Production of fast pions in exclusive neutrino processes

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Single-pion production processes in exclusive neutrino reactions with small momentum transfer to the nucleon, which are due to the scattering of neutrinos on virtual mesons (Reggeons), are studied. In principle, the experimental study of these processes permits the contributions of the different mesons to be separated and information to be obtained on weak meson-pion (Reggeon-pion) transitions.

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1. INTRODUCTION

By now a large amount of experimental data has been gathered in neutrino experiments at high energies so that it is possible to study individual exclusive processes. An interesting class of events discovered in experiments at FNAL and CERN is the production of fast single pions $\nu_\mu \to \mu^-\pi^+$ and $\bar{\nu}_\mu \to \mu^+\pi^-$ without visible traces of a recoil proton or products of target nucleus breakup. These processes are therefore characterized by a small momentum transfer to the nucleon (for a recoil-proton momentum less than 200–300 MeV/c it cannot be detected either from the range in a bubble chamber or from the balancing of the transverse momenta of the muon and pion). This is confirmed by the presence of similar events with a single fast pion and a visible slow recoil proton. It should be emphasized that in a large number of the events found the pion has such a large energy (cases have been observed where the pion energy was 20–30 GeV) that it cannot be explained as neutrino excitation of resonances in the πN system.

To interpret these peripheral events it is natural to assume that they occur as the result of neutrino (antineutrino) scattering on virtual mesons in the reaction

$$\nu A \to \mu\pi A, \tag{1}$$

where A is a nucleon or target nucleus (see Fig. 1 for more details).

At first glance it seems that the main contribution to reaction (1) must come from scattering on virtual pions,

$$\nu_\mu(\bar{\nu}_\mu)+\pi^0\to\mu^\mp+\pi^\pm. \tag{2}$$

However, it should be remembered that for large energies of the pion produced (corresponding to large invariant mass of the πN system) a transition to Regge asymptotic behavior must occur. In other words, in this region process (2) occurs on a Reggeized pion. The rapid falloff of the π trajectory contribution with energy makes it seem that in describing processes (1) the contribution of scattering on ω, ρ, and f mesons corresponding to trajectories which fall off more slowly with energy and also scattering on the Pomeron may be important. Study of the contribution of these processes would make it possible to obtain information on unobservable semileptonic decays of mesons (for example, $\omega\to\pi\mu\nu$) and to study the form factors of the corresponding meson (Reggeon)-pion transitions.

In the present study we estimate the contributions to process (1) from the different virtual mesons (Reggeons) (Sec. 4). We also discuss coherent processes of the production of single pions on nuclei due to the scattering of neutrinos on virtual isoscalar mesons (Sec. 5).

2. PION PRODUCTION IN THE SCATTERING OF NEUTRINOS ON VIRTUAL MESONS

We shall study the class of processes (1) described by the diagram in Fig. 1. Such a diagram with a pion pole in the t channel is usually studied along with s-channel pole diagrams for analyzing reaction (1) at threshold.[1] We are interested in the kinematical region of small momentum transfers in processes at high energies, where the contribution of the diagram in Fig. 1 dominates.[1)] To calculate the contribution of the individual mesons (M) to this diagram it is necessary to know the matrix elements of the weak transitions $M\to\pi$. Except for the case $M=\pi$, such transitions have not been observed experimentally. However, existing theoretical models permit reasonable estimates to be made for them.

Since mesons with zero strangeness have definite G parity, the G-parity selection rules (in the absence of second-class currents) lead to the fact that every such transition is either purely vector or purely axial vector. In Table I we give the variants corresponding to different meson-pion transitions. We note that processes in which pions are produced on isoscalar mesons ω, σ, and f (and also on the Pomeron) will give a coherent contribution to the neutrino production of pions on nuclei.[2)]

We see from Table I that scattering on virtual η and

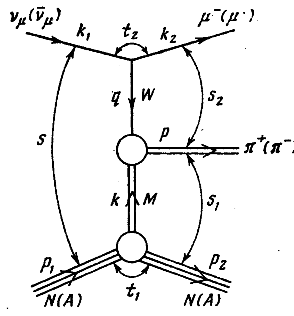

FIG. 1.

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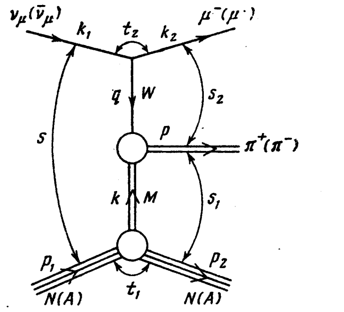

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TABLE I.

Transition	J^P, I^G of initial meson	Charged current: G- transition	Charged current: variant	Neutral current: $C_{\bullet\to\bullet}$- transition	Neutral current: variant
$\pi^0\to\pi$	0^-, 1^-	+	V	+	None
$\rho^0\to\pi$	1^-, 1^+	−	A	−	V
$\omega^0\to\pi$	1^-. 0^-	+	V	−	V
$\sigma^0\to\pi$	0^+. 0^+	−	A	+	A
$A_1^0\to\pi$	1^+. 1^-	+	V	+	None
$D_A^0\to\pi$	1^+. 0^+	−	A	+	A
$\eta^0\to\pi(\eta'\to\pi)$	0^-. 0^+	−	None	+	None
$A_2^0\to\pi$	2^+. 1^-	+	V	+	None
$f^0\to\pi$	2^+. 0^+	−	A	+	A

η' mesons does not contribute to process (1). In fact, the G-parity selection rules require that the transitions $\eta\to\pi$ and $\eta'\to\pi$ originate from the axial current which, on the other hand, is forbidden for transitions between pseudoscalar mesons.

To complete the picture, in Table I we give the transitions $M^0\to\pi^0$ corresponding to the process $\nu A\to\nu\pi^0 A$ due to neutral currents. In this case the choice of variant is specified by the C-parity selection rules.

The structures and constants of weak transitions due to the vector current can be obtained from CVC arguments. We have used the PCAC relations to estimate the constants of the axial transitions (Sec. 3).

The matrix element of reaction (1) is written in the form

$$M=\frac{G}{\sqrt{2}}l_\mu H_\mu, \tag{3}$$

where $l_\mu=\bar{u}_2\gamma_\mu(1+\gamma_5)u_1$ is the charged leptonic current and H_μ is the matrix element of the weak hadronic current. The diagram in Fig. 1 corresponds to the choice of H_μ in the form of a sum of scatterings on all the virtual mesons with the production of a pion:

$$H_\mu=\sum_M T^{(M)}P^{(M)}j_\mu^{(M)}, \tag{4}$$

where $T^{(M)}$ and $P^{(M)}$ symbolically denote the vertex where a virtual meson M is emitted by a nucleon of the target and the meson propagator and $j_\mu^{(M)}=\langle\pi|J_\mu|M\rangle$ is the matrix element of the weak hadronic current between the virtual-meson (M) and pion states.

The nucleon-meson vertices $T^{(M)}$ are determined by the effective constants used in the model of nuclear one-boson-exchange potentials[5] (see Sec. 3).

In describing process (1) the Reggeization of the mesons becomes important for sufficiently large invariant masses of the hadronic πN system. In this case the propagators $P^{(M)}$ must be chosen according to the Regge pole model and the vertices $T^{(M)}$ must be determined from the residues of the corresponding Regge trajectories.

Henceforth we shall restrict ourselves to inclusion of the contributions of the virtual π, σ, ω, ρ, and f mesons.[3)] In this approximation the current H_μ has the form

$$\begin{aligned}H_\mu=&\pm\frac{g_{\pi N}(t_1)(\bar{u}_2\gamma_5u_1)}{t_1-m_\pi^2}f_+(t_1,t_2)(p+k)_\mu\varphi_\pi^+\\&+\frac{g_{\sigma N}(t_1)(\bar{u}_2u_1)}{t_1-m_\sigma^2}f_\sigma(t_1,t_2)(p+k)_\mu\varphi_\pi^++\frac{g_{\omega N}(t_1)(\bar{u}_2\gamma_\nu u_1)}{t_1-m_\omega^2}f_\omega(t_1,t_2)\varepsilon_{\mu\nu\lambda\rho}q_\lambda k_\rho\varphi_\pi^+\\&\pm\frac{g_{\rho N}(t_1)(\bar{u}_2\gamma_\nu u_1)}{t_1-m_\rho^2}[f_{1\rho}(t_1,t_2)g_{\mu\nu}+f_{2\rho}(t_1,t_2)(p+q)_\nu k_\mu]\varphi_\pi^+\\&+\frac{g_{fN}(t_1)(\bar{u}_2\gamma_\nu u_1)(p_1+p_2)_\lambda}{t_1-m_f^2}P_{\sigma\tau}^{\nu\lambda}h_{\mu\sigma\tau}(t_1,t_2)\varphi_\pi^+,\end{aligned} \tag{5}$$

where

$$\begin{aligned}h_{\mu\sigma\tau}=&f_{1f}(t_1,t_2)[g_{\mu\sigma}(p+q)_\tau+g_{\mu\tau}(p+q)_\sigma]\\&+f_{2f}(t_1,t_2)(p+k)_\mu(p+q)_\sigma(p+q)_\tau\end{aligned}$$

and we have omitted terms proportional to q_μ which when contracted with the lepton current give the lepton mass, terms corresponding to vector-meson couplings of the form $(f_V/2m_N)\bar{u}_2\sigma_{\mu\nu}k_\nu u_1$ and proportional to the small momentum transfer to the nucleon, and also the terms corresponding to the tensor-gradient coupling of the f meson to the nucleon.[4)] We now show that the terms corresponding to the exchange of isovector π^0 and ρ^0 mesons have different signs for ν and $\bar{\nu}$ scattering: a plus sign for π^+ production and a minus sign for π^- production. In formula (5) $g_{MN}(t_1)$ are the form factors of the corresponding meson-nucleon couplings, which we shall take at zero momentum transfer; k_1, k_2, p, p_1, and p_2 are the four-momenta of the neutrino, muon, pion, and incident and final nucleons, respectively; $k_1-k_2=q, p_1-p_2=k$ (see Fig. 1); f_+, f_σ, f_ω, $f_{1\rho}$, $f_{2\rho}$, f_{1f}, and f_{2f} are the form factors of the weak transitions $\nu M\to\mu\pi$. We shall neglect the change in these quantities when the mesons go off the mass shell and shall assume that $f(t_1,t_2)\simeq f(0,t_2)\simeq f(m_M^2,t_2)$. The projection operator $P_{\sigma\tau}^{\nu\lambda}$ of the tensor 2^+ meson has the form

$$P_{\sigma\tau}^{\nu\lambda}=\frac{1}{2}\left(\bar{g}_{\nu\sigma}\bar{g}_{\lambda\tau}+\bar{g}_{\nu\tau}\bar{g}_{\lambda\sigma}-\frac{2}{3}\bar{g}_{\nu\lambda}\bar{g}_{\sigma\tau}\right),\quad \bar{g}_{\nu\lambda}=g_{\nu\lambda}-\frac{k_\nu k_\lambda}{M^2},$$

satisfies the conditions

$$k_\nu P_{\sigma\tau}^{\nu\lambda}=k_\lambda P_{\sigma\tau}^{\nu\lambda}=k_\sigma P_{\sigma\tau}^{\nu\lambda}=k_\tau P_{\sigma\tau}^{\nu\lambda}=0,\quad P_{\sigma\tau}^{\lambda\lambda}=P_{\sigma\sigma}^{\lambda\nu}=0,$$

and is symmetric under the interchanges $\lambda\to\nu$, $\sigma\to\tau$ and the pairs $(\nu\lambda)\to(\sigma\tau)$ (see Ref. 8, for example). Introducing the form factors

$$F_{1f}=4(p_1+p_2,q)\frac{g_{fN}f_{1f}}{t_1-m_f^2},\quad F_{2f}=4(p_1+p_2,q)\frac{g_{fN}f_{2f}}{t_1-m_f^2},$$

we can rewrite the term corresponding to the f meson in expression (5) in the form

$$H_\mu^f=\frac{F_{1f}}{2}\left[\bar{u}_2\gamma_\mu u_1+\frac{(\bar{u}_2\hat{q}u_1)}{(p_1+p_2,q)}(p_1+p_2)_\mu\right]+F_{2f}(\bar{u}_2\hat{q}u_1)(p+k)_\mu.$$

We note that the exchange of a high spin (2^+) leads to the appearance of coefficients in the effective form factors F_{1f} and F_{2f} which increase with energy.

Generally speaking, the inclusion of meson Reggeization leads to a considerable change in the form of formula (5). The contribution of the σ meson in the Regge region is not important.[6] On the other hand, at high energies it is possible to have pion production in the scattering of neutrinos on the Pomeron induced by the axial current according to the G-parity selection rules.

In each term (except that corresponding to the f meson) the pole expressions $F_M = g_{MN} f_{M_\pi} / (t_1 - m_M^2)$ and, in the case of the f meson, the effective form factors F_{1f} and F_{2f} become

$$\Phi_R(t_1, t_2, s_2) = \Phi_R^0(t_1, t_2, s_2)\,\eta_M \left(\frac{s_1}{s_0}\right)^{\alpha_M(0)-1},$$

where η_M and α_M are the signature and trajectory of the corresponding Reggeon, $\Phi_R^0(t_1, t_2, s_2)$ is a function determining the weak Reggeon-pion transition, and s_0 is the scale parameter $s_0 = 2M_N E_0$ with $E_0 = 1$ GeV. In addition, in transitions induced by the axial current the covariant structures change and each term in H_μ corresponding to ρ, f, and P trajectories must in general change to the form of a sum

$$\Phi_1(p+k)_\mu + \Phi_2 p_{1\mu} + \Phi_3 q_\mu.$$

When the momentum transfer to the nucleon is small the quantity $p_1 q$ is practically independent of the azimuthal Treiman-Yang angle and for $s \gg M_N^2$ (where M_N is the nucleon mass) is

$$p_1 q = -\frac{s t_2}{2 s_2}, \tag{6}$$

where we have introduced the invariant variables (see Fig. 1)

$$s = (k_1 + p_1)^2,\quad s_1 = (q + p_1)^2 = (p_2 + p)^2,$$
$$s_2 = (k_2 + p)^2,\quad t_1 = k^2,\quad t_2 = q^2.$$

In these variables the differential cross section of reaction (1) has the form (for $s \gg M_N^2$; s_1, $|t_2| \gg |t_1|, m_\pi^2, m_\mu^2$; $|t_1| \ll m_\sigma^2, m_\omega^2, m_\rho^2, m_f^2$)

$$\frac{d^4\sigma}{dt_1\, dt_2\, ds_2\, d\varphi} = \frac{\overline{|M|^2}}{(2\pi)^4 \cdot 32 (s - M_N^2)^2 (s_2 - t_1)}$$

$$\simeq \frac{G^2}{(2\pi)^4 \cdot 8 s^2 s_2}\Big\{ -2 t_1 s_2^2 \left(1 + \frac{t_2}{s_2}\right) |F_\pi|^2 + 2\left(1 + \frac{t_2}{s_2}\right) s^2 \left(1 - \frac{s_2}{s}\right)$$
$$\times |\pm F_\rho + F_f|^2 + 2\left(1 + \frac{t_2}{s_2}\right) s_2^2 |F_\sigma|^2 \tag{7a}$$

$$-2M_N^2 s_2^2 t_2 \left(1 + \frac{t_2}{2 s_2}\right)^2 |F_\omega|^2 + 4\left(1 + \frac{t_2}{s_2}\right) s s_2 \left(1 - \frac{s_2}{2s}\right) \mathrm{Re}\, F_\sigma^+ (\pm F_\rho + F_f)$$
$$+ 8 s \left(1 + \frac{t_2}{2 s_2}\right)\left(1 - \frac{s_2}{2s}\right) \{k_1 p_1 q k\}\, \mathrm{Re}\, F_\omega^+ (\pm F_\rho + F_f) \tag{7b}$$
$$+ 8 s_2 \left(1 + \frac{t_2}{2 s_2}\right) \{k_1 p_1 q k\}\, \mathrm{Re}\, F_\omega^+ F_\sigma$$

$$\pm \{k_1 p_1 q k\}\, \mathrm{Im}\Big[\pm \frac{2 s_2}{2s - s_2} F_{1\rho} F_{1f}^+ \mp 8 \frac{t_2}{s_2}\left(s - \frac{s_2}{2}\right) F_{1\rho} F_{2f}^+ \tag{7c}$$
$$\pm 8 \frac{t_2}{s_2}\left(s - \frac{s_2}{2}\right) F_{2\rho} F_{1f}^+ + 4 (F_{1f} \pm F_{1\rho}) F_\sigma^+ \Big]$$

$$\pm 4 M_N^2 t_2 s_2 \left(1 + \frac{t_2}{2 s_2}\right) \mathrm{Im}\, F_\omega^+ (F_{1f} \pm F_{1\rho}) \Big\}, \tag{7d}$$

where φ is the Treiman-Yang angle, $\{k_1 p_1 q k\} \equiv \varepsilon_{\mu\nu\lambda\sigma} k_{1\mu} p_{1\nu} q_\lambda k_\sigma$,

$$F_\pi = \frac{g_{\pi N} f_+}{t_1 - m_\pi^2},\quad F_\sigma = \frac{2 M_N g_{\sigma N} f_\sigma}{m_\sigma^2},\quad F_\omega = \frac{g_{\omega N} f_\omega}{m_\omega^2},$$

$$F_{1,2\rho} = \frac{g_{\rho N} f_{1,2\rho}}{m^2},\quad F_{1,2f} = 4 (p_1 + p_2, q) \frac{g_{fN} f_{1,2f}}{m_f^2} \tag{8}$$
$$F_\rho = F_{1\rho} - q^2 F_{2\rho},\quad F_f = F_{1f} - q^2 F_{2f}.$$

We note that in the case of scattering on an unpolarized target the pion amplitude does not interfere with the other amplitudes.

In expression (7) the terms (7b) correspond to P-odd effects due to the interference of the vector and axial-vector hadronic currents. These effects manifest themselves in an asymmetry in the Treiman-Yang angle. In the lab frame this term determines the correlation $[(\mathbf{p} \times \mathbf{k}_2) \cdot \mathbf{p}_2]$, which shows up in an asymmetry in the angle between the planes $(\mathbf{p}, \mathbf{k}_2)$ and $(\mathbf{p}, \mathbf{p}_2)$ containing the moment of the pion ($\mathbf{p}$) and muon ($\mathbf{k}_2$) and the pion ($\mathbf{p}$) and recoil nucleon ($\mathbf{p}_2$). To study these effects it is necessary to detect the recoil nucleon.

Terms (7d) and (7c) correspond to the interference of the leptonic vector and axial currents and have different signs in the case of neutrino (upper sign) and antineutrino (lower sign) scattering. Term (7c) corresponds to the asymmetry in the Treiman-Yang angle. Term (7d) also corresponds to the interference of the hadronic vector and axial-vector currents and does not disappear when the Treiman-Yang angle is averaged over. The upper signs in formula (7) and in the following refer to the case of neutrino scattering and the lower signs refer to antineutrino scattering.

Distinguishing the difference between the neutrino and antineutrino cross sections for the production of fast pions therefore allows us to obtain information on the relative signs of the different contributions. The Regge pole theory predicts completely definite relations between the real and imaginary parts of the amplitudes. We note that the difference between the ν and $\bar{\nu}$ cross sections is sensitive to the presence of imaginary parts in the amplitudes which are not related to the Regge behavior: if the non-Regge amplitudes are real, terms (7c) and (7d) become zero in the non-Regge region.

The differential cross section (7) integrated over the Treiman-Yang angle and small momentum transfers t_1 is of interest for analyzing the experimental data. If the momentum of the recoil proton is restricted by the condition $|\mathbf{p}_2| \leqslant \alpha M_N$ ($\alpha < 1$), the integration over t_1 must be carried out between the limits

$$\frac{M_N^2 s_2^2}{s^2} \leqslant |t_1| \leqslant 2(\sqrt{\alpha^2 + 1} - 1) \simeq \alpha^2 M_N^2,\ \alpha \ll 1.$$

The differential cross section thus obtained characterizes both processes (1) with an observable slow recoil nucleon and processes in which the energy of the recoil proton is smaller than the detection threshold. The general form of this cross section is simplified if we change over to the standard variables used to describe deep inelastic scattering, $x = -q^2/2(p_1 q)$ and $y = p_1 q / p_1 k_1$. The conditions for the applicability of the one-meson exchange and Regge pole models also become clear in the plane of the variables (x, y). Taking into account relation (6), we have

$$x \simeq \frac{s_2}{s},\quad y = -\frac{t_2}{s_2} \simeq \frac{2(p_1 q)}{s} \simeq \frac{s_1}{s}. \tag{9}$$

In Fig. 2 we show the plane of the variables x and y. Since $(s_2)_{\max} = (s/M_N)\sqrt{|t_1|_{\max}} \simeq \alpha s$, the kinematical region of reaction (1) for $|t_1| \leqslant \alpha^2 M_N^2$ ($\alpha \ll 1$) consists of the strips $0 \leqslant y \leqslant 1$ and $0 \leqslant x \leqslant \alpha$. The hyperbolas $xy = -(q^2/s)$ ($s \gg M_N^2$) in Fig. 2 correspond to fixed q^2 and the hyperbolas $y(1-x) = s_1/s$ correspond to fixed s_1.

Hyperbola A is $xy = \Lambda^2/s$, where $\Lambda^2 \simeq 1$ GeV^2 arbitrarily divides the (x, y) plane into a region of large q^2

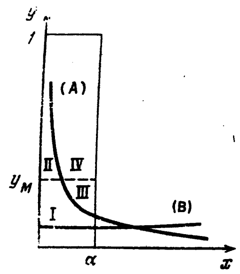

FIG. 2.

(above hyperbola A) in which effects of the virtual meson structure appear in reaction (1) (here q^2 is the dependence of the form factors) and a region of small q^2 in which such effects are unimportant.

Hyperbola B is $y(1-x)=M_R^2/s$, where $M_R^2 \simeq 2$–$3\ \mathrm{GeV}^2$ distinguishes the region (above hyperbola B) in which reaction (1) can proceed via neutrino excitation of resonances in the πN system. Therefore, to experimentally distinguish processes of scattering on virtual mesons it is necessary to choose the region of reaction (1) lying above hyperbola B.

We cannot unambiguously indicate the boundary of the meson and Regge regions in Fig. 2, because of the uncertainties in the theoretical ideas about the transition to Regge behavior of the amplitudes. We certainly can assume that for sufficiently large $y=s_1/s \gtrsim s_R/s=y_R$, with $s_R \simeq 20\ \mathrm{GeV}^2$, the Regge description is valid. If this description is also valid for fairly small y (down to the region of s-channel nucleon resonances), then when there is kinematical separation from the contribution of the resonances of the πN system, reaction (1) above hyperbola B will be determined by the scattering of neutrinos on Reggeons. However, it is possible that Regge asymptotic behavior sets in late enough so that for $y<y_M=s_M/s$, with $s_M \simeq 15\ \mathrm{GeV}^2$ (Ref. 9), the idea about scattering on virtual mesons is justified. In Fig. 2 we show the characteristic regions into which in this case the kinematical region of reaction (1) above the nucleon resonance region (hyperbola B) can be divided. For $s_M/s<y<s_R/s$ there is a changeover from a meson description to a Regge description. Experimental study of this transition region is of great interest, since it would permit important information to be obtained on the physical nature of the transition to Regge asymptotic behavior.

Region I in Fig. 2 ($0 \leqslant x \leqslant \Lambda^2/sy$ and $M_R^2/s(1-x) \leqslant y \leqslant s_M/s$) corresponds to scattering on virtual structureless mesons.

In region III it is important to take into account the structure of the virtual mesons (the q^2 dependence of the form factors). It should be noted that the existence of region III is due to the relation between the parameters Λ^2 and s_M: $\Lambda^2 \leqslant \alpha s_M$; for $\alpha s_M < \Lambda^2$ there is no kinematical region III in reaction (1).

The Regge region $s_R/s \lesssim y \leqslant 1$ also separates into a region of scattering on structureless Reggeons (region II: $s_R/s \lesssim y \leqslant 1$, $0 \leqslant x \leqslant \Lambda^2/sy$) and region IV, where the inclusion of the q^2 dependence of the form factors of the Reggeon-pion transitions is important.

We note that when the momentum transferred to the nucleon is small the variable y is directly related to the observable energy of the final pion: $y=E_\pi/E_\nu$.

The double-differential cross section of reaction (1) $d^2\sigma/dxdy$ has the form (for $s_2 \ll s$)

$$\frac{d^2\sigma}{dx\,dy}=\frac{G^2M_N^2(\alpha^2-x^2)s}{32\pi^3}\left\{(1-y)|F_A|^2+M_N^2x^3ys\left(1-\frac{y}{2}\right)^2|F_\omega|^2\right.$$
$$+\frac{x^2(1-y)}{(\alpha^2-x^2)M_N^2}\left[\ln\frac{\alpha^2+m_\pi^2/M_N^2}{x^2+m_\pi^2/M_N^2}-\frac{m_\pi^2(\alpha^2-x^2)}{M_N^2(\alpha^2+m_\pi^2/M_N^2)(x^2+m_\pi^2/M_N^2)}\right]$$
$$\left.\times\, g_{\pi N}^2f_+^2 \mp 2M_N^2x^2y\left(1-\frac{y}{2}\right)\mathrm{Im}\,F_\omega^+(F_{1f}\pm F_{1\rho})\right\}, \qquad (10)$$

where the transitions due to the axial hadronic current appear in the combinations

$$F_A=F_f\pm F_\rho+xF_\sigma, \qquad (11)$$

and the effective form factors corresponding to $f\pi$ and $\rho\pi$ transitions enter in the form (8).

The contributions of the $\pi\pi$ and $\omega\pi$ transitions and the set of meson-pion transitions due to the axial hadronic current have been separated in expression (10).

The transition to Regge asymptotic behavior is accomplished by the substitution

$$F_\pi\to\Phi_{\pi R},\ F_\omega\to\Phi_{\omega R},$$

and instead of expression (11) for F_A we have

$$F_A\longrightarrow\pm\Phi_{\rho R}^0\eta_\rho\left(\frac{s_1}{s_0}\right)^{\alpha_\rho(0)-1}+\Phi_{fR}^0\eta_f\left(\frac{s_1}{s_0}\right)^{\alpha_f(0)-1}+\Phi_P\eta_P. \qquad (12)$$

In expression (12) we have added the term $\Phi_P\eta_P$ ($\eta_P=i$ is the Pomeron signature) corresponding to the production of pions in neutrino scattering on Pomerons and have omitted the contribution of the σ trajectory, which falls off rapidly with energy.

The separation of the unique combination (11) and (12) in expression (10) is easily understood in terms of the formalism developed in Ref. 10. In fact, at small momentum transfers the main contribution to the matrix element of the axial current in the Regge region is given by the term

$$\bar{u}_2\gamma_\nu^5\, u_1g_{\mu\nu}(\pm\Phi_\rho+\Phi_f+\Phi_P)=\frac{2}{s_1}(\bar{u}_2\hat{q}u_1)\,p_{1\mu}(\pm\Phi_\rho+\Phi_f+\Phi_P)\simeq F_A(p_1+p_2)_\mu, \qquad (13)$$

corresponding in expression (5) to the $F_{1\rho}$ contribution of the ρ meson and the F_{1f} contribution of the f meson.

3. CONSTANTS AND TRANSITION AMPLITUDES

a) Meson-nucleon interaction constants

The values of the constants characterizing the vertices of the meson-nucleon interaction which enter into the expression for the current H_μ (5) are well known for π and σ mesons: $g_{\pi N}^2/4\pi \simeq 15$, $g_{\sigma N}^2/4\pi \simeq 6$. The accuracy with which the vector meson constants are known is considerably worse. We shall take the maximum value of $g_{\rho N}$ obtained assuming complete dominance of the ρ meson in the isovector nucleon form factor: $g_{\rho N}=g_{\rho\pi\pi}=g_\rho$, where $g_{\rho\pi\pi}$ is the constant of the decay $\rho^0\to\pi^+\pi^-$ (see Ref. 11). For the ω-nucleon coupling constant from the quark model we have $g_{\omega N} \simeq 3g_{\rho N}$, which gives $g_{\omega N}^2/4\pi \simeq 25$. To estimate the interaction of the tensor f meson with the nucleon we shall use the quark model and the data on the probability for the decay $f\to 2\pi$. The amplitude of the decay $f^0\to\pi^+\pi^-$ has the form

$$M_{f\to\pi^+\pi^-}=g_{f\pi^+\pi^-}Q_\mu Q_\nu\varphi_1^+\varphi_2^+F_{\mu\nu}, \tag{14}$$

where $Q_\mu=k_{1\mu}-k_{2\mu}$ and $\varphi_1^+(k_1)$ and $\varphi_2^+(k_2)$ are the wave functions (momenta) of the pions. For the matrix element (14) the probability of the decay $f\to\pi^+\pi^-$ is given by the formula

$$\Gamma(f\to\pi^+\pi^-)=g^2_{f\pi^+\pi^-}m_f^5/120\pi, \tag{15}$$

from which using $\Gamma_{f\to\pi^+\pi^-}=(2/3)\Gamma^{\text{tot}}_{f\to2\pi}\simeq 96$ MeV, we find $g^2_{f\pi^+\pi^-}/4\pi\simeq 3/2M_N^2$. Assuming that the constants $g_{f\pi^+\pi^-}$ and g_{fN} are determined by the coherent emission of an f meson by the quarks making up the π meson and the nucleon, we find

$$g_{fN}={}^3/_2 g_{f\pi^+\pi^-},\quad g^2_{fN}/4\pi=27/8M_N^2$$

or for the dimensionless constant $\tilde{g}_{fN}=2M_N g_{fN}$, $\tilde{g}^2_{fN}/4\pi\simeq 14$, which is in agreement with the results of the analysis of the contribution of the f meson in πN and NN scattering at low energies.[7]

b) The constants of weak meson-pion transitions

The value of $f_+(0)$ is determined from the probability of the decay $\pi^+\to\pi^0 e\nu$ and is equal to $\sqrt{2}$. The constant $f_\omega(0)$ can be related to the constant of the decay $\omega\to\pi\gamma$ using the CVC hypothesis. We have

$$f_\omega(0)=(\sqrt{2}/e)\,g_{\omega\pi\gamma}, \tag{16}$$

where $g_{\omega\pi\gamma}$ is the constant characterizing the matrix element of the decay $\omega\to\pi\gamma$,

$$M_{\omega\to\pi\gamma}=g_{\omega\pi\gamma}e_{\mu\nu\lambda\rho}k_\mu\phi_\nu q_\lambda e_\rho.$$

The probability for the decay $\omega\to\pi\gamma$ has the form $\Gamma_{\omega\to\pi\gamma}=g^2_{\omega\pi\gamma}m_\omega^3/96\pi$, from which

$$f_\omega^2(0)=192\pi\Gamma_{\omega\to\pi\gamma}/e^2m_\omega^3. \tag{17}$$

Some information on the q^2 dependence of $f_\omega(q^2)$ in the timelike region can be obtained from the experimental data on the electromagnetic decay $\omega^0\to\pi^0\mu^+\mu^-$ (Ref. 12).

The constant of the axial $\rho\pi$ transition can be obtained using PCAC considerations. In the limit of soft pions we have the following for the matrix element of the weak current:

$$\langle\pi^+|A_\mu^+|\rho^0\rangle\xrightarrow[p_\pi\to 0]{}\frac{i}{f_\pi}\langle 0|[Q_5^-,A_\mu^+]|\rho^0\rangle=\frac{2i}{f_\pi}\langle 0|V_{3\mu}|\rho^0\rangle=\frac{2i}{f_\pi}\frac{m_\rho^2}{g_\rho}\rho_\mu, \tag{18}$$

from which

$$f_{1\rho}(0)=2m_\rho^2/f_\pi g_\rho, \tag{19}$$

for $f_\pi\approx m_\pi$ is the constant of the decay $\pi\to\mu\nu$.

On the other hand, we can use the PCAC hypothesis to relate the divergence of the matrix element of the weak $\rho\pi$ transition to the amplitude of the decay $\rho\to2\pi$:

$$iq_\mu\langle\pi^+|A_\mu^+|\rho^0\rangle=if_\pi\langle\pi\pi|\rho\rangle=if_\pi g_{\rho\pi\pi}Q_\nu\rho_\nu, \tag{20}$$

from which

$$f_\rho(0)=f_{1\rho}(0)=2f_\pi g_{\rho\pi\pi}.$$

Assuming $g_\rho=g_{\rho\pi\pi}$, from relations (19) and (20) we have the familiar KSRF relation[13]

$$f_\pi^2=m_\rho^2/g_\rho^2.$$

Therefore, the estimate of the form factor $f_\rho(0)$ is self-consistent within the framework of the PCAC hypothesis.

PCAC considerations also permit us to obtain restrictions on the value of the constant of the $f\pi$ transition, relating this transition to the amplitude of the decay $f\to\pi^+\pi^-$. In fact, according to PCAC

$$iq_\mu\langle\pi^+|A_\mu^+|f\rangle=f_\pi\langle\pi^+\pi^-|f\rangle,$$

from which we find

$$f_{1f}(0)=f_\pi g_{f\pi^+\pi^-}. \tag{21}$$

For estimates for the σ meson we shall set $f_\sigma(0)=1$.[5]

c) Amplitudes in the Regge region

It is interesting to make an estimate of the amplitude in the Regge region which is independent of assumptions about the residues. The CVC hypotheses can be used to relate the value of $\Phi_{\pi R}$ to the amplitude for photoproduction of charged pions at small momentum transfers[14]; here for the value of $\Phi_{\pi R}$ we obtain

$$\Phi_{\pi R}=F_\pi(s_1/s_0)^{\alpha_\pi(0)-1}.$$

Due to the rapid falloff with energy of the amplitude for scattering on a Reggeized pion the contribution of the latter (together with the contribution of the σ) to the differential cross section (10) will be neglected in the Reggeized region.

The vector dominance hypothesis can be used to relate the amplitude $\Phi_{\omega R}$ characterizing the contribution of the ω trajectory to the strong amplitude of the reaction $\pi N\to\rho N$ at high energies.

The amplitude for $\pi N\to\rho N$ has the form

$$A(\pi N\to\rho N)=\rho_\mu e_{\mu\nu\lambda\rho}(\bar{u}_2\gamma_\nu u_1)q_\lambda k_\rho\Phi^\omega_{\pi\rho}; \tag{22}$$

then because the vector dominance

$$\Phi_{\omega R}=(\sqrt{2}/g_\rho)\Phi^\omega_{\pi\rho}. \tag{23}$$

For the matrix element (22) the differential cross section for the reaction $\pi N\to\rho N$ has the form

$$\frac{d\sigma}{dt}(\pi N\to\rho N)=\frac{t}{16\pi}|\Phi^\omega_{\pi\rho}|^2.$$

The experimental data[15] on the reaction $\pi N\to\rho N$ at small angles on ^{64}Cu nuclei at $E_\pi=150$ GeV permit information to be obtained on the value of $|\Phi^\omega_{\pi\rho}|^2$. According to the results of Ref. 15, the main contribution to the strong amplitude comes from the ω trajectory. For the quantity

$$\frac{|\Phi^0_{\pi\rho}|^2}{16\pi}=\frac{|\Phi^\omega_{\pi\rho}|^2}{16\pi}\frac{s}{s_0}$$

we have

$$\frac{|\Phi^0_{\pi\rho}|^2}{16\pi}\approx\frac{8\cdot10^4}{A^\varkappa}\frac{\text{mb}}{\text{GeV}/c^4}. \tag{24}$$

The value of $\varkappa$ characterizes the effect of the screening of the nucleons and the absorption of mesons in the nucleus. In the absence of these effects $\varkappa=2$. When these effects are taken into account we must have $\varkappa<2$.

As a reasonable estimate we can take $\varkappa = 4/3$.[16]

The PCAC relations permit the combination F_A, Eqs. (11) and (12), due to the axial hadronic current to be related to the elastic πN scattering amplitude. According to (13),

$$q_\mu \langle \pi N | A_\mu | N \rangle = q_\mu F_A (p_1 + p_2)_\mu \simeq s_1 F_A = f_\pi T_{\pi N}, \qquad (25)$$

where $T_{\pi N}$ is the πN scattering amplitude determining the differential cross section for elastic πN scattering:

$$\frac{d\sigma}{dt} = 4\pi \left| \frac{T_{\pi N}}{8\pi s_1} \right|^2.$$

From relation (25) we obtain

$$F_A = (f_\pi / s_1) T_{\pi N}. \qquad (26)$$

4. NUMERICAL ESTIMATES

a) Density of events in the (x, y) plane and estimate of the cross section for pion production on virtual mesons

The above estimates of the constants and amplitudes permit us to obtain the distribution of events with single pions in the (x, y) plane. In the one-meson exchange region from formula (10) (for $x \ll 1$ and $y < s_M/s \ll 1$) we find the following expression for the differential cross section:

$$\frac{d\sigma}{dx dy} = \frac{G^2 M_N^2 (\alpha^2 - x^2) s}{32\pi^3} \{ [\pm F_\rho(0) F_\rho(q^2)$$
$$+ x F_\sigma(0) F_\sigma(q^2) + ys \varphi_f(0) \Phi_f(q^2)]^2 + M_N^2 x^2 ys F_\omega^2(0) F_\omega^2(q^2)$$
$$+ x^2 B(\alpha, \beta, x) \varphi_\pi^2(0) \Phi_\pi^2(q^2) \}, \qquad (27)$$

where according to relations (15)–(17) and (19)–(21) we have

$$F_\rho(0) = \frac{2}{f_\pi}, \quad \varphi_f(0) = \frac{480\pi f_\pi \Gamma^{tot}_{f\to 2\pi}}{m_f^5}, \quad F_\omega^2(0) = \frac{192\pi \Gamma_{\omega\to\pi\gamma} g_{\omega N}^2}{e^2 m_\omega^7}$$

$$B(\alpha, \beta, x) = \frac{1}{\alpha^2 - x^2} \ln \frac{\alpha^2 + \beta^2}{x^2 + \beta^2} - \frac{\beta^2}{(\alpha^2 + \beta^2)(x^2 + \beta^2)},$$

$$\beta = \frac{m_\pi}{M_N}, \quad \varphi_\pi^2(0) = \frac{2 g_{\pi N}^2}{M_N^2}.$$

For $x \to 0$ only the contributions of the ρ and f mesons remain in formula (27):

$$\frac{d\sigma}{dx\, dy} \xrightarrow[x \to 0]{} \frac{G^2 M_N^2 \alpha^2 s}{32\pi^3} \left(\pm \frac{2}{f_\pi} + \frac{480\pi f_\pi \Gamma_{f\to 2\pi}}{m_f^5} ys \right)^2. \qquad (28)$$

In (27), the contribution of each meson is characterized by a certain dependence on x and y, which makes it possible in principle to separate the effects of scattering on each virtual meson and to study the q^2 dependence of the corresponding meson-pion transitions.

It is interesting to check the energy dependence of the processes of scattering on vector and tensor mesons. The growth obtained in formulas (27) and (28) with $s_1 = ys$ is due to the effects of spin exchange in the t channel. The vector dominance and PCAC relations relate F_A and F_ω to the corresponding hadronic processes in which there is no such growth with energy. Therefore, the experimental study of regions I and III (Fig. 2) for $s_1 < s_M \sim 15$ GeV^2 makes it possible to study spin effects in the t channel and to verify the vector dominance and PCAC relations in this kinematical region.

From formula (27) we find the following for the value of the charge-asymmetry effect in the single-meson region:

$$\frac{d\sigma^\nu}{dx\, dy} - \frac{d\sigma^{\bar\nu}}{dx\, dy} = \frac{120 G^2 M_N^2 s^2 \Gamma^{tot}_{f\to 2\pi}}{\pi^2 m_f^5} (\alpha^2 - x^2) y F_\rho(q^2) \Phi_f(q^2). \qquad (29)$$

The increase with energy which is proportional to ys in formula (29) is also related to t-channel spin effects.

The total cross section for fast pion production is the sum $\sigma^{tot} = \sigma_\pi + \sigma_\sigma + \sigma_\omega + \sigma_\rho + \sigma_f \pm \sigma_{\rho f}$, where $\sigma_{\rho f}$ is the term corresponding to interference of the ρ and f mesons, which determines the difference between the neutrino and antineutrino cross sections: $\sigma^\nu - \sigma^{\bar\nu} = 2\sigma_{\rho f}$ (we shall neglect the interference of the ρ and f mesons with the σ meson). Let us assume that all the form factors have the universal q^2 dependence

$$F_M(q^2) = \frac{1}{1 - q^2/\Lambda^2}, \qquad (30)$$

where $\Lambda^2 \simeq 1$ GeV^2; in the limits of small and large $\gamma = \alpha s_M / \Lambda^2$ we obtain simple asymptotic expressions for the total cross sections:

$$\sigma^{tot}(\gamma \ll 1) = \frac{G^2 M_N^2}{32\pi^3} \left\{ s_M \alpha^3 \varphi_\pi^2(0) \left[\frac{2}{9} - \frac{\beta^2 (4\alpha^2 + 5\beta^2)}{3\alpha^2 (\alpha^2 + \beta^2)} + \frac{5}{3} \frac{\beta^3}{\alpha^3} \operatorname{arc\,tg} \frac{\alpha}{\beta} \right] \right.$$
$$+ \frac{2}{15} s_M \alpha^5 F_\sigma^2(0) + \frac{s_M^2 M_N^2 \alpha^6}{24} F_\omega^2(0) + \frac{2}{3} s_M \alpha^3 F_\rho^2(0) + \frac{2}{9} s_M^3 \alpha^3 \varphi_f^2(0)$$
$$\left. \pm \frac{2}{3} s_M^2 \alpha^3 F_\rho(0) \varphi_f(0) \right\}, \qquad (31)$$

$$\sigma^{tot}(\gamma \gg 1) = \frac{G^2 M_N}{32\pi^3} \left\{ \frac{\Lambda^2 \alpha^2 \varphi_\pi^2(0)}{2} \left(\frac{\alpha^2 + 2\beta^2}{\alpha^2 + \beta^2} - 2 \frac{\beta^2}{\alpha^2} \ln \frac{\alpha^2 + \beta^2}{\beta^2} \right) \right.$$
$$+ \frac{\Lambda^2 \alpha^4}{4} F_\sigma^2(0) + \frac{\Lambda^4 \alpha^4}{4} M_N^2 F_\omega^2(0) \left[\ln(1+\gamma) - \frac{\gamma}{1+\gamma} - \frac{3}{4} \right]$$
$$\left. + \Lambda^2 \alpha^2 F_\rho^2(0) \left[\ln(1+\gamma) - \frac{1}{2} \right] + \frac{\Lambda^2 s_M \alpha^2}{2} \varphi_f^2(0) \pm 2\Lambda^2 s_M \alpha^2 F_\rho(0) \varphi_f(0) \right\}. \qquad (32)$$

In Table II we give the numerical estimates of the cross sections for different choices of the parameter s_M and for the case where the momentum transferred to the nucleon is the maximum $|t_1|_{max} \simeq (1-2) m_\pi^2$, which corresponds to $\alpha \simeq 1/7 - 1/5$.

Although the differential cross section (27) increases as s with energy, the size of regions I and III decreases as $1/s$, so the total cross section is constant[6)] and its value together with the ratio of the contributions of the various mesons is important in determining the value of the parameters s_M and Λ^2.

The relative values of the difference of the cross sections $A = (\sigma^\nu - \sigma^{\bar\nu})/(\sigma^\nu + \sigma^{\bar\nu})$ of reaction (1) in ν and $\bar\nu$ beams is determined almost only by the parameter s_M in regions I and III.

TABLE II. Contributions of different mesons to the total cross section in the single-meson region for two choices of the parmeters s_M and α

	σ, cm^2						
	σ_π	σ_σ	σ_ω	σ_ρ	σ_f	$\sigma_{\rho f}$	$A = \frac{\sigma^\nu - \sigma^{\bar\nu}}{\sigma^\nu + \sigma^{\bar\nu}}$
$s_M = 2$ GeV^2, $\alpha = 1/7$	$5\cdot10^{-42}$	$2\cdot10^{-42}$	$6\cdot10^{-43}$	$3\cdot10^{-41}$	$2\cdot10^{-41}$	$4\cdot10^{-41}$	0.8
$s_M = 20$ GeV^2, $\alpha = 1/5$	$8\cdot10^{-41}$	$6\cdot10^{-41}$	$2\cdot10^{-41}$	$3\cdot10^{-40}$	$2\cdot10^{-38}$	10^{-38}	0.5

b) The cross section for pion production on reggeons

Using relations (23) and (25), in the Regge region from formula (10) (for $y > s_R/s$) we find

$$\frac{d\sigma}{dx\,dy}=\frac{G^2M_N^2(\alpha^2-x^2)s}{32\pi^3}\left\{16\pi f_\pi^2(1-y)\left(\frac{d\sigma^\pm}{dt}\Big|_{t=0}\right)\Phi_A^2(q^2)\right.$$
$$+M_N^2x^3ys\left(1-\frac{y}{2}\right)^2\frac{2}{g_\rho^2}|\Phi_{\pi\rho}^{\omega}|^2\tilde{\Phi}_\omega^2(q^2)$$
$$\left.\mp 2M_N^2x^2y\left(1-\frac{y}{2}\right)\frac{\sqrt{2}}{g_\rho}f_\pi\,\mathrm{Re}\,\Phi_{\pi\rho}^{\omega}\sigma_{tot}^{\pi^\pm N}\Phi_A(q^2)\Phi_\omega(q^2)\right\}, \tag{33}$$

where $d\sigma^\pm/dt|_{t=0}$ is the differential cross section for $\pi^\pm N$ scattering

$$\frac{d\sigma^\pm}{dt}\Big|_{t=0}=4\pi\left|\gamma_P\eta_P+\gamma_f\eta_f\left(\frac{ys}{s_0}\right)^{\alpha_f(0)-1}\mp\gamma_\rho\eta_\rho\left(\frac{ys}{s_0}\right)^{\alpha_\rho(0)-1}\right|^2,$$

and γ, η, and $\alpha(0)$ are the residue, signature, and value of $\alpha(0)$ for the P, f, and ρ trajectories. For the axial combination Φ_A we assume the single q^2 dependence of the form factor

$$\tilde{\Phi}_A(q^2)=\Phi_A(q^2)/\Phi_A(0).$$

Numerical estimates show that the main contribution in the Regge region for $s_R/s<y<1$ comes from axial transitions in which the weak Pomeron-pion transition dominates. We see from formula (33) that for $y\to 1$ the contribution of the ω trajectory begins to dominate in the differential cross section. Therefore, study of the distribution in y in the Regge region makes it possible in principle to separate the contributions of the vector and axial-vector currents.

For $x\to 0$, which corresponds to region II in Fig. 2, we have

$$\frac{d\sigma}{dx\,dy}=\frac{G^2M_N^2\alpha^2s}{32\pi^3}16\pi f_\pi^2(1-y)\left(\frac{d\sigma^\mp}{dt}\Big|_{t=0}\right). \tag{34}$$

In the Regge region the difference of the ν and $\bar{\nu}$ cross sections is determined by both the asymmetry due to the interference of the ρ and the Pomeron and by the P-odd asymmetry related to the interference of the leptonic and hadronic vector and axial-vector currents. For the difference of the $\bar{\nu}$ and ν cross sections we have

$$\frac{d\sigma^{\bar\nu}}{dx\,dy}-\frac{d\sigma^{\nu}}{dx\,dy}=\frac{G^2M_N^2(\alpha^2-x^2)s}{32\pi^3}\left\{16\pi f_\pi^2(1-y)\left(\frac{d\sigma^-}{dt}-\frac{d\sigma^+}{dt}\Big|_{t=0}\right)\right.$$
$$\left.\times\Phi_A^2(q^2)+2M_N^2x^2y\left(1-\frac{y}{2}\right)\frac{\sqrt{2}}{g_\rho}\mathrm{Re}\,\Phi_{\pi\rho}^{\omega}(\sigma_{tot}^{+}+\sigma_{tot}^{-})\tilde{\Phi}_A(q^2)\tilde{\Phi}_\omega(q^2)\right\}. \tag{35}$$

Investigation of the q^2 dependence of the differential cross sections (33) and (35) makes it possible to study the form factors of weak Reggeon-pion transitions.

Data on the diffractive dissociation of pions on nucleons[17] indicate that the differential cross section for the dissociation of a pion into a hadronic system of mass M falls off with increasing M as $1/M^4$. Reaction (1) in the Regge region for large q^2 can be viewed in the hadronic block as the inverse of diffractive dissociation of a pion into a system with $M=\sqrt{|q^2|}$ and can be chosen for making estimates of the q^2 dependence of the form factors in the form (30). Then for large $|q^2|=sxy\gg\Lambda^2$ from formula (33) we find

$$\frac{d\sigma}{dx\,dy}=\frac{G^2M_N^2(\alpha^2-x^2)\Lambda^4}{32\pi^3}\left\{\frac{16\pi f_\pi^2}{sx^2y^2}(1-y)\left(\frac{d\sigma^\pm}{dt}\Big|_{t=0}\right)\right.$$
$$+2M_N^2\frac{s_0}{s}\frac{x}{y^2}\left(1-\frac{y}{2}\right)^2\frac{2}{g_\rho^2}|\Phi_{\pi\rho}^{0}|^2\mp 2M_N^2\left(1-\frac{y}{2}\right)$$
$$\left.\times\frac{\sqrt{2}}{g_\rho}\Phi_{\rho\pi}^{0}f_\pi\sigma_{tot}^{\pm}\sqrt{\frac{s_0}{s}}\frac{1}{sy^{3/2}}\right\}. \tag{36}$$

Comparison of formulas (27), (34), and (36) shows that the density of scattering events on mesons and structureless Reggeons increases as s with energy, while the density of scattering events on Reggeons with structure falls off with energy as $1/s$. However, as the energy increases regions I, II, and III in Fig. 2 become smaller while region IV grows, so that the total number of events in regions I, II, and III is constant and independent of energy. The total number of events of neutrino production of pions on Pomerons increases as the logarithm of the energy while the total cross section for the remaining processes of scattering on Reggeons with structure and their interference are constant and are determined substantially by s_R. The total pion production cross section in the Regge region (for $\alpha s_R\gg\Lambda^2$) has the form

$$\sigma^{tot}=\sigma_P+\sigma_\omega\mp\sigma_{\rho P}=\frac{G^2M_N^2}{32\pi^3}\left[\Lambda^2\alpha^2f_\pi^2(\sigma_{tot}^{\pi N})^2\left(\ln\frac{s}{s_R}-1\right)\right.$$
$$\left.+\frac{\Lambda^4\alpha^4s_0|\Phi_{\pi\rho}|^2}{2g_\rho^2s_R}\mp 4\Lambda^2\alpha^2f_\pi^2\sigma_{tot}^{\pi N}\sigma_\rho^{\pi N}(s=s_R)\left(1-\sqrt{\frac{s_R}{s}}\right)\right], \tag{37}$$

where $\sigma_\rho^{\pi N}$ is the contribution of the ρ trajectory to the total πN scattering cross section: $\sigma_{tot}^{\pi^-N}-\sigma_{tot}^{\pi^+N}=2\sigma_\rho^{\pi N}$. For $\Lambda^2\simeq 1\ \mathrm{GeV}^2$, $s=200\ \mathrm{GeV}^2$, $s_R=20\ \mathrm{GeV}^2$, and $\alpha=1/5$, for the individual contributions we will have $\sigma_P\simeq 2\times 10^{-40}\ \mathrm{cm}^2$, $\sigma_\omega\simeq 4\times 10^{-42}\ \mathrm{cm}^2$, and $\sigma^{\bar\nu}-\sigma^{\nu}=3\times 10^{-41}\ \mathrm{cm}^2$. The difference of the total cross sections for producing single fast pions in ν and $\bar{\nu}$ beams changes sign in going to the Regge region. This difference is determined in regions II and IV by the charge-asymmetry effect, which is due to the interference of the ρ and P trajectories, and is 10–15% of the total cross section. The contribution of the interference of the vector and axial-vector currents to the difference of the cross sections is small (<0.1%) and falls off with energy as $1/s$. We see from formula (37) that the total cross section in the Regge region is determined mainly by neutrino (antineutrino) scattering on the Pomeron. Therefore, study of reaction (1) permits information to be obtained on the weak interaction of the Pomeron and its quark structure.

According to formulas (27), (34), and (36), the density of events is maximum in the region of transition to Regge asymptotic behavior for $y\sim s_M/s-s_R/s$ and for $|q^2|\sim\Lambda^2$, so that study of the distributions of events in x and y permits information to be obtained on the space-time picture of Reggeon (virtual meson) production (see the Conclusion). Study of the charge-asymmetry effects appearing in the difference of the neutrino and antineutrino cross sections permits the parameters s_M and s_R determining the transition to Regge asymptotic behavior in the case of lepton-hadron processes to be determined more accurately.

5. PRODUCTION OF SINGLE PIONS ON NUCLEI

In processes of single fast-pion production on nuclei, for sufficiently small t_1 the scattering on σ, f, and the Pomeron is coherent on the entire nucleus and the scattering on π and ρ is determined by the sum of the incoherent contributions of the nucleons of the nucleus. At the vertex where an ω is emitted by a nucleon $g_\omega \bar{u}_2 \gamma_\mu u_1 \omega_\mu$ the interaction with the time component $g_\omega \bar{u}_2 \gamma_0 u_1 \omega_0$ goes to $g_\omega 2M_N \varphi_2^* \varphi_1 \omega_0$ in the nonrelativistic limit (the wave functions φ_1 and φ_2 are normalized to unity) and corresponds to coherent scattering on the nucleus. The interaction of nucleons with the spatial component ω is due to relativistic effects and is not important at the small momentum transfers to the nucleon which we are considering.

The Pauli exclusion principle for the nucleons in the final state can have an important effect on the ratio of the incoherent and coherent contributions of the different mesons to reaction (1) for the production of fast pions with small momentum transfer to the nucleon. The detailed study of incoherent processes taking into account nuclear effects is therefore a separate complicated problem for which numerical counting is necessary.[7)]

The differential cross section for the coherent neutrino production of single pions on nuclei is determined in formulas (7), (10), (27), and (36) by the contributions of σ, ω, f, and the Pomeron. Here the corresponding cross sections must be multiplied by $A_\varkappa$, where $\varkappa = 2$ for coherent scattering neglecting screening of the nucleons and the absorption of pions in the nucleus.

Study of the production of fast pions on nuclei permits the contributions of the ω and f mesons to be separated in the meson region and those of the P, ω, and f trajectories to be separated in the Regge region. It is interesting that the difference of the cross sections for neutrino and antineutrino production of pions on nuclei due to the ρ contribution can practically disappear.

6. CONCLUSION

Space-time picture of the production of virtual mesons and Reggeons

The kinematical region of small momentum transfers to the nucleon corresponds to scattering on the periphery of the nucleus, which in the quark-parton model corresponds to the region of small x. Although the contribution of the region of small x to the total cross section for deep inelastic scattering is small, it is precisely this region that is of great interest from the viewpoint of the problem of the interaction of quarks at large distances. In this region there are strong correlations between partons, and the effects of neutrino scattering on virtual mesons considered in the present study are the manifestation of these correlations.

The idea of scattering on virtual mesons as an effective means of simplifying the description of neutrino scattering on quark-antiquark correlations at the periphery of the nucleon (confinement?) makes it necessary to change the description of these correlations as $s_1 \simeq 2p_1 q = 2M_N q_0$ increases. At characteristic scattering times $\Delta\tau \sim 1/q_0 \gtrsim 1/M_N$ it is as if the correlations manage to be formed into virtual mesons, while at short times the scattering occurs on correlations which have the quantum numbers of the corresponding mesons but which are not actually formed into these mesons. The Regge asymptotic behavior also permits a description of these "brief" correlations which appear in the form of the corresponding Reggeon. Here the parameter s_M (or, rather, $\tau_M = 2M_N/s_M$) that we have introduced which determines the transition from the meson amplitudes to the Regge amplitudes can be interpreted as the time parameter for the formation of a virtual meson.

As q^2 increases the spatial resolution of the parton correlations increases and the appearance of the q^2 dependence of the form factors is due to the transition to distances smaller than the effective size of the mesons. The experimental study of the problem of the q^2 dependence of the form factors of meson-pion and Reggeon-pion transitions (the weak charge radius of the meson and the Reggeon?) is of great interest.

These estimates show that the value of the total cross section for pion production in the scattering of neutrinos on virtual mesons (Reggeons) and the maximum of the density of events in the (x, y) plane are very sensitive to the ratio of the parameters $s_M(s_R)$ and Λ^2, that is, they are in fact determined by the time for meson formation in parton correlations and by the spatial characteristics of the correlations.

It is interesting to use the space-time picture that we have described to study the problem of the existence of a kinematical region of scattering on virtual mesons. In this region the PCAC relation (25) is strongly violated if the estimates (19) and (20) are used for F_A and the experimental data on πN scattering are used for $T_{\pi N}$. This violation can serve as an indication that as in the case of hadron-hadron processes, the transition to the Regge description in reaction (1) occurs directly after the resonance region or even earlier if $s_R < M_R^2$, characterizing the background part of the amplitudes in the resonance region. However, it is possible that violation of the PCAC relation is valid and corresponds to an important difference between lepton-hadron and hadron-hadron processes in the region $s_1 < s_M$ (if $s_M > M_R^2$). In neutrino processes this region is characterized by times $\Delta\tau > \tau_M = 2M_N/s_M$ exceeding the time for the formation of a virtual meson in a hadron at rest. In a collision of two hadrons this process will look like the exchange of a virtual meson only if the characteristic collision time $\Delta\tau \sim 2M_N/s_1$ exceeds the characteristic time for the formation of virtual mesons in the two hadrons. However, this time is increased for the incident hadrons: $\tau_M \to \tau'_M = \tau_M(s_1/2M_N^2)$, which significantly exceeds the characteristic collision time. This can serve as a qualitative argument for a later approach to Regge asymptotic behavior in lepton-hadron processes. Experimental study of reaction (1), especially the charge-asymmetry effects, for $s_1 \lesssim 20$ GeV^2 would permit this possibility to be checked.

It is of great interest to detect the slow recoil nucleon in reaction (1). This would permit the quantity $|t_1|_{max}$ to be measured and thereby allow the study of the properties of parton correlations as functions of x. Detection of the recoil nucleon makes it possible to experimentally study the P-odd asymmetry [see formula (7)] in the Treiman-Yang angle, which would make it possible to establish the relative sign of the different amplitudes. It is therefore extremely important to develop techniques for detecting slow particles in neutrino detectors.

Reaction (1) that was considered in this study is the simplest example of a wide class of neutrino processes occurring at the periphery of the nucleon. Study of such exclusive and inclusive[9] processes would permit information to be obtained on the weak interaction of mesons (Reggeons) and the quark structure of Reggeons and their physical nature, and also would possibly be useful for theoretical and experimental study of the properties of the interaction of quarks at large distances.

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1) Neutrino scattering on virtual pions at high energies was studied in Refs. 2–4.

2) The description of scattering on the σ meson corresponds to the effective description of scattering on an isoscalar 2π systems with mass 550 MeV.

3) We note that the contribution of axial A_1 and D_A mesons can be neglected because the corresponding coupling constants or Regge residues are small.[6] According to the quark model, the A_2-meson-nucleon coupling constant is three times smaller than the corresponding coupling constant of the f meson. We shall therefore neglect the contribution of the virtual A_2 meson compared to the contribution of the f meson.

4) There are indications in the literature that here the coupling constant is small.[7]

5) In the case of the $\sigma\pi$ transition, a relation of the type (20) does not make it possible to estimate $f_\sigma(0)$ because of the uncertainty in the σ-meson width.

6) We stress the fact that the total cross sections become constant at neutrino energies $E_\nu \gg s_M/2M_N$.

7) Coherent neutrino production of π^0 mesons on nuclei due to neutral currents was studied in Ref. 16.

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