The Case against Ghosts in Fundamental Theory

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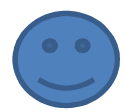
巴黎: 2019年4月26日

Three Quick Questions & Answers
(Witnessing to the heathen)

1. What is a ghost?
   - Particle with negative KE
2. Why should we avoid ghosts?
   - Interacting ghosts would explode the universe!
3. Why do people nonetheless consider ghosts?
   - They want to quantize gravity
   - Stelle (1977) ➔ $R + R^2 + C^2$ is renormalizable
   - Higher ∂`s in C^2 give ghosts!

How Lower Derivatives Work

- Dynamical variable $q(t)$ & Lagrangian $L(q,\dot{q})$
  - $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = \frac{\partial L}{\partial q}$ initial conditions ➔ 2 canonical variables
- Canonical formulation
  - $Q = q$ & $P = \frac{\partial L}{\partial \dot{q}}$ ➔ $\dot{q} = v(Q,P)$ (nondegeneracy)
  - $H(Q,P) = Pv(Q,P) - L(Q, v(Q,P))$
- Hamilton's equations generate time evolution
  - $\dot{Q} = \frac{\partial H}{\partial P} = v + P\frac{\partial v}{\partial P} - \frac{\partial L}{\partial \dot{q}}\frac{\partial v}{\partial P} = v$ 
  - $\dot{P} = -\frac{\partial H}{\partial Q} = -P\frac{\partial v}{\partial Q} + \frac{\partial L}{\partial q} + \frac{\partial L}{\partial \dot{q}}\frac{\partial v}{\partial Q} = \frac{\partial L}{\partial q}$ 
- $H(Q,P)$ can be bounded below

Higher Derivatives (Ostrogradsky 1850)

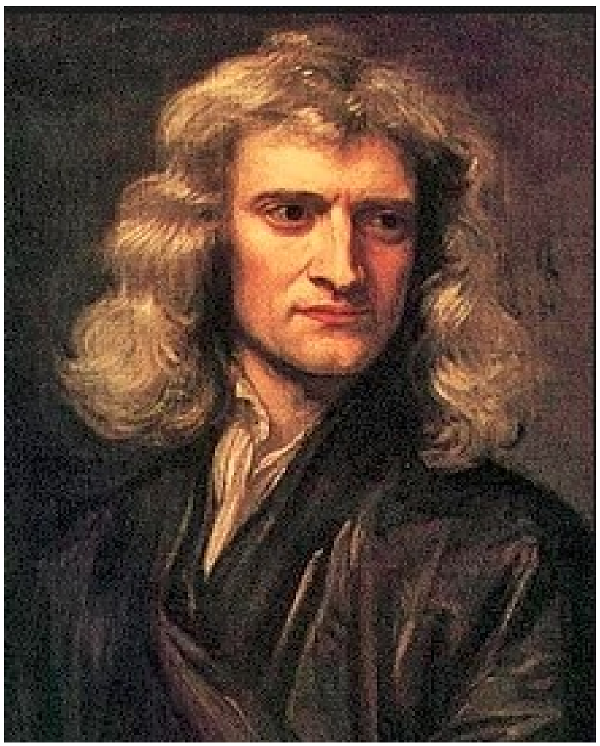
- Lagrangian $L(q,\dot{q},\ddot{q})$
  - $\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{q}} - \frac{d}{dt}\left(\frac{\partial L}{\partial \ddot{q}}\right)\right] = \frac{\partial L}{\partial q}$ 4 initial conditions ➔ 4 canonical coordinates
- Canonical Formulation
  - $Q_1 = q\ ,\ \ P_1 = \frac{\partial L}{\partial \dot{q}} - \frac{d}{dt}\left(\frac{\partial L}{\partial \ddot{q}}\right)\ ,\ \ Q_2 = \dot{q}\ ,\ \ P_2 = \frac{\partial L}{\partial \ddot{q}}$ ➔ $\ddot{q} = a(\vec{Q}, P_2)$ (ND)
  - $H(\vec{Q},\vec{P}) = P_1 Q_2 + P_2 a(\vec{Q},P_2) - L(Q_1, Q_2, a(\vec{Q},P_2))$
- Hamilton's equations generate time evolution
  - $\dot{Q}_1 = \frac{\partial H}{\partial P_1} = Q_2$, $\dot{Q}_2 = \frac{\partial H}{\partial P_2} = a + P_2 \frac{\partial a}{\partial P_2} - \frac{\partial L}{\partial \ddot{q}} \frac{\partial a}{\partial P_2} = a$
  - $\dot{P}_2 = -\frac{\partial H}{\partial Q_2} = -P_1 - P_2 \frac{\partial a}{\partial Q_2} + \frac{\partial L}{\partial \dot{q}} + \frac{\partial L}{\partial \ddot{q}} \frac{\partial a}{\partial Q_2} = \frac{\partial L}{\partial \dot{q}} - P_1$
  - $\dot{P}_1 = -\frac{\partial H}{\partial Q_1} = -P_2 \frac{\partial a}{\partial Q_1} + \frac{\partial L}{\partial q} + \frac{\partial L}{\partial \ddot{q}} \frac{\partial a}{\partial Q_1} = \frac{\partial L}{\partial q}$
- H is **linear** in P_1 ➔ not bounded below (or above)

Why this is bad

- No guaranteed problem without interactions
  - Energy flow from $KE < 0$ to $KE > 0$ excites **both** DoF's
- No guaranteed problem without continuum DoF's
  - Instability is driven by vast d^3k UV phase space
    - Overwhelms even the weakest nonzero coupling
  - Decay is instantaneous
    - $\tau \neq 0$ results only come from imposing a UV cutoff
- Power and simplicity of the result
  - Requires only non-degenerate higher derivatives
    - Non-perturbative &independent of interactions
  - This is the strongest constraint on Fundamental Theory!

Ostrogradsky showed Newton was right about $\vec{F} = m\vec{a}$

Isaac Newton (1687)



Mikhail Ostrogradsky (1850)

$$L = -\frac{gm}{2\omega^2}\ddot{q}^2 + \frac{m}{2}\dot{q}^2 - \frac{m\omega^2}{2}q^2$$

- $q(t) = \sum[C_\pm \cos(k_\pm t) + S_\pm \sin(k_\pm t)]$
  - $k_\pm = \omega\sqrt{\frac{1\mp\sqrt{1-4g}}{2g}}$ carries $\pm$ KE
  - $C_\pm = \frac{k_\mp^2 q_0 + \ddot{q}_0}{k_\mp^2 - k_\pm^2}$ $S_\pm = \frac{k_\mp^2 \dot{q}_0 + \dddot{q}_0}{k_\pm(k_\mp^2 - k_\pm^2)}$
- $H = \frac{m}{2}\sqrt{1-4g}[k_+^2(C_+^2 + S_+^2) - k_-^2(C_-^2 + S_-^2)]$
- $E > 0$ creation and annihilation operators
  - $\alpha \propto C_+ + iS_+$ $\alpha^\dagger \propto C_+ - iS_+$
- $E < 0$ creation and annihilation operators
  - $\beta \propto C_- - iS_-$ $\beta^\dagger \propto C_- + iS_-$

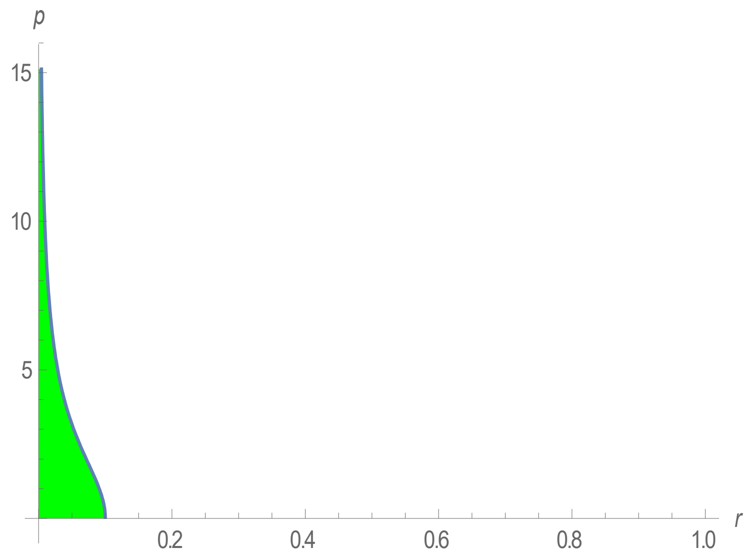
Common Misconceptions 1

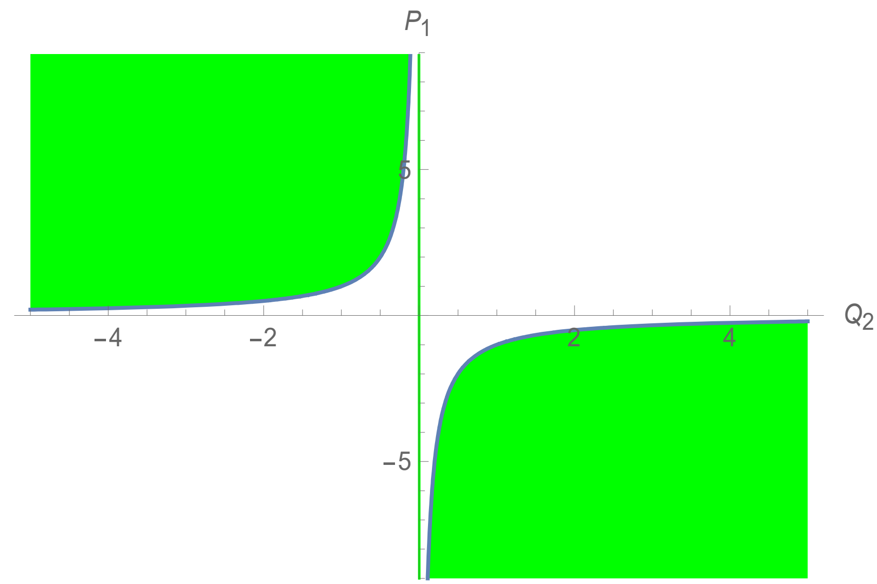
- "No problem for any $q(t) = q_0$"
  - Problem is pathological **time dependence** not special values
- "High mass ghosts decouple at low energies"
  - They actually couple more strongly!
- "No problem if HD's confined to interactions"
  - Problem is non-perturbative
- "Quantization might help"
  - This is a large phase space problem

Quantization can prevent small phase space instabilities, not large ones

$$H = \frac{p^2}{2m} - \frac{e^2}{4\pi\epsilon_0 r} < -E$$

$$H = Q_2 P_1 < -E$$





Common Misconceptions 2

- "Problem is unitarity, not instability"
  - Regards $E < 0\ \ \beta^\dagger(\vec{k})$ as $E > 0$ annihilator
  - $\alpha, \beta|\Omega\rangle = 0$ is normalizable
  - $\alpha, \beta^\dagger|\Omega\rangle = 0$ is **not** normalizable ➔ nonsense
- "No problem from entire functions of ∂^2"
  - Works perturbatively in Euclidean momentum space
  - We live in non-perturbative Minkowski position space
  - Adding derivatives makes the instability worse

Normalizability puts the ``quantum'' in Quantum Mechanics

This is Quantum Mechanics

- SHO ➔ $H = \frac{1}{2m}P^2 + \frac{1}{2}m\omega^2 Q^2$
- $a_{\mp} \equiv \sqrt{\frac{m\omega}{2\hbar}}\left[q \pm \frac{\hbar}{m\omega}\frac{\partial}{\partial q}\right]$
- $\Omega(q) \propto \exp\left[-\frac{m\omega}{2\hbar}q^2\right]$
- $|N\rangle \propto (a_+)^N|\Omega\rangle$
- $H|N\rangle = +(N + \frac{1}{2})\hbar\omega|N\rangle$
- Sum of Hermitian squares has **POSITIVE** spectrum

This is nonsense

- $H|\psi\rangle = E|\psi\rangle$ 2nd order ODE
  - 2 solutions for **every** E
- $\bar{\Omega}(q) \propto \exp\left[+\frac{m\omega}{2\hbar}q^2\right]$
- $|\bar{N}\rangle \propto (a_-)^N|\bar{\Omega}\rangle$k
- $H|\bar{N}\rangle = -(N + \frac{1}{2})\hbar\omega|\bar{N}\rangle$
- Can get any other spectrum
  - Positive, negative, complex . .

Why not just change the norm? ➔Alternate Quantization

- Nothing wrong with alternate quantization for new fields
  - Physics an experimental science
  - Perhaps a new one requires alternate quantization
- But problematic for fields we understand classically
  - Taking $\hbar \to 0$ does not recover the known classical theory
- **Only** data from low energy gravity is classical GR
  - CMB perturbations come from high energy **normal** QGR
- $\lim_{\hbar\to 0} \overline{R + C^2}$ not even a local, metric theory
  - We have a complete catalog of these & it's not there
  - Gives up the solar system, cosmology, black holes, causality . . .
- IF everything worked ➔ START with this and forget HDG

The Case of $f(R)$ Gravity

- HD in $\mathcal{L}_{GR} = R\sqrt{-g}$ is degenerate (partially integrate)
- But $\mathcal{L} = f(R)\sqrt{-g}$ has a nondegenerate HD
    - But no ghosts
- NOT a violation of Ostrogradsky's theorem!
    - $P_1 Q_2$ unbounded below, but also ABOVE
        - ➔ Higher derivative DoF has opposite KE wrt lower derivative DoF
    - $R = (g^{\rho\mu} g^{\sigma\nu} - g^{\rho\sigma} g^{\mu\nu})\partial_\mu \partial_\nu g_{\rho\sigma} +$ lower derivatives
        - Only one metric component carries ∂_0^2
    - This component (Newtonian potential) is a ghost in GR
        - But no problem because constrained
    - So Ostrogradsky **predicts** new $f(R)$ DoF has positive KE

Only Hope is Constraints

- Constraints compromise non-degeneracy
- But only so many gauge symmetries
  - Apply Ostrogradsky to gauge-fixed theory
- Could always try for ad hoc constraints
  - But at odds with interacting QFT
    - Same field carries both $\pm$ DoF's
  - Known cases reduce to lower derivative models
    - E.g., the HD SHO model

Is there **no** way to make sense of higher derivative theories?

- I cannot prove a negative
  - But no one has ever found a legitimate way
- But I urge the application of common sense
  - In 332 years since Newton wrote $\ddot{x} = f(x, \dot{x})$
    - No one has **ever** discovered fundamental HD's
  - Bizarre if this was just an accident
- Perhaps Ostrogradsky's result explains it!

Lessons from Pop Culture

- "You can't always get what you want"
  - Face it: C^2 just isn't viable as a fundamental theory
- "But if you try, sometimes you just might find, that you get what you need"
  - $C\ln(\Box)C$ occurs in Γ_{1loop}
  - Coefficient finite & fixed
  - Stronger in the IR than C^2

Conclusions

- Ostrogradsky Thm is the strongest constraint on fundamental theory
- Need to distinguish effective field theory from fundamental theory
  - Fundamental ghosts present at *all* scales
  - Nonlocal EFT effects stronger than local
- Alternate quantization schemes discard the Correspondence Principle
  - This is not acceptable for gravity!