Monopoles in astroparticle physics

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May 2019

With the creation of physics as a science based on experience, the view was affirmed that the electrical and magnetic properties of bodies differ significantly. This view was clearly expressed by William Gilbert in 1600. The identity of the laws of attraction and repulsion for electric charges and magnetic charges, the poles of magnets, established in 1785 by Charles Coulomb, again raised the question of the similarity of electric and magnetic forces, but by the end of the 18th century it was found out that under laboratory conditions it is impossible to create a body with a non-zero full magnetic charge. The concept of a “magnetically charged substance” was permanently expelled from physics after the work of Ampere in 1820, in which it was proved that a circuit with an electric current creates the same magnetic field as a magnetic dipole. However, by the 1930s, ideas about the sources of magnetic fields starting changing.
The equations of classical electrodynamics formulated by Maxwell connect the electric and magnetic fields with the motion of charged particles. These equations are almost symmetrical with respect to electricity and magnetism. So the equations of electrostatics and magnetostatics are have the form:

$$div\vec{E} = 4\pi\rho; \qquad rot\vec{E} = 0; \qquad div\vec{H} = 0; \qquad rot\vec{H} = 4\pi\vec{j} \tag{1}$$

The simplest case of the appearance of a monopole in theory is obtained by symmetrizing these equations. As Dirac [1] has shown, this leads to the fact that in the presence of a magnetic charge, the electric charge must be quantized.

By analogy with the formula $\vec{E} = \frac{q}{r^3}\vec{r}$, we can define the magnetic charge carried by the magnetic monopole by the formula $\vec{H} = \frac{\mu}{r^3}\vec{r}$. Where μ is the magnetic charge, $\vec{E}$, $\vec{H}$ are the electric and magnetic fields strengths, q is the electric charge, r is the distance to the source of field.
And we also define the vector potential which connected with the magnetic field: $\vec{H} = rot\vec{A}$
In the describing model, the vector potential cannot be specified as a continuous function everywhere outside the magnetic charge. To show

this, we write the flux of the magnetic field through the surfaces Σ_1 and Σ_2 stretched over the contour γ:

$$F_1 = \int_{\Sigma_1} \vec{H} d\vec{S} = \oint\limits_{\gamma} \vec{A} d\vec{r} \qquad F_2 = \int_{\Sigma_2} \vec{H} d\vec{S} = -\oint\limits_{\gamma} \vec{A} d\vec{r} = -F_1 \quad (2)$$

Then the total flow through the surface $\Sigma = \Sigma_1 + \Sigma_2$ is equal to: $F = F_1 + F_2 = 0$ if $\vec{A}(\vec{r})$ is a continuous function $\vec{A} : \Sigma \to TM$. However, in the presence of the source $F = 4\pi\mu \neq 0$. i.e. $\vec{A}$ is not a continuous function on Σ surface.

To show how the presence of a magnetic charge leads to electric quantization, let us use the formulation of quantum mechanics in Feynman Integrals, within the framework of which the amplitude of a particle transition from point 1 to point 2 during time $t_2 - t_1$ is expressed as follows:

$$< 2, t_2 | 1, t_1 >= \int exp(\frac{iS[\vec{r}]}{\hbar}) D[\vec{r}], \qquad S = S_0 + e \int_1^2 \vec{A}(\vec{r}) d\vec{r} \quad (3)$$

Consider the relative phase of two trajectories γ_1 and γ_2:

$$\frac{1}{\hbar}(S[\gamma_1] - S[\gamma_2]) = \frac{e}{\hbar} \oint\limits_{\gamma_1 \cup (-\gamma_2)} \vec{A}(\vec{r}) d\vec{r} \quad (4)$$

Thus, the contribution to the amplitude by these paths is either $\frac{e}{\hbar} F_1$, or $-\frac{e}{\hbar} F_2$. But the amplitude shouldn't depend on this uncertainty. Thus, it is easy to see that the electric charge is determined by the mapping degree $S^1 \to S^1$ (the fundamental group $\pi_1(S^1) = \mathbb{Z}$) due to the requirement of amplitude invariance connected to the change of the relative phase.

$$\frac{e}{\hbar} F_1 - (-\frac{e}{\hbar} F_2) = \frac{e}{\hbar} F = 2\pi n \quad (5)$$

Then

$$e = \frac{n\hbar}{2\mu} \qquad n \in \mathbb{Z} \quad (6)$$

This is the Dirac quantization rule for electric charge in the presence of a magnetic monopole.
In 1974, t'Hooft and Polyakov showed [2] that a magnetic monopole could occur in gauge models of electro-weak interactions. However, the existence of a magnetic monopole solution requires a compact group of uniform symmetry, which includes electromagnetic symmetry, which isn't the case in the framework of Standard Model's Electroweak Interactions.

A magnetic charge, unlike an electric one, is not a Noether, but it has a topological nature. A more detailed study of this phenomenon leads to the concept of the Polyakov-t'Hooft monopoles, which naturally arise in the presence of gauge fields in topologically nontrivial field theory models with spontaneously broken symmetry. [3]
Consider the system, which described by the Lagrangian:

$$\mathscr{L} = \frac{1}{2}(\bigtriangledown_\mu \vec{\phi})^2 - \lambda(\vec{\phi}^2 - a^2)^2 - \frac{1}{4}(\vec{F}_{\mu\nu})^2 \tag{7}$$

$$\vec{F}_{\mu\nu} = \partial_\mu \vec{A}_\nu - \partial_\nu \vec{A}_\mu + g\vec{A}_\mu \times \vec{A}_\mu \tag{8}$$

$\vec{F}_{\mu\nu}$ is the tensor of the gauge field $\vec{A}_\mu$, which is an adjoint representation of the $SO(3)$ gauge group. $\vec{\phi}$ is the isovector Higgs field and $\bigtriangledown_\mu \vec{\phi} = \partial_\mu \vec{\phi} + g\vec{A}_\mu \times \vec{\phi}$. The smallest energy value of such a system is achieved on the field configurations $\vec{\phi} = const$, $|\vec{\phi}| = a^2$. That is, on vacuum spheroids. $SO(3)$ acts transitively at this spheroids but not effectively, i.e., the Ker of the mapping $SO(3) \to S^2$ is nontrivial (it is easy to show that the stabilizer of the group action is $SO(2)$). [5] Thus, the topological space of vacuums is homeomorphic to the quotient space $S^2 = SO(3)/SO(2)$, which is homeomorphic to the two-dimensional sphere S^2_a of radius a in an isotopic space. I.e. the mapping $\vec{\phi} : S^2_\infty \to S^2_a$ on the asymptotics $\vec{r} \to \infty$ is characterized by the second homotopy group $\pi_2(S^2) = \mathbb{Z}$. Thus, the field configuration can be associated with a topological charge Q, which defined as an element $\pi_2(S^2) = \mathbb{Z}$. For example, by calculating the flow through the S^2_∞ vacuum sphere:

$$F = \int_{S^2_\infty} \vec{H} d\vec{\Sigma} = \frac{1}{2}\int_{S^2_\infty} \epsilon_{ijk} \vec{\phi} \vec{F}_{jk} d\Sigma_i = -\frac{4\pi Q}{g} \tag{9}$$

In this case, the homotopy classes will be separated by an infinite potential barrier. Comparing this with the Gauss formula $F = 4\pi\mu$, we conclude that the magnetic charge has a topological nature and is defined as the mapping degree $\vec{\phi} : S^2_\infty \rightarrow S^2_a$ or $\mu = -\frac{Q}{g}$. Because of higher homotopy groups are abelian, we conclude that the magnetic charge is additive.

The considered approach is generalized to the case of arbitrary gauge symmetries and their arbitrary violations. To find out whether magnetic monopoles arises in the theory under consideration, it suffices to consider $\pi_2(G/H)$, where G is the full group of gauge symmetry, H is the group of unviolated symmetries. The criteria for the existence of monopoles in such a model is the nontriviality of $\pi_2(G/H)$.
If G is simply connected, then from consideration of the exact sequence of the bundle $G \rightarrow G/H$ with layer H

$$... \xrightarrow{p^*} \pi_{i+1}(G/H) \xrightarrow{\partial} \pi_i(H) \xrightarrow{i^*} \pi_i(G) \xrightarrow{p^*} \pi_i(G/H) \xrightarrow{\partial} \pi_{i-1}(H) \xrightarrow{i^*} ... \tag{10}$$

and because G is simply connected :

$$0 \xrightarrow{p^*} \pi_2(G/H) \xrightarrow{\partial} \pi_1(H) \xrightarrow{i^*} 0 \tag{11}$$

it follows that $\pi_2(G/H) \approx \pi_1(H)$ [5]. This situation is common for the great unified theories [3]. For example, if electromagnetic and color symmetries are not broken, i.e., $H \approx SU(3) \times U(1)$ then $\pi_1(H) \approx \pi_1(U(1)) \approx \mathbb{Z}$, i.e., these theories predicts the existence of magnetic monopoles. It is easy to verify that in the standard model monopoles are absent because second homotopy group of the bundles base is trivial for the Standard Model gauge groups.

It is important to note that before the phase transition, massive magnetic monopoles simply don't exist, because symmetry has not been violated at this epoch. They arise in the universe after a phase transition.

In the grand unification models, the symmetry of electromagnetic interactions was included in the grand unification (GU) compact symmetry group. So, the existence of magnetic monopoles with Dirac magnetic charge is an inevitable topological consequence of the GU symmetry breaking. The monopoles mass is determined by their vacuum average. The predicted mass of such monopoles was about $m \sim \frac{\Lambda}{e}$, where Λ is the typical magnitude of the GU symmetry breaking. For $\Lambda \sim 10^{15} GeV$, monopoles mass is $m \sim 10^{16} GeV$, which explains the negative results of searches for monopoles on accelerators.

The proton decay ($p^+ \rightarrow e^+ e^- e^+$) with a cross section determined by the proton size ($\sigma \sim 10^{-28} cm^2$) is induced in the magnetic monopoles singular field. This process is explained as follows:

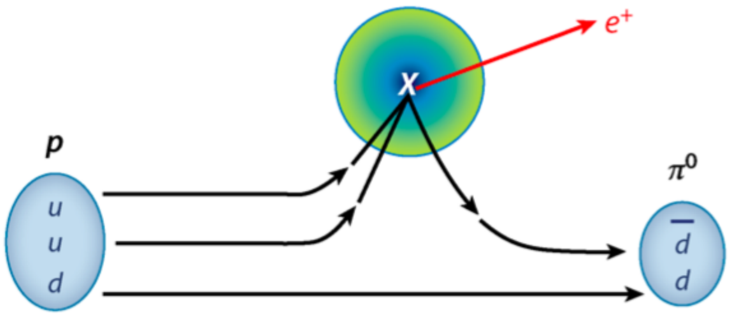
The minimum extension of the standard model should include the direct product of SM gauge groups as a subgroup. The minimal group that includes $SU(3) \times SU(2) \times U(1)$ is the$SU(5)$ group. One can build a minimal great unification model, based on this group.
The fundamental fermions in this model will be included in the Lagrangian in the form of a quintuplet, which has the form:

$$\begin{pmatrix} d_r & d_g & d_b & e^+ & \hat{\nu}_e \end{pmatrix} \tag{12}$$

As well as some decouplet. $SU(5)$ includes 24 generators, 12 of which characterize the standard model gauge interactions mediators, because $SU(3) \times SU(2) \times U(1) \subset SU(5)$. The remaining 12 carriers are unique for this GU model, and they are responsible for the proton decay reaction in the magnetic monopole field.
All 24 mediators can be described using the adjoint representation of $SU(5)$ group:

$$\begin{pmatrix} & & & X_1 & Y_1 \\ & G - 2B/\sqrt{30} & & X_2 & Y_2 \\ & & & X_3 & Y_3 \\ X^1 & X^2 & X^3 & W^3/\sqrt{2} + 3B/\sqrt{30} & W^+ \\ Y^1 & Y^2 & Y^3 & W^- & -W^3/\sqrt{2} + 3B/\sqrt{30} \end{pmatrix} \tag{13}$$

where G is the 3×3 matrix of gluons, B corresponds to the $U(1)$ gauge boson, $B + W^3$ gives the photon and the Z boson states.
It is easy to show that the forms builded of the fermions quintuplet



and decuplet, as well as the gauge adjoint representation of the $SU(5)$ group, affect all possible standart model vertices. However, in the $SU(5)$ theory the amount of vertices turn out to be larger than they were in the SM.
One of the new vertices is a vertex describing the X boson - quarks interaction:

$$\begin{pmatrix} d_r & d_g & d_b & e^+ & \hat{\nu}_e \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & 0 & X_1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} d_r \\ d_g \\ d_b \\ e^+ \\ \hat{\nu}_e \end{pmatrix} = d_r \cdot X_1 \cdot e^+ \quad (14)$$

Showed vertex clearly leads to the baryon number conservation violation. Similar vertices for X_2 and X_3 will differ from this one only in the mediators color. Similarly, the Y boson is responsible for the transformation of quarks into neutrinos. Another new vertice in this theory is $u \cdot X \cdot u$

To explain the low intensity of proton decay intermediate bosons X and Y are assumed to be very massive (their masses greater than 10^{14} GeV). Therefore, it is impossible to observe them directly in any imaginable experiments with the modern development of technology. However, in a singular field of a massive monopole, such a decay becomes possible due to the sufficient mass of the magnetic monopole. A picture illustrating this process is shown at the top. So the process

$$\begin{array}{ll} p^+ + M \rightarrow e^+ + M + & \pi^0 \\ & \hookrightarrow \; e^+ e^- \end{array} \quad (15)$$

Describes the possibility of proton decay ($p^+ \to e^+ e^- e^+$) in the presence of a magnetic monopole.

According to the hot Universe model and modern cosmological models after the hitting, particles with a mass of m of any kind should be in equilibrium at $T > m$ if the interaction of these particles is strong enough to satisfy equilibrium conditions with plasma and radiation. This means that the rate of the reactions σv is large enough to satisfy condition $n(T)(\sigma v) > \Gamma$. where $n(T)$ is the number of particles density at the temperature T and $\Gamma \sim \frac{T^2}{m_{pl}}$ is the cosmological expansion rate. When the temperature drops to $T < m$ At the temperature $T\ T_f$, when the particles interaction rate is compared with the cosmological expansion rate, the particles get out of equilibrium. As a result, the particles are freezed, and their relative concentration does not changes anymore, because the decreasing of temperature leads to the fact that the rate of expansion begins to exceed the rate of annihilation of monopole-antimonopol pairs, which should have led to relic magnetic monopoles concentration freezing.

The annihilation cross section is determined by the Coulomb attraction of magnetic charges. The rate of annihilation of the monopole and antimonopole can be obtain in the diffusion approximation whith considering the diffusion of particles with a magnetic charge $-\mu$ to an absorbing sphere with a radius $a < \Gamma_0$ and with a magnetic charge $+\mu$. [4]

Then for the magnetic monopoles mass $m \sim 10^{16} GeV$ - the magnetic monopoles density turns out to be 16 orders of magnitude greater than the baryon density. In order to solve the problem of monopoles overproduction, an inflationary cosmological model was proposed, in which the initial concentration of monopoles was strongly suppressed.

The considered mechanism of the topological formation of monopoles in the process of phase transition occurs in such a way that first the monopole field is formed in space and only then the singularity of this field is localized. It is clear that the local field generated in the phase transition has no orientation. Thus, together with monopoles and antimonopoles, magnetic field loops can appear, forming the primary magnetic fields structure in the Universe.

Conclusion

Magnetic monopoles play an important role in fundamental physics and cosmology. Being an integral part of great unification theories, they (their experimental discovery) are the indicators of their trueness. Also they are gives an important constraints on the parameters of these theories. The contradictions arising in such theories, connecting with the monopoles overproduction in the early universe, are one of correctness indications of inflation theory, within the framework of which these contradictions can be resolved.

The magnetic charge of a monopole naturally turns out to be quantized; in addition, its presence inevitably leads to the quantization of the electric charge, the quantum nature of which is not completely clear today.

An important indirect experimental indication of the presence of magnetic monopoles would be the detection of the proton decay, the intensity of which greatly increases in the presence of magnetic monopoles.

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