EXPLAINING THE DENSITY PROFILE OF DARK MATTER HALO

Dong-Biao Kang, VIA, January, 2020

Density profile of dark matter halo

- Simulations of "isolated" equilibrium cold dark matter halo

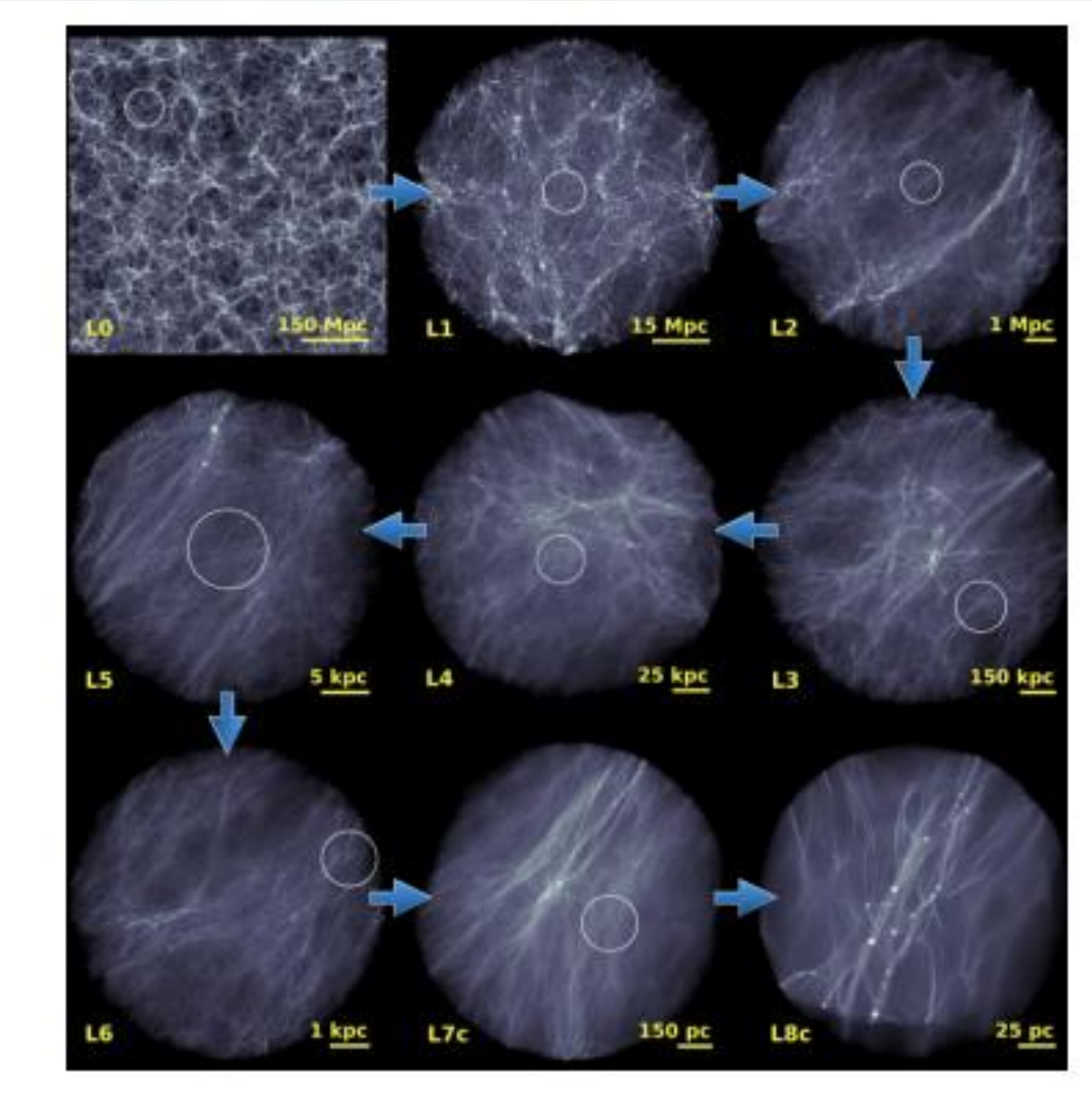
$$\rho(r) = \rho_s r_s^3 / r(r + r_s)^2,$$

Navarro et al, 1997

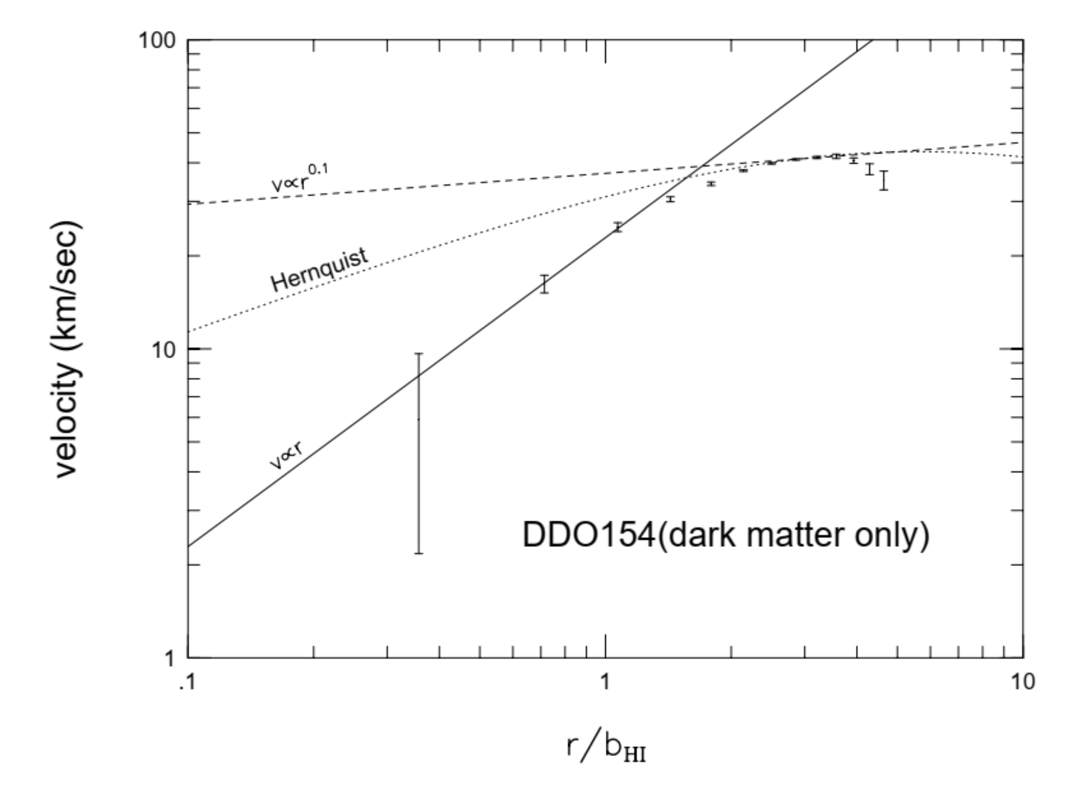
$$\rho(r) = \rho_{-2} \exp[-2\alpha^{-1}((r/r_{-2})^{\alpha} - 1)],$$

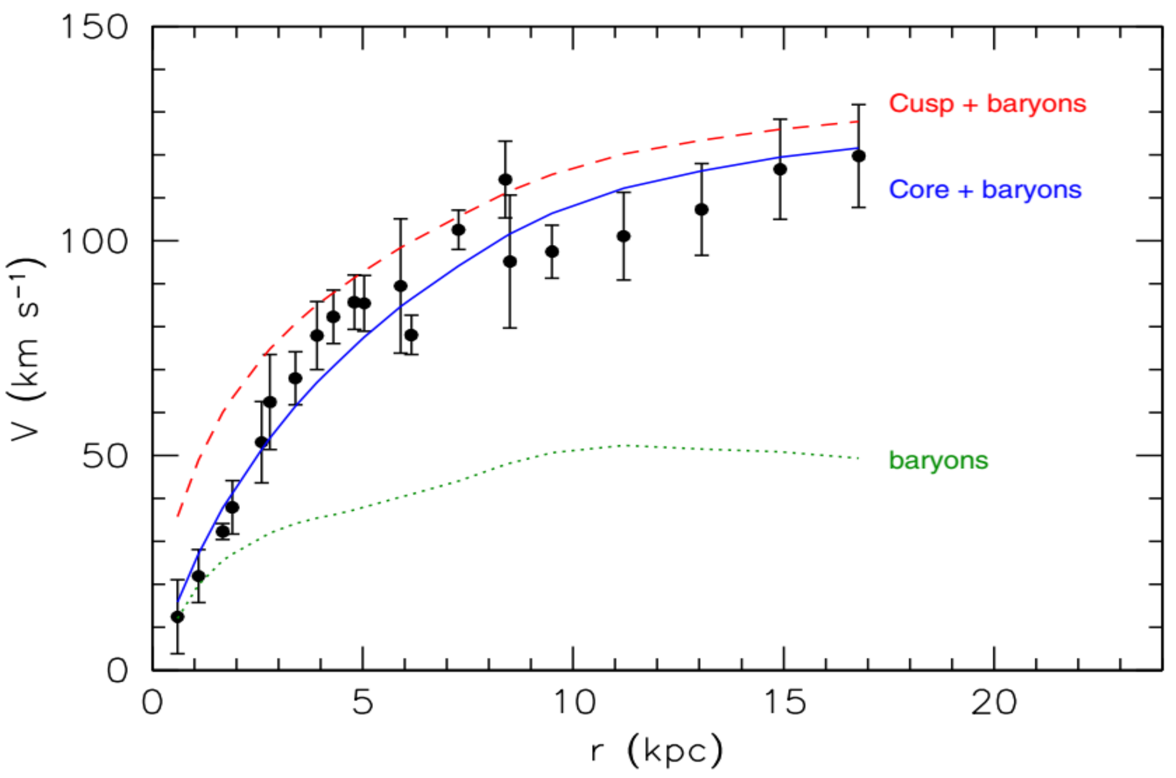
Einasto, 1965

- Very recently, Wang et al 2019 in their simulations shows this universality can be extended to the halos over twenty orders of magnitude in mass.



◦ Observations, core-cusp problem

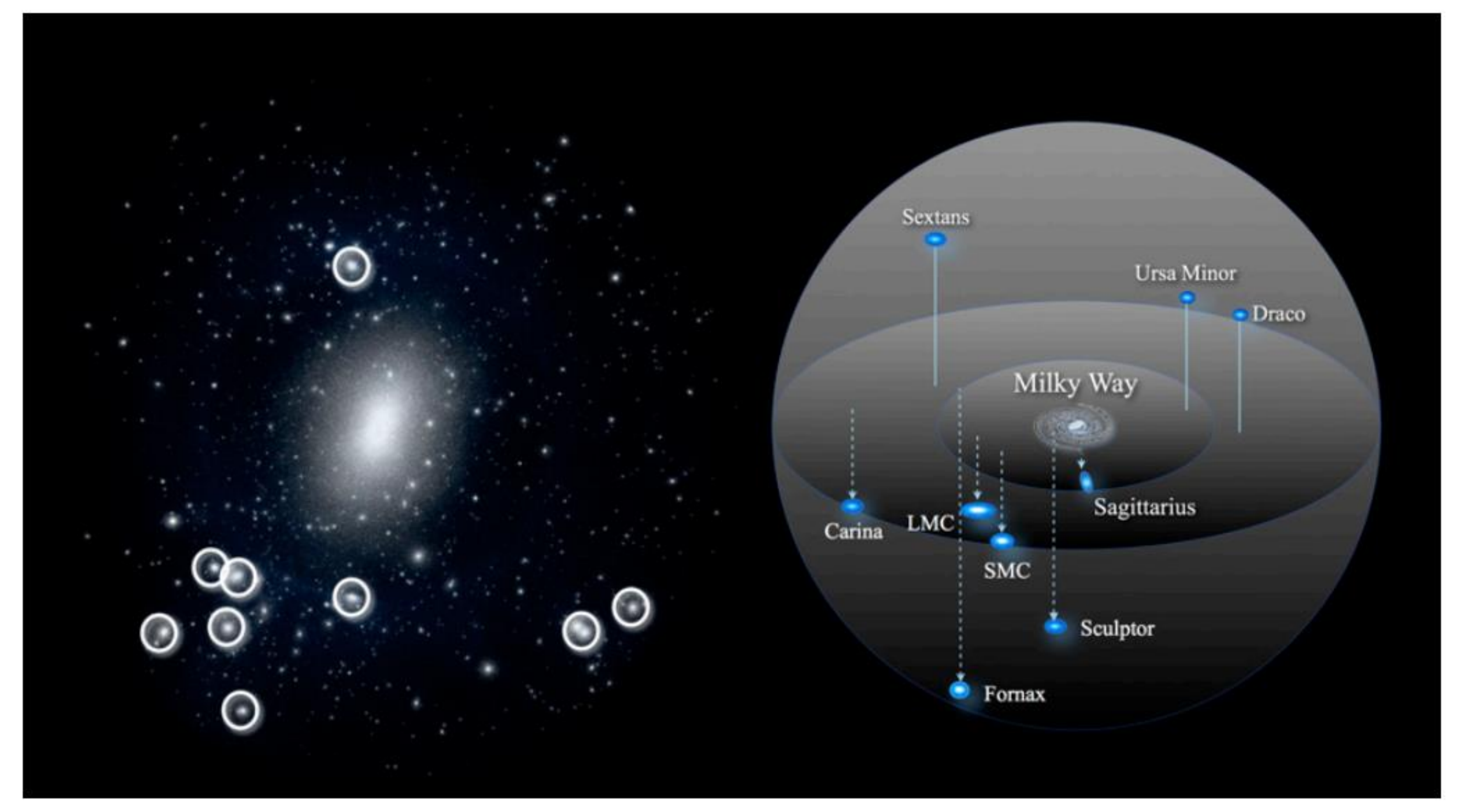




Flores and Primack, 1994

Kuzio et al , 2008

Other small scale controversies



◦ Solutions to the core-cusp problem

◦ Baryonic physics:

the supernova
feedback of a vigorous

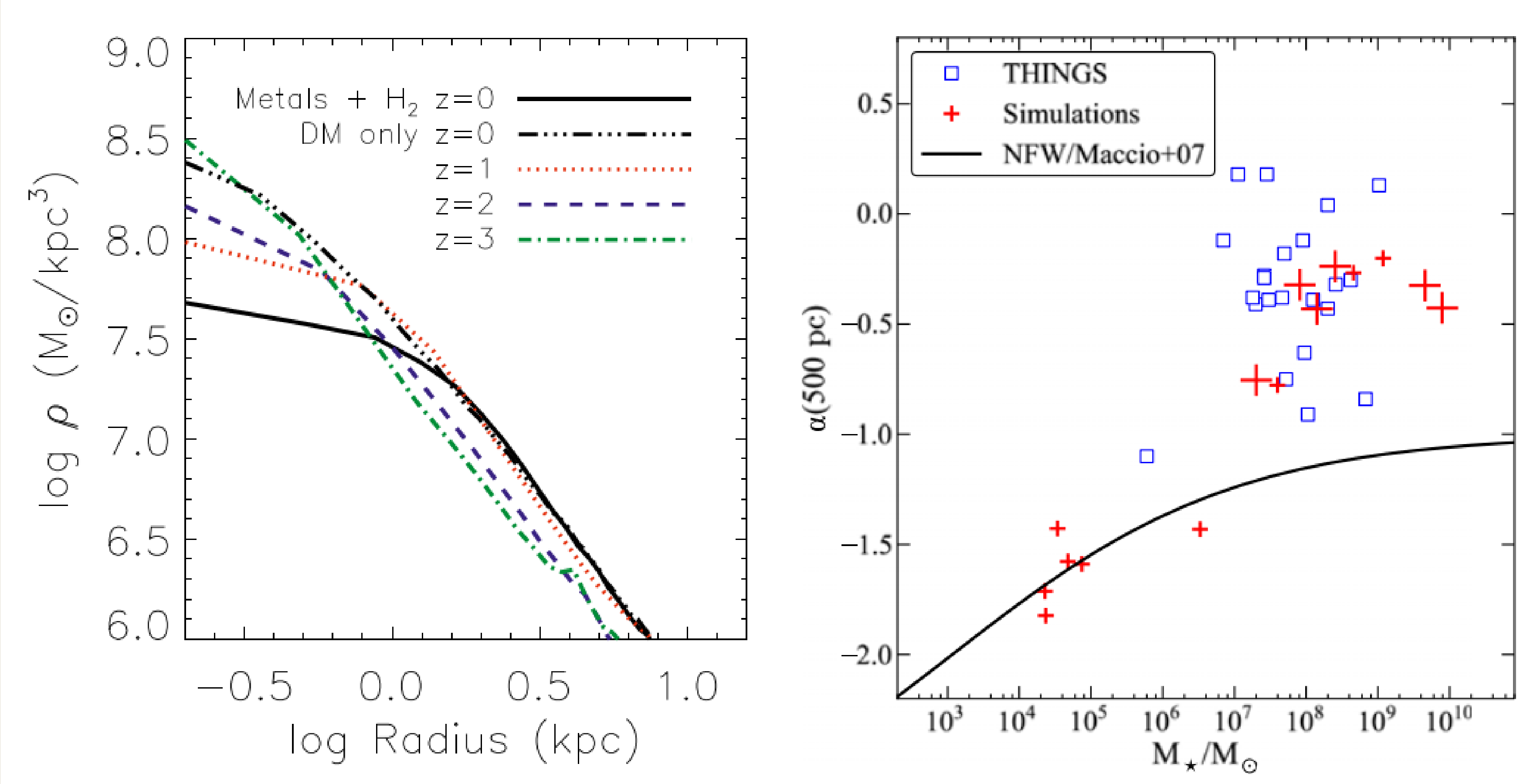
starburst blows out a

substantial fraction
of the baryonic material.

Other schemes include

Dynamical friction,

Triaxiality, off-set, etc.

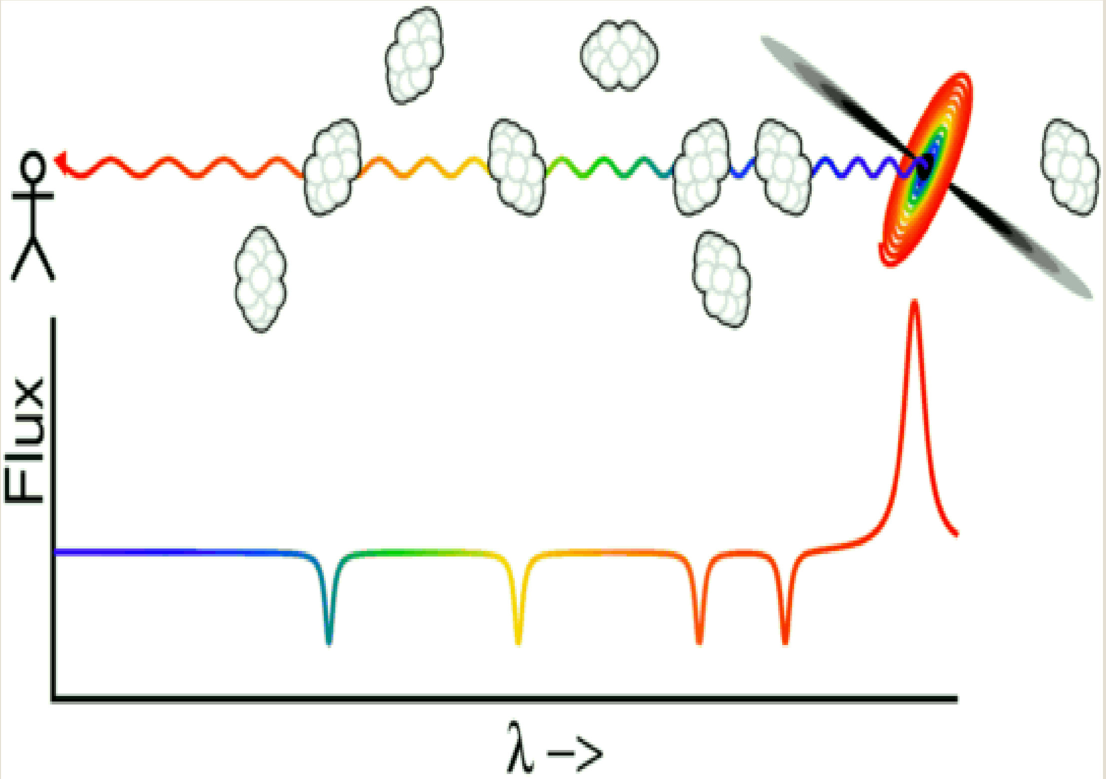


- Particle physics
- Warm dark matter

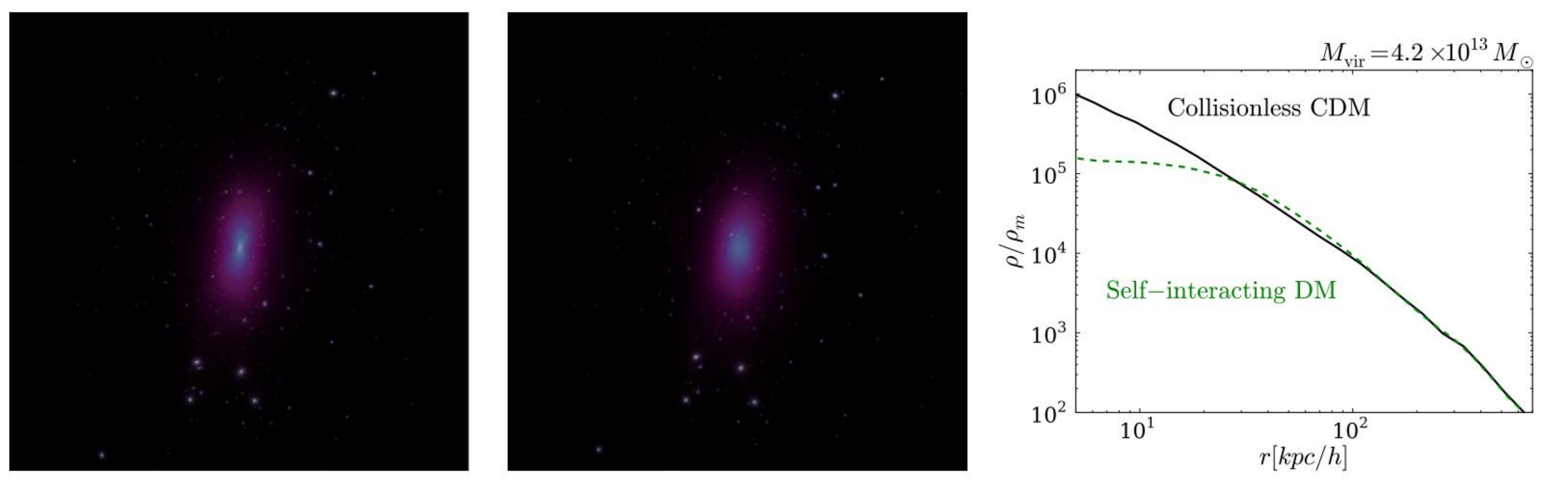
If the core radius in simulations is comparable,

m<0.1kev

which can not be consistent with observations of Lyman alpha forest



- Self-interacting dark matter (SIDM)



However, core seems almost universal for observations (arXiv:2002.12192), cusp is almost universal for simulations,

why both are universal?

The baryonic solution seems not to be the universal mechanism transferring cusp to core

While the SIDM model changes the nature of dark matter and its universality to fit

the core radius with different galaxies should also be further justified.

Explanations for the density profile

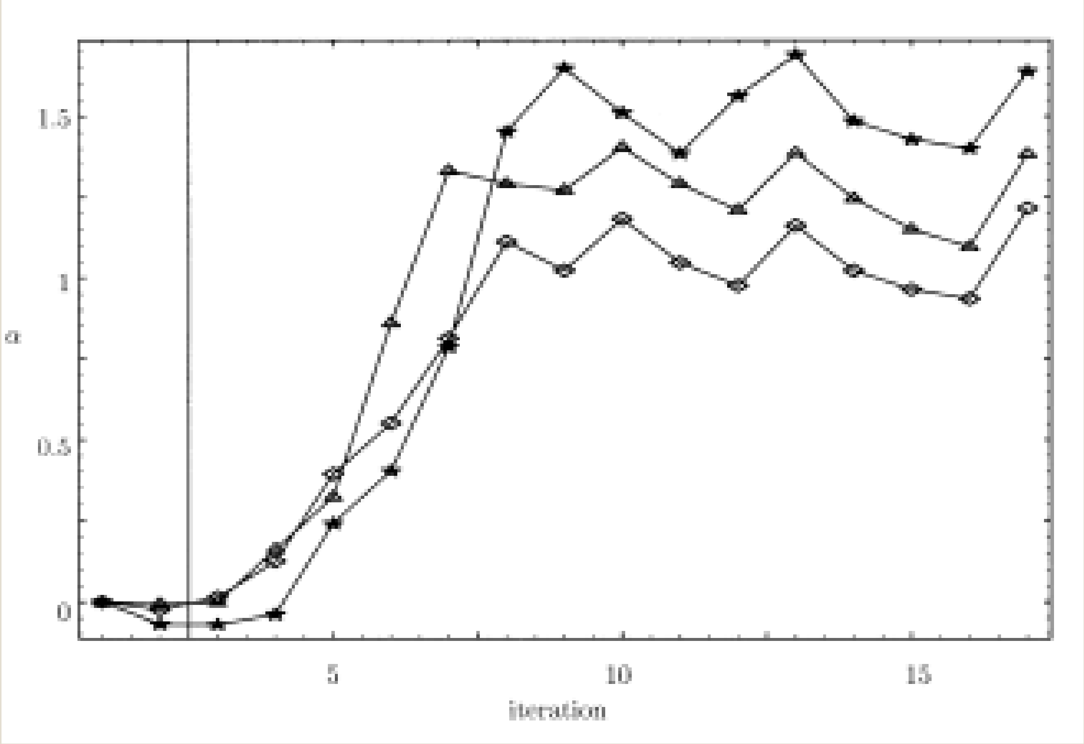
- hierarchical clustering, Syer& White, 1998

  it is a fixed point in the process of repeated mergers

  the density should vary as $r^{-\alpha}$

  $\alpha \simeq 3(3+n)/(5+n)$

  initial fluctuations $P(k) \sim k^n$

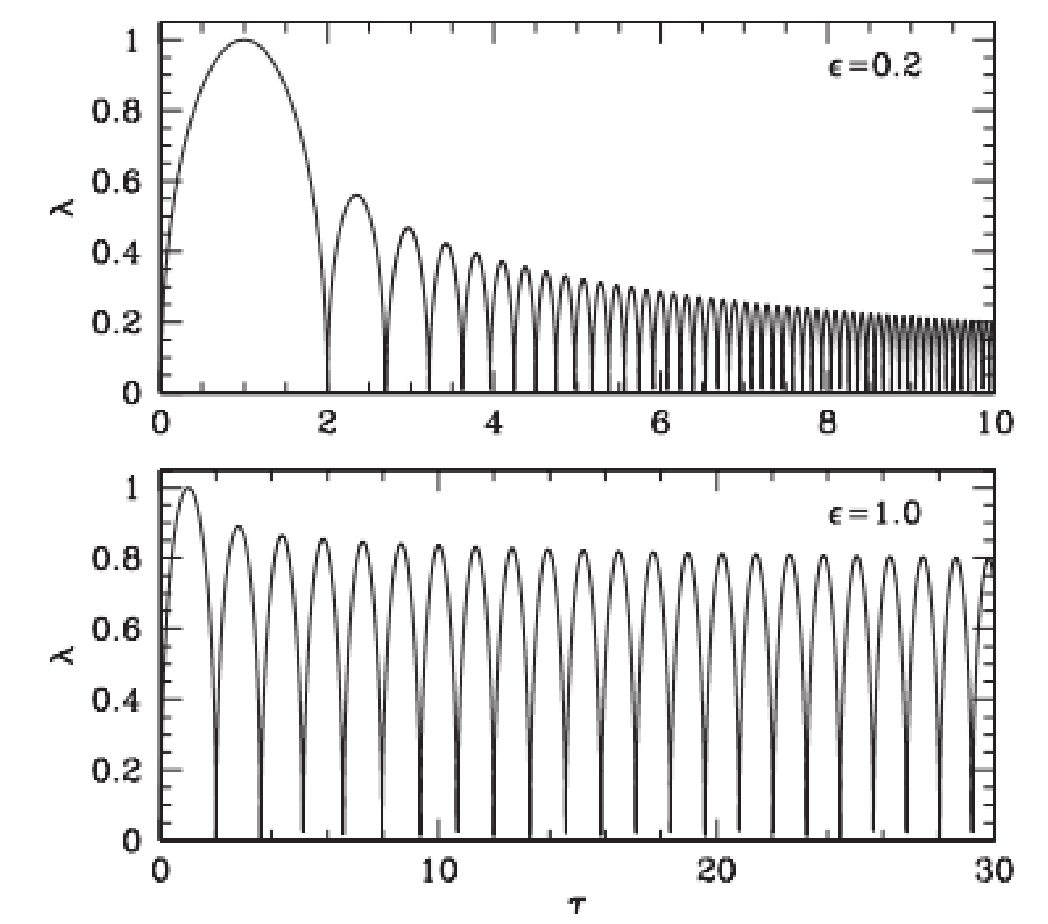


- Self-similar solution of collisionless collapse

$$\frac{d^2 r}{dt^2} = -\frac{GM(r,t)}{r^2}.$$

$$\delta_i \equiv \frac{\delta M_i}{M_i} = \left(\frac{M_i}{M_*}\right)^{-\varepsilon},$$

$$\gamma = 2, \qquad \text{for } \varepsilon \leq \tfrac{2}{3};$$
$$\gamma = \tfrac{9\varepsilon}{(1+3\varepsilon)}, \quad \text{for } \varepsilon > \tfrac{2}{3}.$$



Berschinger, 1985

Statistical mechanics for self-gravitating system

- Properties:
- Thermodynamical limit (N=∞)

two-body relaxation timescale $$t_{\rm relax} \simeq \frac{0.1N}{\ln N} t_{\rm cross}.$$

violent relaxation $$t_v \sim \left(\frac{3\pi}{32G\bar{\rho}}\right)^{1/2}.$$

- Non-additivity of energy

E≠E1+E2，violates the requirement of canonical ensemble

- In-equivalence of ensembles

Cv>0 in the canonical; Cv<0 in the microcanonical, E=K+V=-K=-3NkT/2

- Ergodicity breaking (Mukamel et al.2005)

Mean-Field model

- One-particle distribution f(x, v,t)
- Boltzmann-Gibbs entropy

$$S = -N \int \mathrm{d}^3\mathbf{x}\mathrm{d}^3\mathbf{v}\, f \ln f$$

With fixed mass and energy, we have isothermal solution with infinite mass

(Binney&Tremaine2008)

- Proposals
- Tsallis statistics , based on the non-additivity

$$S = -\int \frac{f^q - 1}{q - 1} \mathrm{d}\tau$$

where q is a parameter, when q=1, S recovers to the Boltzmann-Gibbs form

at equilibirium the equation of state is polytropic ,

$$p = \rho^\gamma, \gamma = \frac{5q-3}{3q-1}$$

which has finite mass but not consistent with observations and simulations

(Tsallis 1988)

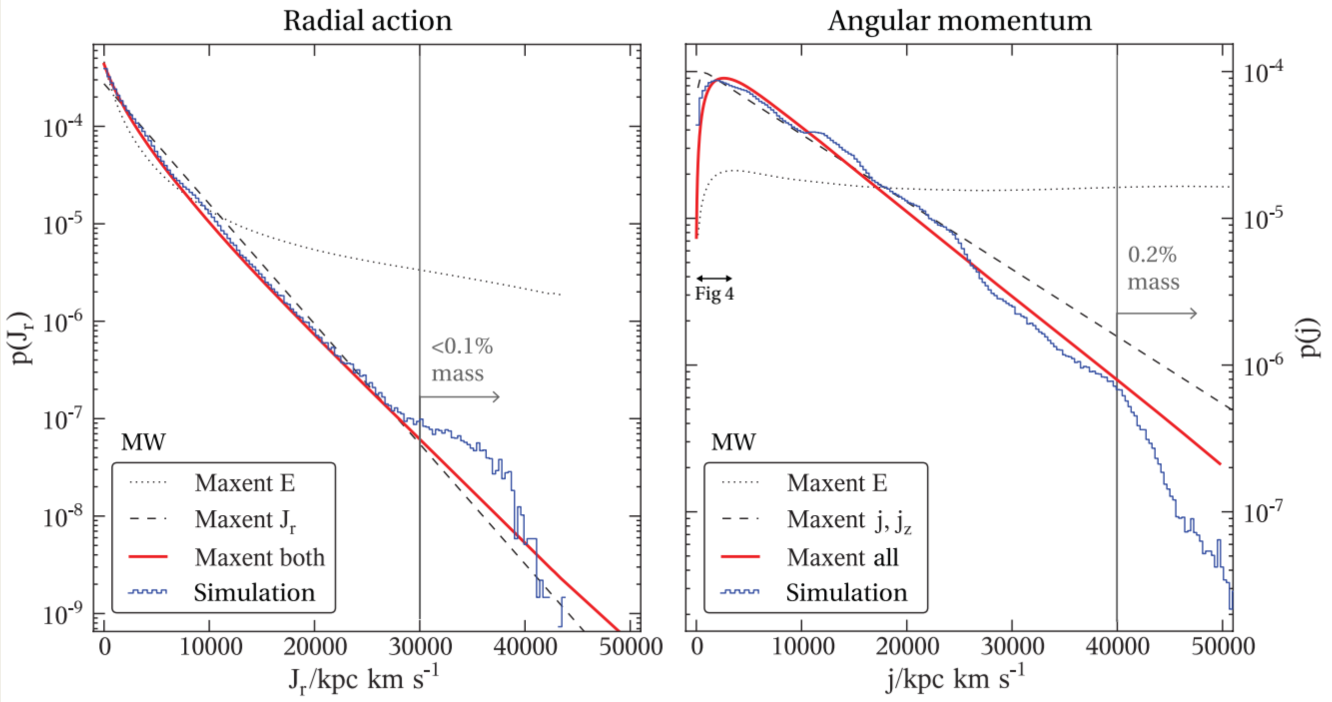
- Incomplete relaxation
- such as (Potzen & Governato 2013), which suggests

$$J_r = \frac{1}{\pi}\int_{r_{\min}}^{r_{\max}} \sqrt{2E - 2\Phi(r;t) - j^2/r^2}\,\mathrm{d}r.$$

The ensemble average of $\boldsymbol{J} = (J_r,\, j,\, j_z)$

is conserved if the potential changes slowly in time

$$f(\boldsymbol{J}) \propto \exp\left(-\boldsymbol{\beta}\cdot\boldsymbol{J} - \beta_E E(\boldsymbol{J})\right),$$



- Our previous works in the microcanonical ensemble

Based on (White & Narayan 1987)'s work on self-gravitating gas,

for collisional system or collision-less system on the coarsely graining ,

In spherical coordinate we assume

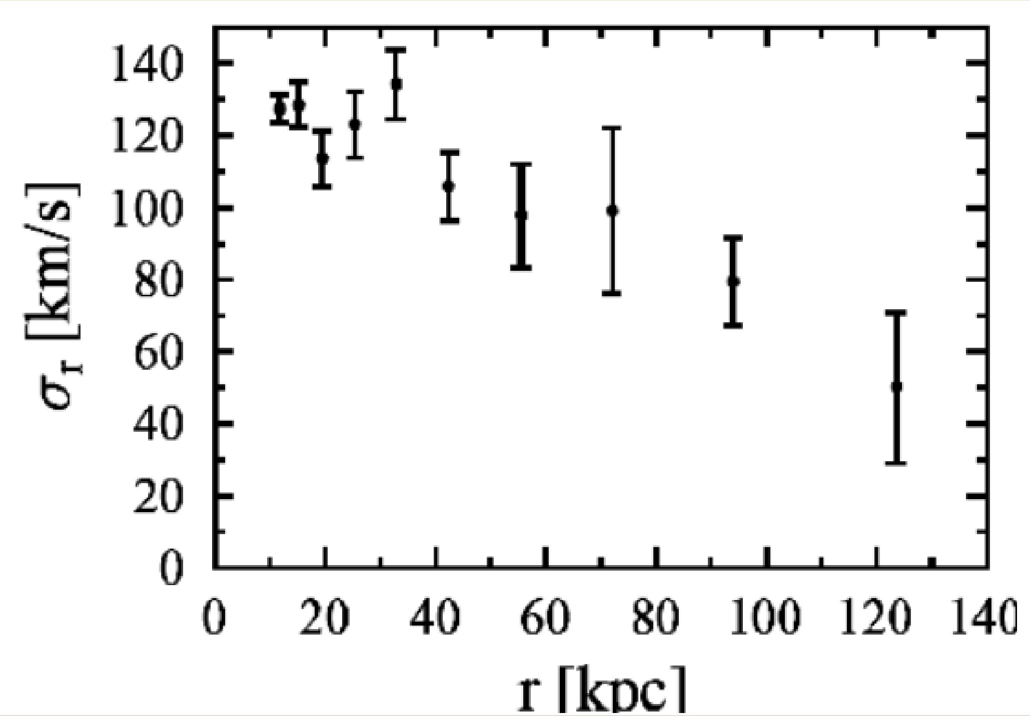
$$f(r,v) = \frac{1}{(\sqrt{2\pi})^3}\frac{\rho(r)}{\sigma^3(r)}e^{-\frac{v^2}{2\sigma(r)^2}}$$

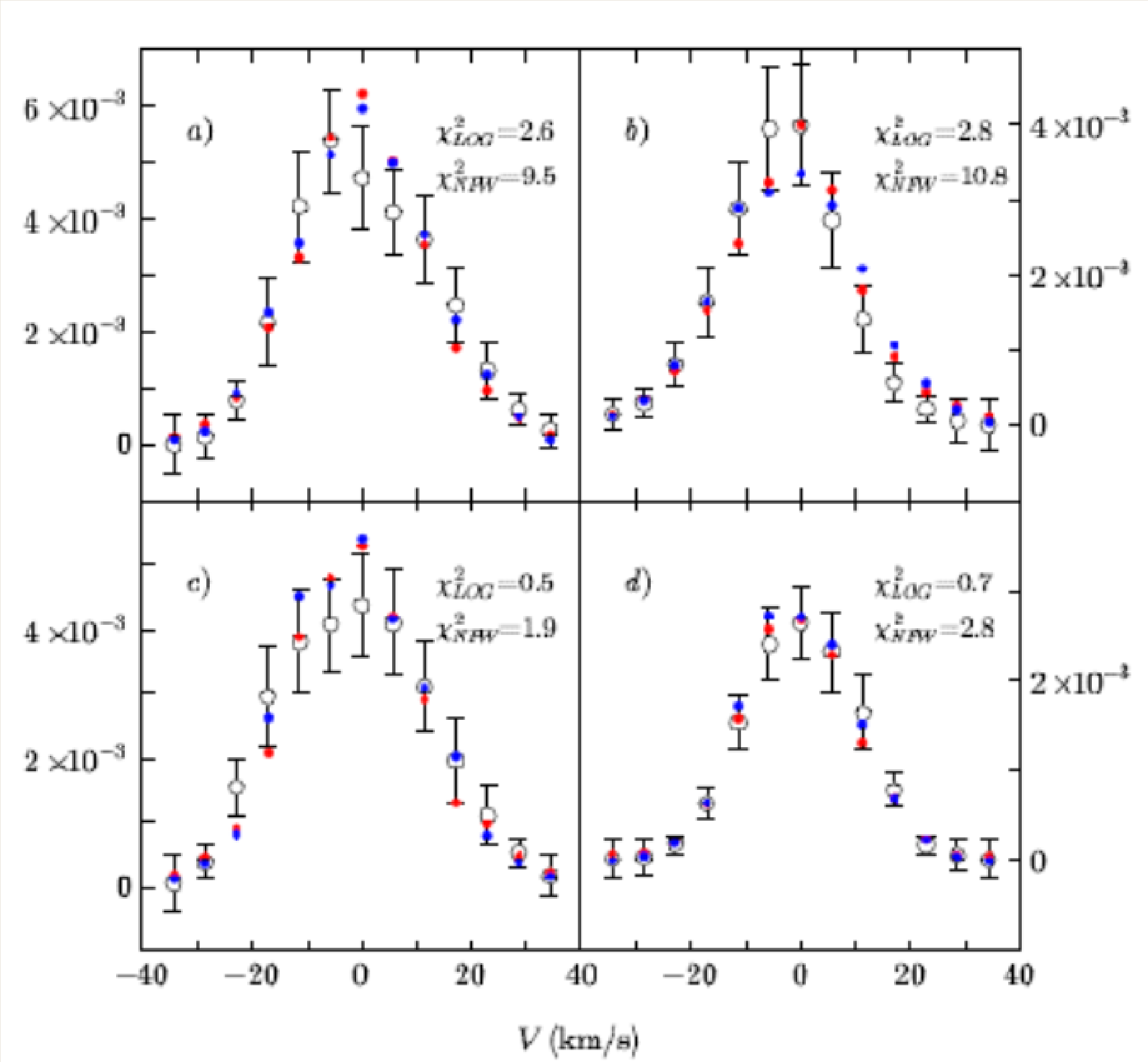
Which can be supported by observations.

-

(Kang 2012)

Observation supports





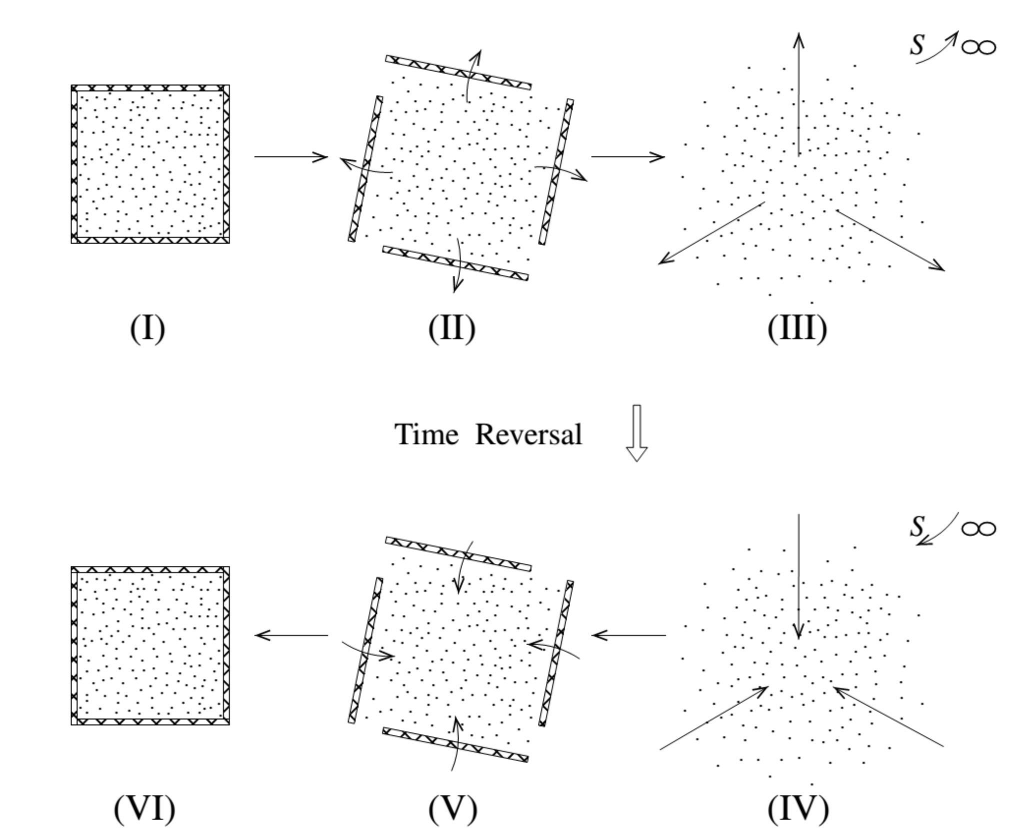
$$S_{\mathrm{t}} = \int_0^\infty 4\pi r^2 \rho s \, \mathrm{d}r = \int_0^\infty 4\pi r^2 \rho \ln(p^{3/2}\rho^{-5/2}) \, \mathrm{d}r.$$

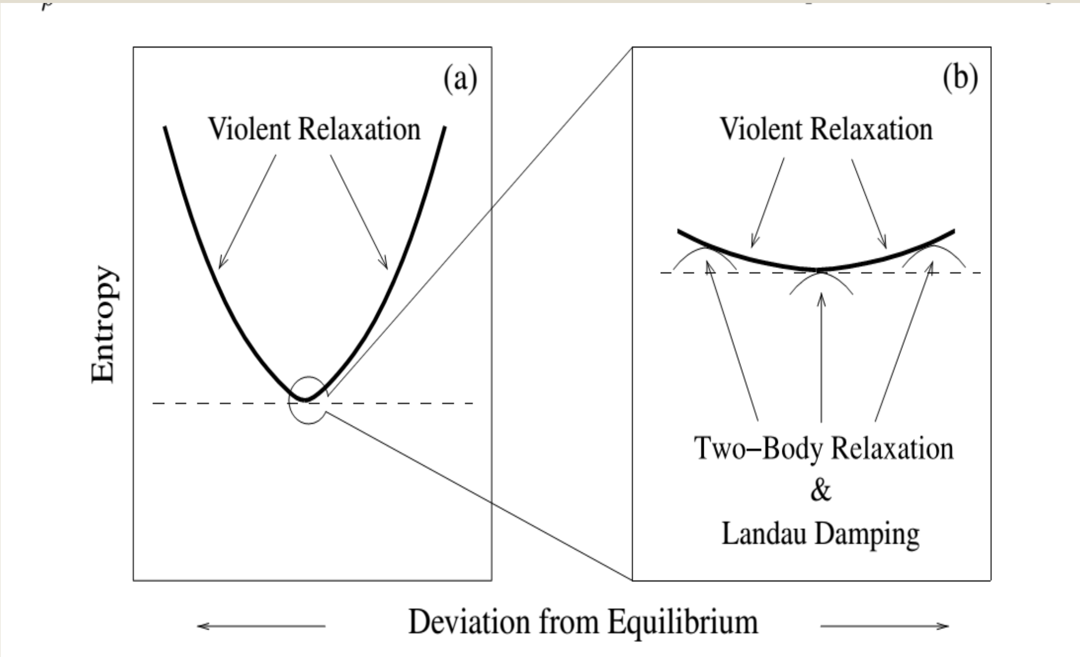
$$M_{\mathrm{t}} = \int_0^\infty 4\pi r^2 \rho(r) \, \mathrm{d}r.$$

$$\begin{aligned} E_{\mathrm{V}} &= -\int \frac{G\rho(\boldsymbol{r})\rho(\boldsymbol{r}')}{2|\boldsymbol{r}-\boldsymbol{r}'|} \, \mathrm{d}V \mathrm{d}V' \\ &= -4\pi G \int_0^\infty M(r)\rho(r) r \, \mathrm{d}r. \end{aligned}$$

-

(He&Kang2010)

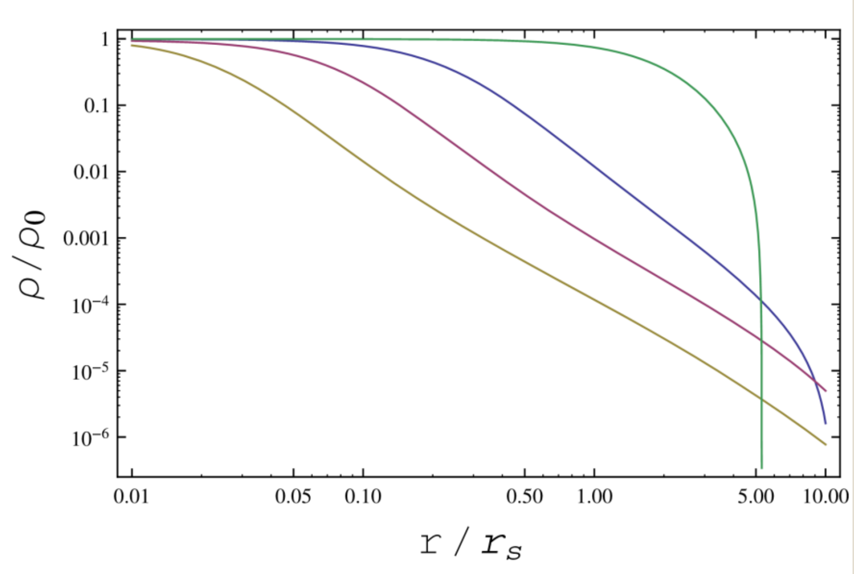

S ∞
(I)
(II)
(III)
Time Reversal
S ∞
(VI)
(V)
(IV)


(a)
Violent Relaxation
Entropy
(b)
Violent Relaxation
Two–Body Relaxation
&
Landau Damping
Deviation from Equilibrium

$$\rho = \lambda P + \mu P^{3/5}$$

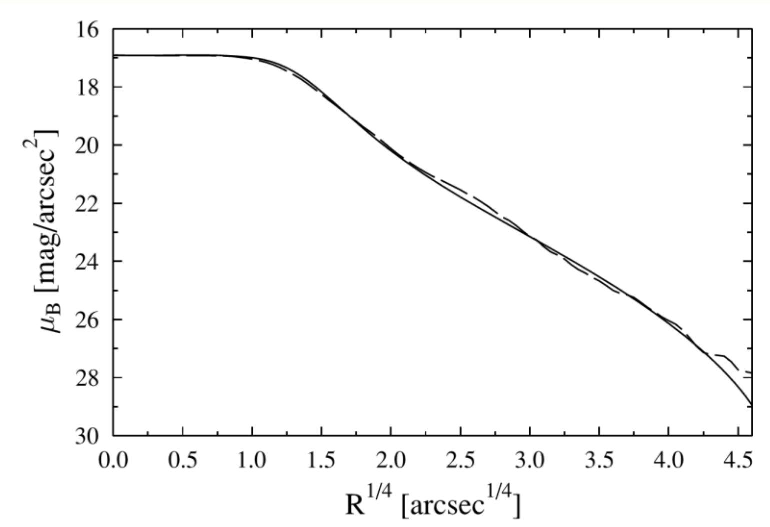
Which formly can be obtained by maximizing the entropy

$$S[f] = S[f_1, f_2] = \int \left(-f_1 \ln f_1 - \frac{f_2^q - f_2}{q-1} \right) \mathrm{d}\tau,$$

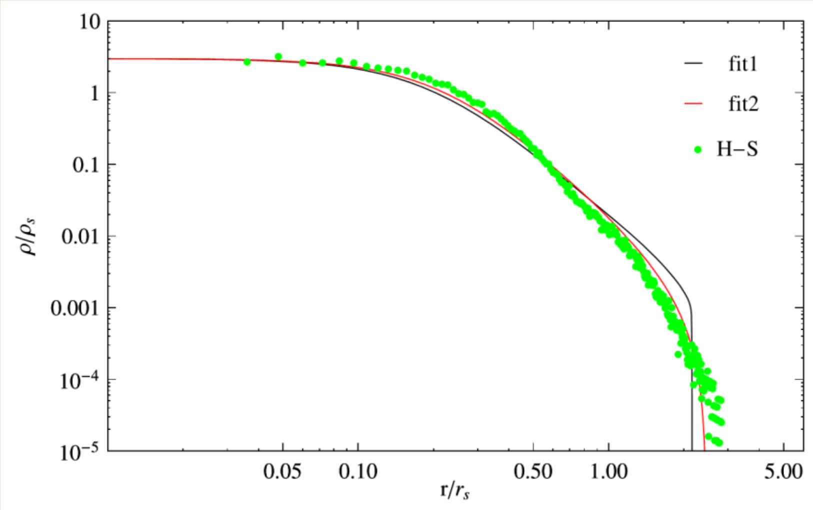


-

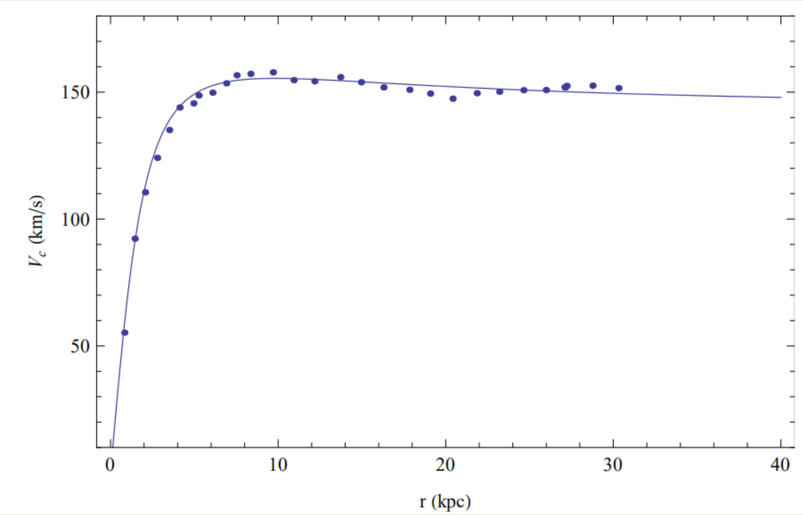
(Kang&He2011)



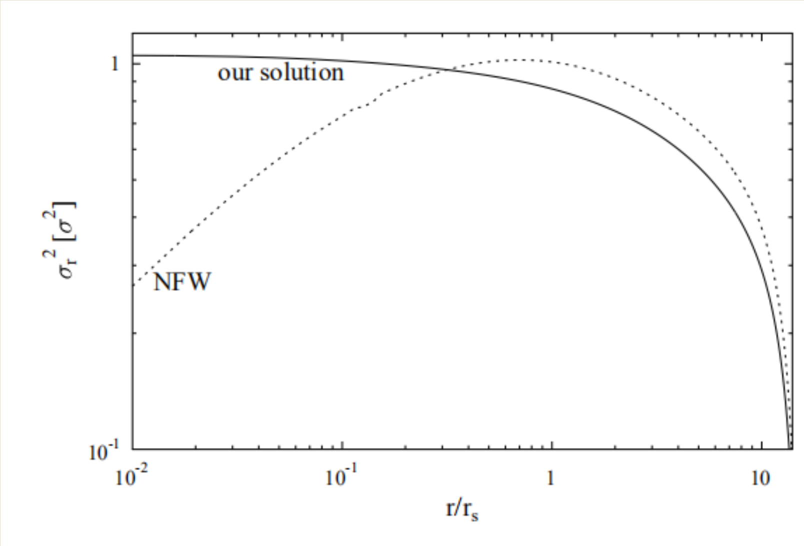
μ_B [mag/arcsec^2]
$R^{1/4}$ [arcsec$^{1/4}$]
16
18
20
22
24
26
28
30
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5



ρ/ρ_s
r/r_s
fit1
fit2
H−S
10
1
0.1
0.01
0.001
10^{-4}
10^{-5}
0.05
0.10
0.50
1.00
5.00



V_c (km/s)
r (kpc)
50
100
150
0
10
20
30
40



$\sigma_r^2\ [\sigma^2]$
r/r_s
our solution
NFW
1
10^{-1}
10^{-2}
10^{-1}
1
10

Thermodynamics stability

$$\delta^2 S_t = \frac{1}{2}\int\left[\left(m\frac{G}{2kTr^2}\right)(\delta M)^2 - \frac{10\pi r^2}{m\rho}(\delta\rho)^2 - \frac{6\pi r^2\rho}{mP^2}(\delta P)^2\right]\mathrm{d}r$$

$$E = -\frac{3}{2}kT[N - \alpha(kT)^n A],$$

$$C_V = \frac{\partial E}{\partial T} = \frac{3}{2}k[(1+n)\alpha(kT)^n A - N],$$

New work in the canonical ensemble

- Landau-Ginzburg theory

describing the long range correlation of fluctuations from the equilibrium state in an approximate fashion:

$$C(r) = \overline{[\rho(\mathbf{r}) - \bar{\rho}][\rho(0) - \bar{\rho}]}$$

$$\Delta F = F - \bar{F} = \int \{a_1(\rho - \bar{\rho}) + \frac{a_2}{2}(\rho - \bar{\rho})^2 + \frac{b}{2}(\nabla\rho)^2\} \mathrm{d}^3 r.$$

To ensure that the system is stable equilibrium state at $\rho = \bar{\rho}$, $\delta\rho = 0$
a2 and b are required to be positive

Two approaches

- One is

$$\rho(\mathbf{r}) - \bar{\rho} = \int \rho_{\mathbf{k}} \mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}} \mathrm{d}^3 k,$$

$$\Delta F = \frac{1}{2} \int \mathrm{d}^3 k \, |\rho_k|^2 \left(a_2 + b k^2\right)$$

$$w = w_0 e^{-\Delta F / k_B T},$$

- density perturbation field is Gaussian with power spectrum

$$P(k) = \frac{k_{\mathrm{B}} T}{a_2 + b k^2}$$

$$C(r) = \frac{k_{\mathrm{B}}T}{(2\pi)^3} \int \frac{\mathrm{e}^{\mathbf{i}\mathbf{k}\cdot\mathbf{r}}}{a_2 + bk^2} \mathrm{d}^3k = \frac{k_B T}{4\pi b} \frac{e^{-\frac{r}{\xi}}}{r}.$$

where

$$\xi = \sqrt{\frac{b}{a_2}}$$

two-point correlation function $C(r) \propto 1/r$ for $r \ll \xi$, while $C(r) \propto e^{\frac{-r}{\xi}}$ for $r \gg \xi$.

- The other is

$$F(\rho, T) = G + h\rho$$

$$dF = -SdT + hdM$$

$$h(\mathbf{r}) = \frac{\delta F}{\delta \rho(\mathbf{r})}$$

$$h(\mathbf{r}) = a_2\phi(\mathbf{r}) - b\nabla^2\phi(\mathbf{r})$$

$$\phi(r) = \rho(r) - \bar{\rho}.$$

In spherical coordinate, If h(r)=0, the roots of the general solution are

$$\frac{c_1 e^{-r/\xi}}{r} + \frac{c_2 e^{r/\xi}}{r}$$

After test we speculate If h(r) is shallower than r^{-4} or can be written as $\delta(r)g(r)$, the r^{-1} cusp can be always obtained

- If h(r) is localized, such as

$$h(\mathbf{r}) = h_0\delta(\mathbf{r})$$

$$\phi(r) = \frac{h_0}{4\pi b}\frac{e^{-r/\xi}}{r}.$$

- Including a term in the Hamiltonian

$$-\int d^3r\rho(\mathbf{r})h(\mathbf{r})$$

$$\overline{\rho(\mathbf{r})} = \frac{\mathrm{Tr}\rho(\mathbf{r})\exp\left\{-\beta\left[H_0 - \int \mathrm{d}^3r'h\left(\mathbf{r}'\right)\rho\left(\mathbf{r}'\right)\right]\right\}}{\mathrm{Tr}\exp\left\{-\beta\left[H_0 - \int \mathrm{d}^3r'h\left(\mathbf{r}'\right)\rho\left(\mathbf{r}'\right)\right]\right\}}$$

$$\begin{aligned}\frac{\delta\overline{\rho(\mathbf{r})}}{\delta h(0)} &= \phi(\mathbf{r})/h_0 = \beta\overline{\rho(\mathbf{r})\rho(0)} - \beta\overline{\rho(\mathbf{r})}\ \overline{\rho(0)} \\ &= \beta\overline{[\rho(\mathbf{r}) - \bar{\rho}][\rho(0) - \bar{\rho}]} \propto C(r)\end{aligned}$$

- We assume the density distribution of the equilibrium background is , $\rho(r)$

in the first approach we can obtain

$$h(r,r') = a_2(\rho(r,r') - \overline{\rho(r)}) - b\nabla^2(\rho(r,r') - \overline{\rho(r)})$$

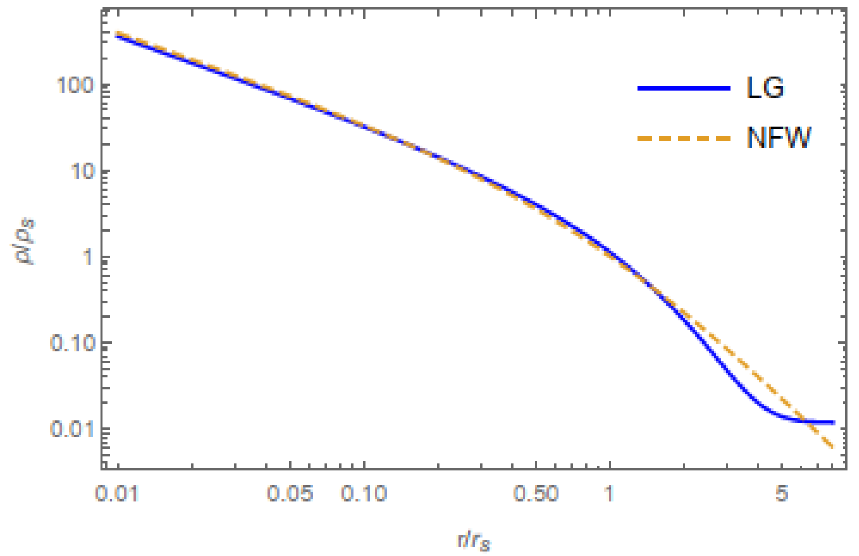
The perturbations here will be $h_0\delta(r-r')$, which may be an ideal model for galaxy merger, gravity instability and etc.

$$\rho(r,r') = \rho(|r-r'|) \propto \frac{1}{|r-r'|} \; for \; r \to 0$$

LG theory predicts the behavior of Gaussian power

spectrum at large enough wave number

$$P(k) \propto k^{-2},$$

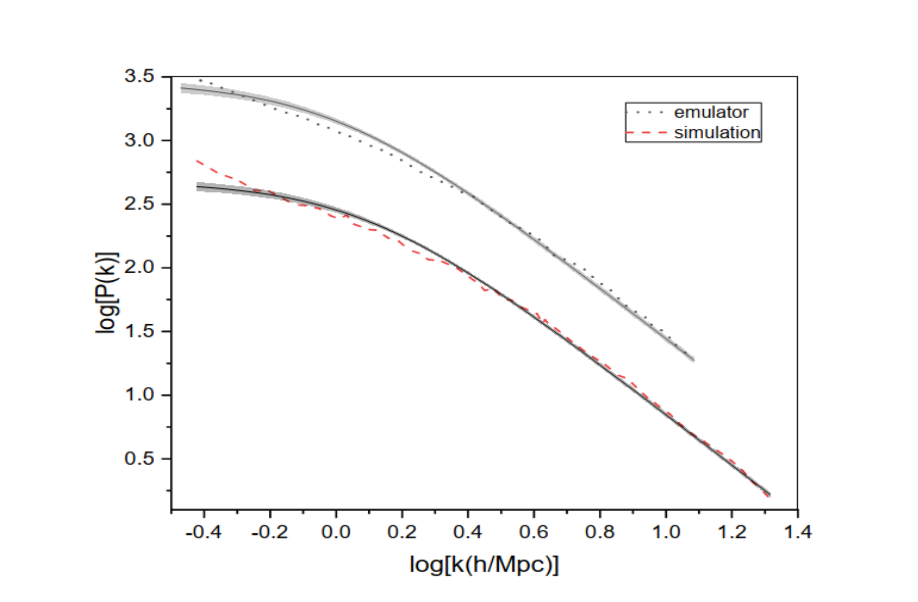


Which will be compared with observations and

simulations. But if k is too large, the density field will

not be Gaussian, so here we mainly fit the P(k)

near the galaxy cluster scale.



- Discussions

Density can be the order parameter in the LG theory, because it can reflect the symmetry of the system.

In the background cosmology the pressure of the matter is always to be zero, which in fact has equivalently assumed the matter is at equilibrium, and here we just accept this assumption to explain the density profile of the main halo.

This work seems to be inconsistent with simulations of SIDM and HDM, which will be further investigated

Conclusion

Compared to solve the core-cusp problem, we mainly focus on explaining the universality of these density profiles of self-gravitating system

Our previous works in the microcanonical ensemble more support the SIDM and observations of the core；while the work in the canonical ensemble can explain the cusp.

The new work by LG theory in the canonical ensemble can easily produce the r^{-1} cusp, but not easily a core.

Thanks for your attention!