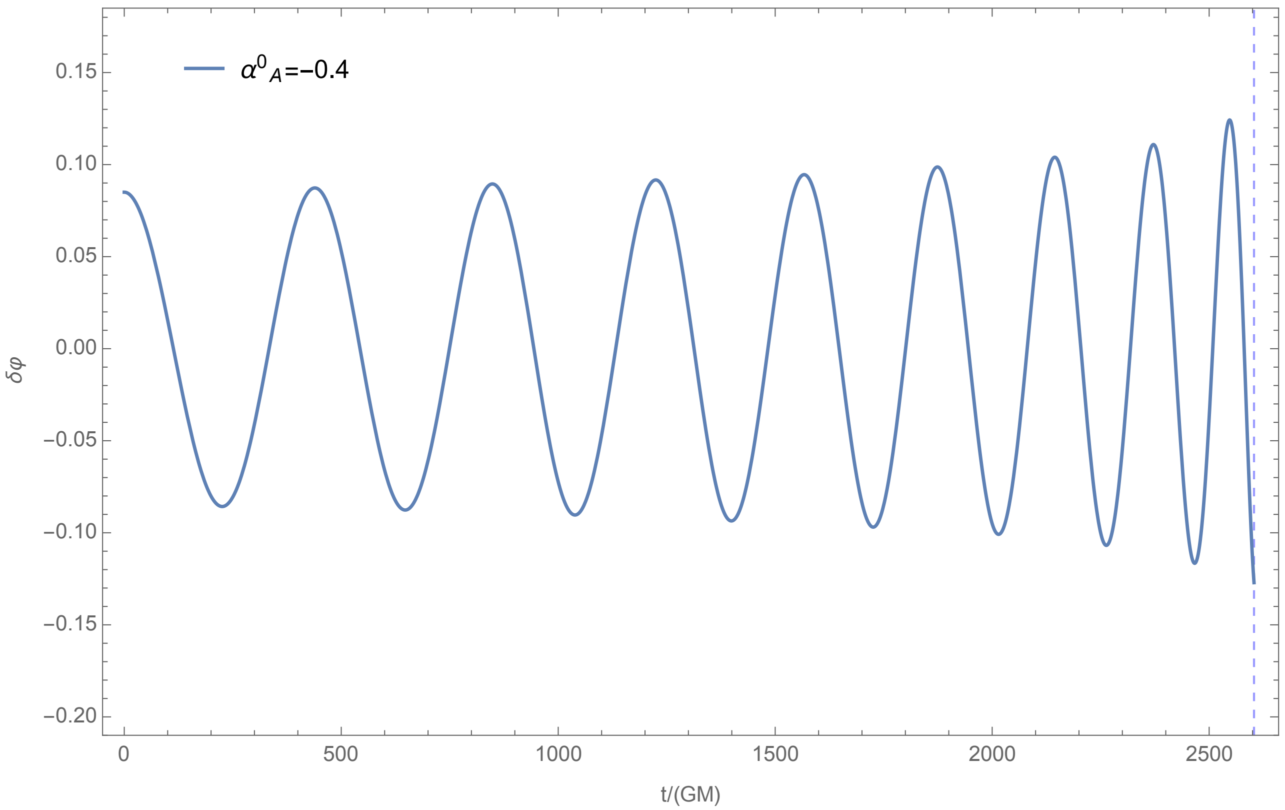
$$\boxed{\begin{aligned} h^{TT}_{ij} &= \frac{2G}{3}\frac{\mathscr{P}_{ij}{}^{kl}\ddot{\mathscr{Q}}_{kl}}{x} \quad \text{with} \quad \mathscr{Q}^{ij} = \sum_A m^0_A \left(3x^i_A x^j_A - \delta^{ij} x^2_A\right) \\ \delta\varphi &= \varphi - \varphi_\odot = -G\frac{n_i \dot{\mathscr{D}}^i_S}{x} \quad \text{with} \quad \mathscr{D}^i_S = \sum_A m^0_A \alpha^0_A x^i_A \end{aligned}}$$

Analytical waveforms for an inspiralling ESGB BH binary

$$h = \left(G_{AB}M\dot{\phi}\right)^{2/3}\cos(2\phi)$$



$$\delta\varphi = (1/4)(\alpha_A^0 - \alpha_B^0)\left(G_{AB}M\dot{\phi}\right)^{1/3}\cos(\phi)$$



- On this example, the scalar amplitude is **numerically comparable** to the tensor one.
- However, its contribution is numerically **lowered** by $|\alpha_\odot| \lesssim 10^{-2}$ in the **solar system**
- Observed frequency : $f = \dot{\phi}/(\pi\mathscr{A}_\odot)$

Recap

- Remarkably, the EOB approach can be extended beyond general relativity. In **ESGB** and **scalar-tensor gravity:**

$$A^{2\mathrm{PN}}(u) = \mathscr{P}^1_5[A^{Taylor}_{5\mathrm{PN}} + 2\epsilon_{1\mathrm{PN}}u^2 + (\epsilon^0_{2\mathrm{PN}} + \nu\,\epsilon^\nu_{2\mathrm{PN}})u^3]$$

- Also works in **Einstein-Maxwell-dilaton** (EMD) theories at 1PN: [FLJ 18]

$$I_{\mathrm{EMD}} = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - e^{-2a\varphi}F^{\mu\nu}F_{\mu\nu}\right)$$

- The ST and EMD examples suggest a generic **"parametrized EOB" (PEOB)** ansatz:

$$\boxed{A^{\mathrm{PEOB}}(u) = \mathscr{P}^1_5[A^{Taylor}_{5\mathrm{PN}} + 2(\epsilon^0_{1\mathrm{PN}} + \nu\,\epsilon^\nu_{1\mathrm{PN}})u^2 + (\epsilon^0_{2\mathrm{PN}} + \nu\,\epsilon^\nu_{2\mathrm{PN}})u^3]}$$

- We generalized Eardley's sensitivites $m_A(\varphi)$ to **hairy black holes**, and shed light on the role of the cosmological environment φ_0 of a binary on its dynamics.

- Necessity to observe sources emitting from a **large range of redshifts**, using **LISA**?

Future developments

- Pole in the scalar coupling α_A^0 predicted by Padé approximants: to be confirmed and interpreted **using numerical BH solutions.**

- **Skeletonize "scalarized" black holes** to include them in the EOB framework. [Silva et al. 17]

- Refine our waveforms using **higher PN order** Lagrangians and fluxes; e.g., ST-ESGB at 3PN [Bernard 18]

- Match our waveforms to the **quasi-normal modes** of the final black hole [Brito-Pacilio 2018]

New on arXiv:

Numerical relativity is crucial to further explore the strong field regime near merger & calibrate EOB templates.

- Existing work in ESGB **in the small Gauss-Bonnet coupling α limit** [Witek et al. 19, Okounkova 20];
- To be extended to the full, **non-perturbative theory**?

→ d+1 formalism in ESGB gravity [FLJ-Berti, arXiv:2004.00003] [see also Witek et al. 20]

Thank you for your attention.