Dark side of modified gravity?

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Main Points

1. Introduction
2. Field equations for $f(R)$ gravity
3. Conformal transformation to the Einstein frame
4. Chameleon mechanism and screening
5. Scalaron as a dark matter particle
6. Scalaron interaction with the massless and massive vector and fermionic fields
7. Decay widths and constraints on scalaron mass
8. Observations and theory
9. Summary

What is this talk about?

In this talk, I will discuss the geometrical effects of dark matter without actually invoking the dark matter, *per se*, in the standard form. It will be done by modifying the Einstein-Hilbert action, especially in $f(R)$ model and the new scalar field would be discussed under a conformal transformation from the Jordan frame to the Einstein frame.
I will also show that the mass of the scalaron undergoes a chameleon mechanism and becomes large in the high curvature regions to cause a screening of the fifth force. Then, I would discuss the cosmological conditions on various features of the scalaron to show how it can act as a dark matter particle.

The modified gravity action

$$\mathcal{A} = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} f(R) + \mathcal{A}_m(g_{\mu\nu}, \Psi_m) \tag{1}$$

where $\kappa^2 = 8\pi G$ and $\mathcal{A}_m$ is the action of the matter part with matter field Ψ_m.

- The spacetime as homogeneous, isotropic and spatially flat.
- Given by the Friedmann-Lemaitre-Robertson-Walker (FLRW) spacetime as

$$ds^2 = -dt^2 + a^2(t)[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)] \tag{2}$$

$a(t)$ is time dependent scale factor and the speed of light $c = 1$.

The field equations

- We use the metric formalism in which connections $\Gamma^{\alpha}_{\beta\gamma}$ are defined in terms of the metric tensor $g_{\mu\nu}$.
- Varying the action with respect to $g_{\mu\nu}$, the field equations are given by

$$
\begin{aligned}
&F(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} \\
&-\nabla_{\mu}\nabla\nu F(R) + g_{\mu\nu}\Box F(R) = \kappa^2 T_{\mu\nu},
\end{aligned}
\tag{3}
$$

  where $F(R) \equiv \frac{\partial f}{\partial R}$ and $T_{\mu\nu}$ is the energy-momentum tensor for matter.
- The trace of field equations (3) is given by

$$
3\Box F(R) + F(R)R - 2f(R) = \kappa^2 T \tag{4}
$$

Conformal transformation from Jordan frame to Einstein frame

We rewrite the action in the form

$$\mathcal{A} = \int \sqrt{-g}\left(\frac{1}{2\kappa^2}F(R)R - U\right) d^4x + \mathcal{A}_m, \tag{5}$$

where

$$U = \frac{F(R)R - f(R)}{2\kappa^2}. \tag{6}$$

We switch over to the Einstein frame to see the real effects of dark matter in form of the scalar degree of freedom. It is possible to derive an action in the Einstein frame under the conformal transformation

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}. \tag{7}$$

New Ricci scalar degree

$$R = \Omega^2(\tilde{R} + 6\tilde{\Box}\omega - 6\tilde{g}^{\mu\nu}\partial_\mu\omega\partial_\nu\omega), \quad (8)$$

where

$$\omega \equiv \ln\Omega, \partial_\mu\omega \equiv \frac{\partial\omega}{\partial\tilde{x}^\mu}, \tilde{\Box}\omega \equiv \frac{1}{\sqrt{-\tilde{g}}}\partial_\mu(\sqrt{-\tilde{g}}\tilde{g}^{\mu\nu}\partial_\nu\omega). \quad (9)$$

Propagation of R in a new degree of freedom.

The action under the conformal transformation is transformed as

$$\mathcal{A} = \int d^4x\sqrt{-\tilde{g}}\times \left[\frac{1}{2\kappa^2}F\Omega^{-2}(\tilde{R}+6\tilde{\Box}\omega-6\tilde{g}^{\mu\nu}\partial_\mu\omega\partial_\nu\omega)-\Omega^{-4}U\right] + \mathcal{A}_m. \quad (10)$$

The two fold dilatonic coupling in $\mathcal{A}_m$:

- Coupling of ϕ with matter fields through Lagrangian.
- Through $\sqrt{-\tilde{g}}$.

Over to scalar field: Conformal degree of freedom

The linear action in $\tilde{R}$ can be written by choosing

$$\Omega^2 = F. \tag{11}$$

A new scalar field ϕ defined by

$$\kappa\phi \equiv \sqrt{\frac{3}{2}} \ln F. \tag{12}$$

Using the relations (11) and (12), the action in Einstein frame is found as

$$\mathcal{A} = \int d^4x \sqrt{-\tilde{g}} \left[\frac{1}{2\kappa^2}\tilde{R} - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) \right] + \mathcal{A}_m. \tag{13}$$

Geometric potential

$$V(\phi) = \frac{U}{F^2} = \frac{FR - f}{2\kappa^2 F^2} \tag{14}$$

stands as the potential term of the scalar degree of freedom in a general $f(R)$ model.
But this is not all! You have contributions from the SM that effectively draws it to a minimum.

Fixing the form of f(R)

- Constant tangential velocity condition on the motion of a test particle in the stable orbits of the spiral galaxies (few hundred km/s).
- $f(R)$ can be given by $f(R) = R^{1+\delta}$, where $\delta << 1$ is related to the tangential velocity.
- To solve the problem of dark matter, only very small deviation from general relativistic theory is required: *delta* $\sim 10^{-6}$.
- This shows up scalaron as a “BUILT-IN” field, NOT an *ADDITIONAL* one !

In the Einstein frame the scalar field ϕ is coupled with non-relativistic matter. This coupling has the relation

$$\Omega^2 = F = e^{-2Q\kappa\phi}, \tag{15}$$

where Q is the strength of coupling. Now, from equation (12) and equation (15), Q is given by

$$Q = -\frac{1}{\sqrt{6}}. \tag{16}$$

The scalar field arise in the $f(R)$ model given by

$$f(R) = \frac{R^{1+\delta}}{(R_c)^{\delta}} \tag{17}$$

where R_c is a constant having unit of the Ricci scalar R and δ is a small parameter of the model.

Connection of Ricci scalar to scalar field

-

$$F = (1+\delta)\frac{R^{\delta}}{(R_c)^{\delta}}. \tag{18}$$

- For $Q = -\frac{1}{\sqrt{6}}$, the Ricci scalar R in terms of scalar field ϕ is given as

$$R = R_c \left[\frac{e^{\sqrt{2/3}\kappa\phi}}{1+\delta}\right]^{\frac{1}{\delta}}. \tag{19}$$

- The field equations for R EQUIVALENTLY transform to the field equations of ϕ.

The variation of the action with respect to ϕ yields the equation of motion of the scalar field as

$$\tilde{\Box}\phi = V'(\phi) + \frac{\kappa}{\sqrt{6}}\tilde{T} \tag{20}$$

where $\tilde{\Box}\phi = \frac{1}{\sqrt{-\tilde{g}}}\partial_\mu(\sqrt{-\tilde{g}}\tilde{g}^{\mu\nu}\partial_\nu\phi)$, $V'(\phi) = \frac{dV}{d\phi}$ and $\tilde{T} = \tilde{g}^{\mu\nu}\tilde{T}_{\mu\nu}$.
Equation (20) can also be written as

$$\tilde{\Box}\phi = V'_{eff}(\phi) \tag{21}$$

where $V'_{eff}(\phi) = V'(\phi) + \frac{\kappa}{\sqrt{6}}\tilde{T}$.

The effective potential

$$V_{eff}(\phi) = \frac{\delta R_c}{2\kappa^2(1+\delta)^{\frac{(1+\delta)}{\delta}}} e^{\sqrt{\frac{2}{3}}\frac{(1-\delta)}{\delta}\kappa\phi} + \frac{1}{4}\rho e^{\frac{-4\kappa\phi}{\sqrt{6}}} \tag{22}$$

- The first part at ϕ_{min} yields the DE.
- The second part is quite important as DM!

Leading to a minimum

Finding value of the scalar field ϕ at which $V_{eff}(\phi)$ is minimum, $\frac{dV_{eff}}{d\phi}$ given as

$$V'_{eff}(\phi) = \frac{R_c}{\sqrt{6}\kappa}\frac{(1-\delta)}{(1+\delta)^{\frac{1+\delta}{\delta}}} e^{\sqrt{\frac{2}{3}}\frac{(1-\delta)}{\delta}\kappa\phi} - \frac{\kappa}{\sqrt{6}}\rho e^{\frac{-4\kappa\phi}{\sqrt{6}}} \tag{23}$$

For $V'_{eff}(\phi) = 0$, ϕ at the minimum of $V_{eff}(\phi)$ given by,

$$\phi_{min} = \sqrt{\frac{3}{2}}\frac{1}{\kappa}\ln\left[(1+\delta)\left(\frac{\kappa^2\rho}{R_c(1-\delta)}\right)^{\frac{\delta}{1+\delta}}\right] \tag{24}$$

Scalaron mass

For the calculation of the mass of the scalar field, we have

$$V''_{eff}(\phi) = \frac{R_c}{3}\frac{(1-\delta)^2}{\delta(1+\delta)^{\frac{1+\delta}{\delta}}}e^{\sqrt{\frac{2}{3}}\frac{(1-\delta)}{\delta}\kappa\phi} + \frac{2\kappa^2}{3}\rho e^{\frac{-4\kappa\phi}{\sqrt{6}}} \tag{25}$$

which for the value of ϕ_{min} becomes

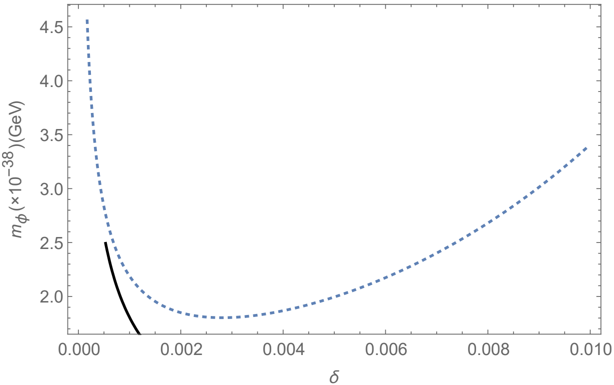
$$V''_{eff}(\phi_{min}) = \frac{(1-\delta)^{\frac{2\delta}{1+\delta}}}{3\delta(1+\delta)}(R_c)^{\frac{2\delta}{\delta+1}}(\kappa^2\rho)^{\frac{1-\delta}{1+\delta}}. \tag{26}$$

Thus, mass of the scalaron ($= V''_{eff}(\phi_{min})$) is clearly given by

$$m_\phi^2 = \frac{(1-\delta)^{\frac{2\delta}{1+\delta}}}{3\delta(1+\delta)}(R_c)^{\frac{2\delta}{\delta+1}}(\kappa^2\rho)^{\frac{1-\delta}{1+\delta}}. \tag{27}$$

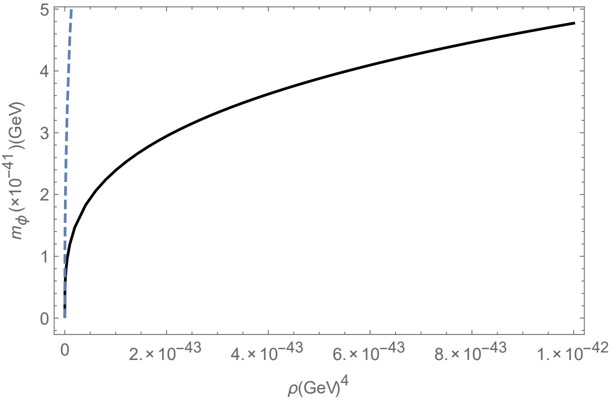
Scalaron potential $V(\phi_{min}) = M^2_{planck}\Lambda$ can be identified as the dark energy.

Scalaron mass with model parameter δ



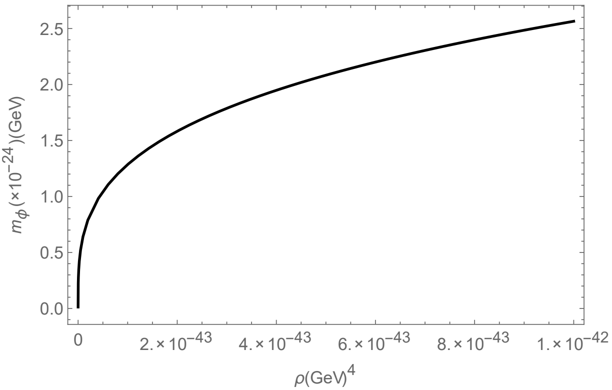
Plot for the variation of the scalaron mass m_ϕ with parameter δ. Here, the black curve corresponds to $R_c = \Lambda$ (value of cosmological constant) and the dotted curve to $R_c = 1$. The value of the energy density of matter at the galactic scale is $\rho = 4 \times 10^{-42}(GeV)^4$ for both curves.

m_ϕ with ρ , $R_c = \Lambda$



The variation of scalaron mass m_ϕ with the energy density ρ of matter. Here, black and dashed curves correspond to $\delta = 0.25$ and $\delta = 0.10$, respectively, and $R_c = \Lambda$ (value of the cosmological constant).

m_ϕ with ρ, $\delta = 0.25$ and $R_c = 1$



Plot for the variation of scalaron mass m_ϕ with the energy density ρ of matter corresponding to $\delta = 0.25$ and $R_c = 1$.

What do we find?

- The mass of the scalar field depends upon the energy density of matter.
- The Compton wavelength becomes too small and the fifth force is almost screened.
- This is consistent with the local galactic and solar system constraints.
- However, this size puts a constraint on the mass of the scalaron to act as a dark matter particle and the mass should not be too high. Interaction with SM possible.

- When $R_c = \Lambda$, the scalaron mass decreases with increasing δ, while in case of $R_c = 1$, m_ϕ decreases more sharply initially for smaller values of δ and then increases for its larger values for the given energy density of matter ρ at the galactic scale.
- Since δ substantially determines the form of the $f(R)$ model, therefore, we have the model dependent mass of the scalaron. It tends to infinity as δ tends to zero and our model approaches the standard general relativistic description.
- Even a small change of δ makes a large difference for such dependence of the mass on the energy density background.

- The equation of motion of the scalar field

$$\frac{d^2\phi}{d\tilde{t}^2} + 3\tilde{H}\frac{d\phi}{d\tilde{t}} + V'_{eff}(\phi) = 0. \quad (28)$$

  where $\tilde{H}$ is the Hubble parameter in the Einstein frame.
- When $\tilde{H} = 0$, the energy of the system is conserved and the oscillations are periodic.
- When $\tilde{H} \neq 0$, then it acts to produces a dissipative force against the oscillations of the scalar field.
- If the Hubble parameter $\tilde{H}$ varies slowly (adiabatically) with time during the time period T of the oscillation such that

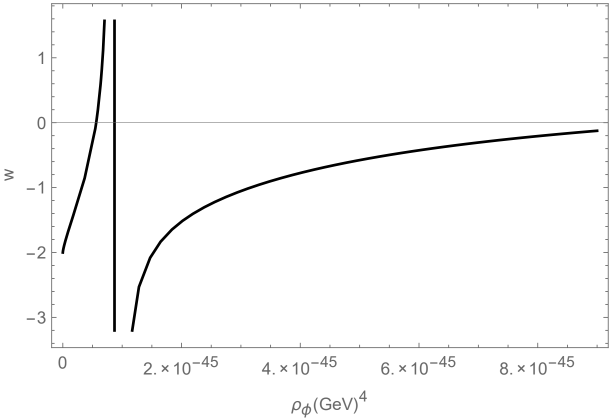
$$\tilde{H} \ll \nu \quad (29)$$

  The rate of energy loss is proportional to $\tilde{H}$.

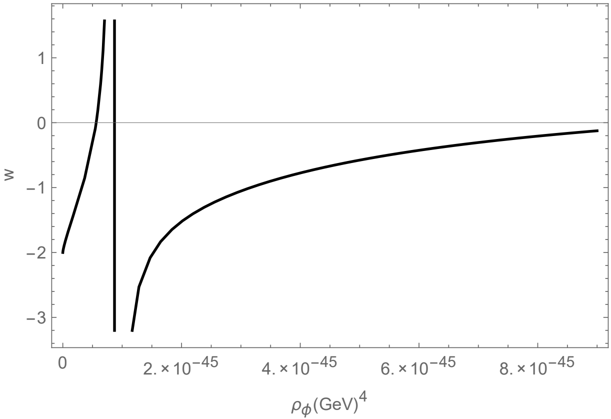
$$J = \oint p d\phi = 2 \int_{\phi_1}^{\phi_2} \sqrt{2(\rho_\phi - V_{eff}(\phi))} d\phi \tag{30}$$

where p is the momentum and ρ_ϕ is the energy density of scalar field. ϕ_1 and ϕ_2 are the values of ϕ at which $V_{eff}(\phi_1) = V_{eff}(\phi_2) = \rho_\phi$. The equation of state w of the scalar field is defined as

$$w = -1 + \frac{J}{\rho_\phi} \frac{1}{dJ/d\rho_\phi} \tag{31}$$



Plot for the variation of equation of state *w* with the energy density ρ_ϕ of scalar field corresponding to $\delta = 0.20$.



Plot for the variation of equation of state *w* with the energy density ρ_ϕ of scalar field corresponding to $\delta = 0.20$.

Coupling to massless/massive vector field

$$L_V(g^{\mu\nu}, A_\mu) = -\frac{1}{4} e^{4\sqrt{1/6}\kappa\phi} \tilde{g}^{\alpha\mu} \tilde{g}^{\beta\nu} F_{\alpha\beta} F \mu\nu. \tag{32}$$

This does not couple to the scalaron at the classical level. But due to scale anomaly, at the quantum level it does.
With mass term,

$$L_{V-mass}(g^{\mu\nu}, A_\mu) = -\frac{1}{2} m_V^2 e^{2\sqrt{1/6}\kappa\phi} \tilde{g}^{\mu\nu} A_\mu A_\nu. \tag{33}$$

$$\mathcal{A}_m = \int d^4x \sqrt{-\tilde{g}} [L_{V-mass}(\tilde{g}_{\mu\nu}, A_\mu) + L_{V-\phi}(\tilde{g}_{\mu\nu}, A_\mu, \phi)] \tag{34}$$

where

$$L_{V-\phi}(\tilde{g}_{\mu\nu}, A_\mu, \phi) = -\frac{1}{2} m_V^2 (e^{-2\sqrt{1/6}\kappa\phi} - 1) \tilde{g}^{\mu\nu} A_\mu A_\nu. \tag{35}$$

...and coupling to fermions

In curved spacetime,

$$L_F(\gamma^\mu, \psi) = i\bar{\psi}(x)\gamma^\mu \nabla_\mu \psi(x) \tag{36}$$

Coupling appears explicitly!
However, we can recover the action

$$\mathcal{A}_m = \int d^4x \sqrt{-\tilde{g}}\, i\bar{\psi}'(x)\tilde{\gamma}^\mu \tilde{\nabla}_\mu \psi'(x) \tag{37}$$

by canonical transformation as

$$\psi \to \psi' = e^{-3/2\sqrt{1/6}\kappa\phi}\psi \tag{38}$$

For massive fermion, the conformal Lagrangian has additional ϕ dependent terms as

$$L_{F-\phi}(\psi', \phi) = \frac{\kappa\phi}{\sqrt{6}}.m_F\bar{\psi}'\psi' + O(\kappa^2\phi^2) \tag{39}$$

Scalaron as dark matter particle

Background $\phi = \phi_{min} + \hat{\phi}$, $\rho_{EW} = (100 GeV)^4$

$$\tilde{\Box}\phi_{min} = V'(\phi_{min}) - \frac{\kappa}{\sqrt{6}} e^{\frac{-4\kappa\phi_{min}}{\sqrt{6}}} \rho_{EW} \tag{40}$$

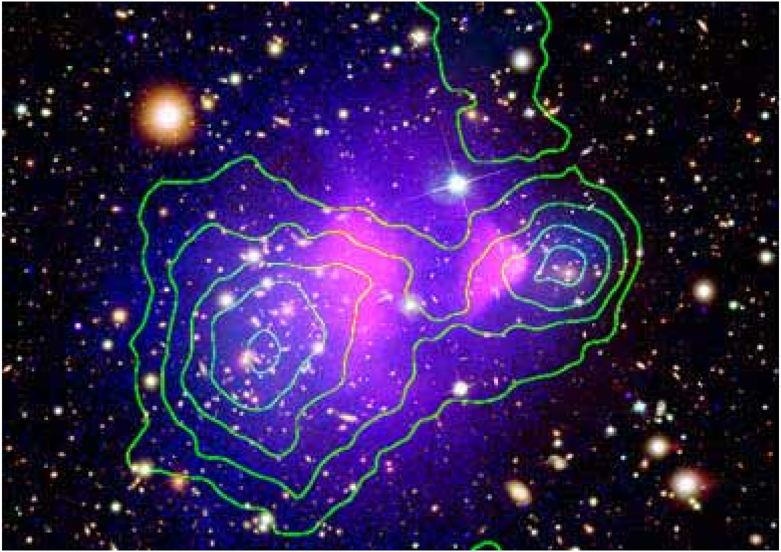
Mass of scalar dark matter particle in our model

$$m_\phi^2 = \frac{(1-\delta)^{\frac{2\delta}{1+\delta}}}{3\delta(1+\delta)} (R_c)^{\frac{2\delta}{\delta+1}} (\kappa^2 \rho_{EW})^{\frac{1-\delta}{1+\delta}}. \tag{41}$$

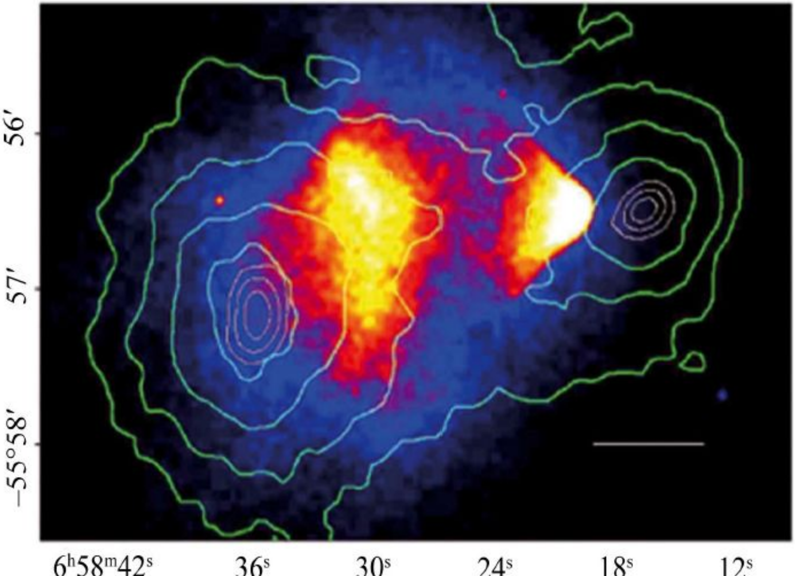
- Dark matter particle appears and interacts as $\hat{\phi}$.
- The temperature dependent velocity field of T^μ_μ of SM sector set an anisotropy in m_ϕ and self-interaction of scalaron dark matter.
- MAY EXPLAIN THE OBSERVED OFFSET BETWEEN BARYONIC HOT GAS AND DM IN CLUSTERS!

Bullet Cluster 1E0657-56

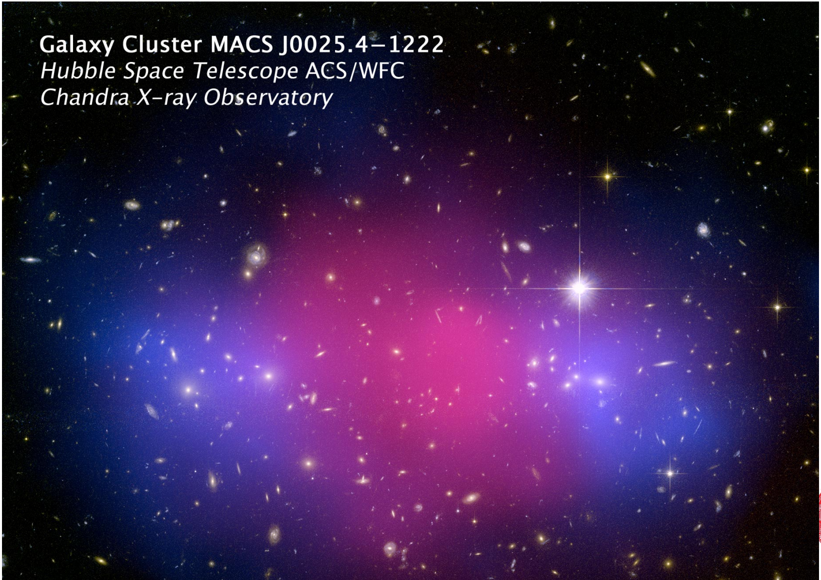
Dark matter mapping



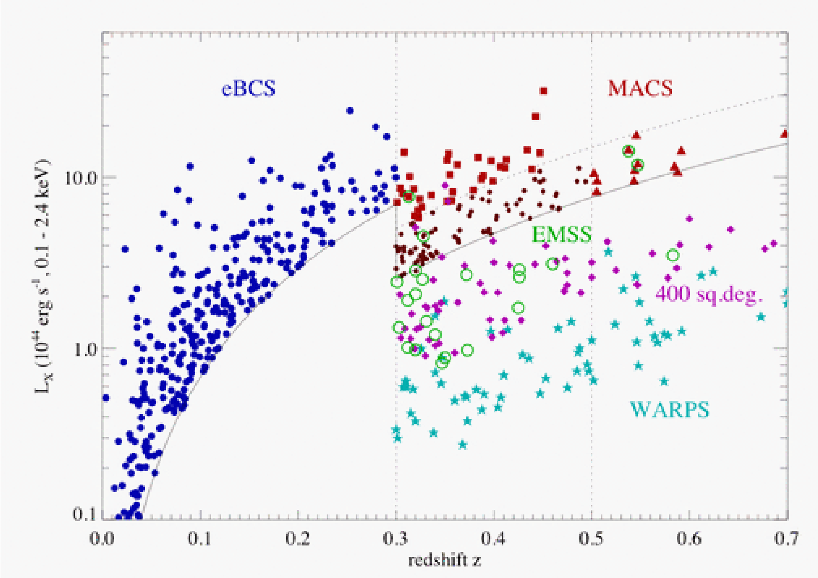
1E0657-56: The offset



MACS J0025.4-1222



MACS : $z > 0.3$



eBCS
MACS
EMSS
400 sq.deg.
WARPS
L_X (10^44 erg s^-1, 0.1 - 2.4 keV)
10.0
1.0
0.1
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
redshift z

Chameleonic decay

- The mass of the scalar field depends upon the energy density of standard matter.
- Anisotropic in the velocity fields of standard matter. May explain the DM-Hot gas offset in Clusters!
- For a very small $\delta \sim 10^{-6}$, at the electro-weak scales $\rho_{EW} \sim (100 GeV)^4$, $m_\phi \sim 10^{-3}$ eV,
- At the solar system scales $\rho_\odot \sim 10^{19} eV^4$, scalaron must be very light as $\sim 10^{-16}$ eV. PPTA (2018).
- Such particle will not decay soon. Has very WEAK coupling to FERMION and VECTOR fields

Decay widths into $F\bar{F}$

Tree level:

$$L_{\phi F\bar{F}} = \frac{\kappa\phi}{\sqrt{6}} \sum m_F \bar{\psi}'_F \psi'_F. \qquad (42)$$

$$\Gamma = \frac{N^{(F)} m_\phi m_F^2 \kappa^2}{48\pi} \left(1 - \frac{4m_F^2}{m_\phi^2}\right)^{3/2}. \qquad (43)$$

$N^{(F)} = 1$ Leptons,
$N^{(F)} = 3$ Quarks

Decay into Diphotons and Digluons

- Scale anomaly
- One-loop: the Yukawa vertices/ or involving W bosons in the standard matter perturbations.

For massless vector fields,

$$L_{anomaly} = -\frac{g_V^2}{8(4\pi)^2}\left(\frac{3}{2}\sqrt{1/6}\kappa\phi\right)F^2_{\mu\nu}(V). \tag{44}$$

$$L_{\phi WW} = \frac{2\kappa\phi}{\sqrt{6}} m_W^2 W^+_\mu W^{\mu -}. \tag{45}$$

The net contributions to Γ increase with m_ϕ.

- Bound from cosmological age $m_\phi \leq 0.24 GeV$. Our model suggests $m_\phi \sim 10^{-16}$eV.
  BKY-MMV arxiv: 1811.03964 [gr-qc]
- Current bounds: $m_\phi \sim 10^{-22}$eV. Porayko *et al.* Phys Rev D 98, 102002, (2018) from Parkes Pulsar Timing Array (PPTA).

Dynamical dark matter in an evolving background!

- Scalaron with $m_\phi > 0.23 GeV$ must all decay till now.
- $m_\phi < 0.23 GeV$ must survive.
- Scalaron had a mass spectrum in the early universe due to density perturbations in the background.
- The inhomogeneities would be co-extensive, and co-moving with the cosmic evolution.
- However, evolving m_ϕ would change the self-interaction in scalaron fields. More so in the massive SM backgrounds.
- They produce the observable consequences.

SUMMARY

- Modified gravity $f(R)$ EQUIVALENT to Eistein's theory with BUILT-IN scalar field.
- The scalaron couples to matter through Lagrangain AND gravity.
- It shows a kind of dilatonic coupling. Mass increases with the massive background as in Clusters.
- It can be explained in form of dark matter as the oscillating field.
- Scalaron potential $V(\phi_{min}) = M^2_{Planck}\Lambda$ can be identified as the dark energy.
- Scalaron is light enough to last at least as long as the age of the universe.
- Within the solar system, our results match with the PPTA and other observations.
- The current observations point to an ultra-light scalar dark matter.
- The model parameters are to be tuned with astrophysical/cosmological observations.

THANK YOU!