Universal relations for neutron stars with modern microscopic equations of state of nuclear matter

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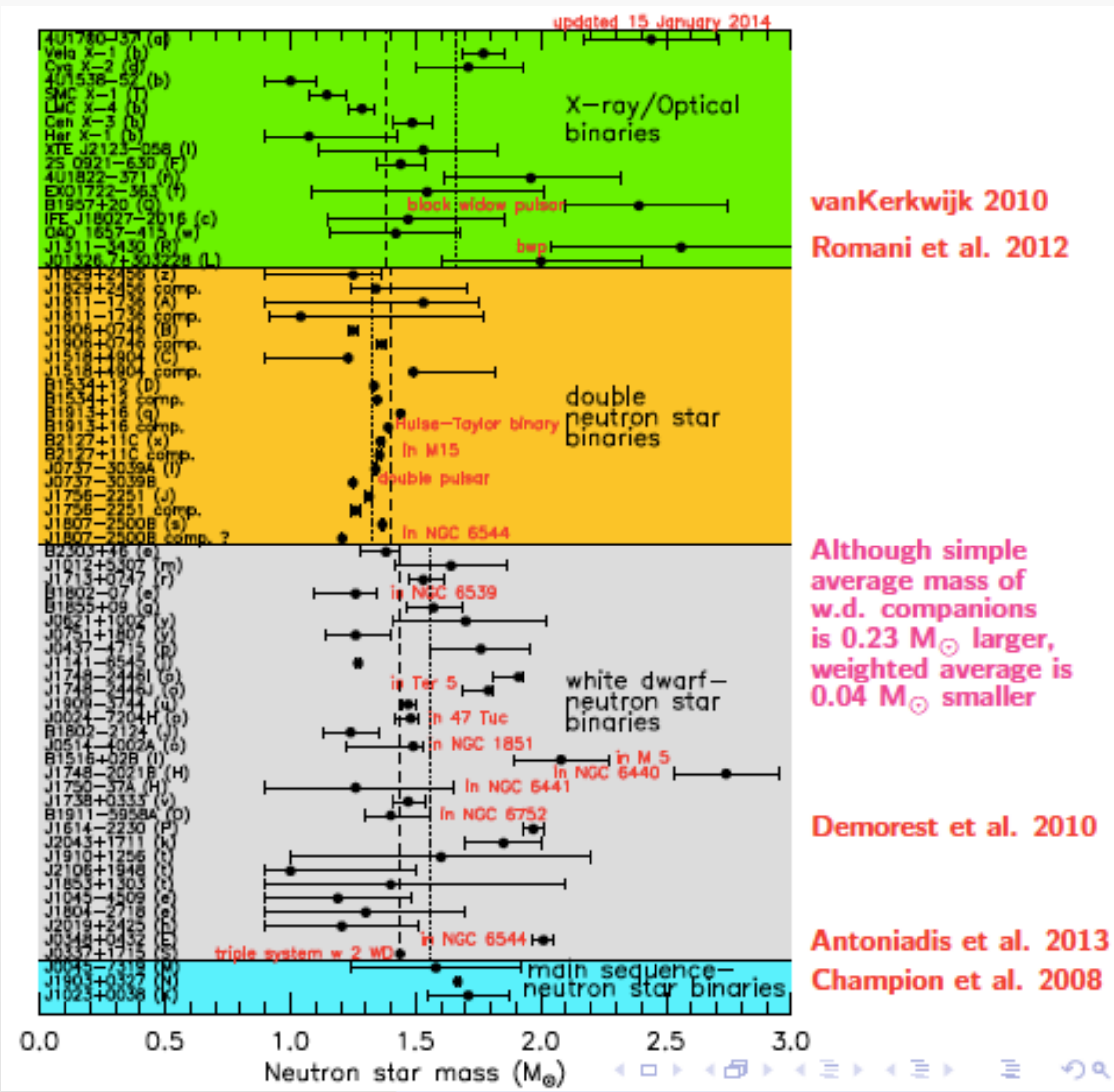
based on arXiv:1809.04315

Virtual Institute of Astroparticle Physics (VIA), Paris, December 7th, 2018

Neutron Stars in a Nutshell

J. Lattimer



- Highly relativistic objects
    - Typical masses $1<M<2\ M_0$

      $M_0 \approx 2\text{x}10^{33}$ g
    - Typical radius $10<R<14$ km … probably smaller …
    - Huge compactness

      $M/R \approx 0.15(M/M_0)\ (R/10\ \text{km})^{-1}$
- Average density $\simeq 4 \text{ x } 10^{14}$ g cm^{-3}

  ~a few times the nuclear density at saturation

  $\rho_0 \sim 2\text{x}10^{14}$ g cm^{-3}

Schematic view of a neutron star

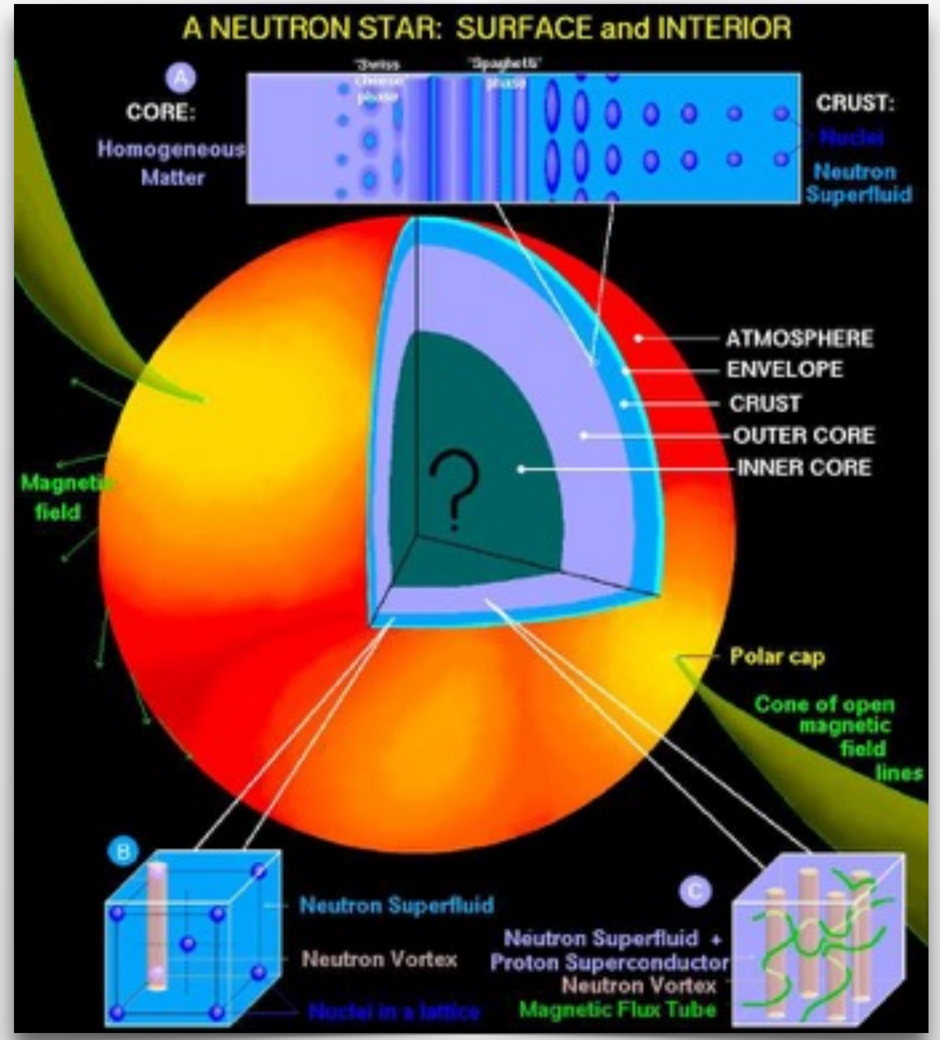
1 km

Outer crust. ^{56}Fe ions immersed in an electron gas.

Inner crust. Electrons are beta-captured by nuclei, —> more and more neutron-rich —> neutron drip. A gas of free neutron along with electrons and nuclei. At increasing density, nuclear matter sets in.

Outer core. Asymmetric nuclear matter above saturation density. Mainly composed by neutrons, protons, electrons and muons. Its exact composition depends on the nuclear matter Equation of State (EoS).

Inner core. The most unknown region. "Exotic matter" . Hyperons ? Kaons ? Quarks ?



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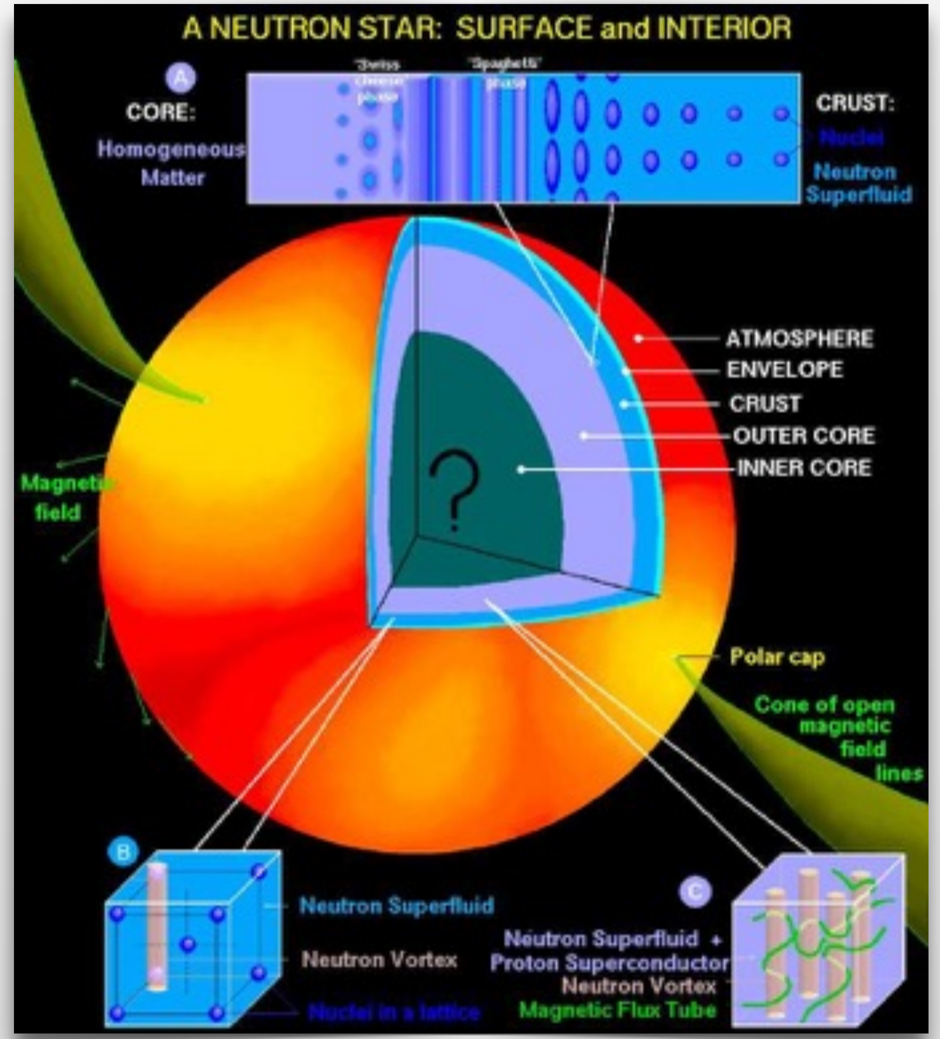
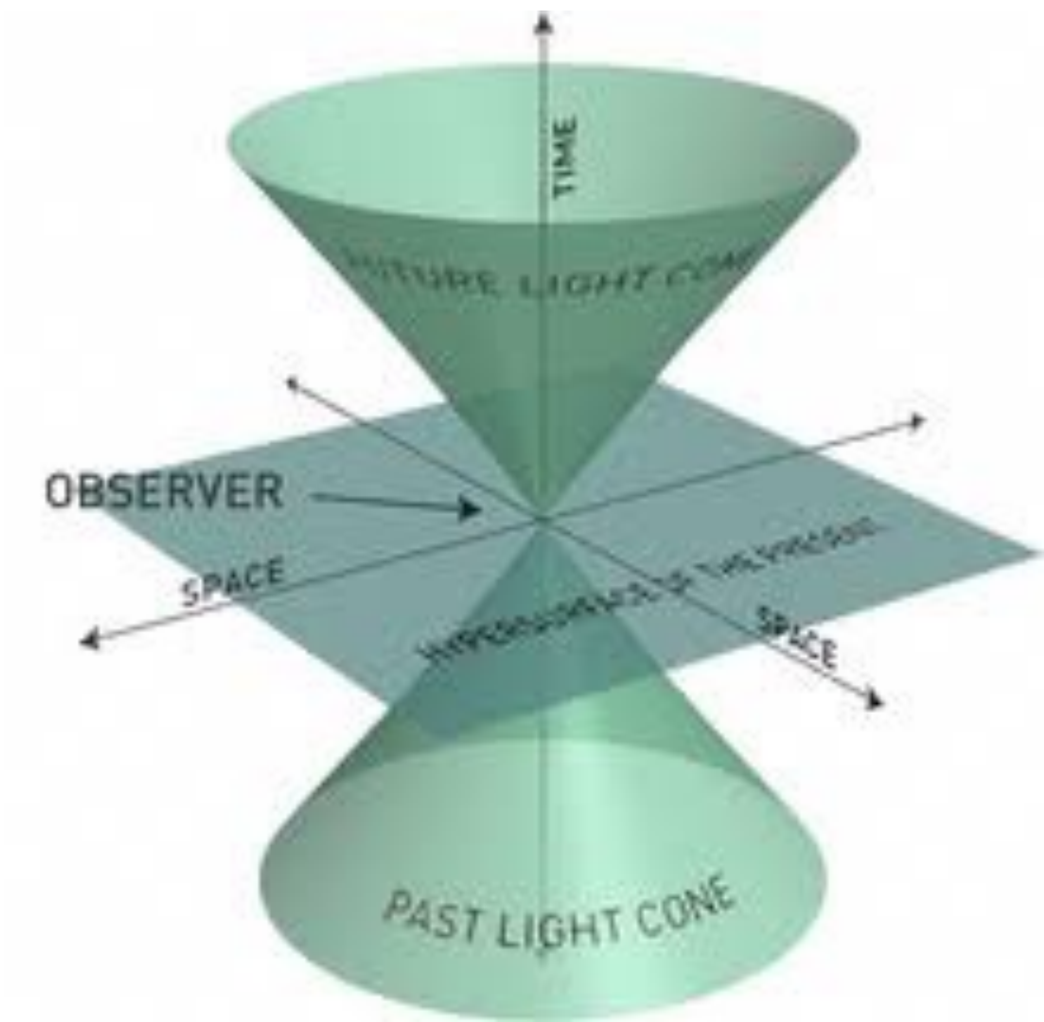


☞ **EoS in the crust is known reasonably well.**

☞ **EoS in the outer core is not very certain.**

☞ **EoS in the inner core is a mystery.**

Neutron Star Structure Equations



$$\boxed{R_{ik} - \frac{1}{2} g_{ik} R = \frac{8\pi G}{c^4} T_{ik}}$$ **Einstein Equations**

R_{ik} = Ricci curvature tensor; $R = R_i^i$ = scalar curvature

$T_{ik} = (P+E)\, u_i u_k - P\, g_{ik}$ = energy-momentum tensor

($E = \rho c^2$, u^i = 4-velocity, g_{ik} = metric tensor)

Einstein Equations for a Star

$$\begin{cases}
(1) \quad \dfrac{dP}{dr} = -\dfrac{G\rho m}{r^2}\left(1+\dfrac{P}{\rho c^2}\right)\left(1+\dfrac{4\pi r^3 P}{mc^2}\right)\left(1-\dfrac{2Gm}{rc^2}\right)^{-1} \\
(2) \quad \dfrac{dm}{dr} = 4\pi r^2 \rho \\
(3) \quad \dfrac{d\Phi}{dr} = -\dfrac{1}{\rho c^2}\dfrac{dP}{dr}\left(1+\dfrac{P}{\rho c^2}\right)^{-1}
\end{cases}$$

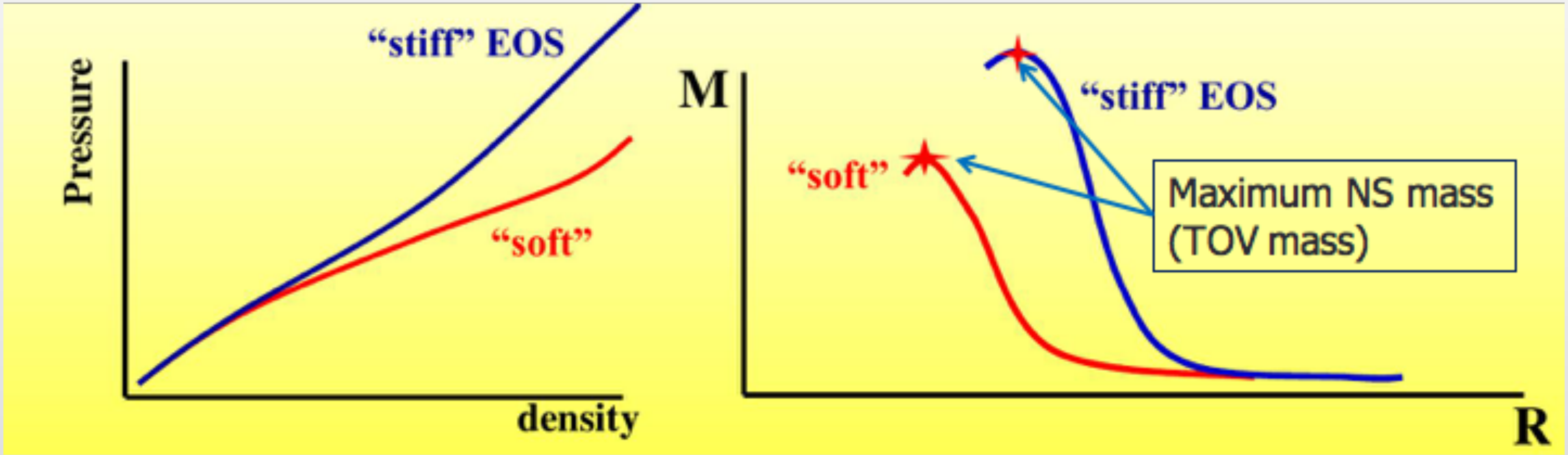
Tolman-Oppenheimer-Volkoff (1939)

P=P(ρ) is the only input needed !

Imposing boundary conditions : $\begin{matrix} \rho(0) = \rho_c \\ m(0) = 0 \end{matrix} \Rightarrow \begin{matrix} R,\ p(R) = 0 \\ M = m(R) \end{matrix}$

Need an Equation of State
$P = P(\rho)$

Given an EoS, compute sequence of equilibrium models:
the Mass-Radius relation



The construction of the EoS : two possible philosophies

Phenomenological approaches

Based on effective density-dependent NN force with parameters fitted on nuclei properties.

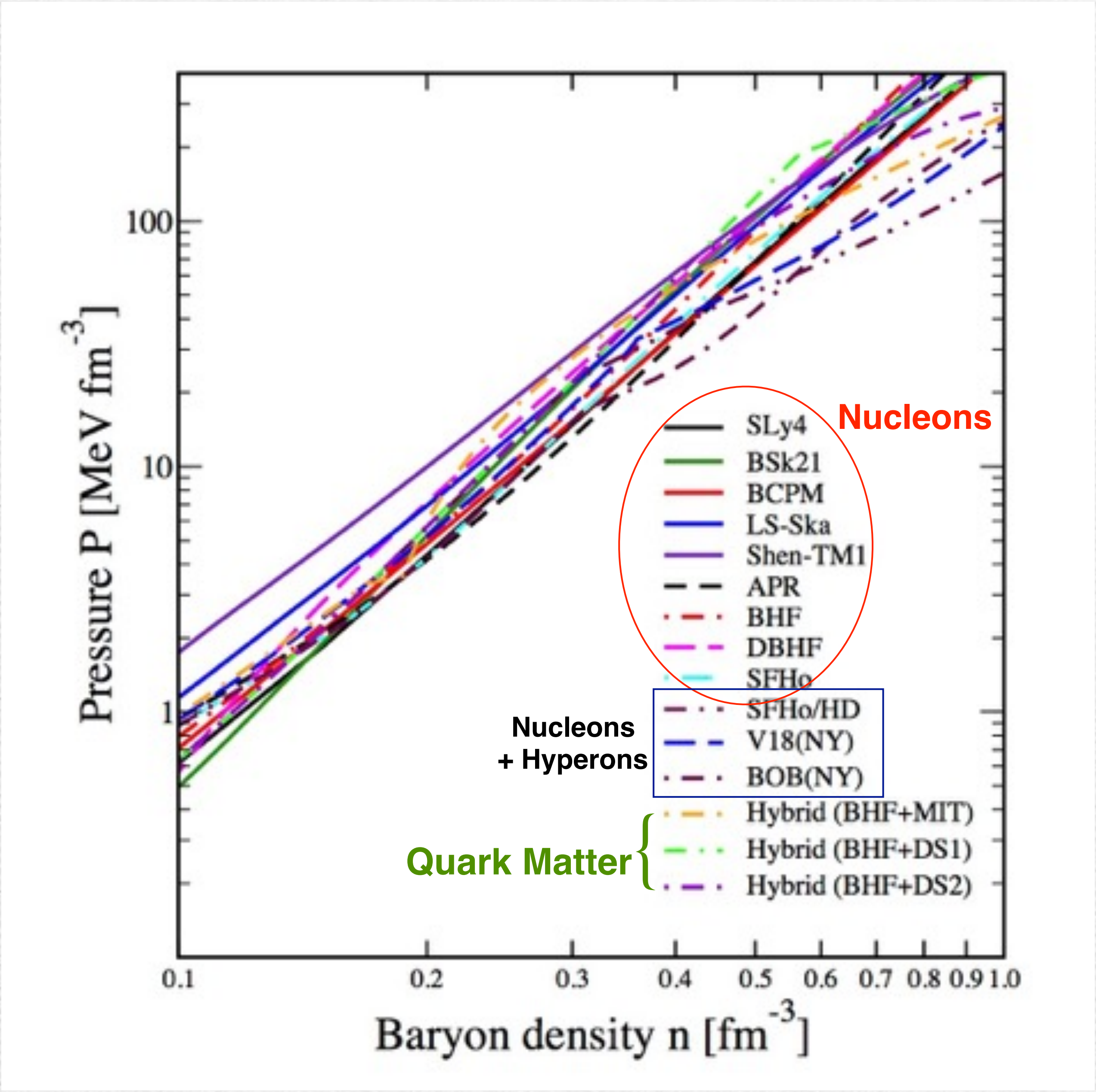
- Liquid Drop models
  - ✧ BPS Baym et al, *ApJ* ***170****, 299 (1971)*
  - ✧ BBP Baym et al., *NPA* ***175****, 225 (1971)*
  - ✧ LS Lattimer&Swesty, *NPA* ***535****, 331 (1991)*
  - ✧ DH Douchin&Haensel, *A&A* ***380****, 151 (2001)*
- TF + RMF
  - ✧ Shen et al., *NPA* ***637****, 435 (1998)*
- ETFSI + Eff. Skyrme force
  - ✧ BSk Goriely et al.,*PRC* ***82****, 035804 (2010)*
- Hartree-Fock
  - ✧ NV Negele&Vautherin, *NPA* ***207****, 298 (1973)*
  - ✧ RMF Serot&Walecka, *Adv. NP 16, 1 (1986)*
  - ✧ RHF Boussy et al., *PRL* ***55****, 1731 (1985)*
  - ✧ QMC Guichon et al., *NPA* ***814****, 66 (2008)*
- Statistical models
  - ✧ NSE Raduta&Gulminelli. *PRC* ***82****, 065801 (2010)*
  - ✧ HS Hempel&Schaffner-Bielich, *NPA* ***837****, 210 (2010)*

Ab initio approaches

The nuclear problem is solved starting from the two- and three-body realistic nucleon interaction.

- Diagrammatic
  - ✧ BBG Day, *RMP****39****, 719 (1967)*
  - ✧ SCGF Kadanoff&Baym, Quantum Statistical Mechanics (1962)
  - ✧ DBHF Ter Haar&Malfiet, *Phys. Rep.* ***149****, 207 (1987);*
- Variational
  - ✧ APR *Akmal et al.*, *PRC* ***58****, 1804 (1998)*
  - ✧ FHNC Fantoni&Rosati, *Nuovo Cimento A****20****, 179 (1974)*
  - ✧ CBF Fabrocini&Fantoni, *PLB* ***298****, 263(1993)*
  - ✧ LOCV Owen et al., *NPA* ***277****, 45 (1978)*
- Monte Carlo
  - ✧ VMC Wiringa, *PRC****43****, 1585 (1991)*
  - ✧ GFMC Carlson, *PRC****68****, 025802 (2003)*
  - ✧ AFDMC Schmidt&Fantoni, *PLB****446****, 99 (1999)*

A large set of possible EoS (nucleons, hyperons, quark matter …)



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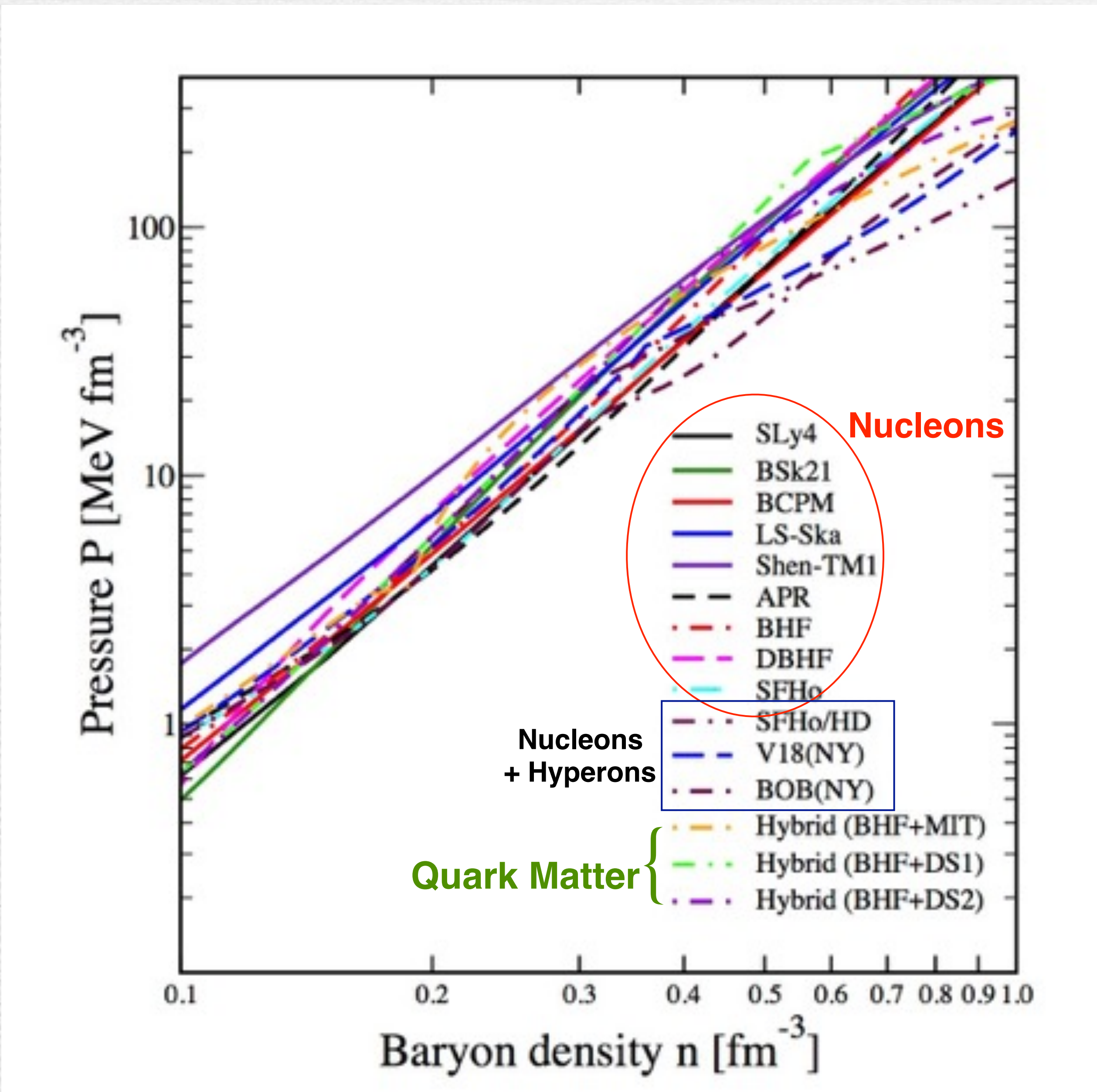
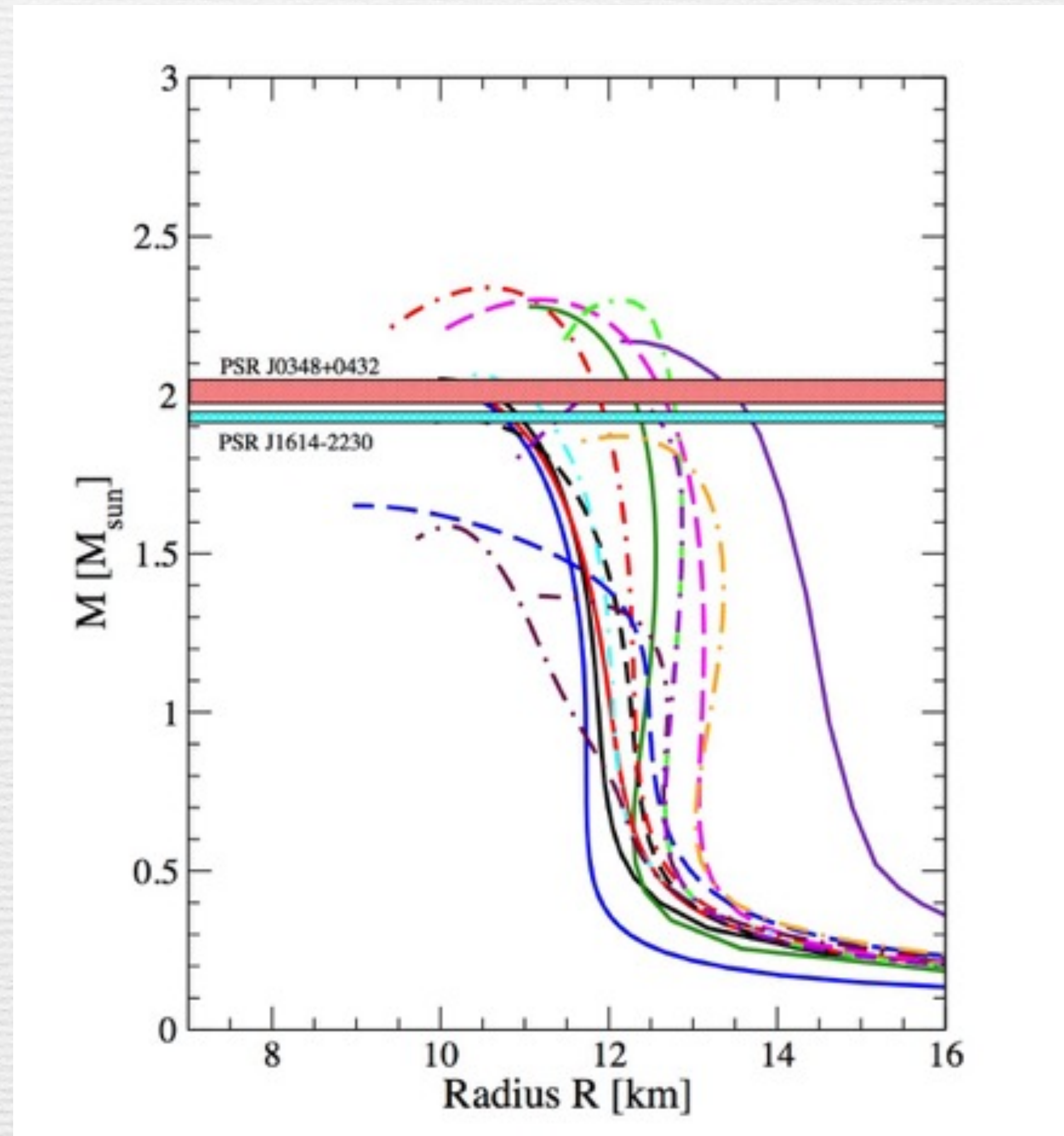
A large variety of mass-radius relations

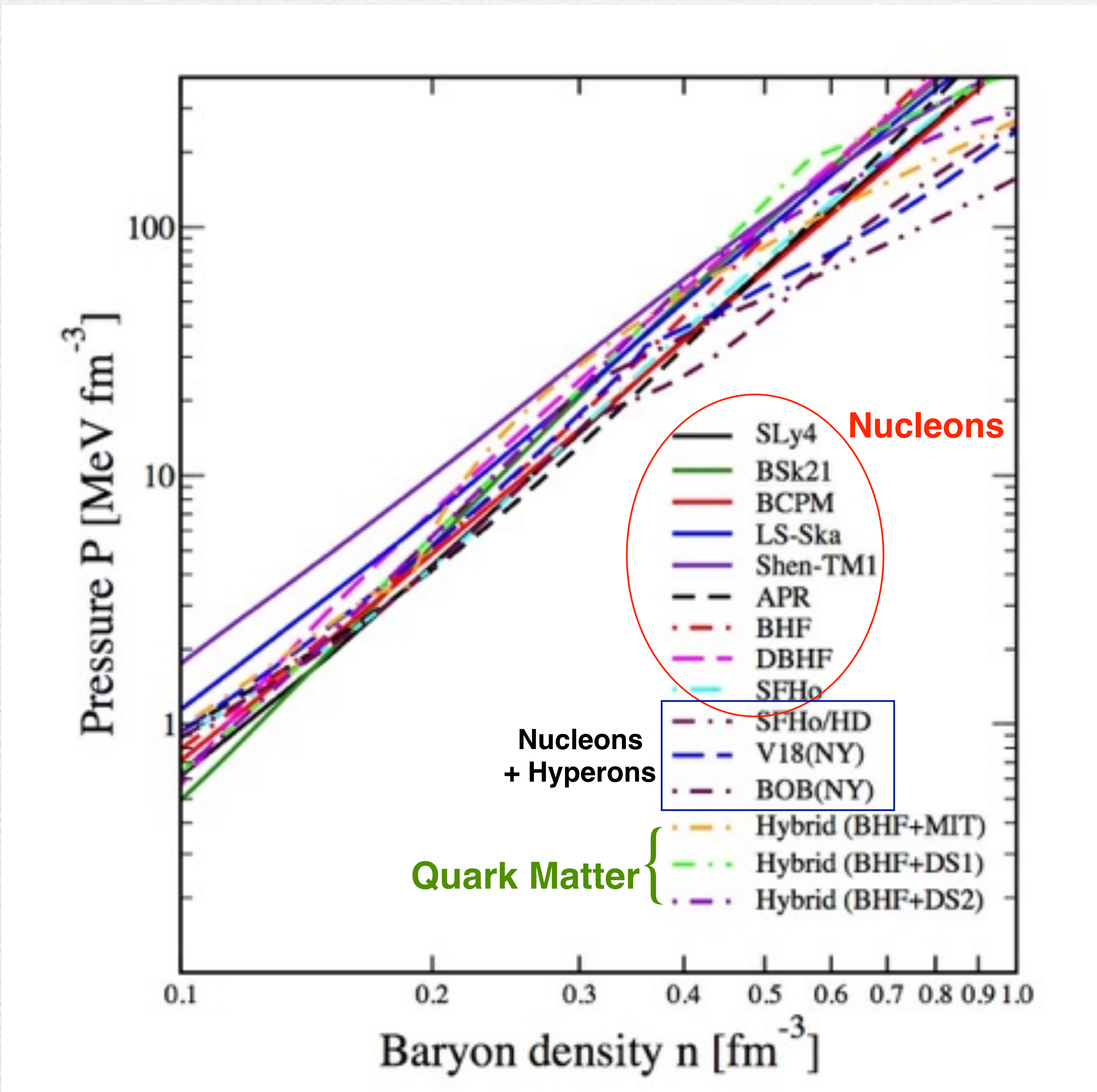


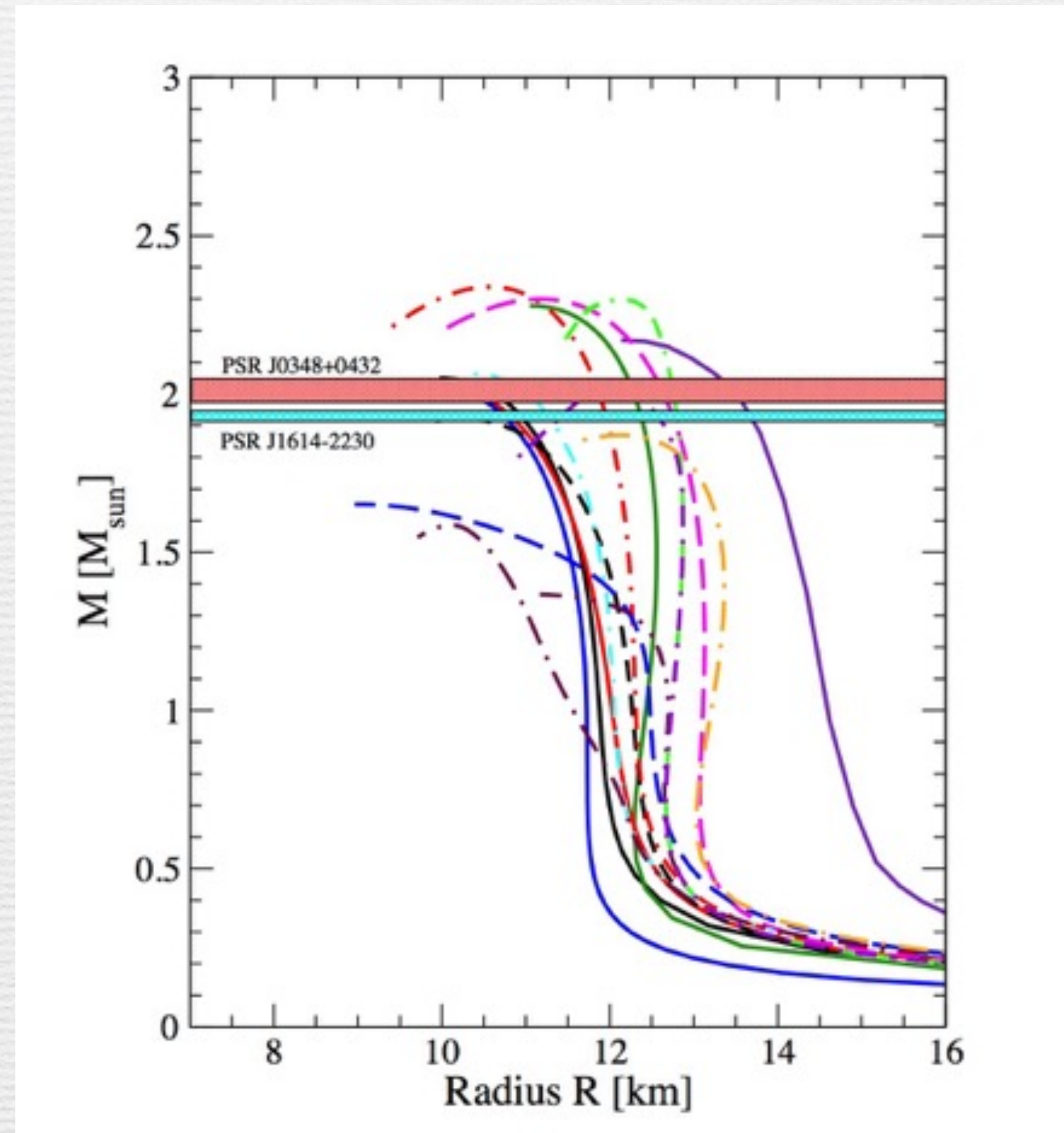


A large set of possible EoS (nucleons, hyperons, quark matter …)

☞

A large variety of mass-radius relations

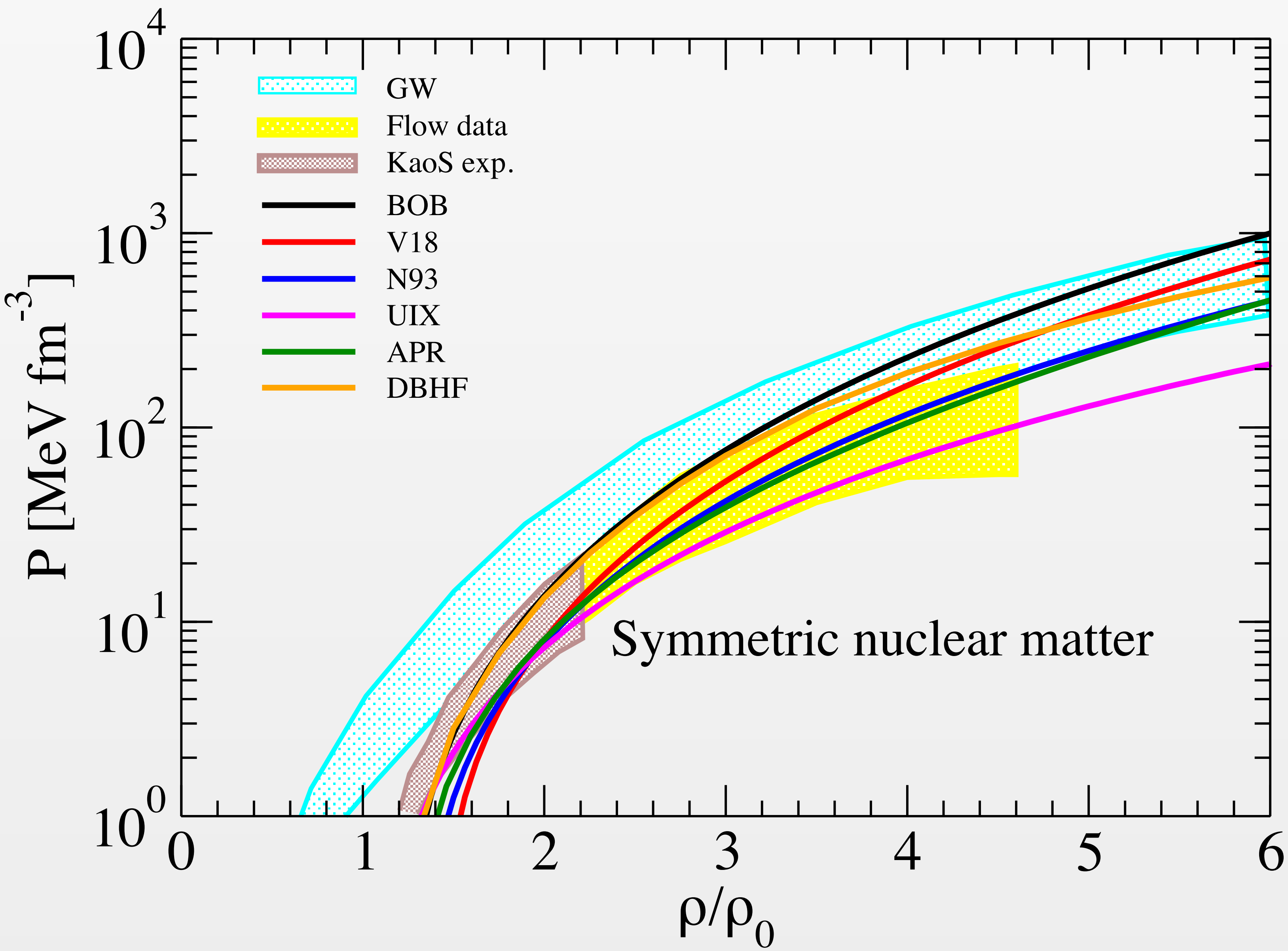




More in F. B. and A. Fantina,
White Book of the NewCompstar Action,
arXiv:1804.03020

SUPRA-SATURATION DENSITY : CONSTRAINTS FROM HEAVY ION REACTIONS and GWs

- Transverse flow measurements in Au + Au collisions at E/A=0.5 to 10 GeV
- Pressure determined from simulations based on the Boltzmann-Uehling Uhlenbeck transport theory
- Flow data exclude very repulsive and very soft equations of state
- GW data complementary to heavy ion data



Heavy ion data from P. Danielewicz, Science (2002)
GW data from Abbott et al., PRL (2018)

The EoS : where do we stand ?

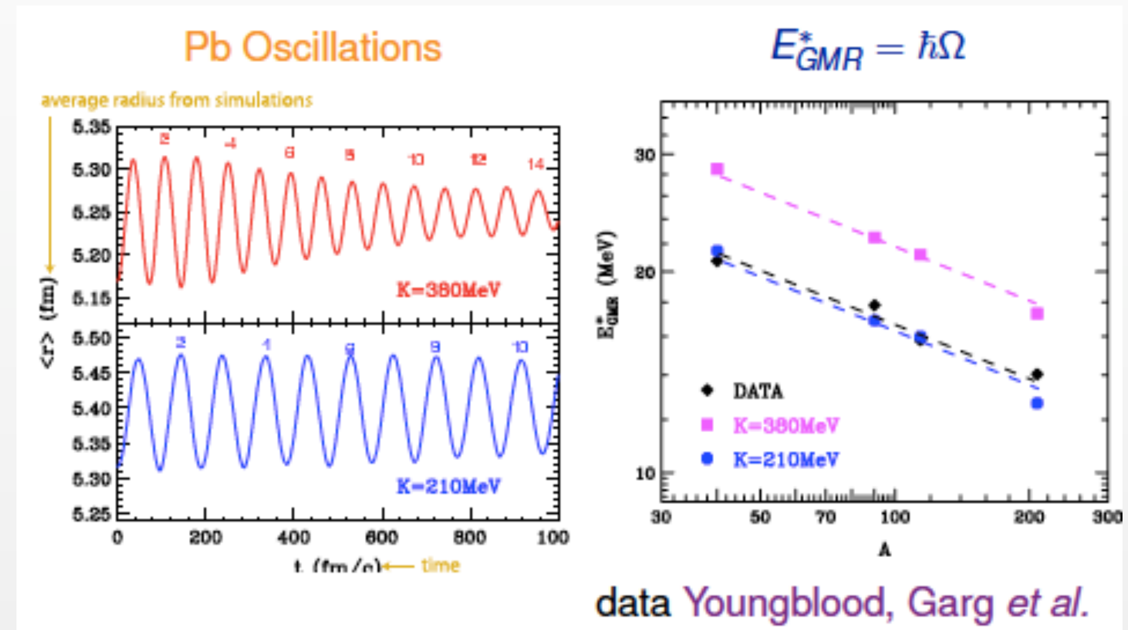
Close to saturation density $\rho_0 = 2 \times 10^{14} g/cm^3$

- ☞ Structure properties of about 3339 nuclides.
- ☞ Compressibility from Giant Monopole Resonance.

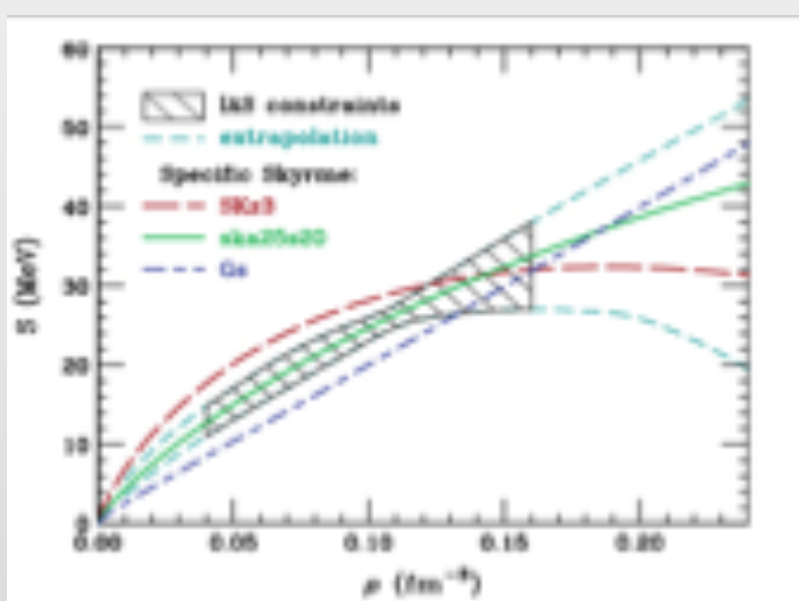
J.P. Blaizot, (1980), $K_\infty = 210 \pm 30\,\mathrm{MeV}$

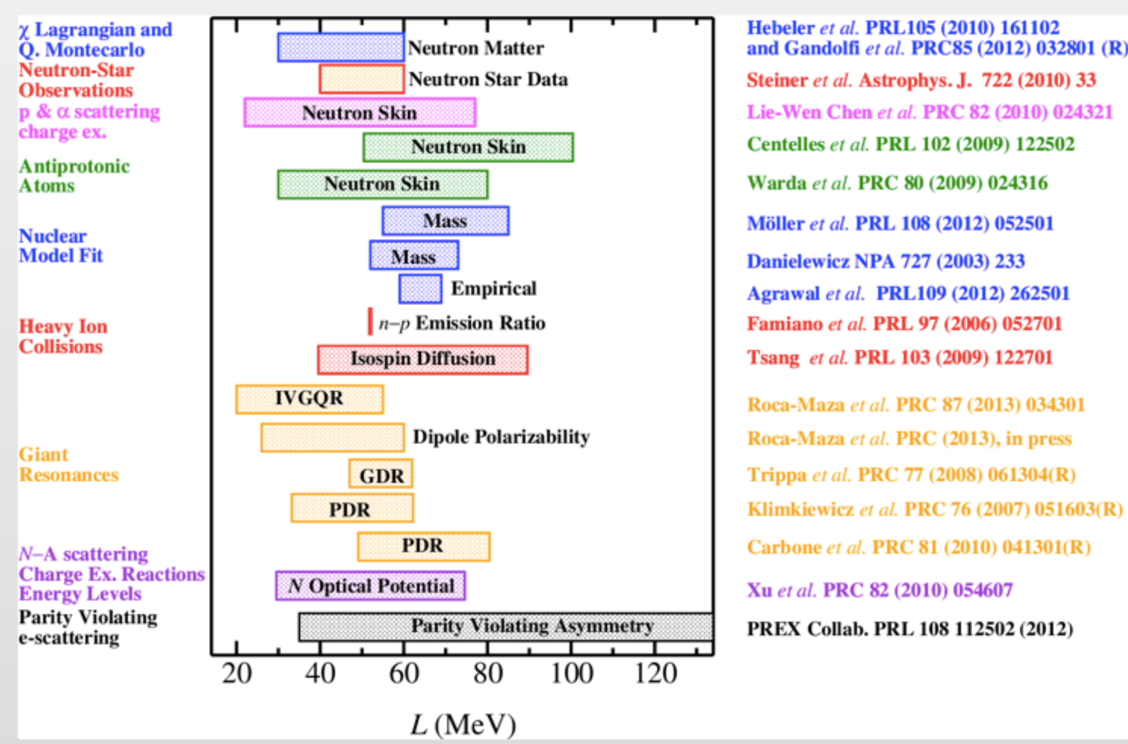
G. Colo'et al., (2004), $K_\infty = 240 \pm 10\,\mathrm{MeV}$

J. Piekarewicz, (2004), $K_\infty = 248 \pm 8\,\mathrm{MeV}$



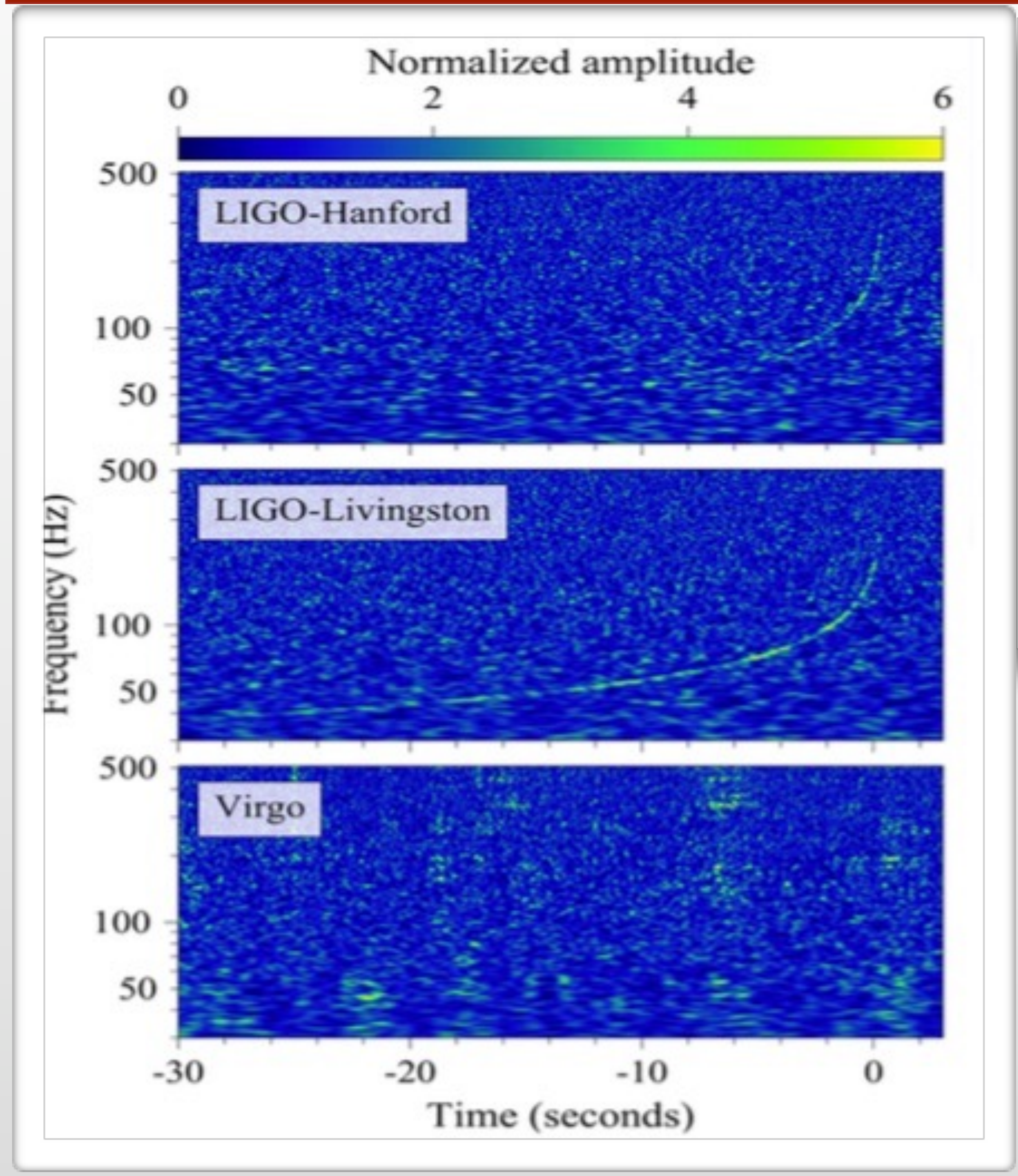
- ☞ Symmetry energy S_0 and its slope L.

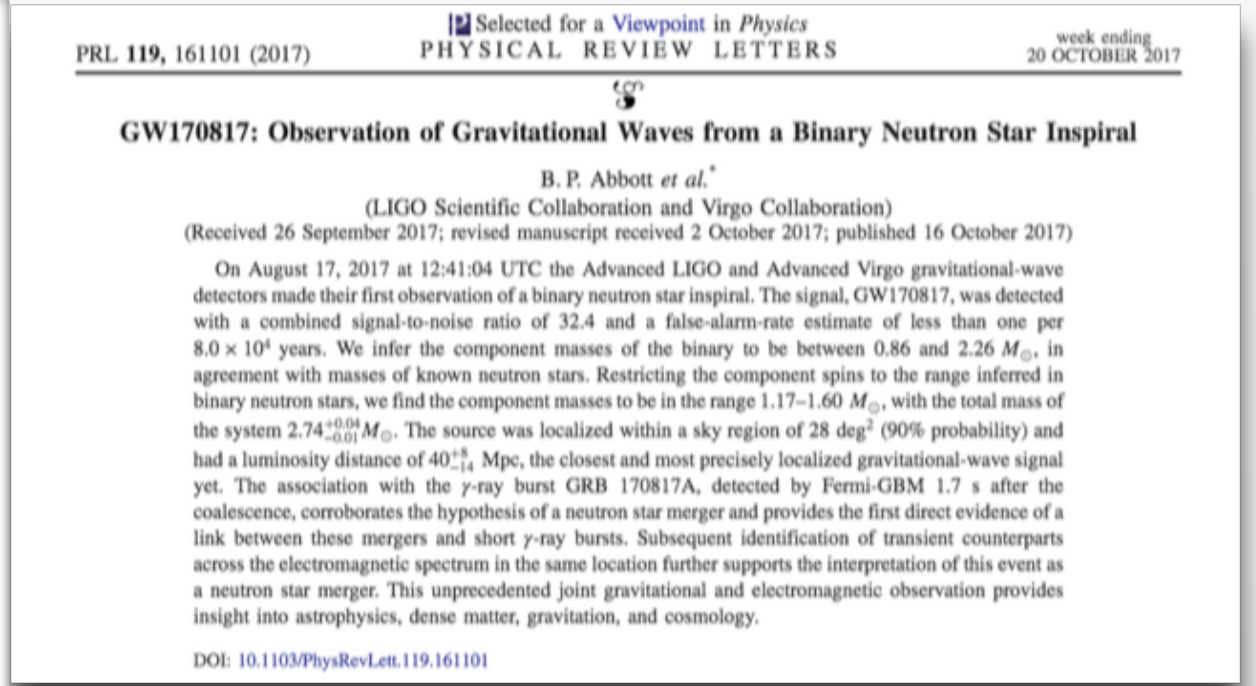




The dawn of multi-messenger astronomy

On August 17, 2017, the LIGO-VIRGO detector network
observed a gravitational-wave signal from the inspiral of
two low-mass compact objects consistent with a binary neutron star (BNS) merger.





Selected for a Viewpoint in *Physics*

PRL **119,** 161101 (2017) PHYSICAL REVIEW LETTERS week ending 20 OCTOBER 2017

GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral

B. P. Abbott *et al.**

(LIGO Scientific Collaboration and Virgo Collaboration)

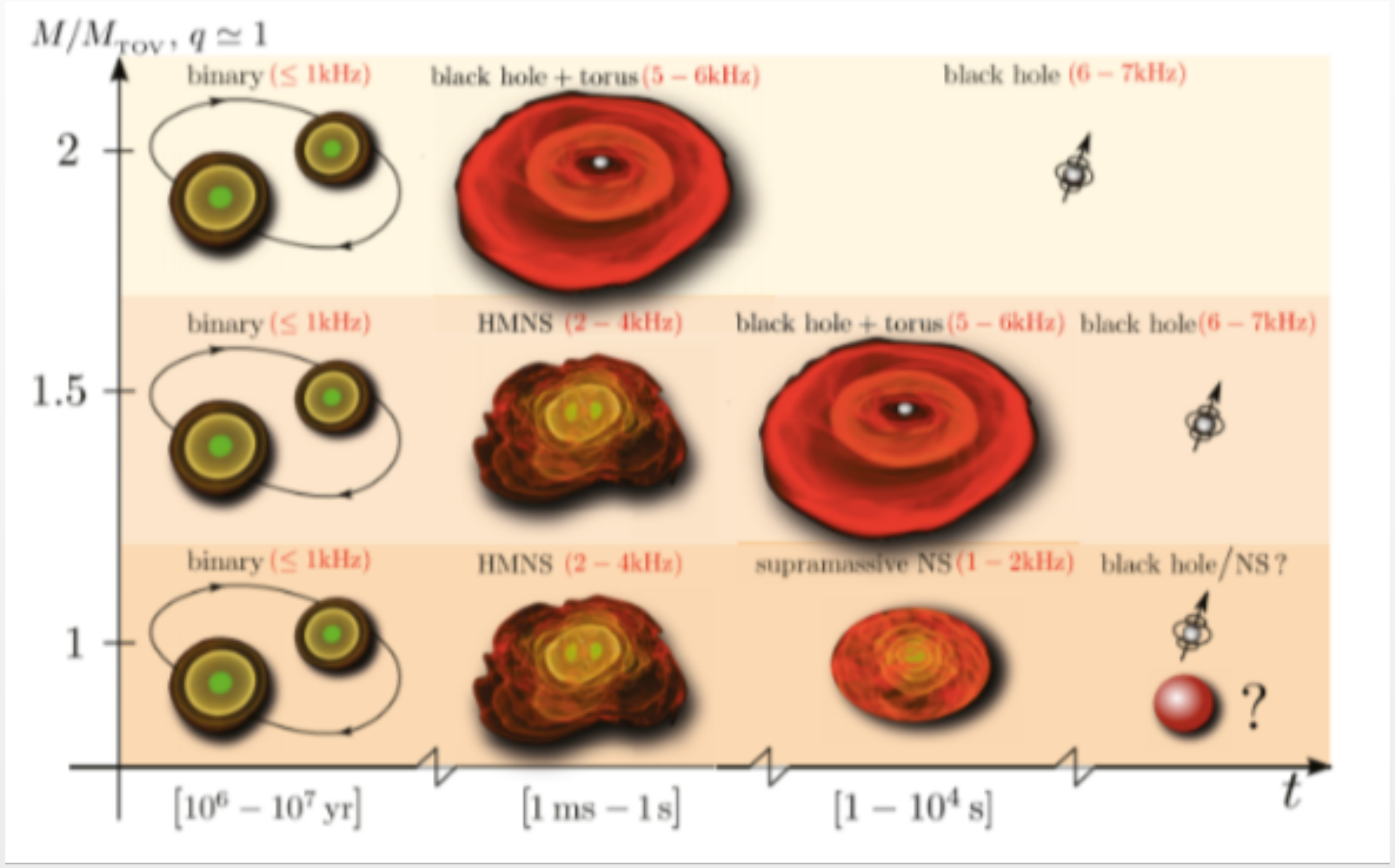
(Received 26 September 2017; revised manuscript received 2 October 2017; published 16 October 2017)

On August 17, 2017 at 12:41:04 UTC the Advanced LIGO and Advanced Virgo gravitational-wave detectors made their first observation of a binary neutron star inspiral. The signal, GW170817, was detected with a combined signal-to-noise ratio of 32.4 and a false-alarm-rate estimate of less than one per 8.0×10^4 years. We infer the component masses of the binary to be between 0.86 and 2.26 $M_\odot$, in agreement with masses of known neutron stars. Restricting the component spins to the range inferred in binary neutron stars, we find the component masses to be in the range 1.17–1.60 $M_\odot$, with the total mass of the system $2.74^{+0.04}_{-0.01} M_\odot$. The source was localized within a sky region of 28 deg^2 (90% probability) and had a luminosity distance of 40^{+8}_{-14} Mpc, the closest and most precisely localized gravitational-wave signal yet. The association with the γ-ray burst GRB 170817A, detected by Fermi-GBM 1.7 s after the coalescence, corroborates the hypothesis of a neutron star merger and provides the first direct evidence of a link between these mergers and short γ-ray bursts. Subsequent identification of transient counterparts across the electromagnetic spectrum in the same location further supports the interpretation of this event as a neutron star merger. This unprecedented joint gravitational and electromagnetic observation provides insight into astrophysics, dense matter, gravitation, and cosmology.

DOI: 10.1103/PhysRevLett.119.161101

About 60 groups/collaborations participated to the investigations of GW170817, GRB170817A, AT2017fgo

Role of the EoS during NS-NS merger



- **Inspiral** decay of the orbital separation with progressive reduction of the orbit. GW emission. Strong tidal forces depending on the compactness M/R, i.e. EoS.
- **Merger** Duration and fate depend on EoS and total mass. Stiffer EoS —-> larger supported mass —-> collapse to BH delayed or avoided.
- **Post-merger** Remnant size and frequency of the dominant oscillation mode dependent on the EoS.

☞ NS mergers as valuable probe for testing the EoS !!!!

L. Baiotti and L. Rezzolla, Rep. Prog. Phys. (2017), arXiv:1607.03540

Inspiral phase of GW170817 :

Tidal deformability λ and Love numbers

The Newtonian Theory of Tides :

The Love numbers were introduced by August E. H. Love in 1911 : they are a set of dimensionless parameters which measure the rigidity of a planetary body and show how its shape changes in response to an external tidal potential.

These numbers can be generalized for stars in General Relativity.
In particular, we are interested in one of these numbers, which connects the tidal field with the quadrupolar deformation of the star.

The Love number k_2

Solve in GR together with the TOV eqs. for the pressure p and the enclosed mass m

$$k_2 = \frac{8}{5} \frac{\beta^5 z}{6\beta(2-y_R) + 6\beta^2(5y_R - 8) + 4\beta^3(13 - 11y_R) + 4\beta^4(3y_R - 2) + 8\beta^5(1+y_R) + 3z\log(1-2\beta)},$$

$$z \equiv (1-2\beta^2)[2 - y_R + 2\beta(y_R - 1)]$$

$$\frac{dp}{dr} = -\frac{m\epsilon}{r^2} \frac{(1+p/\epsilon)(1+4\pi r^3 p/m)}{1-m/r},$$

$$\frac{dm}{dr} = 4\pi r^2 \epsilon,$$

ε being the mass-energy density

$$\frac{dy}{dr} = -\frac{y^2}{r} - \frac{y-6}{r-2m} - rQ,$$

$$Q \equiv 4\pi \frac{(5-y)\epsilon + (9+y)p + (\epsilon+p)/c_s^2}{1-2m/r} - \left[\frac{2(m+4\pi r^3 p)}{r(r-2m)}\right]^2,$$

with $c_s^2 = d\epsilon/dp$ and the EOS $\epsilon(p)$ as input.

$$k_2 = \frac{3G}{2R^5}\lambda$$

The Love number k_2
depends crucially on the compactness β=M/R, hence on the EoS.

Abbott et al., PRL 119, 161101 (2017)

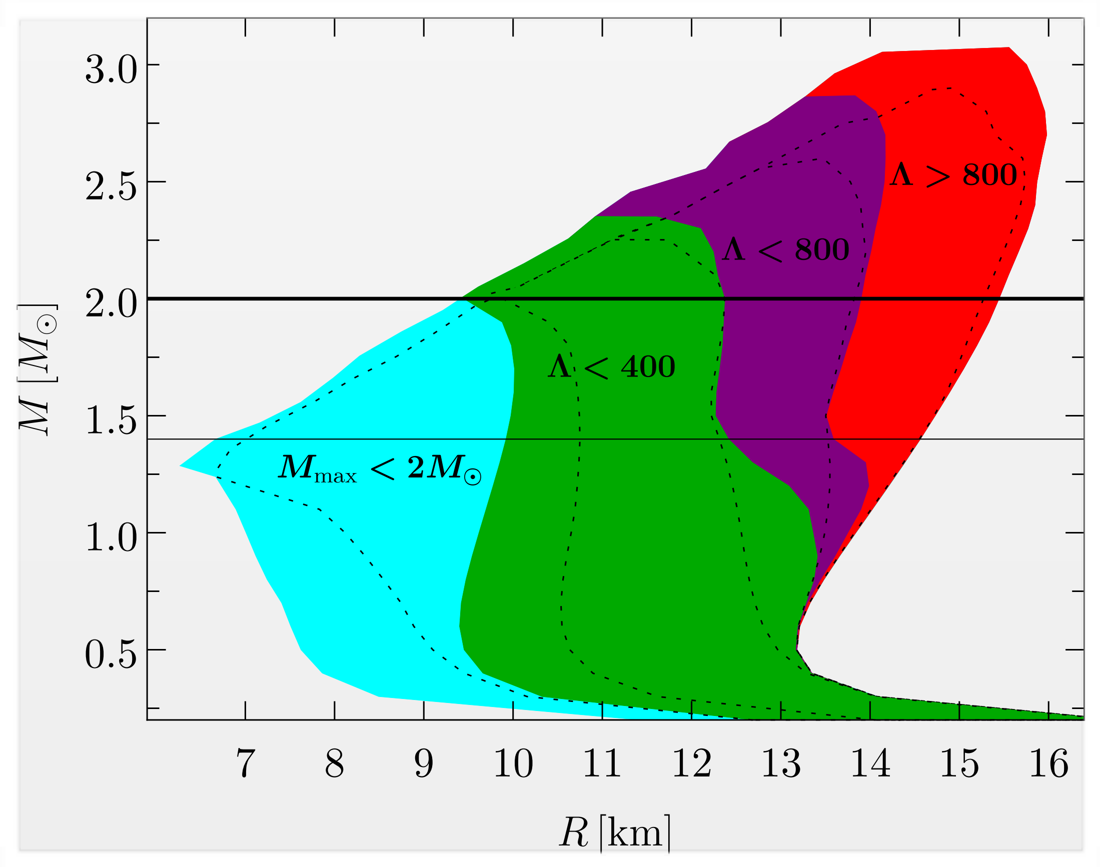
Constraints from GW170817 and the kilonova signal AT2017gfo:

The tidal deformability $\Lambda=\lambda/M^5$

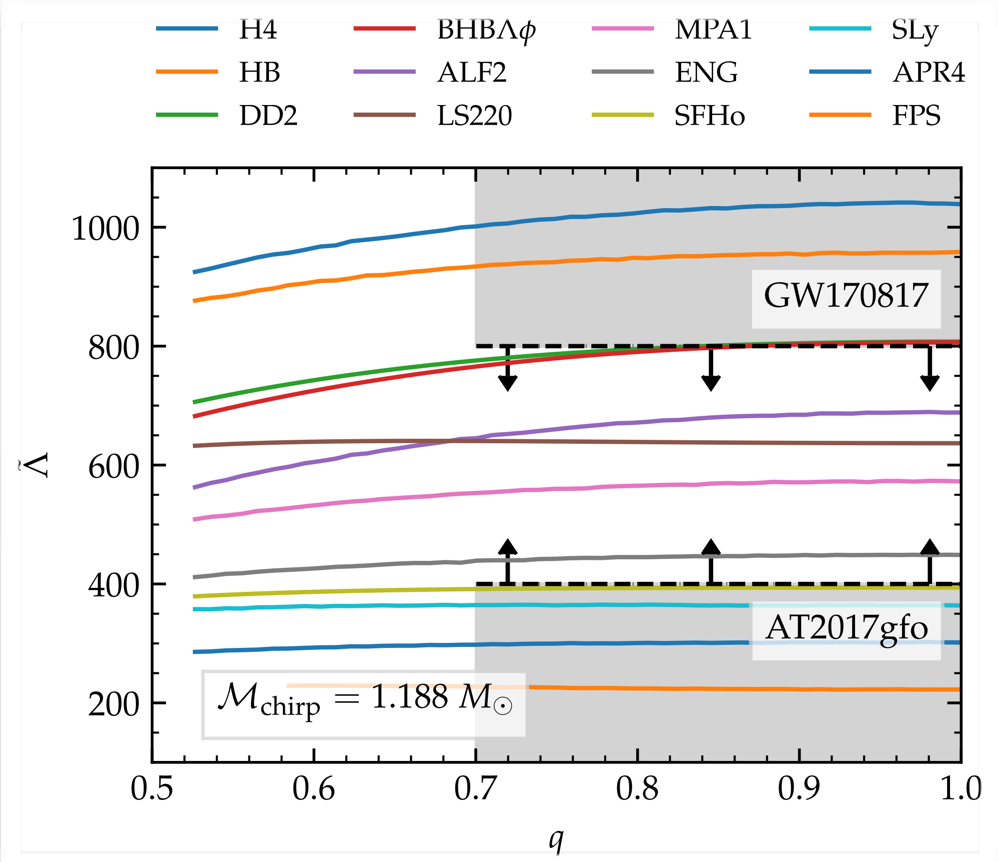
$\Lambda_{1.4} < 800$ at 90% confidence level

$$\tilde{\Lambda} = \frac{16}{13}\left\{\frac{(M_1 + 12M_2)M_1^4}{(M_1+M_2)^5}\Lambda_1 + (1 \leftrightarrow 2)\right\} > 400$$

Annala et al., PRL 120, 172703 (2018)



Radice et al., ApJ 852, L29 (2018)



- Very stiff EoS are excluded (large radii)
- Limit for the radius $R_{1.4} < 13.6$ km

Most et al., arXiv:1803.00549
Lim et al., arXiv:1803.02803

$R_{1.4} \gtrsim 12$ km

Fattoyev, PREX experiment (neutron skin), PRL 108, 112502 (2012)

Summary (I)

- Observational data about masses : larger than 2 solar mass
- Constraints from low energy nuclear collisions close to saturation density : soft EoS
- Constraints from heavy ion reactions close and slightly above saturation density : not too stiff EoS
- Controversial observational data about radii (strong dependence on the model for atmosphere)
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All data taken together are not yet constraining enough the Equation of State

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Likely measurement of the moment of inertia $I \propto MR^2$, PSR J0737−3039A
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How to relate M, R, I, Λ … ?

I-Love-Q Universal relations for Neutron Stars

- Universal relations have been shown to exist which establish a link among various quantities, in particular the NS spin-induced quadrupole moment, moment of inertia, and tidal deformability.

These three quantities give interesting physical information:

- the moment of inertia quantifies how fast a NS can spin assumining a fixed angular momentum
- the spin-induced quadrupole moment describes the NS shape deviates from sphericity due to rotation
- the Love number quantifies how much deformable the NS is.

K. Yagi, N. Yunes, Science 241, 2013;   PRD 88, 2013

- Universal means that these relations are independent on the Equation of State of matter in the neutron star interior.

- Since the I-Love-Q relations are EoS independent, measuring one member of the trio gives information on the other two, even if not accessible by observations.

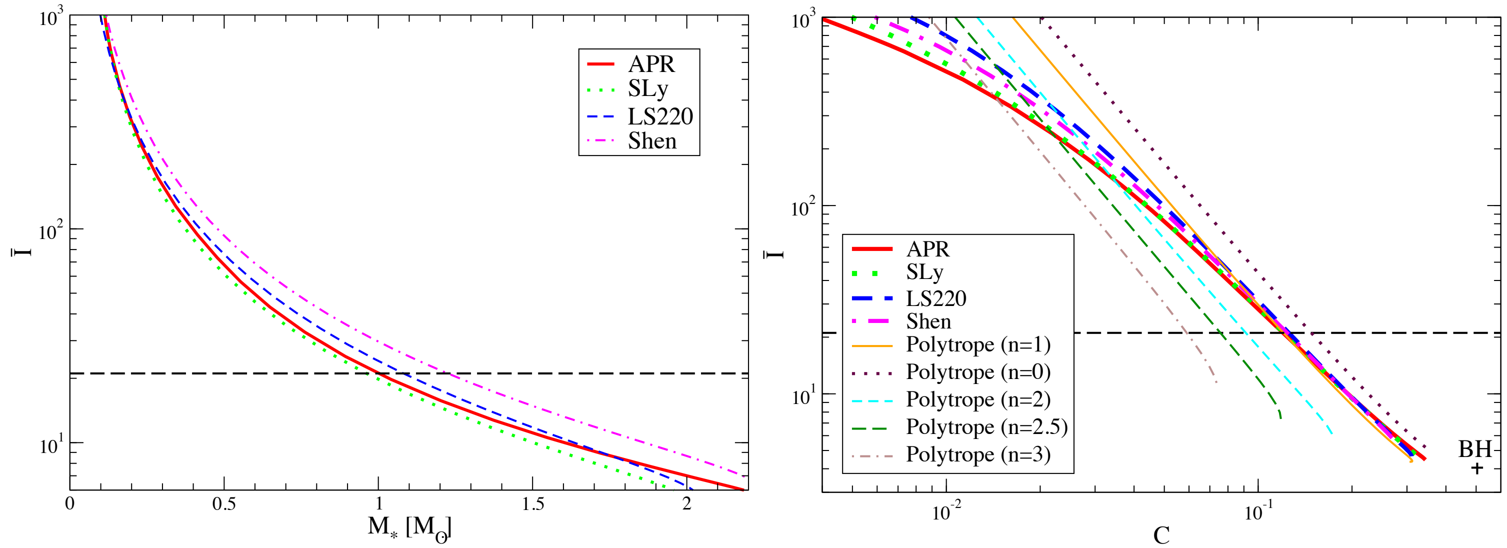

FIG. 6. (Color Online) Dimensionless moment of inertia $\bar{I}$, defined in Eq. (26), as functions of M_* (left) and C (right) for various EoSs. The horizontal dashed lines at $\bar{I} = 21.1$ correspond to $M_* = 1M_\odot$ for the APR EoS; NSs below this line have higher M_* and C. The solid cross indicates the value of $\bar{I}$ for a BH. Observe that the $\bar{I}$ curves for realistic EoSs approach each other as C increases, and moreover they approach the BH limit $\bar{I}_{\rm BH} = 4$ as $C \to 0.5$.

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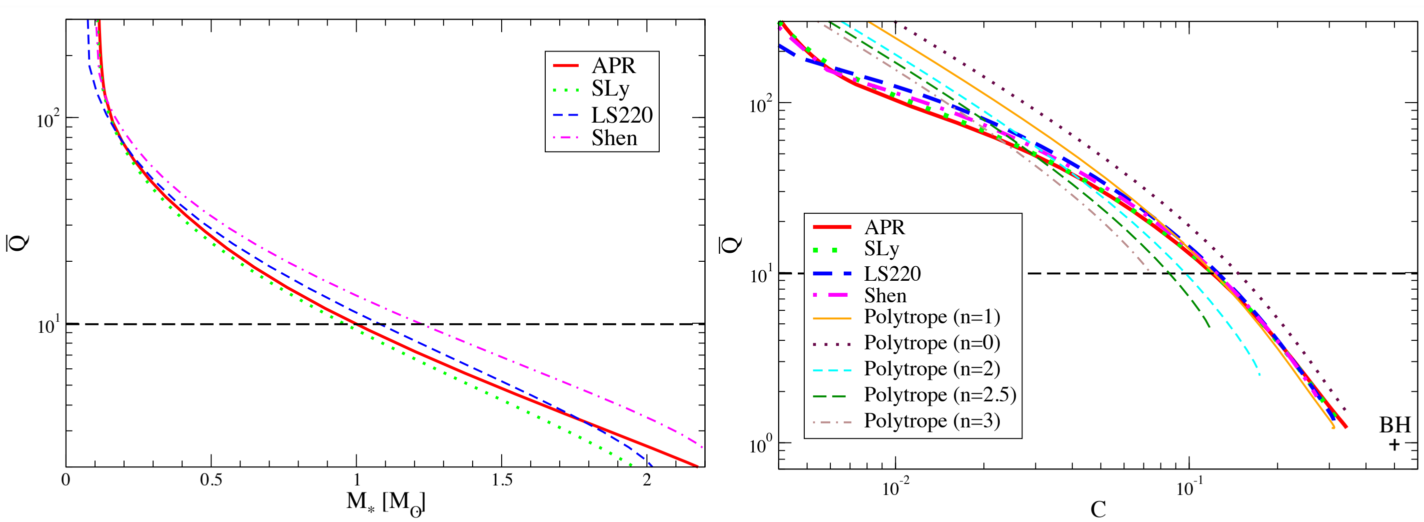


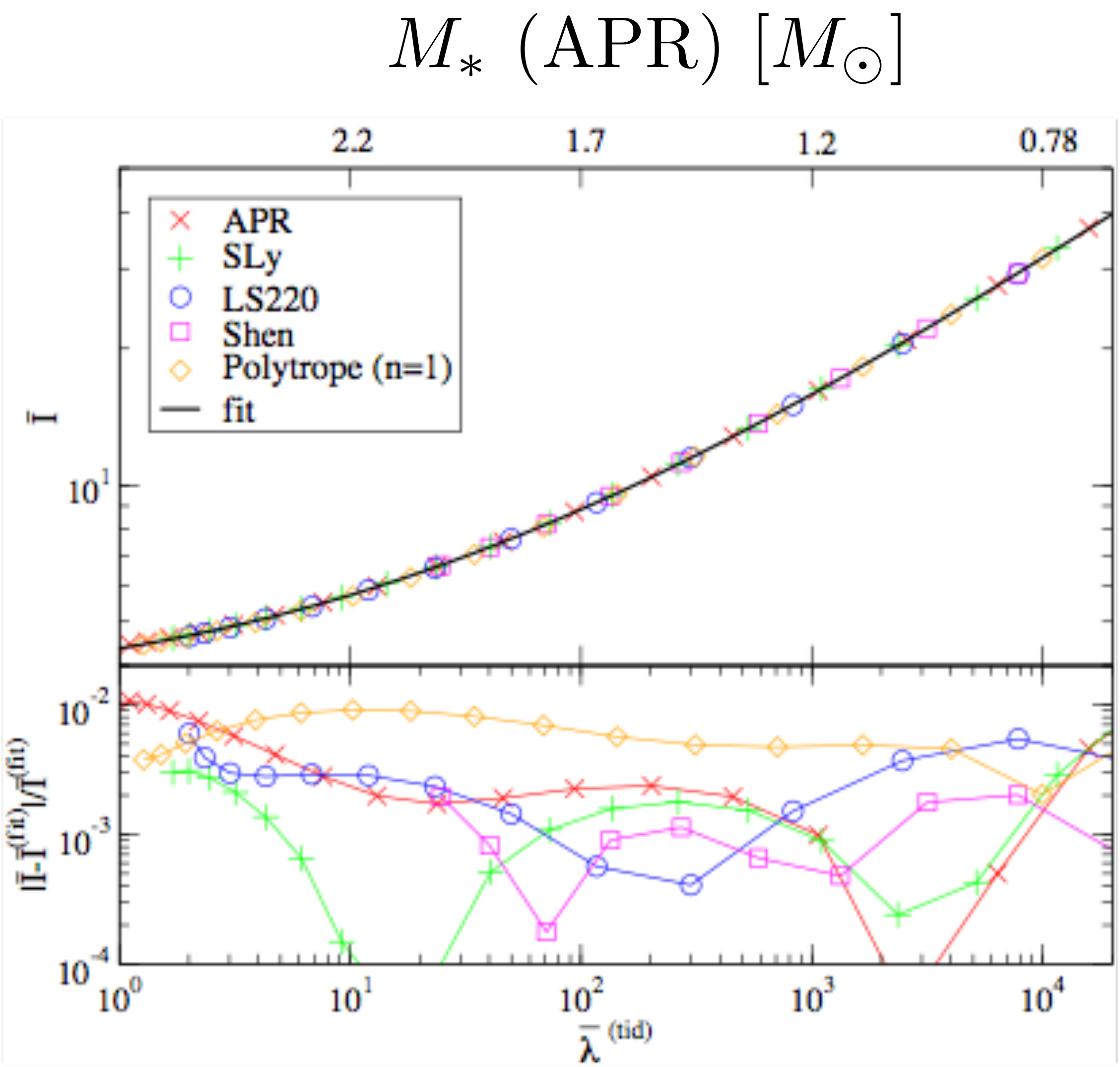
FIG. 7. (Color Online) Dimensionless quadrupole moment $\bar{Q}$, defined in Eq. (40), as functions of M_* (left) and C (right) for various EoSs. The horizontal dashed lines at $\bar{Q} = 9.89$ correspond to a NS with $M_* = 1M_\odot$; curves below this line have higher M_* and C. Observe that the $\bar{Q}$ curves for realistic EoSs approach each other as C increases, and moreover, they approach the BH limit $\bar{Q}_{\rm BH} = 1$ as $C \to 0.5$.

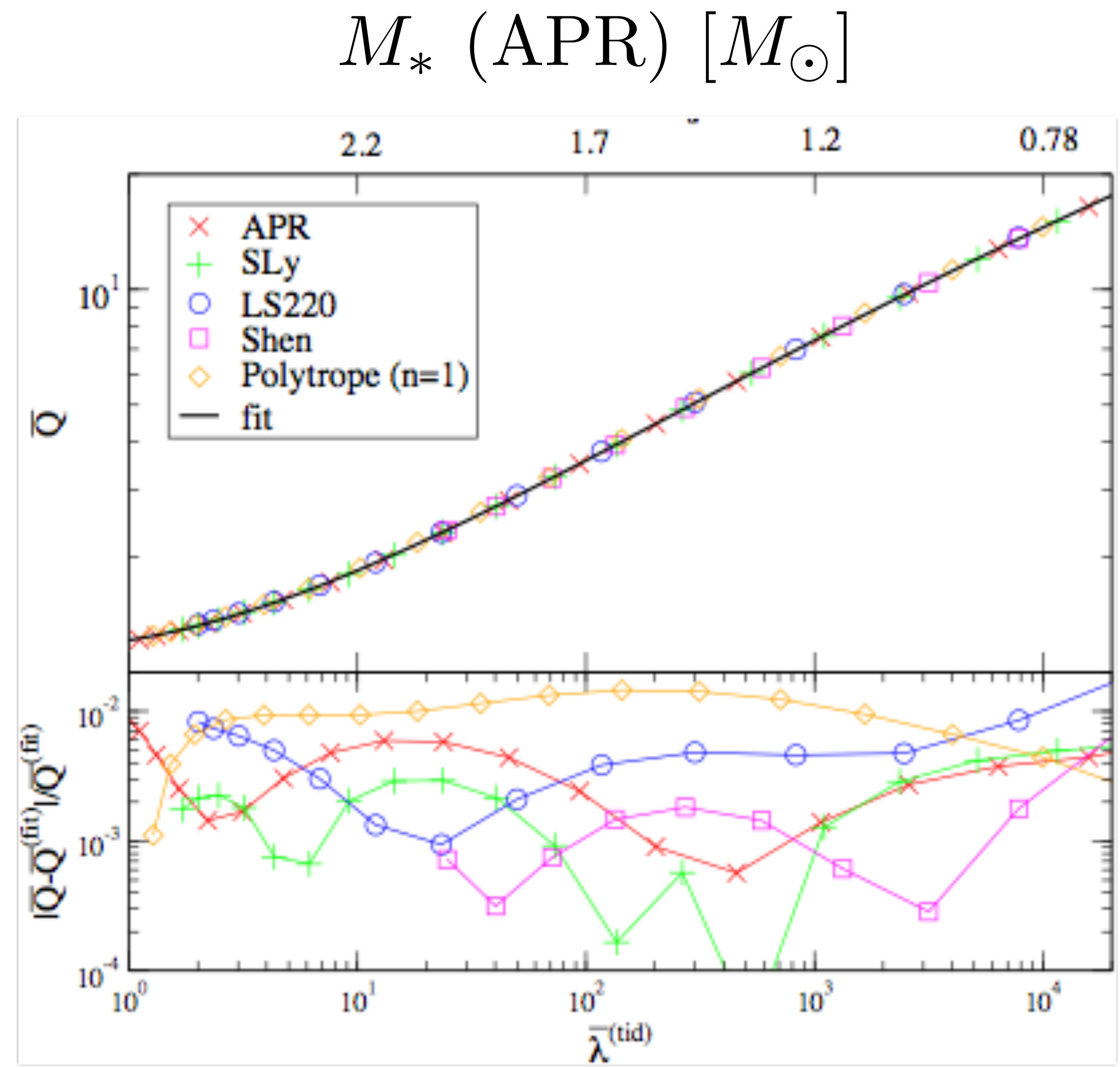
K. Yagi, N. Yunes PRD 88, 2013

the I-M and I-C relations depend on the EoS

Same for Q-M, Q-C and λ-M , λ-C

C= stellar compactness = M/R





K. Yagi, N. Yunes PRD 88, 2013

$\bar{I} = I/M_*^3$ $\bar{\lambda}^{(tid)} = \lambda/M_*^5$ $\bar{Q} = Q^{(rot)}/[M_*^3(S/M_*^2)^2]$

$$\ln y_i = a_i + b_i \ln x_i + c_i(\ln x_i)^2 + d_i(\ln x_i)^3 + e_i(\ln x_i)^4$$

y_i	x_i	a_i	b_i	c_i	d_i	e_i
$\bar{I}$	$\bar{\lambda}^{(tid)}$	1.47	0.0817	0.0149	2.87×10^{-4}	-3.64×10^{-5}
$\bar{I}$	$\bar{Q}$	1.35	0.697	-0.143	9.94×10^{-2}	-1.24×10^{-2}
$\bar{Q}$	$\bar{\lambda}^{(tid)}$	0.194	0.0936	0.0474	-4.21×10^{-3}	1.23×10^{-4}

Neutron star global observables : Love number k_2 & moment of inertia $I=J/\Omega$

$$
\begin{aligned}
\frac{I}{MR^2} &= \frac{1}{2\beta}\frac{w_R}{3+w_R}, \quad w_R = \frac{r}{\omega}\frac{d\omega}{dr}\Big|_{r=R} \\
k_2 &= \frac{3}{2}\frac{\lambda}{R^5} = \frac{3}{2}\beta^5\Lambda \\
&= \frac{8}{5}\frac{\beta^5 z}{6\beta(2-y_R)+6\beta^2(5y_R-8)+4\beta^3(13-11y_R)+4\beta^4(3y_R-2)+8\beta^5(1+y_R)+3z\log(1-2\beta)}, \\
&\quad z \equiv (1-2\beta^2)[2-y_R+2\beta(y_R-1)]
\end{aligned}
$$

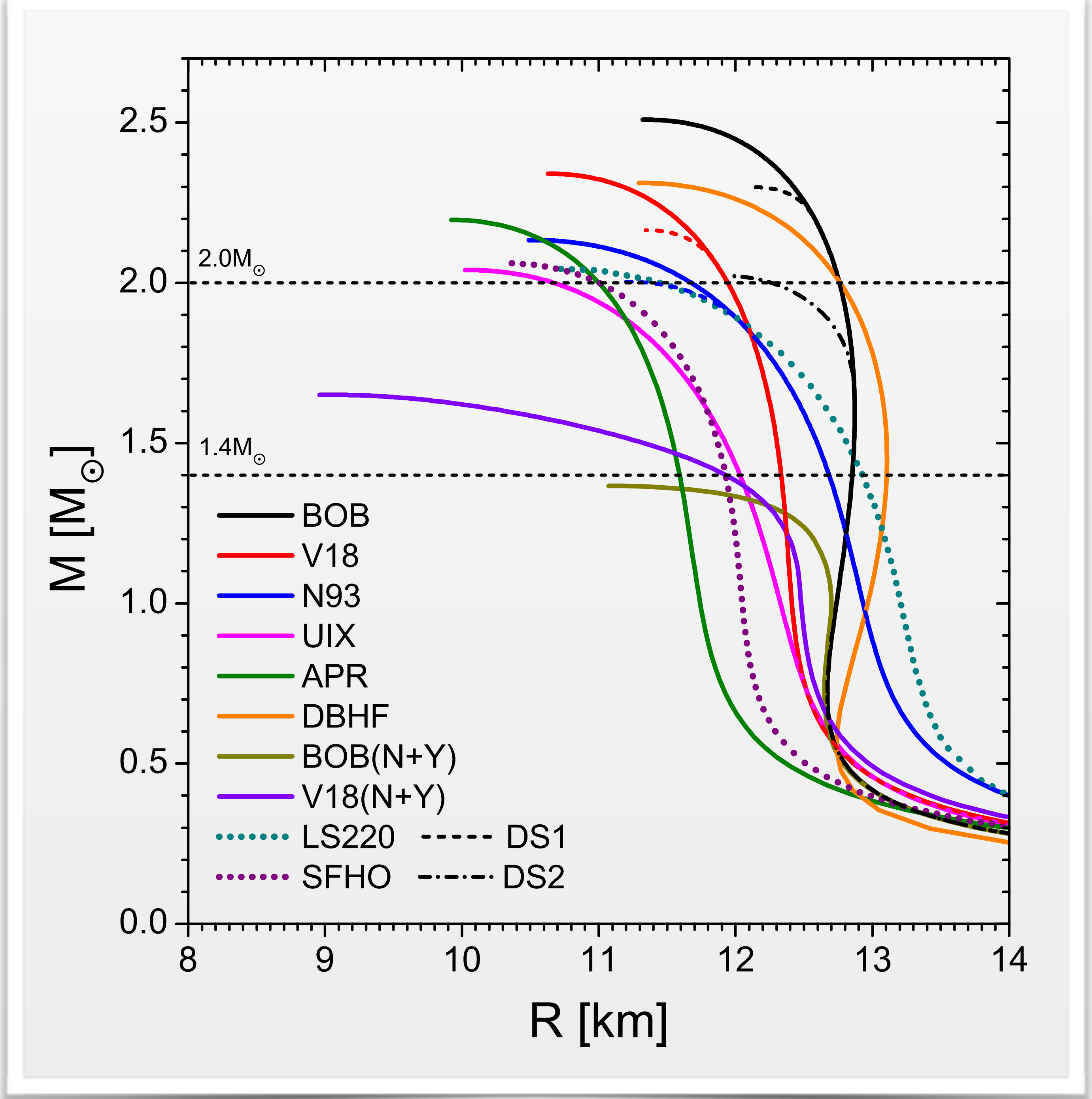
to solve in GR with the TOV eqs. for the pressure p and the enclosed mass m

$$
\begin{aligned}
\frac{dp}{dr} &= -\frac{m\epsilon}{r^2}\frac{(1+p/\epsilon)\left(1+4\pi r^3 p/m\right)}{1-m/r}, \\
\frac{dm}{dr} &= 4\pi r^2\epsilon, \\
\frac{dw}{dr} &= \frac{4\pi r(\epsilon+p)(4+w)}{1-2m/r} - \frac{w(3+w)}{r}, \\
\frac{dy}{dr} &= -\frac{y^2}{r} - \frac{y-6}{r-2m} - rQ, \\
Q &\equiv 4\pi\frac{(5-y)\epsilon+(9+y)p+(\epsilon+p)/c_s^2}{1-2m/r} - \left[\frac{2(m+4\pi r^3 p)}{r(r-2m)}\right]^2
\end{aligned}
$$

with $c_s^2 = d\epsilon/dp$ and the EOS $\epsilon(p)$ as input.

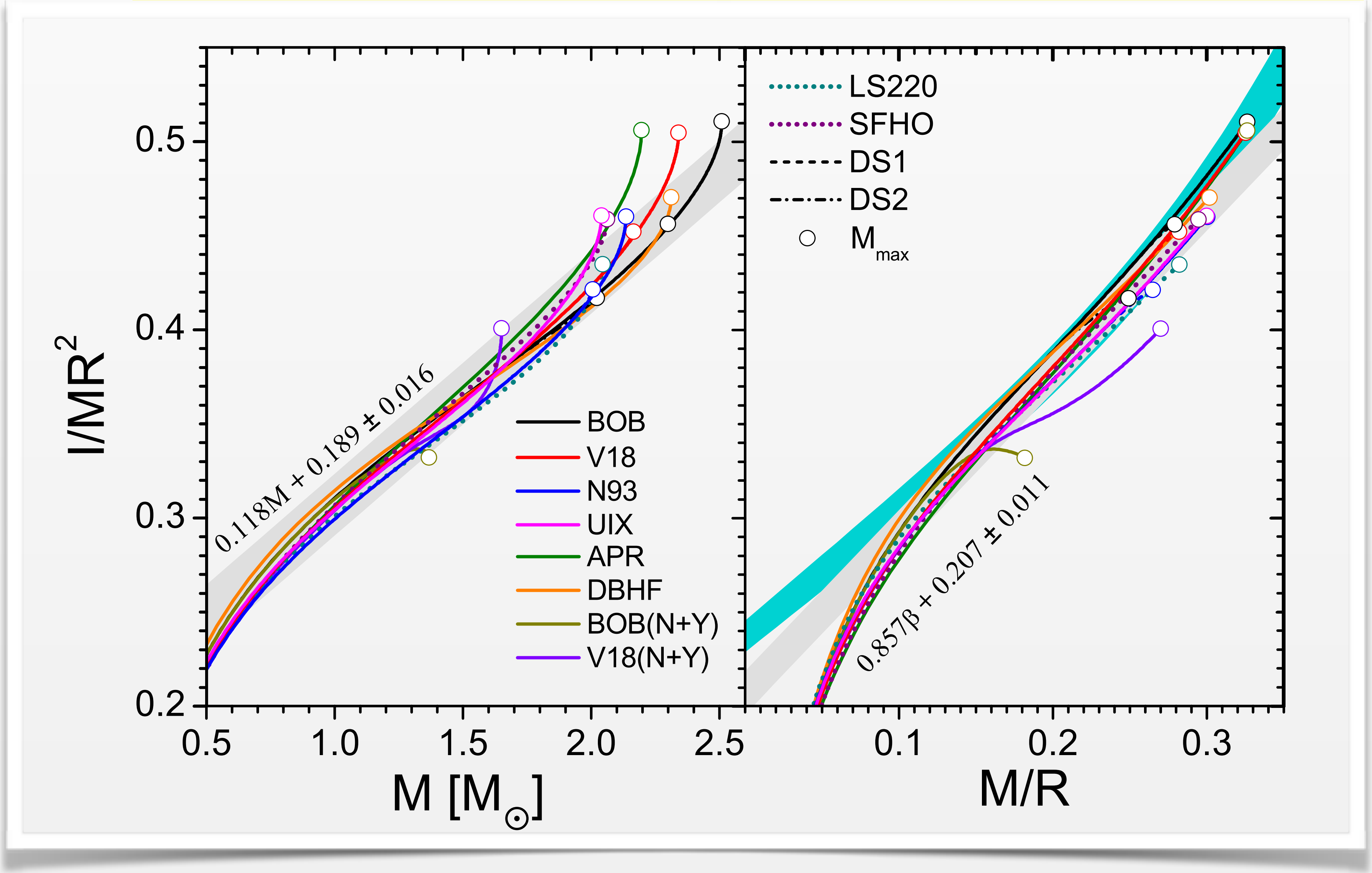
Mass-Radius relations

- Microscopic non-relativistic EoS : BHF with Bonn B, V18, N93, UIX
- Variational : APR
- Microscopic relativistic EoS : DBHF
- Microscopic EoS with hyperons : BOB(N+Y), V18(N+Y)
- Phenomenological EoS : LS220, SFHO
- Hybrid EoS : BHF with Bonn B and Dyson-Schwinger EoS for QM. DS1 and DS2.
- All give maximum masses above 2M except the ones with hyperons.



The moment of inertia

more in arXiv:1809.04315



Not valid for large masses >2.

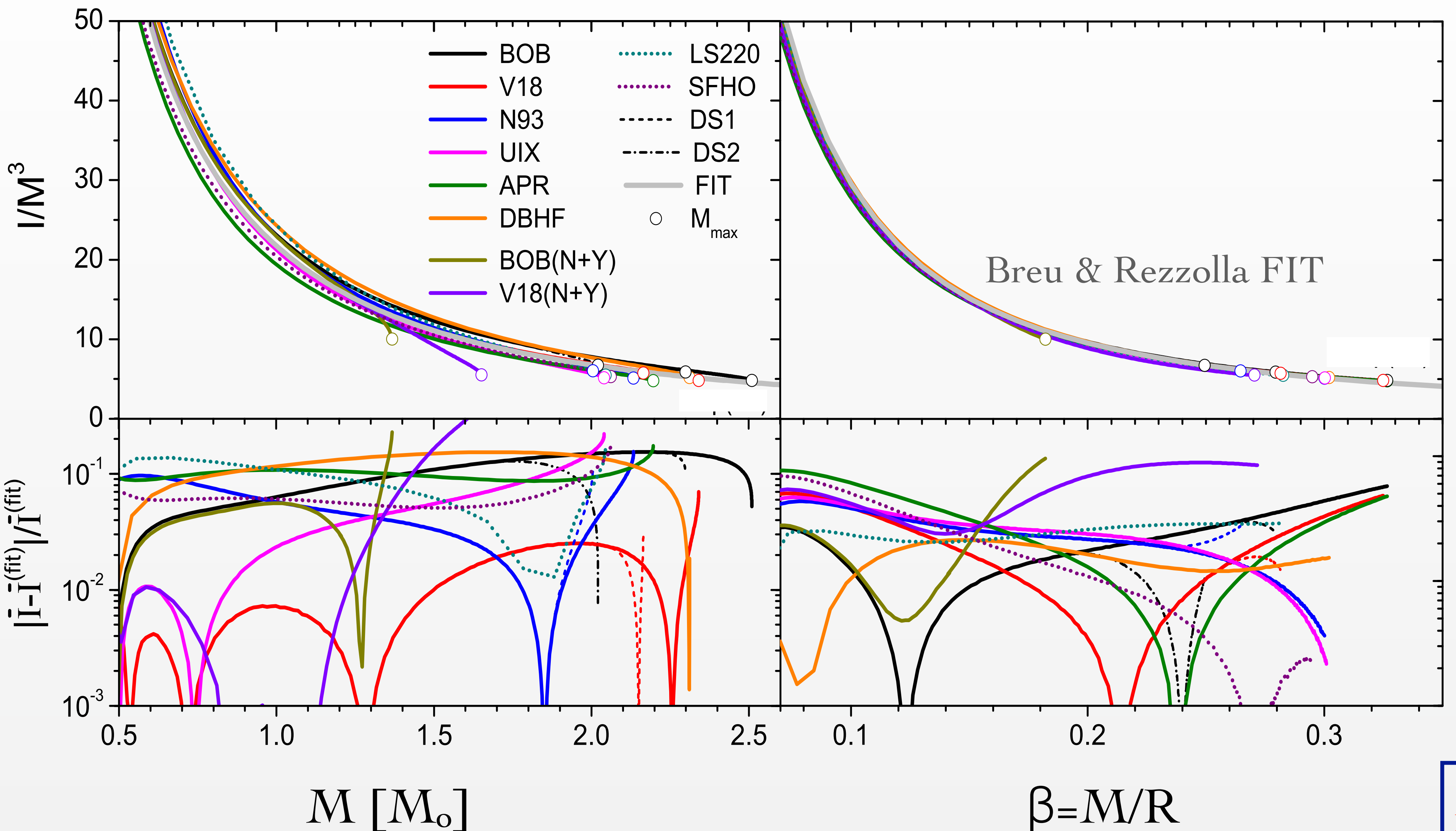
Not valid for hyperonic stars.

For fixed M and I :

$$\frac{\delta R}{R} \approx \frac{1}{2}\frac{\delta f}{f} \approx \frac{0.016}{0.4} < 4\%$$

Blue band : Lattimer fit (ApJ550, 426 (2001))

$$\frac{I}{MR^2} \approx (0.237 \pm 0.008)[1 + 2.844\beta + 18.91\beta^4]$$



Breu&Rezzolla, MNRAS 459 (2016)

$$\frac{I}{M^3} \equiv 0.8134\,\beta^{-1} + 0.2101\,\beta^{-2} + 0.003175\,\beta^{-3} - 0.0002717\,\beta^{-4}$$

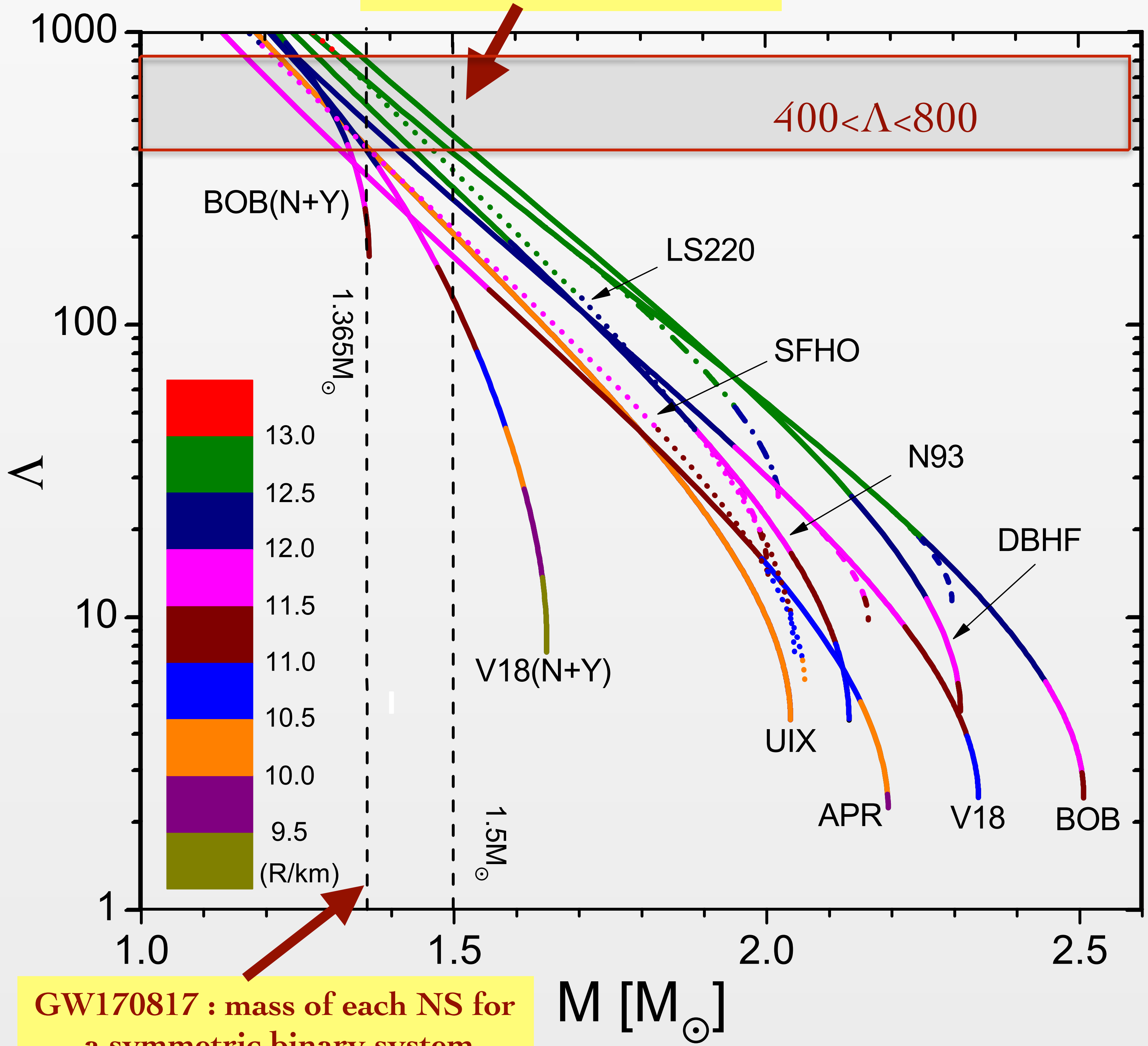
$$\frac{I}{M^3} \equiv 1.0334\,M^{-1} + 30.7271\,M^{-2} - 12.8839\,M^{-3} + 2.8841\,M^{-4}$$

Deviations from the universal fits about a few percent

Constraining the EoS

Correlations between M, R and Λ

GW170817 : limit derived in Annala et al



GW170817 : mass of each NS for a symmetric binary system

- Fixed chirp mass

$$\mathcal{M}_c = \frac{(M_1 M_2)^{3/5}}{(M_1+M_2)^{1/5}} = 1.188\, M_\odot$$

$$q = \frac{M_2}{M_1} = 0.7 - 1$$

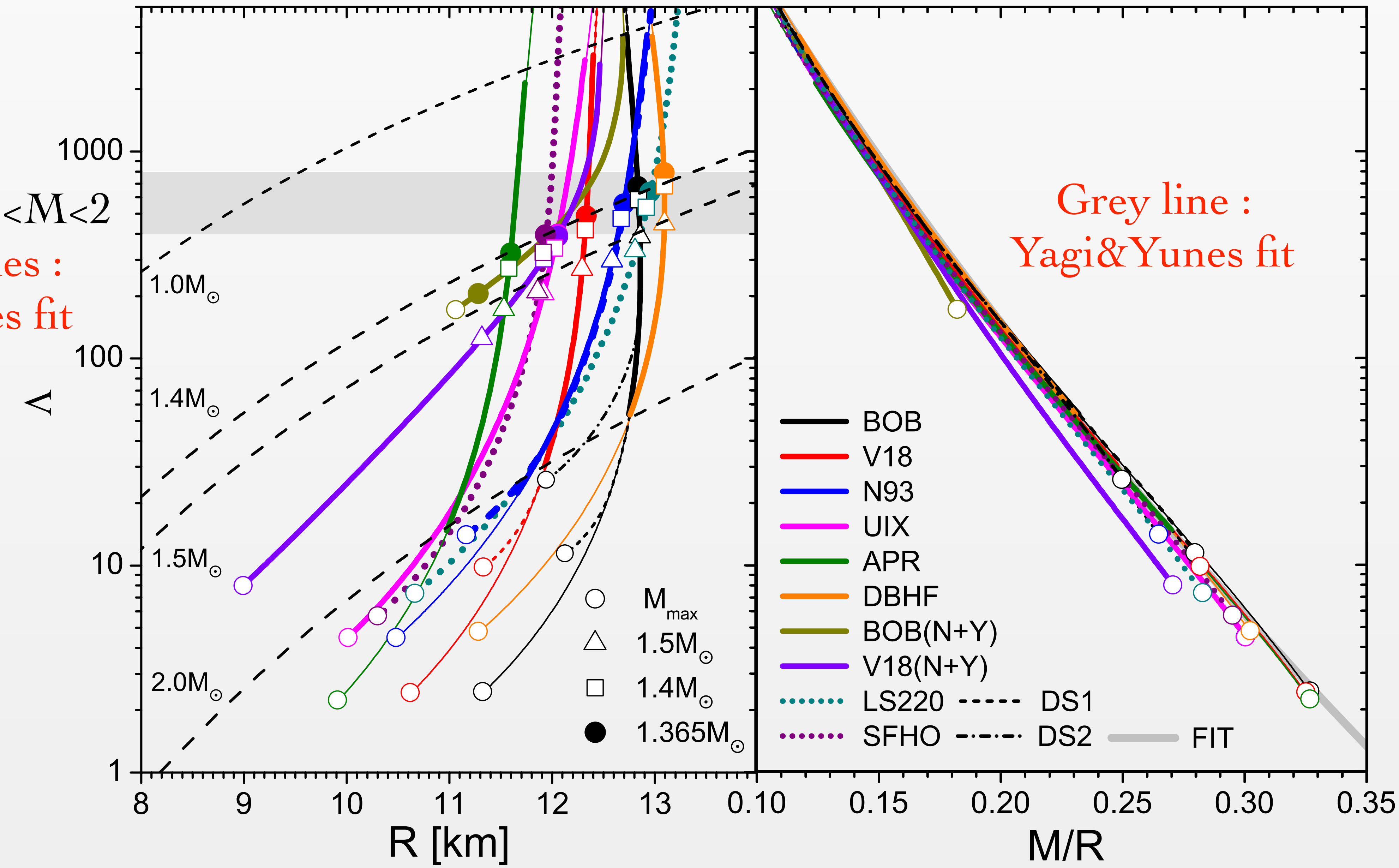
- The conditions $M_1 = M_2 = 1.365$ M_0 and 400<Λ<800 imply **12<R<13 km**
- Compatible EoS : V18(N+Y), UIX, V18,N93, BOB(N), DBHF, LS220, DS1, DS2.
- Not compatible : APR, BOB(N +Y), and SFHO (marginally).

Selection of the EoS !

Universal Relations : Λ vs. ______

Solid lines : 1<M<2

Dashed lines : Yagi&Yunes fit



Yagi & Yunes (2017)

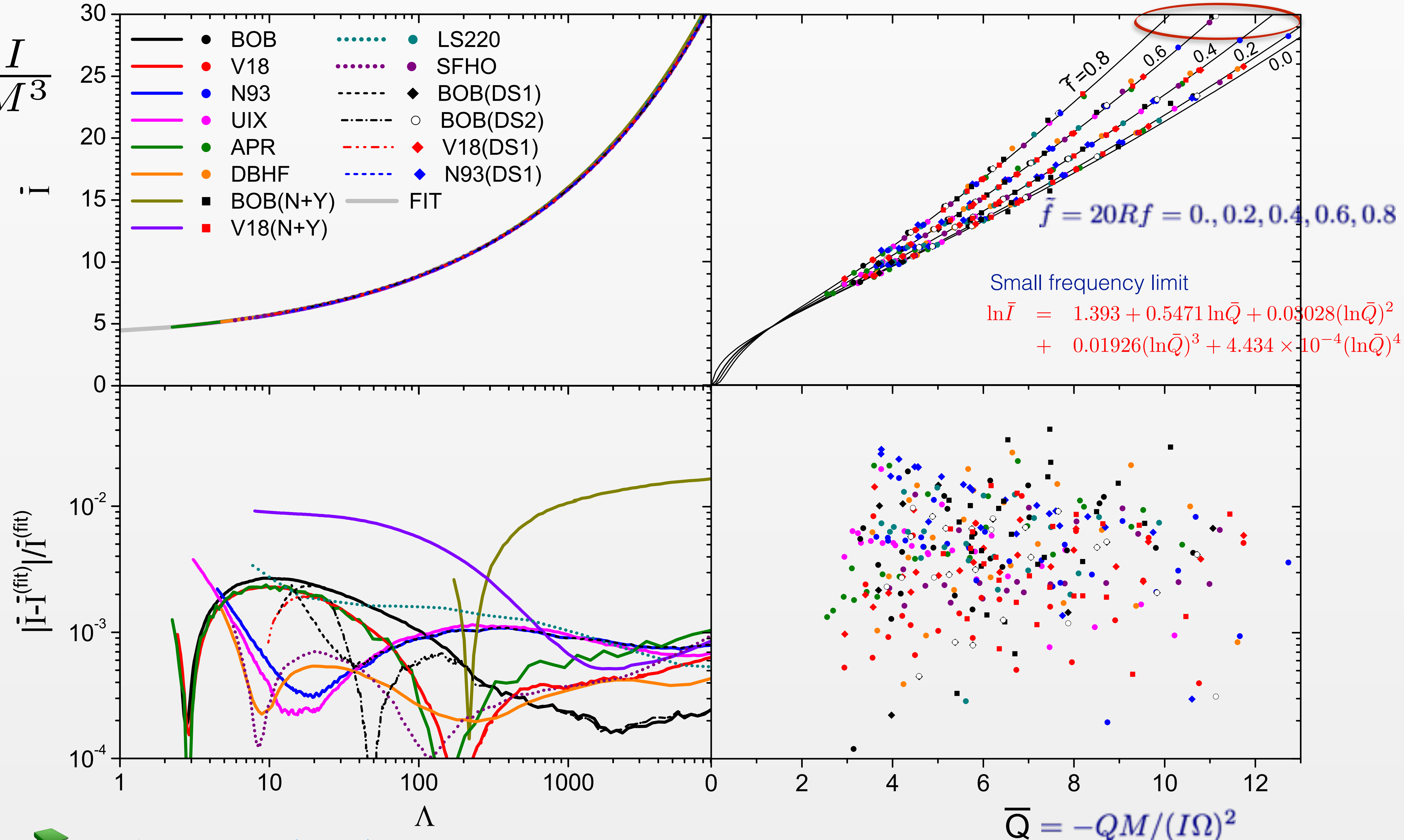
$$\beta = 0.36 - 0.0355 \ln\Lambda + 0.000705 \, (\ln\Lambda)^2$$

The fit does not work for the hyperonic EoS !

Universal Relations : I vs. Λ

Chakrabarti et al., PRL (2014)

$$\bar{I} = \frac{I}{M^3}$$



Yagi & Yunes (2017)

$$\ln(I/M^3) = 1.496 + 0.05951\ln\Lambda + 0.02238(\ln\Lambda)^2 - 6.953\times10^{-4}(\ln\Lambda)^3 + 8.345\times10^{-6}(\ln\Lambda)^4$$

Dots : generalized fits by Chakrabarti et al.

Fractional errors smaller than 1%, except for hyperons !

Fractional errors smaller than 5%

Conclusions

- A large set of microscopic and phenomenological EoS are compatible with the several constraints on the EoS from nuclear structure, heavy-ion collisions, and maximum observed mass but still not enough constraining ….

GW170817 event has added further constraints $\longrightarrow$ the tidal deformability.

- GW170817 is compatible with the merging of two nucleonic neutron stars with a microscopic EoS with maximum mass >2M
- Universal relations I-Love-Q are fulfilled by hadronic and hybrid stars. Strong deviations exhibited by hyperonic EoS.
- More refined constraints from future GW merger events.
- High-precision telescopes, NICER, ATHENA+, SKA are expected to improve our knowledge of the NS mass-radius relation.

In the inspiral phase, the space-time metric inside one star is modified by the influence of the second star, differently from region to region. This breaks the spherical symmetry and determines a deformation of the NS.

Onset of an induced quadrupole moment Q_{in} :

To linear order in the external tidal field C_{ij} the "tidal-induced" quadrupole moment Q_{ij} can be written $Q_{ij} = \lambda \, C_{ij}$

λ, the tidal deformability, is related to the l=2 tidal Love number

$$k_2 = \frac{3G}{2R^5}\lambda$$ ☞ It depends on the EoS !