The neutrino mass in elementary-particle physics and in big bang cosmology

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Some theoretical aspects of a nonzero value for the neutrino rest mass and its possible implications for physics are discussed. The nature of the neutrino mass is analyzed, as well as the physical consequences that may derive from the existence of new helicity states for the neutrino or from lepton charge nonconservation if the mass is of Dirac or Majorana character, respectively. Massive neutrinos are examined in the context of grand unified theories combining the weak, strong, and electromagnetic interactions. Searches for neutrino-mass effects in β decay and for neutrino oscillations are reviewed. Several astrophysical effects of the neutrino mass are described: solar-neutrino oscillations, the decay of primordial neutrinos, the feasibility of detecting massive primordial neutrinos experimentally. The predictions of big bang theory regarding the neutrino number density in the universe are analyzed, and a discussion is given of the influence neutrino oscillations might have on the neutrino density and on cosmological nucleosynthesis.

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INTRODUCTION

This review celebrates an anniversary—that of a particle, one whose story is closely intertwined with the dramatic process of deciphering the most fundamental laws of nature. Fifty years ago our particle was born at the tip of Wolfgang Pauli's pen, in hopes of salvaging the laws of energy and angular-momentum conservation in β decay.[1)] A quarter century later the nonconservation of parity in weak interactions was discovered: β-decay phenomena were found to violate the fundamental left–right symmetry of inanimate nature. An opportunity was at hand to link this breakdown of symmetry to the idea that the mass of every neutrino is precisely zero. In this way a new kind of fundamental symmetry could be laid down; it was taken to be a cornerstone of physics in microcosm. But now the neutrino on the eve of its fiftieth birthday brings us yet another surprise: delicate measurements of the β-decay spectrum recently carried out by a group at the Institute of Theoretical and Experimental Physics in Moscow[1] indicate that the rest mass of the neutrino is not quite zero after all.

If the results of these experiments should be confirmed, the neutrino once again will radically alter our concepts regarding not only the work in microcosm, but even more so, the macrocosm—the universe as a whole.

Actually more than three decades have passed since physicists and astrophysicists first realized that neutrinos might play a major role in the fate of stars during their terminal evolutionary stages. When the interior of a star eventually heats up to very high temperatures, the neutrinos that are formed will freely escape to infinite distance, carrying along energy (which the observer will scarcely be able to record) and dooming the stellar core to swift collapse. Readers who are sensitive to the fine points of the Russian language may appreciate the name "Urca" that George Gamow bestowed on this process.[2)] Today physicists in several countries are building installations designed to

[1)] At that time artificial radioactivity was still unknown. A continuous spectrum of electrons emitted by RaE nuclei was observed.

[2)] Neutrinos steal energy not only from individual nuclei but from the stars themselves. Gamow likens them to habitual criminals.

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detect bursts of neutrinos emitted by exploding stars—super-novae.

Neutrinos have become a tool for studying the physical conditions in the interior of the sun. The chlorine-argon method of neutrino detection which Pontecorvo had proposed[2] is being applied to solar neutrinos in the widely publicized experiment of Raymond Davis.[3,4] After Arno A. Penzias and Robert W. Wilson discovered the microwave background radiation, thereby confirming Gamow's big bang model of the universe, it became apparent that our universe is permeated not only with photons but also with neutrinos. The average number of these neutrinos per cubic centimeter is predicted to be approximately the same as the number of photons, 500 particles $/\mathrm{cm}^3$, or 10^8–10^9 times the mean number density of atoms. The 3 °K temperature of the primordial photons corresponds to a very low mean photon energy, 10^{-3} eV, and accordingly to a low photon mass density, $\rho=\varepsilon/c^2 \approx 10^{-34}\ \mathrm{g/cm^3}$. If neutrinos are massless, the neutrino density would be just as low; but if neutrinos have a mass in excess of 10 eV, it is they which would determine the mean density of matter in the universe and the age of the universe. Accepting that the age of the universe exceeds the 4.6×10^9 yr age of the earth and the solar system, Gershteĭn and one of the authors[5] in 1966 placed an upper limit on the mass of a neutrino, relying on cosmological arguments: $m_{\nu_e}<200$ eV, $m_{\nu_\mu}<200$ eV (the latter bound is still far lower than any accelerator experiment can provide). That investigation was the first to make use of the fact that in the theory of the big bang universe the neutrino number density need not be small.

Astrophysicists have long been concerned with the "missing mass" paradox encountered in clusters of galaxies. Perhaps this paradox may be resolved by finite-mass, or "massive," neutrinos.

There are several lines of evidence suggesting that ordinary matter in the universe has a low density, about 0.02–0.05 of the critical value. This circumstance would imply that the universe is hyperbolic, an idea which does not in itself conflict with anything. But a hyperbolic world of low mass density would pose major difficulties for the theory of galaxy formation. The heavy-neutrino theory can resolve these difficulties on a simple, qualitative level, and this advantage may lead astrophysicists to put some faith in neutrinos with a mass of about 30 eV.

If neutrinos do have a finite mass, our concept of the universe as a whole may undergo the most radical change: the world may turn out to be flat, or it may be closed. In fact, it was with the theory of a closed universe that Einstein began in 1917 and Friedmann in 1922.

One notes with interest that a year before the experimental developments of 1980, Peebles[6] had written of the desirability from a cosmological standpoint of introducing some sort of material—"massonium"—that cannot interact with ordinary matter and radiation. Peebles, to be sure, raised this option not simply in light of arguments based on astronomical observations: he was influenced as well by many papers[7-11] which had discussed massive neutrinos.[3)] Yet the mere fact that the question of massonium was posed at all is significant.

For neutrinos to have a mass would, then, be highly desirable for cosmology. But how legitimate are massive neutrinos from the viewpoint of elementary-particle theory? What might be the nature of the neutrino mass, and how would it tie in with the fundamental principles of microscopic physics? What experimental or observational effects might be anticipated? These are the questions that we shall explore in this review.

1. THE NEUTRINO MASS AND FUNDAMENTAL PROPERTIES OF ELEMENTARY PARTICLES

a) Particle mass and left-right transitions

The significance of a finite rest mass for neutrinos can only be comprehended by comparing this idea with the picture that was generally accepted until very recently. The first elementary particle to be discovered was the electron. The electron proved to have an intrinsic angular momentum of $\hbar/2$, or in other words, a spin of 1/2.

A theory of particles with spin 1/2 was developed by Dirac. In atomic physics Dirac's equation is usually written in the form

$$i\frac{\partial\chi_1}{\partial t}=(\mathbf{p}\sigma)\,\chi_2+m\chi_1,\qquad i\frac{\partial\chi_2}{\partial t}=(\mathbf{p}\sigma)\,\chi_1-m\chi_2, \tag{1.1}$$

with $\chi_2=0$ for a particle at rest. The state of an electron at rest is described by a two-component wave function χ_1; the two components correspond to opposite spin states of the electron, "up" and "down." The two components of χ_2, properly written χ_2^*, describe a positron at rest.

But there is an alternative version, convenient for describing a relativistic electron. One may take Dirac's equation to be

$$i\frac{\partial\psi}{\partial t}=(\sigma\mathbf{p})\,\psi+m\varphi,\qquad i\frac{\partial\varphi}{\partial t}=-(\sigma\mathbf{p})\,\varphi+m\psi. \tag{1.2}$$

The wave functions χ_1,χ_2 are related to the wave functions ψ,φ by

$$\chi_1=\frac{1}{\sqrt{2}}(\psi+\varphi),\quad \chi_2=\frac{1}{\sqrt{2}}(\psi-\varphi)$$

so that

$$\psi=\frac{1}{\sqrt{2}}(\chi_1+\chi_2),\quad \varphi=\frac{1}{\sqrt{2}}(\chi_1-\chi_2).$$

To describe the state of a free electron in a continuous spectrum we superpose the two functions:

$$\begin{vmatrix}\psi(\mathbf{x},t)\\ \varphi(\mathbf{x},t)\end{vmatrix}=\begin{vmatrix}\psi_0\\ \varphi_0\end{vmatrix}e^{-iEt+i\mathbf{p}\mathbf{x}},$$

and from Eqs. (1.2) we obtain

3) Gravitationally bound objects consisting of massive neutrinos had earlier been considered by Markov[12,13] and Bludman.[14]

$$E\psi = (\sigma p)\,\psi + m\varphi,$$
$$E\varphi = -(\sigma p)\,\varphi + m\psi. \qquad (1.3)$$

For specified $\mathbf{p}$ and E, the state ψ_0, like φ_0, is determined by a single (complex) number.

In the ultrarelativistic approximation (as $m/E \to 0$) the components of ψ and φ become independent. Such a representation is of practical value[15] for describing the behavior of ultrarelativistic electrons and positrons in an electric or magnetic field, in the presence of scattering, and so on. If a particle with an initial state $\psi_0 \neq 0, \varphi_0 = 0$ is emitted, then as time passes and evolution begins the state will remain the same. If the particle is an ultrarelativistic electron, then electromagnetic interactions, as well as gravitational ones, will not alter this fact. The functions φ, ψ represent eigenfunctions of the quantum-mechanical helicity operator (the projection of the spin, $\sigma\mathbf{p}$, on the direction of motion). If the electron spin σ is aligned with the direction of motion $\mathbf{p}$, then $\sigma\mathbf{p} = |\mathbf{p}|$, that is, the particle will travel in the sense of a right-handed screw, and this case will be described by the wave function ψ, but if the spin is oppositely directed, with $\sigma\mathbf{p} = -|\mathbf{p}|$, as in a left-handed screw, the wave function φ will apply, and ψ will be zero. According to Dirac's equation we may say that transitions $\varphi \to \psi$ will take place, provided $m \neq 0$.

In Dirac's equation the $\varphi \to \psi$ transitions describe a term with mass; that is, in relativistic quantum mechanics the mass of a particle determines the probability of a left-right transition[4)] for particles of spin 1/2. The higher the particle energy, the lower will be the probability of a reversal of its helicity: in the case of a relativistic particle with energy $E \gg m$ the amplitude of the $\varphi \to \psi$ transition will be proportional to m/E, so that helicity will be preserved to within m/E in the amplitude [the transition probability will be proportional to $(m/E)^2$].

In an electromagnetic field the functions φ, ψ will not transform into each other: if electrons change their direction of travel in an electromagnetic field, say by moving between the poles of a magnet, there will be a compensating change in the sense of spin. Only if a small anomalous magnetic moment is present (different from the value implied by the Dirac equation), as predicted by quantum electrodynamics, will the spin of an electron moving in the field (say rotating in an orbit) be reversed.

The Dirac equation also yielded solutions describing the state of an electron with negative energy ($E < -mc^2$). Analysis of these solutions led to the prediction that antiparticles ought to exist. If the corresponding states were free, then the states of electrons in atoms would be unstable against transitions to such states, which clearly is contrary to experiment. Dirac proposed that states of negative energy be considered filled; then transitions to them would be forbidden by the Pauli principle. The electrons occupying states of negative energy should not induce any gravitational or electromagnetic effects. In other words, the answer to the problem of negative-energy states lay in a new interpretation of the electron vacuum: the vacuum was thought of as a "sea" of fully populated negative-energy states (the current picture of a physical vacuum is described more thoroughly in a separate review by one of us[18]). The development of a vacancy with negative energy ($E < -mc^2$), that is, the absence of a negatively charged electron, should then manifest itself as the presence of a positively charged electron (a positron) of energy $E_+ = -E$. The prediction was made that new physical states ought to exist, taking the form of "holes" in the electron-vacuum sea. Along with particles (electrons) the theory predicted antiparticles (positrons). And the ensuing discovery of the positron became a brilliant confirmation of Dirac's theory.

The Dirac equation for positrons analogous to Eqs. (1.3) may be written as

$$E_+\bar{\psi} = -(\sigma p)\,\bar{\psi} + m\bar{\varphi},$$
$$E_+\bar{\varphi} = (\sigma p)\,\bar{\varphi} + m\bar{\psi}, \qquad (1.4)$$

so that in the case of the positron the helical state $\bar{\psi}, \bar{\varphi}$ have changed roles: a left-polarized positron is described by the wave function $\bar{\psi}$, and a right-polarized positron by the wave function $\bar{\varphi}$. The states $\bar{\psi}, \bar{\varphi}$ are the states of an antiparticle corresponding to the states ψ, φ of a particle. If an electron-positron pair is formed in some electromagnetic process, then the state of the pair will be described either by the state $\psi\bar{\psi}$ or by the state $\varphi\bar{\varphi}$. In the ultrarelativistic limit one may say that ψ, φ describe two different kinds of charged particles (electrons); correspondingly, there are also two kinds of positrons, $\bar{\psi}$ and $\bar{\varphi}$.

In two-particle electromagnetic decay of a stationary meson with spin 1, such as $\rho^0 \to e^+e^-$, the electron and positron have opposing helicities and fly off in opposite directions, but they rotate in the same sense; the total angular momentum of the pair is 1, as required by the laws for conservation of total angular momentum. But in the analogous decay of a particle of spin 0, say π^0

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4) How can one visualize these transitions? If a free particle is in motion its angular momentum will remain constant. One may picture the particle[16,17] as moving at speed c in one direction most of the time (a fraction $1-\varkappa$) and in the opposite direction for a small part $\varkappa$ of the time. As a consequence its mean velocity will be different from c. But the departure of the velocity from c reflects the fact that $m \neq 0$; $v = c(1 - m^2/E^2)^{1/2}$, and $\varkappa \sim m^2/E^2$ if $m^2/E^2 \ll 1$. The motion of a free relativisitic particle of spin 1/2 has the character of a Zitterbewegung, or oscillatory motion, wherein the instantaneous velocity is always c but the mean velocity is pc/E. The mean velocity differs from the instantaneous value because of discrete velocity jumps occurring at very high frequency, so high that the Zitterbewegung is unobservable in any realistic experiment. This terminology was introduced because a formal calculation of the eigenvalue of the Dirac-particle velocity gives $|v| \equiv c$; but the velocity does not commute with the energy, giving rise to the graphic if none too accurate concept of velocity jumps. This picture is just as intuitive, and just as inaccurate, as the notion that the electrons in atoms undergo orbital motion. In view of the fundamental principles of quantum mechanics neither orbital motion of electrons in atoms nor jumps in the velocity of a free electron can produce radiation, as one would have supposed on the basis of classical, nonquantum representations.

$\to e^+e^-$, conservation of angular momentum requires that the total momentum of the pair be 0; that is, an electron and positron flying off in opposite directions will have the same helicity. The state of the pair should be described either as $\varphi\bar{\psi}$ or as $\bar{\varphi}\psi$; in both cases either the electron or the positron will have an "improper" helicity[19] relative to the partner formed along with it. If electrons had zero mass, then conservation of helicity would be strict and decay would be forbidden (only pairs corresponding to states $\psi\bar{\psi}$ or $\varphi\bar{\varphi}$ could be formed). When the nonzero electron mass is taken into account, one finds that decay can take place through $\psi \rightarrow \varphi$ and $\bar{\psi} \rightarrow \bar{\varphi}$ transitions, whose amplitude will be proportional to $m_e/E_+ = m_e/E_- = 2m_e/m_\pi$ $(E_+ + E_- = m_\pi, E_- = E_+)$. Hence the probability of the $\pi^0 \to e^+e^-$ decay will be diminished by a factor $(m_e/m_\pi)^2$.

The same line of argument regarding helicity explains why two-particle weak lepton decay $\pi^+ \to e^+\nu_e$ is observed to be suppressed relative to $\pi^+ \to \mu^+\nu_\mu$ decay. The spin of $\pi^\pm$ particles is zero, and the electron or muon μ^- (positron or μ^+) formed in such a decay should have an "improper" helicity. Hence the decay probability will be proportional to $(m_e/m_\pi)^2$, $(m_\mu/m_\pi)^2$, and the ratio of the $\pi \to e\nu, \pi \to \mu\nu$ decay probabilities will be $\approx (m_e/m_\mu)^2 \approx 2 \cdot 10^{-4}$. Throughout this discussion we have made use of properties of the weak interaction (its resemblance to electromagnetic interaction); these will be examined in some detail below.

b) Parity nonconservation and helicity of the neutrino

Originally, before the nonconservation of parity in weak interactions was discovered, it seemed obvious that even if the neutrino mass were stricly zero, the neutrino would have two helical states, left and right, emitted in β decay with equal probability. Neutrino theory was assumed to be entirely analogous to electron theory, with the sole exception that $m_\nu = 0$ and that left-right transitions of a freely moving particle were completely forbidden. However, the puzzle of K decays (an identical particle, a K^+-meson, was found to decay into both 2π and 3π states, possessing opposite parity) cast doubt on the conservation of parity—the fundamental symmetry of left and right in inanimate nature.

Such symmetry implies that when a process undergoes mirror reflection either it will not be changed at all or it will transform into another process which likewise operates in nature. For example, the mirror image of β^+ decay in which a left-polarized neutrino is formed would be a process in which β^+ decay yields a right-polarized neutrino ψ_ν. Parity conservation demanded that the two processes occur with equal probability. Accordingly, the emission of right- and left-polarized positrons ought to be equally probable.

The decay of a K meson represents a special case of weak interaction: even though K and π mesons are hadrons, decay of K cannot be induced by strong interaction. In 1956 Lee and Yang[20] put forward the hypothesis that parity may fail to be conserved in weak interaction—not merely in the decay of K but in all weak-interaction processes. A corollary of this hypothesis was the prediction that the electron and the neutrino emitted in a β decay should have a preferred helicity.

Analysis of the weak interaction responsible for β-decay revealed that the electron emitted in $n \to p + e^- + \bar{\nu}_e$ decay will always be left-polarized; that is, the electron emitted will always possess only a φ component with left helicity (or in positron decay, the positron emitted will possess only a $\bar{\varphi}$ component with right helicity). In electromagnetic interaction φ, ψ are equivalent; but in β decay the ψ component can appear a second time for the electron, owing to its mass, and the probability that this component will appear is proportional to $(m/E)^2$. In just the same way β decay will yield only one kind of neutrino: in β^- decay only right-polarized antineutrinos $\bar{\nu}_{eR}(\bar{\varphi}_\nu)$ will be emitted, and in β^+ decay only left-polarized neutrinos $\nu_{eL}(\varphi_\nu)$.

Correspondingly, the process inverse to β^- decay,

$$\bar{\nu}_e + p \to n + e^+, \tag{1.5}$$

first observed in the laboratory by Reines and Cowan[21,22] more than 25 years ago, gives rise only to right-polarized antineutrinos $\bar{\nu}_{eR}(\bar{\varphi}_\nu)$, while the process

$$\nu_e + n \to p + e^- \tag{1.6}$$

should produce left-polarized neutrinos $\nu_{eL}(\varphi_\nu)$.

The fact that a definite polarization is singled out in the β decay means that in weak interaction the symmetry of physical processes with respect to mirror reflection is violated to a maximum extent.

If neutrinos have zero rest mass, then transitions $\varphi_\nu \rightarrow \psi_\nu$ would not occur, and in weak interaction neither direct or indirect (owing to the mass; thus there would be no absorption) right neutrinos ψ_ν or left antineutrinos $\bar{\psi}_\nu$ would be created. Such massless-neutrino states could not arise at all in our inverse (apart from the possibility of gravitational interaction[23]). Indeed, all phenomena observed in the laboratory are compatible with the premise that such states simply do not exist.[5)] The table of leptons would acquire an asymmetric form with three types of particles and three types of antiparticles:

$$e^-_L \equiv \varphi_e, \quad e^-_R \equiv \psi_e, \quad \nu_{eL} \equiv \varphi_\nu;$$
$$e^+_R \equiv \bar{\varphi}_e, \quad e^+_L \equiv \bar{\psi}_e, \quad \bar{\nu}_{eR} \equiv \bar{\varphi}_\nu,$$

5) The suggestion has been made[24,26] that the fundamental symmetry of nature is CP symmetry, that is, a symmetry with respect to simultaneous replacement of a particle by an antiparticle and mirror reflection. In such a transformation, left neutrinos φ_ν would transform into right antineutrinos $\bar{\varphi}_\nu$. CP symmetry has led to an attractive two-component theory of neutrinos[25,27,28]: the neutrino mass m_ν would be zero, and only the φ_ν, $\bar{\varphi}_\nu$ states would exist. The 1964 discovery[29] of CP symmetry violation in laboratory decays of neutral K mesons did not shake the two-component theory. One cannot decide from the CP violation whether the neutrino mass is zero or not. But the violation of CP invariance does play a paramount role for cosmology: the fact that our universe consists of *matter* (atoms, protons, neutrons) with no matching quantity of antimatter is presumably a consequence of the fact that the laws of nature are *not* CP invariant—that is, an outcome of the asymmetry in the interaction of particles and antiparticles.

of which ν_L would participate in weak, e_L in weak and electromagnetic, and e_R only in electromagnetic interactions. Furthermore, the electron would have an $e_L e_R$ interaction resulting from its mass (because of the electron mass, transitions $\varphi_e \rightarrow \psi_e$ are possible).

In the Weinberg-Salam theory,[30,31] the coexistence of a heavy neutral boson (its exchange can induce weak neutral-current interaction) and a photon has the effect that e_R can participate directly in weak interaction, but only in the interaction of neutral currents, not in that of charged currents—that is, not in β decay! The asymmetric form of the lepton table remains unchanged in this theory. Moreover, the Z^0 boson has now carried the asymmetry over into atomic physics (two reviews[32,33] have been given in this journal[6)]; see also some current investigations[36,37]). Because of exchange of a Z^0-boson with a nucleus, e_R will be scattered by the nucleus, but in a different manner from e_L. Interaction of an electron with the nucleus through Z^0 will interfere with the analogous interaction through a photon, that is, with electromagnetic (Coulomb) interaction.

c) A matter universe?

If $m_\nu \neq 0$, then the states $\nu_R(\psi_\nu), \bar{\nu}_L(\bar{\psi}_\nu)$ cannot be rejected, and the neutrino mass would permit the transformations $\varphi_\nu \rightarrow \psi_\nu, \bar{\psi}_\nu \rightarrow \bar{\varphi}_\nu$. This is the case of the ordinary "Dirac neutrino." It might seem that we are reverting here to the situation that prevailed before 1956, to the old ideas before the nonconservation of parity in weak interactions was discovered. But today we know that the components $\psi_\nu, \bar{\psi}_\nu$ fail to participate in any of the processes studied in the laboratory. We do return to the old ensemble of particles, but not to the same concepts regarding their interaction. The table of particles becomes symmetric, but the interaction asymmetry discovered in 1956 remains.

A finite mass for the neutrino, then, will revive the $\nu_R(\psi_\nu), \bar{\nu}_L(\bar{\psi}_\nu)$ particles (the right neutrino and the left antineutrino) which had been buried in 1956. These particles are subject to neither strong, nor weak, nor electromagnetic interaction. All they have is gravitation and a tiny mass which governs their transformation into $\varphi_\nu, \bar{\varphi}_\nu$ and allows them to interact very weakly (far more so than by what is traditionally called the weak interaction) with the other particles involved in such neutrino processes[7)] as

$$\underbrace{\psi_\nu + n \rightarrow \varphi_\nu}_{\text{mass}} + n \rightarrow \varphi_e + p$$

The probability of such a process will amount to a small fraction [proportional to $(m_\nu/E_\nu)^2$] of the probability of the analogous process for φ_ν having the same energy E_ν. Since the neutrinos emitted in the laboratory are ultrarelativistic, with

$$E_\nu \gg m_\nu,$$

it becomes quite understandable why $\psi_\nu, \bar{\psi}_\nu$ have thus far remained unobserved. Are we faced with a blind alley on our road to knowledge or a chink opening into a new world? What properties would that world have?

Previously, between 1956 and 1980, when it was widely believed that $m_\nu = 0$, there was some discussion[20,38] of the possibility of a "mirror world," a universe in which the right components $\psi_\nu, \bar{\psi}_\nu$ would take part in mirror interactions with mirror electrons and mirror nucleons. Ordinary particles, it was suggested,[38] would not participate in these mirror interactions, whereas the mirror particles would not undergo ordinary weak, strong, and electromagnetic interaction. Hence the mirror world, symmetric to our own, could be perceived by us only gravitationally.

The mirror world would not represent another universe in a spatial sense but a different set of fundamental particles. The objects all around us, consisting of ordinary nucleons and electrons, would coexist in one and the same space with objects made up of mirror particles.

Okun'[39,40] has recently discussed an analogous situation in the context of the possible existence of new types of long-range forces (tetons, Y particles). Might the minute neutrino mass represent a slender little thread leading us into the world beyond the mirror? Incidentally, the mirror world may have properties quite distinct from those of our own universe (as Alice discovered in Lewis Carroll's *Through the Looking-Glass*). Is there any hope of gleaning some information on these properties? Could there perhaps already be some astronomical clues[41] pointing to the existence of a mirror universe?

d) The Majorana neutrino

But there is another possibility as well. We limit attention to particles presently known. One may postulate the neutrinos and antineutrinos represent the same particle. Right antineutrinos would represent the deficient right neutrino states; that is, instead of $\varphi_\nu \rightarrow \psi_\nu$ transitions the finite neutrino mass would result in $\varphi_\nu \rightarrow \bar{\varphi}_\nu$ transitions. For electrons, such a relationship is not possible: the law of electric charge conservation would be violated in $e^+ \rightarrow e^-$ transitions. But neutrinos have no electric charge, and $\nu \rightarrow \bar{\nu}$ transitions simply mean that neutrinos have no specific lepton charge either.

Let us consider somewhat more fully the question of the conservation of lepton charge (or charges). Electric-charge conservation has two aspects: a) in all elementary-particle processes the sums of the initial- and final-particle charges are equal; b) a dynamical aspect, whereby the electric charge involves interaction with photons. Charged particles constitute a

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6) Zel'dovich[34] and Bludman[35] have written qualitatively (not suggesting any specific mechanism or estimating the magnitude of the effect) about the possibility that parity might be violated in atoms due to weak interaction of the electrons with the nucleus.

7) Generally speaking, the direct process $\psi_\nu + n \rightarrow \psi_e + p$ could operate by means of right-handed currents, whose existence has not been demonstrated. Experiment, though, sets only an upper limit on the corresponding constant ($G_R < 0.1\ G_F$).

source of a long-range Coulomb field. The nonconservation of electric charge would call for an instantaneous rearrangement of the Coulomb field, and that is not possible.[42-44] But lepton charge, on the contrary, like baryon charge, does not have this second, dynamical, aspect; such charges are not coupled to a massless field. Baryon-charge conservation represents a simple description of the fact that in the laboratory we do not observe, for example, $p \to e^+ + \gamma$ decay. Since $\mu^+ \to 2e^+ + e^-$ and $\mu^+ \to e^+ + \gamma$ do not occur either, it would be natural to introduce two lepton charges (electronic and muonic). A third intrinsic charge (the τ-lepton charge) would be attributable to the τ lepton and its neutrino.

The concept of lepton charge[45,46] (or charges) arose prior to the parity nonconservation, when ν was considered a four-component particle, like e; it was postulated that a particle, the neutrino, exists with statistical weight 2 (two directions of spin), and an antiparticle, the antineutrino, with statistical weight 2. The concept of charge was retained in Landau's two-component theory. Experiments in which neutrino-producing processes were detected confirmed the idea of charges (not just one charge but two-electron, three-electron, muon, τ-lepton charges). Hence if neutrinos do have a rest mass, one may revert to the massive Dirac neutrino and three charges.

However, the violation of P invariance in neutrino interactions enables one to renounce the concept of charge without coming into conflict with experiment. The role of charge is played by the helicity. In the $m_\nu = 0$ theory the equivalence is complete. In the case $m_\nu \neq 0$ the version denying charge conservation differs from the helicity concept in that the total number of neutrinos is not conserved: some of the neutrinos may transform into antineutrinos. A chain of processes such as the following will then be possible:

$$\begin{array}{l} n + n \to p + e^- + n + \bar{\nu}_e \\ \qquad\qquad\qquad\updownarrow \text{mass} \\ p + e^- + n + \nu_e \to p + e^- + p + e^- \to 2p + 2e^-, \end{array} \tag{1.7}$$

so that in nuclei the so-called double neutrinoless decay $2n \to 2p + 2e^-$ may take place.

In practice the nuclei which have been considered are Se^{82}, Te^{130}, Te^{132}; these are stable against ordinary β decay. The double β-decay process might take place in such nuclei either with direct participation of two neutrinos [the process (1.7)] or because of the fact that Δ resonance (with a mass of 1230 MeV) can be excited within the nucleus. The process

$$n \to \Delta^{++} + e^- + e^- \tag{1.8}$$

(followed by $\Delta^{++} + n \to 2p$) is possible (Fig. 1), or the process (after the virtual process $2n \to \Delta^- + p$)

$$\Delta^- \to p + e^- + e^-. \tag{1.9}$$

In quark language, both types of processes, (1.7) and (1.8), (1.9), correspond to the same quark process (Fig. 2):

$$\begin{array}{l} d \to u + e + \bar{\nu}_e, \\ \qquad\qquad\updownarrow \text{mass} \\ d + u + e + \nu_e \to 2u + 2e, \end{array}$$

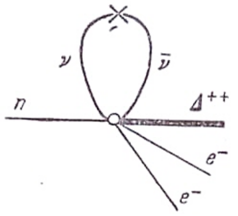

FIG. 1. The Feynman diagram for the process $n \to \Delta^{++} + e^- + e^-$.

except that in the process (1.7) the initial d quarks belong to different nucleons, while in the process (1.8), (1.9) they belong to the same nucleon (Δ isobars).

The rate of these processes is determined by the $\bar{\nu}_e \to \nu_e$ transition ($\varphi_\nu \to \bar{\varphi}_\nu$), whose probability is proportional to $(m_\nu/E_\nu)^2$.

Double β decay of the nuclei Se^{82}, Te^{130}, Te^{128} may also be accompanied by the emission of two neutrinos, through the processes

$$n + n \to p + p + e + e + \bar{\nu}_e + \bar{\nu}_e \tag{1.10}$$

or

$$n \to \Delta^{++} + 2e + 2\bar{\nu}_e. \tag{1.11}$$

A Japanese group[47] has recently claimed, from analysis of experimental data[48] on double β decay of Te^{130} and Te^{128}, that the latter decay is determined primarily by the process (1.7). The halflife of Te^{128}, as estimated from the observed excess of its decay product Xe^{128} in ancient rocks,[8)] would seem to imply[47] that $m_\nu \approx 30$ eV.

Particles which at the same time are antiparticles are said to be truly neutral. Thus far the only truly neutral particles known are bosons (particles with integer spin), including the π^0 meson (spin 0), photon, ω^0 meson, ρ^0 meson (spin 1), f^0 meson (spin 2), and graviton (also spin 2, but massless).

If the Japanese results[47] should be confirmed, then neutrinos would represent intrinsic antiparticles and would become the first instance of truly neutral fermions (that is, truly neutral particles with half-integer spin). Majorana developed a theory for such fer-

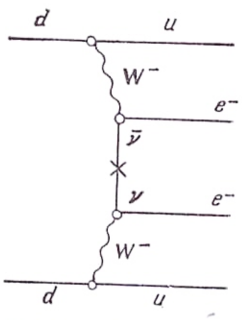

FIG. 2. The Feynman diagram for the quark process resulting in double β decay.

8) Direct experiments have recently been performed[49-53] to search for double β decay of Te^{130}.

mions, so they are called Majorana particles. Thus our second alternative is that neutrinos are Majorana particles, and the neutrino mass would characterize $\varphi_\nu \to \bar{\varphi}_\nu$ transitions. The discovery of the neutrino mass opens the door to a new property of particles presently known; it does not change the number of particles.

When β^- decay occurs a longitudinally (right-) polarized particle "ν" is emitted, with a definite projection of spin onto momentum. If the rest mass of "ν" is small its velocity will be close to c. This particle can give rise to the reaction "ν" $+p \to n + e^+$. The probability that it will lead to a reaction "ν" $+n \to p + e^-$ on a stationary neutron, while low, is not zero. Consider, however, an ultrarelativistic neutron (or proton) rushing in pursuit of a neutrino. Now we would need $E_n/m_n \gg E_\nu/m_\nu$. In a reference frame coupled to that neutron (proton), "ν" would become left-polarized, and the probabilities of the reactions "ν" $\to e^-$, "ν" $\to e^+$ would change places. We should emphasize that if the velocity were exactly equal to c, such a thought-experiment would not be possible. The concept of helicity as a strictly defined internal property of a particle is associated with $v = c$, that is, with a mass $m = 0$. In our example the helicity of the particle emitted in β-decay is nearly complete, but not quite 100% complete, precisely because $v \neq c$.

e) Are superheavy particles inevitable?

The question of whether the neutrino mass is of Dirac or Majorana nature[9)] awaits a theoretical and an experimental decision, but there is one question common to both alternatives for the neutrino mass: Why is it so small?

This question, as a matter of fact, parallels another query: Why is the proton stable? Modern "grand unified theories" seeking to combine the strong, weak, and electromagnetic interaction respond to this last question as follows: The stability of the proton is not absolute! The fact that the lightest baryon is effectively stable reflects the approximate conservation of "baryon charge": in every process that has been studied where baryons participate, the total number of baryons less the total number of antibaryons is conserved.

But unified-theory models start from the fundamental unified nature of all fields—from the assumption that a symmetry exists between particles (an approximate, not an exact, symmetry, because the particle masses differ). Unified theories also incorporate a symmetry of interactions, carried by vector bosons. The term "interaction" refers not only to a change in particle trajectories (say the path of an electron moving in a magnetic field); it also includes transmutations: $e \to \nu, p \to n$. Here too symmetry is violated; the quantitative differences in the interaction probabilities reflect the differences of the intervening bosons in mass. For example, the interaction inducing $\nu \to e$ transitions is "weak," since the W boson responsible for such a transition has a large mass.

In unified theory quarks and leptons are combined, and in principle one would think that almost anything can be transformed into anything—in particular, that transitions with baryon-number nonconservation ($2q \to \bar{q} + 1$) can occur, making the proton unstable: $p \to \pi^0 + e^+ \to 2\gamma + e^+$. The decay probability is low, due to the large mass of the intermediate bosons, the X boson,[10)] exciting such transitions. The mass of the X boson is determined in the theory by the energy scale M_X on which complete symmetry of the forces of nature is realized (see Okun's review of unified theories in this journal[54]). If current theoretical ideas regarding the strong, weak, and electromagnetic interactions are extrapolated to the ultrahigh-energy domain, one can predict that $M_X \approx 10^{14}$–10^{16} GeV. Grand unified theory implies in a natural way that the proton should have a long lifetime: $\gtrsim 10^{30}$ yr.

The unified-theory models developed by analogy with the Weinberg-Salam theory[30,31] represent a successful (Nobel prize!) example of gauge theory unifying the weak and electromagnetic interactions. In this theory the energy scale

$$\lambda \sim 300 \text{ GeV} \sim 300 m_p \quad (M_W \sim e\lambda \sim 100 \text{ GeV})$$

characterizes the distance over which the distinction between these two interactions should disappear. The same quantity acts as a scale for measuring the mass of particles ($m \sim \alpha M_W \sim e\alpha\lambda$).

In the Weinberg-Salam theory proper, the neutrino mass need not be specified, for it arises as a reflection of the higher stages of the theory, resulting from transitions with supermassive virtual intermediate states. In this case

$$m_\nu \sim \frac{\lambda^2}{M_X}. \tag{1.12}$$

The estimate[11)] (1.12) refers both to a Majorana and to a Dirac neutrino mass; in both cases it determines the energy threshold of the $\varphi_\nu \to \bar{\varphi}_\nu$ or $\varphi_\nu \to \psi_\nu$ transition. We are not concerned here with an energy barrier in space, so that the transition probability contains no exponentially small factors. The transition probability is reduced because the transition takes place through a superheavy virtual intermediate state; the same will be true of the proton decay probability as well.

[9)] A mixed version is also possible; see Sec. 2.

[10)] X bosons possess fractional electric charge (4/3, 1/3) and color. Hence quantum chromodynamics condemns them forever: fractionally charged particles with color cannot be observed in a free state. The possibility that fractionally charged composite ("white") particles, or "fractons," might exist is explored by one of us in a recent letter.[53]

[11)] Numerically we obtain $m_\nu \approx 1$ eV. If we were to set $m_\nu \approx m_W^2/M_X$ with $M_X \approx 10^{15}$ GeV, we would have $m_\nu \approx 10^{-2}$ eV. In some theories with right-handed currents the neutrino mass is related to the mass of the corresponding lepton by $m_\nu \approx m_l^2/m_R$, where m_R denotes the mass of the boson responsible for interaction of the right currents. For theoretical estimates of the neutrino mass, see Okun's review[54] and some recent work at CERN.[55-57]

We see, then, that a *very light neutrino* might signal the existence of a world of *superheavy particles*. And one and the same energy threshold might lie along the roads of proton decay and conversion of left into right neutrinos.

2. THE NEUTRINO MASS AND EXPERIMENT

a) Beta-decay

That a finite neutrino mass might affect the spectrum of electrons in β decay was suggested as long ago as 1933 by Enrico Fermi,[58] when he laid down the foundations of weak-interaction theory.[12)]

First of all, if the neutrino has a nonvanishing mass $m_\nu \neq 0$, the limiting β-spectrum energy will change. In this event the maximum energy of the electrons in β-decay will be $E_0 - m_\nu c^2$, rather than simply the mass difference E_0 between the initial and final nuclei, as would be true if $m_\nu = 0$. To distinguish this effect would require a very precise, independent measurement of the quantity E_0. If $m_\nu \approx 10-100$ eV, the mass differential of tritium and He^3 would have to be established to one or two orders higher accuracy than is given by modern mass spectroscopy. Nevertheless, the task of measuring the difference in mass between the singly charged He^3 ion and tritium is a perfectly realistic one. If this problem could be solved, our field of search for neutrino-mass effects would be broadened.

A nonzero neutrino mass should alter the form of the β-spectrum near the maximum electron energy $E_0 - m_\nu$. In fact, the kinetic energy E_ν of the neutrinos emitted in β decay is related to the kinetic energy E of the electrons by $E + E_\nu = E_0 - m_\nu c^2$, and if $E_0 - m_\nu - E \approx E_\nu \sim m_\nu c^2$ the neutrinos emitted will become nonrelativistic. In the ultrarelativistic limit ($E_\nu \gg m_\nu c^2$), $E_\nu \approx c p_\nu$, while in the nonrelativistic limit ($E_\nu \ll m_\nu c^2$), $E_\nu = p_\nu^2/2m_\nu$. The change in the dependence of the kinetic energy of a neutrino upon its momentum as one passes from the region $E_0 - E \gg m_\nu c^2$ to $E_0 - E \sim m_\nu c^2$ should, according to the laws of energy and momentum conservation, produce a characteristic distortion of the electron spectrum by $1 - [m_\nu^2/2(E_0 - E)^2]$, $E_0 - E \gg m_\nu c^2$ (Fig. 3). If the neutrino mass is small enough, this distortion of the β-spectrum would take place in a very narrow energy interval near maximum energy.[13)] According to the law of β decay, the probability that the energy of the decay electrons will have a value near E_0 is low. One must therefore achieve a maximum depression of the background, so as to come as close as possible in the measurements to the limiting energy $E_0 - m_\nu$ and thereby expand the interval of measurement in which the distortion of the spectrum would exceed the statistical error of detection and become perceptible. That is why fifty years of work on β decay was needed before experimental evidence would be obtained for a nonzero neutrino mass.

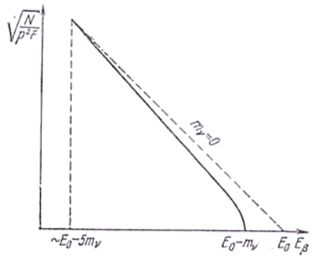

FIG. 3. The Kurie plot (idealized) for the spectrum of the β decay $H^3 \to He^3 + e^- + \bar{\nu}_e$. *Solid curve*, the case $m_\nu \neq 0$; *dashed line*, $m_\nu = 0$.

In their experiment Lyubimov, Novikov, Nozik, Tret'yakov, and Kozik[1] measured the spectrum of the tritium β decay $T \to {}^3He + e + \bar{\nu}_e$. To distinguish neutrino-mass effects it is desirable for the quantity E_0 to be kept as small as possible in order that the relative proportion of β decays in which the electron energy lies in the "mass sensitive" interval of the spectrum may be enhanced. This was the reason for the choice of tritium, whose β decay has a minimal value for E_0 ($\approx$18 keV). In its free state tritium is gaseous, so in order to raise the tritium density in the source the complex organic compound valine $(CH_3)_2CHCH(NH_2)CO_2H$ was used, with two of the eleven hydrogen atoms in its molecule replaced by tritium, on the average. The β-spectrometer employed in these measurements at the Institute of Theoretical and Experimental Physics had the advantage that the background was 15 times lower than in the best prior analyses for the same statistics and resolution. As a result the mass-sensitive interval could be expanded by a factor of $\approx$2.5, and the authors thereby hoped to improve the existing upper limit on m_ν. However, they maintain that their analysis demonstrates the premise of zero neutrino mass to be incompatible with the experimental data.[1]

In this attempt to distinguish neutrino-mass effects, uncertainties arose because of the complex structure of the levels comprising the excited states of valine, the corrections required for the form of the resolution function, and so on. The ITEP authors conclude from their data[1] that despite all these uncertainties the neutrino mass evidently exceeds 14 eV. Thus they believe that whatever hypothesis may be invoked in analyzing

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12) Early experiments on the form of the β-spectrum near its upper limit[59-68] (see also a 1944 review by Grinberg[69] and the book by Allen[70]) enabled the first constraints on the neutrino mass to be imposed during the 1930s and 1940s. The history of these beginning efforts to determine the neutrino mass in β decay is very dramatic. The disparity observed[60-62] between theoretical predictions[71] and the experimental data was initially interpreted as a possible neutrino-mass effect. Such an interpretation of the experiments[61,62] provided various values for the neutrino mass ($0.3\,m_e$, $0.8 m_e$), inferred from decay of differing nuclei, Zavel'skiĭ[63] showed that the theoretical description[71] of β decay was itself incorrect, and the experimental data could only set an upper bound on the neutrino mass. Subsequent analysis[64,65] of ThC and RaE β decays yielded the limits $m_\nu < 0.02-0.01\, m_e$, which were much sharpened by measurements[66-68] of the β decay spectrum of tritium: $m_\nu < 0.002-0.001 m_e$.

13) Roughly speaking, the relation $\Delta E \cdot \Delta m = \text{const}$ will hold equally well, where ΔE represents the mass-sensitive interval and Δm is the accuracy in the mass measurement.

the events observed, and for any systematic errors they (the authors[1]) can envisage, their experimental data point to a nonzero rest mass for the neutrino.

As to the actual value of the neutrino rest mass, this question proves to be more complicated. The ITEP experiment does not yield a unique value; the authors quote an admissable range[1] of

$$14 < m_\nu < 46 \text{ eV}. \tag{2.1}$$

In view of the possible mixing of the various kinds of neutrinos (see below), we would point out that even though the ITEP data were derived from β decay in which electron-type (anti-) neutrinos $\bar{\nu}_e$ were formed, the experimental results[1] cannot be interpreted as a measurement of the "mass of the electron neutrino." The $\bar{\nu}_e$ state, while possessing a definite (electronic) lepton charge, might not have a definite mass, but may be a superposition[14)] of neutrino states with differing masses, say $m_1 > m_2 > m_3$. In this event the β-spectrum ought to change in a more complicated fashion[58,72] near E_0 (Fig. 4).

The measurements[1] were not accurate enough to distinguish a superposition of the effects of several masses from the case where the distortion of the spectrum would have been due to a particular mass m_{ν_e} for $\bar{\nu}_e$. However, as the authors themselves remark,[1] analysis of their data assuming several mass parameters (say two: m_1, m_2) leaves room for the possibility that $m_1 \approx 30$ eV and $m_2 \ll m_1$; and the proportion of the $\bar{\nu}_e$ in the state with mass m_1 could actually be even smaller[15)] than 50%. As we shall see presently, that possibility could have some interesting implications for astrophysics.

b) Leptons and lepton charges

At present, along with the electron e and the positron e^+, two other types of leptons (and their antiparticles) are known, $\mu^\pm$ and $\tau^\pm$, with masses

$$m_\mu = 105 \text{ MeV}, \quad m_\tau = 1760 \text{ MeV},$$

compared with $m_e = 0.511$ MeV. In their properties μ and τ resemble the electron, and on the whole it is not at all clear why they exist. The term *lepton* comes from the Greek λεπτόσ (small, light), and if it was an appropriate name for the relatively light e and μ, then in the case of τ one cannot evade the adjective "heavy." Presumably with each type of charged lepton there is associated a corresponding neutrino.

Experiment tells us that the neutrino formed together with μ in the process

$$\pi^\pm \to \mu^\pm + \nu_\mu \bar{\nu}_\mu \tag{2.2}$$

will, upon interacting with a nucleon, yield

$$\bar{\nu}_\mu + p \to \mu^+ + n, \quad \nu_\mu + n \to \bar{\mu} + p \tag{2.3}$$

but not

$$\bar{\nu}_\mu + p \not\to e^+ + n, \quad \nu_\mu + n \not\to \bar{e} + p \tag{2.4}$$

so that ν_μ, ν_e represent distinct particles.

Direct experimental proof for the existence of the τ neutrino, such as the $\nu_\tau + n \to p + \tau^-$ reaction, is presently lacking. Thus far the properties of ν_τ can only be assessed indirectly, from the energy it would carry off in the decay of a τ lepton. In this manner the limit $m_{\nu_\tau} < 250$ MeV has been set on the ν_τ mass.[16)]

If the τ lepton were to decay not into ν_τ but into ν_e or ν_μ, then there would be a strong nonconservation of lepton charge, and τ leptons ought to be formed in ν_e and ν_μ beams, say by the process $\nu_\mu + n \to \tau + p$. Such reactions are not observed in the laboratory: accordingly, one may set a limit on the probability of the weak processes that would induce ν_μ $(\nu_e) \to \tau$ transitions, and hence on the probability that the τ lepton will decay into ν_e or ν_μ.

Information on the total decay probability of the τ lepton is indefinite at present, but refinement of the data will likely allow us to preclude the possibility of such decays. The current experimental restrictions on the properties of τ and ν_τ are described in another review in this journal.[74]

The absence of $\nu_\mu \to e$ or $\nu_e \to \mu$ processes has several different interpretations. According to one of these, each type of charged lepton and associated neutrino would consist of its own kind of matter, and separate electron-, muon-, and τ-type charges would be preserved. In the early 1950's one of the authors[45] as well as Konopinski and Mahmoud[46] proposed an alternate interpretation, whereby $e^-, \mu^+, \nu_e, \bar{\nu}_\mu$ would all have the same lepton charge in common, so that ν_μ, possessing the opposite charge, could not transform into e^-. In such a scheme, lepton-number conservation would not forbid the reaction $\bar{\nu}_\mu + n \to e + p$; but in the absence of right-handed currents this reaction is excluded by helicity arguments: left currents cannot turn right

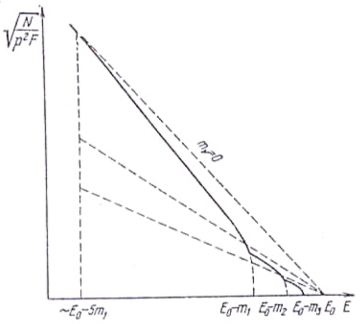

FIG. 4. The Kurie plot (idealized) for the spectrum of the β decay $H^3 \to He^3 + e^- + \bar{\nu}_e$ in the case where the neutrino has several masses: $m_1 > m_2 > m_3 \neq 0$.

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14) See the discussion of oscillations in Sec. 2c.

15) We are indebted to V. A. Lyubimov and V. Z. Nozik for calling our attention to this circumstance.

16) Existing data do not rule out, for example, the possibility that $m_{\nu_\tau} \approx 200$ MeV and that ν_τ is unstable with, say, a lifetime of order $10^{-4} - 10^{-2}$ sec. It is worth recalling that the most stringent constraints on the mass of stable ν_μ, ν_τ are set by cosmology.[5,73] In the event of unstable neutrinos, however, the cosmological limits would be much weakened (see the review by Dolgov and one of us[73]).

neutrinos into left electrons.

Lepton-number (lepton charge) conservation has been verified to within a few percent, so there is no reason to believe that the corresponding conservation laws do not have just as absolute a character as the conservation of electric charge.

Several current theories classify particles in such a way that the electron and its neutrino fall into the same group (or family) of particles as the pair of quarks from which the proton and the neutron are built. Muons and muon neutrinos (neutrettos) are combined with the strange and charmed quarks that make up the corresponding strange hadrons and hadrons with charm. Finally, a third family includes the charged τ meson and (by assumption) its neutrino, along with two even heavier quarks. These two flavors of quarks, belonging to hadrons, should give some heavier particles still. Only particles with the b (bottom) quark have yet been discovered; there is no direct evidence for the existence of the sixth and heaviest kind of quark.

The particles in the second and third families are heavier than those in the first family, so they will decay, transforming into first-family particles. This decay will take place according to the laws of weak interaction, giving a lifetime of order 10^{-10}–10^{-6} sec for the strange particles and the muon. The lifetime of the third-family particles is far shorter. Thus the question of why muons and τ leptons should exist is replaced by the question of why three families of particles should exist.[17] To a first approximation in which the neutrino has negligible mass, the three types of neutrinos would not transform into one another, unlike the transmutations of heavy quarks into light quarks. But one cannot be dogmatic about the stability of the neutrino!

c) Neutrino oscillations

The phenomenon of oscillations has been thoroughly studied in the $K^0,\bar{K}^0$ meson system. These two mesons have opposite strangeness, that is, the specific charge determined by the number of strange (s) quarks or anti-s quarks ($\bar{s}$) making up the hadron. K^0 consists of $\bar{s}$ and d quarks; $\bar{K}^0$, of s and $\bar{d}$. In the processes of electromagnetic and strong interaction, s quarks can vanish only through $s\bar{s}$ pair annihilation, so that the number of s quarks less the number of anti-s quarks ($\bar{s}$) will be conserved. But in weak-interaction processes quarks can disappear individually, turning into u and d quarks in accord with baryon-charge conservation, such as by the process $\bar{s}+d\to\bar{u}+u$ or $s+\bar{d}\to u+\bar{u}$. Thus weak interaction can excite the transitions

$$K^0=(\bar{s}d)\to(u\bar{u})\to(s\bar{d})=\bar{K}^0. \tag{2.5}$$

Because of such transitions, $K^0,\bar{K}^0$ states with definite strangeness are not states with a definite value for the mass. The states with a well-defined mass (and lifetime) are $K_1=(K^0+\bar{K}^0)/\sqrt{2}$, $K_2=(K^0-\bar{K}^0)/\sqrt{2}$, and the corresponding masses $m_{K_1}\neq m_{K_2}$. If a K^0 meson of energy E, say, is formed in some hadron process, that is, the state $K^0=(K_1+K_2)/\sqrt{2}$, where K_1,K_2 have energy E and momenta $p_1=\sqrt{E^2-m_{K_1}^2}$, $p_2=\sqrt{E^2-m_{K_2}^2}$, then at a distance x from the source we will have the state

$$K(x)=\frac{K_1(0)}{\sqrt{2}}e^{ip_1x}+\frac{K^2(0)}{\sqrt{2}}e^{ip_2x}$$
$$=\frac{1}{2}K^0(e^{ip_1x}+e^{ip_2x})+\frac{1}{2}\overline{K^0}(e^{ip_1x}-e^{ip_2x}). \tag{2.6}$$

An originally pure K^0 beam will acquire a $\bar{K}^0$ contaminant. The proportion of $\bar{K}^0$ will depend on the distance x:

$$P(\overline{K}_0)=\frac{1}{2}[1-\cos(p_1-p_2)x]=\sin^2\frac{(p_1-p_2)x}{2}=\sin^2\frac{\delta m_K^2 x}{4E},$$

where

$$\delta m_K^2=m_{K_1}^2-m_{K_2}^2,$$

and we have made use of the fact that for $E\gg m_{K_{1,2}}$, $p_{1,2}\approx E-m_{K_{1,2}}^2/2E$. The $\bar{K}^0$ component of the K^0 beam will oscillate as x varies. We have deliberately simplified here the description of the oscillations in the $K^0,\bar{K}^0$ system (actually one ought to take into account the difference in the lifetimes of K_1,K_2 and the effects of violation of CP invariance) so as to emphasize the resemblance of neutrino oscillations to the $K^0,\bar{K}^0$ oscillations.

Pontecorvo,[76] as early as 1957, was the first to point out that if lepton charge[18] is not strictly conserved and if the neutrino has a small but finite rest mass, then neutrino oscillations could arise, by analogy to the $K^0,\bar{K}^0$ oscillations. In order for neutrino oscillations to develop it is important that both conditions be satisfied simultaneously. Merely to have nonconservation of electronic or muonic charge, which might have induced such processes as $\mu^+\to 2e^++e^-$, is not enough; in order for oscillations to occur it is essential that the proper time $c^2\tau^2=c^2t^2-x^2$ of the particle be different from zero, that is, a nonzero rest mass is required.

If the neutrino does have a rest mass, then the neutrino states with a definite lepton number (ν_e,ν_μ,ν_τ) will not necessarily possess a well defined mass.

Suppose that two quantum states, say ν_e,ν_μ, represent a superposition of states ν_1,ν_2 with definite masses m_1,m_2:

$$|\nu_e\rangle=\sin\alpha\cdot|\nu_1\rangle+\cos\alpha\cdot|\nu_2\rangle,$$
$$|\nu_\mu\rangle=\cos\alpha\cdot|\nu_1\rangle-\sin\alpha\cdot|\nu_2\rangle. \tag{2.7}$$

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[17] Perhaps the answer may have something to do with the fact that CP violation is a general consequence of models with three families of quarks.[75] Three quark families are *necessary* if we wish to associate the baryon asymmetry of the universe with the CP-asymmetry of baryon creation. For the cosmological constraints on the number of families, see the review by Dolgov and Zel'dovich[73] and Sec. 4 of the present review.

[18] As formulated today, the three different lepton charges electron-, muon-, tau-type) are not strictly conserved. Incidentally, we would mention that publications abroad regularly refer to later work,[77-79] overlooking the 1957–1958 papers[76] which establish Pontecorvo's absolute priority in this matter.

Then if any process (say in β^+ decay) taking place at the point $x=0$ a neutrino ν_e should be formed, the energy of ν_e will be fixed (since the decay energy is known), and $m_1 \neq m_2$ will imply that the momenta $p_1 \neq p_2$. Hence at distance x from the source we will observe a superposition of plane waves:

$$|\nu(x)\rangle = \sin\alpha e^{ip_1x}|\nu_1\rangle + \cos\alpha e^{ip_2x}|\nu_2\rangle. \tag{2.8}$$

For given energy $E \gg m_1, m_2$ we will have (in a system of units with $\hbar = c = 1$)

so that the neutrino state $\nu(x)$ will be described by a plane wave $A(x)e^{ip_1x}$ with the oscillating amplitude

$$A(x) = \sin\alpha \cdot |\nu_1\rangle + \cos\alpha \cdot e^{i(m_1^2 - m_2^2)x/2E}|\nu_2\rangle. \tag{2.9}$$

In the case of neutrinos we are able to record only states having a definite lepton charge ($\nu_e, \nu_\mu, \ldots$), and the probability of recording such states will depend on the distance; for example, the probability of detecting ν_e at distance x from the ν_e source will be proportional to

(2.10)

Observable oscillation effects will arise, with the quantity $L = 4\pi E/(m_1^2 - m_2^2)$ describing the length of the oscillations [at that distance $P(\nu_e)$ will reach a minimum value $P(L) = 1 - \sin^2 2\alpha$; beyond, $P(2L) = 1$, and so on].

If the source or the detector is large compared with the oscillation length, the contribution of the various oscillations will be averaged out, so that the mean flux should be $\overline{P} = 1 - \frac{1}{2}\sin^2 2\alpha$. If $\alpha = \pi/4$ we would have $P(L) = 0$ and $\overline{P} = \frac{1}{2}$.

The question of what states will arise in the oscillation process ties in with the nature of the neutrino mass. If the neutrino has a pure Dirac or pure Majorana mass, oscillations $\nu_e \to \nu_\mu(\overline{\nu}_e \to \overline{\nu}_\mu)$ or $\nu_e \to \nu_\tau(\overline{\nu}_e \to \overline{\nu}_\tau)$ would arise. Transitions of the type $\nu_{eL} \to \nu_{\mu R}$, which are possible in the case of a Majorana mass, will be suppressed because of the approximate (to within a factor of order m_ν/E_ν) conservation of helicity. But if the neutrino mass is of both Dirac and Majorana character, that is, if there exist $\psi_\nu, \overline{\psi}_\nu$ states of right neutrinos and left antineutrinos as well as off-diagonal mass transitions $\varphi \to \overline{\varphi}$ and $\varphi \to \psi$, then it would also be possible for ν–$\overline{\nu}$ transitions to occur,[72,76,80-82] in states practically inaccessible to the detectors now available [for example, $\nu_{eL} \to \overline{\nu}_{\mu L}(\varphi_{\nu_e} \to \psi_{\nu\mu})$].

3. ASTROPHYSICAL IMPLICATIONS OF MASSIVE NEUTRINOS

a) Solar neutrinos and laboratory evidence of oscillations

The hope of detecting neutrino oscillations has become a matter of urgency in light of the problem of neutrinos arriving from the sun.[3,4] Nuclear processes in the solar interior will produce only electron-type neutrinos. If neutrino oscillations occur, some of these ν_e may be converted into ν_μ or ν_τ. Neutrino detectors placed on the earth would only be able to record ν_e, as the detection technique relies on the reaction $\nu_e + Cl^{37} \to e + Ar^{37}$. The energy of solar neutrinos is far below threshold for production of μ and τ; the reactions $\nu_{\mu(\tau)} + Cl^{37} \to \mu(\tau) + Ar^{37}$ would not be induced by ν_μ, ν_τ of such low energy.

Because of the oscillations the neutrino flux that is recorded will be lower than predicted by theory,[19)] so one might have expected that neutrino oscillations would explain the observed deficiency[3,4] in the solar-neutrino flux. But the mere fact that the flux appears to be lower than indicated by theory cannot be taken as proof that oscillations are occurring: the deficiency might equally well be due to other astrophysical or experimental factors. It is important to emphasize that whatever those factors may be, there is no reason to question our basic ideas about the physics of processes in the sun; there can be no doubt that the prime source of solar energy is thermonuclear synthesis in the sun's interior. Confirmation is afforded simply by the fact that we do observe neutrinos from the sun. But if laboratory experiment should indicate that oscillations do exist, then a depressed level of neutrino flux would be a natural consequence of the oscillations.

We would point out that if the oscillation length is long enough ($L \approx 10^{10}$–10^{11} cm, corresponding to $\delta m^2 \approx 10^{-8}$–$10^{-9}$ eV^2), longer than the diameter of the earth, then observations of solar neutrinos might exhibit the Pomeranchuk effect—a periodic fluctuation in the signal due to the ellipticity of the earth's orbit around the sun. In this event sensitive solar-neutrino detectors might serve as a valuable complement to laboratory facilities in the research for neutrino oscillations. When the Pomeranchuk effect is being sought in solar-neutrino experiments, one has to separate out the contribution of monochromatic neutrinos from the reactions $e^- + Be^7 \to \nu_e + Li^7$ or $p + e^- + p \to d + \nu_e$. According to Ehrlich[83] there may even now be evidence in Davis's observations[3,4] for a Pomeranchuk effect caused by oscillations of monochromatic neutrinos with $\delta m^2 \approx 5 \times 10^{-10}$ eV.^{2} However, this interpretation can hardly be considered firmly established at the present time.

Some laboratory evidence of possible neutrino oscillations has recently been announced.[84,85] What is the basis for these claims?

The fragments (A, Z) of uranium fission in a reactor are neutron-excess isotopes unstable against β decay. Electron-type antineutrinos will therefore be produced in the reactor by the decays

$$(A, Z) \to (A, Z+1) + e^- + \overline{\nu}_e,$$
$$n \to p + e^- + \nu_e. \tag{3.1}$$

The $\overline{\nu}_e$ flux from the reactor can be recorded by means of the inverse β-decay reaction

19) If neutrinos have a mass and mass difference of the order of electron volts, complete averaging should occur over astronomical distances: a Dirac or Majorana mass could not depress the flux by more than a factor of three, or a mixed Dirac and Majorana mass by more than a factor of six.

$$\bar{\nu}_e + p \to n + e^+. \tag{3.2}$$

At the same time a neutron will be recorded from the reaction $n + Cd \to \gamma + \cdots$ as well as a positron from Cherenkov radiation and subsequent annihilation by the process $e^+e^- \to 2\gamma$. The notable feature is that by measuring the energy of the positron one can unambiguously recover the energy of the antineutrinos that have been recorded:

$$\bar{\nu}_e \text{ energy} = e^+ \text{ energy} + (m_n - m_{e^+} - m_p)c^2$$
$$= e^+ \text{ energy} + 2.3 \text{ MeV}. \tag{3.3}$$

The measured ν spectrum can be compared with the theoretical spectrum. Experiments were performed over two distances from the reactor, 6 m and 11 m, and in the opinion of the authors[84,85] the dependence of the number of events on E/L is oscillatory in character[20)] (Fig. 5).

Splitting of the deuteron d can take place in $\bar{\nu}_e$ beams from a reactor:

$$\text{(A)}\quad \bar{\nu}_e + d \to e^+ + n + n. \tag{3.4}$$

The deuteron can also be split without e^+ formation, owing to neutral currents:

$$\text{(B)}\quad \bar{\nu}_e + d \to \bar{\nu}_e + p + n. \tag{3.5}$$

If oscillations give rise to $\bar{\nu}_e \to \bar{\nu}_\mu$ or $\bar{\nu}_e \to \bar{\nu}_\tau$ transitions, the number of reactions (A) should diminish. The reactions (B) will take place for $\bar{\nu}_\mu, \bar{\nu}_\tau$ with the same probability as for $\bar{\nu}_e$; hence if $\bar{\nu}_e \to \bar{\nu}_\mu, \bar{\nu}_e \to \bar{\nu}_\tau$ oscillations are present the number of reactions (B) would remain unchanged.

This number could be reduced if transitions to mirror states (Sec. 1c) exist: $\varphi_\nu \to \psi_\nu$, because reaction (B) has negligible probability for mirror states, but there should be a compensating drop in the number of reactions (A); hence whatever the detailed properties of the oscillations may be, the ratio R of the number of (A) reactions to the number of (B) reactions may serve as an indicator for the existence of oscillations.

However, it is quite difficult to measure R accurately. In order to discriminate the reaction (B) experimentally one would have to record the final neutron coming from the disintegration of the deuteron, while in the case of reaction (A) two neutrons should be recorded simultaneously. Since not all neutrons will be detected, substantial uncertainty arises in determining the number of reactions (A) and the ratio R, and it is aggravated by the differing interaction of the nucleons in the final states of reactions (A) and (B). Accordingly, the manifestation of any oscillation effects becomes more complicated.

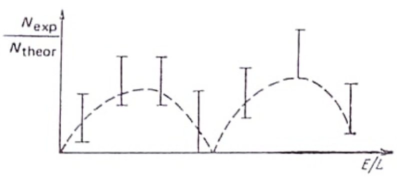

FIG. 5. Oscillations in the ratio of the observed and predicted number of events.

Feynman and Vogel[86] have criticized on these grounds the oscillation data[84,85] that rely on measurement of the ratio R. They show that the data contain an inherent inconsistency, and from an independent analysis of the data they conclude that the apparent evidence for oscillations merely reflects the inaccuracy of the measurements. But new experiments are being planned, and before very long we shall probably learn whether or not neutrino oscillations exist.

There have been some interesting proposals in this regard to search for neutrino-oscillation effects in ν_e scattering.[87,88] Electron neutrinos should be scattered by electrons both through exchange of a charged $W^\pm$ boson and through exchange of a neutral Z^0 boson. But ν_μ, ν_τ can be scattered by an electron only through Z^0 exchange. The cross section for $\nu_e e$ scattering is predicted to be about six times the cross sections for scattering of ν_μ, ν_τ by an electron. If $\nu_e \to \nu_\mu$ and $\nu_e \to \nu_\tau$ oscillations exist, then monochromatic neutrino beams should exhibit oscillations in the νe scattering cross section because of those transformations. If electron neutrinos ν_e exist, their oscillations into ν_μ or ν_τ would diminish the cross section for electron scattering. Conversely, oscillations of ν_μ or ν_τ into ν_e would enlarge the cross section for neutrino–electron interaction. In nonmonochromatic beams the oscillation effects would be averaged out, decreasing (in ν_e beams) or increasing (in ν_μ or ν_τ beams) the effective cross section for interaction with electrons.

In the absence of oscillations, the cross sections for scattering of antineutrinos $\bar{\nu}_e$ and $\bar{\nu}_\mu$ (or $\bar{\nu}_\tau$) would differ by only a factor of three. Thus neutrino-oscillation effects should be manifested more weakly in the scattering of antineutrinos by electrons than in the scattering of neutrinos.

In experiments involving neutrinos from the sun or antineutrinos from reactions the experiments are concerned with electron-type neutrinos and antineutrinos. It is to this species of neutrino (antineutrino) that the suspected evidence of neutrino oscillations refers.

The chief sources of neutrinos in accelerator experiments are decays of π and K mesons, in which the neutrinos produced are primarily of another type: muon neutrinos ν_μ and antineutrinos $\bar{\nu}_\mu$. Experiments with muon neutrinos and antineutrinos show no sign that any oscillations exist, and the experimental data set limits on the parameters of $\nu_\mu(\bar{\nu}_\mu)$ oscillations. This absence of observable oscillation effects for muon neutrinos and antineutrinos presumably either indicates a weak degree of mixing between muon neutrinos (antineutrinos) and other types of neutrinos (antineutrinos), with a

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20)The possible relationship between the observed baryon asymmetry of the universe and the interaction parameters of elementary particles, first discussed more than a decade ago by Sakharov[125] and Kuz'min[126] and later by Ignat'ev *et al.*,[127] is now regarded as the most noteworthy cosmological implication of models for grand unification of the strong, weak, and electromagnetic interactions (see two previous reviews[54,73]).

small value of $\sin 2\alpha$ and hence a small oscillation amplitude, or else it implies a small value for δm^2 and thereby a long oscillation length.

The most stringent constraints on the value of δm^2 (assuming strong mixing of neutrinos) can be obtained from experiments with cosmic neutrinos.[89,90] Interactions of cosmic protons in the atmosphere will induce formation of charged π and K mesons, whose decay products will be "atmospheric" muons and muon neutrinos. Since muon production in such decays will inevitably be accompanied by production of muon neutrinos, one can calculate[91] from the observed atmospheric-muon flux a value for the flux of resultant atmospheric neutrinos. Neutrinos can pass straight through the solid earth and actually come out on the opposite side. The interactions of such muon neutrinos rising from below will yield fast muons, and these have been recorded with the facility at the Baksan Valley Neutrino Observatory of the Institute of Nuclear Research, USSR Academy of Sciences. From their observations of the upward passage of a fast muon from below, the experimenters there[92] conclude that the observed number of events is consistent with the predictions of theory. If $\nu_\mu \to \nu_e$ or $\nu_\mu \to \nu_\tau$ oscillations with $\delta m^2 > 0.01$ eV^2 existed (assuming maximum mixing, $\sin 2\alpha \approx 1$), then the ν_μ flux entering the neutrino telescope "from below" would be lower than the calculated value (since some of the ν_μ would be converted into ν_e or ν_τ), and thus fewer fast muons would be observed. The good agreement between the expected and observed number of events therefore enables a limit to be set for reactions of the type $\nu_\mu \to$ all: $\delta m^2 < 0.01$ eV^2 (for maximum mixing). This upper limit depends significantly, however, on the premise that the mixing angle is large; if the mixing should be small there would be no such limit.

b) Instability of the neutrino

The failure of lepton charge to be conserved if the neutrino has a nonzero mass not only may manifest itself as an instability of the states ν_e, ν_μ, ν_τ with definite lepton charge (neutrino oscillations); in addition, neutrino states of definite mass (say ν_1, ν_2, ν_3) may decay, for example, through electromagnetic decay

$$\nu_1 \to \nu_2 + \gamma \tag{3.6}$$

of a heavier neutrino ν_1 to a lighter neutrino ν_2. Estimates indicate[93-95] that the probability of such decay is proportional to $W \sim G_F^2 \alpha m_\nu^5 \sum_l (m_l^2/m_W^2)^2$, and if $m_{\nu_1} \approx 30$ eV (with $m_{\nu_1} \gg m_{\nu_2}$) the lifetime of ν_1 would be $\tau \approx 5 \times 10^{36}$ sec $\approx 10^{29}$ yr. Thus the neutrino lifetime predicted by theory exceeds the age of the universe ($\sim 10^{10}$ yr) by 19 orders of magnitude.

The neutrino decay probability is very low because the neutrino mass is small, and the decay probability (3.6) is proportional to m_ν^5. Furthermore, there is an extra suppression factor $\sum_l (m_l^2/m_W^2)$ arising from the orthogonality of the matrix for mixing of neutrino states. The point is that the process (3.6) operates through the virtual transitions $\nu_1 \to l + W \to l + W + \gamma \to \nu_2 + \gamma$, and the transitions $\nu_1 \to l + W$ and $l + W \to \nu_2$ are proportional to the elements a_{1l}, a_{l2} of the mixing matrix. In the limit $m_W \to \infty$, the amplitude of the decay $\nu_1 \to \nu_2 + \gamma$ will be determined by the sum $\sum_l a_{1l} a_{l2}$ over all charged leptons, which vanishes since the mixing matrix is orthogonal.

Indeed, if there are two kinds of neutrinos [see Eqs. (2.7)] we will have $a_{1e} = \cos\alpha, a_{1\mu} = -\sin\alpha, a_{e2} = \sin\alpha, a_{\mu 2} = \cos\alpha$, and $a_{1e}a_{e2} + a_{1\mu}a_{\mu 2} = \sin\alpha \cdot \cos\alpha - \sin\alpha \times \cos\alpha = 0$. The decay (3.6) will have a nonzero amplitude only in the first order with respect to m_l^2/m_W^2: the sum

$$\sum_l a_{1l} a_{l2} \frac{m_l^2}{m_W^2} \neq 0. \tag{3.7}$$

Thus in the case of two neutrino types

$$\sin\alpha \cdot \cos\alpha \frac{m_e^2}{m_W^2} - \sin\alpha \cdot \cos\alpha \frac{m_\mu^2}{m_W^2} \approx -\frac{m_\mu^2}{m_W^2} \sin\alpha \cdot \cos\alpha. \tag{3.8}$$

The sum $\sum_l a_{1l} a_{l2} (m_l^2/m_W^2)$, and thereby the amplitude of the decay (3.6), will be dominated by the contribution of the heaviest charged lepton.

With so low a probability, the process (3.6) will be practically unobservable under laboratory conditions.

Lately there has been wide discussion[95-98] of the chances of detecting the electromagnetic radiation that would be generated in decays (3.6) of primordial neutrinos. Cosmology makes the definitive prediction (see below) that the mean number density of primordial neutrinos at the present epoch should be ≈ 150 cm^{-3}. Unfortunately, the theoretical estimate we have mentioned for the lifetime of a neutrino leaves little room for hope that radiation from decays (3.6) can be recorded. The intensity of the photons arising from decays (3.6) of primordial neutrinos would be 10–12 orders of magnitude weaker than the observed ultraviolet background. Nonetheless, the eagerness literally to "see" the neutrino sea [to observe decays (3.6) of primordial neutrinos] has grown so strong that desperate measures have been undertaken to lower the theoretical value for the lifetime of a neutrino, even to the extent of applying theoretical ideas that are not very well founded. At any rate, on the basis of attempts to observe ultraviolet background radiation one can now maintain[98] that the lifetime of a nuetrino of mass 10–100 eV against decay (3.6) to a photon of energy in the range $5 < \varepsilon_\gamma < 50$ eV ought to be at least 10^{22}–10^{23} sec ($\sim 10^{15}$ yr).

If neutrinos have a Dirac mass they should possess a small magnetic moment[99-101]: $\mu_\nu \sim e G_F m_\nu \sim 10^{-13} \mu_b$ (for $m_\nu = 30$ eV).

One will recall that as a charged Dirac particle moves through a magnetic field, the trajectory and the magnetic moment of the particle will turn in the same sense, so that its helicity will remain unchanged. A neutrino has no electric charge. In a magnetic field its direction of travel will be constant; and if the neutrino is moving through a strong magnetic field its spin may precess.[102,103] An ordinary left-handed neutrino $\nu_L(\varphi_\nu)$ may in a strong magnetic field transform into a "sterile" right-handed neutrino $\nu_R(\psi_\nu)$ which is practically incapable of interacting with matter.

The neutrino magnetic moment prediced by theory is small, much smaller than the experimental and astrophysical limits that can be set upon it.[104-106] so that helicity reversal could hardly be expected to occur in interstellar magnetic fields or even in the magnetic field of the sun.[107] Such an effect might, however, arise in the strong magnetic field of a newly formed neutron star,[95,102] diminishing the observable neutrino pulse that would accompany the stellar collapse: half of the neutrinos produced in the collapse would be converted into undetectable sterile neutrino states. Unfortunately, the uncertainties inherent in theoretical predictions of the magnitude of the neutrino flux make a successful identification of this effect highly prolematical.

c) Nonrelativistic neutrinos?

If the neutrino mass is about 30 eV, then in any laboratory experiment we would encounter mainly ultrarelativistic neutrinos. Even with a finite mass, neutrinos would behave as if they were massless. Only in those experiments of the Moscow group[1] that recorded an electron of energy between E_{max} and $E_{max} - m_\nu c^2$ was the velocity of the neutrino lower than 200,000 km/sec. That is just why it is so hard to search for neutrino mass effects. But according to the theory of the big bang, or "hot," universe (see Sec. 4), primordial neutrinos would become nonrelativistic as the expansion of the universe proceeds, and they should be nonrelativistic today.

A recent analysis of the cosmological evolution of finite-mass neutrinos[108-110] has shown that nonrelativistic primordial neutrinos ought to be distributed irregularly, condensing on the scales of galaxies and clusters of galaxies; as a result the density of such neutrinos in our Galaxy should be 10^7-10^8 cm^{-3}. So intense a background might manifest itself in some fashion in its interactions with matter.

At first glance it might seem that we should turn to the effects of interaction at high energies. The neutrino–matter interaction probability will rise with neutrino energy or, if the massive neutrinos are at rest, with the energy of the incident particles. This is just the situation we have in the Galaxy: the neutrinos are nonrelativistic, almost at rest, but cosmic rays contain particles of ultrahigh energy, and we might have had some prospect of recording bursts due to interaction of cosmic rays with the neutrino sea. However, while the neutrino number density in the Galaxy should exceed the density of interstellar gas atoms by seven or eight orders of magnitude, the probability of strong interaction between cosmic rays and atoms so far surpasses the probability of their weak interaction with neutrinos that such flashes in the ambient medium would be practically unobservable,[111] compared with the background of cosmic ray–gas interaction effects.

But it turns out that an effect should exist which is directly related to the circumstance that the neutrinos are nonrelativistic. This effect was outlined 10 years ago by the Soviet astrophysicist V. F. Shvartsman. Neutrinos in the Galaxy should have a small momentum (their velocity is only ≈300 km/sec), and accordingly a long wavelength: $\lambda_\nu \approx h/p_\nu \sim 10^{-2}$ cm; and the neutrinos will be scattered coherently by the atoms within a volume λ_ν^3. As the momentum p_ν decreases the cross section for elastic scattering of a nonrelativistic neutrino by an individual electron or nucleon will approach the constant value $\sigma_0 \approx G_F^2 m_\nu^2$. In coherent interaction the cross section is proportional to the square of the number N of particles in the volume λ_ν^3, so the interaction cross section per particle will be proportional to $\sigma_0 N$. In other words, at nonrelativistic energies the probability of neutrino–matter interaction will be enhanced!

Matter should here be considered porous or powdery, with a characteristic pore or particle size of order λ_ν. When they undergo coherent scattering the neutrinos will transfer their momentum to matter. The transfer of momentum will constitute a force. Waves in the neutrino sea will push matter with that force. If a body is at rest relative to the neutrino sea, the mean force upon it will be zero; but motion in the sea, such as the motion of the earth or the solar system, will induce a transfer of directed momentum. Experiments could best be conducted with pellets of diameter λ_ν or with platelets of thickness λ_ν, for these would show the maximum effect. One might study the periodic shift in signals induced by the earth's diurnal rotation. One could measure the oscillations of a torsion balance with spherules made of materials so selected as to increase the angular momentum. In principle, one would be able to determine his own motion through the neutrino sea.

Massive neutrinos can transfer their momentum to electrons. It would be worthwhile considering the coherent effects of interactions between neutrinos and electrons in a superconducting state—in particular, the possibility of generating quantized vortices.

If the neutrinos have a velocity $v \approx 300$ km/sec, a mass $m_\nu \approx 30$ eV, and a flux density of $\approx 10^{15}$ cm^{-2} $\times$ sec^{-1}, the acceleration imparted by the neutrino sea to a body moving within it would be $\approx 10^{-19}$ cm/sec^2. (In 1 g of matter this much acceleration would be produced, on the average, by 10^4 neutrino collisions per second.) Fantastic as it may seem, such an effect might actually be observable. The state of the art in gathering experimental data from measurements of small accelerations is now at a level of $\approx 10^{-15}-10^{-16}$ cm/sec^2 (a decade ago[112] it was only 10^{-13} cm/sec^2), expected to be achieved in a program, presently in the planning stage,[113] for testing the principle of equivalence in the space environment. There is no fundamental reason[114,115] why the measurement of arbitrarily small accelerations of a macroscopic oscillator should be precluded. Perhaps in the decades to come we can perform a direct experiment to unveil the secret of the missing mass!

4. NEUTRINOS IN THE BIG BANG

Today our cosmological perception rests on two key facts: the observed expansion of the universe and the existence of thermal electromagnetic background radiation. The ideas about the evolution of the universe based on these facts lead to an inescapable conclusion: in the remote past, some 10–20 billion years ago, the universe was very hot, so hot that the radiation density much exceeded the density of matter. According to this picture, at very early stages, just a small fraction of a second from the start of the expansion, temperatures would have been well above 1 GeV and radiation would have been in equilibrium with pairs of all sorts of particles and antiparticles—leptons and quarks. Their density in the universe would have conformed to thermodynamic equilibrium. Theories regarding this era in the evolution of the universe are intimately bound up with the very latest ideas of elementary-particle theory.

The relationship between elementary-particle theory and the theory of the big bang universe has been examined in detail in the recent review by Dolgov and one of us.[73]

In our present review we single out the question of how neutrinos would have behaved in the hot universe—what their density and composition should have been, their type and momentum distributions, their helicity. The problem of reciprocal influence of the neutrinos and their finite mass upon astronomical phenomena will not be treated in this paper. It is a subject of its own, calling on concepts, facts, methods that are far indeed from the field of elementary-particle theory. We hope that this theme will be addressed in the pages of *Uspekhi* in the very near future.

To discuss the behavior of neutrinos in the big bang is a necessary underpinning for the cosmology of the neutrino world. Let us proceed, then, to describe the evolution of neutrinos in the early universe.

As the universe began to expand the temperature would have dropped and pairs of heavier particles would have been transformed into lighter ones. The temperature decline would have been accompanied by a decrease in the density of particles, and naturally the rates of their interaction processes would have fallen as well.

When T had dropped to ≈3 MeV the rates of neutrino weak-interaction processes would have become lower than the expansion rate of the universe. Neutrinos would no longer interact with radiation. The mechanism serving to maintain thermodynamic equilibrium would be shut off. As we shall see below, however, this does not mean that equilibrium would itself soon be *violated.*

At the epoch of decoupling in the universe, along with the neutrinos, photons, and a small (10^{-10}–10^{-8}) admixture of nucleons in equilibrium with the radiation, electrons and positrons would have been present. When T reached ≈1 MeV, in the first second of expansion, the expansion rate would have begun to exceed the rate of the β processes $n \rightarrow p$, which had established an equilibrium ratio of neutron and proton densities. The ratio of the neutron population to the proton population would have been hardened, remaining unchanged as the temperature continued to fall. This is an important property for the subsequent cosmological nucleosynthesis, but since the number density of nucleons would have been low, processes involving them would not have affected the ratio of neutrino and photon densities.

During the era when $T \lesssim 0.5$ MeV, electron–positron annihilation would take place. The universe would retain its photons, neutrinos, and a small (10^{-10}–10^{-8}) proportion of electrons and nucleons. For the first three minutes of expansion the nucleons would take part in nuclear reactions, culminating in conversion of a substantial fraction of the neutrons to ^{4}He nuclei. In the intermediate stages of this transformation, D, T, and He^3 would be produced. By the time the cosmological nucleosynthesis terminated, a portion (10^{-6}–10^{-3}) of the D and T + He^3 would not yet have been consumed, and the residual density of these isotopes would depend strongly on the density of all matter. The bulk of the nucleons, 70–80%, would remain in the form of protons. At 3000°K temperature, some tens of millennia after the start of the expansion, the protons would recombine with the electrons. The hot phase would end, and matter would cease to interact with radiation.

a) Mass of neutrinos and their residual density in the universe

The theory of the big bang universe offers a definitive prediction of the residual neutrino density. Gershshtein and one of us[5] were the first, in 1966, to point out this fact. In the early universe, according to the theory, neutrinos were in equilibrium with radiation. During this era the temperature far exceeded the neutrino rest mass: the neutrino rest mass was ultrarelativistic, and the neutrino/photon population ratio was determined by the ratio of statistical factors. When $T \approx 3$ MeV the neutrinos, remaining ultrarelativistic, ceased to interact with the radiation, but the neutrino/photon density ratio remained the same. Because of neutral-current interactions all types of neutrinos were decoupled (from plasma and radiation) nearly simultaneously. Somewhat later, when $T \lesssim 0.2$ MeV, electron–positron annihilation occurred. These particles surrendered energy (properly speaking, entropy) to the photons. The number of photons per unit comoving volume increased, whereas the number of neutrinos stayed the same. Thenceforth, for all $t > 10$ sec, the neutrino/photon population ratio has remained unchanged to the present day.

One may visualize the change in the ratios of the neutrino, photon, and electron–positron pair densities as follows. The transitions $\gamma \rightarrow e^+e^- \rightarrow \nu\bar{\nu}$ may be regarded as tubes connecting vessels that contain photons, electron–positron pairs, and neutrinos, as depicted in Fig. 6. As the universe expands the energy of the neutrinos will diminish, as will the neutrino interactions, and the tube joining the neutrino vessel to the other two will, as it were, be tied off. The interaction of the neutrinos with the rest of the particles will cease. But if the levels in the vessels had been approximately equal, and then these vessels all ex-

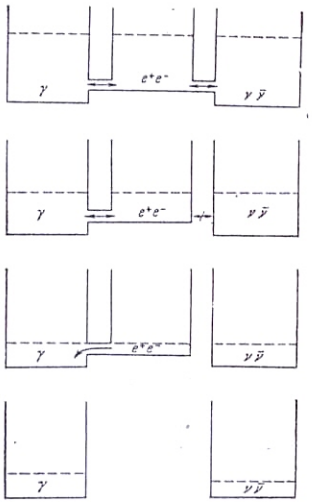

FIG. 6.

panded concurrently (for they actually are not different vessels but different gases in the same vessel, a single universe), the equality in the number of particles would be preserved. Measuring the number of photons, and knowing that there was once an era when photons and neutrinos were in equilibrium, we may infer—and evidently with confidence—that there are approximately as many primordial neutrinos as primordial photons. There will be a small quantiative difference, because the bottom of the vessel with electron–positron pairs is at a higher level than the bottom of the photon vessel (when $T < m_e$, electron–positron annihilation will set in), so everything contained in the electron–positron vessel will flow into the photon vessel. Since communication with the vessel has been closed off, the T_γ/T_ν ratio will rise from 1 to $(11/4)^{1/3}$, while the ratio of the total number of photons to the number of neutrinos and antineutrinos ($\nu_L + \bar{\nu}_R$) of the same type will decrease from 3/4 to 3/11 (see below).

If the neutrino is a Majorana particle, the calculation of the residual density of neutrinos having a small ($m_\nu \ll m_e$) but finite mass agrees completely with the calculation for massless neutrinos.[73]

For the number density of neutrinos and antineutrinos one obtains the relation

$$n_\nu + n_{\bar{\nu}} = \frac{3}{4} \cdot \frac{4}{11} n_\gamma. \qquad (4.1)$$

The factor 3/4 enters because the neutrino gas and the photon gas are described by Fermi-Dirac and Bose-Einstein statistics, respectively (a derivation is given in the recent book by Okun'[116]), while the factor 4/11 represents the increase in the photon population due to electron–positron annihilation.

Equation (4.1) holds for all types of neutrinos, but it takes into account only left neutrinos φ_ν and the right anitneutrinos $\bar{\varphi}_\nu$. There is no doubt that Eq. (4.1) is applicable, provided the neutrino is a Majorana particle.

If, however, the neutrino is a Dirac particle, then right neutrino states ψ_ν and left antineutrino states $\bar{\psi}_\nu$ will exist: these states too should be present in the universe. But it turns out that the number density of these states should be lower than the $\varphi_\nu, \bar{\varphi}_\nu$ density by at least an order of magnitude[73,117,118] (unless the mass includes Majorana terms, that is, unless lepton charge is conserved; see Sec. 4b).

If there are no right-handed currents (if the processes $\psi_\nu + e_R \to \psi_\nu + e_R$ or $\psi_\nu \bar{\psi}_\nu \to e_L^+ e_R^-$ are absent), then ψ_ν, $\bar{\psi}_\nu$ will interact with other particles only through $\psi \to \varphi (\psi \to \varphi)$ transitions arising from the nonzero neutrino rest mass. For ultrarelativistic particles ($E_\nu \gg m_\nu$) the probability of such transitions will be depressed in proportion to m_ν^2/E_ν^2; hence $\psi_\nu, \bar{\psi}_\nu$ will have a very weak interaction.

The weaker the interaction of a particle, the sooner will it emerge from equilibrium with radiation (the higher the value of T). At temperatures $T > 0.1$–1 GeV a great many kinds of particles would have been in equilibrium with radiation: $\mu^+\mu^-$ pairs, gluons, all types of quarks, and so on. Particles emerging from equilibrium with radiation would retain the same proportion of thermal energy (property, entropy) which they had according to the equipartition law before they left equilibrium. Subsequent transformations of heavy leptons, binding of quarks and gluons into hadrons, hadron annihilation, and $\mu^+\mu^-$ pair annihilation would all serve to raise the fraction of thermal energy allocable to electron–positron pairs, photons, and ordinary neutrinos. Hence their temperature, and accordingly their density, would become higher than the temperature (and density) of particles that left equilibrium prior to those transformations.

In our vessel scheme (Fig. 7) this circumstance means that $\nu_R(\psi_\nu)$ and $\bar{\nu}_L(\bar{\psi}_\nu)$ fill a separate vessel, which becomes decoupled from the radiation long before the vessel with $\nu_R(\varphi_\nu)$ and $\bar{\nu}_R(\bar{\varphi}_\nu)$ does. At the epoch when the first vessel is decoupled, many more kinds of particles would have been in equilibrium; corresponding to that epoch we should draw in the diagram numerous vessels whose bottoms lie well above the bottom of the vessel containing e^+e^- pairs. These "shallower" vessels would contain various types of quarks, gluons, and so on. Once the $\nu_R(\psi_\nu), \bar{\nu}_L(\bar{\psi}_\nu)$ vessel has been decoupled from the other vessels, the contents of all the shallower vessels would flow into the deeper ones, until only the vessels communicating

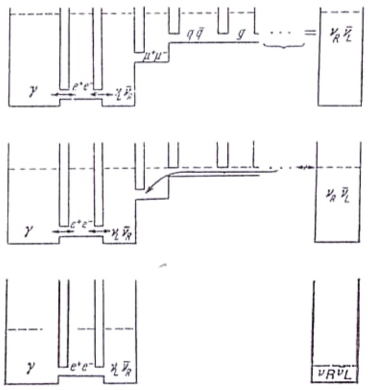

FIG. 7. $P(\nu_e) = 1 - \sin^2 2\alpha \cdot \sin^2 \left(\frac{m_1^2 - m_2^2}{4E} x \right)$.

with the neutrinos, photons, and e^+e^- pairs remain. Since the contents of all the shallower vessels poured into these deep vessels after the disconnection of the $\nu_R(\psi_\nu), \bar{\nu}_L(\bar{\psi}_\nu)$ vessel, the level in the deep vessels clearly will be substantially higher than the level in the $\nu_R(\psi_\nu), \bar{\nu}_L(\bar{\psi}_\nu)$ vessel (by a factor of 10–100); consequently the density of $\nu_R(\psi_\nu)$ and $\bar{\nu}_L(\bar{\psi}_\nu)$ will be lower by the same amount, if they are present at all. The $\nu_R(\psi_\nu), \bar{\nu}_L(\bar{\psi}_\nu)$ density could be comparable with the $\nu_L(\varphi_\nu), \bar{\nu}_R(\bar{\varphi}_\nu)$ density only if a mirror universe were to exist linked with $\nu_R(\psi_\nu), \bar{\nu}_L(\bar{\psi}_\nu)$ and having a large enough population of particles; the level in the $\psi_\nu, \bar{\psi}_\nu$ vessel could then be significantly raised. Apart from this possibility, we may regard Eq. (4.1) as valid to within 10% or better, even if right neutrinos and left antineutrinos should actually exist.

b) Neutrino density in the universe; neutrino oscillations

If the neutrino mass is purely Majorana or purely Dirac, then the possible neutrino-oscillation effects described in Sec. 2c would not alter the estimates for the residual neutrino density obtained in Sec. 4a. In both cases the oscillations would link similar components of different types of neutrinos and antineutrinos: $\varphi - \varphi$ (or $\bar{\varphi} - \bar{\varphi}$) in the case of Majorana neutrinos; $\varphi - \varphi, \psi - \psi$ ($\bar{\varphi} - \bar{\varphi}, \bar{\psi} - \bar{\psi}$) in the case of Dirac neutrinos. Since similar components of different neutrino types would have the same residual density, the densities of the different neutrino types would remain unchanged when the oscillations between similar components "turn on." Nor would there be any change in the density ratio of the ψ ($\bar{\psi}$) and φ ($\bar{\varphi}$) components if the neutrinos are Dirac in character.

A fundamentally new situation[117,119] will arise in the more general case of Dirac neutrinos with a Majorana mass, for transitions of the type $\varphi - \bar{\psi}, \bar{\varphi} - \psi$ will then become possible as well. If such transitions should come into play before the left neutrinos (right antineutrinos) are split away from the radiation [that is, if the oscillation length $L \sim E_\nu/\delta m^2 \sim T/\delta m^2$ becomes shorter than the mean free path $l \sim 1/n_i\sigma_{\nu_i} \sim 1/T^3 G_F^2 T^2$ of φ ($\bar{\varphi}$) neutrinos for $T \gtrsim 3$ MeV, corresponding to $\delta m^2 > 10^{-7}$ eV^2], then an efficient mechanism would develop to restore the equilibrium of the ψ ($\bar{\psi}$) components with the radiation. The density of the ψ ($\bar{\psi}$) components would conform to thermodynamic equilibrium. They would emerge from equilibrium a second time only when the weak-interaction rate of the φ ($\bar{\varphi}$) components becomes lower than the rate of expansion. The ψ ($\bar{\psi}$) and φ ($\bar{\varphi}$) components will simultaneously come out of equilibrium with the radiation; consequently their residual densities will be equal. Therefore type $\varphi - \bar{\psi}$ ($\bar{\varphi} - \psi$) oscillations with $\delta m^2 > 10^{-7}$ eV^2 should have the effect that even in the absence of right currents the φ, ψ states will have the same residual density; and as a result the residual density of the various species of neutrinos will acquire twice the value given by Eq. (4.1).

For $\psi - \bar{\varphi}, \bar{\varphi} - \psi$ transitions in the event that $\delta m^2 < 10^{-7}$ eV^2, the oscillations between left neutrinos and left antineutrinos (or right antineutrinos and right neutrinos) will turn on after the left neutrinos φ and right antineutrinos $\bar{\varphi}$ have come out of equilibrium with radiation, so that the combined density of the φ ($\bar{\varphi}$) and ψ ($\bar{\psi}$) components will be unchanged. On the other hand. $\varphi - \bar{\psi}$ and $\bar{\varphi} - \psi$ transitions will alter the ratio between the densities of the φ ($\bar{\varphi}$) and ψ ($\bar{\psi}$) components: before the oscillations turn on (in this case the mean free path of the left neutrinos φ or right antineutrinos $\bar{\varphi}$ will coincide with the horizon: $l \sim m_{\mathrm{pl}}/T^2$) the ψ ($\bar{\psi}$) density would have been (Sec. 4a) at least an order of magnitude below the φ ($\bar{\varphi}$) density, while after the oscillations come into play the ψ ($\bar{\psi}$), φ ($\bar{\varphi}$) densities would become equal.

The oscillation effects just described result from the fact that as the universe expands the temperature will drop, shortening the oscillation length $L \sim T/\delta m^2$, whereas the mean free path $l \sim 1/G^2 T^5$ (or, when $T < 3$ MeV, the horizon $l_{\mathrm{h}} \sim m_{\mathrm{Pl}}/T^2$) will lengthen. Hence shortly after the oscillation turns on (when $l \sim L$ or, in the era $T < 3$ MeV, when $l_{\mathrm{h}} \sim L$), the oscillation length will become much shorter than the horizon: the effects of the oscillations will be averaged out.

Naturally one can speak of the oscillations "turning on" only in a conventional sense, because a neutrino gas will always contain neutrinos of low energy $E \ll T$, which will begin to oscillate long before the condition $l \approx L$ (or $l_{\mathrm{h}} \approx L$) starts to be satisfied for neutrinos of energy $E \sim T$. But since these low-energy neutrinos comprise a small fraction of all the neutrinos, with the distribution peaking at a higher energy $E \sim 3T$, it is advisable to consider the oscillation effects specifically for neutrinos whose energy is close to the peak of the distribution.

Petcov and one of us[119] have called attention to a delicate oscillation effect, $\nu_{eL} - \bar{\psi}$ and $\bar{\nu}_{eR} - \psi$ with $\delta m^2 \approx 10^{-8}$–$10^{-10}$ eV^2, which would manifest itself directly at the epoch when oscillations turn on for $l_{\mathrm{h}} \sim L$. Oscillations of this type would in fact come into play during the era $T \sim 1$ MeV (the period of neutron hardening), when the expansion rate begins to surpass the reaction rate of the weak interactions

$$\bar{\nu}_e + p \rightleftarrows e^+ + n,$$
$$\nu_e + n \rightleftarrows e^- + p$$

which serve to establish the equilibrium density ratio of neutrons and protons. The phenomenology of neutrino oscillations indicates that if the neutrinos involved have more than two different components (φ or ψ), the effects of CP-symmetry violation may appear in the oscillations. If CP violation were to occur in $\nu_e, \bar{\nu}_e$ oscillations, then the oscillations $\nu_{eL} - \bar{\psi}$ and $\bar{\nu}_{eR} - \psi$ would turn on separately: during the era of oscillation turn-on, a substantial [compared with the total $\nu_{eL}(\varphi_{\nu_e})$ density] excess of ν_{eL} over $\bar{\nu}_{eR}$ or excess of $\bar{\nu}_{eR}$ over ν_{eL} could develop. The development of such an excess during the era of neutron hardening might significantly alter the ratio between the rates of the weak $n \to p$ and $p \to n$ reactions, thereby causing a substantial decrease ($n_{\nu_e} > n_{\bar{\nu}_e}$) or increase ($n_{\nu_e} < n_{\bar{\nu}_e}$) in the hardened density of neutrons.

c) Neutrino mass and cosmological nucleosynthesis

Let us recall the accepted classical scenario of nucleosynthesis in the theory of a big bang universe with massless left neutrinos and right antineutrinos; we have alluded to this picture before.

A considerable fraction of the hardened neutrons would have been converted by subsequent nuclear transformations into He^4 nuclei. Hence the He^4 abundance observed today would serve as a sensitive indicator of the conditions under which the neutron hardening occurred. This circumstance has been invoked to place a well-recognized cosmological limit on the admissible number of kinds of light particles, as originally proposed by Shvartsman[120] (see also Steigman *et al.*[121,122] and the previous review[73]). The higher the density of relativistic particles during the neutron-hardening era, the faster would have been the expansion at that time; neutrons would have been hardened earlier (at a higher temperature T_{hd}), the hardened neutron density would have been higher $\{$since $(n/p)_{hd} \sim \exp[(m_p - m_n)/T_{hd}]\}$, and more He^4 would have been formed. Thus the observed He^4 abundance enabled an upper bound to be set on the density of relativistic particles in the neutron-hardening period and, in particular, a limit on the number of kinds of neutrinos.

How would this picture change in the differing versions of the neutrino theory?

In work that was done a decade or more ago it was thought that the lèpton number (or numbers) would be conserved—that one might specify the chemical potential of the electron-type neutrino and directly shift the equilibrium between neutrons and protons. The decidedly untenable hypothesis of a cold universe[123] represents an extreme instance of that mode of operation. In the cold-universe hypothesis the neutrino chemical potential μ was selected such that $n/p \equiv 0$; nothing but protons would have been present.

In 1967 Wagoner, Fowler, and Hoyle[124] examined the dependence of the He^4 density on the value of μ. They showed that if $\mu/kT \ll 1$ the neutrino chemical potential would not affect the nucleosynthesis. At present a value $\mu/kT \sim 1$ seems unlikely, from the standpoint both of grand unified theories and of the observed baryon asymmetry of the universe, which suggests[20)] that $\mu/kT \lesssim 10^{-8}$.

Setting aside possible effects from the neutrino chemical potential, let us summarize the possible influence of a finite neutrino mass upon the nucleosynthesis of helium:

1. For Majorana neutrinos the standard scenario would not change.

2. For Dirac neutrinos, with a Dirac mass but with right currents or a Majorana mass absent, the picture would not differ quantitatively[73,117,118] from the standard one.

3. For neutrinos with Dirac mass and right currents, a large quantity of "mirror" particles, or a Majorana mass inducing $\bar{\psi} \to \psi$, $\psi \to \bar{\varphi}$ oscillations with $\delta m^2 > 10^{-7}$ eV^2, the number of neutrino components in the universe would be doubled in comparison with the standard picture. The density of the resultant helium would be enhanced as well.[73] In big bang model universes with a low ($\lesssim 1.5 \cdot 10^{-31}$ g/cm^3) matter density the helium abundance would be diminished, but this possibility cannot be ruled out on the basis of the observed helium abundance.

4. For neutrinos with both Dirac and Majorana mass, but with right currents or with $\bar{\psi} \to \varphi$ or $\psi \to \bar{\varphi}$ oscillations having $\delta m^2 > 10^{-7}$ eV^2 absent, the total neutrino density quantitatively would remain nearly unchanged compared with the standard model, so that the expansion rate during the neutron-hardening era would also remain the same, but:

5. The effects of CP violation in nonequilibrium oscillations ($\bar{\psi} \to \varphi$ or $\psi \to \bar{\varphi}$) of neutrinos having $10^{-10} < \delta m^2 < 10^{-8}$ eV^2 might seriously influence the density of the helium that is formed.

In principle, then, massive neutrinos could significantly alter the course of cosmological nucleosynthesis. But any such changes would result from the effects of right currents and (or) oscillations with $\delta m^2 > 10^{-10}$ eV^2, effects which (in principle!) are accessible to observation in the laboratory or in studies of neutrinos arriving from the sun. The theory of the big bang universe retains its link with performance of experiments on neutrinos!

CONCLUSION

Let us pay homage, in closing, to our anniversary hero: in just half a century the neutrino has gone from an elusive entity to a foundation stone of our existence. Its tiny mass gives it a weight of the highest order on the cosmic scale. A neutrino revolution has taken place, affecting the most fundamental principles of the universe in which we dwell. This revolution in fact marks a turning point in our approach to physical phenomena. And if we are long ruled by Occam's razor—"all surplus is cut off," "nothing exists unless it is necessary"—today we are no longer threatened by its cold steel. For whatever is not banned can happen. Doesn't the forbidden eventually turn out to be allowed, when new evidence is brought to bear?

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