Inflation and Geometry

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APC

March 31st 2020

I Inflation: a reminder

II The physics of inflation?

III Curved field space and geometrical destabilization of inflation

IV Strongly non-geodesic motion & new EFTs

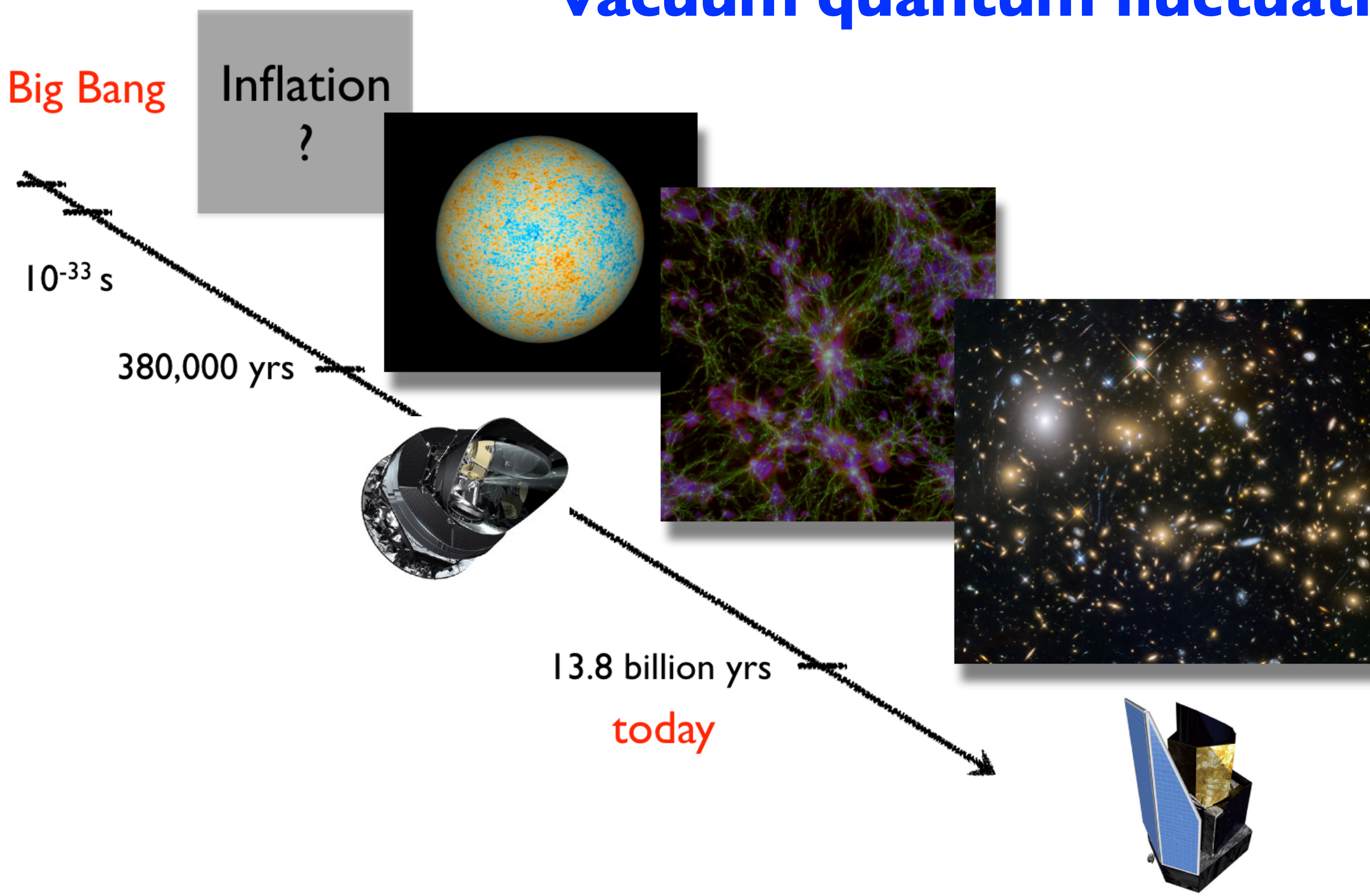
V Revisiting non-Gaussianity in multifield inflation with curved field space

I

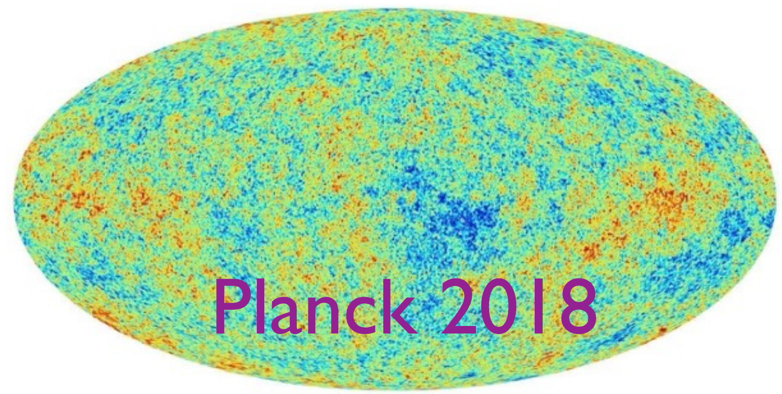
Inflation: a reminder

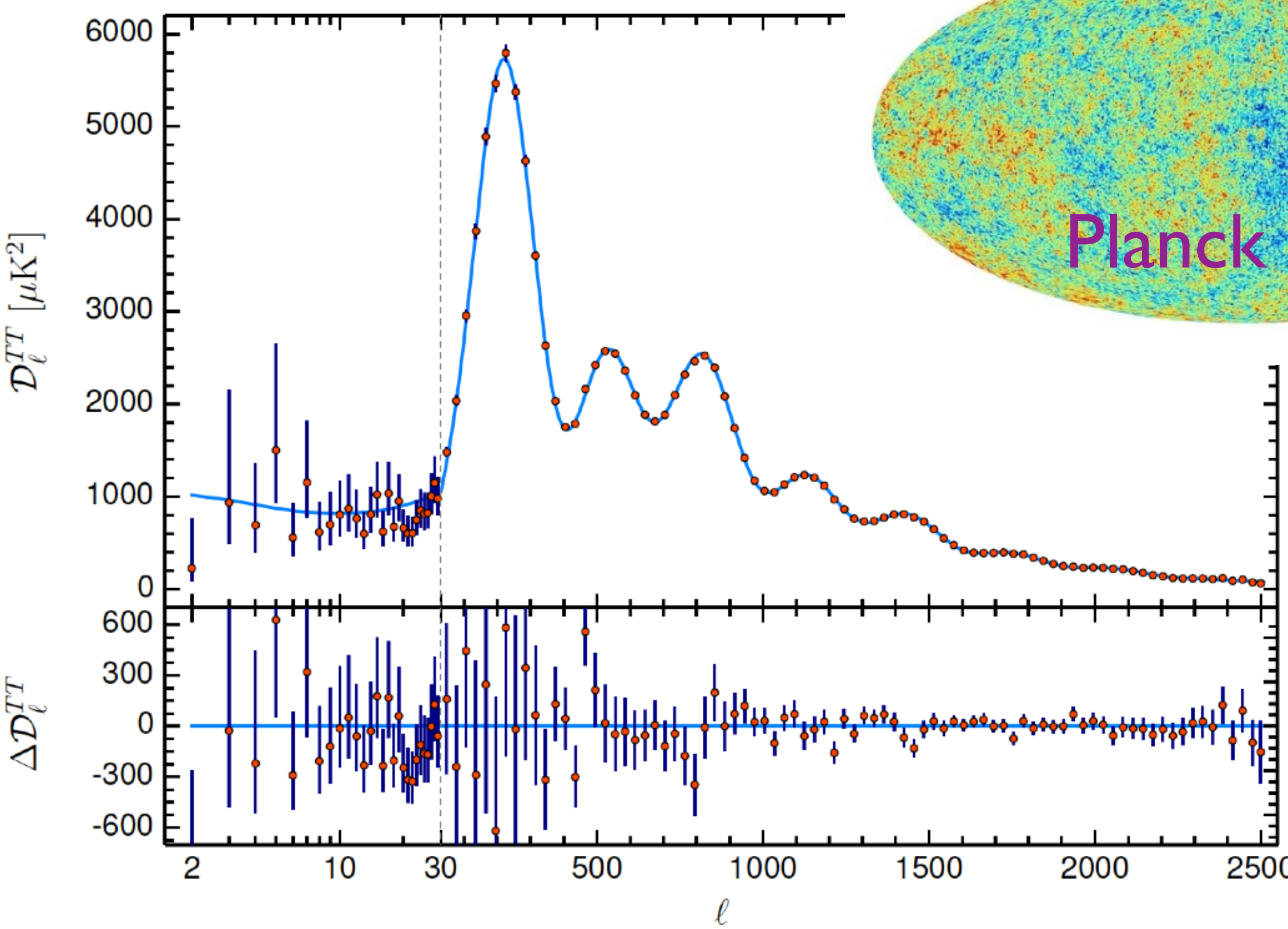
Inflation: a period of accelerated expansion before the radiation era that solves the problems of the Hot Big bang model

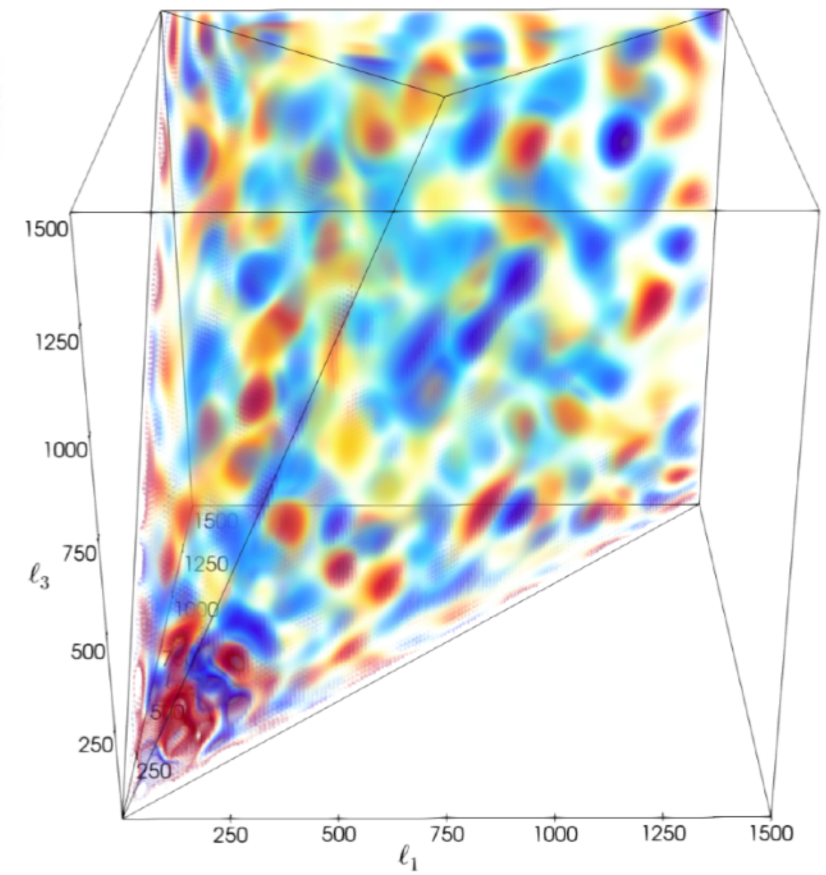
Structures in universe emerge from vacuum quantum fluctuations!



Primordial universe: observations







Density fluctuations: Superhorizon - adiabatic
almost scale-invariant - Gaussian

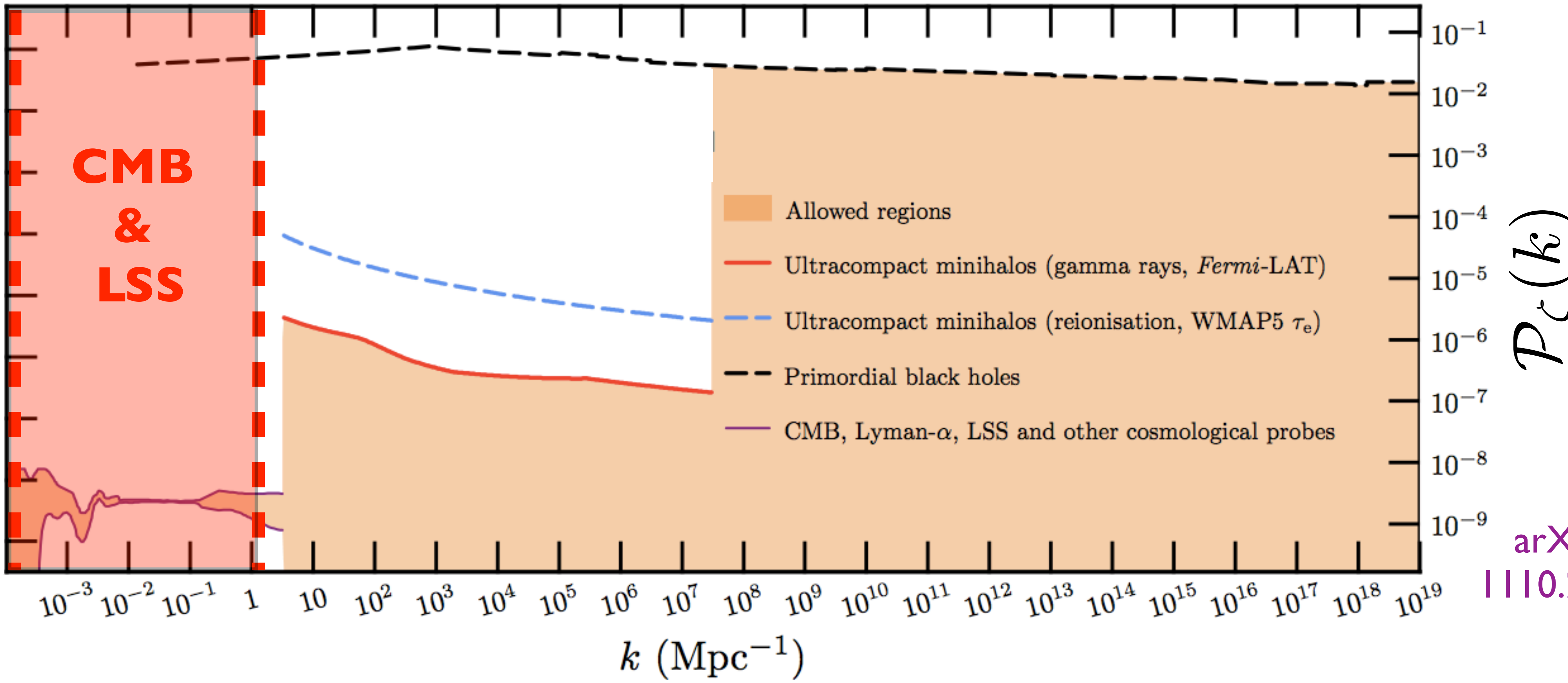
Simplest fit to data: single-field slow-roll inflation

Adiabaticity

Automatic after single-field inflation:
all components inherit inhomogeneities of the unique inflaton field.

Thermal equilibrium reached during reheating after multifield inflation does the same.

Almost scale invariance



arXiv: 1110.2484

$$\Delta N = \Delta \ln(a) \sim 50$$

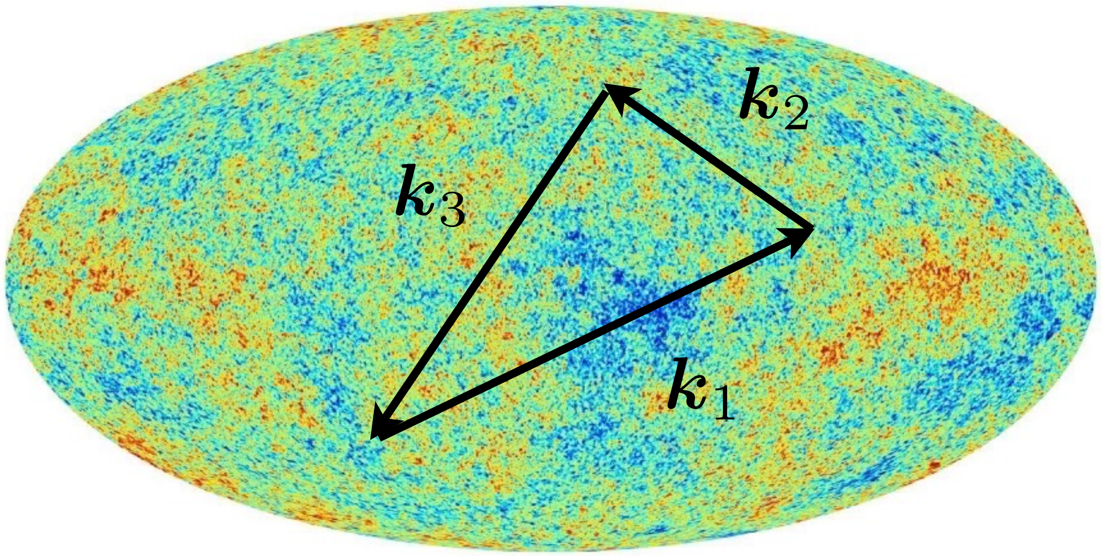
$$\mathcal{P}_\zeta(k) = \underbrace{A_s(k_\star)}_{\sim 10^{-9}} \left(\frac{k}{k_\star}\right)^{n_s(k_\star)-1}$$

$$n_s = 0.9649 \pm 0.0042 \ (68\%\mathrm{CL})$$

Planck 2018

Non-Gaussianity

$$\frac{\delta T}{T} \sim \zeta \sim 10^{-5}$$



$$\zeta \sim \zeta_G \left(1 + f_{\rm NL} \zeta_G\right)$$

- Current Planck constraints: $|f_{\rm NL}| \lesssim \mathcal{O}(10)$

Gaussianity already tested to better than 0.1%

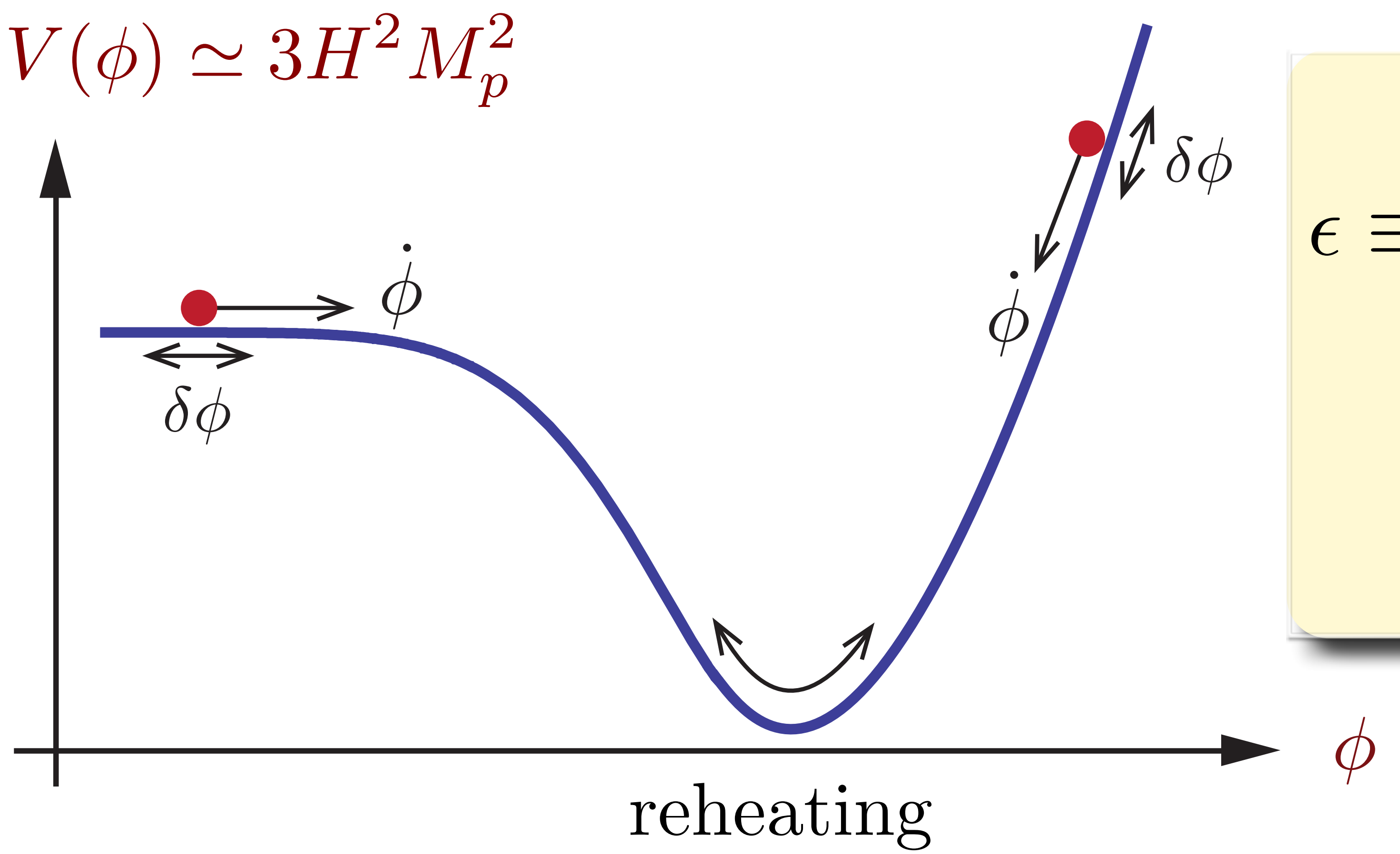
- Slow-roll single field prediction:

$$f_{\rm NL} \sim (n_s - 1) \sim 10^{-2}$$ Maldacena (03)

Single-field slow-roll inflation

- Scalar field with flat potential in Planck units

$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - V(\phi) \right]$$



$$\epsilon \equiv -\frac{\dot{H}}{H^2} \simeq \frac{M_{\rm Pl}^2}{2} \left(\frac{V_{,\phi}}{V} \right)^2 \ll 1$$

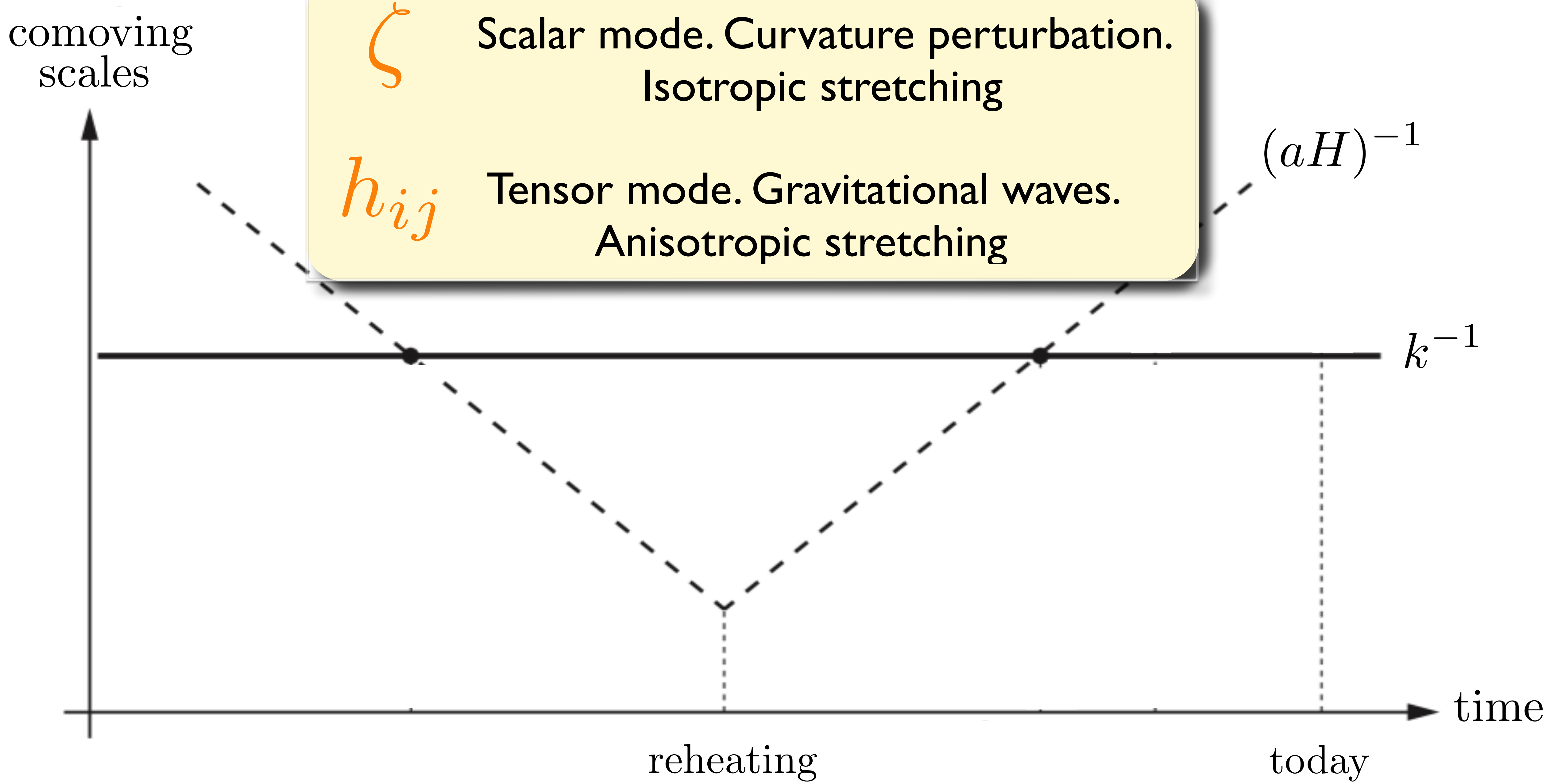
$$\eta \equiv M_{\rm pl}^2 \frac{V_{,\phi\phi}}{V} \ll 1$$

Inflation for fluctuations

$$g_{ij} = a^2(t)e^{2\zeta}\left[e^{h}\right]_{ij} \qquad \mathrm{Tr}[h_{ij}] = 0$$

ζ Scalar mode. Curvature perturbation. Isotropic stretching

h_{ij} Tensor mode. Gravitational waves. Anisotropic stretching



Inflation

Hot Big-Bang

Inflation for fluctuations

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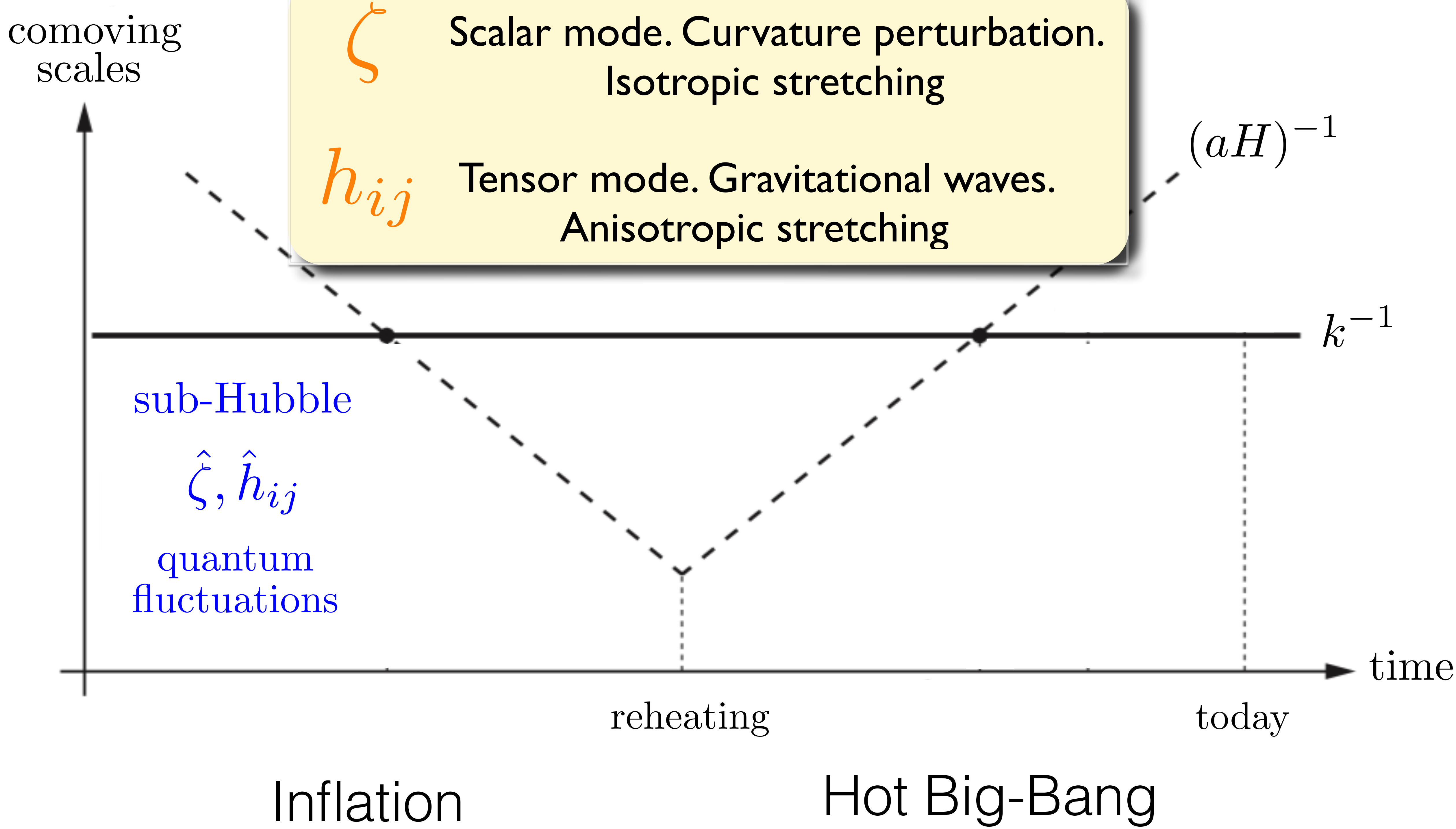

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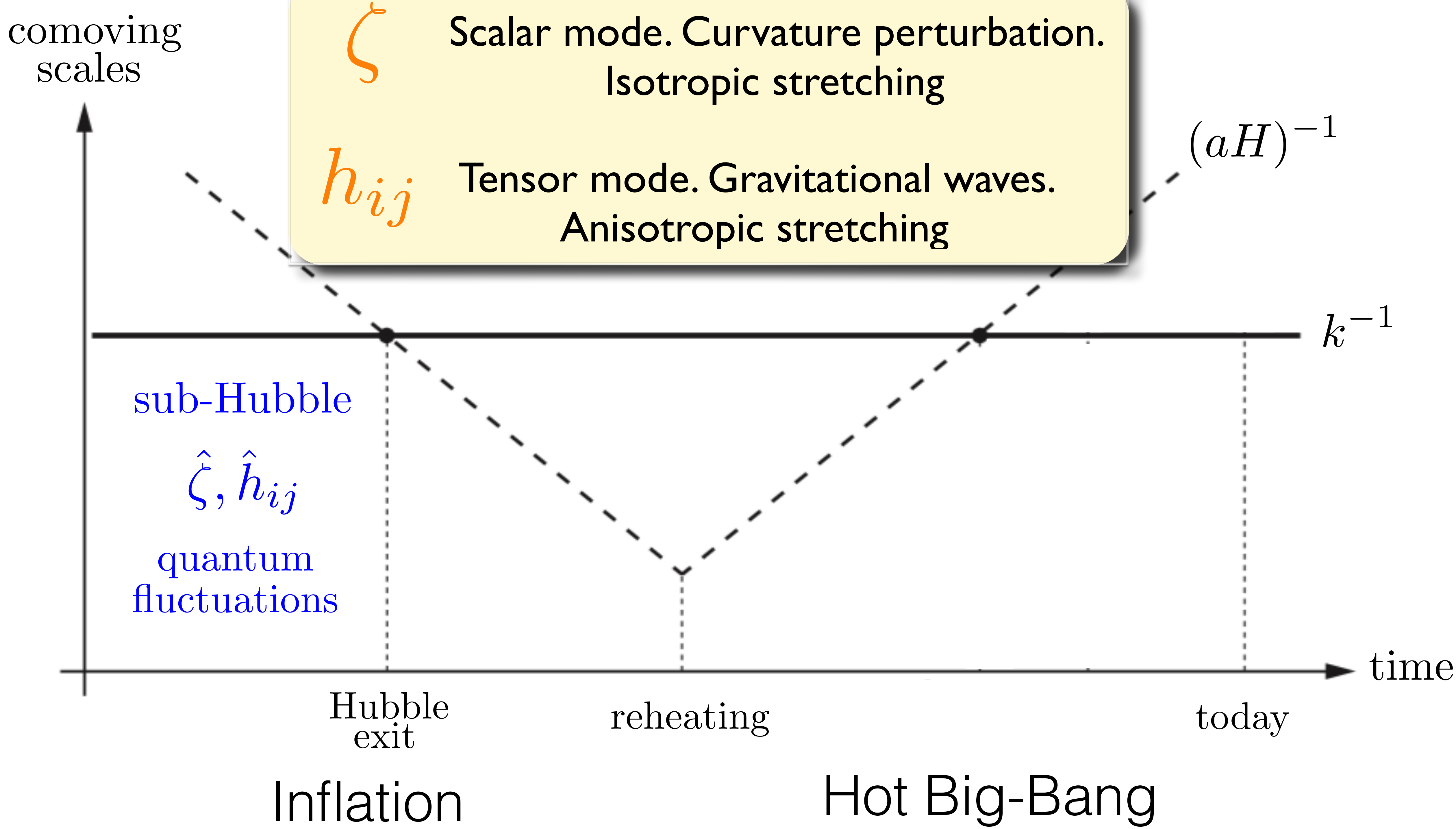

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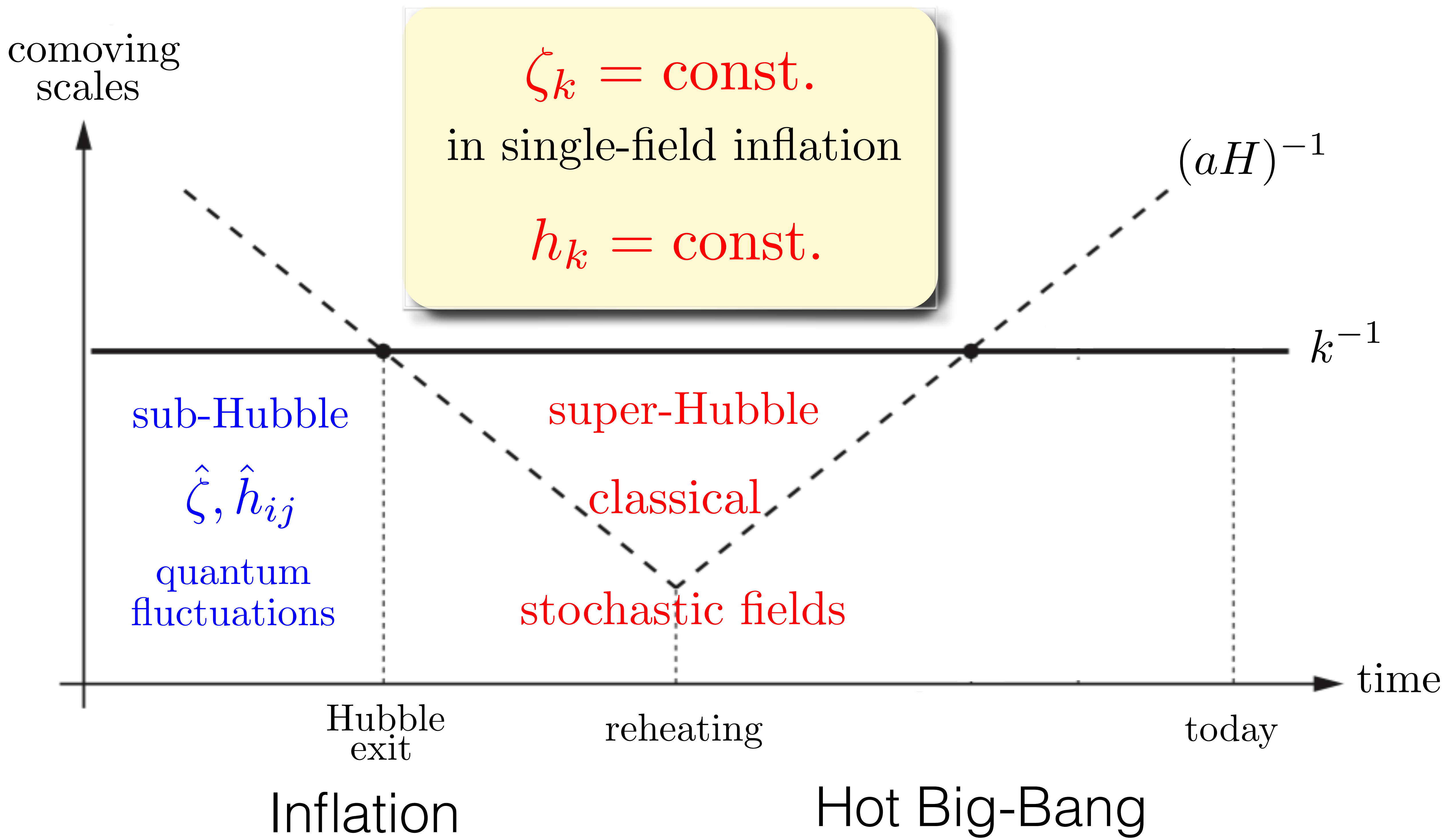

Inflation for fluctuations

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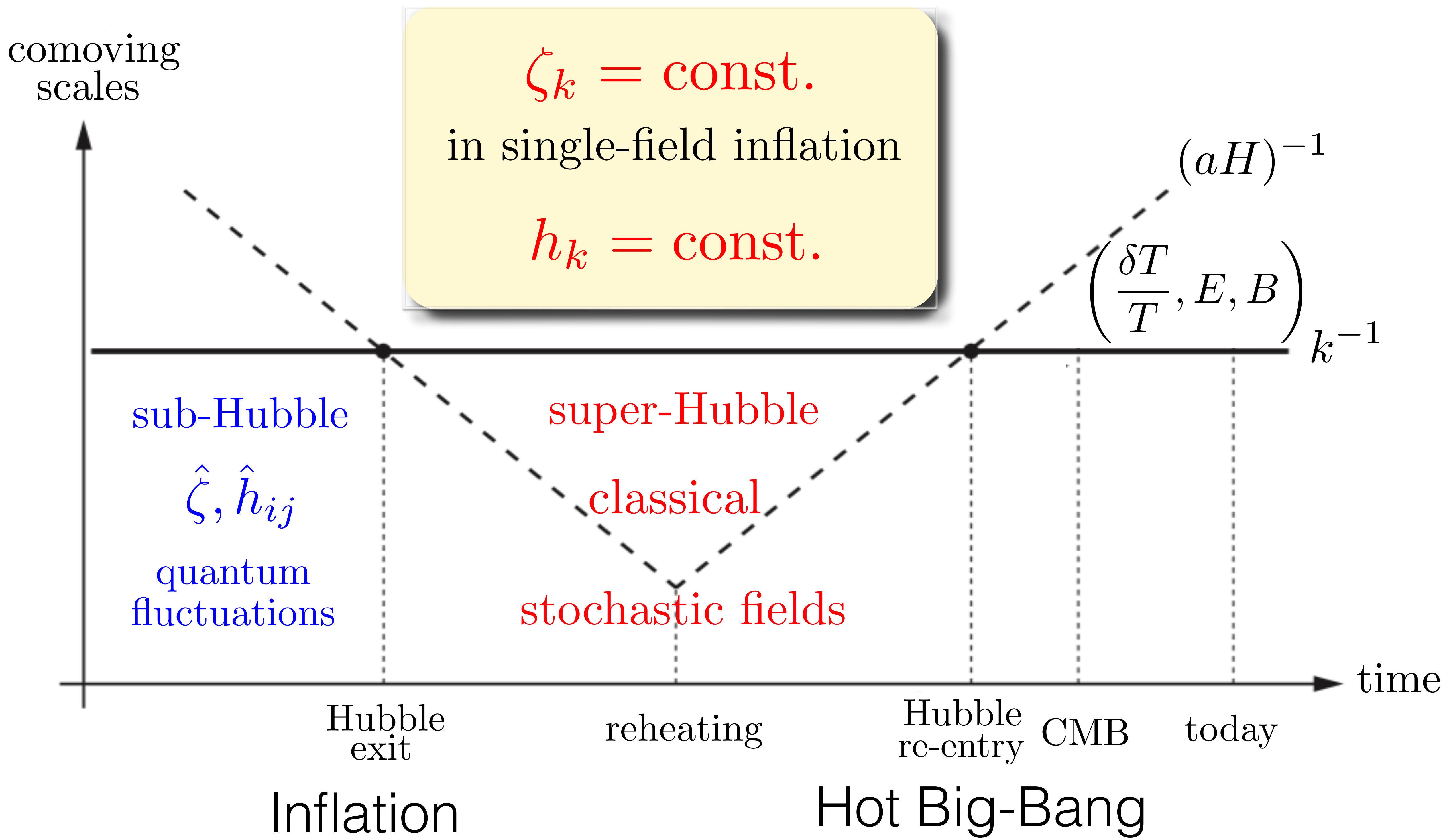
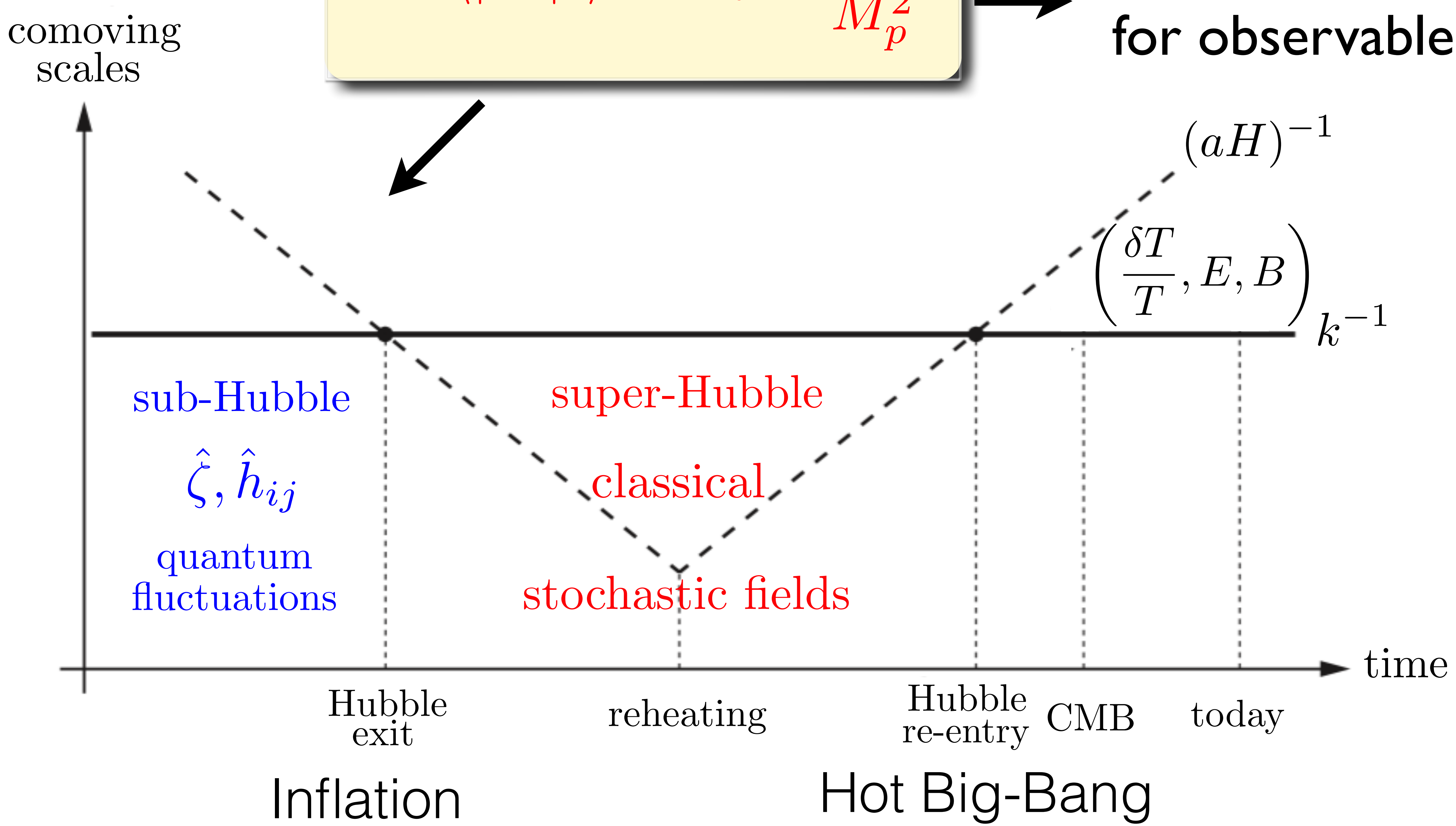


Inflation for fluctuations

$$k^3 \langle |h_k|^2 \rangle \sim \mathcal{P}_t \sim \frac{H^2}{M_p^2}$$

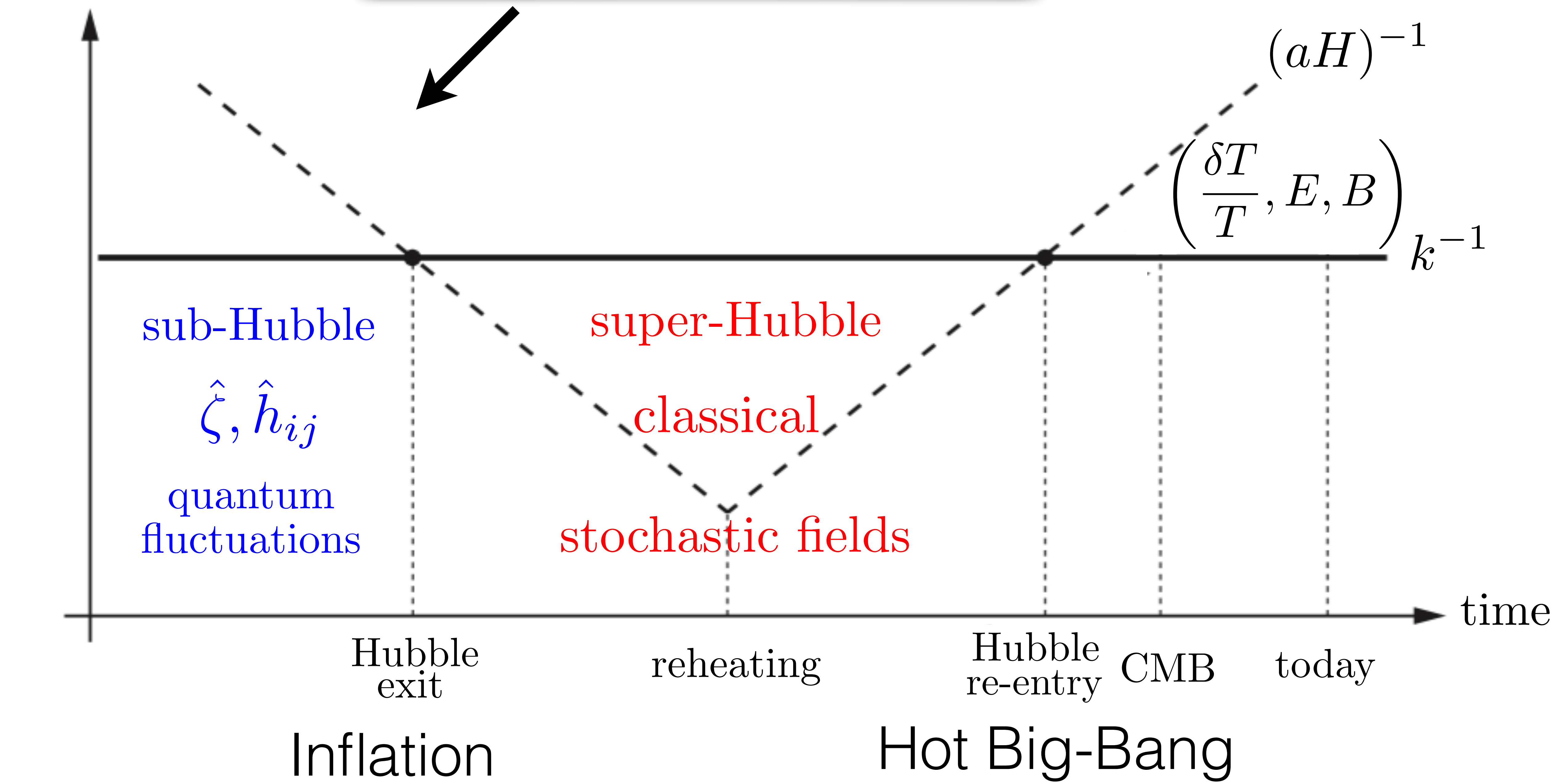
→ Prediction for observables



Inflation for fluctuations

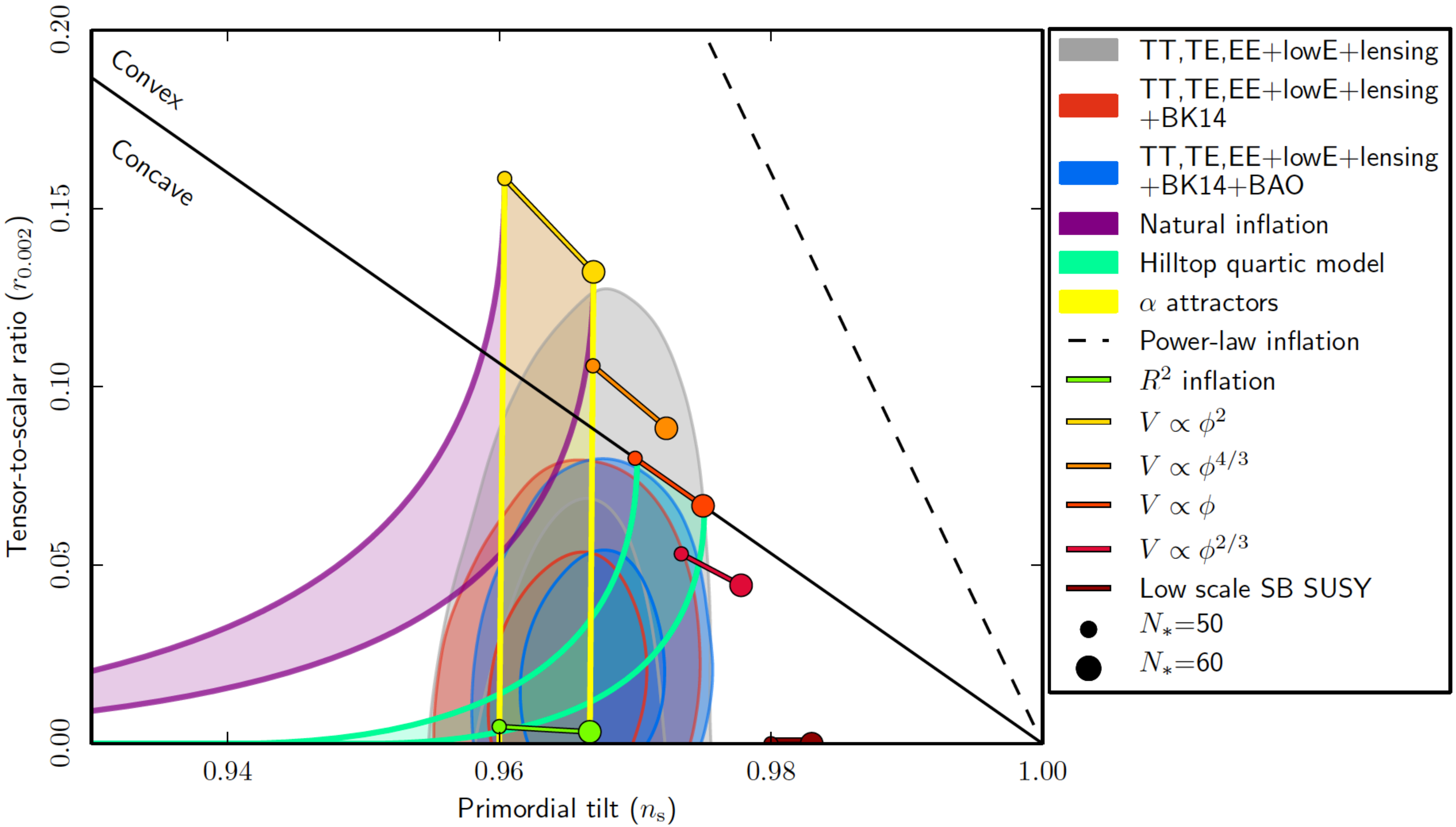
$$k^3\langle|\zeta_k|^2\rangle \sim \mathcal{P}_\zeta \sim \frac{1}{\epsilon}\frac{H^2}{M_{\rm Pl}^2}$$

→ Prediction for observables



$$r = \frac{\mathcal{P}_t(k_\star)}{\mathcal{P}_\zeta(k_\star)} = 16\epsilon_\star$$

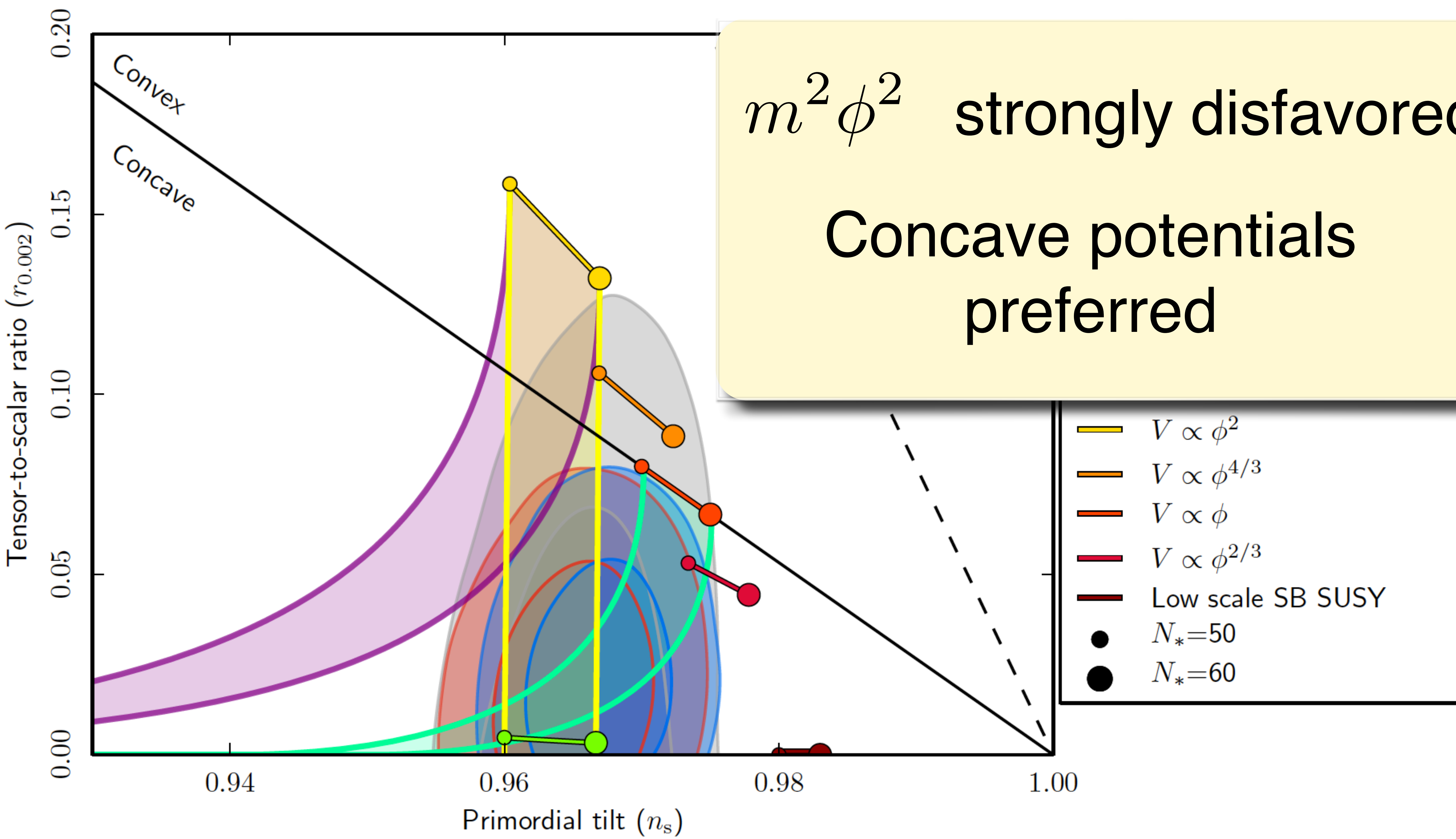
The Planck ns-r plane



$$n_s - 1 = 2\eta_\star - 6\epsilon_\star$$

$$r = \frac{\mathcal{P}_t(k_\star)}{\mathcal{P}_\zeta(k_\star)} = 16\epsilon_\star$$

The Planck ns-r plane



$m^2\phi^2$ strongly disfavored

Concave potentials preferred

$$n_s - 1 = 2\eta_\star - 6\epsilon_\star$$

II

The physics of inflation?

Physics of inflation?

- So far, merely phenomenological description

Single-field slow-roll models:

- decoupled from the rest of physics
- lack UV completions

Physics of inflation?

Primordial universe: invaluable observational probe of high-energy physics

What is the inflaton?

Origin of its potential?

Which extension of the Standard Model?

At which energy inflation occured?

How did it transfert is energy to Standard Model particles?

The only degree of freedom?

Coupling to other fields?

…

Physics of inflation?

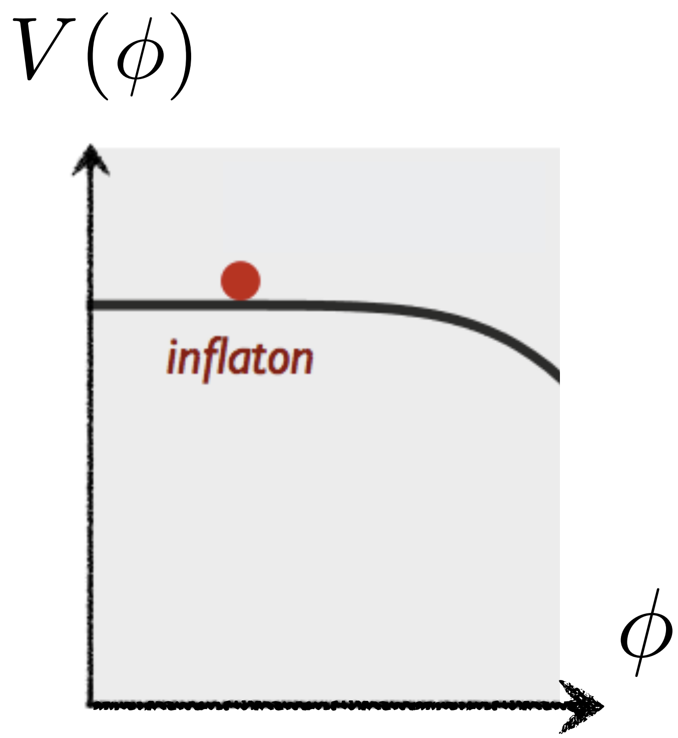
• Inflation is sensitive to the physics at the Planck scale

• Physical understanding and candidate physical theories motivate much more complicated dynamics than the simplest scenarios.

• Single-field slow-roll: at best emergent approximate description

• Cosmologists seek deviations to it in motivated manner

The Eta problem

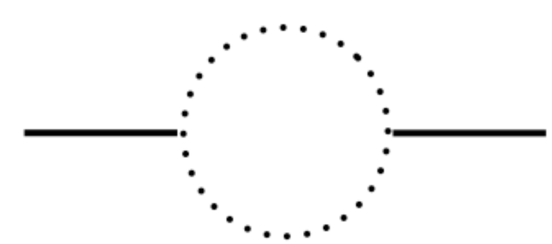


$$\epsilon \simeq \frac{M_{\rm Pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2 \ll 1$$

$$\eta \equiv M_{\rm pl}^2 \frac{V_{,\phi\phi}}{V} \ll 1$$

Why is the inflaton so light? $\quad \eta \approx \frac{m_\phi^2}{H^2} \ll 1$

like the Higgs
hierarchy problem



$$m_\phi^2 \sim \Lambda_{\rm uv}^2 \gg H^2$$

UV-sensitivity of inflation

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V_0(\phi) + \sum_\delta \frac{\mathcal{O}_\delta(\phi)}{M^{\delta-4}}$$

Slow-roll action

Corrections to the low-energy effective potential

Unless forbidden by symmetry: $\Delta V \sim V_0(\phi)\dfrac{\phi^2}{M^2}$

$$\boxed{\Delta m_\phi^2 \sim \frac{V_0}{M^2} \sim H^2\left(\frac{M_{\rm Pl}}{M}\right)^2}$$

$$\Delta\eta \gtrsim 1$$

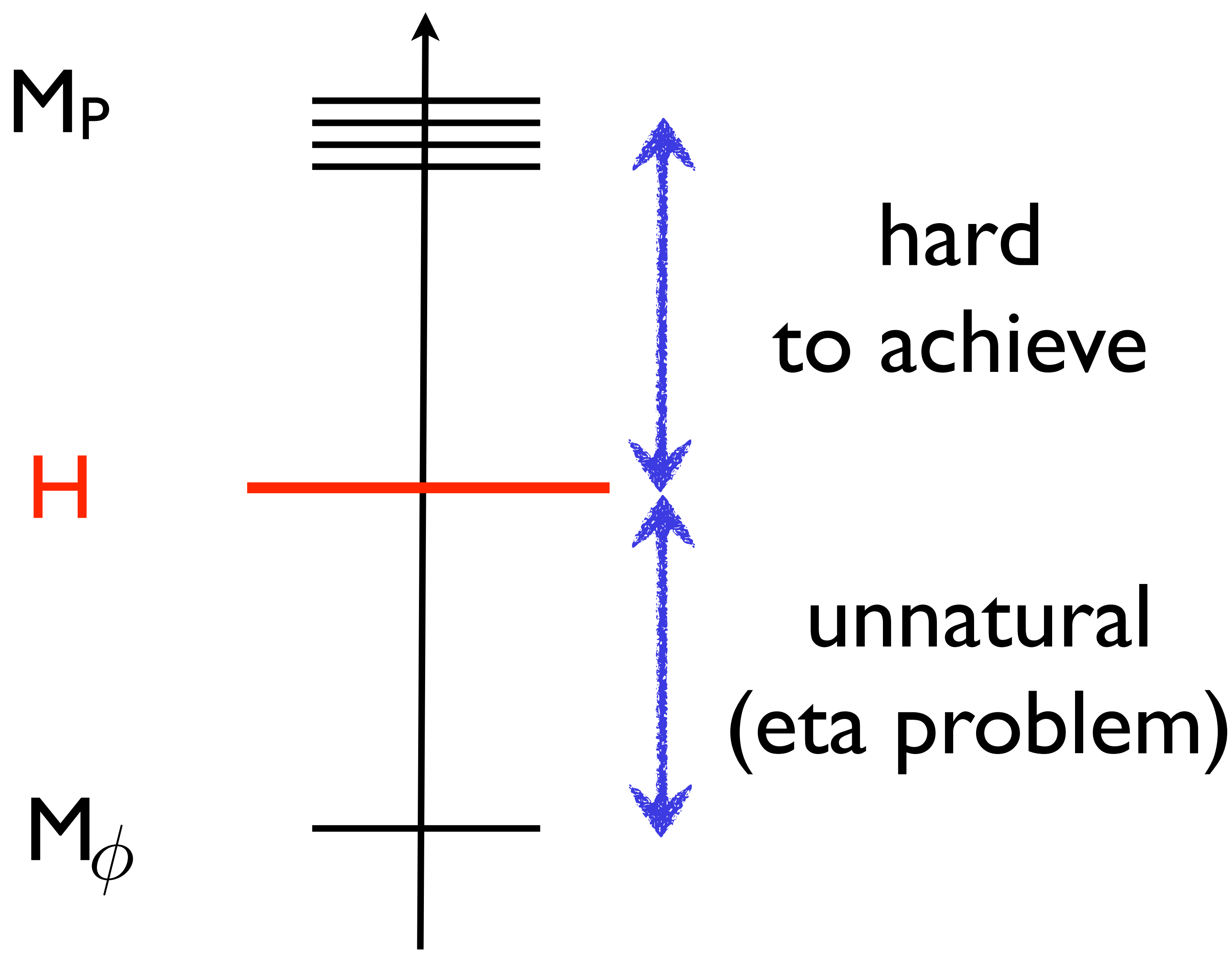
Sensitivity of inflation to Planck-suppressed operators

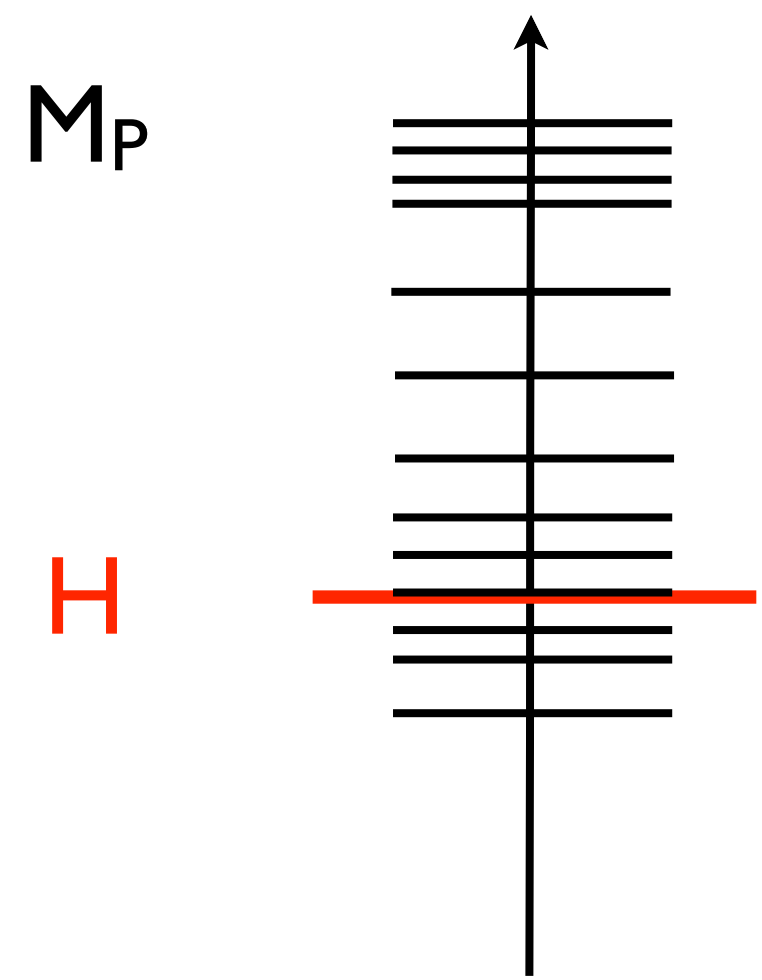
Guidance from UV complete theories

Supergravity, string theory: many degrees of freedom

<u>Hope</u>: light inflaton,
Planck-mass moduli

<u>Find</u>: many fields
of different masses





Guidance from UV complete theories

Supergravity, string theory: many degrees of freedom

Hope: light inflaton, Planck-mass moduli

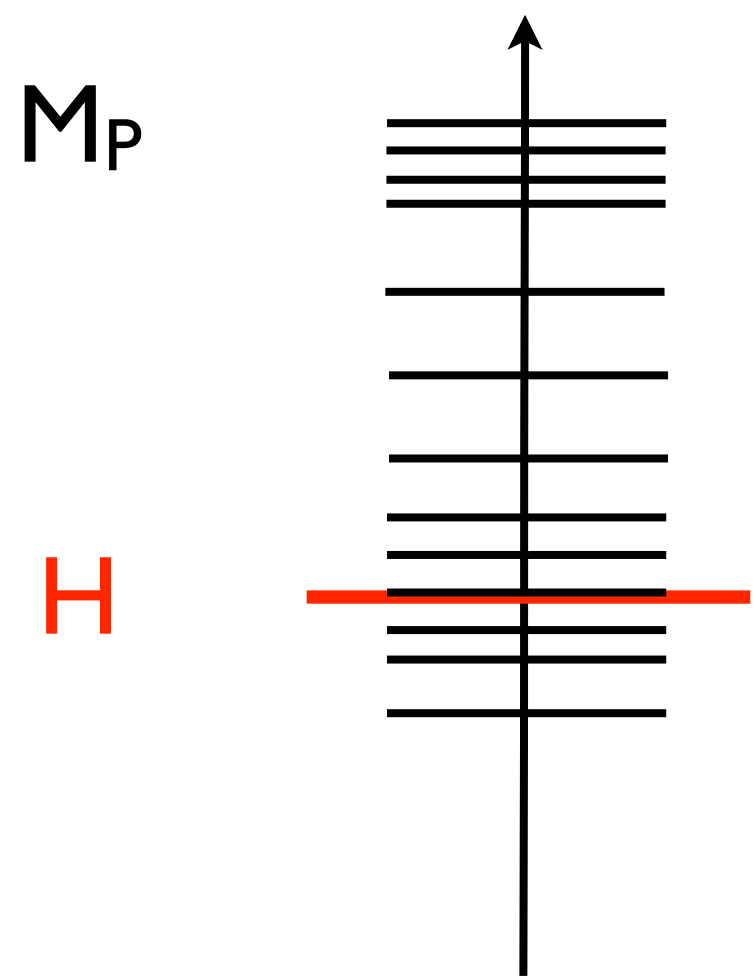
Find: many fields of different masses

Multiple (light and heavy) degrees of freedom:
zeta not conserved

Steep potentials

Large couplings

Time-dependent masses and couplings



Guidance from UV complete theories

Heavy fields substantially modify the slow-roll picture

$$\mathcal{L}(\phi, \chi) \xrightarrow[\text{Integrating out}]{\chi \text{ heavy}} \tilde{\mathcal{L}}(\phi) \neq \mathcal{L}_{\mathrm{slow-roll}}$$

$$\mathcal{L} \supset \frac{\epsilon}{c_s^2}\left(\dot{\zeta}^2 - c_s^2 \frac{(\partial\zeta)^2}{a^2}\right)$$

Reduced 'speed of sound' of fluctuations ...

$$+\left(\frac{1-c_s^2}{H}\right)\dot{\zeta}\frac{(\partial\zeta)^2}{a^2}$$

... comes with derivative interactions

Guidance from UV complete theories

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Reduced 'speed of sound' of fluctuations ...

$$+\left(\frac{1-c_s^2}{H}\right)\dot{\zeta}\frac{(\partial\zeta)^2}{a^2}$$

... comes with derivative interactions

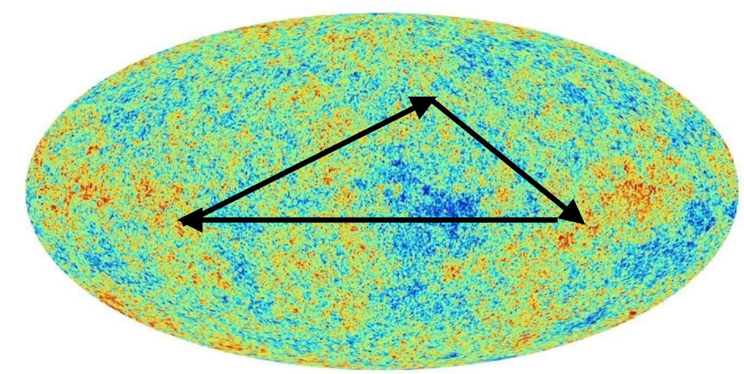
… + signatures of particle production not captured by single-field EFT

Observational program

Primordial gravitational waves and refined power spectrum

Primordial non-Gaussianities

Energy scale of inflation
up to 10^{16} GeV



Super-Planckian field displacement:
Quantum Gravity is required

Rule out all simplest
models of inflation

Beyond simplest models

$$f_{\rm NL} = \mathcal{O}(\epsilon, \eta) \sim 10^{-2}$$ Maldacena (03)

UNDER HYPOTHESES

- Single field
- Standard kinetic term
- Slow-roll
- Initial vacuum state
- Einstein gravity

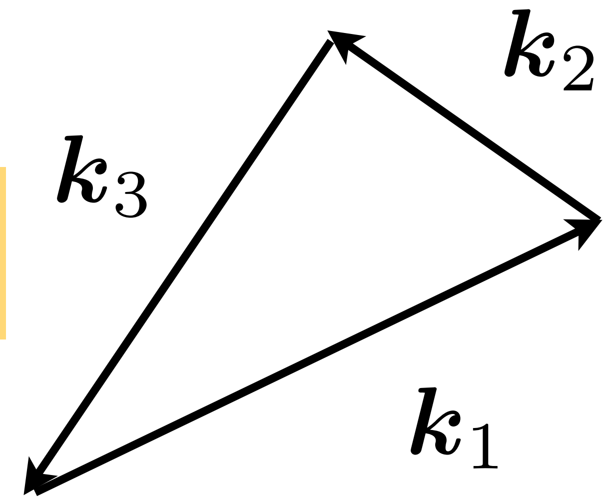
Violating any of these assumptions in general leads to observably large NGs.

$$f_{\rm NL} \gtrsim \mathcal{O}(1)$$

and we have a dictionary between physical effects and types of non-Gaussianities

The bispectrum

$$\langle \zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\zeta(\boldsymbol{k}_3)\rangle = (2\pi)^7 \delta(\sum_{i=1}^{3} \boldsymbol{k}_i)\mathcal{P}_\zeta^2 \frac{S(k_1,k_2,k_3)}{(k_1 k_2 k_3)^2}$$



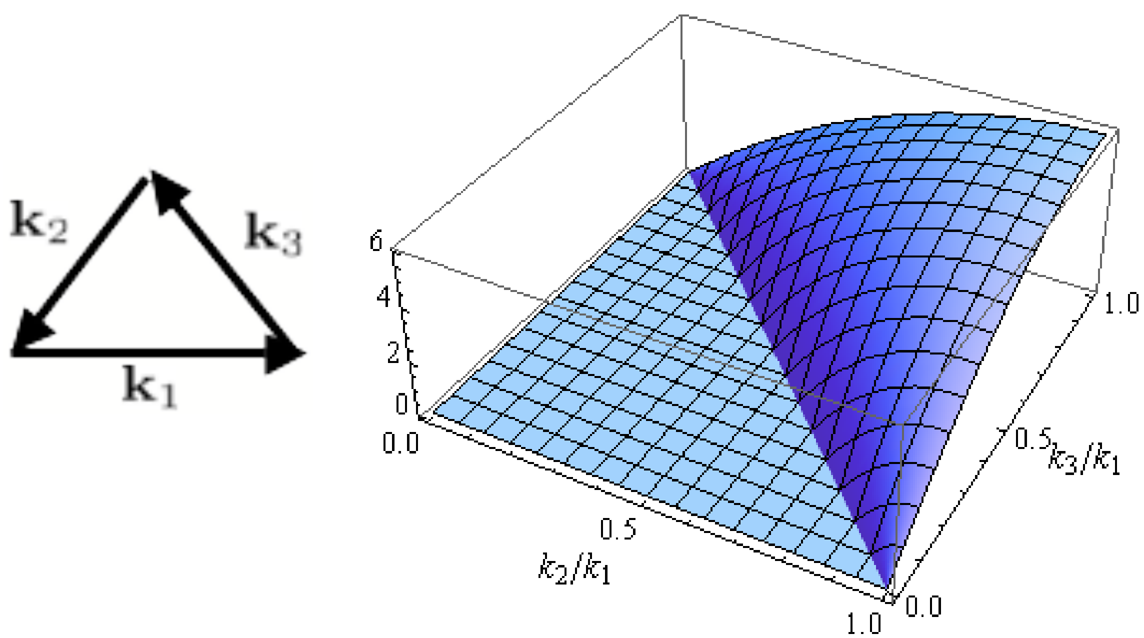
- $f_{NL} \sim S$ dimensionless measure of the **amplitude** of the bispectrum

- **Shape** (dependence on the configuration of triangles)

- **Scale-dependence** (growing or shrinking on small scales?)

Inflationary physics and non-Gaussian shapes

Equilateral type (quantum)



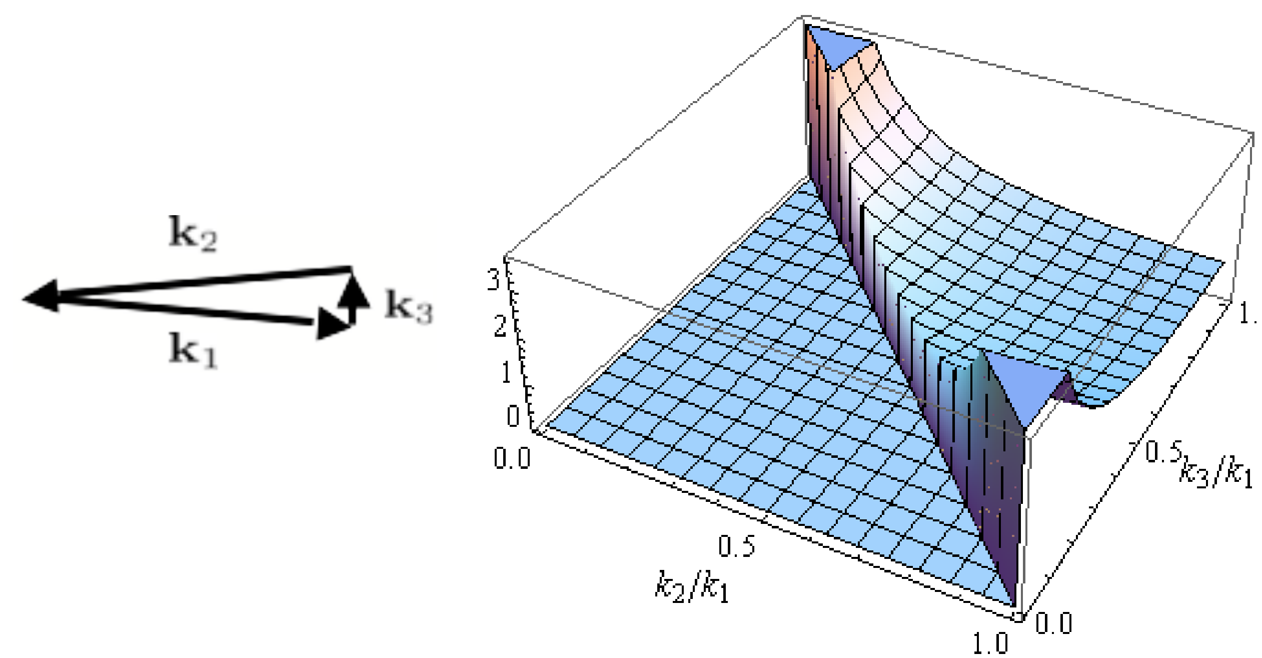
$$f_{\rm NL}^{\rm eq} = -26 \pm 47$$

Planck 18

Heavy field and
derivative interactions

$$f_{\rm NL}^{\rm eq} \sim \frac{1}{c_s^2} - 1 \qquad c_s \geq 0.021$$

Local type (classical)

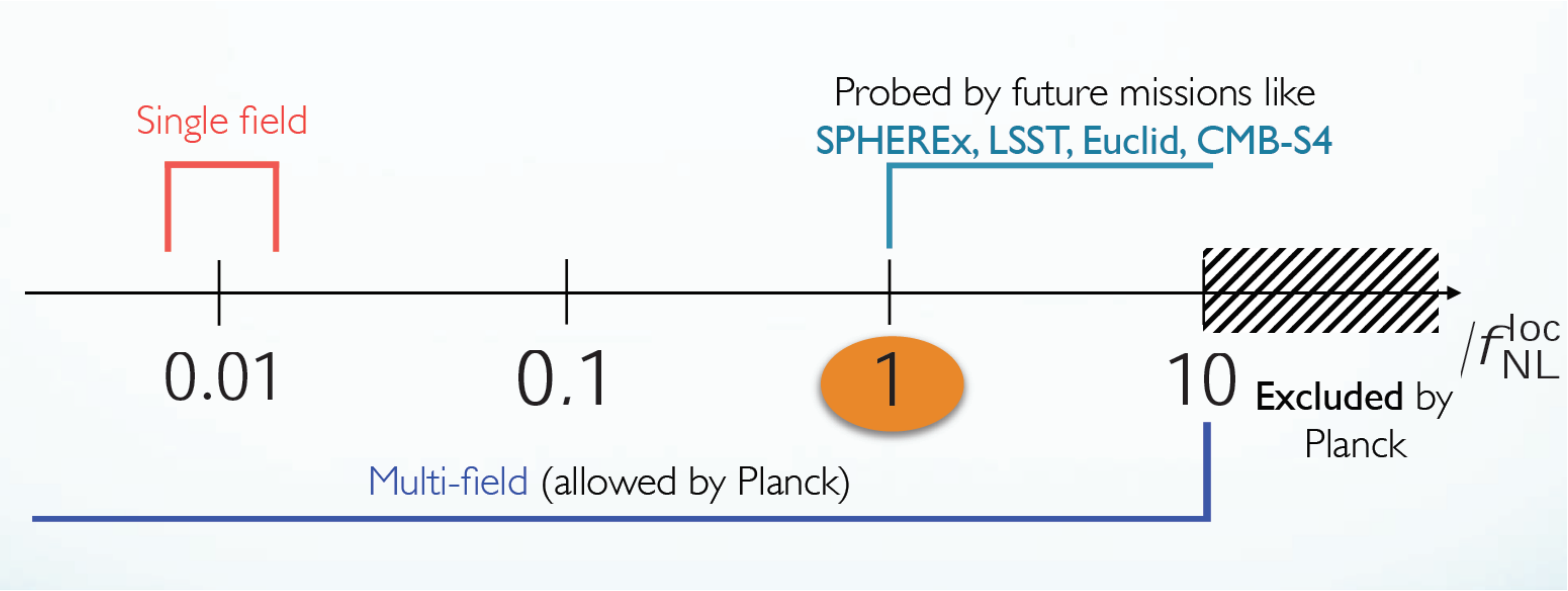


$$f_{\rm NL}^{\rm loc} = -0.9 \pm 5.1$$

Planck 18

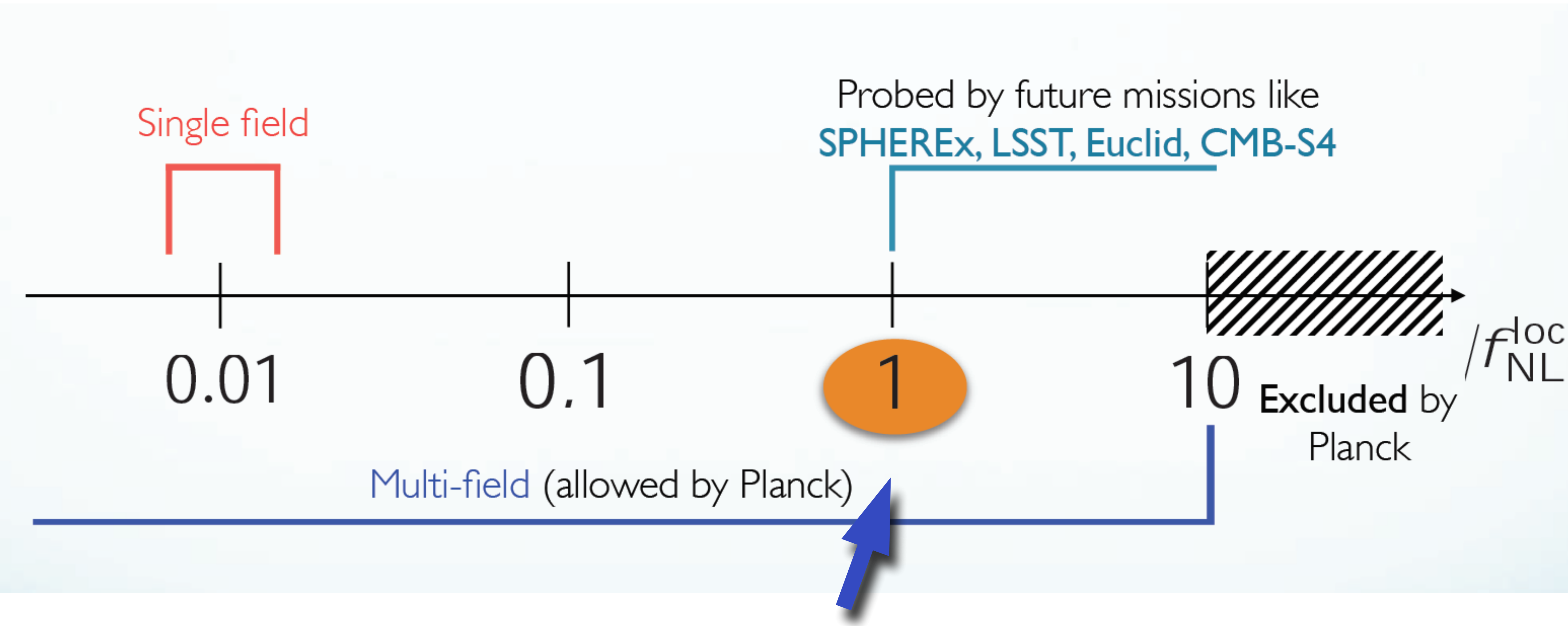
Light degrees of freedom
beyond the inflaton

Prospects

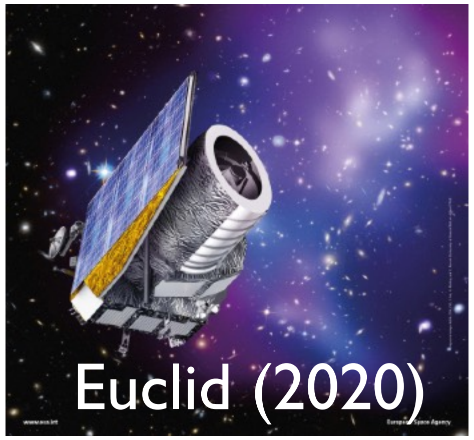


Single field
Probed by future missions like
SPHEREx, LSST, Euclid, CMB-S4
0.01
0.1
1
10
Excluded by
Planck
$|f_{NL}^{loc}|$
Multi-field (allowed by Planck)

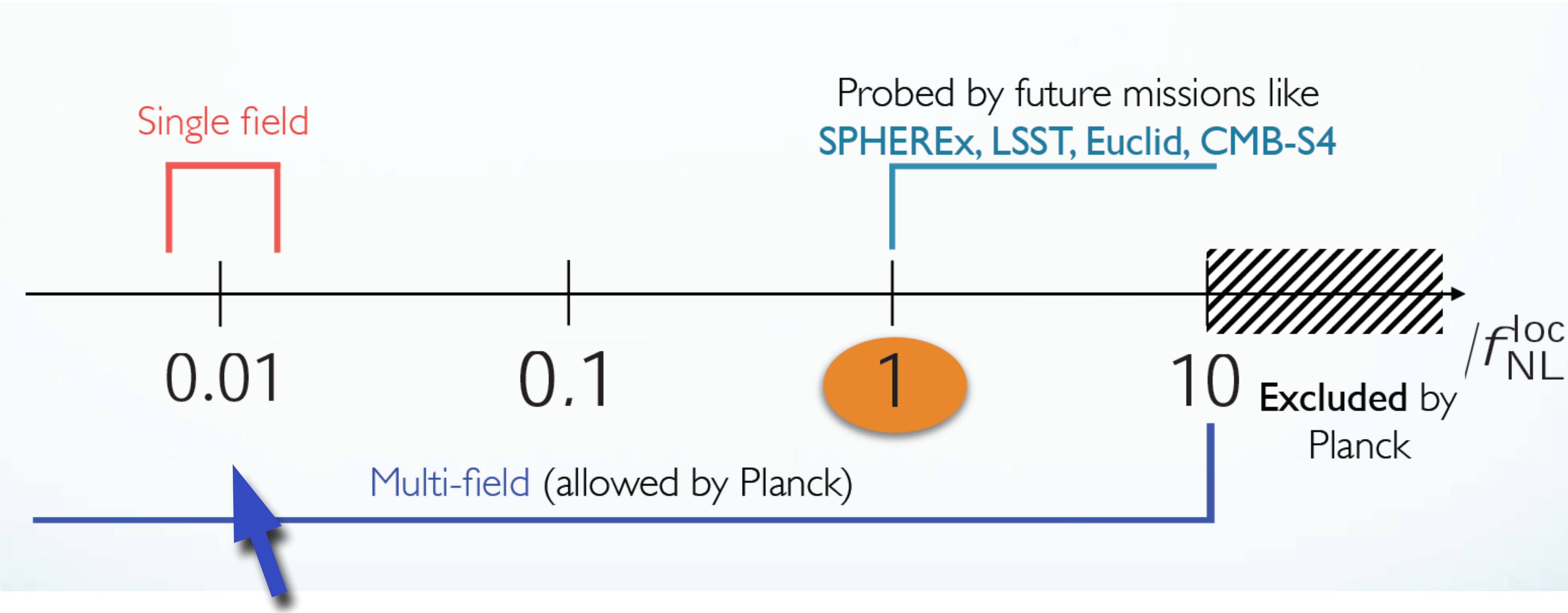
Prospects

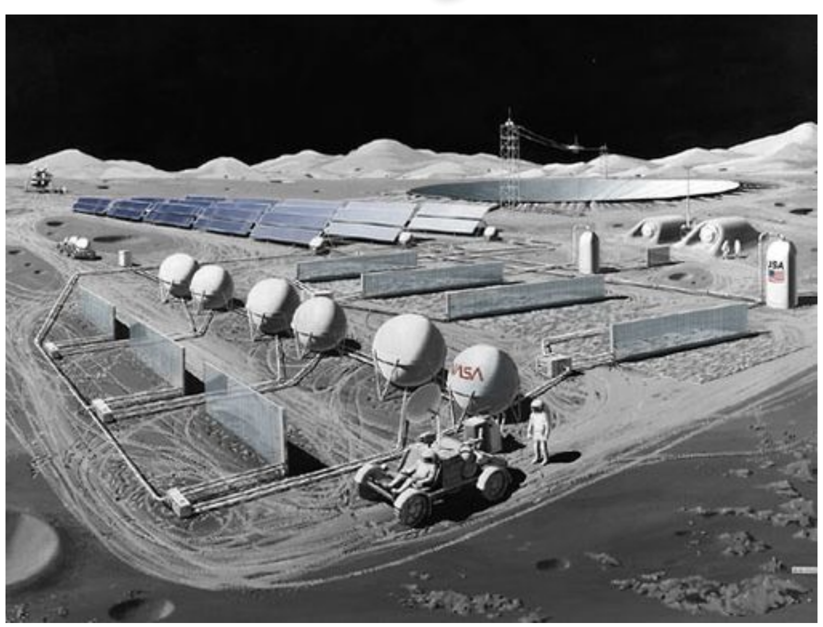


Huge efforts to reach this sensitivity with large-scale structure survey and scale-dependent bias


Euclid (2020)

Prospects



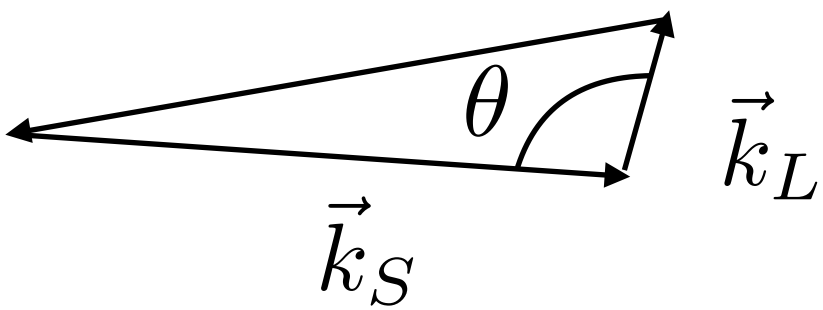


21cm emission from hydrogen
clouds during dark ages

radio-astronomy
from the far side of the moon!

Cosmological collider physics, aka NGs in soft limits

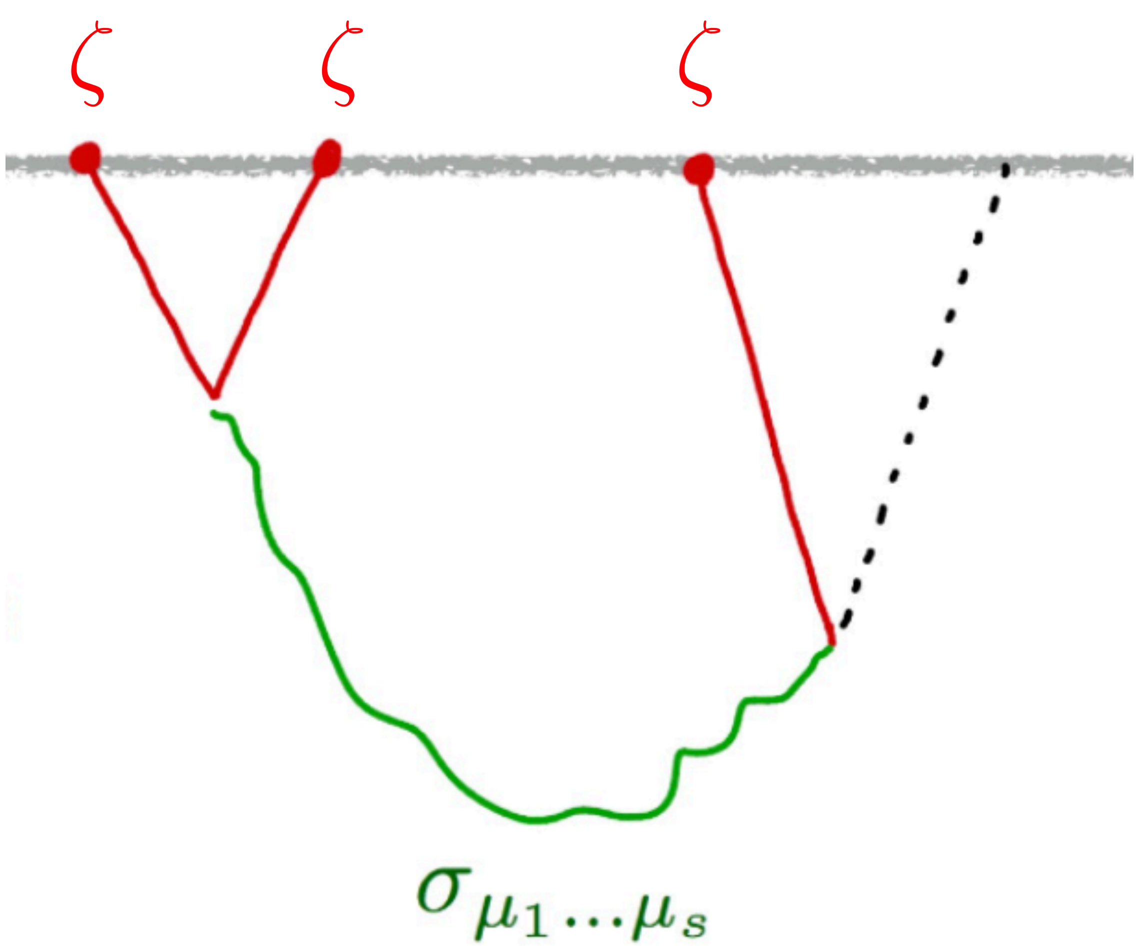
3pt



$$\lim_{k_L\to 0}\langle\zeta_{\vec{k}_L}\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\rangle \propto \left(\frac{k_L}{k_S}\right)^{3/2}\cos\left[\frac{M}{H}\ln\left(\frac{k_L}{k_S}\right)+\delta\right]P_S(\cos\theta)$$

Mass & Spin

of heavy exchanged particle



Chen, Wang 2009
Baumann Green 2011
Noumi, Yamaguchi, Yokohama 2012
Arkani-Hamed, Maldacena 2015
Lee, Bauman, Pimentel 2016
Arkani-Hamed, Baumann, Lee, Pimentel 2018
…

Cosmological collider physics, aka NGs in soft limits

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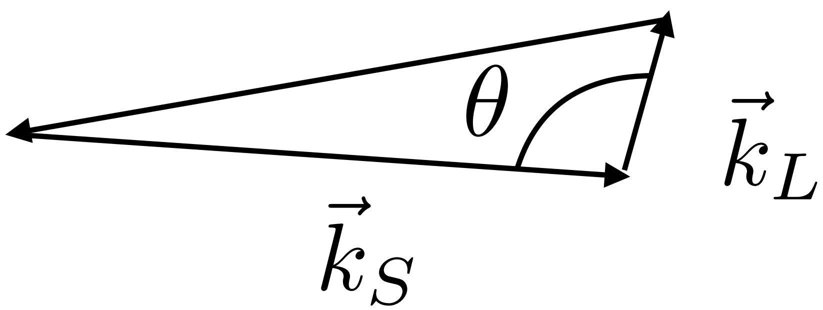

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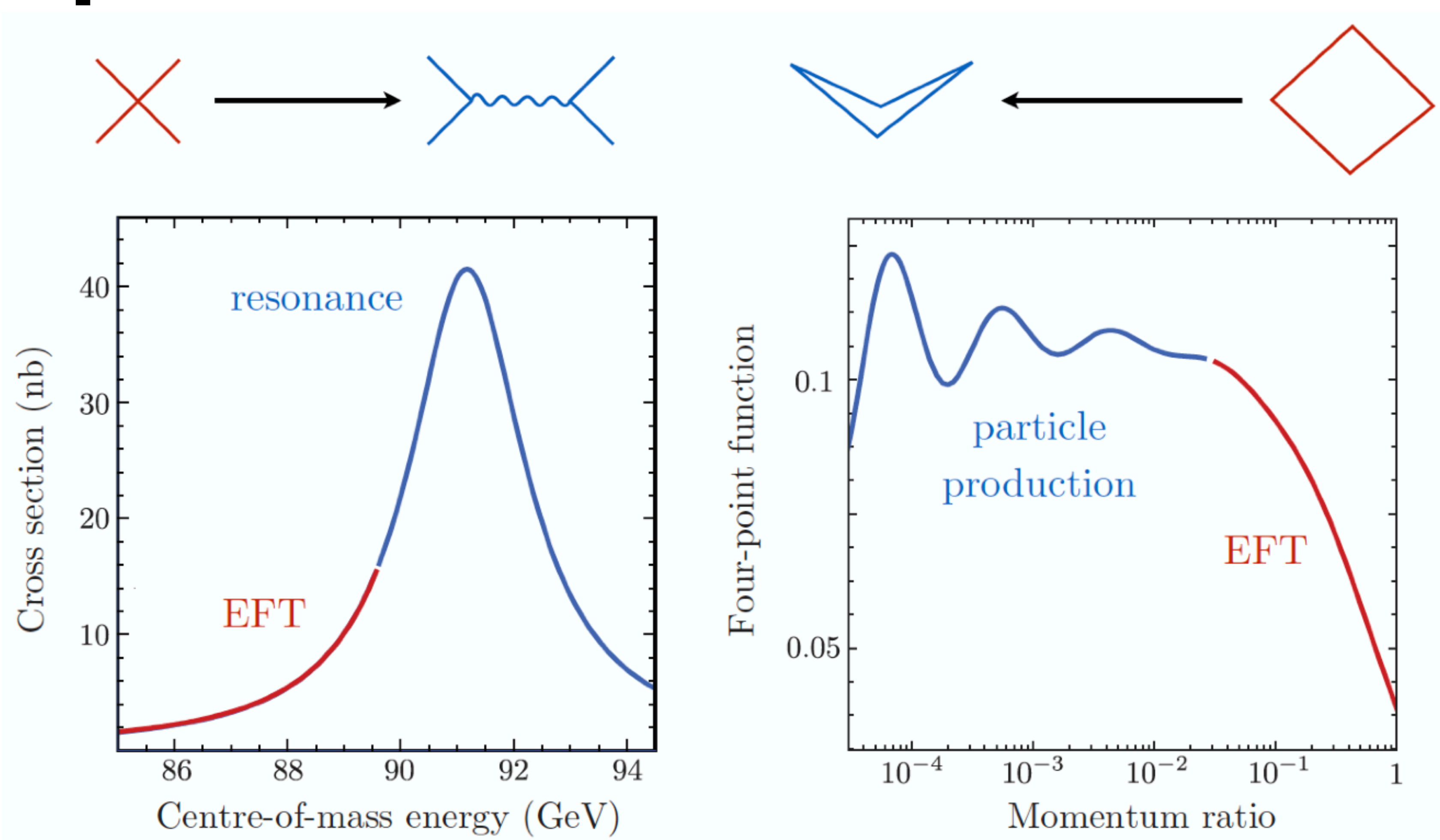
Mass & Spin

of heavy exchanged particle

4pt



Credit: 1811.00024

Chen, Wang 2009
Baumann Green 2011
Noumi, Yamaguchi, Yokohama 2012
Arkani-Hamed, Maldacena 2015
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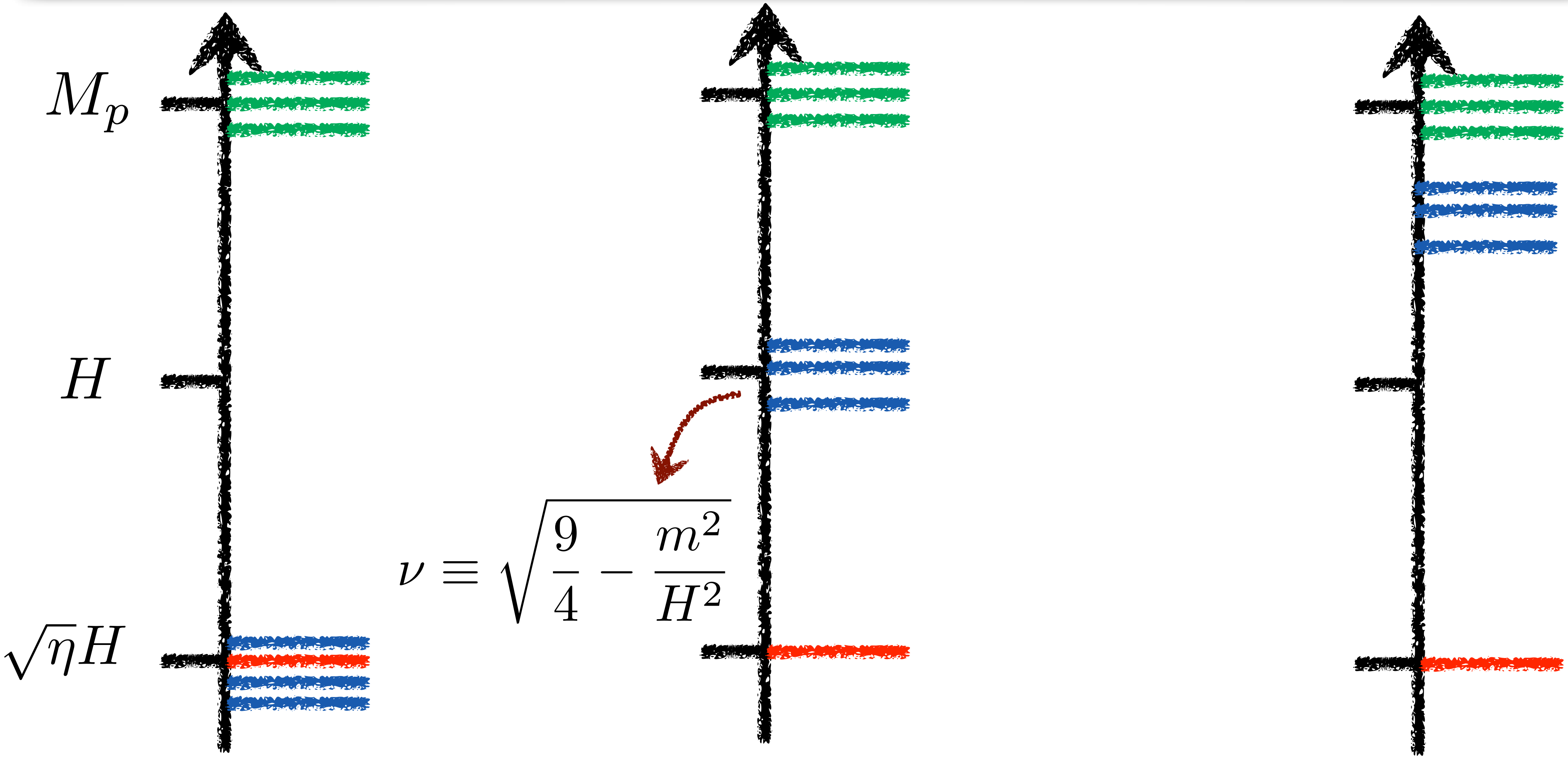
…

Non-Gaussianity as a particle detector



$$\nu \equiv \sqrt{\frac{9}{4} - \frac{m^2}{H^2}}$$

	Light dof	Quasi-single-field	Heavy dof
$\lim_{k_L \to 0} k_L^3 \langle \zeta_{\vec{k}_L} \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle$	1	$\left(\frac{k_L}{k_S}\right)^{\frac{3}{2}-\nu}$	$\left(\frac{k_L}{k_S}\right)^{\frac{3}{2}} \cos\left[\lvert\nu\rvert \ln\left(\frac{k_L}{k_S}\right) + \delta\right]$

Physics of inflation?

- Single-field slow-roll: at best emergent approximate description

- Cosmologists seek deviations to it in motivated manner

- Inflation provides us with a cosmological collider

Looking for new physics…

…is looking for multifield effects

III

Curved field space
and geometrical destabilization of inflation

Beyond toy-models: multifield inflation

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(-\frac{1}{2}G_{IJ}(\phi^K)\partial_\mu\phi^I\partial^\mu\phi^J - V(\phi^I)\right)$$

Curved field space is generic

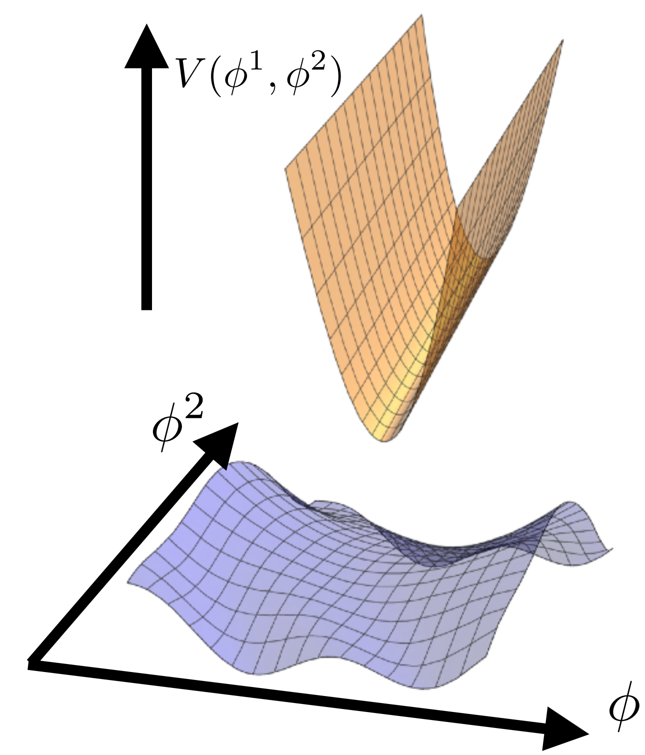
Top-down (e.g. sugra) or bottom-up (EFT)

Studied for a long time, but impact and consequences not fully appreciated: (e.g.)

- Landscape studies with random potential

- conditions for successful inflation usually (wrongly) formulated only in terms of Hessian of potential

$$\nabla^I\nabla_J V/V$$



What to expect?

Potential:

$$V \sim \bar{V} + V'\delta\phi + \frac{1}{2}V''\delta\phi^2 + \frac{1}{6}V'''\delta\phi^3 + \ldots$$

Height	Slope & Curvature	Self interactions
$\to r$	$\to (n_s, r)$	$\to f_{\rm NL}$

What to expect?

Potential:

$$V \sim \bar{V} + V'\delta\phi + \frac{1}{2}V''\delta\phi^2 + \frac{1}{6}V'''\delta\phi^3 + \ldots$$

Height	Slope & Curvature	Self interactions
$\rightarrow r$	$\rightarrow (n_s, r)$	$\rightarrow f_{\rm NL}$

Geometry:

$$G \sim \delta + 0 + R_{\rm fs}\delta\phi^2 + \nabla R_{\rm fs}\delta\phi^3 + \ldots$$

in local inertial frame

What to expect?

Potential:

$$V \sim \bar{V} + V'\delta\phi + \frac{1}{2}V''\delta\phi^2 + \frac{1}{6}V'''\delta\phi^3 + \dots$$

Height $\rightarrow r$

Slope & Curvature $\rightarrow (n_s, r)$

Self interactions $\rightarrow f_{\rm NL}$

Geometry:

$$G \sim \delta + 0 + R_{\rm fs}\delta\phi^2 + \nabla R_{\rm fs}\delta\phi^3 + \dots$$

in local inertial frame

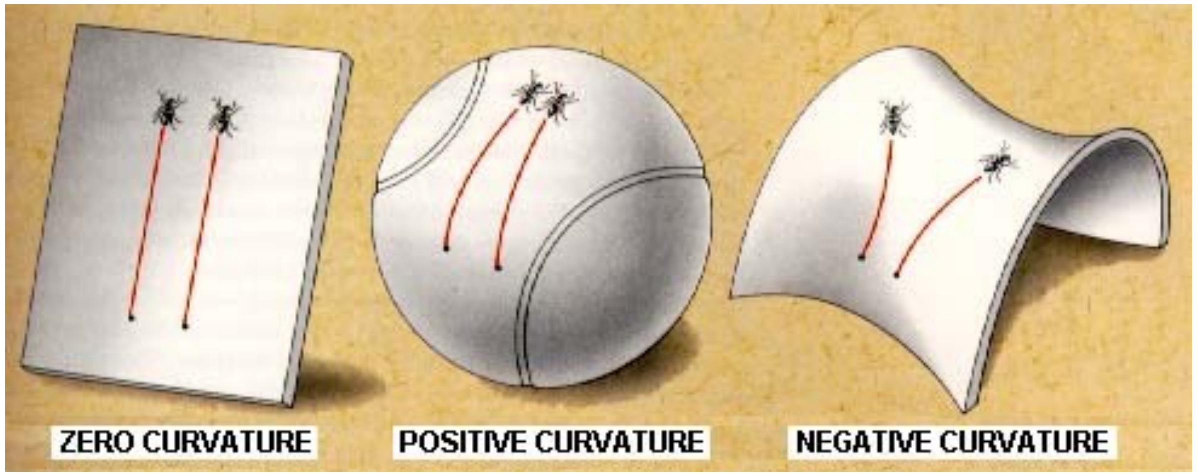
Riemann curvature and derivatives

Field space curvature $\sim 1/M^2$ $\quad H \lesssim M \lesssim M_{\rm Pl}$

Impacts: stability of background, linear fluctuations, NGs

Geometrical destabilization of inflation

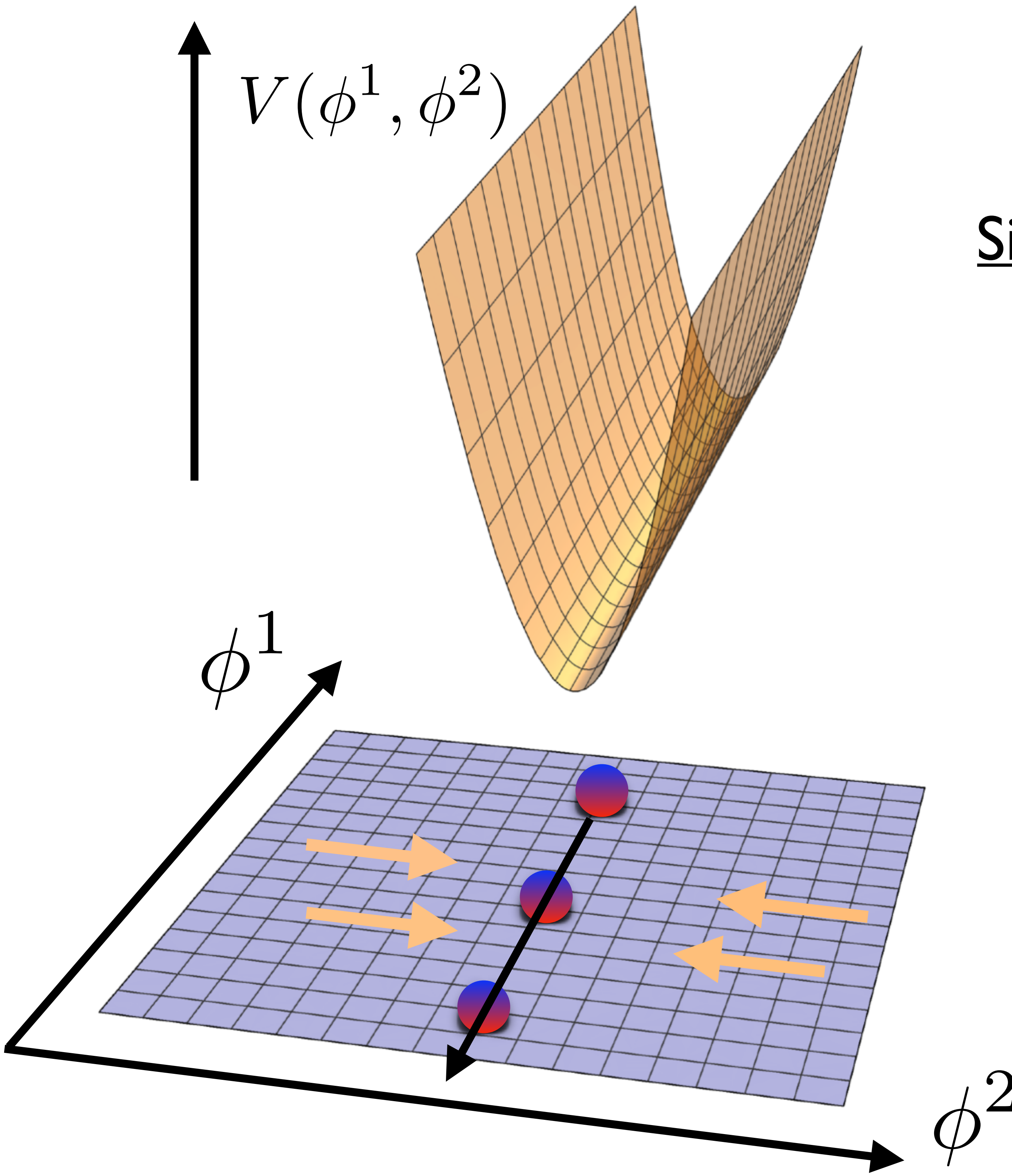
Initially neighboring geodesics tend to fall away from each other in the presence of negative curvature.



This effect applies during inflation,
it easily overcomes the effect of the potential,
and can destabilize inflationary trajectories.

Basic mechanism

Renaux-Petel, Turzynski, 16
PRL Editors' Highlight



<u>Simplest 'realistic' models (hope):</u>

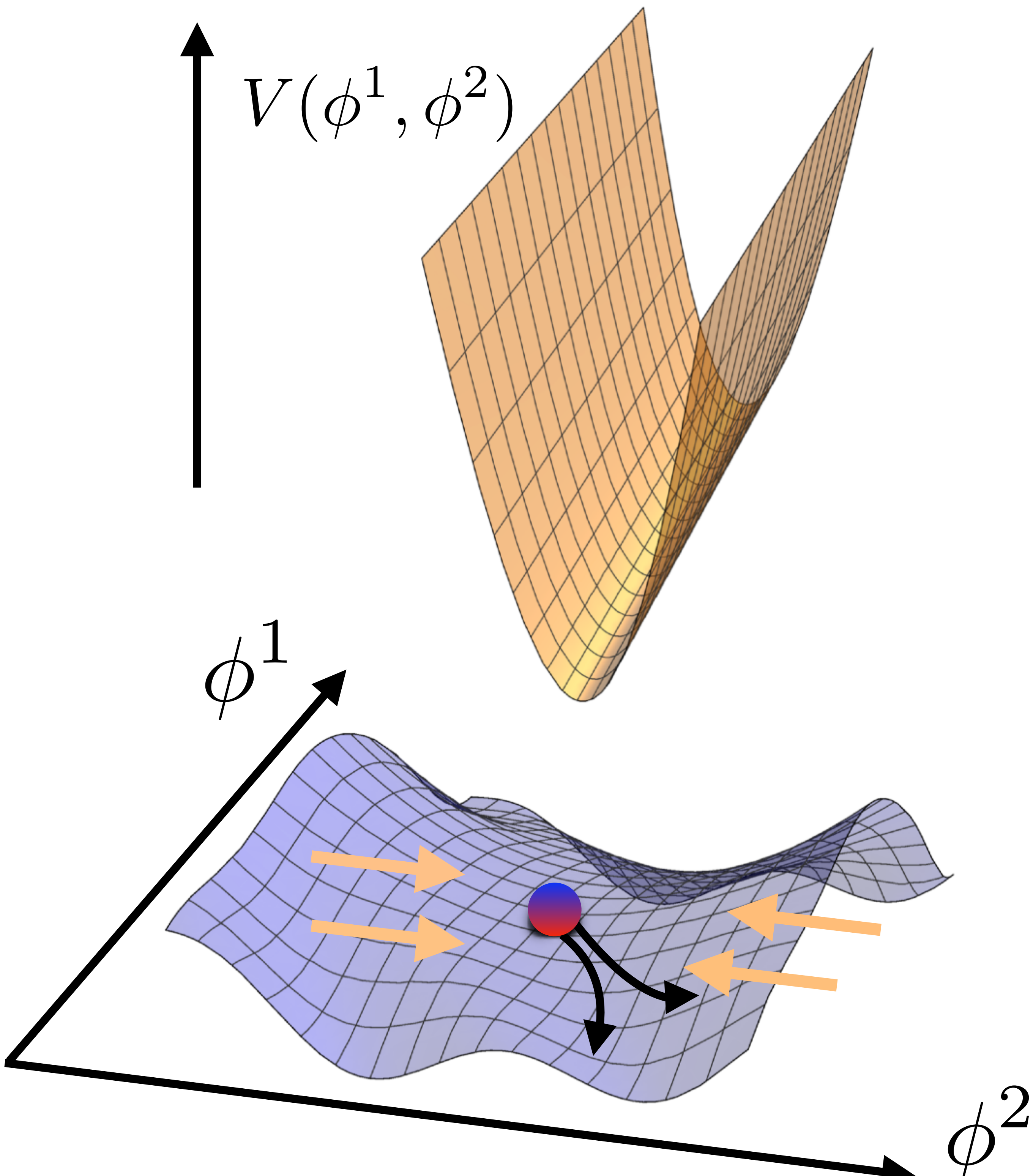
Light inflaton
+
Extra heavy fields

Effective
single-field dynamics

(valley with steep walls)

Basic mechanism

Renaux-Petel, Turzynski, 16
PRL Editors' Highlight



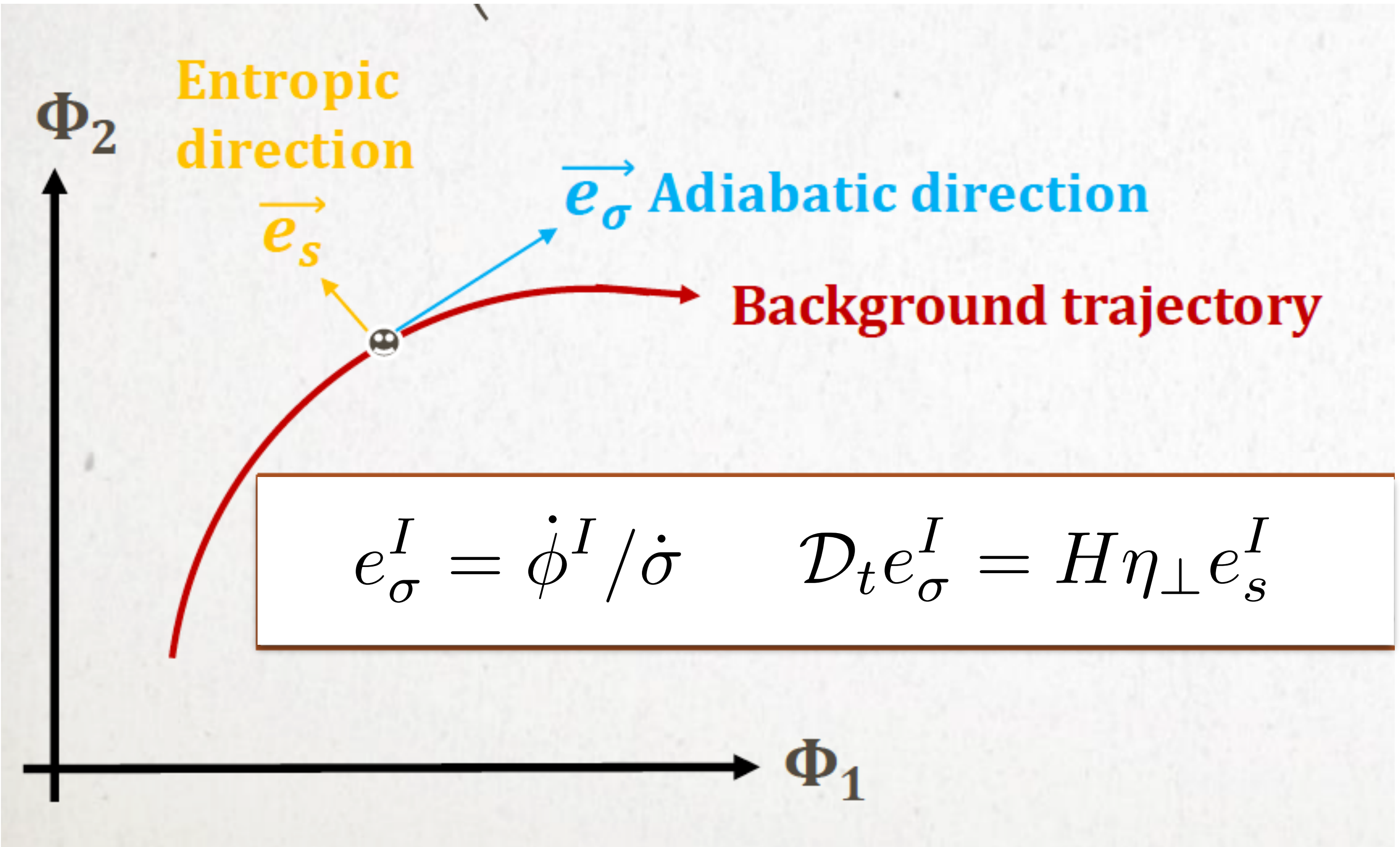
<u>More realistic:</u>

Light inflaton
+
Extra +/- heavy fields
+
Curved field space

Competing effects of potential and geometry

Geometrical instability

Two-field models (simplicity)

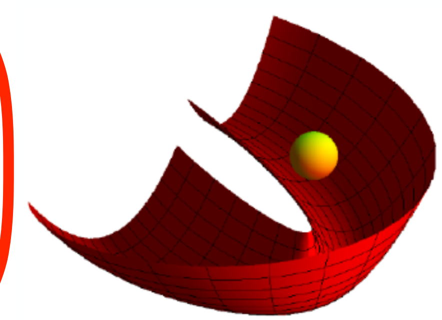


$$e^I_\sigma = \dot{\phi}^I/\dot{\sigma} \qquad \mathcal{D}_t e^I_\sigma = H\eta_\perp e^I_s$$

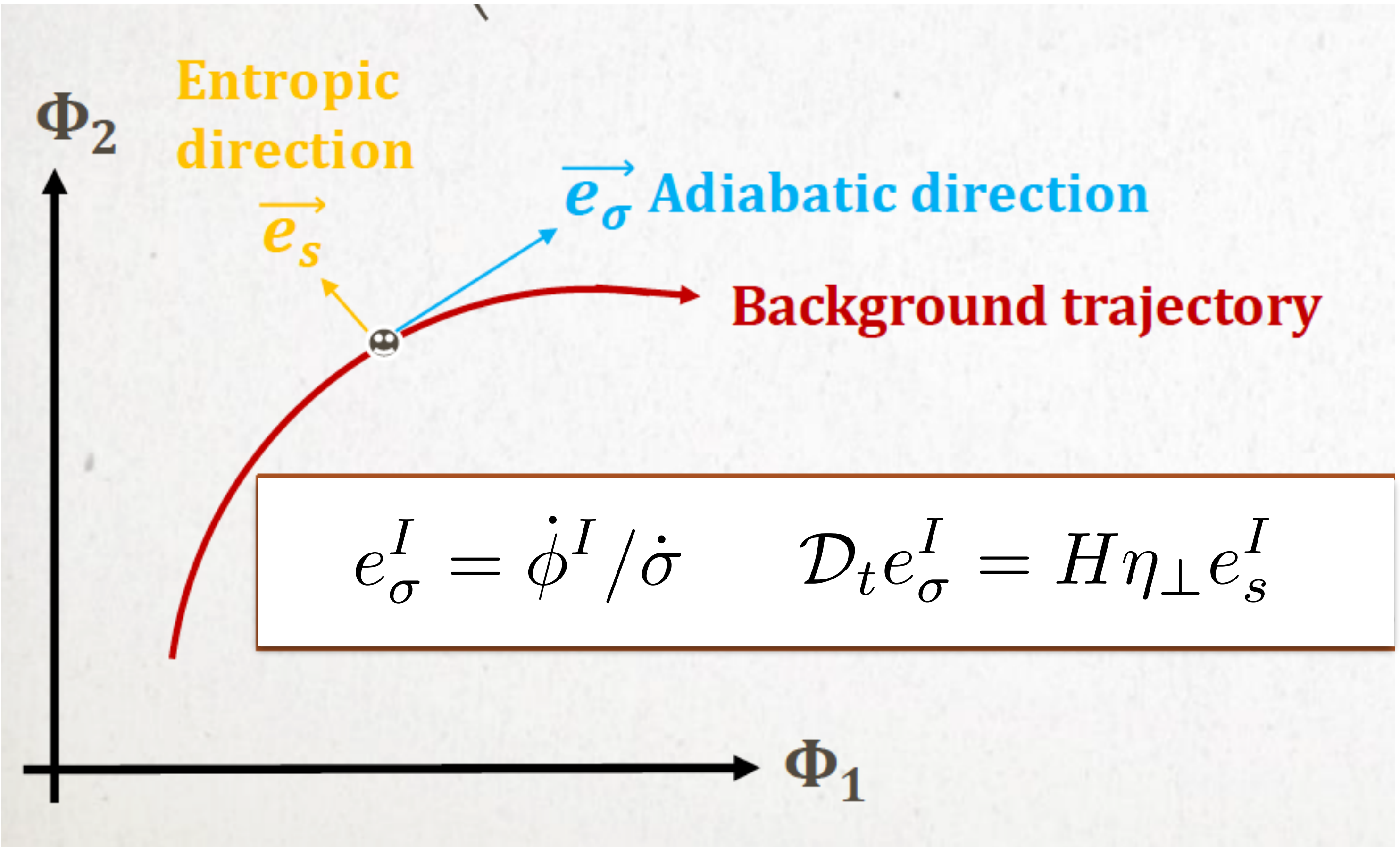
$$\dot{\sigma}^2 = G_{IJ}\dot{\phi}^I\dot{\phi}^J = 2M_{\rm Pl}^2 H^2 \epsilon$$

adiabatic and entropic vectors: local orthonormal frame adapted to dynamics

$\eta_\perp$ = dimensionless perpendicular acceleration = deviation from geodesic motion $= -\dfrac{V_s}{H\dot{\sigma}}$



Two-field models (simplicity)



$$e^I_\sigma = \dot{\phi}^I/\dot{\sigma} \qquad \mathcal{D}_t e^I_\sigma = H\eta_\perp e^I_s$$

$$\dot{\sigma}^2 = G_{IJ}\dot{\phi}^I\dot{\phi}^J = 2M^2_{\mathrm{Pl}}H^2\epsilon$$

adiabatic and entropic vectors: local orthonormal frame adapted to dynamics

$$\phi^I(t,\mathbf{x}) = \phi^I(t) + Q^I(t,\mathbf{x})$$

Adiabatic curvature perturbation

$$\zeta = \frac{H}{\dot{\sigma}} e_{\sigma I} Q^I$$

$$Q_s = e_{sI} Q^I$$

entropic fluctuation ⟷ multifield effects

Quadratic Lagrangian

$$\mathcal{L}^{(2)} = a^3 \left[M_{\rm Pl}^2 \epsilon \left(\dot{\zeta}^2 - \frac{(\partial \zeta)^2}{a^2} \right) + \underline{2 \dot{\sigma} \eta_\perp \dot{\zeta} Q_s} + \frac{1}{2} \left(\dot{Q_s}^2 - \frac{(\partial Q_s)^2}{a^2} - m_s^2 Q_s^2 \right) \right]$$

Langlois, Renaux-Petel 2008

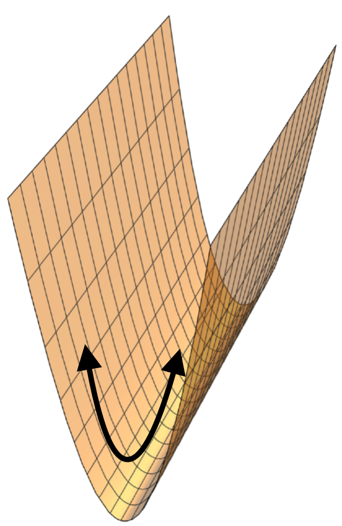
coupling via bending

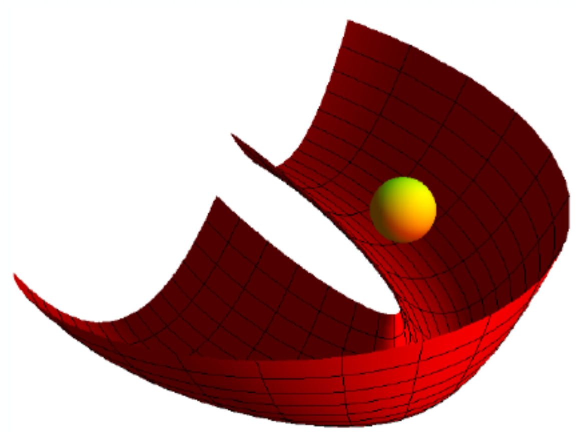
$$m_s^2 \equiv V_{;ss} - H^2 \eta_\perp^2 + \epsilon H^2 M_{\rm Pl}^2 R^{\text{field space}}$$

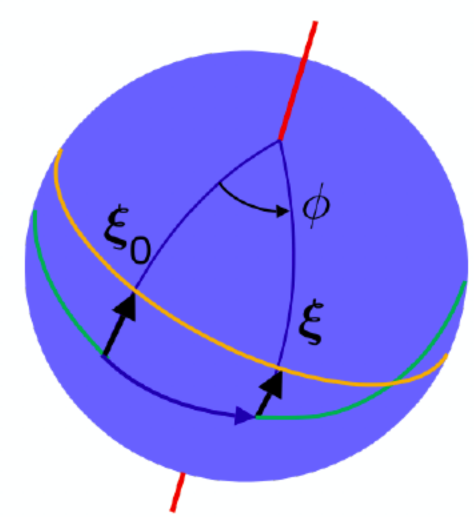
Hessian of the potential

Deviation of geodesic

Field-space curvature







Super-Hubble regime

super-Hubble evolution
of the entropic field

$$\ddot{Q}_s + 3H\dot{Q}_s + m^2_{s(\mathrm{eff})} Q_s = 0$$

Effective entropic mass squared:

$$\frac{m^2_{s(\mathrm{eff})}}{H^2} \equiv \frac{V_{;ss}}{H^2} + 3\eta_\perp^2 + \epsilon R^{\mathrm{field\,space}} M_{\mathrm{Pl}}^2$$

Hessian
contribution

bending
contribution

'geometrical'
contribution

Geometrical destabilization

$$\frac{m_{s(\mathrm{eff})}^2}{H^2} \equiv \frac{V_{;ss}}{H^2} + 3\eta_\perp^2 + \epsilon\, R^{\mathrm{field\,space}} M_{\mathrm{Pl}}^2$$

Hessian contribution

bending contribution

'geometrical' contribution

When the geometrical contribution is negative and large enough, it can **render the entropic fluctuation tachyonic, even with a large mass in the static vacuum**: unstable background

Geometrical destabilization

Necessary condition (2-field): $\boldsymbol{R^{\text{field space}} < 0}$

$$\boxed{R^{\text{field space}} M_{\rm Pl}^2 \sim \left(M_{\rm Pl}/M\right)^2}$$ generically $\gg 1$

Let us consider for instance

$$\underline{M = \mathcal{O}(10^{-2}, 10^{-3}) M_{\rm Pl}}$$

(string scale, KK scale, GUT scale...)

Even for $\dfrac{V_{;ss}}{H^2} \sim 100$

The effective mass becomes tachyonic when:

$\epsilon \to \epsilon_c = 10^{-4}$ or 10^{-2}

Minimal realization

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2\left(1 + 2\frac{\chi^2}{M^2}\right) - V(\phi) - \frac{1}{2}(\partial\chi)^2 - \frac{1}{2}m_h^2\chi^2$$

- Slow-roll model of inflation, with inflaton ϕ
- Heavy field χ with $m_h^2 \gg H^2$
- Simple dimension 6 operator suppressed by a **mass scale of new physics** $M \gg H$
- Generally expected from EFT view point

Minimal realization

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 \left(1 + 2\frac{\chi^2}{M^2}\right) - V(\phi) - \frac{1}{2}(\partial\chi)^2 - \frac{1}{2}m_h^2\chi^2$$

- Higher-orders in chi suppressed near the inflationary valley

- Does correspond to lots of models in the literature, in which it is sometimes said : «chi is stabilized by a large mass» so let us put chi=0 (consistently with the equations of motion)

$$\frac{m^2_{s(\mathrm{eff})}}{H^2} = \frac{m_h^2}{H^2} - 4\,\epsilon(t)\left(\frac{M_{\mathrm{Pl}}}{M}\right)^2 \quad \text{along} \quad \chi = 0$$

- The inflationary trajectory becomes unstable after $\epsilon \longrightarrow \epsilon_{\mathrm{c}}$

Similarity with the eta-problem

$$\mathcal{L}_{\text{eff}}[\phi^I] = \mathcal{L}_l[\phi^I] + \sum_i c_i \frac{\mathcal{O}_i[\phi^I, \partial\phi^I, \dots]}{M^{\delta_i - 4}}$$

Slow-roll action

Corrections to the low-energy effective action

Correction to kinetic terms

$$\boxed{\Delta\mathcal{L} \sim (\partial\phi)^2 \frac{\chi^2}{M^2}}$$

$$\Rightarrow \quad \Delta m_\chi^2 \sim \frac{(\partial\phi)^2}{M^2} \sim \epsilon H^2 \left(\frac{M_P}{M}\right)^2$$

$M \ll M_P \Rightarrow \epsilon_c \ll 1$ Geometrical destabilization of inflation

Similarity with the eta-problem

$$\mathcal{L}_{\rm eff}[\phi^I] = \mathcal{L}_l[\phi^I] + \sum_i c_i \frac{\mathcal{O}_i[\phi^I, \partial\phi^I, \ldots]}{M^{\delta_i - 4}}$$

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$M \simeq M_P \rightarrow \epsilon_c \sim 1$ Important for end of inflation/reheating

Turzynski et al, Sfakianakis et al 2018

Similarity with the eta-problem

$$\mathcal{L}_{\rm eff}[\phi^I] = \mathcal{L}_l[\phi^I] + \sum_i c_i \frac{\mathcal{O}_i[\phi^I, \partial\phi^I, \ldots]}{M^{\delta_i - 4}}$$

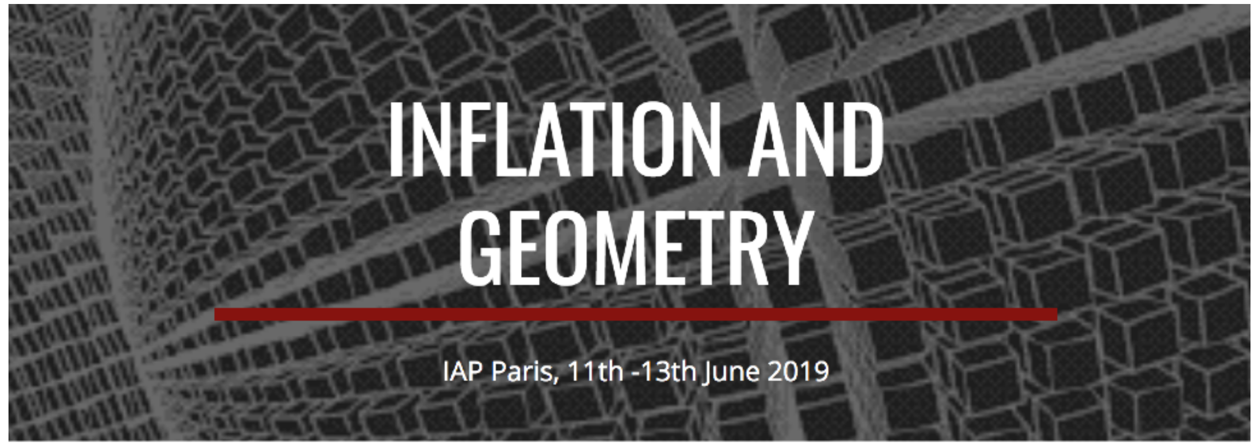
Slow-roll action

Corrections to the low-energy effective action

Even more obstacles than previously thought in realizing a stable phase of inflation

Same origin as eta problem, but symmetry cannot help

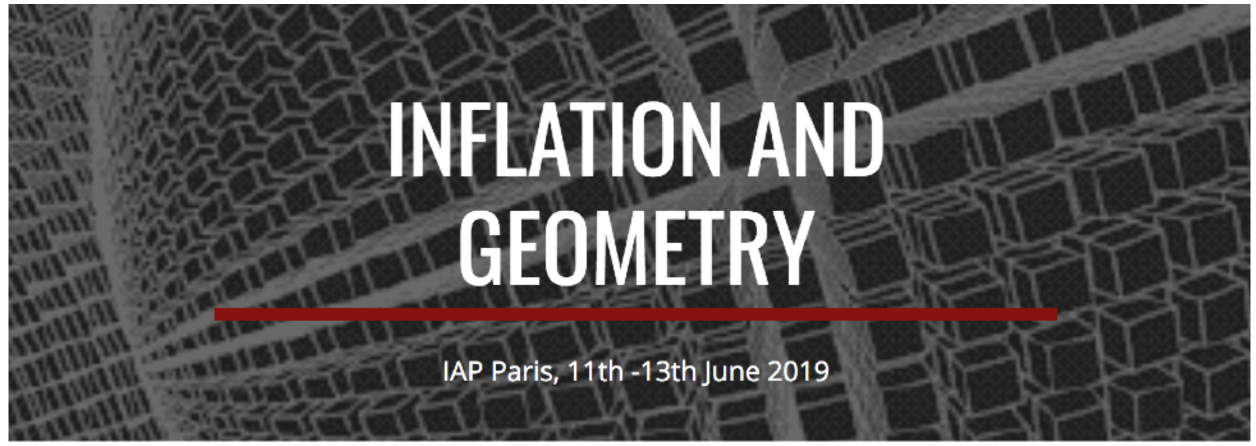
Geometry is as important as potential



Workshop
IAP, June 2019

active field 2015-now

Achucarro, Brown, Bjorkmo, Christodoulidis, Ferreira,
Fumagalli, Garcia-Saenz, Kallosh, Linde, Marsh, Mizuno,
Mukohyama, Palma, Pinol, Renaux-Petel
Roest, Ronayne, Sfakianakis, Sypsas,
Wang, Welling, Zavalla …



Workshop
IAP, June 2019

Strongly non-geodesic motion & new EFTs

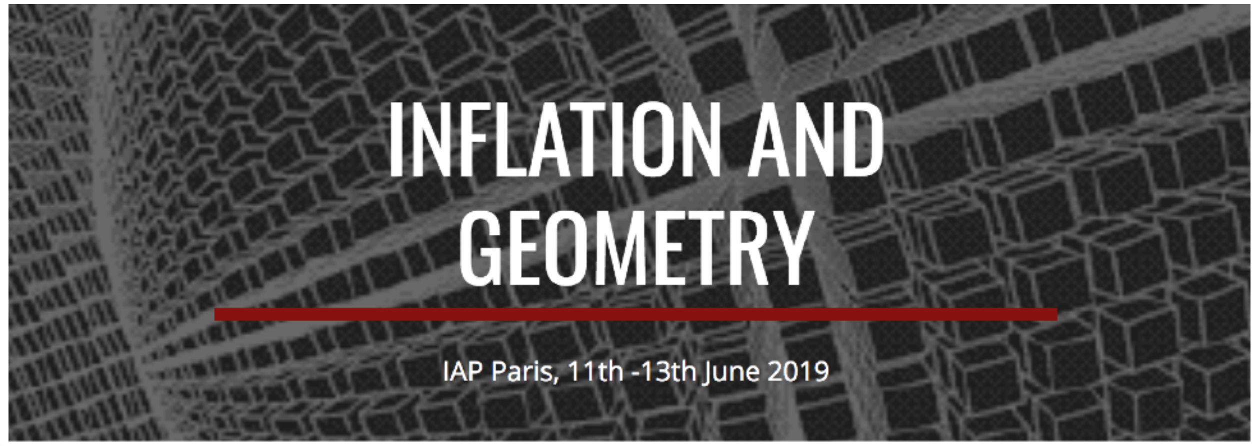
Imprints of massive fields in curved field-space

Geometrical destabilization of inflation

Multifield alpha-attractors & orbital inflation

Curved field space effects during (p)reheating

Higgs vacuum (in)stability during inflation



Workshop
IAP, June 2019

Strongly non-geodesic motion & new EFTs

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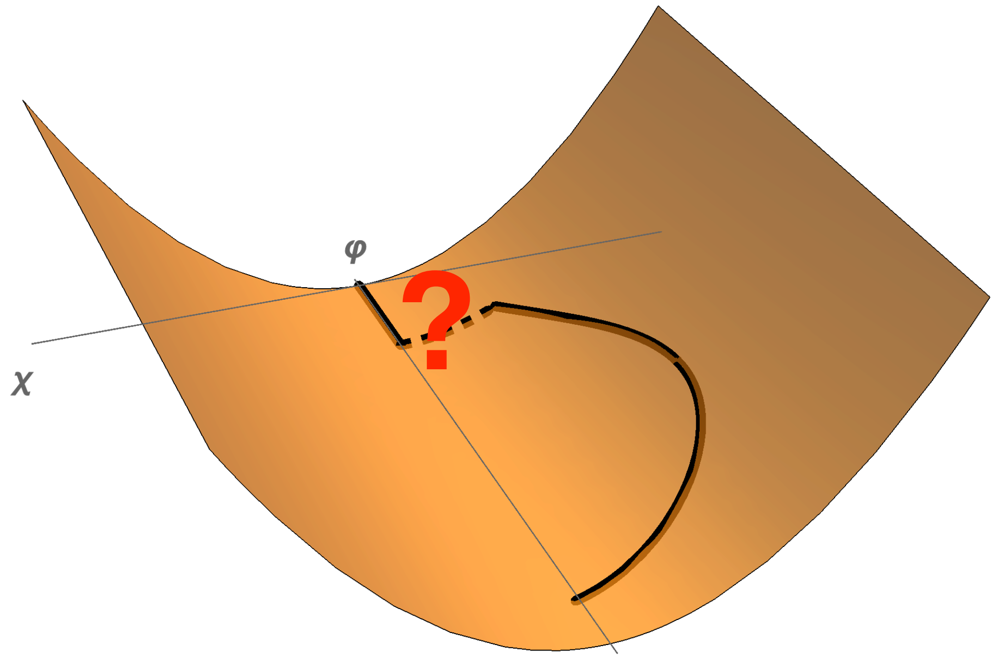
IV

Strongly non-geodesic motion & new EFTs

Fate of the instability?



Inhomogeneities dominate

↓

Premature end of inflation

Renaux-Petel, Turzynski, Vennin, 2017

OR

Inhomogeneities are shut off

↓

Second sidetracked phase of inflation

Garcia-Saenz, Renaux-Petel, Ronayne, 2018

Backreaction of fluctuations

Analytical

Similarities with hybrid inflation

+ Stochastic analysis

Grocholski, Kalinowski, Kolanowski, Renaux-Petel, Turzynski, Vennin 2019

Numerical

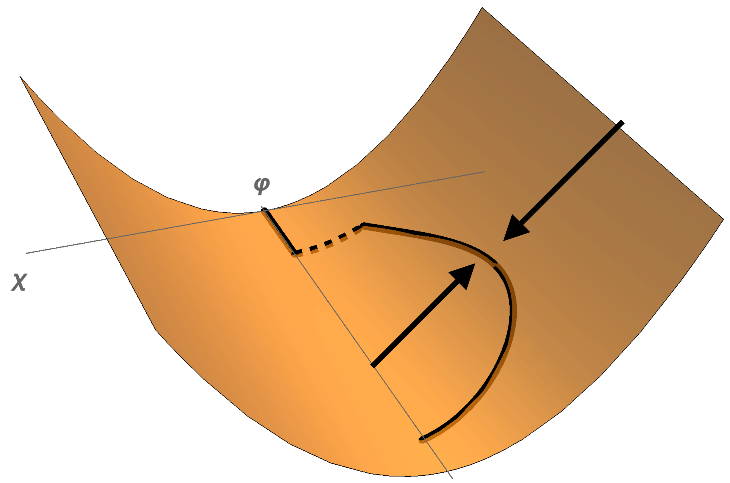
Lattice simulations

Caravano, Ronayne, Renaux-Petel, Turzynski, Wieczorek

Instability shut off before perturbations reach nonlinear regime

Fate: sidetracked phase

Sidetracked inflation: background



Characteristic features
seen in minimal realization

$$\ddot{\chi} + 3H\dot{\chi} - 2\frac{\dot{\varphi}^2}{M^2}\chi + V_{,\chi} = 0$$

Additional field: at minimum of effective potential, depends on kinetic energy of inflaton

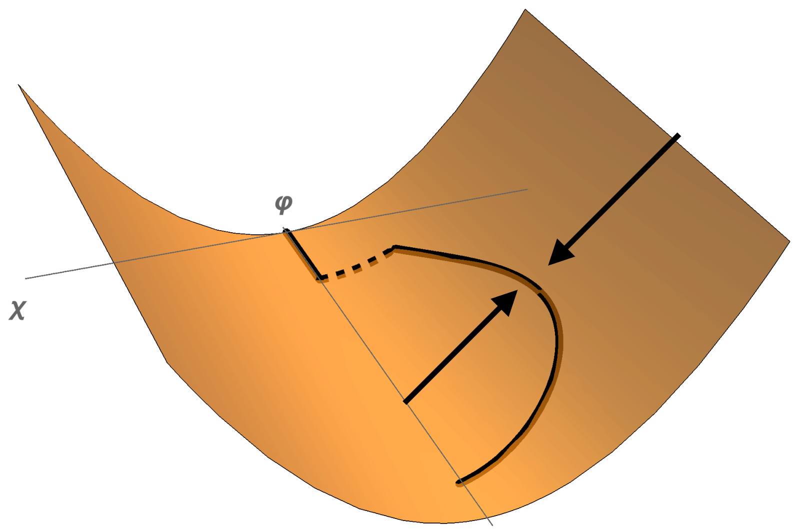
see 'gelaton' Tolley, Wyman 2009

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{4\chi}{M^2\left(1 + \frac{2\chi^2}{M^2}\right)}\dot{\chi}\dot{\varphi} + \frac{V_{,\varphi}}{1 + \frac{2\chi^2}{M^2}} = 0$$

Inflaton: 'standard' but
modified effective potential: flattened compared to original V

see also Dong et al 2011, McAllister et al 2014, Flauger et al 2014

Sidetracked inflation: background



Competition potential vs geometry:

$$\eta_\perp^2 = \mathcal{O}\left(\frac{m_h^2}{H^2}\right) \gg 1$$

Strongly non-geodesic motion

Requirement for sidetracked inflation:
flat potentials wrt curvature scale

$$M\frac{V_{,\varphi}}{V} \ll 1\,, \quad M\frac{V_{,\varphi\varphi}}{V_{,\varphi}} \ll 1$$

Interesting for UV embeddings?

Generic: inflation with
steep potential in Planck units
iff strongly non-geodesic motion

$$\epsilon = \frac{\frac{M_{\rm Pl}^2 (\nabla V)^2}{2V^2}}{1+\eta_\perp^2/9}$$

Hetz and Palma 2016
Achucarro and Palma 2018

Mass scales for perturbations

Super-Hubble entropic mass

$$\frac{m^2_{s\,(\mathrm{eff})}}{H^2} = \mathcal{O}\left(\frac{m_h^2}{H^2}\right)$$

Stability of background, and adiabatic limit reached

'Sub-Hubble' entropic mass

$$\ddot{Q}_s + 3H\dot{Q}_s + \left(\frac{k^2}{a^2} + m_s^2\right) Q_s = -2\dot{\sigma}\eta_\perp\dot{\zeta}$$

$$m_s^2 \equiv V_{;ss} - H^2\eta_\perp^2 + \epsilon\, H^2\, R^{\mathrm{field\,space}} M_{\mathrm{Pl}}^2$$

Depending on potential/geometry

$$\frac{m_s^2}{H^2} \ll 1 \quad \text{or} \quad \frac{m_s^2}{H^2} \gg 1$$

Integrating out entropic fluctuations

Single field EFT for adiabatic fluctuation

Well studied (many authors)

$\lvert m_s^2 \rvert \ll H^2$	$m_s^2 \gg H^2,\ m_s^2 > 0$
Quadratic dispersion relation	Reduced speed of sound
$\omega(k) \propto k^2$	$\omega = c_s k,\ \dfrac{1}{c_s^2} = 1 + \dfrac{4H^2\eta_\perp^2}{m_s^2}$

Equilateral NGs

Integrating out entropic fluctuations

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$$|m_s^2| \ll H^2$$

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$$\omega(k) \propto k^2$$

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Equilateral NGs

Exotic

$$|m_s^2| \gg H^2,\ m_s^2 < 0$$

Imaginary sound speed

$$\frac{1}{c_s^2} = \frac{m_{s\,(\mathrm{eff})}^2}{m_s^2} < 0$$

Transient tachyonic instability

Inflation with strongly non-geodesic motion

Strongly non-geodesic $\eta_\perp^2 \gg 1$

$\xrightarrow[\text{e.g. by potential}]{\text{unless stabilization}}$

Stable background but transient instability of entropic fluctuations $m_s^2 < 0$ $\left|\frac{m_s^2}{H^2}\right| \gg 1$

Inflation with strongly non-geodesic motion

Strongly non-geodesic $\eta_\perp^2 \gg 1$

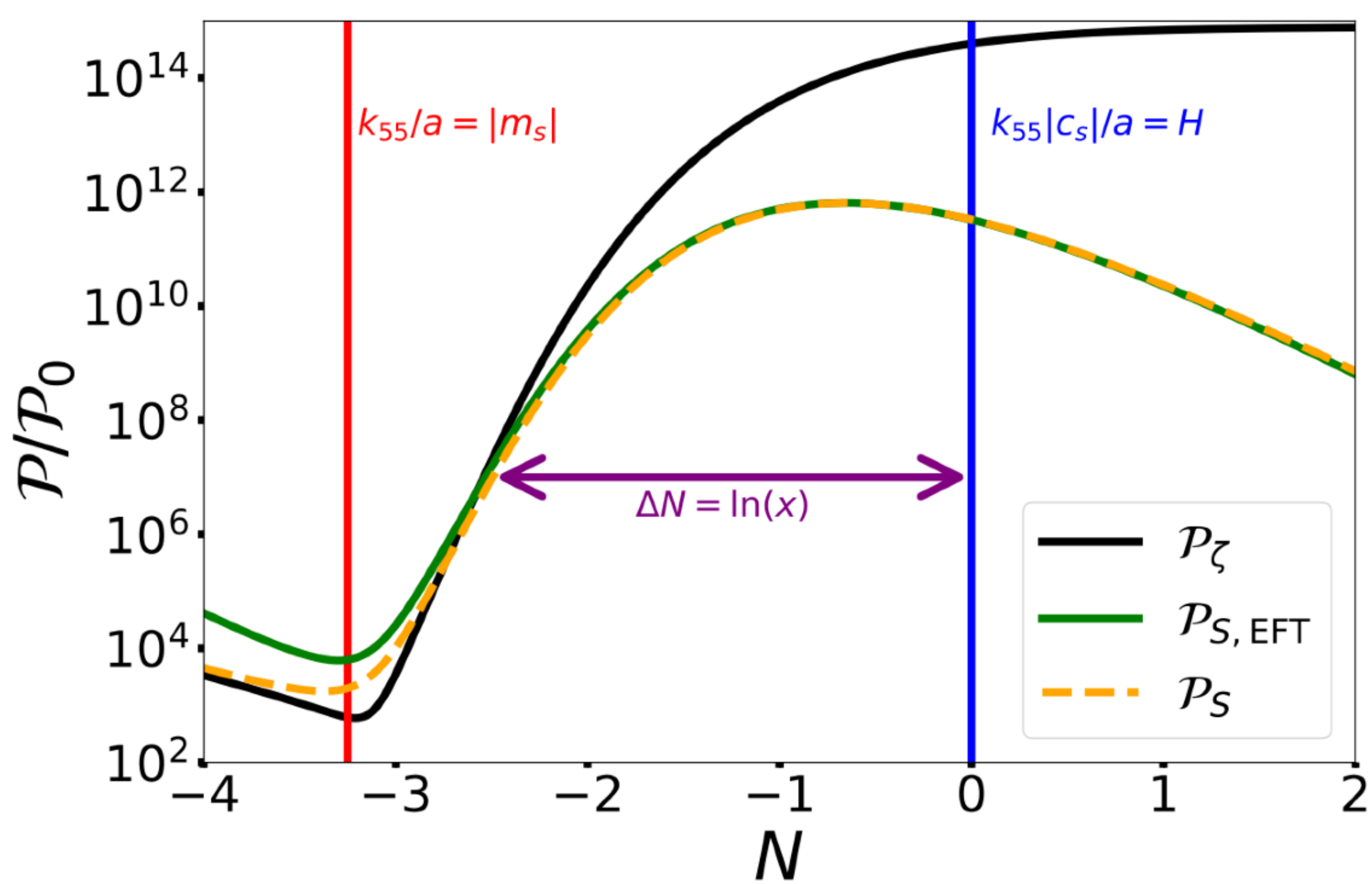
unless stabilization

e.g. by potential

Stable background but transient instability of entropic fluctuations $m_s^2 < 0$ $\left|\frac{m_s^2}{H^2}\right| \gg 1$

Transferred to observable curvature perturbation

$$\mathcal{P}_\zeta / \mathcal{P}_0 \sim e^{2x} \gg 1$$



$$\mathcal{P}_0 = H^2/(8\pi^2 \epsilon M_{\rm Pl}^2)_{k=aH}$$

Cremonini et al 2010,
Brown 2018,
Renaux-Petel et al 2018, 2019,
Marsh et al 2019

EFT with imaginary sound speed

$$S^{\rm eff} = \int d\tau d^3x\, a^2 \epsilon M_{\rm Pl}^2 \left[\frac{\zeta'^2}{c_s^2} - (\vec{\nabla}\zeta)^2\right] + \int d\tau d^3x\, \frac{a\,\epsilon M_{\rm Pl}^2}{H}\left(\frac{1}{c_s^2}-1\right)\left[\zeta'(\vec{\nabla}\zeta)^2 + \frac{A}{c_s^2}\zeta'^3\right]$$

Standard action, but unusual mode function

EFT valid for $k|c_s|/a < xH$

$$\zeta_k(\tau) = \left(\frac{2\pi^2}{k^3}\right)^{1/2} \alpha\left(e^{k|c_s|\tau + x}(k|c_s|\tau - 1) - \rho e^{i\psi} e^{-(k|c_s|\tau+x)}(k|c_s|\tau+1)\right)$$

exponentially growing (real) exponentially decaying:

$$\langle \zeta_{\boldsymbol{k}_1}(t)\zeta_{\boldsymbol{k}_2}(t)\zeta_{\boldsymbol{k}_3}(t)\rangle = 2\,\mathrm{Im}\left[\int_{-\infty(1-i\epsilon)}^{t} \mathrm{d}t'\, \langle 0|\zeta_{\boldsymbol{k}_1}(t)\zeta_{\boldsymbol{k}_2}(t)\zeta_{\boldsymbol{k}_3}(t) H_{(3)}(t')|0\rangle\right]$$

NGs come from interactions between growing and decaying modes

Quantization condition

$$2\alpha^2\rho\sin(\psi)|c_s| = \mathcal{P}_0$$

$\longrightarrow$ Unambiguous predictions for bispectrum

Garcia-Saenz, Renaux-Petel 2018

EFT with imaginary sound speed

$$f_{NL}^{\mathrm{eq}} \simeq \frac{10}{9}\left(\frac{1}{|c_s|^2}+1\right)\left(\frac{13A}{6}-\frac{5}{24}\right)$$

Enhancement in flattened configurations, like non Bunch-Davies vacuum

$$f_{NL}^{\mathrm{flat}} \simeq \frac{1}{192}\left(\frac{1}{|c_s|^2}+1\right)\times\left(39(A-1)+12x^2+4(A+1)x^3\right)$$

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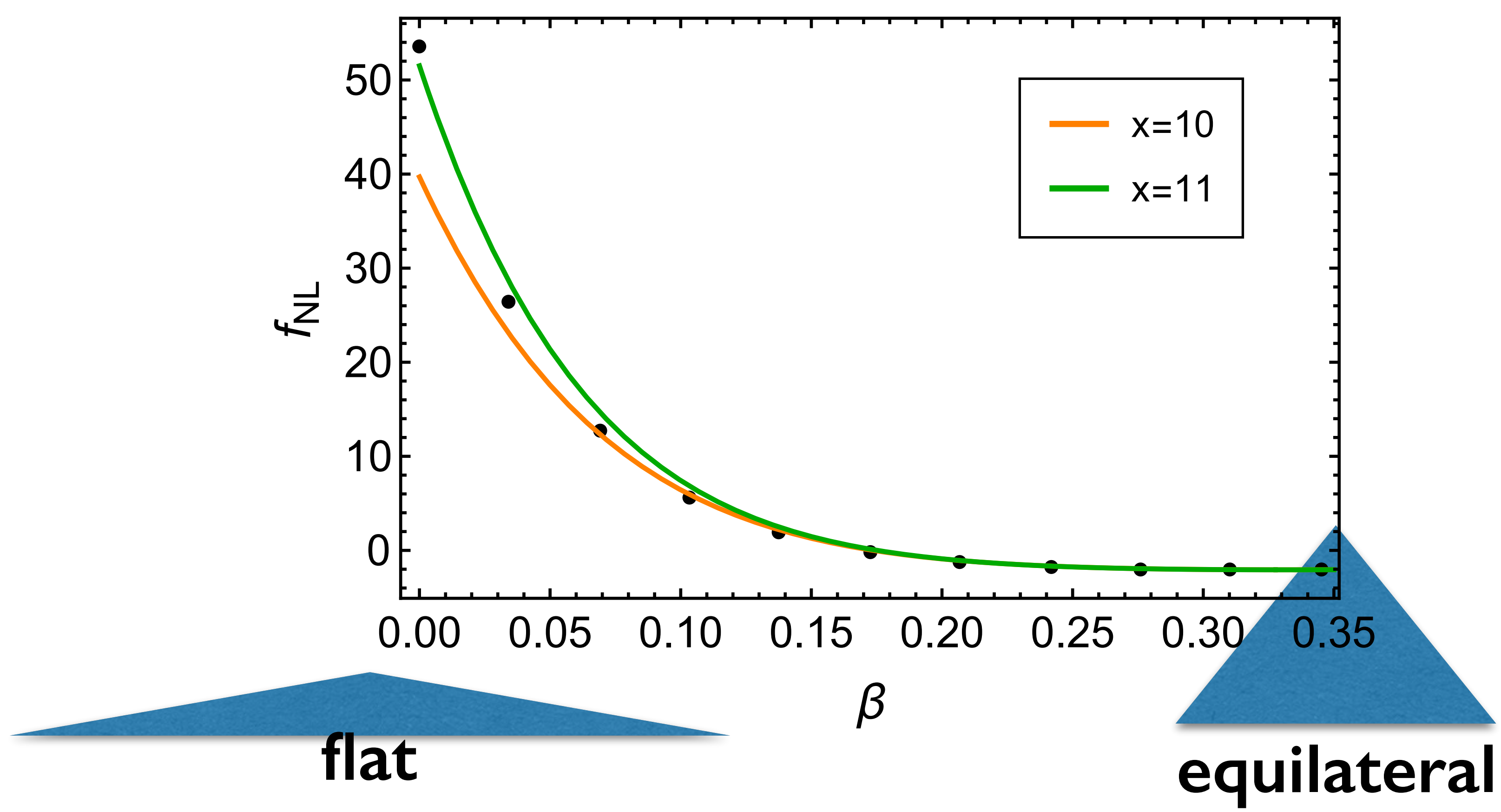
EFT

vs

first-principle two-field numerical computation

with PyTransport Mulryne & Ronayne



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EFT

vs

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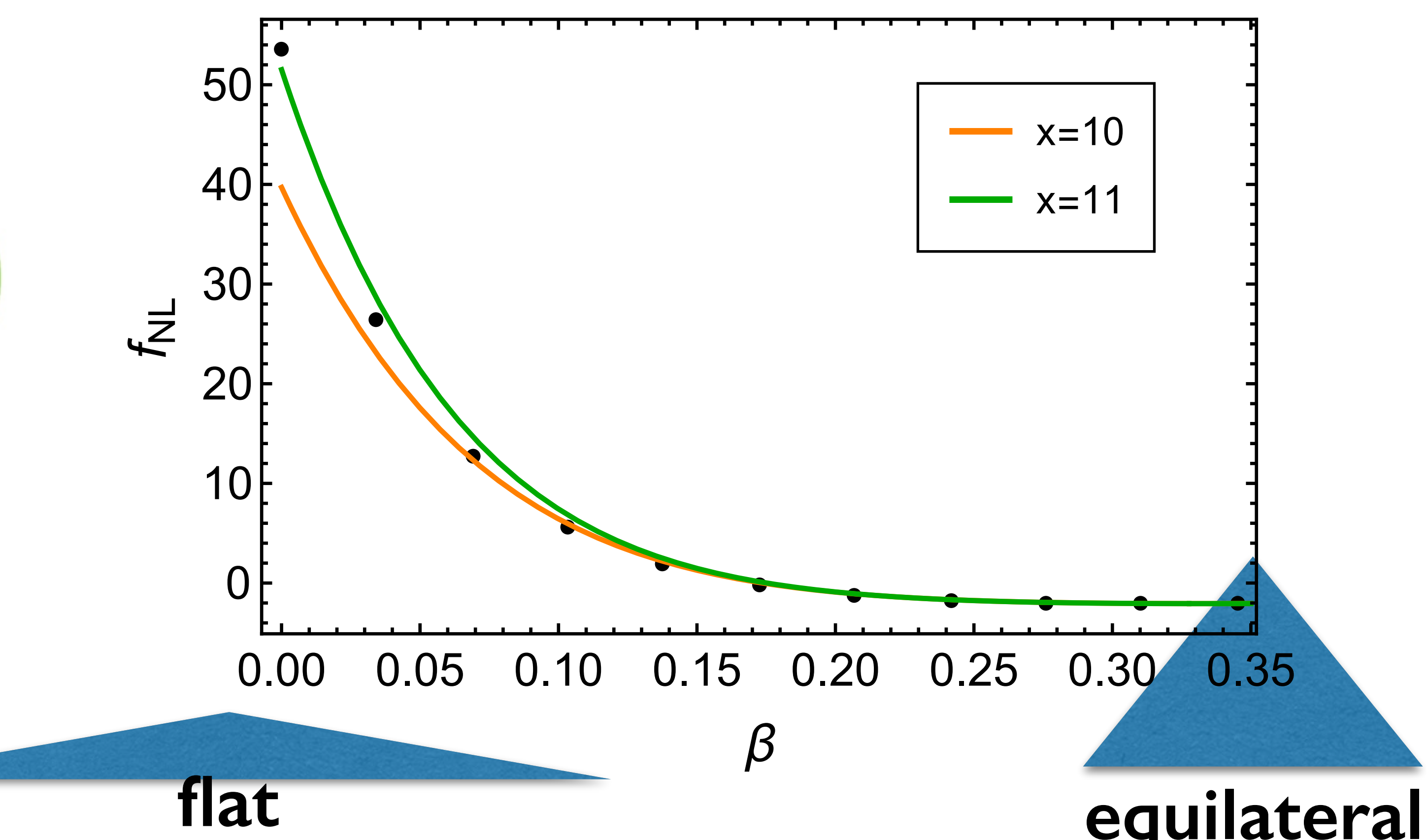
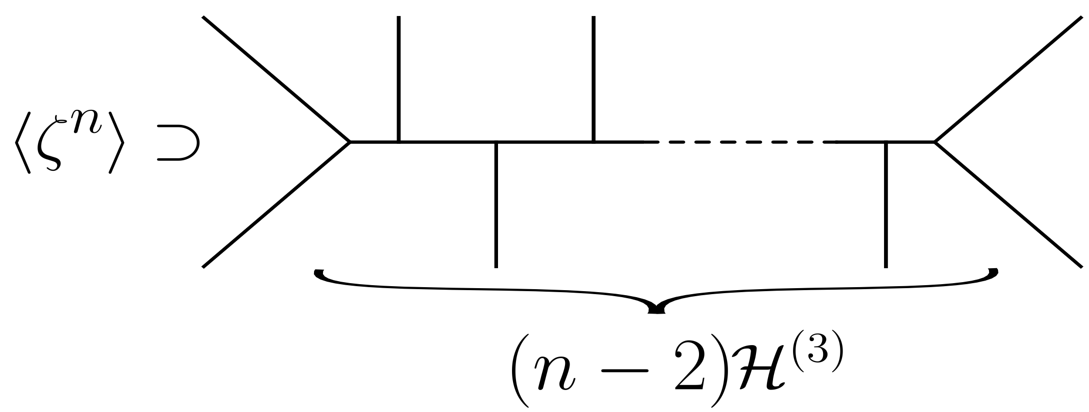
with PyTransport
Mulryne & Ronayne



EFT with imaginary sound speed

n-point functions also enhanced in flattened configurations



$$\boxed{\frac{\langle \zeta^n \rangle}{\langle \zeta^2 \rangle^{n-1}} \sim \left(f_{NL}^{\mathrm{flat}}\right)^{n-2}}$$

Fumagalli, Garcia-Saenz, Pinol, Renaux-Petel, Ronayne PRL 2019

Powerful model-independent constraints on non-standard inflationary attractors (e.g. hyperinflation, sidetracked inflation)

Similarity with gauge field coupled to the inflaton   Marsh et al 2019

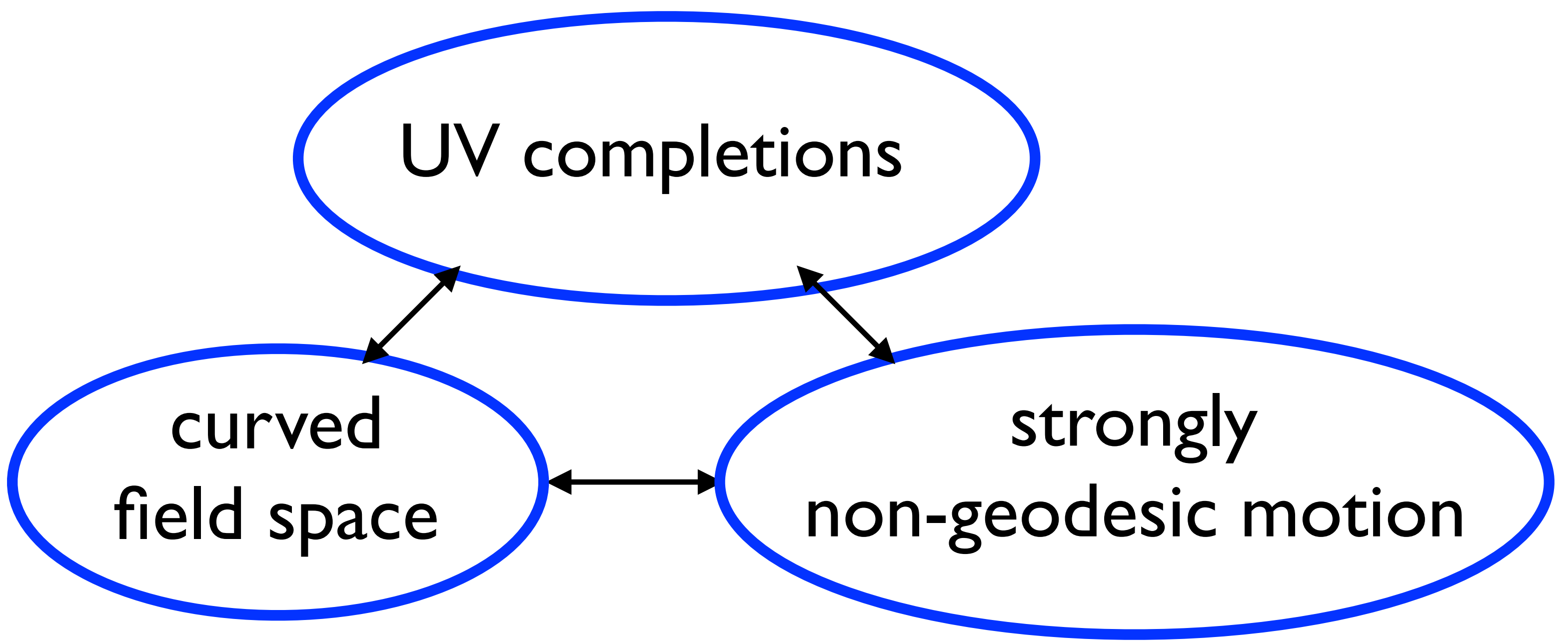
Mechanism to generate PBHs   Fumagalli, Renaux-Petel, Ronayne, Witkowski, to appear

V

Revisiting non-Gaussianity in multifield inflation with curved field space

Motivation

Theoretical and observational motivations for NGs
+ Many recent developments in multifield inflation



Need for revisiting non-Gaussianities in multifield inflation

Generalize Maldacena's computation to multifield inflation

Additional motivation

Extension of Maldacena's computation to multifield inflation

Garcia-Saenz, Pinol, Renaux-Petel, 2019

byproduct: NG signature of heavy fields in curved field space

$$S_{\rm EFT} = \int \mathrm{d}t\, \mathrm{d}^3x\, a^3 M_{\rm Pl}^2 \frac{\epsilon}{c_s^2}\left(\dot\zeta^2 - c_s^2 \frac{(\partial\zeta)^2}{a^2}\right)$$

$$+ \int \mathrm{d}t\, \mathrm{d}^3x\, a^3 M_{\rm Pl}^2 \frac{\epsilon}{H}\left(\frac{1}{c_s^2} - 1\right)\left[\dot\zeta \frac{(\partial\zeta)^2}{a^2} + \frac{A}{c_s^2}\dot\zeta^3\right] + \dots$$

reduced sound speed

depends on UV completion

Revisiting non-Gaussianity in multifield inflation

Garcia-Saenz, Pinol, Renaux-Petel, 2019

$$c_s^2 > 0 \qquad f_{\rm NL}^{\rm eq,orth} = \left(\frac{1}{c_s^2} - 1\right)\left(\mathcal{O}(1) + \mathcal{O}(1) A\right)$$

$$A = -\frac{1}{2}(1+c_s^2) - \frac{1}{6}(1-c_s^2)\left(\frac{\kappa V_{sss}}{m_s^2} + \frac{\kappa \epsilon H^2 M_{\rm Pl}^2 R_{{\rm fs},s}}{m_s^2}\right)$$

$$+\frac{2}{3}(1+2c_s^2)\frac{\epsilon H^2 R_{\rm fs} M_{\rm Pl}^2}{m_s^2}$$

Self coupling

Derivative of scalar curvature

Scalar curvature

$$\kappa = \sqrt{2\epsilon} M_{\rm Pl}/\eta_\perp$$

bending radius of trajectory

Direct impact of geometry on observables

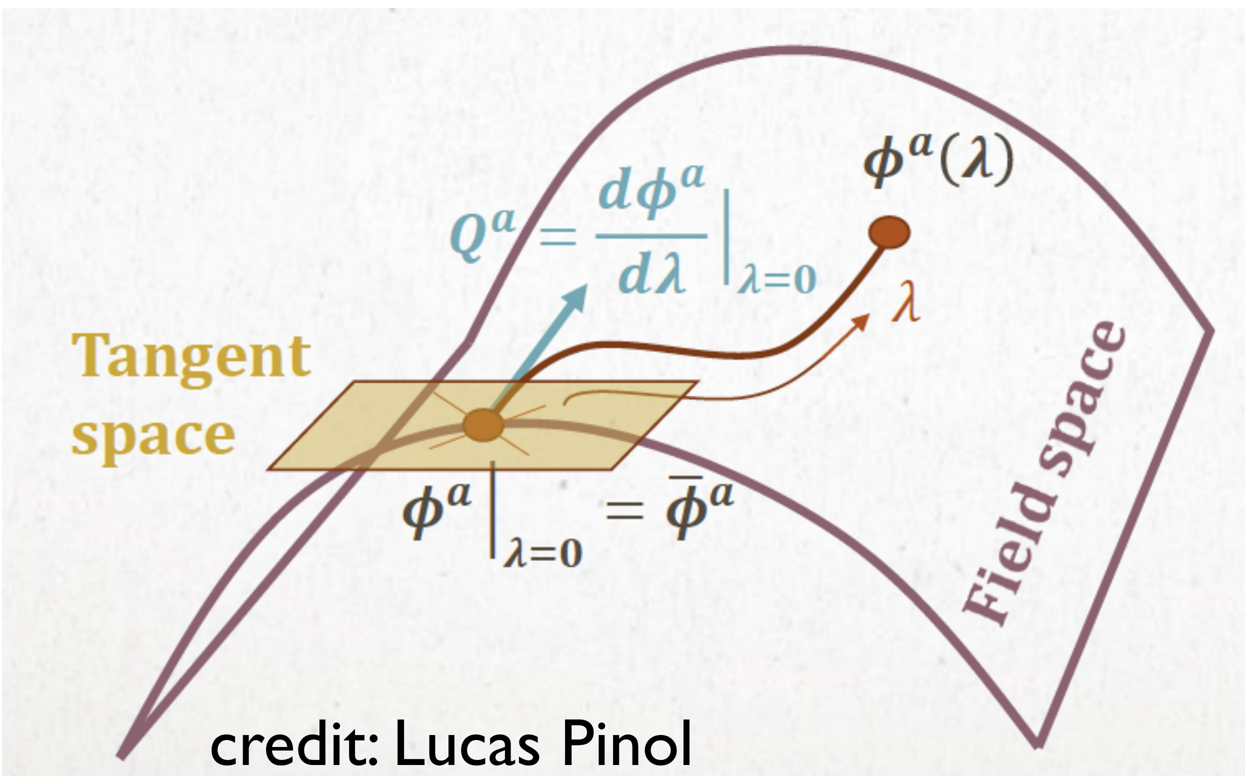
Beautiful result!

$$\mathcal{L}^{(3)} = M_{\rm Pl}^2\, a^3 \left[\epsilon(\epsilon-\eta)\dot{\zeta}^2\zeta + \epsilon(\epsilon+\eta)\zeta\frac{(\partial\zeta)^2}{a^2} + \left(\frac{\epsilon}{2}-2\right)\frac{1}{a^4}\left(\partial\zeta\right)\left(\partial\chi\right)\partial^2\chi + \frac{\epsilon}{4a^4}\partial^2\zeta(\partial\chi)^2\right]$$
$$+ a^3\left[\frac{1}{2}m_s^2(\epsilon+\mu_s)\zeta\mathcal{F}^2 + (2\epsilon-\eta-2\lambda_\perp)\,\dot{\sigma}\eta_\perp\zeta\dot{\zeta}\mathcal{F} + \frac{\dot{\sigma}\eta_\perp}{a^2 H}\,\mathcal{F}(\partial\zeta)^2\right.$$
$$- \frac{\dot{\sigma}\eta_\perp}{H}\,\dot{\zeta}^2\mathcal{F} - \frac{1}{H}\left(H^2\eta_\perp^2 - \epsilon M_{\rm Pl}^2 H^2 R_{\rm fs}\right)\dot{\zeta}\mathcal{F}^2 - \frac{1}{6}\left(V_{;sss} - 2\dot{\sigma}H\eta_\perp R_{\rm fs} + \epsilon M_{\rm Pl}^2 H^2 R_{{\rm fs},s}\right)\mathcal{F}^3$$
$$\left. + \frac{1}{2}\epsilon\zeta\left(\dot{\mathcal{F}}^2 + \frac{(\partial\mathcal{F})^2}{a^2}\right) - \frac{1}{a^2}\dot{\mathcal{F}}\partial\mathcal{F}\partial\chi\right] + \mathcal{D}$$

- Exact and compact
- Size of couplings reflect the genuine size of NGs
- Convenient for integrating out massive entropic fluctuations
- Impact of boundary terms minimized

Covariant formulation

Beyond liner order, field fluctuations do not transform covariantly under field redefinitions



Non-perturbative and geometrical definition of covariant fluctuations Q^I

Gong and Tanaka, 2011

$$\delta\phi^I = Q^I - \frac{1}{2}\Gamma^I_{JK}Q^J Q^K + \frac{1}{6}(2\Gamma^I_{LM}\Gamma^M_{JK} - \Gamma^I_{JK,L})Q^J Q^K Q^L + \mathcal{O}(Q^4)$$

Gauche choice

$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + h_{ij}(\mathrm{d}x^i + N^i\mathrm{d}t)(\mathrm{d}x^j + N^j\mathrm{d}t)$$

Common choice: spatially flat gauge

$$h_{ij}^{\mathrm{flat}} = a^2(t)\delta_{ij}$$

dofs: all Q^I

intuitive, perturbations of metric negligible in decoupling limit for large fNL, less computations

not directly related to observables, symmetries less clear, results complex and physics somewhat hidden

Gauche choice

Here: 'comoving' gauge

$$h_{ij}^{\text{comoving}} = a^2 \underline{e^{2\zeta}} \delta_{ij} \quad \text{and} \quad \underline{e_{\sigma I} Q^I = 0}$$

no adiabatic field fluctuation

dofs: ζ and $\mathcal{F} \equiv e_{sI} Q^I$: entropic fluctuation

long computations not to have misleading results,
but it is now done!

direct observable, symmetries manifest,
simple results amenable to analytical approximations

Brute force cubic Lagrangian

$$\mathcal{L}^{(3)} = a^3\Big\{M_{\rm Pl}^2\Big[\epsilon\left(3\zeta-\frac{\dot\zeta}{H}\right)\dot\zeta^2-\frac{\epsilon}{a^2}\zeta(\partial\zeta)^2+\frac{1}{2a^4}\left(3\zeta-\frac{\dot\zeta}{H}\right)\left(\partial_i\partial_j\theta\partial_i\partial_j\theta-(\partial^2\theta)^2\right)$$
$$-\frac{2}{a^4}\partial_i\zeta\partial_i\theta\partial^2\theta\Big]+\dot\sigma\eta_\perp\left(6\zeta-\frac{\dot\zeta}{H}\right)\dot\zeta\mathcal{F}+\frac{1}{2}\left(3\zeta-\frac{\dot\zeta}{H}\right)\dot{\mathcal{F}}^2-\frac{1}{2a^2}\left(\zeta+\frac{\dot\zeta}{H}\right)(\partial\mathcal{F})^2$$
$$-\frac{1}{a^2}\dot{\mathcal{F}}\partial\mathcal{F}\partial\theta-\frac{1}{2H}\left(m_s^2+2H^2\eta_\perp^2-2\epsilon M_P^2H^2R_{\rm fs}\right)\dot\zeta\mathcal{F}^2-\frac{3}{2}m_s^2\zeta\mathcal{F}^2$$
$$-\frac{1}{6}\left(V_{;sss}-2\dot\sigma H\eta_\perp R_{\rm fs}+\epsilon M_{\rm Pl}^2H^2R_{{\rm fs},s}\right)\mathcal{F}^3\Big\}+\mathcal{D}_0\,,$$

… Expansion is necessary but not sufficient …

$\mathcal{L}_3 \supset \mathcal{O}(\epsilon^0,\epsilon)\zeta^3$ is misleading

Genuine size (purely adiabatic) $\mathcal{L}_3 \sim \mathcal{O}(\epsilon^2)\zeta^3$

Task: render this explicit in the more complicated multifield situation

Manipulating the cubic Lagrangian

Principles:

Arroja and Tanaka 2011
Burrage, Ribeiro, Seery, 2011

One performs (many many) integrations by part

One can use freely the linear eoms $\frac{\delta S^{(2)}}{\delta \zeta} = 0$ and $\frac{\delta S^{(2)}}{\delta \mathcal{F}} = 0$

Temporal boundary terms (total derivative) contribute to correlation functions and should be kept

Result

$$\mathcal{L}^{(3)} = M_{\rm Pl}^2\, a^3 \left[\epsilon(\epsilon-\eta)\dot{\zeta}^2\zeta + \epsilon(\epsilon+\eta)\zeta\frac{(\partial\zeta)^2}{a^2} + \left(\frac{\epsilon}{2}-2\right)\frac{1}{a^4}\left(\partial\zeta\right)\left(\partial\chi\right)\partial^2\chi + \frac{\epsilon}{4a^4}\partial^2\zeta(\partial\chi)^2\right]$$
$$+ a^3\left[\frac{1}{2}m_s^2(\epsilon+\mu_s)\zeta\mathcal{F}^2 + (2\epsilon-\eta-2\lambda_\perp)\,\dot{\sigma}\eta_\perp\zeta\dot{\zeta}\mathcal{F} + \frac{\dot{\sigma}\eta_\perp}{a^2 H}\,\mathcal{F}(\partial\zeta)^2\right.$$
$$-\frac{\dot{\sigma}\eta_\perp}{H}\,\dot{\zeta}^2\mathcal{F} - \frac{1}{H}\left(H^2\eta_\perp^2 - \epsilon M_{\rm Pl}^2 H^2 R_{\rm fs}\right)\dot{\zeta}\mathcal{F}^2 - \frac{1}{6}\left(V_{;sss} - 2\dot{\sigma}H\eta_\perp R_{\rm fs} + \epsilon M_{\rm Pl}^2 H^2 R_{{\rm fs},s}\right)\mathcal{F}^3$$
$$\left. + \frac{1}{2}\epsilon\zeta\left(\dot{\mathcal{F}}^2 + \frac{(\partial\mathcal{F})^2}{a^2}\right) - \frac{1}{a^2}\dot{\mathcal{F}}\partial\mathcal{F}\partial\chi\right] + \mathcal{D}$$

$$\epsilon = -\dot{H}/H^2, \eta = \dot{\epsilon}/(H\epsilon), \lambda_\perp = \dot{\eta}_\perp/(H\eta_\perp), \mu_s = (m_s^2)^{\cdot}/(Hm_s^2)$$

$$\frac{1}{a^2}\,\partial^2\chi = \epsilon\dot{\zeta} + \frac{\dot{\sigma}\eta_\perp}{M_{\rm Pl}^2}\,\mathcal{F}$$

Result

$$\mathcal{L}^{(3)} = \boxed{M_{\rm Pl}^2\, a^3 \left[\epsilon(\epsilon-\eta)\dot\zeta^2\zeta + \epsilon(\epsilon+\eta)\zeta\frac{(\partial\zeta)^2}{a^2} + \left(\frac{\epsilon}{2}-2\right)\frac{1}{a^4}\left(\partial\zeta\right)\left(\partial\chi\right)\partial^2\chi + \frac{\epsilon}{4a^4}\partial^2\zeta(\partial\chi)^2\right]}$$

$$+ a^3\left[\frac{1}{2}m_s^2(\epsilon+\mu_s)\zeta\mathcal{F}^2 + (2\epsilon-\eta-2\lambda_\perp)\dot\sigma\eta_\perp\zeta\dot{\mathcal{F}} + \frac{\dot\sigma\eta_\perp}{a^2 H}\mathcal{F}(\partial\zeta)^2\right.$$

like Maldacena (but chi is multifield)

$$-\frac{\dot\sigma\eta_\perp}{H}\dot\zeta^2\mathcal{F} - \frac{1}{H}\left(H^2\eta_\perp^2 - \epsilon M_{\rm Pl}^2 H^2 R_{\rm fs}\right)\dot\zeta\mathcal{F}^2 - \frac{1}{6}\left(V_{;sss} - 2\dot\sigma H\eta_\perp R_{\rm fs} + \epsilon M_{\rm Pl}^2 H^2 R_{{\rm fs},s}\right)\mathcal{F}^3$$

$$\left. + \frac{1}{2}\epsilon\zeta\left(\dot{\mathcal{F}}^2 + \frac{(\partial\mathcal{F})^2}{a^2}\right) - \frac{1}{a^2}\dot{\mathcal{F}}\partial\mathcal{F}\partial\chi\right] + \mathcal{D}$$

$$\epsilon = -\dot H/H^2,\ \eta = \dot\epsilon/(H\epsilon),\ \lambda_\perp = \dot\eta_\perp/(H\eta_\perp),\ \mu_s = (m_s^2)^{\cdot}/(Hm_s^2)$$

Result

$$\mathcal{L}^{(3)} = M_{\rm Pl}^2\, a^3 \left[\epsilon(\epsilon-\eta)\dot{\zeta}^2\zeta + \epsilon(\epsilon+\eta)\zeta\frac{(\partial\zeta)^2}{a^2} + \left(\frac{\epsilon}{2}-2\right)\frac{1}{a^4}\left(\partial\zeta\right)\left(\partial\chi\right)\partial^2\chi + \frac{\epsilon}{4a^4}\partial^2\zeta(\partial\chi)^2 \right]$$

'new multifield' interactions

$$+\, a^3 \left[\frac{1}{2}m_s^2(\epsilon+\mu_s)\zeta\mathcal{F}^2 + (2\epsilon-\eta-2\lambda_\perp)\,\dot{\sigma}\eta_\perp\zeta\dot{\zeta}\mathcal{F} + \frac{\dot{\sigma}\eta_\perp}{a^2 H}\,\mathcal{F}(\partial\zeta)^2 \right.$$

$$- \frac{\dot{\sigma}\eta_\perp}{H}\,\dot{\zeta}^2\mathcal{F} - \frac{1}{H}\left(H^2\eta_\perp^2 - \epsilon M_{\rm Pl}^2 H^2 R_{\rm fs}\right)\dot{\zeta}\mathcal{F}^2 - \frac{1}{6}\left(V_{;sss} - 2\dot{\sigma}H\eta_\perp R_{\rm fs} + \epsilon M_{\rm Pl}^2 H^2 R_{{\rm fs},s}\right)\mathcal{F}^3$$

$$\left. + \frac{1}{2}\epsilon\zeta\left(\dot{\mathcal{F}}^2 + \frac{(\partial\mathcal{F})^2}{a^2}\right) - \frac{1}{a^2}\dot{\mathcal{F}}\partial\mathcal{F}\partial\chi \right] + \mathcal{D}$$

Result

$$\mathcal{L}^{(3)} = M_{\rm Pl}^2\, a^3 \left[\epsilon(\epsilon-\eta)\dot{\zeta}^2\zeta + \epsilon(\epsilon+\eta)\zeta\frac{(\partial\zeta)^2}{a^2} + \left(\frac{\epsilon}{2}-2\right)\frac{1}{a^4}\left(\partial\zeta\right)\left(\partial\chi\right)\partial^2\chi + \frac{\epsilon}{4a^4}\partial^2\zeta(\partial\chi)^2\right]$$

$$+ a^3\left[\frac{1}{2}m_s^2(\epsilon+\mu_s)\zeta\mathcal{F}^2 + \left(2\epsilon-\eta-2\lambda_\perp\right)\dot{\sigma}\eta_\perp\zeta\dot{\zeta}\mathcal{F} + \frac{\dot{\sigma}\eta_\perp}{a^2 H}\mathcal{F}(\partial\zeta)^2\right.$$

$$- \frac{\dot{\sigma}\eta_\perp}{H}\dot{\zeta}^2\mathcal{F} - \frac{1}{H}\left(H^2\eta_\perp^2 - \epsilon M_{\rm Pl}^2 H^2 R_{\rm fs}\right)\dot{\zeta}\mathcal{F}^2 - \frac{1}{6}\left(V_{;sss} - 2\dot{\sigma}H\eta_\perp R_{\rm fs} + \epsilon M_{\rm Pl}^2 H^2 R_{{\rm fs},s}\right)\mathcal{F}^3$$

$$\left. + \frac{1}{2}\epsilon\zeta\left(\dot{\mathcal{F}}^2 + \frac{(\partial\mathcal{F})^2}{a^2}\right) - \frac{1}{a^2}\dot{\mathcal{F}}\partial\mathcal{F}\partial\chi\right] \boxed{+\,\mathcal{D}}$$

boundary terms

Result

$$\mathcal{L}^{(3)} = M_{\rm Pl}^2\, a^3 \left[\epsilon(\epsilon-\eta)\dot{\zeta}^2\zeta + \epsilon(\epsilon+\eta)\zeta\frac{(\partial\zeta)^2}{a^2} + \left(\frac{\epsilon}{2}-2\right)\frac{1}{a^4}\left(\partial\zeta\right)\left(\partial\chi\right)\partial^2\chi + \frac{\epsilon}{4a^4}\partial^2\zeta(\partial\chi)^2\right]$$
$$+\, a^3\left[\frac{1}{2}m_s^2(\epsilon+\mu_s)\zeta\mathcal{F}^2 + (2\epsilon-\eta-2\lambda_\perp)\,\dot{\sigma}\eta_\perp\zeta\dot{\zeta}\mathcal{F} + \frac{\dot{\sigma}\eta_\perp}{a^2H}\,\mathcal{F}(\partial\zeta)^2\right.$$
$$-\frac{\dot{\sigma}\eta_\perp}{H}\,\dot{\zeta}^2\mathcal{F} - \frac{1}{H}\left(H^2\eta_\perp^2 - \epsilon M_{\rm Pl}^2 H^2 R_{\rm fs}\right)\dot{\zeta}\mathcal{F}^2 - \frac{1}{6}\left(V_{;sss} - 2\dot{\sigma}H\eta_\perp R_{\rm fs} + \epsilon M_{\rm Pl}^2 H^2 R_{{\rm fs},s}\right)\mathcal{F}^3$$
$$\left. + \frac{1}{2}\epsilon\zeta\left(\dot{\mathcal{F}}^2 + \frac{(\partial\mathcal{F})^2}{a^2}\right) - \frac{1}{a^2}\dot{\mathcal{F}}\partial\mathcal{F}\partial\chi\right] + \mathcal{D}$$

Exact and compact

Size of couplings reflect the genuine size of NGs

Convenient for integrating out massive entropic fluctuations

Impact of boundary terms minimized

Integrating out heavy entropic fluctuations

$$\left(m_s^2 - \Box\right)\mathcal{F} = 2\dot\sigma\eta_\perp\dot\zeta \quad \Rightarrow \quad \boxed{\mathcal{F}_{\rm LO} = \frac{2\dot\sigma\eta_\perp}{m_s^2}\,\dot\zeta}$$

Adiabaticity conditions $\quad \dfrac{X^{(n)}}{m_s^n X} \ll 1$ — Cespedes et al, 2012

Predictivity condition $\quad \dfrac{H^2}{m_s^2 c_s^2} \ll 1$ — Cremonini et al 2010, Baumann and Green 2011

Effective quadratic action: Achucarro, Palma et al

$$\mathcal{L}^{(2)}_{\rm EFT} = a^3 M_{\rm Pl}^2 \frac{\epsilon}{c_s^2}\left(\dot\zeta^2 - c_s^2\frac{(\partial\zeta)^2}{a^2}\right) \qquad \frac{1}{c_s^2} - 1 \equiv \frac{4H^2\eta_\perp^2}{m_s^2}$$

Including all interactions

$$\mathcal{L}^{(3)}_{\rm EFT,bulk} = M_{\rm Pl}^2\, a^3\, \frac{\epsilon}{c_s^2} \left(\begin{array}{c} f_0 c_s^2 \dfrac{\dot\zeta}{H} \dfrac{(\partial\zeta)^2}{a^2} + \\ \dfrac{f_1}{H}\dot\zeta^3 + \\ f_2 \dot\zeta^2\zeta + \\ f_3 c_s^2 \zeta \dfrac{(\partial\zeta)^2}{a^2} + \\ f_4 \dot\zeta \partial_i \partial^{-2} \dot\zeta \partial_i \zeta + \\ f_5 \partial^2\zeta (\partial_i \partial^{-2} \dot\zeta)^2 \end{array} \right) \quad \text{with} \quad \left(\begin{array}{l} f_0 = \left(\dfrac{1}{c_s^2} - 1\right) \\ f_1 = \left(\dfrac{1}{c_s^2} - 1\right) A \\ f_2 = \epsilon - \eta + 2s \\ f_3 = \epsilon + \eta \\ f_4 = \dfrac{\epsilon}{2c_s^2}(\epsilon - 4) \\ f_5 = \dfrac{\epsilon^2}{4c_s^2} \end{array} \right)$$

A is a UV-dependent parameter, independent of cs in general

Recovering canonical single-field limit

$c_s^2 \to 1$

$$\mathcal{L}^{(3)}_{\mathrm{EFT,bulk}} = M_{\mathrm{Pl}}^2\, a^3 \frac{\epsilon}{c_s^2} \left(\cancel{f_0 c_s^2 \frac{\dot\zeta}{H} \frac{(\partial\zeta)^2}{a^2}} + \cancel{\frac{f_1}{H}\dot\zeta^3} + f_2 \dot\zeta^2 \zeta + f_3 c_s^2 \zeta \frac{(\partial\zeta)^2}{a^2} + f_4 \dot\zeta \partial_i \partial^{-2} \dot\zeta \partial_i \zeta + f_5 \partial^2 \zeta (\partial_i \partial^{-2} \dot\zeta)^2 \right)$$

with

$$\left(\begin{array}{l} f_0 = \left(\frac{1}{c_s^2} - 1\right) \\ f_1 = \left(\frac{1}{c_s^2} - 1\right) A \\ f_2 = \epsilon - \eta \\ f_3 = \epsilon + \eta \\ f_4 = \frac{\epsilon}{2}(\epsilon - 4) \\ f_5 = \frac{\epsilon^2}{4} \end{array} \right.$$

Maldacena's result

$f_{\mathrm{NL}} = \mathcal{O}(\epsilon, \eta)$

Recovering the EFT of inflation

Cheung et al 2008

$\boldsymbol{\epsilon, \eta, s \to 0}$

$$\mathcal{L}^{(3)}_{\rm EFT,bulk} = M_{\rm Pl}^2\, a^3 \frac{\epsilon}{c_s^2}\left(f_0 c_s^2 \frac{\dot\zeta}{H}\frac{(\partial\zeta)^2}{a^2} + \frac{f_1}{H}\dot\zeta^3 + \cancel{f_2\dot\zeta^2\zeta} + \cancel{f_3 c_s^2\zeta\frac{(\partial\zeta)^2}{a^2}} + \cancel{f_4\dot\zeta\partial_i\partial^{-2}\dot\zeta\partial_i\zeta} + \cancel{f_5\partial^2\zeta(\partial_i\partial^{-2}\dot\zeta)^2}\right)$$

with

$$f_0 = \left(\frac{1}{c_s^2} - 1\right)$$

$$f_1 = \left(\frac{1}{c_s^2} - 1\right) A$$

$$f_2 = \epsilon - \eta + 2s$$

$$f_3 = \epsilon + \eta$$

$$f_4 = \frac{\epsilon}{2c_s^2}(\epsilon - 4)$$

$$f_5 = \frac{\epsilon^2}{4c_s^2}$$

Decoupling limit

$$\boldsymbol{f_{\rm NL} \sim \frac{1}{c_s^2} - 1}$$

A is a UV-dependent parameter, independent of cs in general

Recovering k-inflation/P(X)

Chen et al 2008
Burrage et al 2011

Redundancy of operators

P(X) result

$$\mathcal{L}^{(3)}_{\rm EFT,bulk} = M_{\rm Pl}^2\, a^3\, \frac{\epsilon}{c_s^2} \left(f_0 c_s^2 \frac{\dot\zeta}{H}\frac{(\partial\zeta)^2}{a^2} + \frac{f_1}{H}\dot\zeta^3 + f_2 \dot\zeta^2\zeta + f_3 c_s^2 \zeta \frac{(\partial\zeta)^2}{a^2} + f_4 \dot\zeta \partial_i \partial^{-2}\dot\zeta \partial_i \zeta + f_5 \partial^2\zeta (\partial_i \partial^{-2}\dot\zeta)^2 \right)$$

with

$$f_1 = \left(\frac{1}{c_s^2} - 1\right)(1+A)$$

$$f_2 = -3\left(\frac{1}{c_s^2} - 1\right) + \frac{\epsilon - \eta}{c_s^2}$$

$$f_3 = \frac{1}{c_s^2} - 1 + \frac{\epsilon + \eta - 2s}{c_s^2}$$

$$f_4 = \frac{\epsilon}{2c_s^2}(\epsilon - 4)$$

$$f_5 = \frac{\epsilon^2}{4c_s^2}$$

A is a UV-dependent parameter, independent of cs in general

Recovering k-inflation/P(X)

Redundancy of operators

$$\mathcal{L}^{(3)}_{\mathrm{EFT,bulk}} = M_{\mathrm{Pl}}^2\, a^3 \frac{\epsilon}{c_s^2} \left(f_0 c_s^2 \frac{\dot\zeta}{H} \frac{(\partial\zeta)^2}{a^2} + \frac{f_1}{H}\dot\zeta^3 + f_2 \dot\zeta^2 \zeta + \right)$$

P(X) result

$$f_1 = \left(\frac{1}{c_s^2} - 1\right)(1+A)$$

$$f_2 = -3\left(\frac{1}{c_s^2} - 1\right) + \frac{\epsilon - \eta}{c_s^2}$$

upon identifications

$$\left(\frac{1}{c_s^2} - 1\right)^{\mathrm{P(X)}} = \frac{2XP_{,XX}}{P_{,X}} \quad \leftrightarrow \quad \left(\frac{1}{c_s^2} - 1\right)^{\mathrm{two-field}} = \frac{4H^2\eta_\perp^2}{m_s^2}$$

$$2\frac{\lambda}{\Sigma} = \frac{2X^2P_{,XX} + 4/3X^3P_{,XXX}}{XP_{,X} + 2X^2P_{,XX}} \quad \leftrightarrow \quad -\left(\frac{1}{c_s^2} - 1\right)A$$

Consistency check

$$\mathcal{L} = -\frac{1}{2}e^{2b(\chi)}(\partial\phi)^2 - \frac{1}{2}(\partial\chi)^2 - V(\phi,\chi)$$

Gelaton
Tolley and Wyman 2010

Under conditions, the full field chi can be integrated out

$$\mathcal{L}^{\mathrm{EFT}} = e^{2b(\chi_\star(\phi,X))}X - V(\phi,\chi_\star(\phi,X)) = P(X,\phi)$$

Agreement: same P(X) and
explicit observables in terms of V and b

In our work: chi fluctuations are integrated out
about two-field background, and P(X)-like cubic action

Revisiting non-Gaussianity in multifield inflation

Multifield cubic action in terms of directly
observable curvature perturbation

+ Genuine size of interactions made manifest

Exact, compact, unifies previous results
Starting point for many applications

First application: single-field EFT
when heavy entropic fluctuation

+ Observable signatures of curved field space

Conclusions

- Inflation: leading paradigm for primordial cosmology

- Formidable laboratory for very high-energy physics

- Quest of primordial GWs and non-Gaussianities

Energy scale of inflation

particle detector

- Recent developments about field space geometry: theoretical challenges and new observational perspectives

*Historical perspective**

*obviously schematic

1980's first models: potential chosen at will

1990's eta-problem: must ensure flatness of potential in QFT

2000's multiple fields: often 'standard kinetic terms for simplicity' or geometrical effects overlooked

One can not afford that anymore!

Today:

must ensure control over potential + geometry

New theoretical challenges and
new observational perspectives

Thank you!

Back-up slides

Expanding the action up to cubic order

In the following, N=2 and $e_{sI}Q^I \equiv \mathcal{F}$

Sufficient to solve the constraints at linear order

$$N = 1 + \frac{\dot{\zeta}}{H} \qquad N^i = \delta^{ij}\partial_j\theta/a^2$$

$$\theta = -\frac{\zeta}{H} + \chi \qquad \boxed{\frac{1}{a^2}\,\partial^2\chi = \epsilon\dot{\zeta} + \frac{\dot{\sigma}\eta_\perp}{M_{\rm Pl}^2}\,\mathcal{F}}$$

Expanding the action up to cubic order

In the following, N=2 and $e_{sI}Q^I \equiv \mathcal{F}$

Sufficient to solve the constraints at linear order

$$N = 1 + \frac{\dot{\zeta}}{H} \qquad N^i = \delta^{ij}\partial_j\theta/a^2$$

$$\theta = -\frac{\zeta}{H} + \chi \qquad \boxed{\frac{1}{a^2}\,\partial^2\chi = \epsilon\dot{\zeta} + \frac{\dot{\sigma}\eta_\perp}{M_{\rm Pl}^2}\,\mathcal{F}}$$

$$-\Psi/H \qquad \partial^2\Psi/a^2 = \delta\rho_{\rm com}/(2M_{\rm Pl}^2)$$

Bardeen potential `Poisson equation'

Impact of boundary terms

$$
\begin{aligned}
B_\zeta(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) - B_{\zeta,\mathrm{bulk}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = \\
&-a(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)P_{\zeta\mathcal{F}}(k_1)P_\zeta(k_2) && + 5\,\mathrm{perms.}\\
&-2\,b(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)P_\zeta(k_1)P_\zeta(k_2) && + 2\,\mathrm{perms.}\\
&-2\,c(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)P_\zeta(k_3)P_{\zeta p_\zeta}(k_2) && + 5\,\mathrm{perms.}
\end{aligned}
$$

with

$$
\begin{aligned}
a(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) &= \frac{\dot{\sigma}\eta_\perp}{\epsilon H M_{\mathrm{Pl}}^2}\\
b(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) &= \frac{1}{8a^2H^2}\left(k_1^2+k_2^2-(\boldsymbol{k}_1\cdot\hat{\boldsymbol{k}}_3)^2-(\boldsymbol{k}_2\cdot\hat{\boldsymbol{k}}_3)^2\right)\\
c(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) &= \frac{1}{8a^3\epsilon H M_{\mathrm{Pl}}^2}\left(-2+\epsilon\left(1-(\hat{\boldsymbol{k}}_1\cdot\hat{\boldsymbol{k}}_2)^2\right)\right)\,,
\end{aligned}
$$

Impact of boundary terms

$$B_\zeta(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) - B_{\zeta,\mathrm{bulk}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) =$$

$$-a(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)P_{\zeta\mathcal{F}}(k_1)P_\zeta(k_2) \qquad + 5\,\mathrm{perms.}$$

$$\cancel{-2\,b(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)P_\zeta(k_1)P_\zeta(k_2) \qquad + 2\,\mathrm{perms.}}$$

$$\cancel{-2\,c(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)P_\zeta(k_3)P_{\zeta p_\zeta}(k_2) \qquad + 5\,\mathrm{perms.}}$$

with $$a(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = \frac{\dot{\sigma}\eta_\perp}{\epsilon H M_{\mathrm{Pl}}^2}$$

- last 2 terms negligible on super-Hubble scale, independently of the conservation or not of zeta

- first term relevant only if entropic modes do not decay

Integrating out heavy entropic fluctuations

Effective cubic action:

$$\mathcal{L}^{(3)} = M_{\rm Pl}^2\, a^3 \left[\epsilon(\epsilon-\eta)\dot{\zeta}^2\zeta + \epsilon(\epsilon+\eta)\zeta\frac{(\partial\zeta)^2}{a^2} + \left(\frac{\epsilon}{2}-2\right)\frac{1}{a^4}\left(\partial\zeta\right)\left(\partial\chi\right)\partial^2\chi + \frac{\epsilon}{4a^4}\partial^2\zeta(\partial\chi)^2\right]$$

$$+\, a^3\left[\frac{1}{2}m_s^2(\epsilon+\mu_s)\zeta\mathcal{F}^2 + \left(2\epsilon-\eta-2\lambda_\perp\right)\dot{\sigma}\eta_\perp\zeta\dot{\zeta}\mathcal{F} + \frac{\dot{\sigma}\eta_\perp}{a^2 H}\,\mathcal{F}(\partial\zeta)^2\right.$$

$$-\,\frac{\dot{\sigma}\eta_\perp}{H}\,\dot{\zeta}^2\mathcal{F} - \frac{1}{H}\left(H^2\eta_\perp^2 - \epsilon M_{\rm Pl}^2 H^2 R_{\rm fs}\right)\dot{\zeta}\mathcal{F}^2 - \frac{1}{6}\left(V_{;sss} - 2\dot{\sigma}H\eta_\perp R_{\rm fs} + \epsilon M_{\rm Pl}^2 H^2 R_{{\rm fs},s}\right)\mathcal{F}^3$$

$$\left. +\, \frac{1}{2}\epsilon\zeta\left(\dot{\mathcal{F}}^2 + \frac{(\partial\mathcal{F})^2}{a^2}\right) - \frac{1}{a^2}\dot{\mathcal{F}}\partial\mathcal{F}\partial\chi\right] + \mathcal{D}$$

Integrating out heavy entropic fluctuations

Effective cubic action:

$$\mathcal{L}^{(3)} = M_{\rm Pl}^2\, a^3 \left[\epsilon(\epsilon-\eta)\dot\zeta^2\zeta + \epsilon(\epsilon+\eta)\zeta\frac{(\partial\zeta)^2}{a^2} + \left(\frac{\epsilon}{2}-2\right)\frac{1}{a^4}\left(\partial\zeta\right)\left(\partial\chi\right)\partial^2\chi + \frac{\epsilon}{4a^4}\partial^2\zeta(\partial\chi)^2 \right]$$

$$\dot\zeta\partial_i\partial^{-2}\dot\zeta\partial_i\zeta \qquad \partial^2\zeta(\partial_i\partial^{-2}\dot\zeta)^2$$

$$\boxed{+\, a^3\left[\frac{1}{2}m_s^2(\epsilon+\mu_s)\zeta\mathcal{F}^2 + (2\epsilon-\eta-2\lambda_\perp)\,\dot\sigma\eta_\perp\zeta\dot\zeta\mathcal{F} + \frac{\dot\sigma\eta_\perp}{a^2H}\,\mathcal{F}(\partial\zeta)^2\right.} \longrightarrow \dot\zeta^2\zeta\,,\,\dot\zeta(\partial\zeta)^2$$

$$\boxed{-\,\frac{\dot\sigma\eta_\perp}{H}\,\dot\zeta^2\mathcal{F} - \frac{1}{H}\left(H^2\eta_\perp^2 - \epsilon M_{\rm Pl}^2H^2R_{\rm fs}\right)\dot\zeta\mathcal{F}^2 - \frac{1}{6}\left(V_{;sss} - 2\dot\sigma H\eta_\perp R_{\rm fs} + \epsilon M_{\rm Pl}^2H^2R_{{\rm fs},s}\right)\mathcal{F}^3} \longrightarrow \dot\zeta^3$$

$$\cancel{+\,\frac{1}{2}\epsilon\zeta\left(\dot{\mathcal{F}}^2 + \frac{(\partial\mathcal{F})^2}{a^2}\right) - \frac{1}{a^2}\dot{\mathcal{F}}\partial\mathcal{F}\partial\chi}\Bigg] + \mathcal{D}$$

negligible