

Testing the no-hair theorem with GW150914

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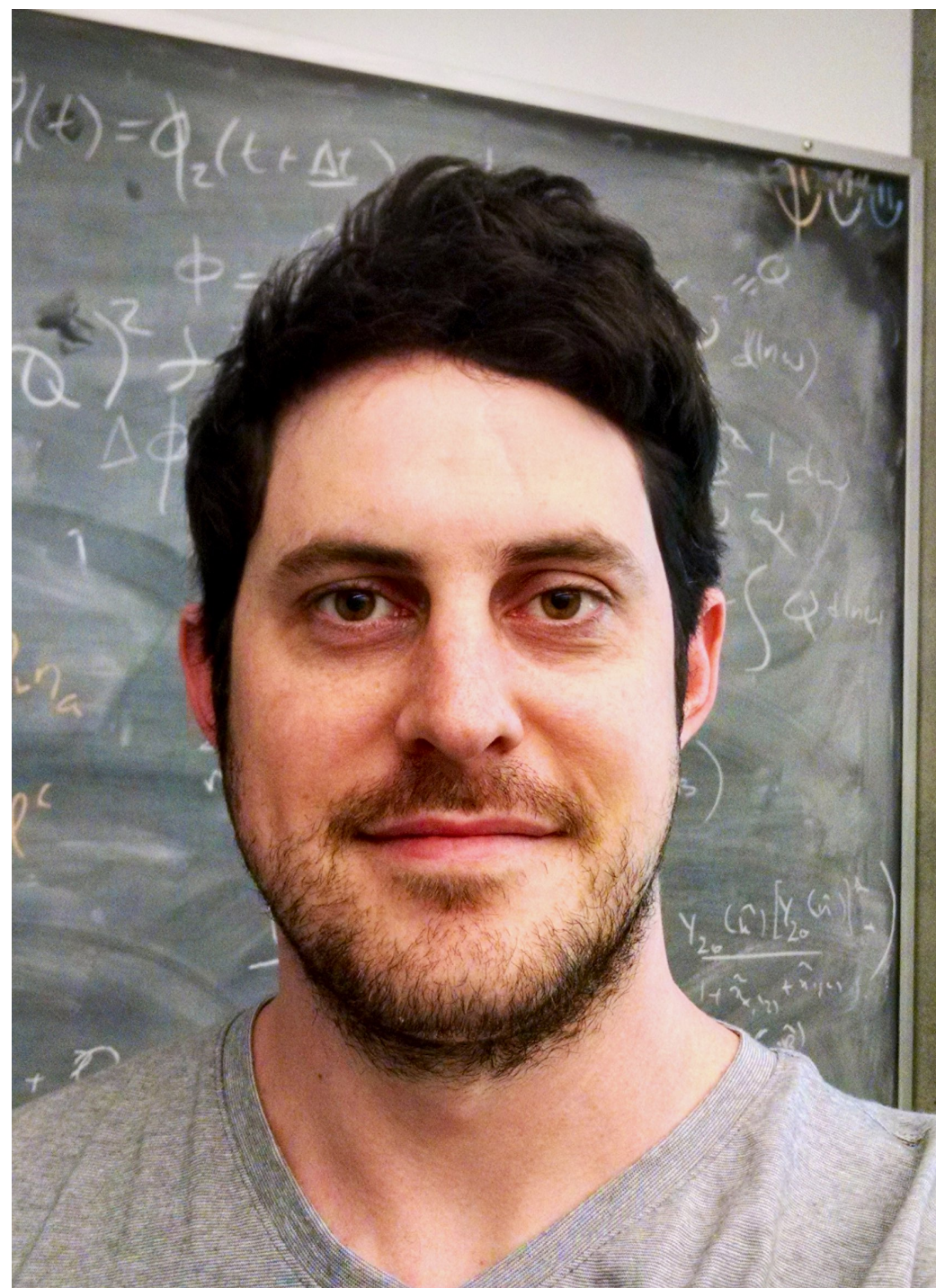
[\[arXiv:1903.08284\]](#) [\[arXiv:1905.00869\]](#)



GW150914 simulated by SXS

collaborators

Matt Giesler



Caltech

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Flatiron, Stony Brook

Mark Scheel



Caltech

Saul Teukolsky



Caltech, Cornell

[\[arXiv:1903.08284\]](#)

[\[arXiv:1905.00869\]](#)

bottom line

we have observed the GW150914 remnant **ringing**
this allows us to begin testing the **no-hair** theorem
much to gain from new **LIGO & Virgo** observations

more specific
bottom line

with **overtones**, the ringdown starts at the GW peak
we identify **two modes** in GW150914, check consistency
BH **spectroscopy** within reach of present instruments

GW = gravitational wave; BH = black hole

no-hair theorem

no-hair theorem

in general relativity (GR), a BH has only three properties

mass
spin
charge

no charge for *astrophysical* BHs

spacetime described by the *Kerr* metric

if not true, not a BH in GR!



units (an interlude)

mass

M

usually quoted in solar masses M_{\odot}

will often use geometric units

$$G = c = 1$$

and measure time in units of M

e.g. $1M_{\odot} \sim 5 \mu s$

spin

χ

dimensionless spin

$$\chi = ac^2/(GM)$$

Kerr parameter (length)

$$a = J/(Mc)$$

for angular momentum J

no-hair theorem

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growing hair

black holes may possess other properties
in **theories beyond GR**

for example, could acquire *scalar charge*
in Horndeski gravity e.g. [[arXiv:1711.07431](#)]

quantum corrections could endow
a BH with *soft hair* e.g. [[arXiv:1601.00921](#)]

if in a massive scalar field, BHs could
grow hair through **superradiance**



growing hair

in GR, may have *extreme compact objects* (ECOs) that act as **BH mimickers**

these are **horizonless** compact objects that *look* like Kerr from afar

multiple examples, including gravastars, fuzzballs and wormholes

best understood: **boson stars** and (maybe) anisotropic stars



looking for hair

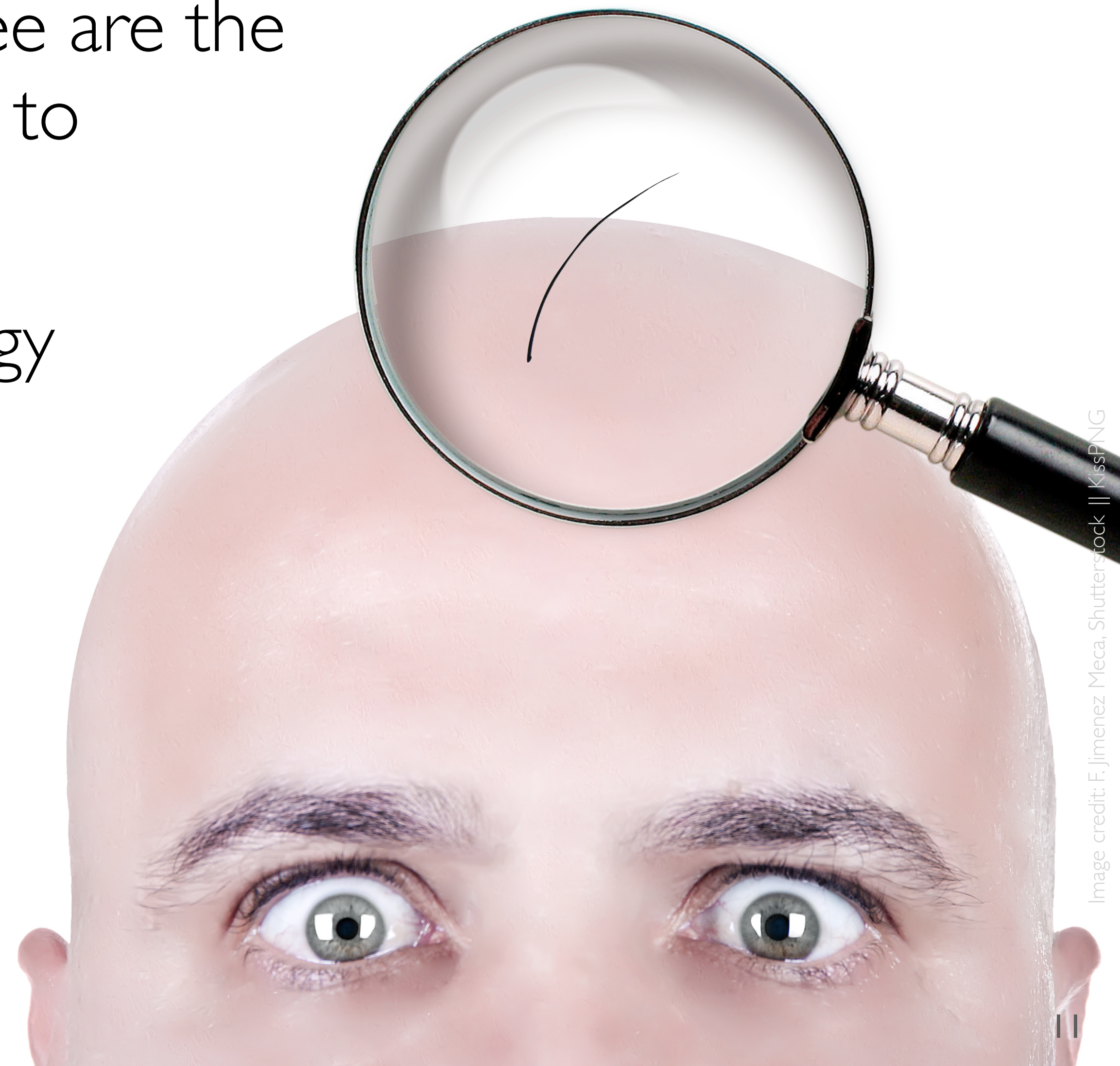
making sure that compact objects we see are the Kerr BHs predicted by GR is *paramount* to fundamental physics

consequences for: gravitation, high-energy physics, cosmology, etc.

however, there are strict restrictions on the properties of ECOs and their phenomenology

hair will be small and hard to find!

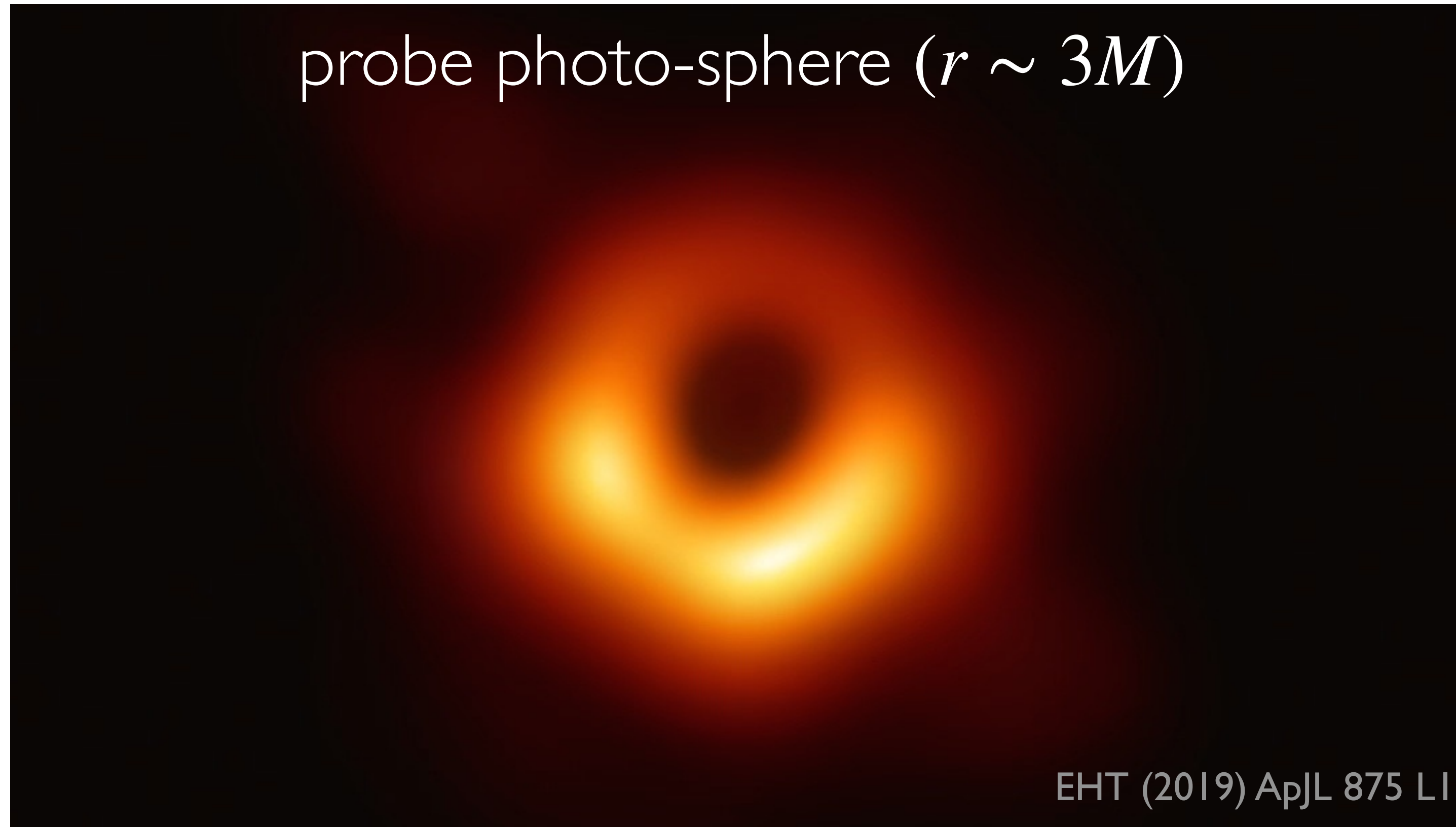
(most likely and if it exists)



looking for hair

BH images with very long baseline interferometry

probe photo-sphere ($r \sim 3M$)

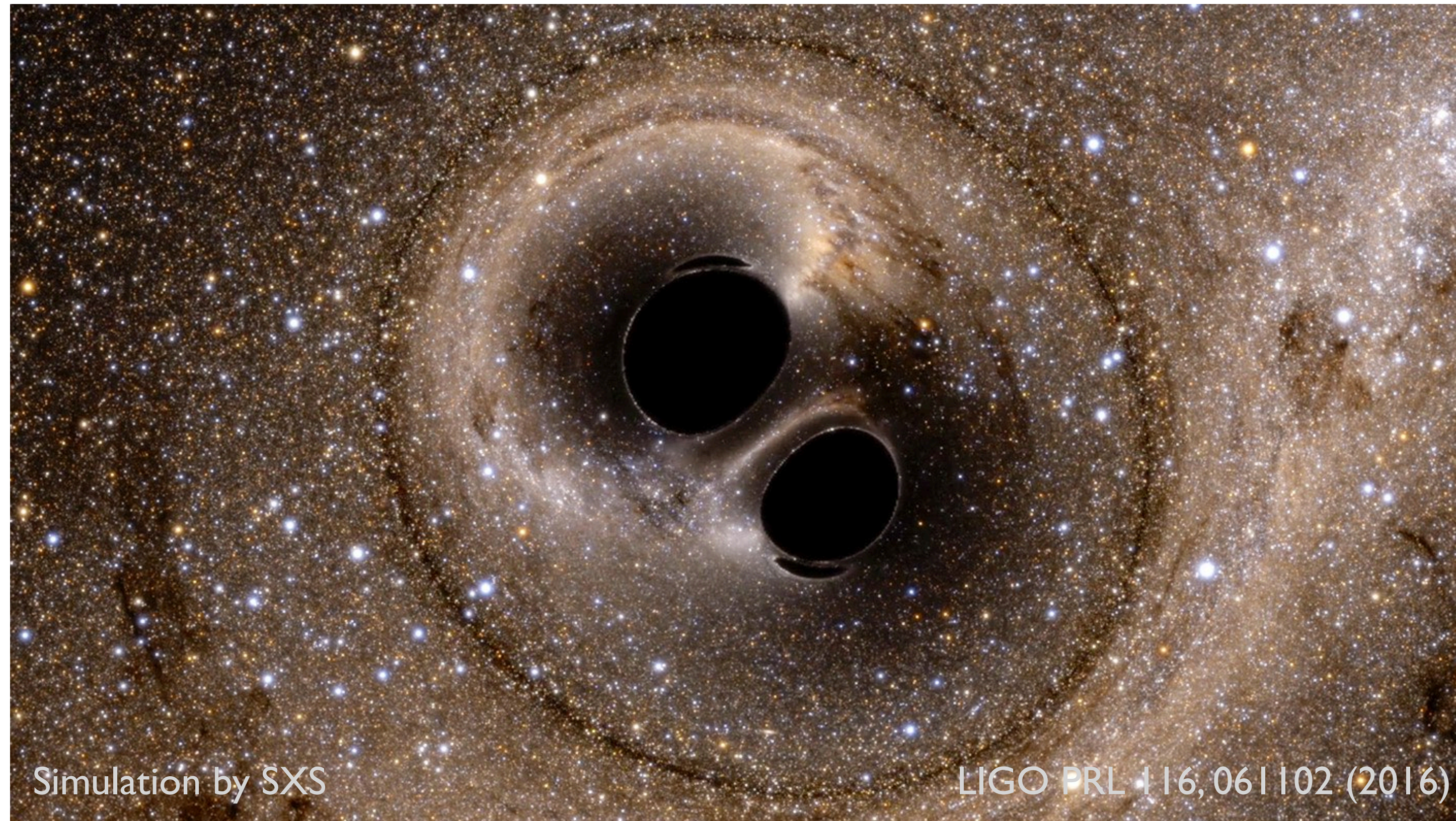


deviation from Kerr would have to be large

ECOs tend to have same light-ring as Kerr + hard to decouple astrophysics

looking for hair

gravitational wave observations



most likely the cleanest probe of strong field

black-hole spectroscopy

black-hole ringdown

a perturbed black hole rings like a bell

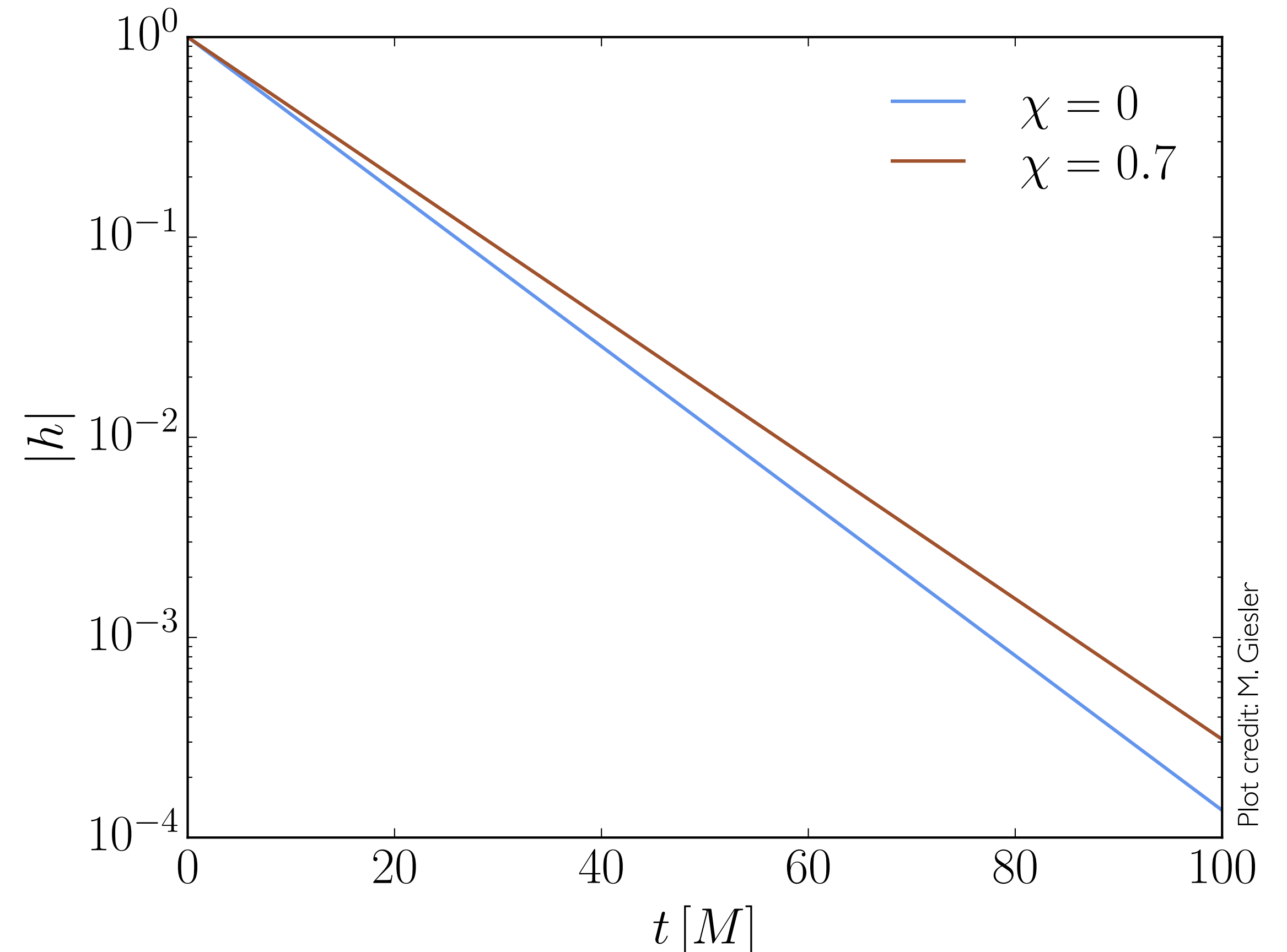
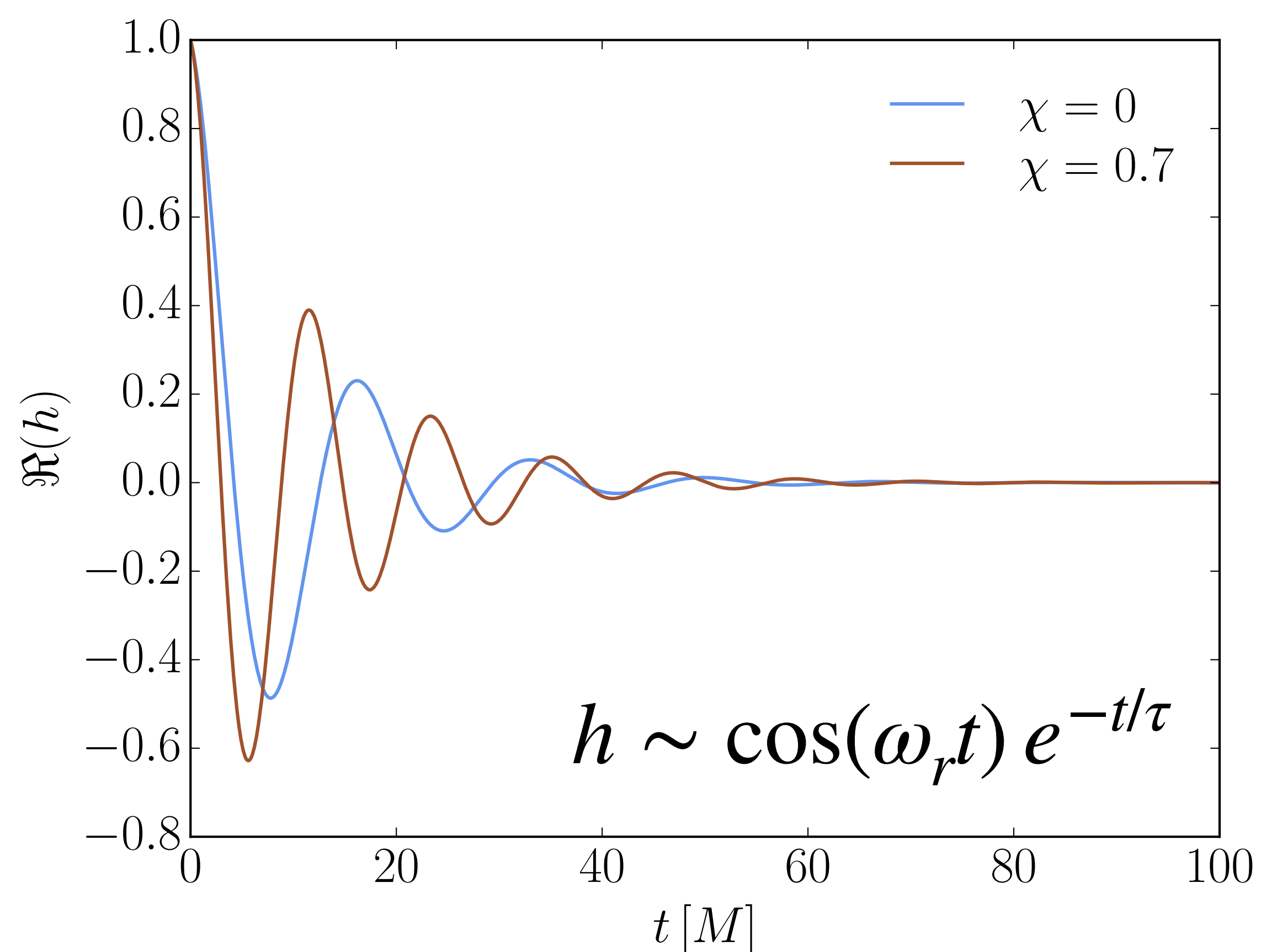


for Kerr, spectrum depends only on mass and spin
just like, for a bell, it depends on size and shape

black-hole ringdown

radiation takes form of damped sinusoids: $e^{-i\omega t}$

$$\omega = \omega_r + i\omega_i \quad \omega_i = -1/\tau < 0$$

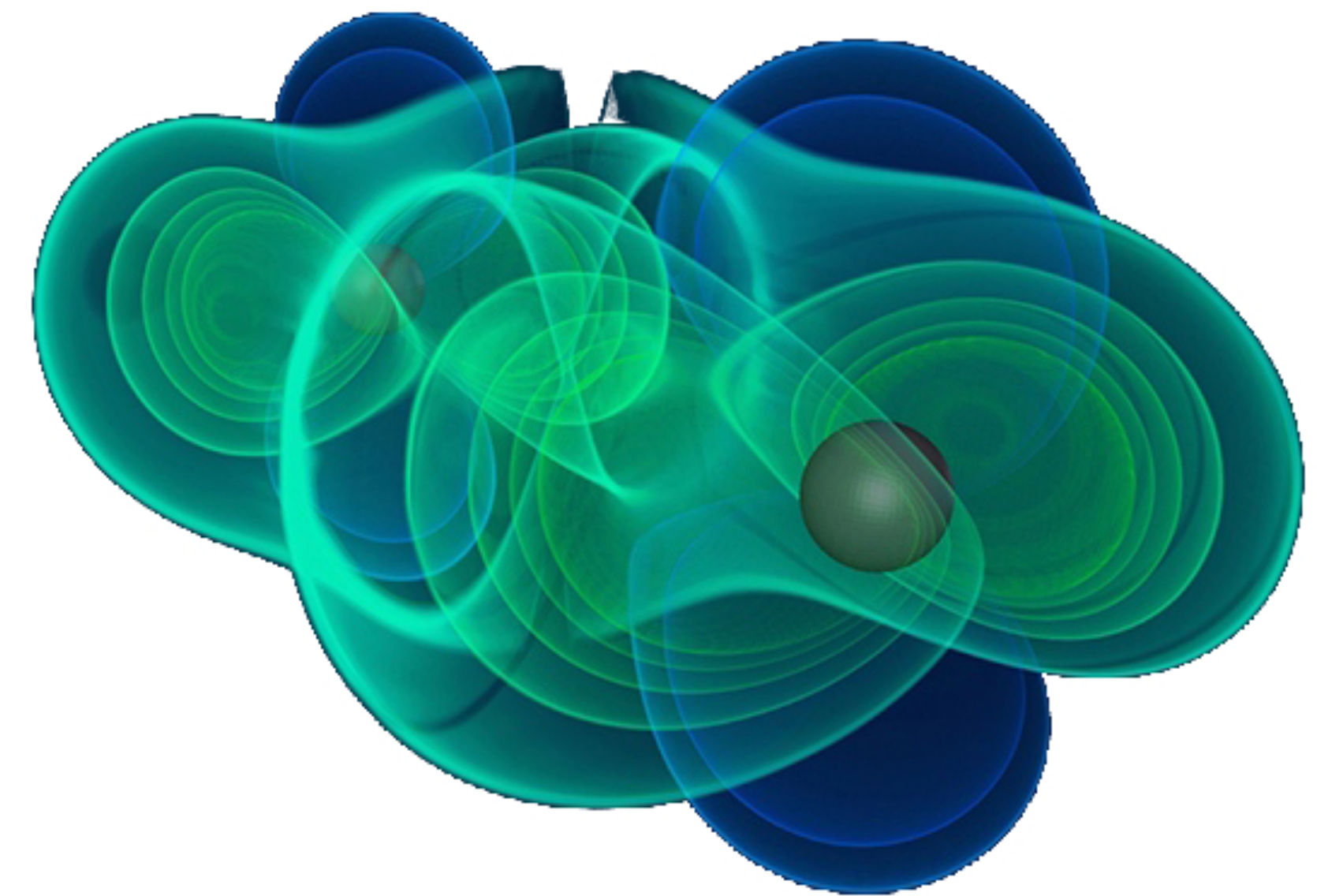
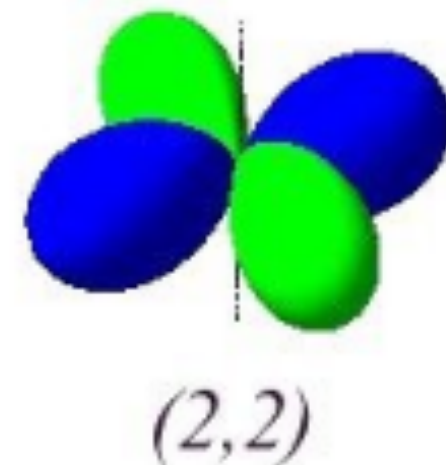
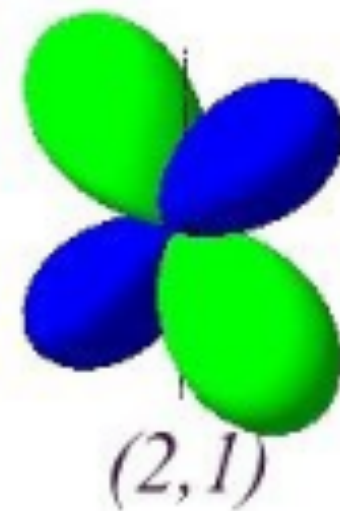
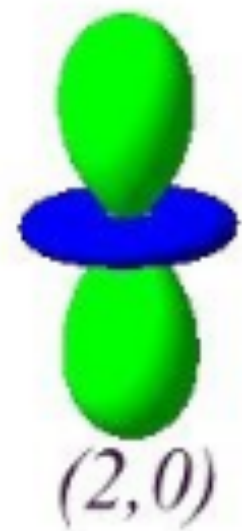


black-hole ringdown

strain is a sum over multiple angular modes

$$h = \sum h_{\ell m} Y_{\ell m}$$

spherical harmonics, e.g.



GW emission pattern from BH binary

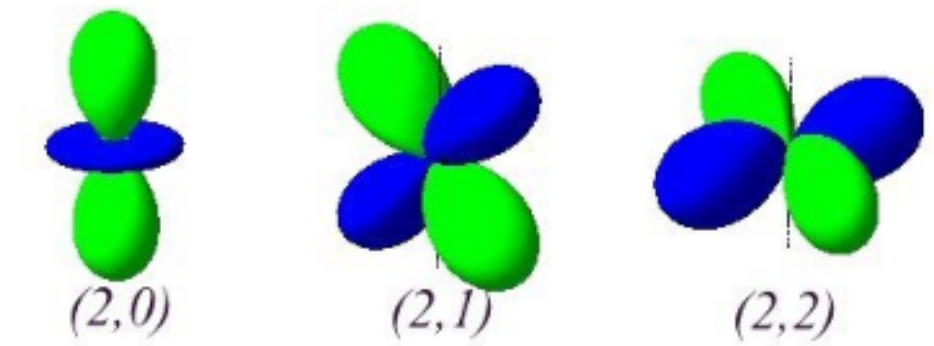
Credit: Max Planck Institute for Gravitational Physics /Institute for
Theoretical Physics, Frankfurt/Zuse Institute Berlin

ringdown overtones

For a given (ℓ, m) :

$$h_{\ell m} = \sum_n^N C_{\ell mn} e^{-i\omega_{\ell mn} t}$$

angular mode, e.g.



- Kerr: $\omega_{\ell mn} = \omega_{\ell mn}(M_f, \chi_f)$
- n is the overtone index, similar to radial quantum number n_r
- Higher n , lower frequencies
- $n > 0$ often ignored, considered subdominant
- n sorts modes by their decay times, not by their relative importance

black-hole spectroscopy

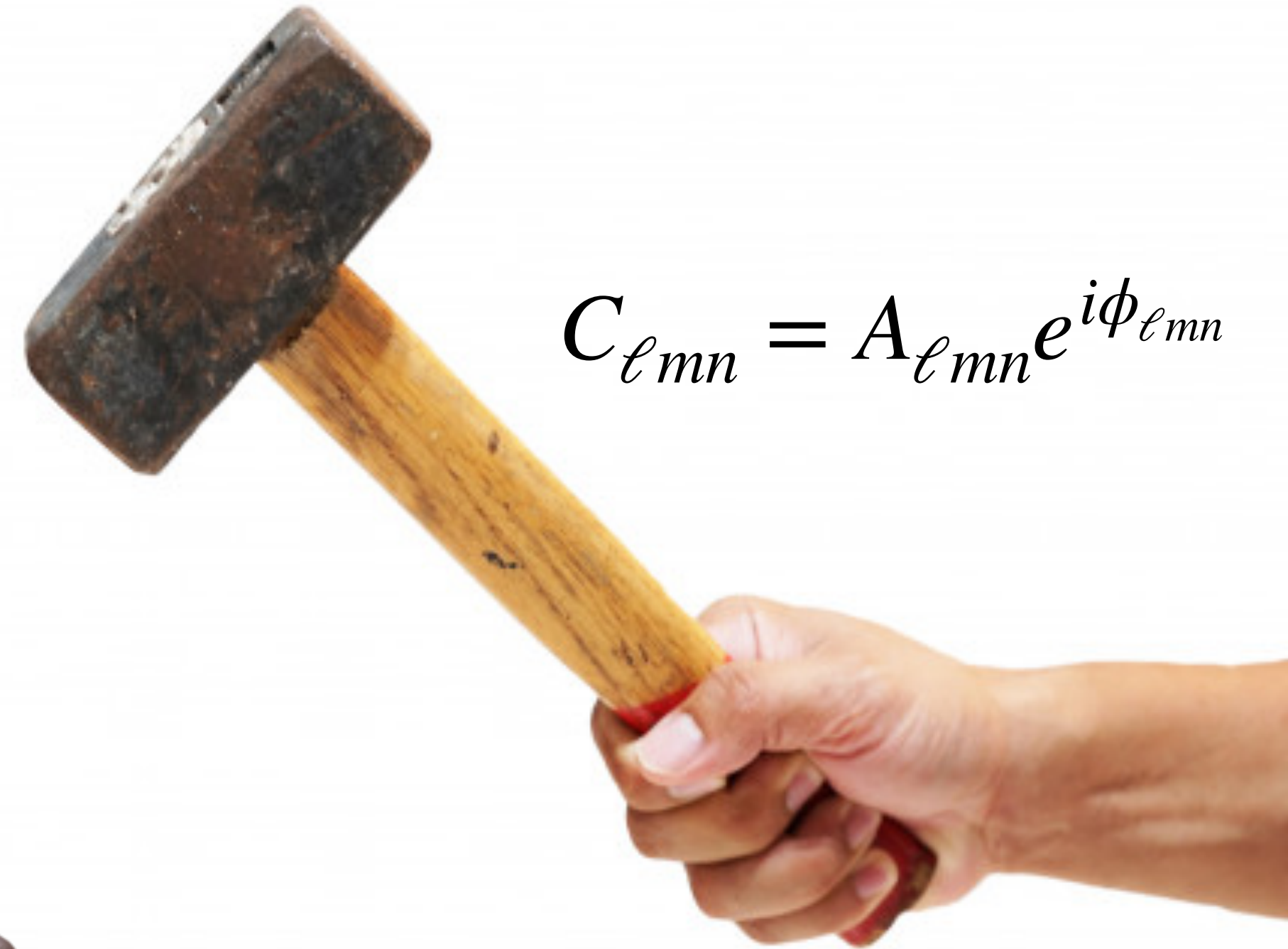
learn about BH properties
from its ringing

$$h_{\ell m} = \sum C_{\ell mn} e^{-i\omega_{\ell mn} t}$$

measure different modes,
as in *atom spectroscopy*



$$\omega_{\ell mn}(M, \chi)$$



$$C_{\ell mn} = A_{\ell mn} e^{i\phi_{\ell mn}}$$

$\omega_{\ell mn}$: intrinsic geometry

$C_{\ell mn}$: initial conditions

black-hole spectroscopy

for a Kerr BH, **full spectrum predicted** by mass and spin

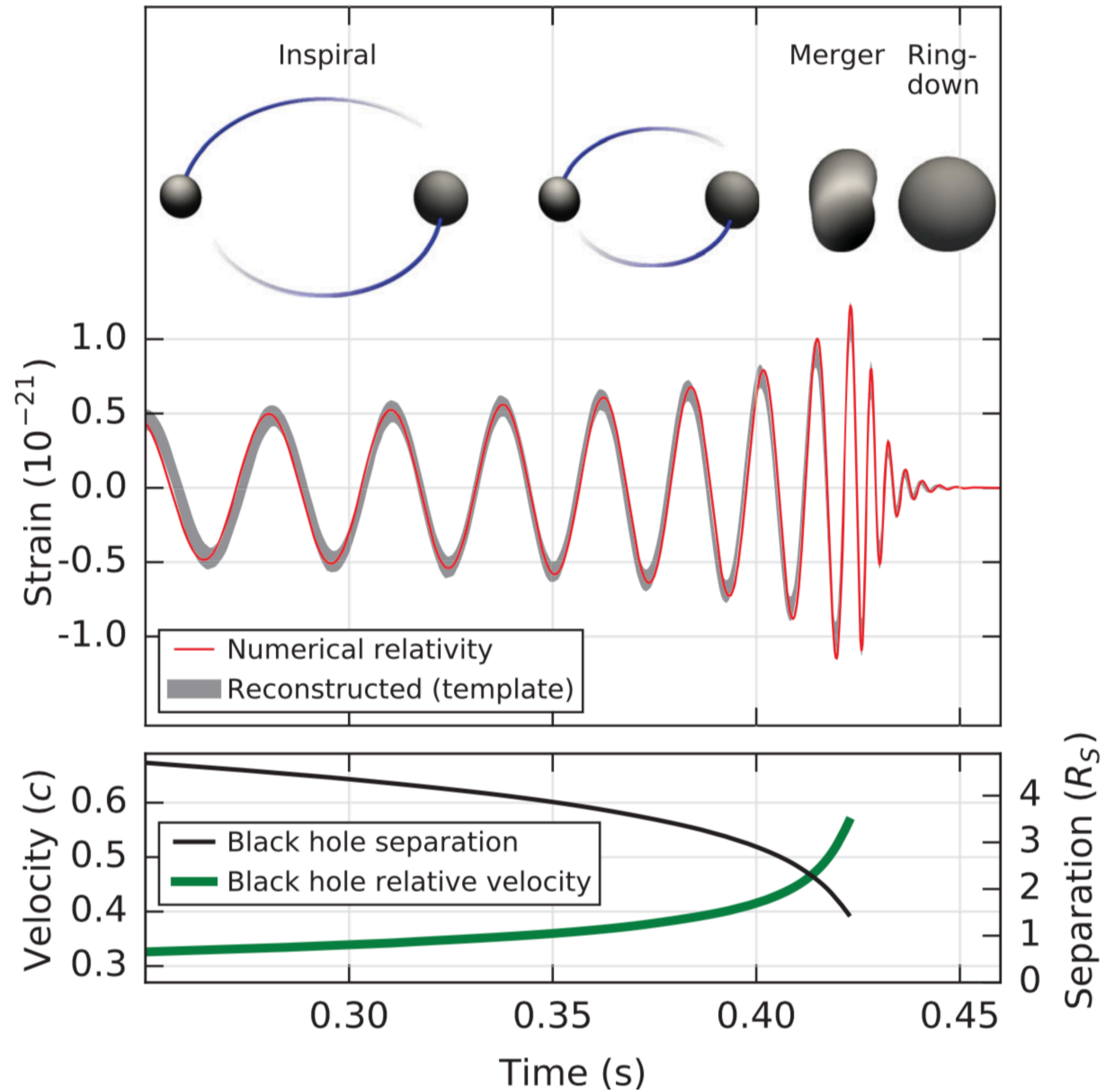
an **object with hair may deviate** from this prediction

e.g. beyond GR in MOG [[arXiv:1711.03199](#)] and CS [[Okounkova et al. \(in prep\)](#)]

measuring the ringing of an ECO is a **null test for Kerr**

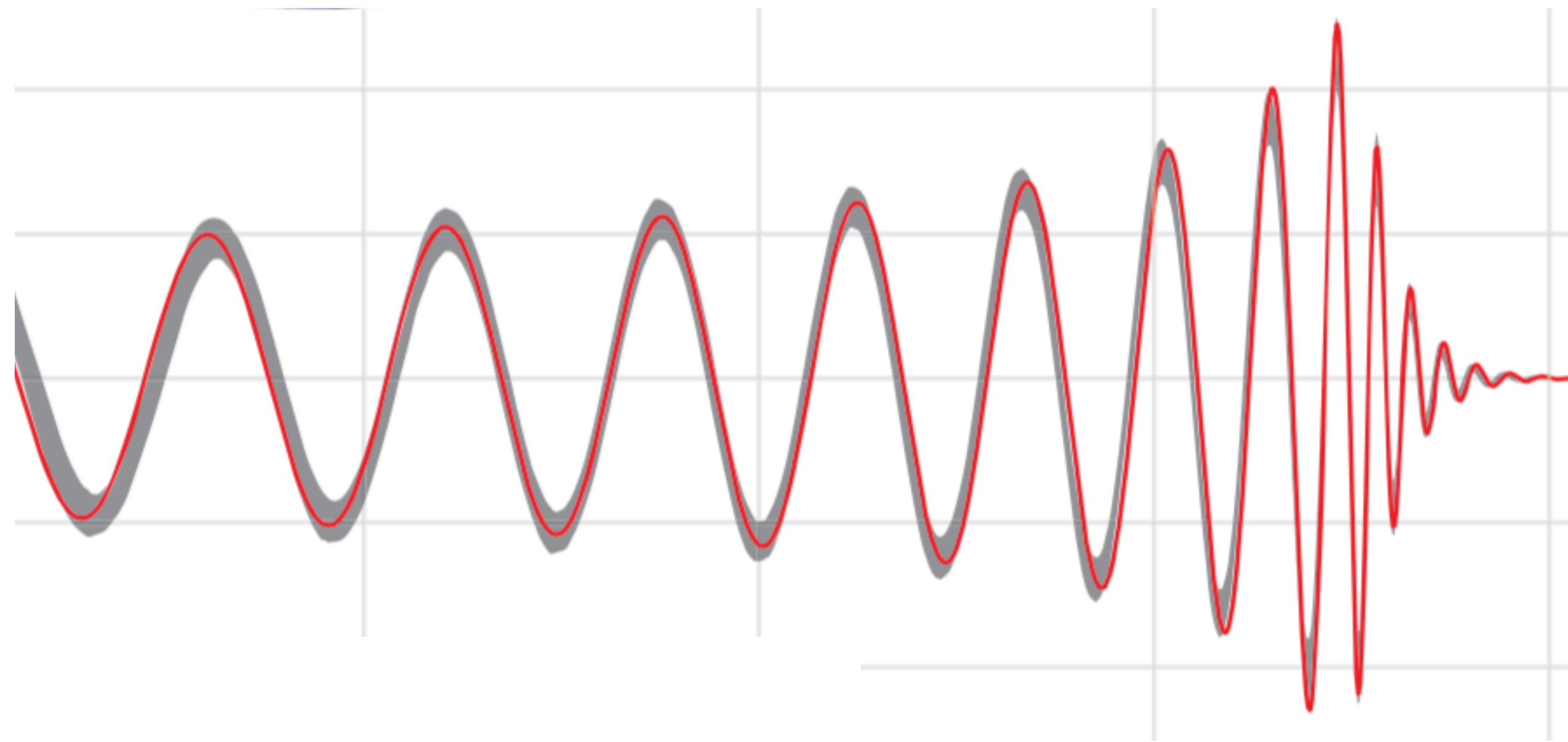
how can we observe a perturbed black hole?





black-hole spectroscopy

extract multiple ringdown modes from data and compare



but when does the ringdown start?

ringdown in GW150914

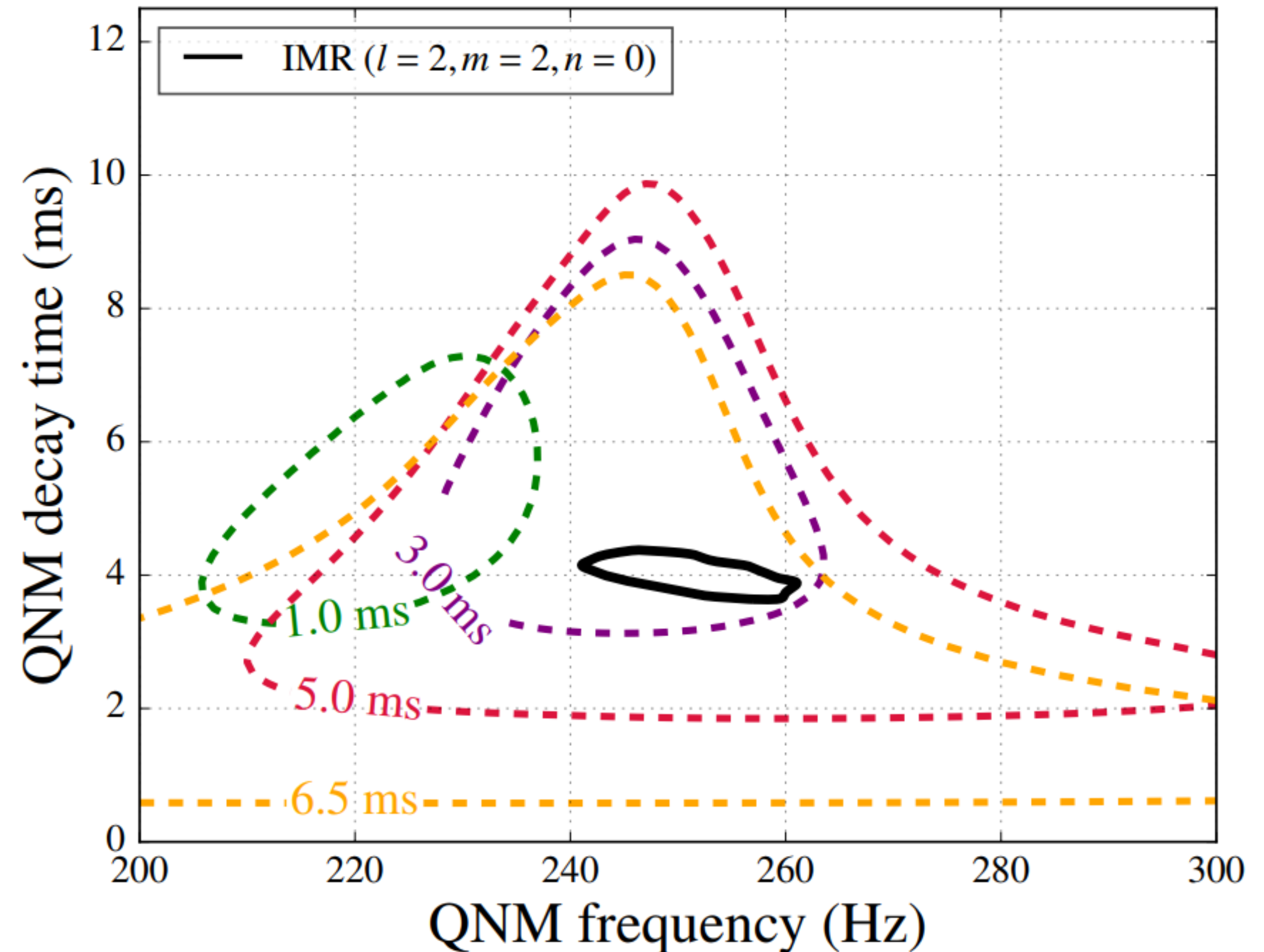
arxiv:1602.03841

look for **single damped sinusoid**

very sensitive to **start time!**

discrepancy attributed to **nonlinearities**

believed spectroscopy would have to wait for **better detectors**



ringdown in GW150914

arxiv:1602.03841

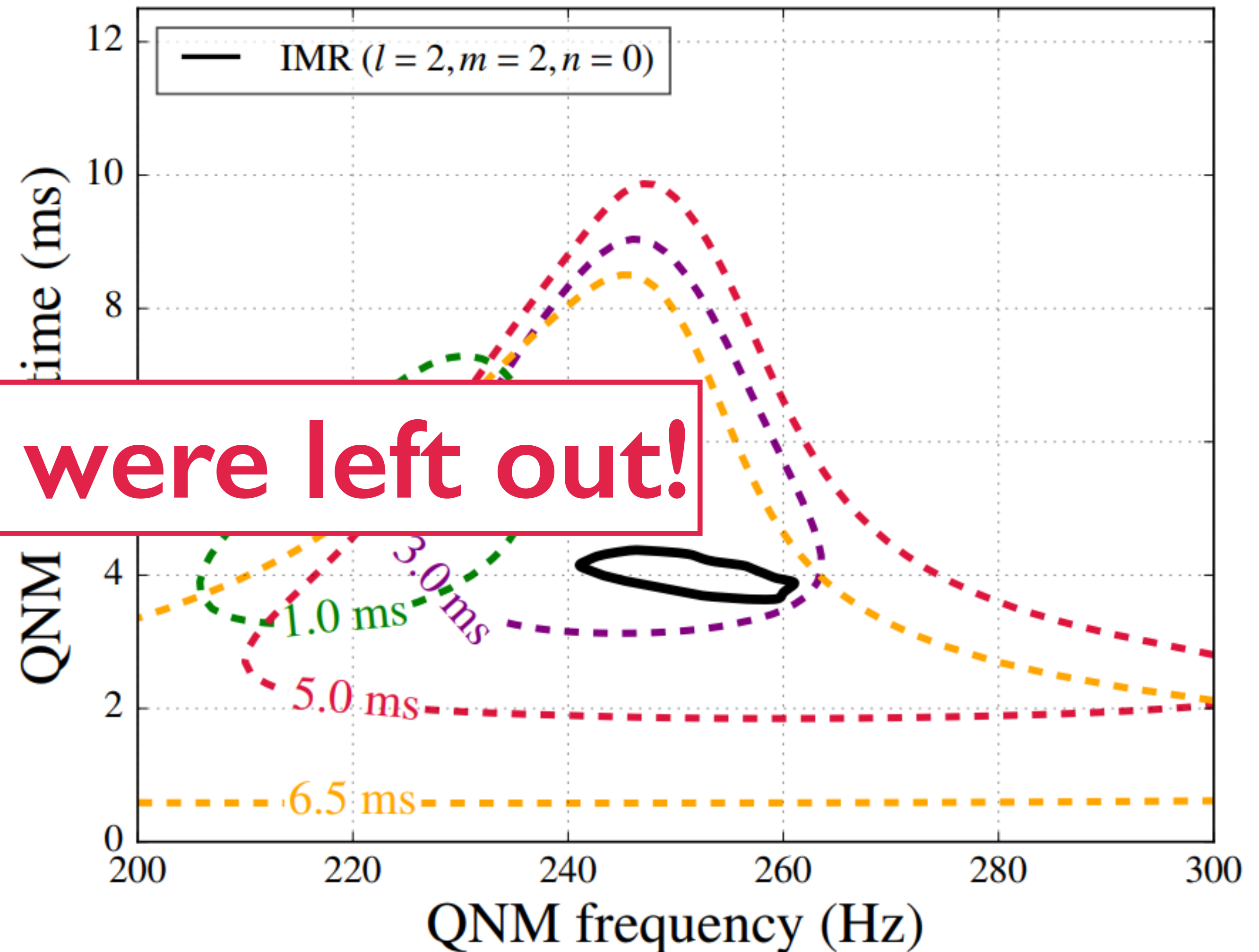
look for **single damped sinusoid**

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discrepancy a
to **nonlinearities**

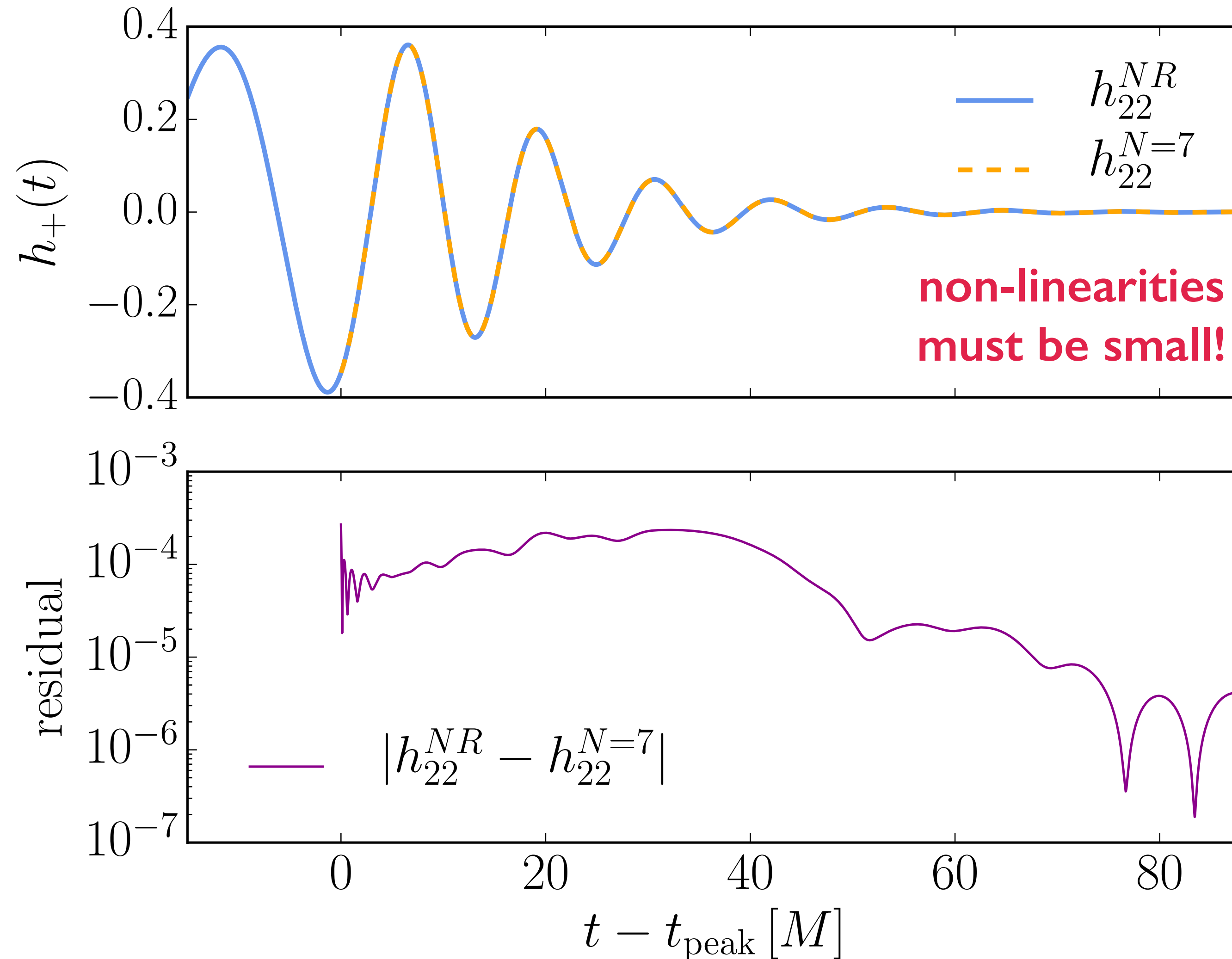
believed spectroscopy
would have to wait for
better detectors

overtones were left out!



ringdown starts at the peak!

(or even slightly before)



fit earlier with more overtones

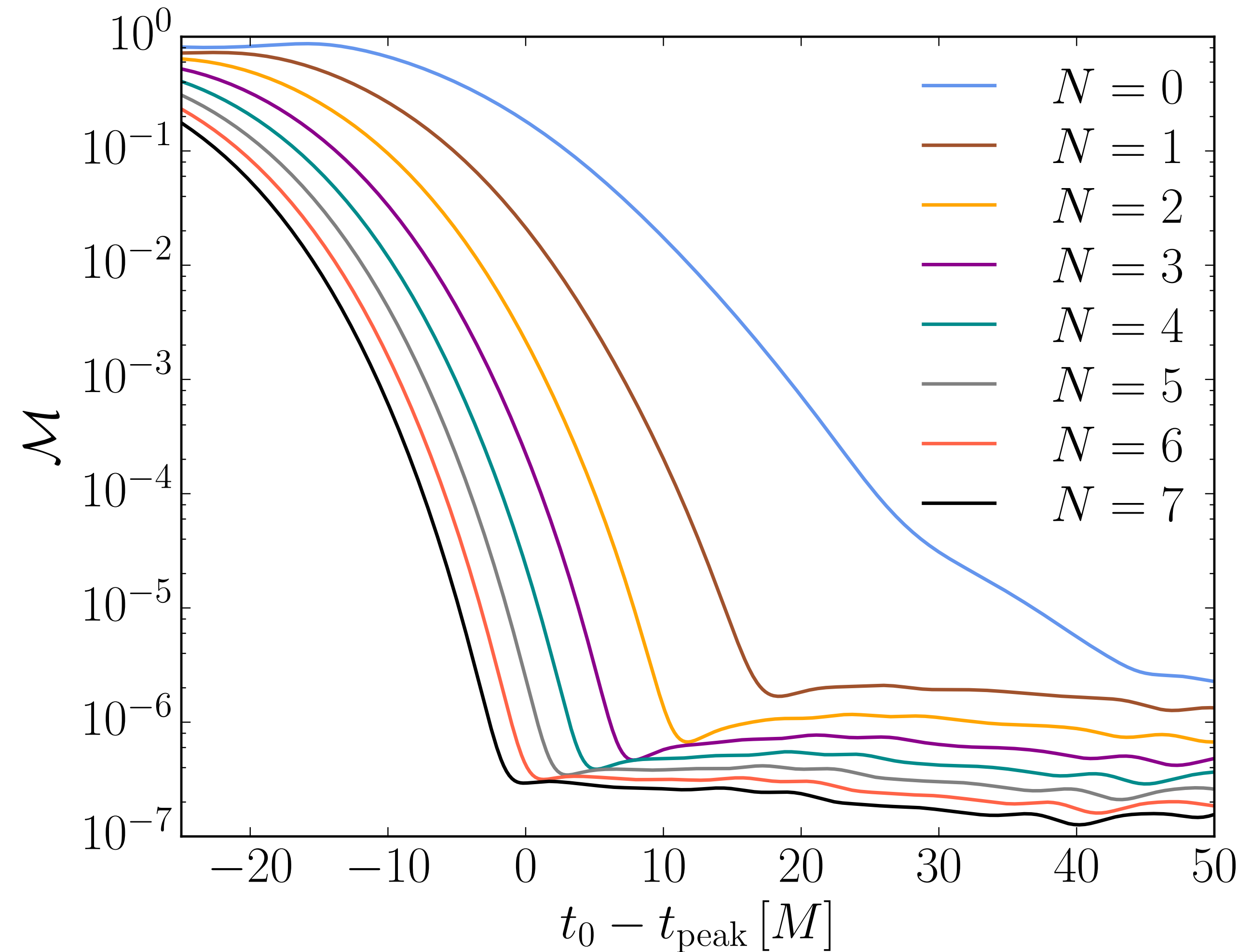
$$h_{22} = \sum_n^N C_{22n} e^{-i\omega_{22n}(t-t_0)}$$

C_{22n} from least-squares, ω_{22n} from (M_f^{NR}, χ_f^{NR})

$$\mathcal{M} = 1 - \frac{\langle h_{22}^{NR}, h_{22}^N \rangle}{\sqrt{\langle h_{22}^{NR}, h_{22}^{NR} \rangle \langle h_{22}^N, h_{22}^N \rangle}}$$

$$\langle x(t), y(t) \rangle = \int_{t_0}^T x(t) \overline{y(t)} dt$$

lower \mathcal{M} = better fit



overtones dominate early times

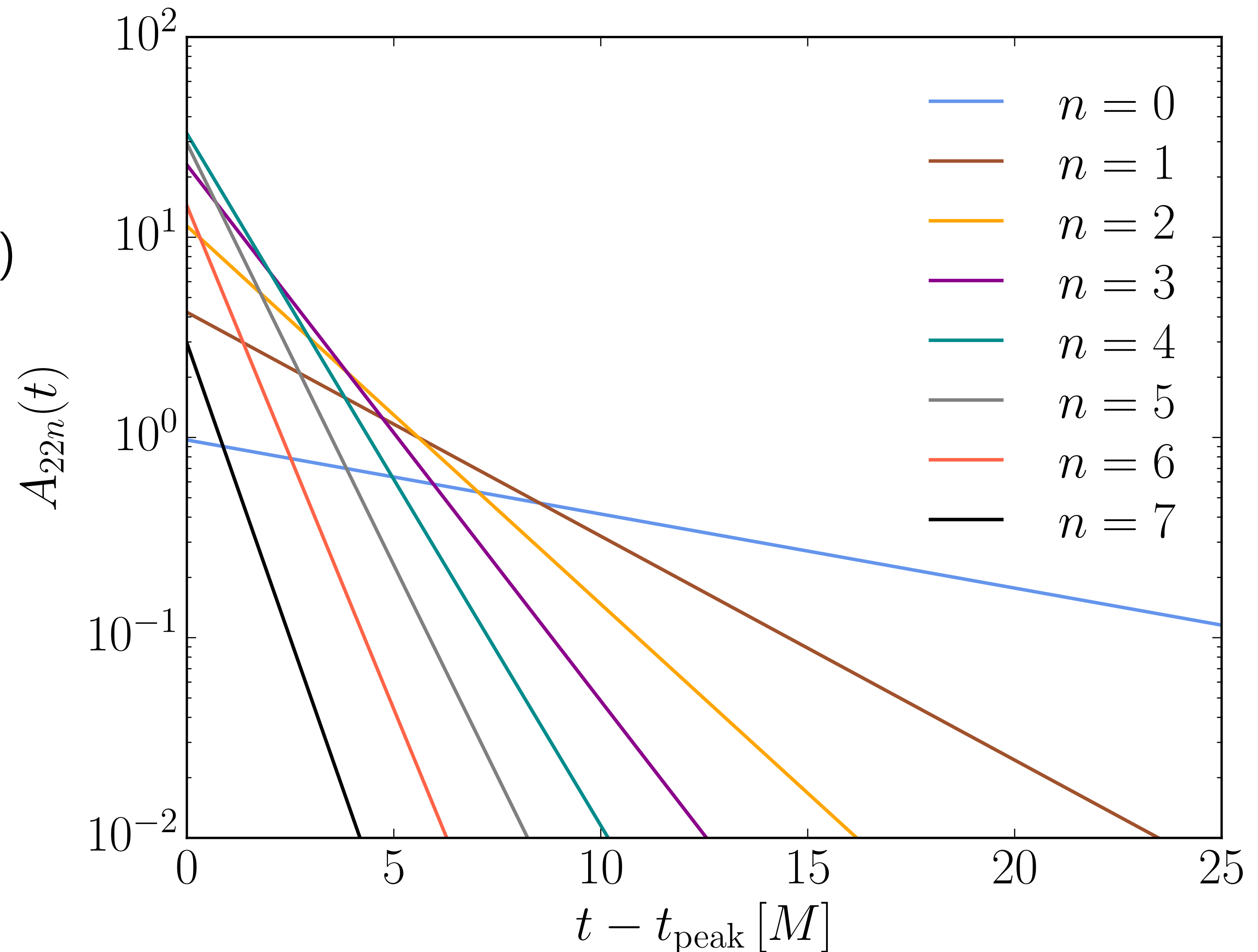
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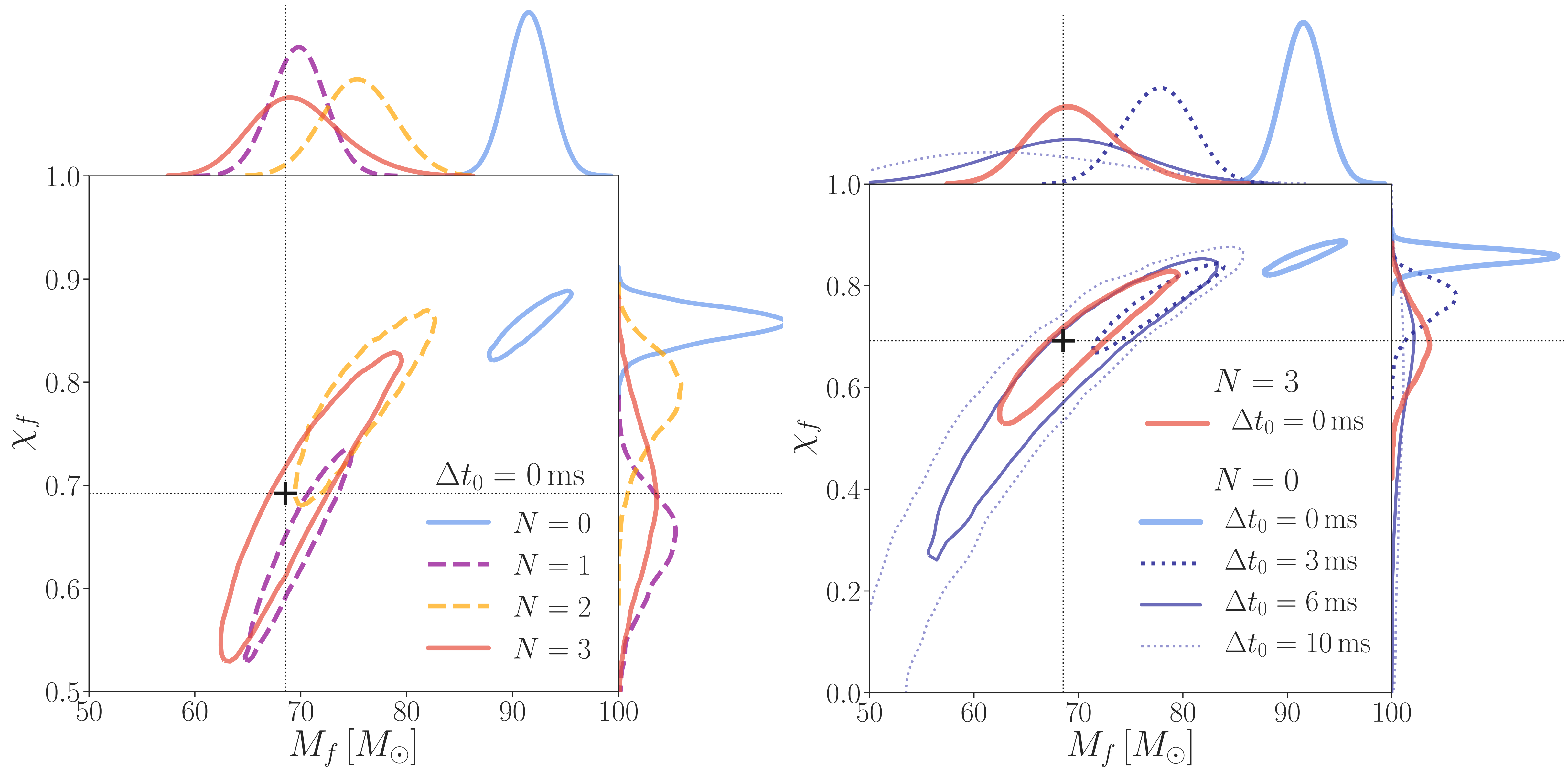
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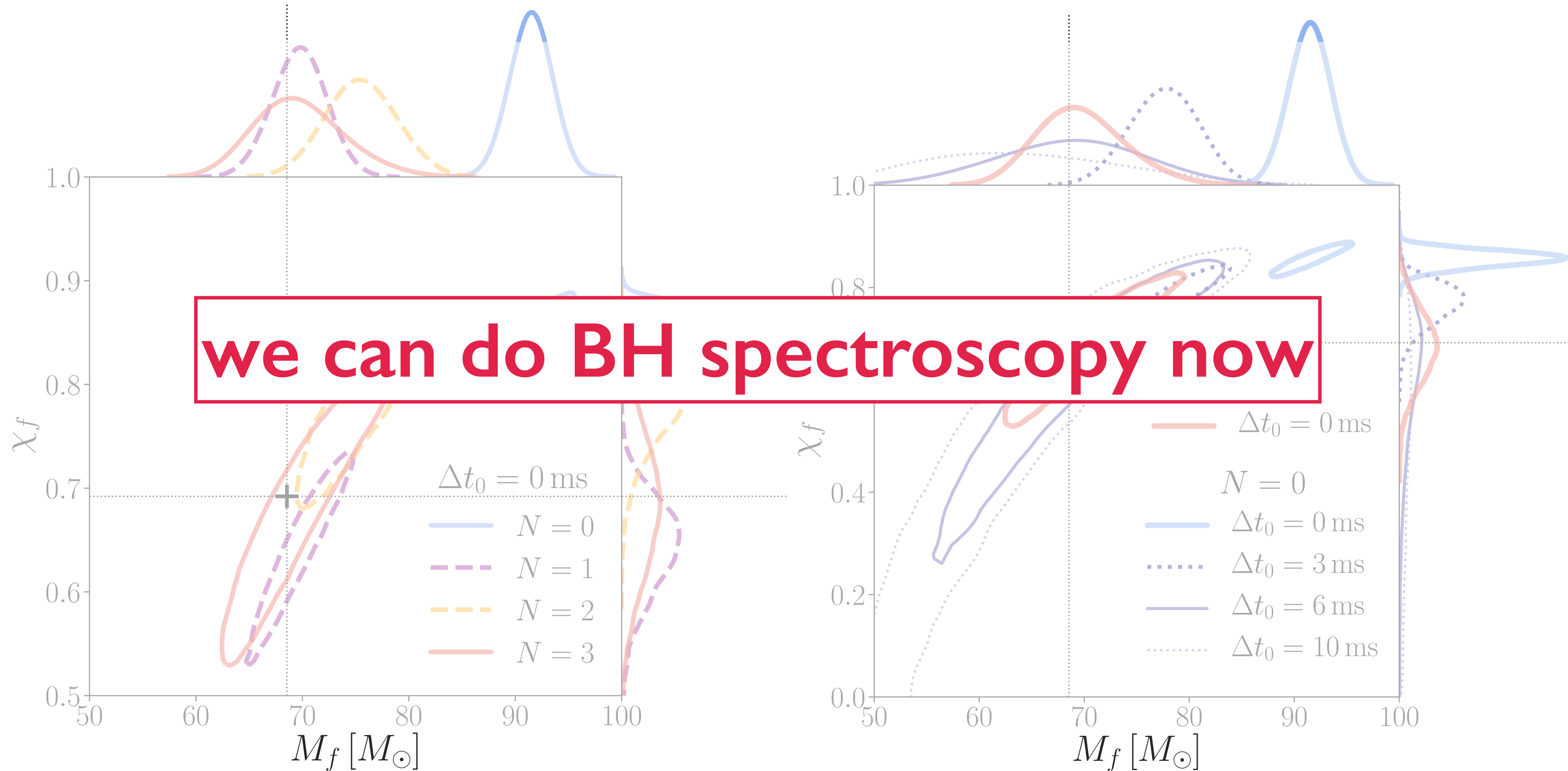
lower \mathcal{M} = better fit



simulated LIGO data



simulated LIGO data



results for GW150914

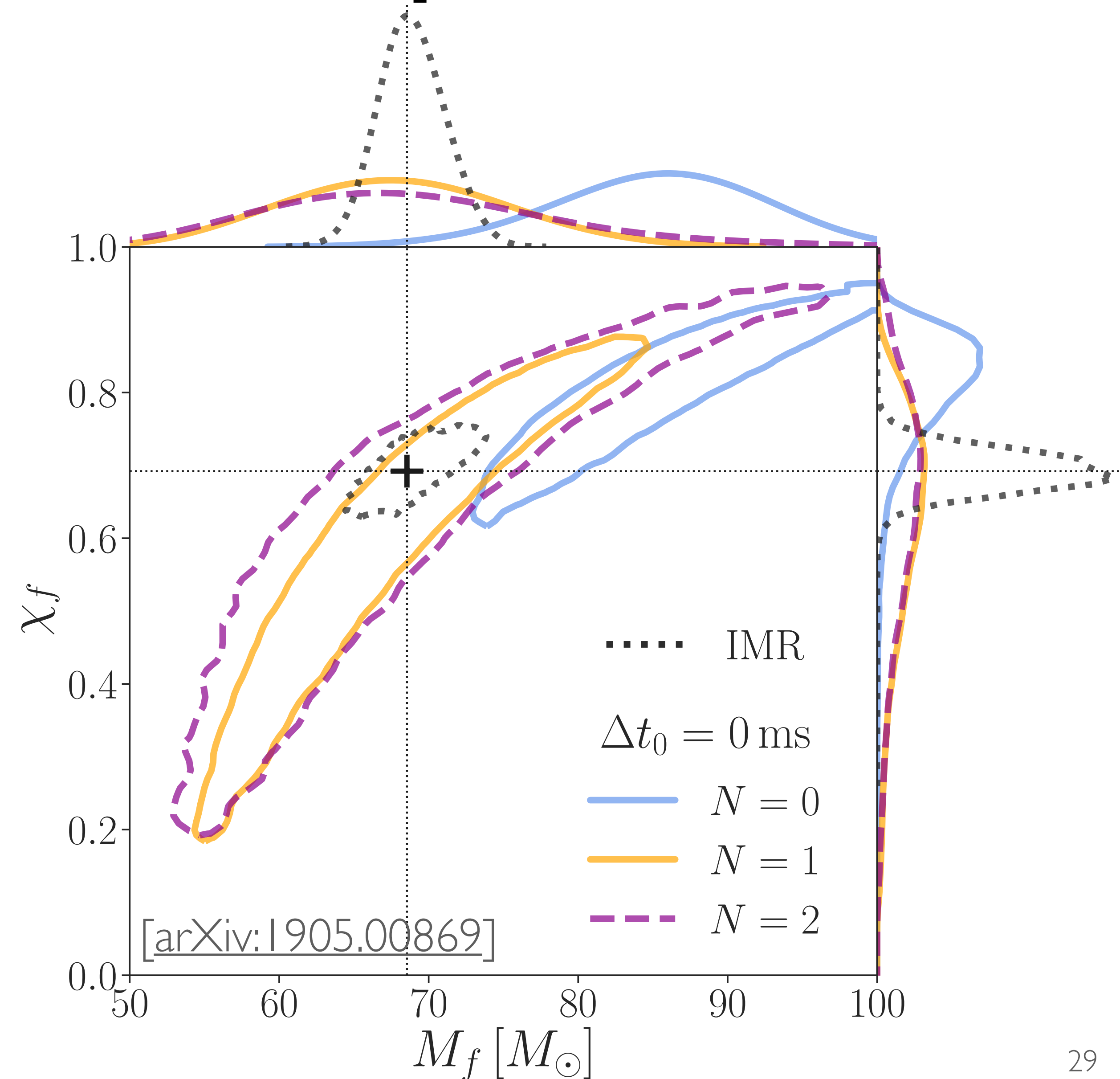
GW150914 at the peak

we identify at least **two ringdown modes** in GW150914

that is the least-damped mode plus **at least one overtone** ($N=1$)

with $N=1$, can **measure remnant mass and spin** at the peak

result in agreement with measurement from full waveform

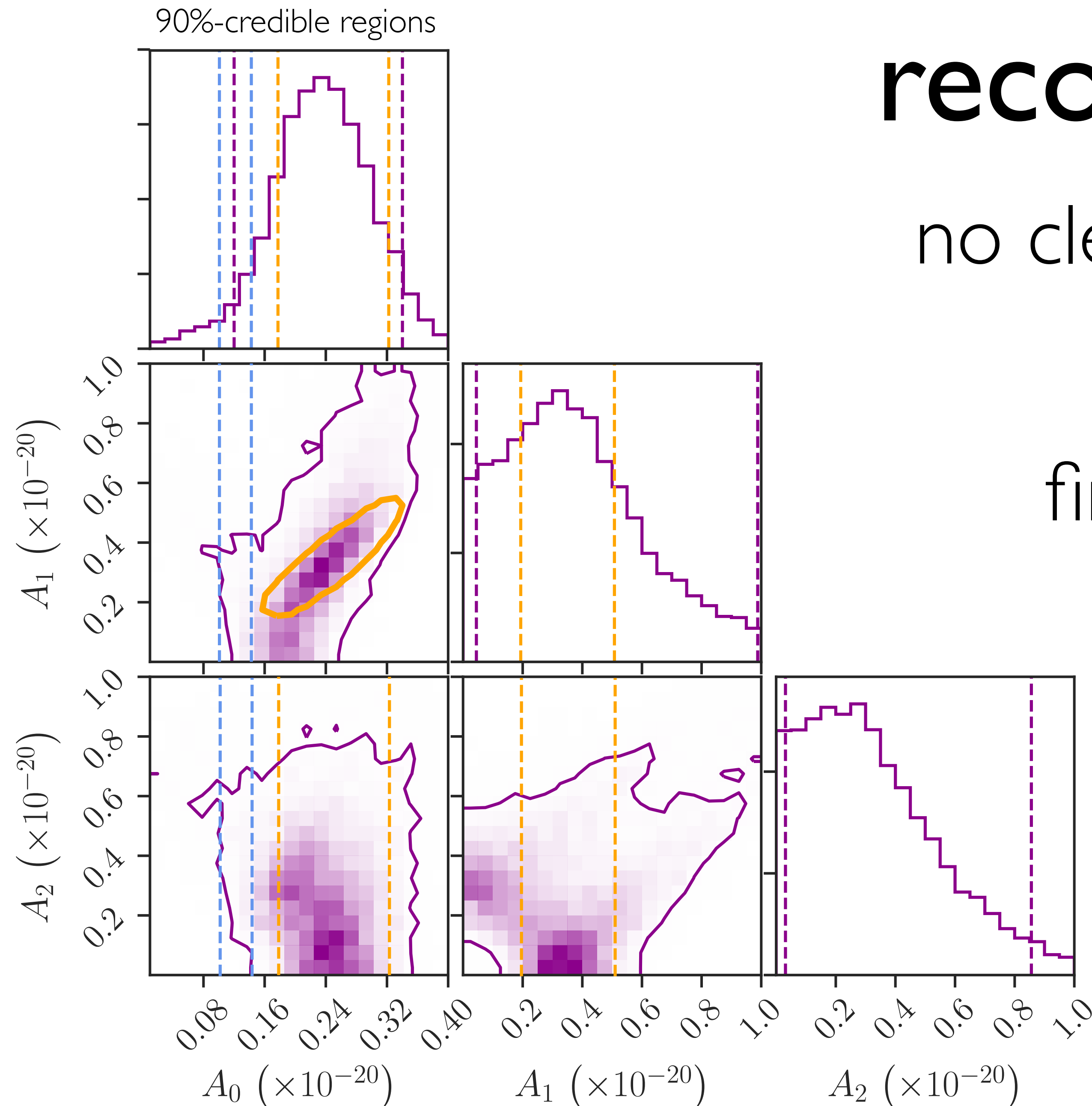


recovered amplitudes

no clear gain from 2nd overtone

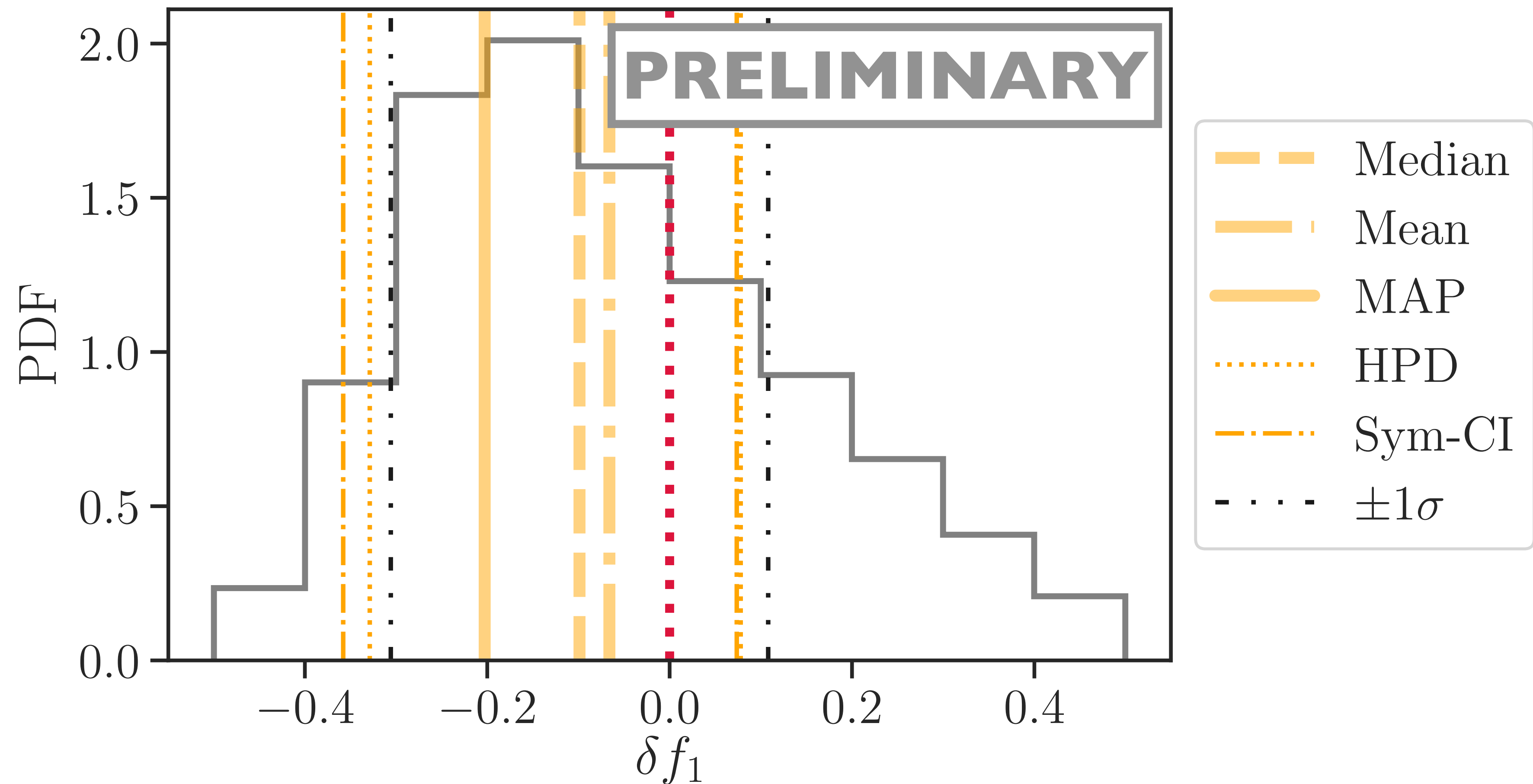
posterior for N=2 favors
first overtone, in agreement
with **posterior for N=1**

posterior for N=0
misses the mark, as
expected at the peak



frequency measurement

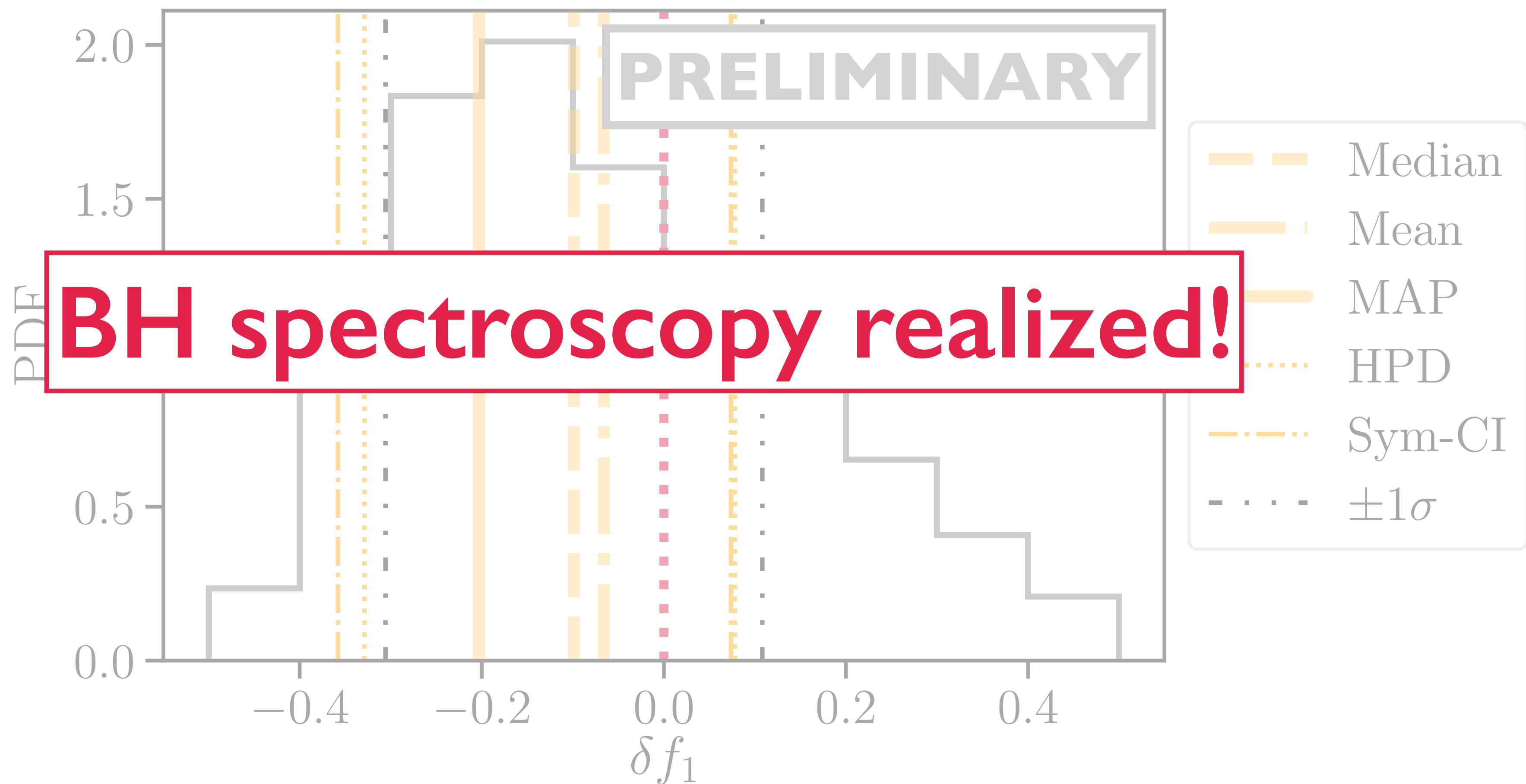
allow 1st overtone frequency to vary around GR value



agreement with Kerr better than $\sim 20\%$ at 1σ

frequency measurement

allow 1st overtone frequency to vary around GR value



agreement with Kerr better than $\sim 20\%$ at 1σ

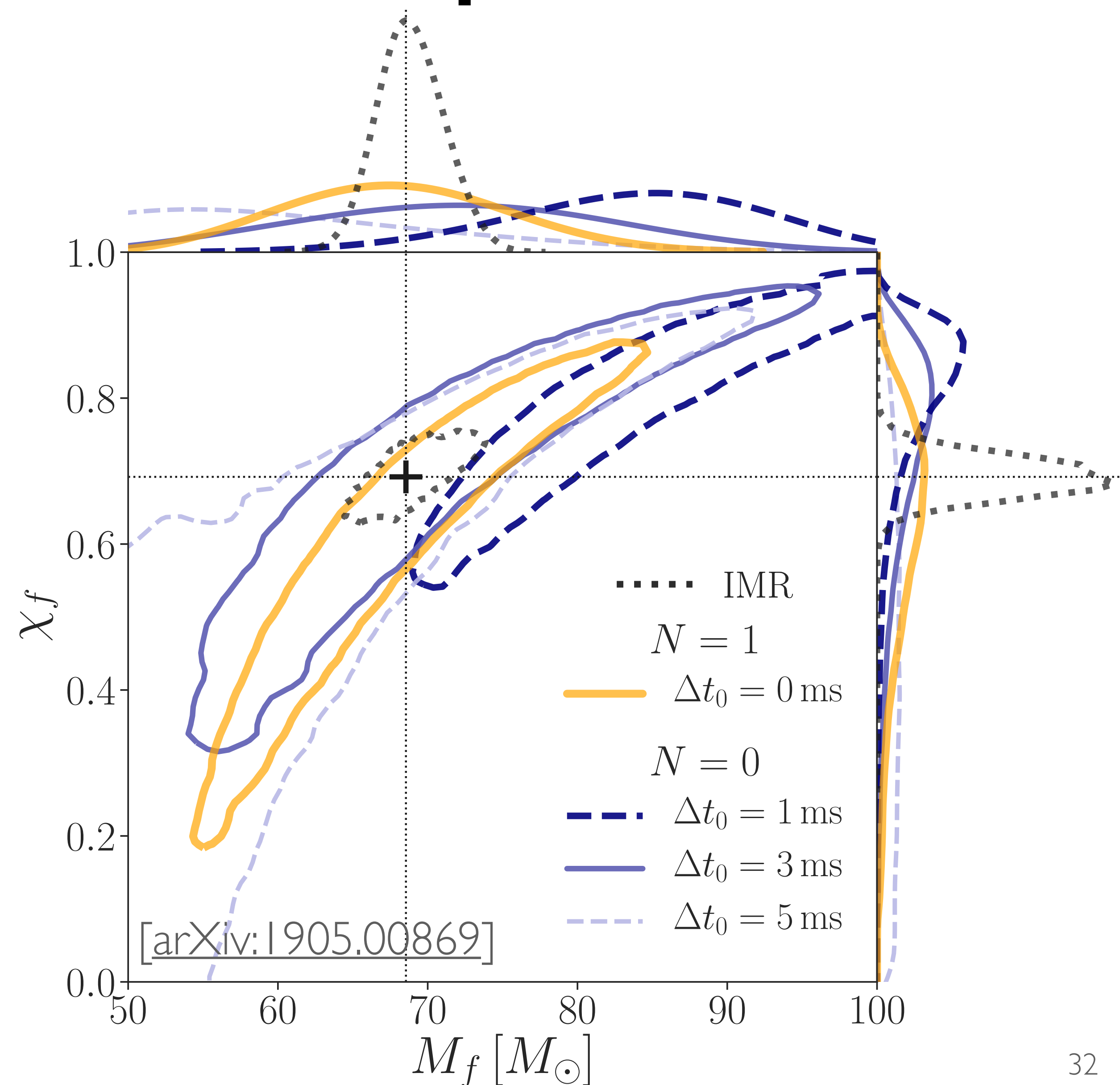
GW150914 after the peak

measurement at the peak with $N=1$
consistent with $N=0$ at later times

overtone result is slightly better
constrained (higher signal-to-noise ratio)

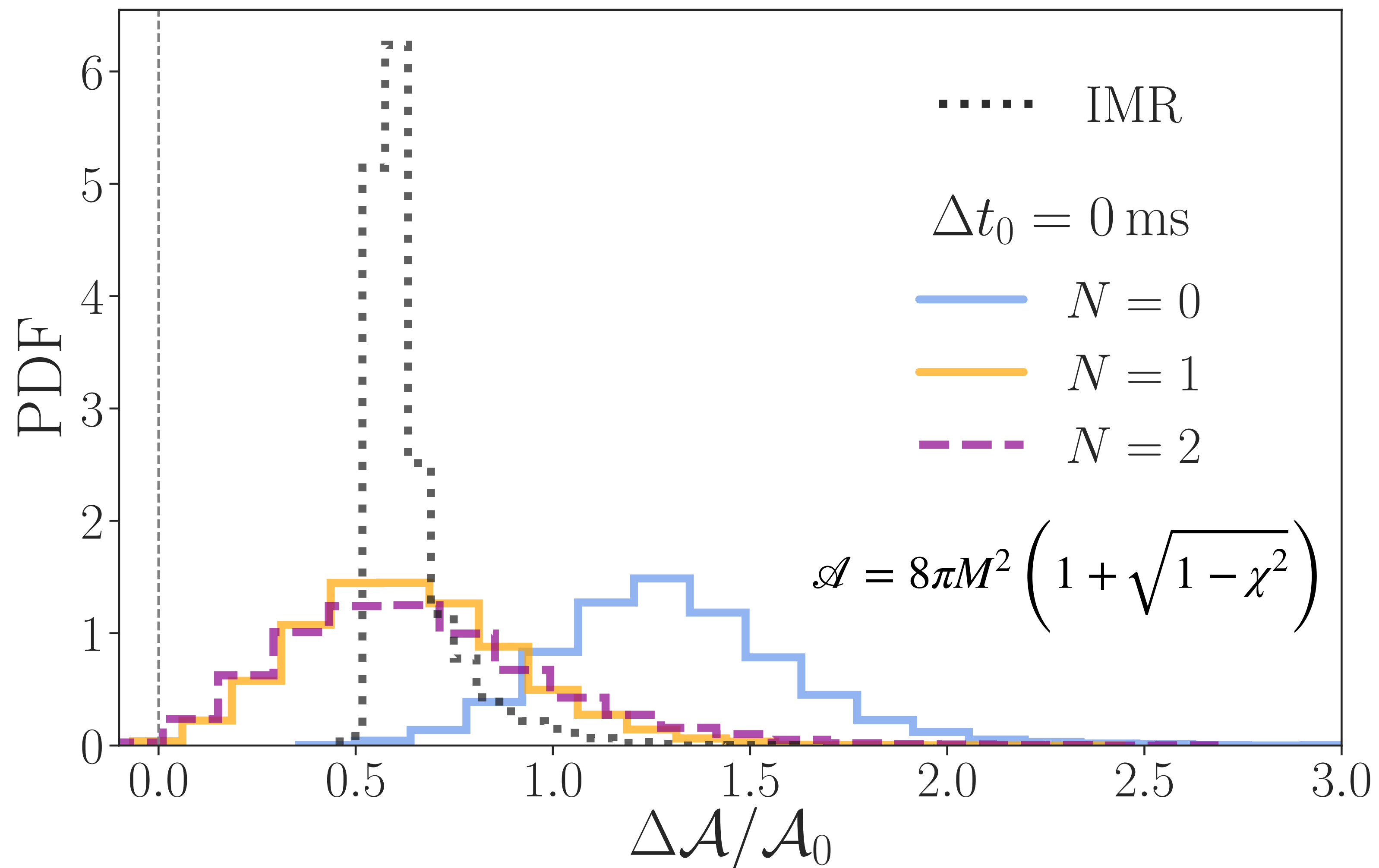
best measured combination of mass
and spin **agrees to $\sim 10\%$**

agreement between measurements
represents a **no-hair test**



area law

can recast measurement at the peak in terms of BH area



summary

with **overtones**, the ringdown starts at the GW peak

we identify **two modes** in GW150914

we check their consistency and measure frequency,

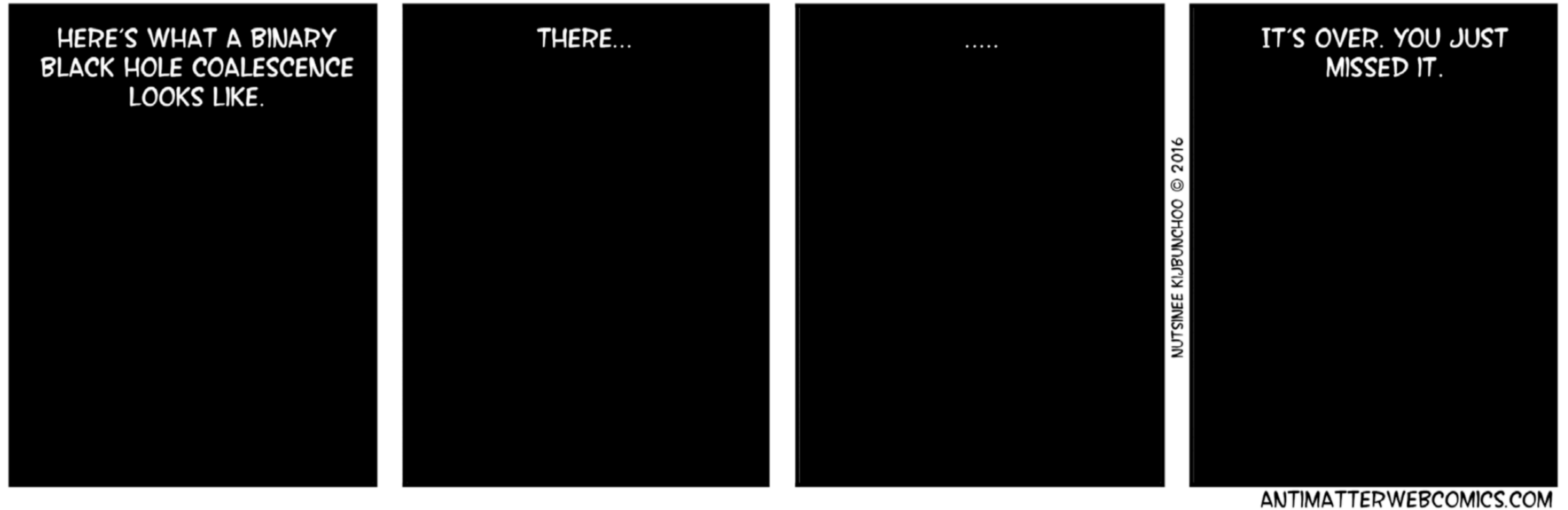
testing the no-hair theorem with GW150914

BH **spectroscopy** within reach of present instruments

much to gain from new **LIGO & Virgo** observations

the future is bright!

(not literally)



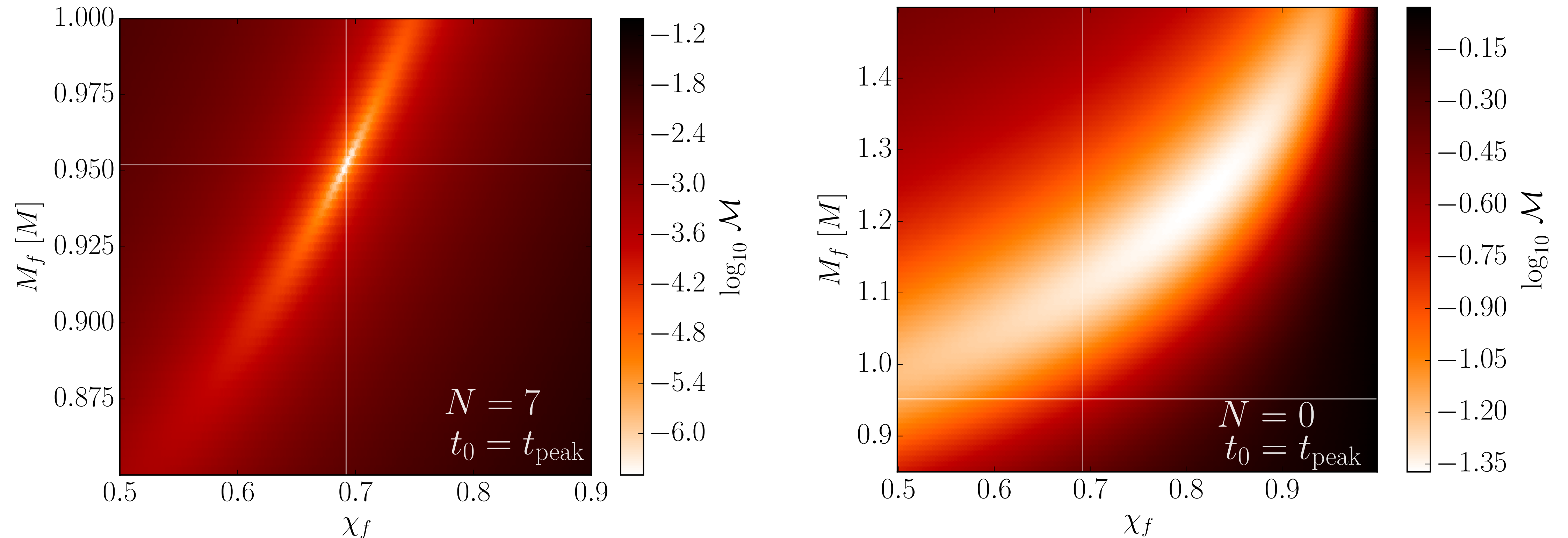
Nutsinee Kijbunchoo

thanks!

extra

extracting remnant properties

let BH mass and spin vary in fit



- $N = 7$ at peak \rightarrow accurate M_f and χ_f
- Fundamental biased by overtones around peak

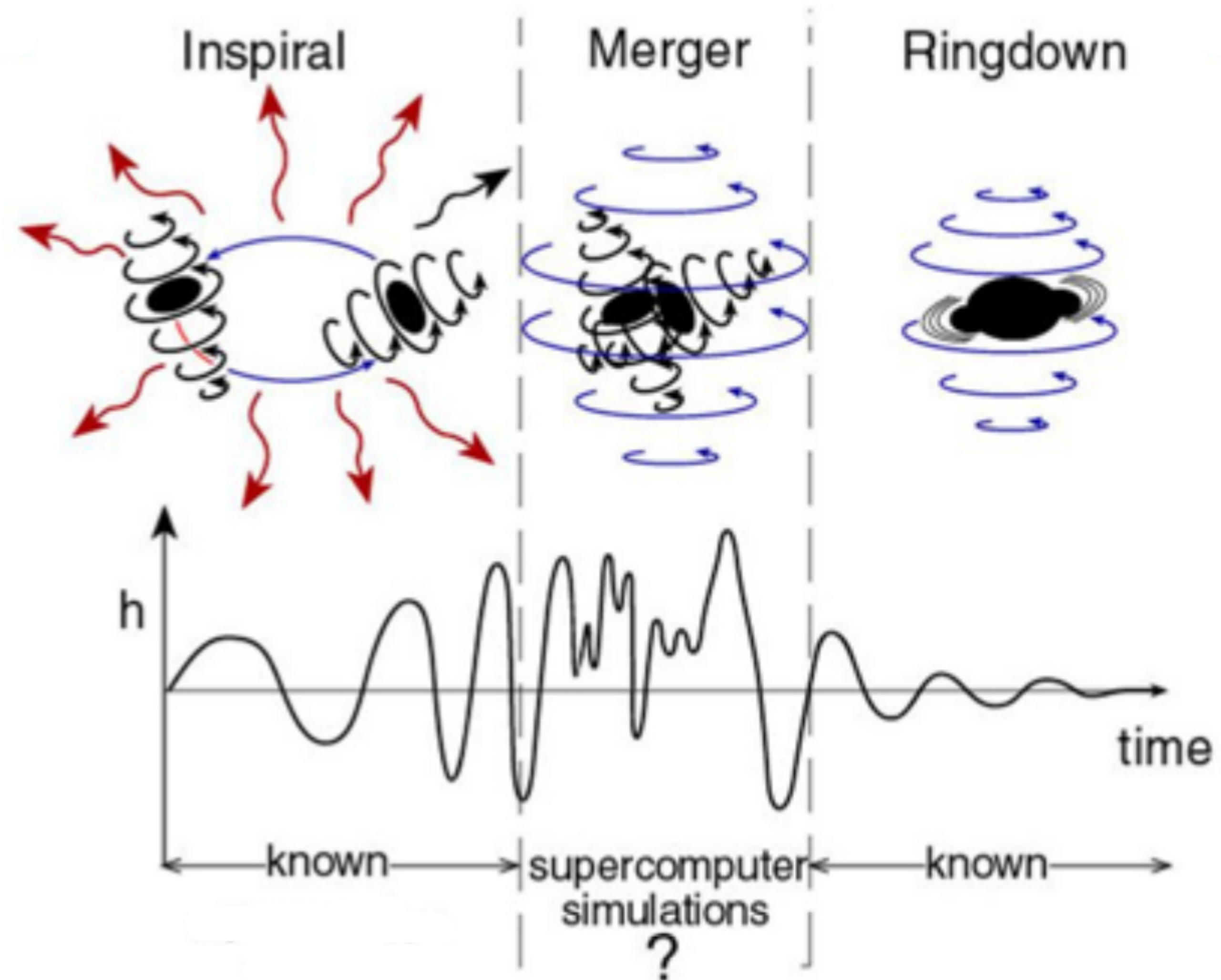
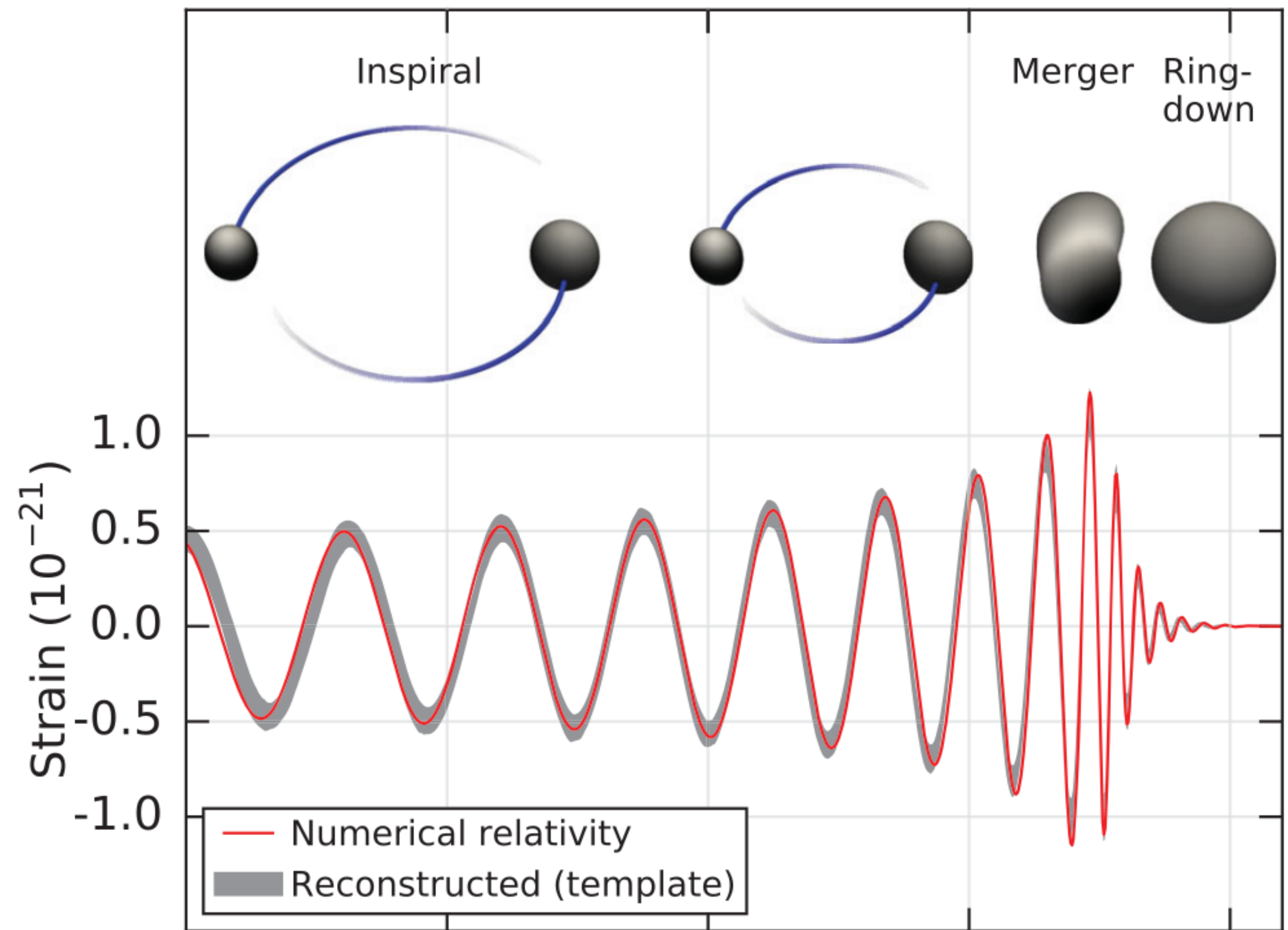


Diagram by Kip Thorne

overtone summary

ringdown starts at the **peak**
(maybe earlier)

overtone can reduce
inference uncertainty and bias
this enable first **tests of no-hair
theorem**



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