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Heterotic string model with gauge symmetry E8xE8'

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1 Introduction

The Standard Model of particle physics is the theory describing three of the four known fundamental forces (the electromagnetic, weak, and strong interactions, and not including the gravitational force) in the universe, as well as classifying all known elementary particles. String theory is a candidate for theory of everything that describes all fundamental forces and forms of matter.

In string theory, particles are presented as vibrational modes of strings with a size of about 10^{-33} cm (the Planck scale). These strings have certain vibrational modes that determine the quantum numbers, such as mass and electric charge. Thus, different vibrational states of the same string can be particles of different types.

Superstring theory includes supersymmetry which allows to involve into consideration also fermion strings. There are five versions of superstring theories.

The type I string has one supersymmetry in the ten-dimensional spacetime (16 supercharges). This theory is special in the spacetime that it is based on unoriented open and closed strings, while the rest of theories are based on oriented closed strings.

The type II string theories have two supersymmetries in the ten-dimensional space-time (32 supercharges).There are actually two kinds of type II strings called type IIA and type IIB. The difference between these theories is mainly that the IIA theory is non-chiral (parity conserving) while the IIB theory is chiral (parity violating).

16 or 32 supercharges refer to the number of unbroken supersymmetries. In ten dimensions the minimal spinor is Majorana–Weyl (MW) and has 16 real components, so the conserved supersymmetry charges (or "supercharges") correspond to just one MW spinor in three cases (type I, HE, and HO). Type II superstrings have two MW supercharges, with opposite chirality in the IIA case and the same chirality in the IIB case.

The heterotic string theories are based on a peculiar hybrid of a type I superstring and a bosonic string. There are two kinds of heterotic strings differing in their ten-dimensional gauge groups: the heterotic $E_8 \times E_8$ string and the heterotic SO(32) string. The name heterotic SO(32) is slightly inaccurate since among the SO(32) Lie groups, string theory singles out a quotient Spin(32)/Z2 that is not equivalent to SO(32)(quotient group or factor group is a mathematical group obtained by aggregating similar elements of a larger group using an equivalence relation that preserves the group structure). The dangerous Lorentz and Yang- Mills chiral anomalies cancel if and only if the gauge group of the theory is SO(32) or $E_8 \times E_8'$. The $E_8 \times E_8'$ version of this theory offers the best phenomenological prospects for reproducing the real world.

2 Compactification and symmetry breaking

Initially, symmetry is supposed between ordinary world(E8) and mirror world(E_8'). 26 dimensional strings are reduced to 10-dimensional gravity with the $E_8 \times E_8'$ gauge group by compactification of 16 internal dimensions on the torus.

In order to make contact between the string theories and real world 10-dimensional spacetime should to be the form of $M^4 \times K$, where M^4 is four-dimensional Minkowski space and K is a compact six-dimensional manifold. Gauge symmetry is broken by compactification into Calabi-Yau manifolds or orbifolds. One can choose a non-trivial background Yang-Mills connection on manifold, for example V. Such connection is not invariant under E_8 gauge transformations and thus will spontaneosly break some gauge symmetry, at the natural scale of compactification. The remaining unbroken group at low energies is the commutant in E_8 of the holonomy group of V . Simple group theory implies that to realize the GUT groups E_6, $SO(10)$ and $SU(5)$, the holonomy of V must be $SU(3)$, $SU(4)$ or $SU(5)$ respectively.

Calabi-Yau manifolds are the Kähler manifolds which admit a Ricci flat metric(so that they have SU(3) holonomy). Compactification on Calabi–Yau n-folds are important because they leave some of the original supersymmetry unbroken(if the holonomy is the full SU(3)). These Calabi-Yau compactifications, produce for each manifold K, a consistent string vacuum, for which the gauge group is no larger than $E_6 \times E_8'$, and N = 1 supersymmetry is preserved. In general Calabi-Yau manifolds have many free parameters (moduli – the vacuum expectation values of massless scalar fields) which determine their size and shape. This leads to the existence of a huge number of false vacuums, this problem is known as the landscape of string theory.

Hausdorff space, separated space or T_2 space is a topological space where for any two distinct points there exists a neighbourhood of each which is disjoint from the neighbourhood of the other.

An n-dimensional orbifold is a Hausdorff topological space X, called the underlying space, with a covering by a collection of open sets U_i, closed under finite intersection. For each U_i, an open subset V_i of R^n, invariant under a faithful linear action of a finite group Γ_i; a continuous map φ_i of V_i onto U_i invariant under Γ_i, called an orbifold chart, which defines a homeomorphism between V_i/Γ_i and U_i.

Orbifold T_6/G is the quotient space of T_6 (6-torus) by a finite isometry group G which acts with fixed points. The quotient space X/$\sim$ of a topological space X and an equivalence relation $\sim$ on X is the set of equivalence classes of points in X (under the equivalence relation $\sim$) together with the following topology given to subsets of X/$\sim$: a subset U of X/$\sim$ is called open if and only if $U_{([x]\in U)}[x] \subset X$ is open in X. Quotient spaces are also called factor spaces. We compactify on tori divided by the action of a discrete group. This allow us to break gauge group and arrive at different low-energy predictions.

The mechanism of gauge symmetry breaking as a result of compactification on Calabi – Yau manifolds or orbifolds leads to the prediction of homotopically stable solutions with a mass $m_\alpha = r_c/\alpha'$, where r_c – radius of compactification, α' – string tension.

The homotopically stable particles are "sterile" with respect to the charges of strong, weak, and electromagnetic interactions and participate only in the gravitational interaction. Therefore the possibilities of verifying the existence of these particles are related exclusively to their cosmological manifestations.

After compactification E_8 is broken to E_6 and further E_6 is broken to the Standard Model group.

If there exists a discrete symmetry group, Z, which acts freely on

K, one can consider the smaller manifold K/Z. E_6 break down to the observed low energy gauge group if K/Z is multiple connected, one can allow flux of the unbroken E_6 to run through it, with no change in the vacuum energy. The net effect is that, when we go around a hole in the manifold through which some flux runs, we must perform a nontrivial gauge transformation on the charged degrees of freedom. Such noncontractible Wilson loops act like Higgs bosons, breaking E_6 down to the largest subgroup that commutes with all of them. By this mechanism one can, without generating a cosmological constant, find vacua whose unbroken low energy gauge group is, say, SU(3) x SU(2) x U(1) x (typically, an extra U(1) or two). Moreover, there exists a natural reason for the existence of massless Higgs bosons which are weak isospin doublets (and could be responsible for the electro-weak breaking at a TeV), without accompanying color triplets.

E_8' symmetry can be used to break N=1 supersymmetry. Below the compactification scale it yields a strong, confining gauge theory like QCD, but without light matter fields. In general this sector would be totally unobservable to us, consisting of very heavy glueballs, which would only interact with our sector with gravitational strength at low energies. However there could very well exist in this sector a gluiono condensate which can serve as source for supersymmetry breaking. E_8' is broken to $E_7 \times U(6)$ in one of the model.[11]

In some models E_8' remains unbroken and symmetry between "ordinary" and "mirror" world is broken, and it leads to existence of "shadow" world which interacts with ordinary matter only by gravity.

3 Generations of fermions

The Eulerian characteristic of the topology of compactified 6 dimensions determines the number of generations of quarks and leptons.

To calculate the number of generations predicted by the theory, we must first calculate the number of massless particles. The 10-dimensional Klein-Gordon operator:

$\Box_{10}\psi = (\Box_4 + \Box_6)\psi = 0$

In general, $\Box_6$ will have eigenvalues denoted by m^2, that is, $\Box_6\psi_m = m^2\psi_m$ so that wave equation becomes

$(\Box_4 + m^2)\psi_m = 0$

We are interested in the massless sector in four dimensions, so we want to keep only the zero eigenvalues of the $\Box_6$ operator. Thus,

$\Box_4\psi = \Box_6\psi$

It means that the four-dimensional fermions are massless. And it also means that ψ is a harmonic form in six dimensions. Therefore the number of massless modes in four dimensions will be related to the number of harmonic forms that we can write for the six dimensional manifold. Thus, topological arguments alone should give the number of generations. It is expected that the number of generations is a topological number because of the Dirac index theorem as a special case of the Atiyah–Singer index theorem, which connect Dirac index and topological index.[18] We know that the solutions of the Dirac equation can, in general, have zero modes:

$i\not{D}\psi = 0$

In fact, the index of this operator is equal 10 the difference between the positive and negative chiralilies of the zero modes:

$Index(\not{D}) = n_+ - n_-$

But the Dirac index is also equal to the generation number, because we will be considering only fermions of one specific chirality. Thus, the precise relation between the generation number and the Dirac index, or the Euler number, is

Generation number $= \frac{1}{2}|\chi(M)|$

The generation number can be reduced by considering nonsimply connected manifolds. The original manifold M_0 is divided by a discrete symmetry group G that acts freely on the manifold (i.e., no fixed points), yielding a manifold M. If the number of discrete generators of G is $n(G)$, then the Euler number of the original manifold divided by the discrete group G is

$\chi(M) = \frac{\chi(M_0)}{n(G)}$

where

$M = \frac{M_0}{G}$

For example, one will consider a hypersurface Q in CP^4, for which Euler charactestic in the case of the Calabi-Yau 3-fold $\chi(Q) = -200$ ([17]) and the discrete symmetry group $F = Z_5 \times Z_5$, which has $N(F) =$ 25 generators.

Thus, the Euler number of this new manifold $K = \frac{Q}{F}$ is

$\chi(M) = \frac{\chi(Q)}{N(F)} = \frac{-200}{25} = -8$

which predicts four generations.

A typical construction of Calabi-Yau manifold gives a very large Euler number. Three dimensional algebraic manifold with $c_1 = 0$ and the Euler charachteristic $\chi = 6$ was constructed by Tian and Yau.[14]

In other case spin(10) group can arise from the spontaneous breaking of the observable sector E_8 group by an SU(4) gauge instanton on an internal Calabi–Yau threefold. The Spin(10) group is then broken by a Wilson line(or Wilson loop – constant gauge-background fields corresponding to the non-contractible loops of the torus underlying the orbifold)) to a gauge group containing $SU(3)_C \times SU(2)_L \times U(1)_Y$ as a factor. To achieve this, the Calabi–Yau manifold must have, minimally, a fundamental group $Z_3 \times Z_3$. In this case appears three families of quarks and leptons, each with a right-handed neutrino.

Also three generation can be obtained in Z_3 orbifold models with two Wilson lines.

4 Inflation

There are two different versions of string inflation. In the first version, modular inflation, the inflaton field is associated with one of the moduli. It's based on the model, called KKLT.[15] The main idea of KKLT model is to find a supersymmetric AdS minimum taking into account nonperturbative effects, and then uplift this minimum to dS state by adding the positive energy density contribution of branes. The position of the dS minimum and the value of the cosmological constant there depend on the quantized values of fluxes in the bulk and on the branes. [16]

In the second version, the inflaton is related to the distance between branes moving in the compactified space. In the paper [12] is considered heterotic string theory compactified on a Calabi-Yau threefold. One of the orbifold fixed planes considered as to the visible brane (or the visible sector), another one is as to the hidden brane (or the hidden sector). Such compactifications also allow five-branes wrapped on holomorphic cycles in the Calabi-Yau manifold and parallel to the orbifold fixed planes.

Inflation can be studied within the context of D-branes. Under certain conditions the D-brane modulus can be treated as an inflaton. The five-brane moduli can be stabilized in a non-supersymmetric anti de Sitter minimum. The potential energy has one more minimum when the five-brane coincides with the visible brane. A heterotic M-theory vacuum can contain several five-branes wrapped on non-isolated genus zero or higher genus curves in a Calabi-Yau threefold. Those which are located relatively far away from the visible brane will be stabilized. On the other hand, those which are located close enough to the visible brane will roll towards it and, eventually, collide with it.

These five-branes can be stabilized as well by balancing the supergravity potential energy against the Fayet-Iliopoulos terms induced by an anomalous U(1) gauge group in the hidden sector. The Fayet-Iliopoulos D-term is a D-term in a supersymmetric theory obtained from a vector superfield V simply by an integral over all of superspace: $S_{FI} = \xi \int d^4\theta V$. Because a natural trace must be a part of the expression, the action only exists for U(1) vector superfields. In terms of the components, it is proportional simply to the last auxiliary D-term of the superfield V. It means that the corresponding D that appears in D-flatness conditions (and whose square enters the ordinary potential) is additively shifted by ξ , the coefficient.

It is also possible to create a positive potential satisfying the slow roll conditions and treat the five-brane translational modulus as an inflaton.

Inflation takes place when the five-brane approaches the visible brane. However, this potential has one negative feature. It has a vanishing first derivative when the five-brane coincides with the visible brane. This means that it takes infinite time for the branes to collide.

This also means that the primordial fluctuations will become infinite. On the other hand, at very short distances, one expect the appearance of new light states, which can provide escape from inflation. It's supposed that the new states are particles and the moduli space is describable by the superpotential $W = W(\Phi, X)$. When the five-brane gets very close to the visible sector, the fields Φ become tachyonic and begin to roll downhill. Since no slow roll conditions on Φ are satisfied, this terminates inflation.

After inflation, the five-brane hits the visible brane and disappears through a small instanton transition. The new system of moduli does not contain the five-brane but has extra vector bundle moduli. Unlike the five-brane modulus, these moduli can be stabilized. The new system of moduli can be stabilized in a vacuum with a positive cosmological constant which can be fine tuned to be very small and consistent with observations.

5 Baryogenesis. Affleck – Dine – Linde mechanism

Baryon asymmetry can be explained by the processes with the B- and CP-violation.

Affleck – Dyne – Linde mechanism is considered in the model which is consistent supersymmetry. New hypothetical scalar fields (SUSY-partners of ordinary particles) carries the baryon(lepton) number. Through interactions with the inflaton field CP-violating and B-violating effects can be introduced. As the scalar particles decay to fermions, the net baryon number the scalars carry can be converted into an ordinary baryon excess. The Affleck – Dine – Linde mechanism is an example of non-thermal baryogenesis.

6 Baryon asymmetry as a consequence of the existence of the "Shadow" world

Baryon asymmetry also may occur due to the decay of shadow particles in the early Universe under the condition unbroken E_8'. Shadow hadrons arise during gluodynamic confinement, and they are nonrelativistic massive particles with a mass of the order of $M \gg 10^2 GeV$, which dominate in the Universe beginning with $t_M \sim m_p/M$ and up to the time of their decay $t_D \sim m_p^4/M^5$.

During this period, the ordinary particles, whose masses are negligibly small in comparison with the shadow hadrons, are ultrarelativistic; their contribution to the cosmological density falls as a^{-1} (a is the scale factor) and after the end of the shadow hadrons era is $1/a^4 \sim (M/m_p)^2$ from the initial one. Because of this suppression at $t > t_D$ the products of shadow hadrons decay dominate in the Universe.

Shadow hadrons can only decay in a gravitational way, so their lifetime is quite long. The decays of shadow particles can be gravitational (decay into gravitons and gravitino), and can also decay into ordinary particles. In the latter case, baryon asymmetry may occur due to the effects of a CP-violation in four-particle decays of shadow particles with a violation of the baryon number

$T \to G \to q\bar{q} \to qqql$, $\tilde{T} \to \tilde{G} \to q_s\bar{q} \to q_sqql$

In this way, the observed matter originated from decays of the shadow hadrons, with the unbroken E8' group. [13]

7 Dark matter

After symmetry breaking between ordinary and mirror world appears "shadow" world which consist 248 fields of matter and of interactions which are connected with ordinary particles only by gravity. They may be candidates for the dark matter.

Contribution to the dark matter can give also 4th generation of heavy neutrino and neutralino which appears in the supersymmetry theories.

8 Conclusions

The model of heterotic string with gauge group $E_8 \times E_8'$ was considered in this thesis. It's looks like phenomenologically attractice unified theory. Gauge group is broken down to a group of Standart Model by compactification of extra six dimensions into Calabi-Yau manifolds or orbifolds. It can leads to existence of new interactions and generations of fermions, number of which can be different and even extremely large for some models. But compactification may be tuned in the way in which appears only three generations that corresponds to observations. Shadow world which appears under condition unbroken E_8' symmetry can create a baryon asymmetry in early Universe. Also shadow particles can be the candidate in dark matter.

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