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Marie Sklodowska-Curie actions

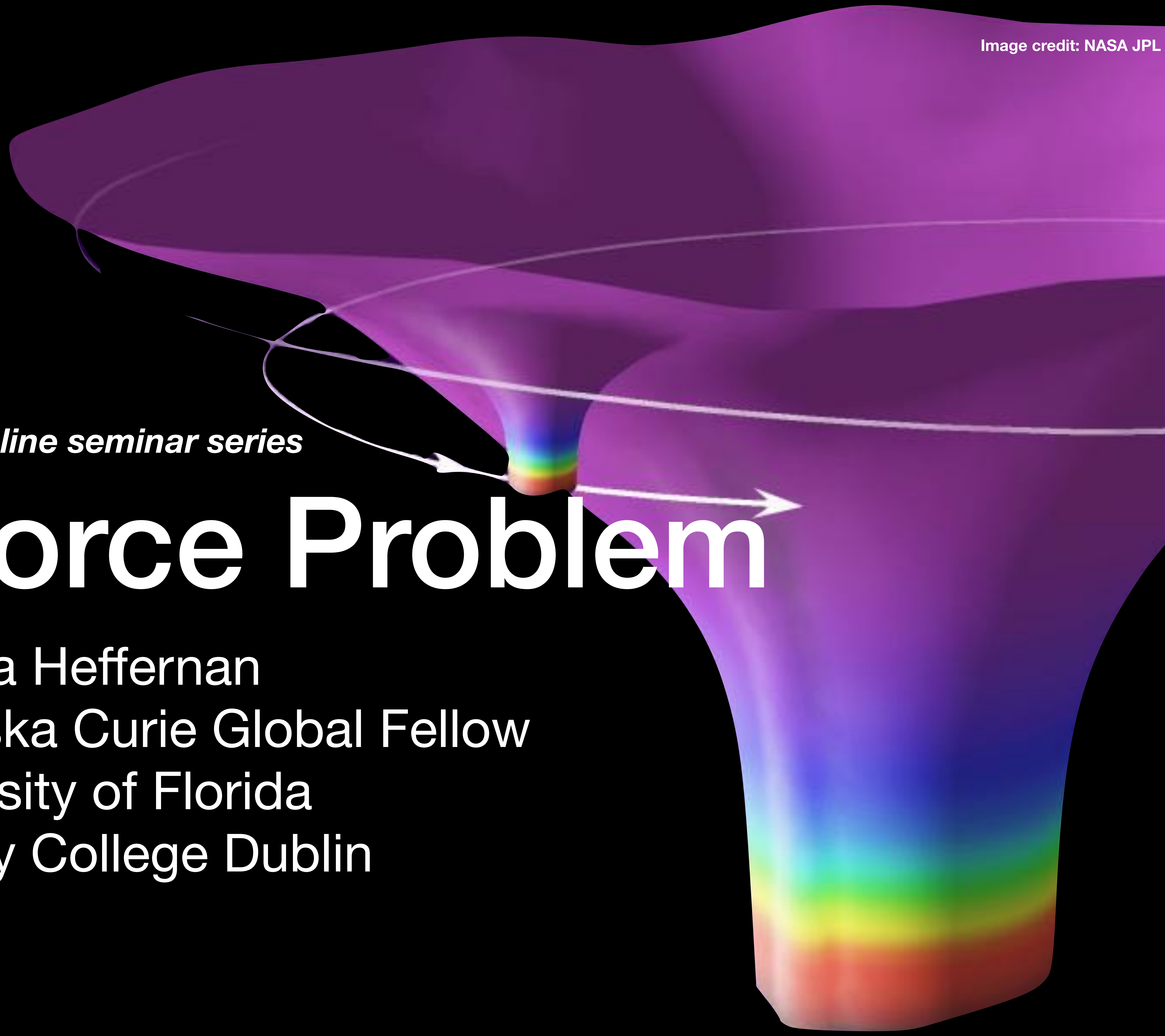


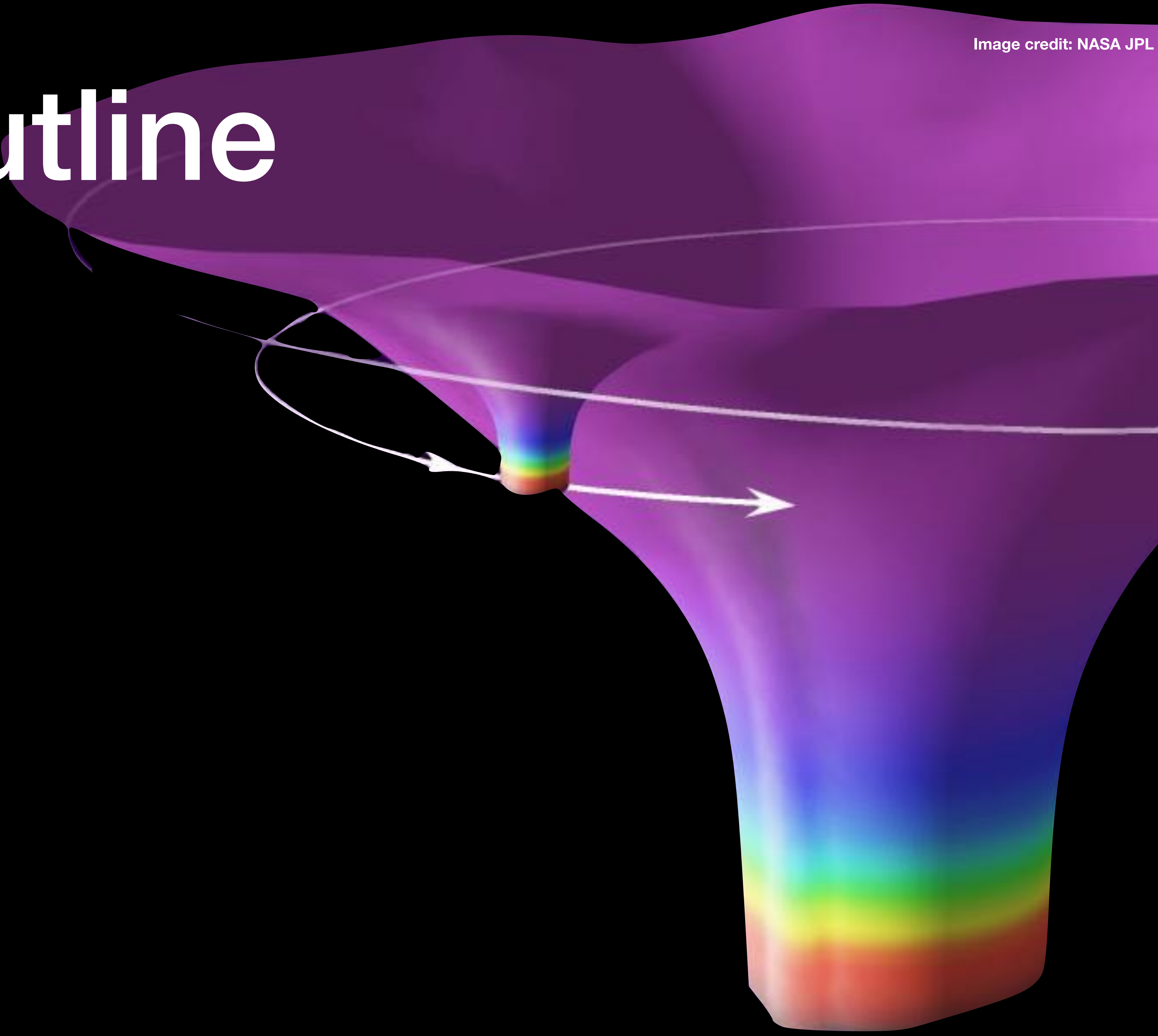
Image credit: NASA JPL

Virtual Institute of Astroparticle Physics online seminar series

The Self-Force Problem

Anna Heffernan
Marie Skłodowska Curie Global Fellow
University of Florida
University College Dublin

Outline



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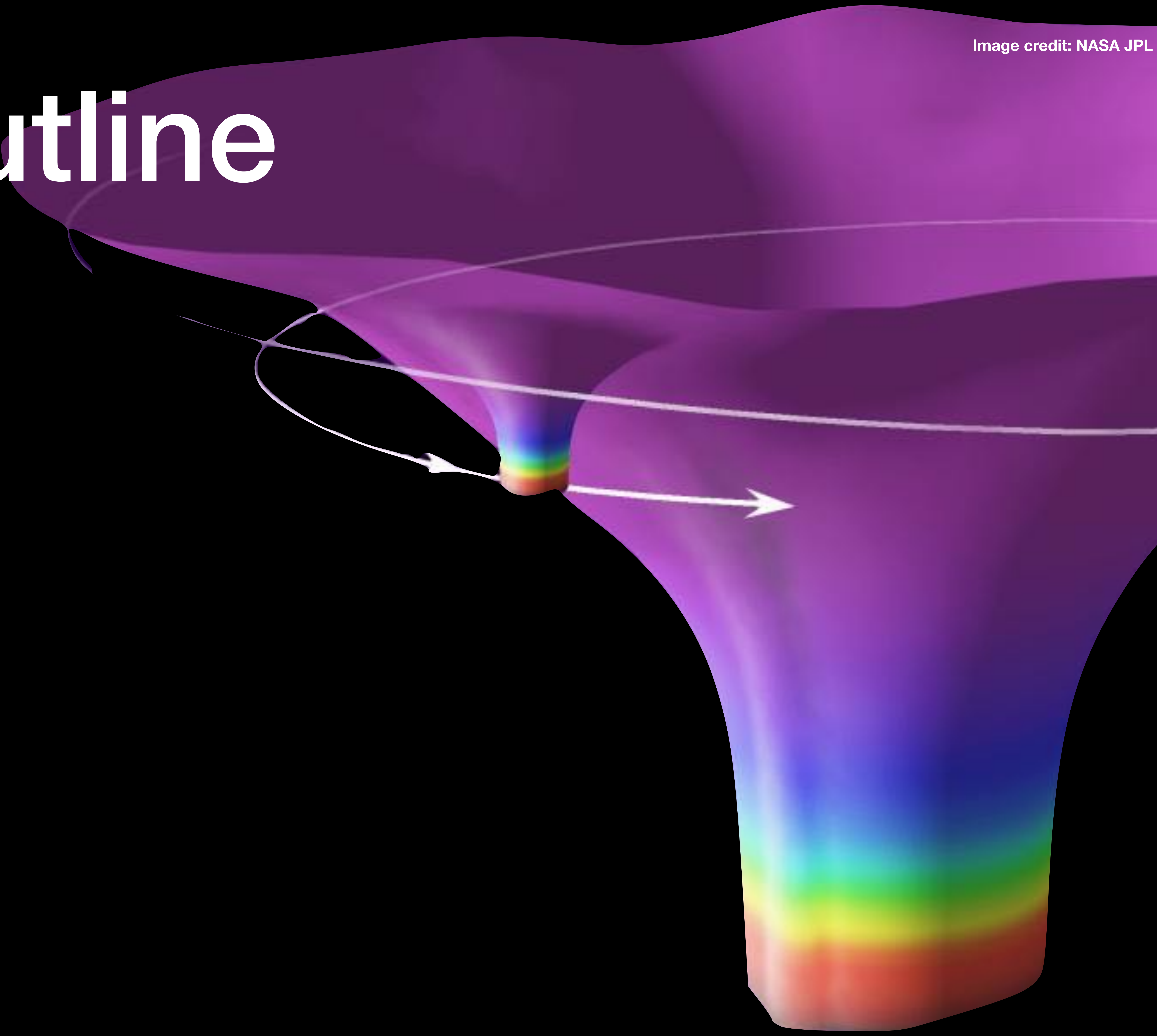
Outline

- The two body problem



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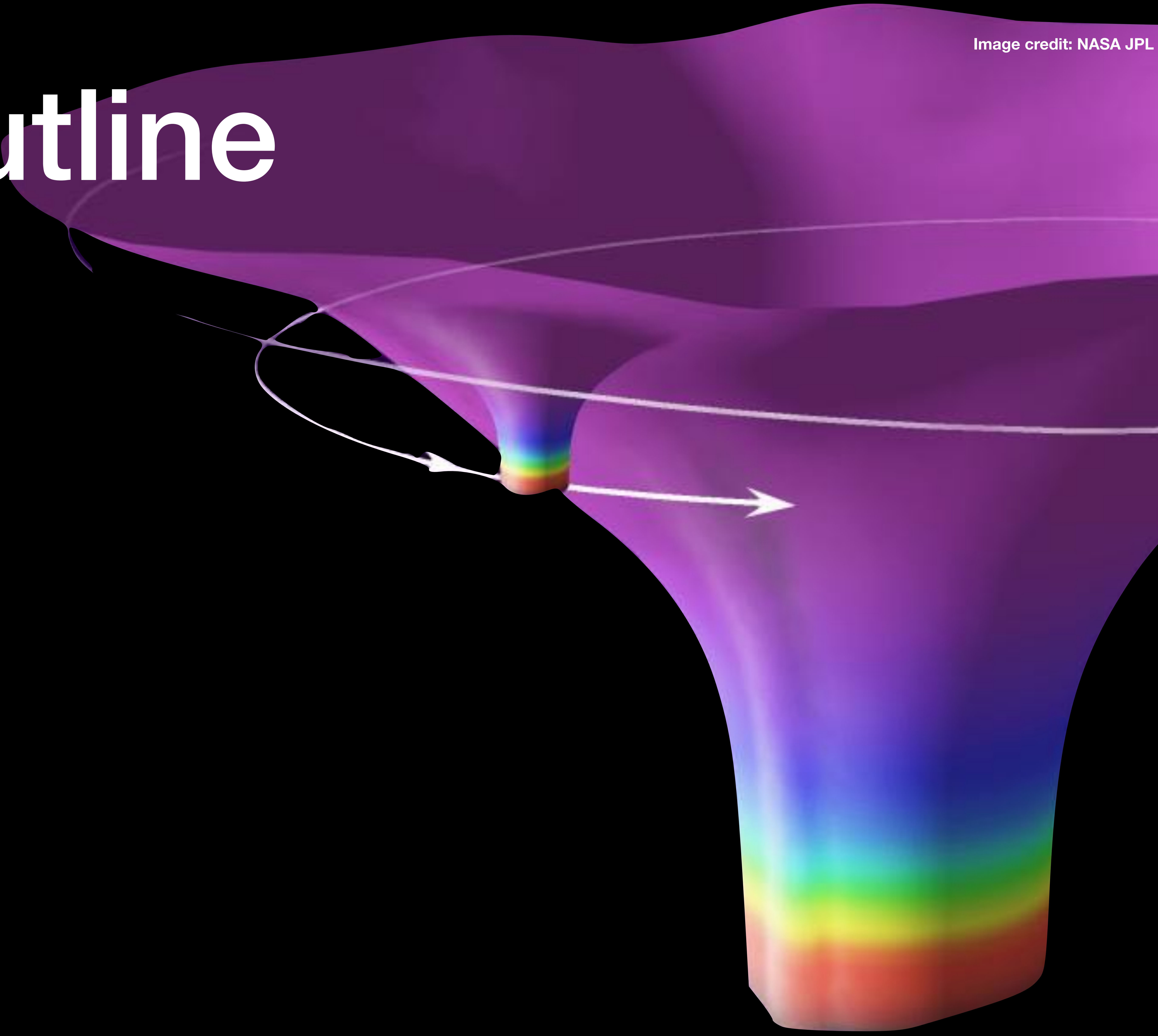
Outline

- The two body problem
- What is the self-force?



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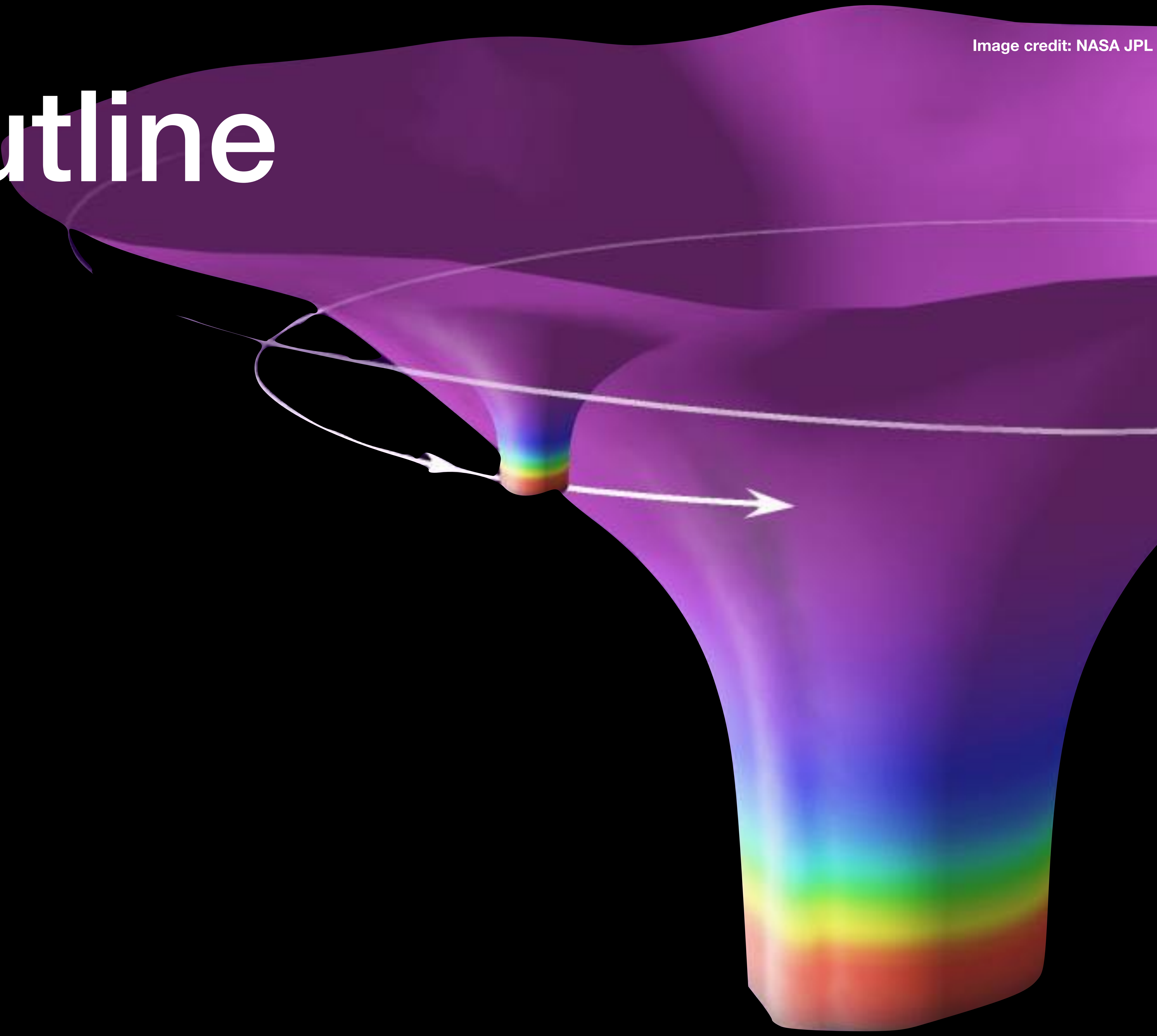
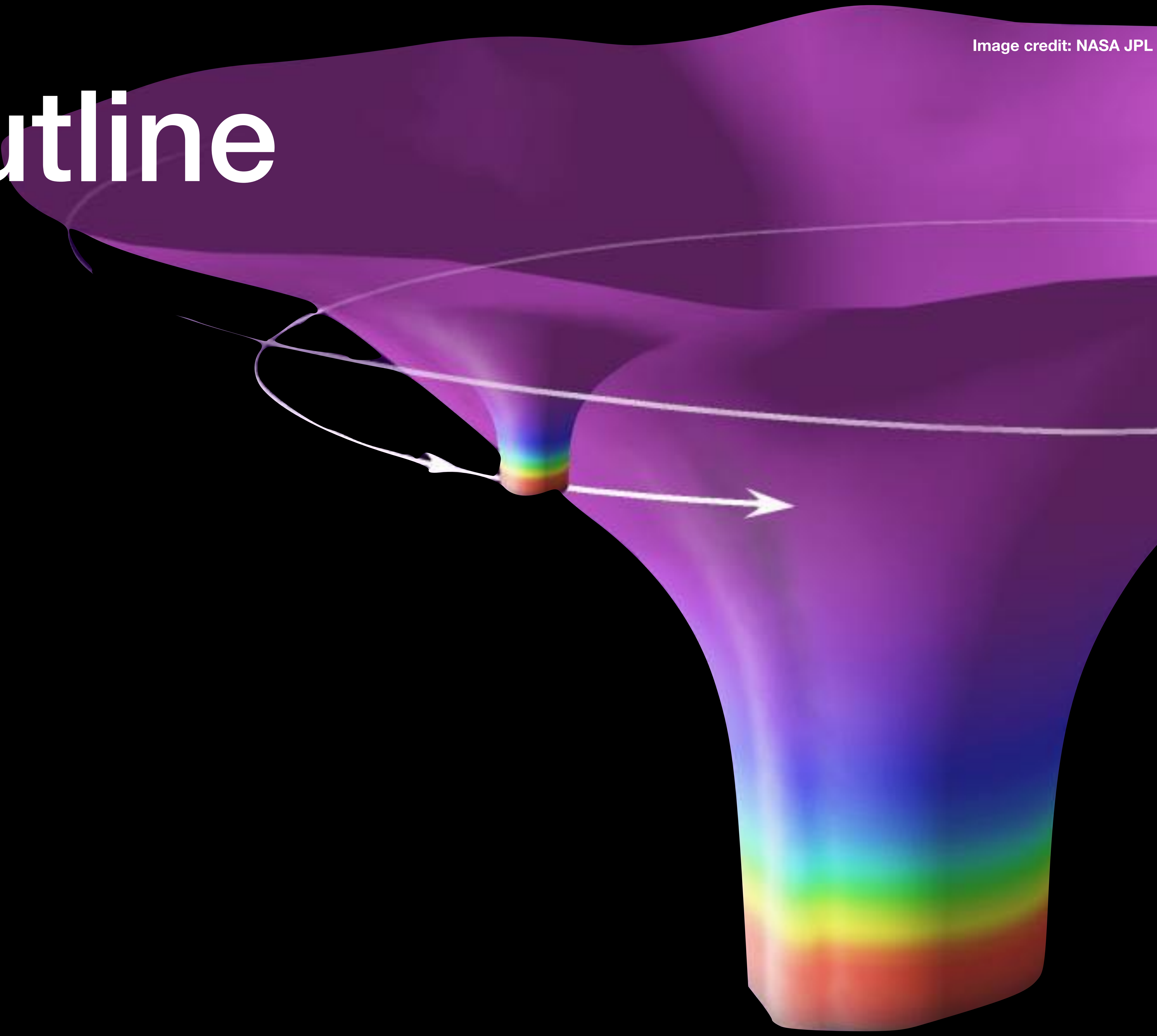


Image credit: NASA JPL

- The two body problem
- What is the self-force?
- Why should you care?

Outline

- The two body problem
- What is the self-force?
- Why should you care?
- Regularisation



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Outline

- The two body problem
- What is the self-force?
- Why should you care?
- Regularisation
- Methods

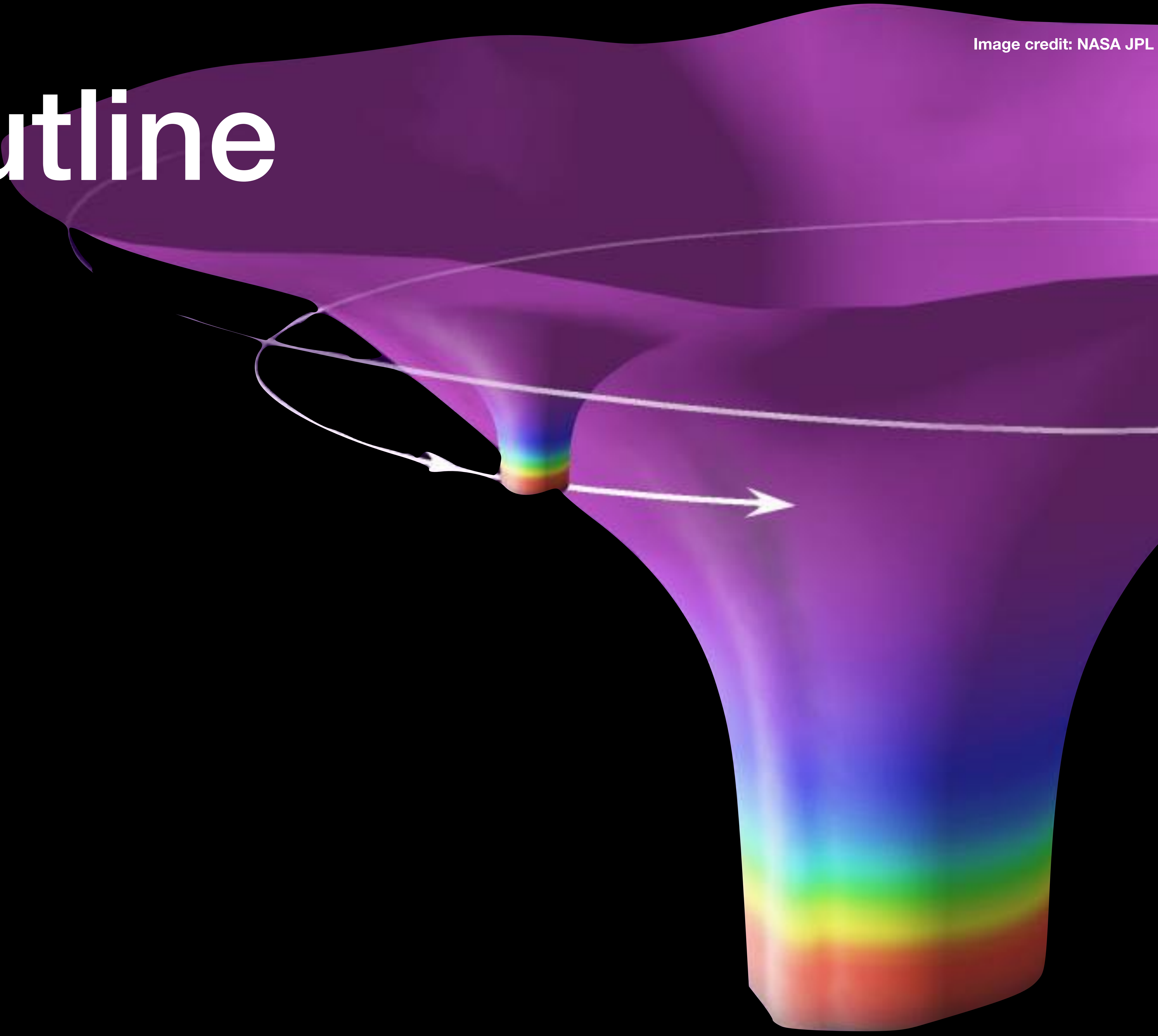

Image credit: NASA JPL

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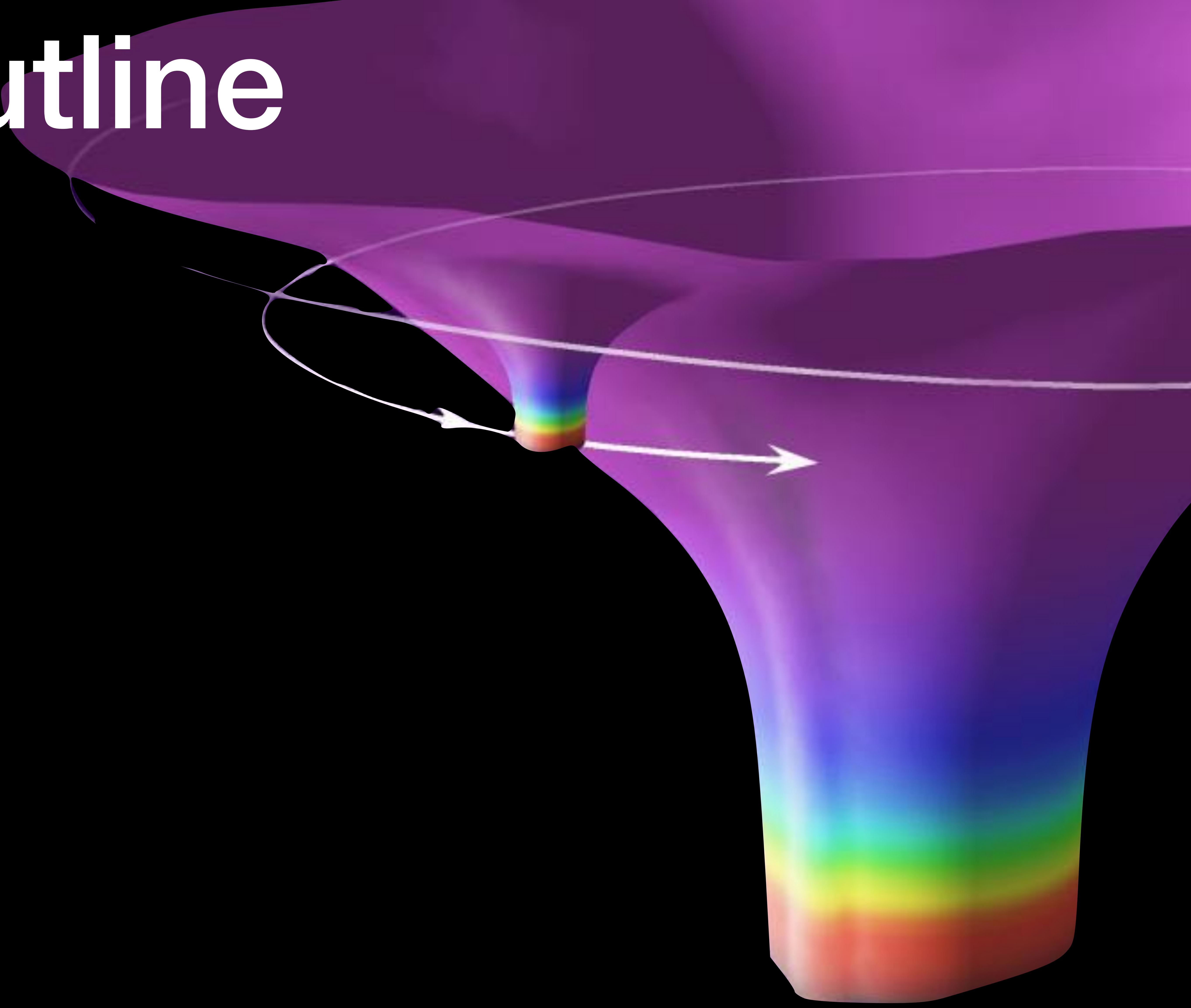
Outline

- The two body problem
- What is the self-force?
- Why should you care?
- Regularisation
- Methods
- Key areas of interest



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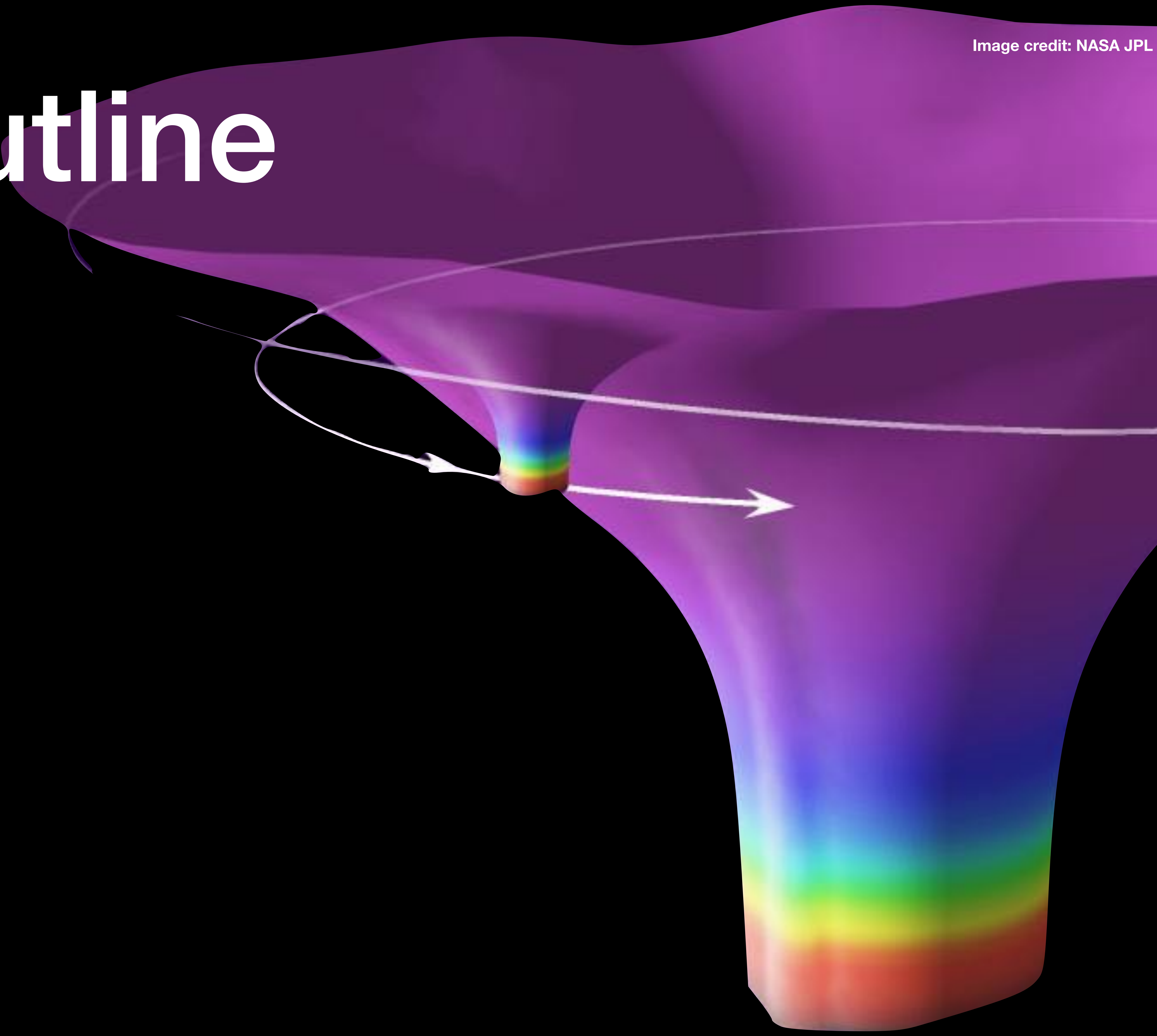

Image credit: NASA JPL

- The two body problem
- What is the self-force?
- Why should you care?
- Regularisation
- Methods
- Key areas of interest
- Ongoing research

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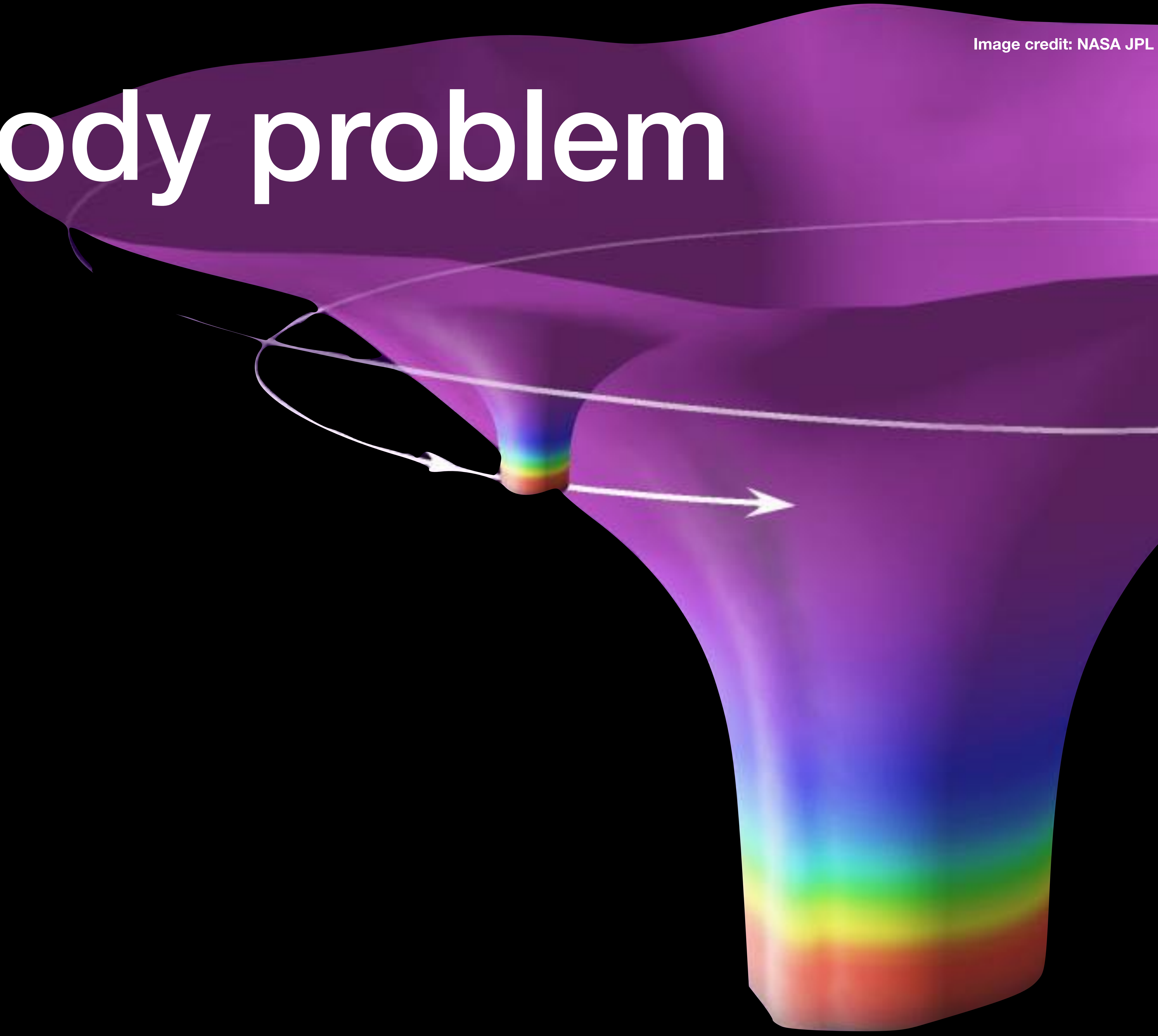
The two body problem

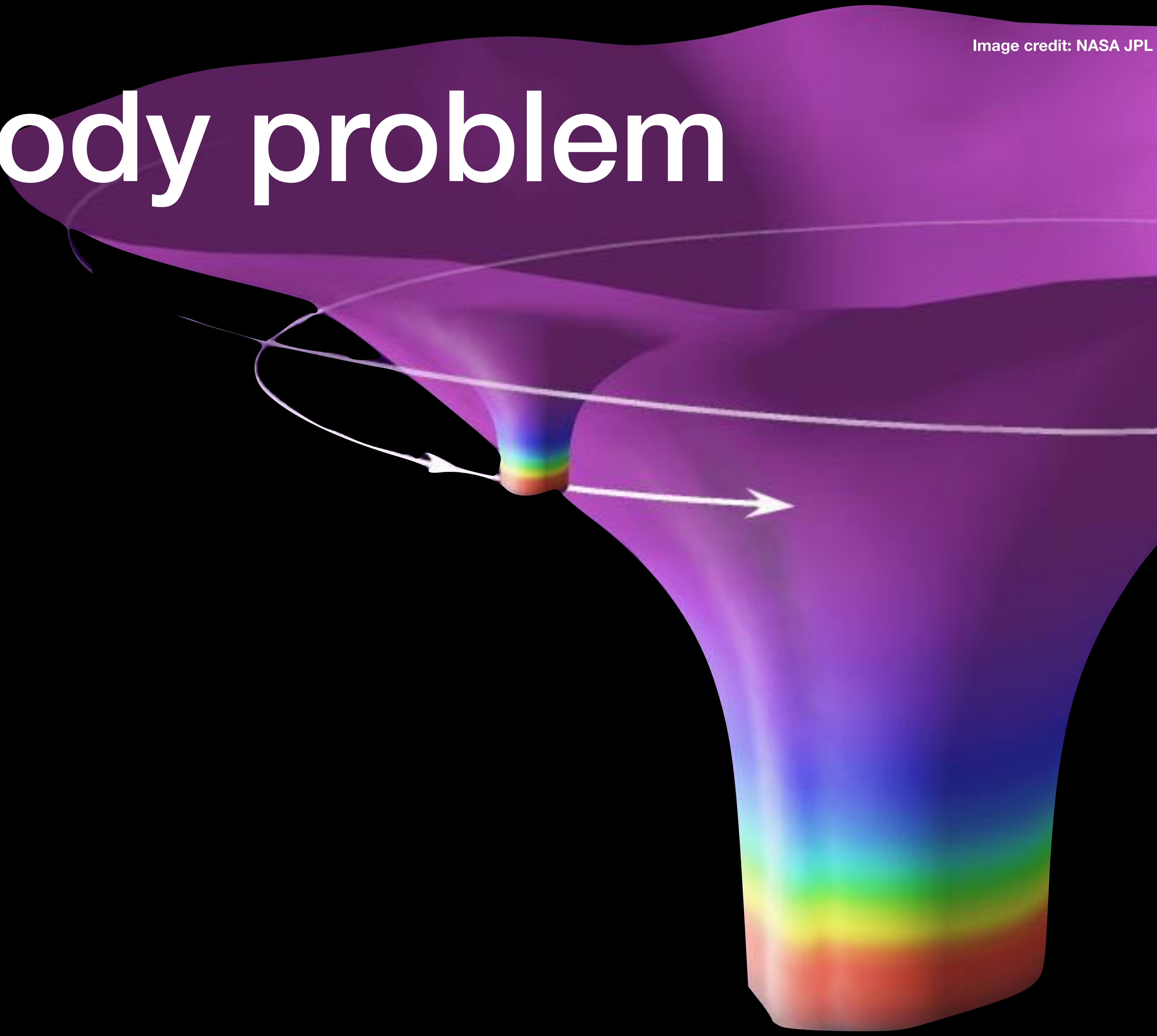
Image credit: NASA JPL



The two body problem

Image credit: NASA JPL

- **Newton's flat space**



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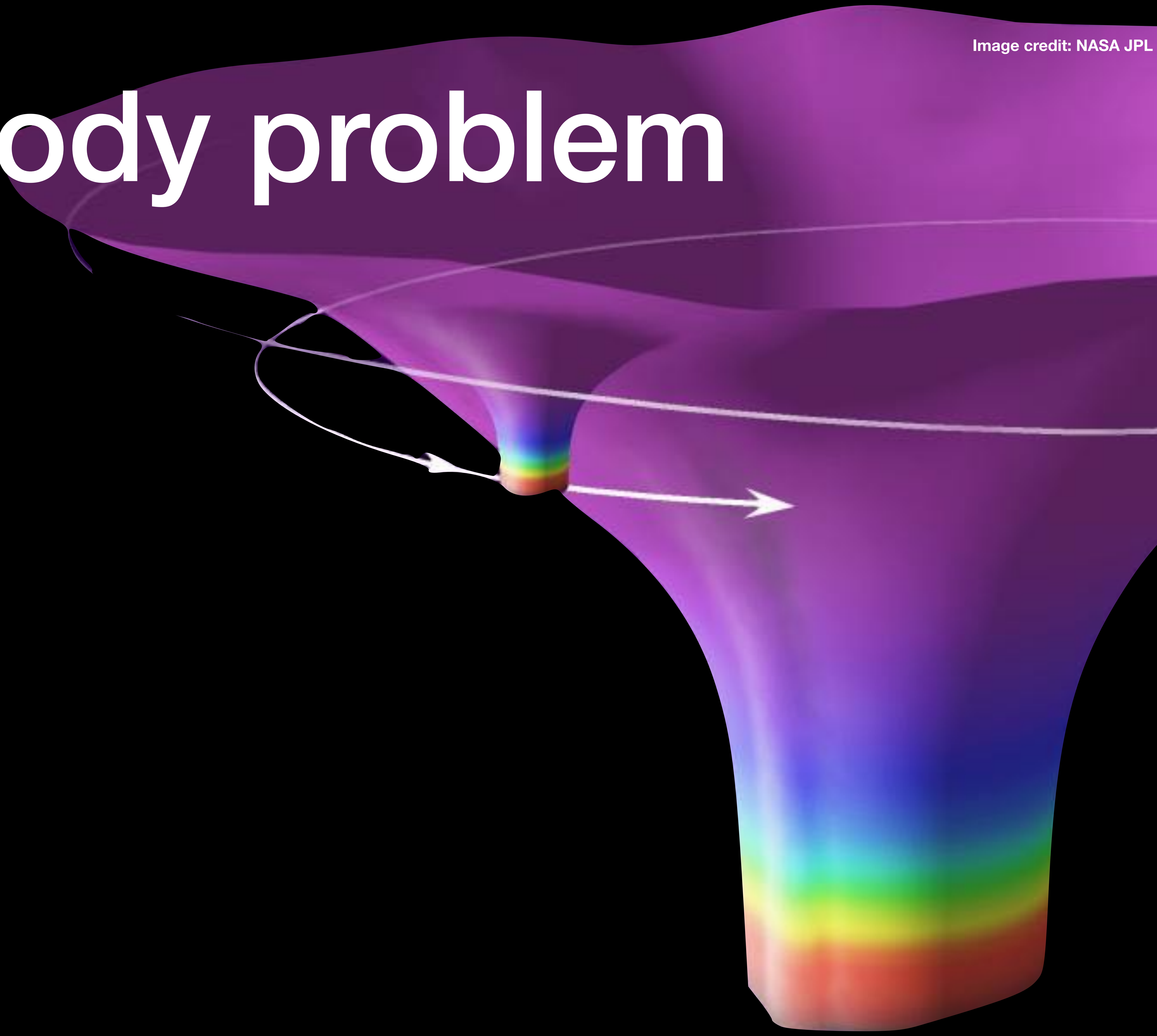

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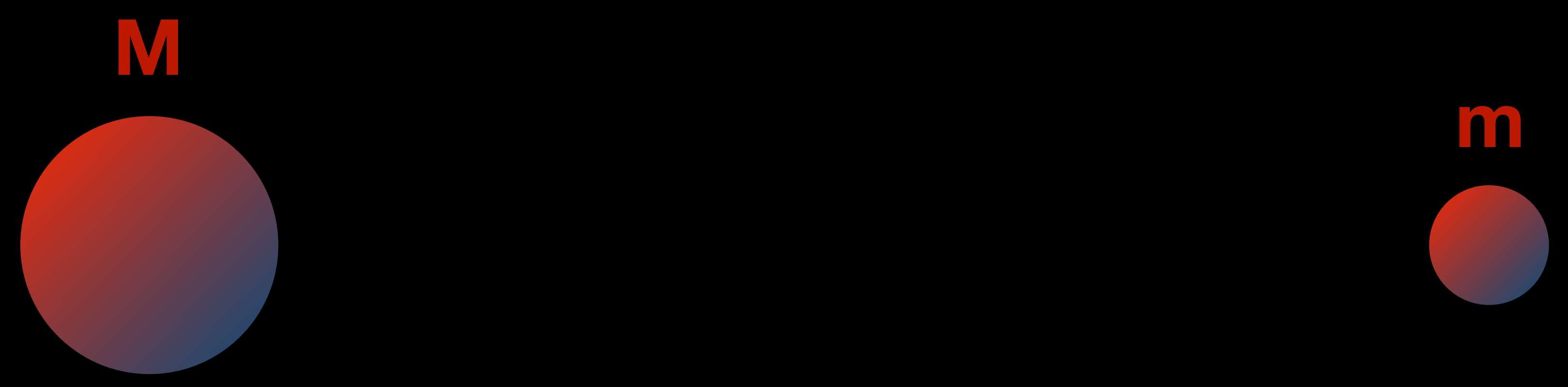

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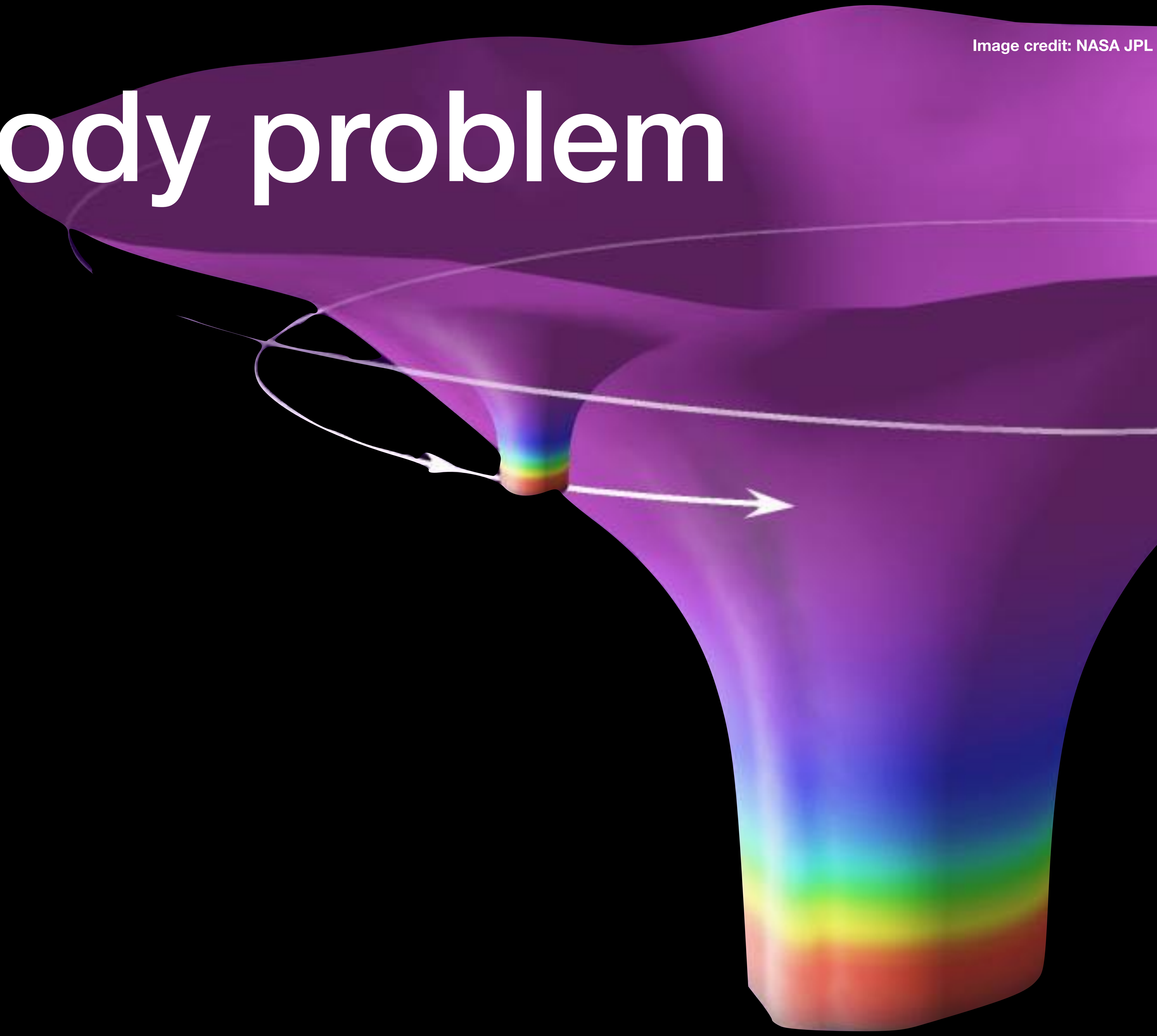

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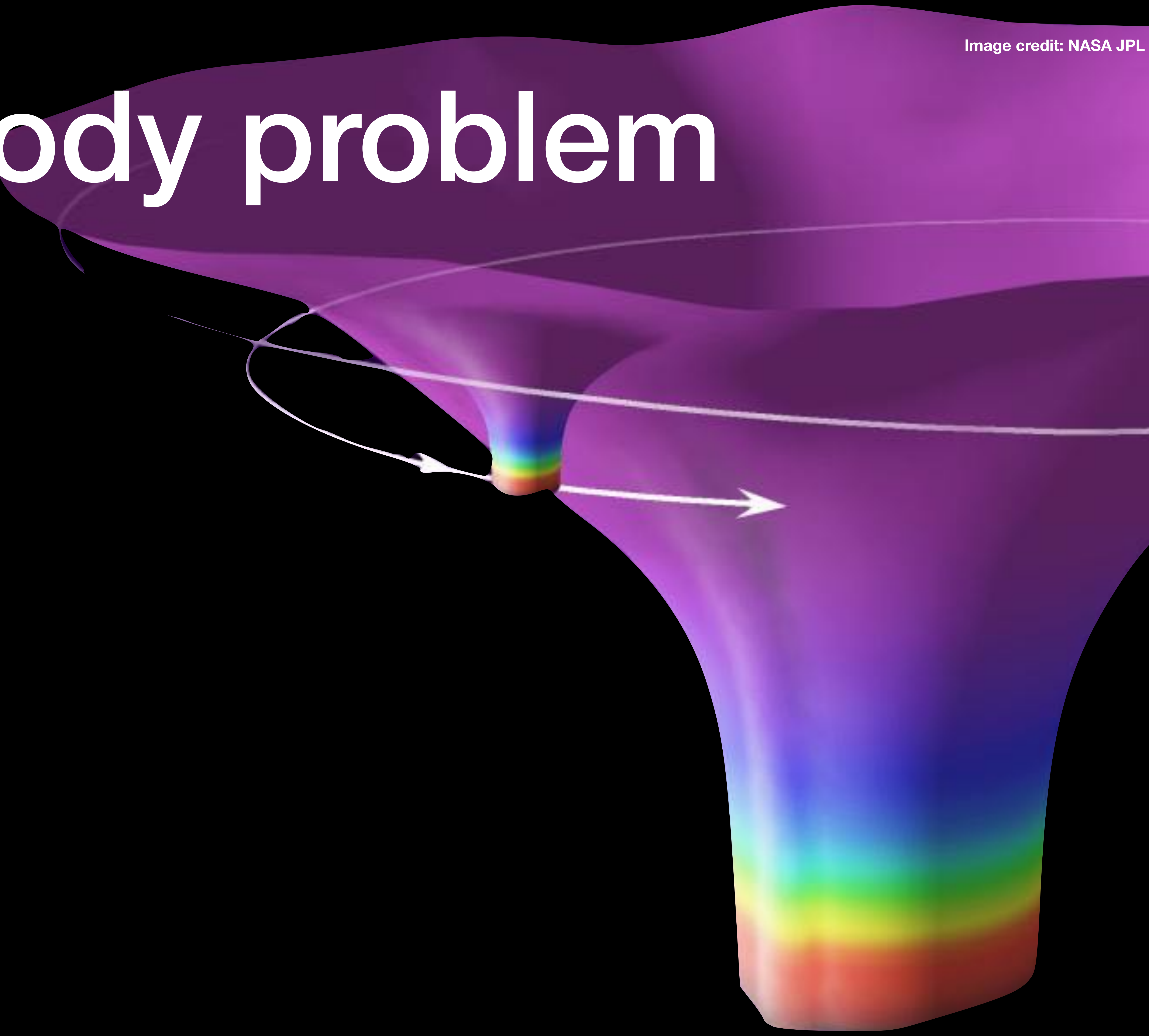

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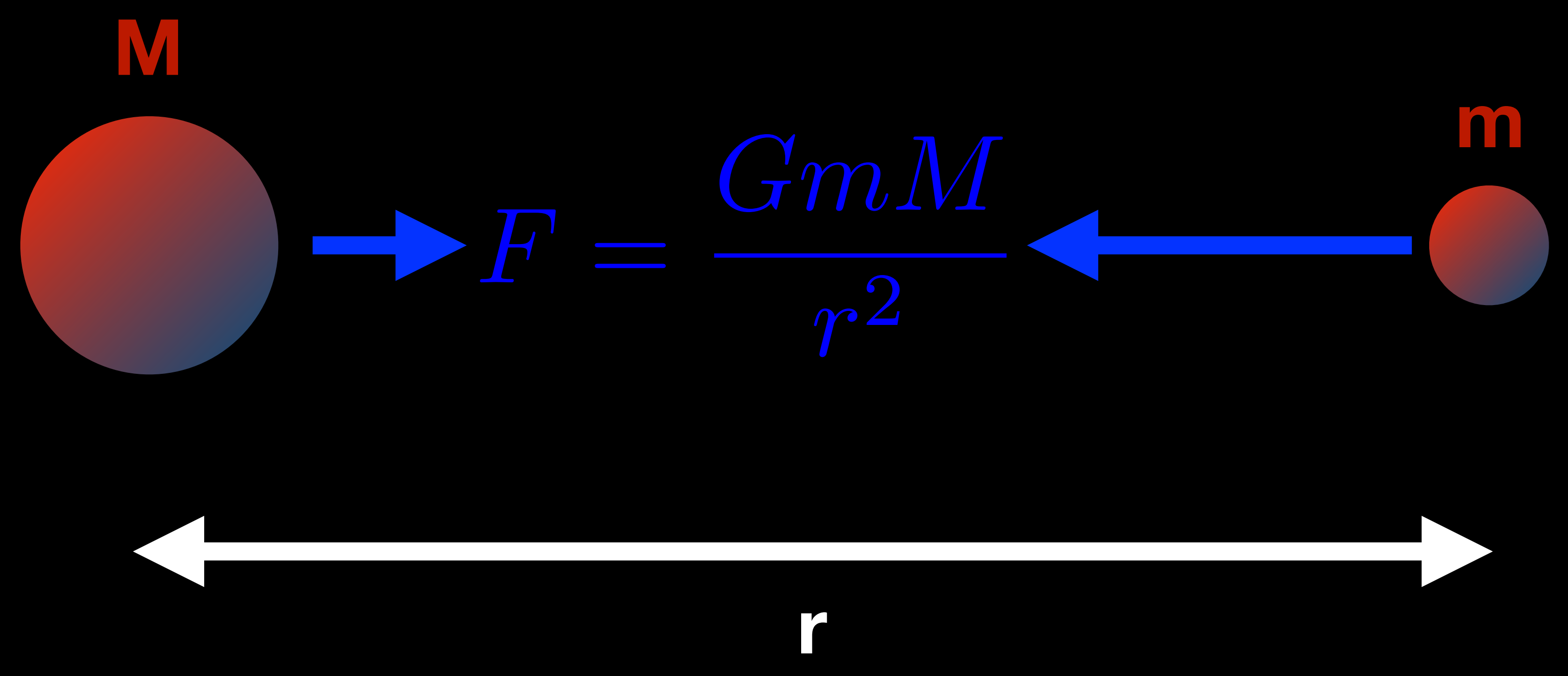

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The two body problem

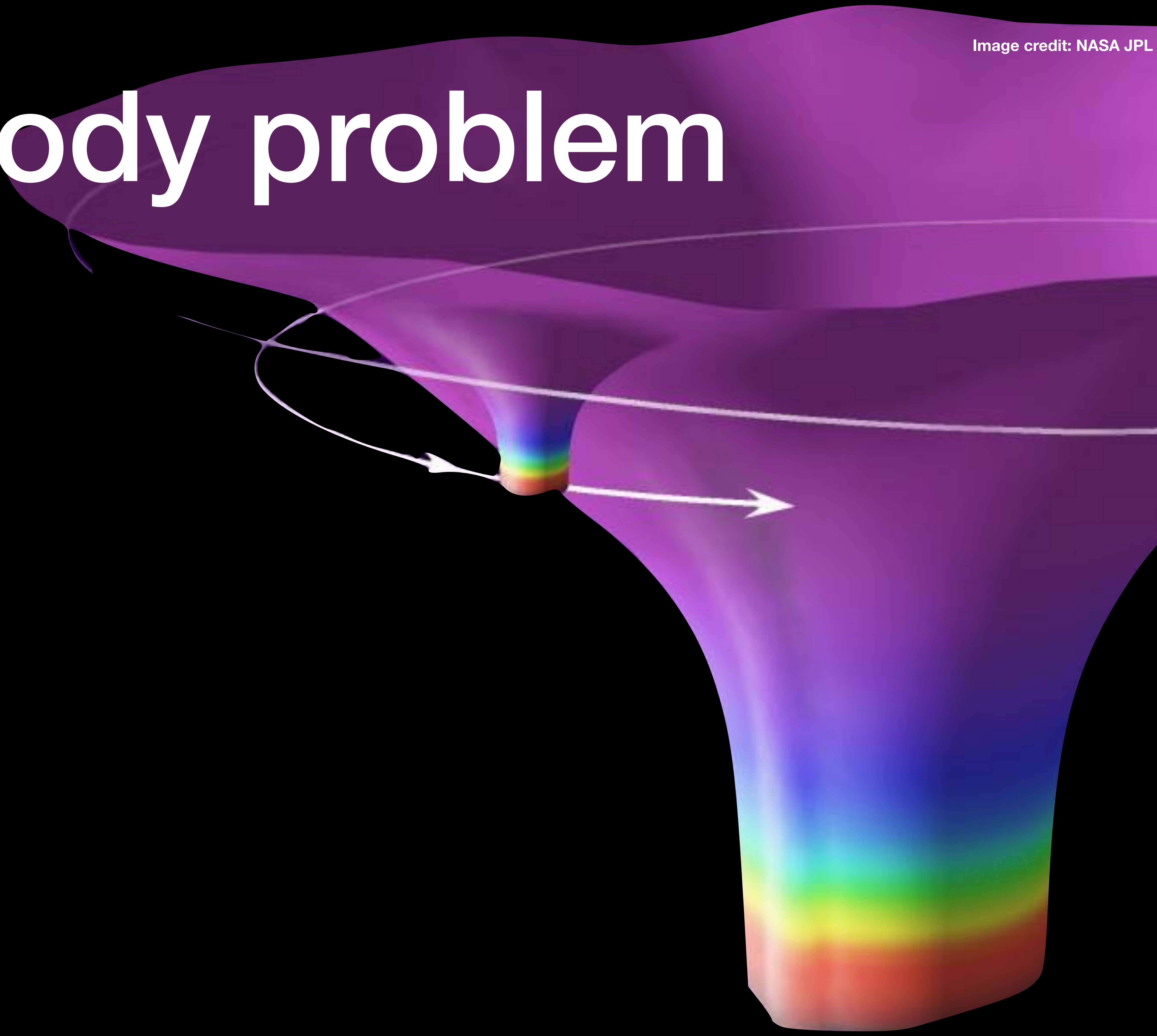


Image credit: NASA JPL

- **Newton's flat space**

M

m

$$F = \frac{GmM}{r^2}$$

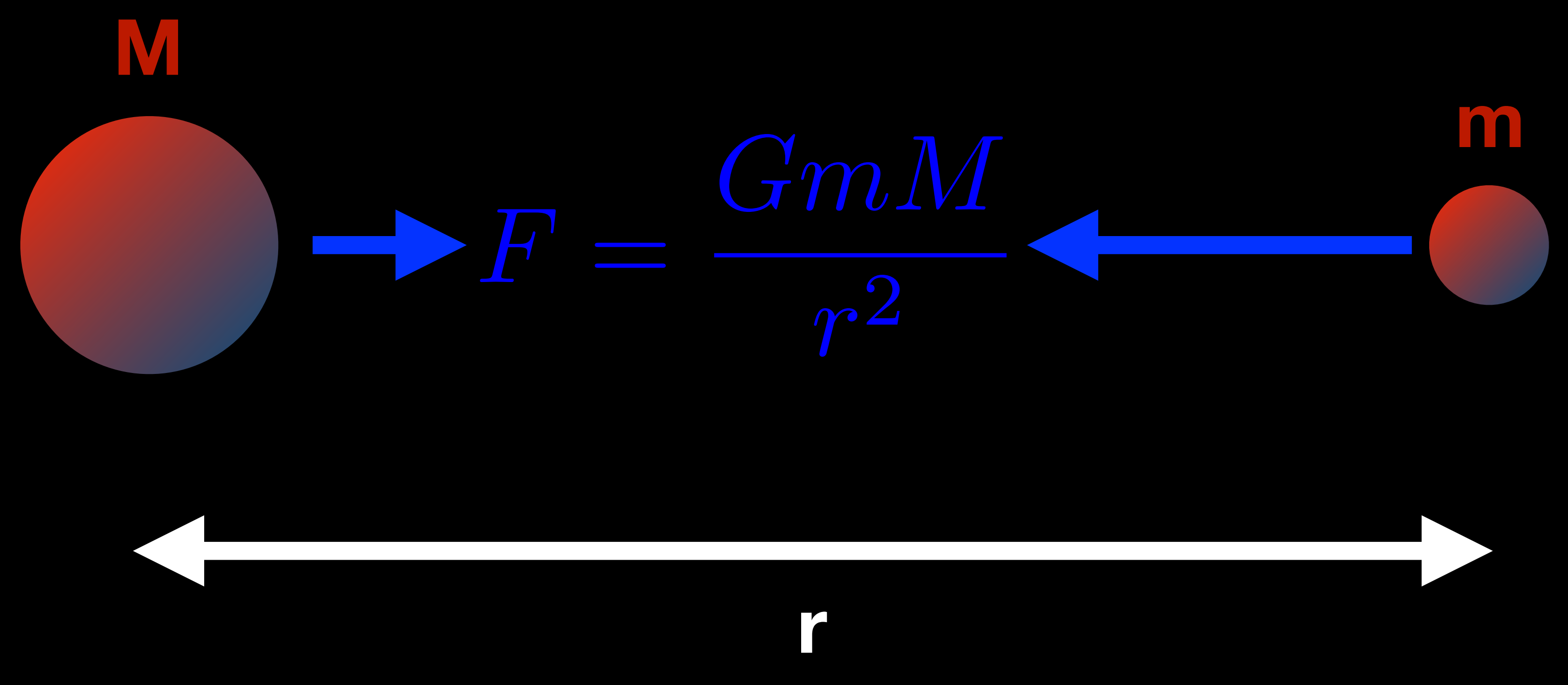
r

$$E = \int F dr,$$

$$L = r \times \int F dt.$$

The two body problem

- **Newton's flat space**



$$E = \int F dr,$$

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- **Einstein's general relativity**

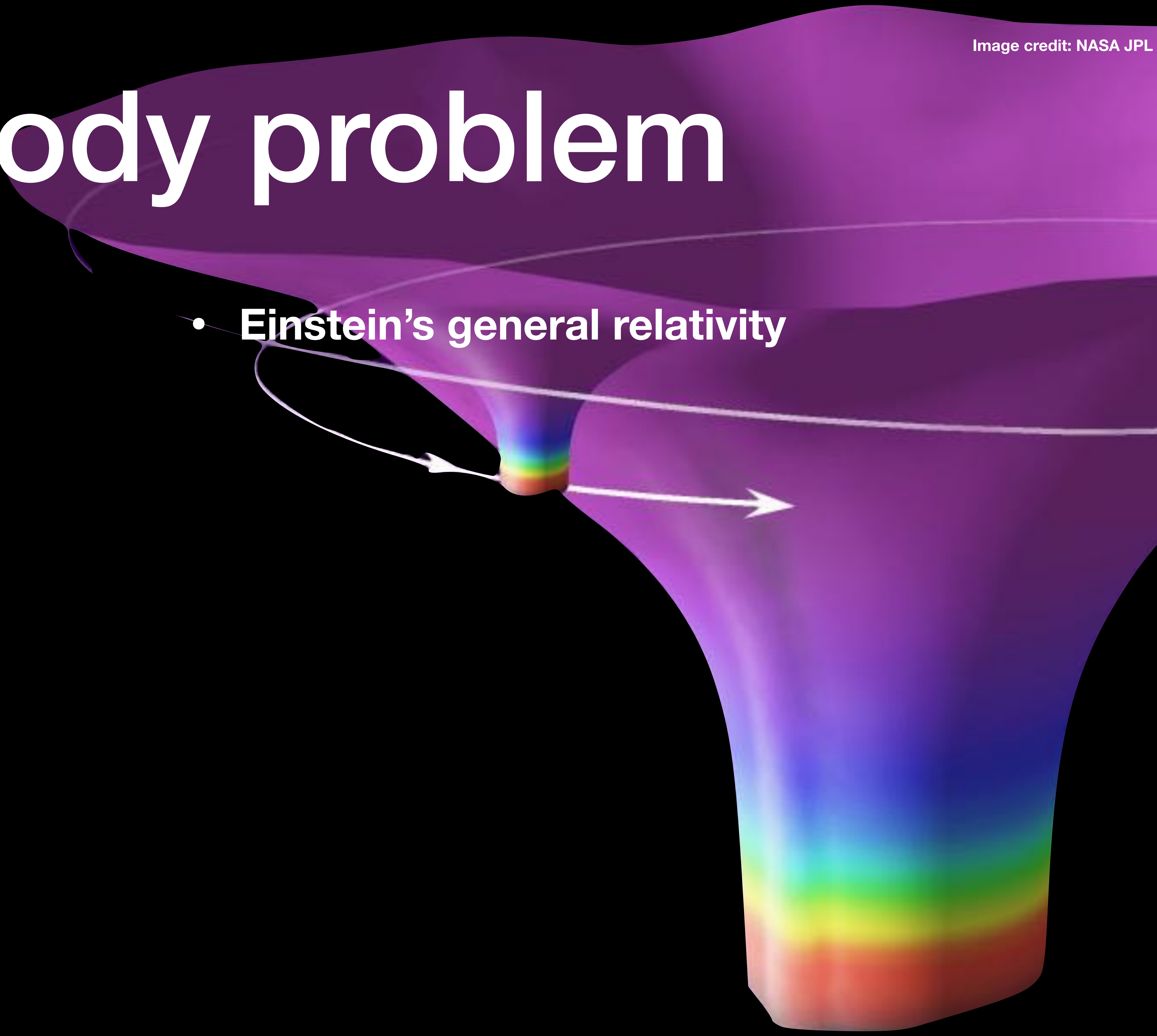

Image credit: NASA JPL

The two body problem

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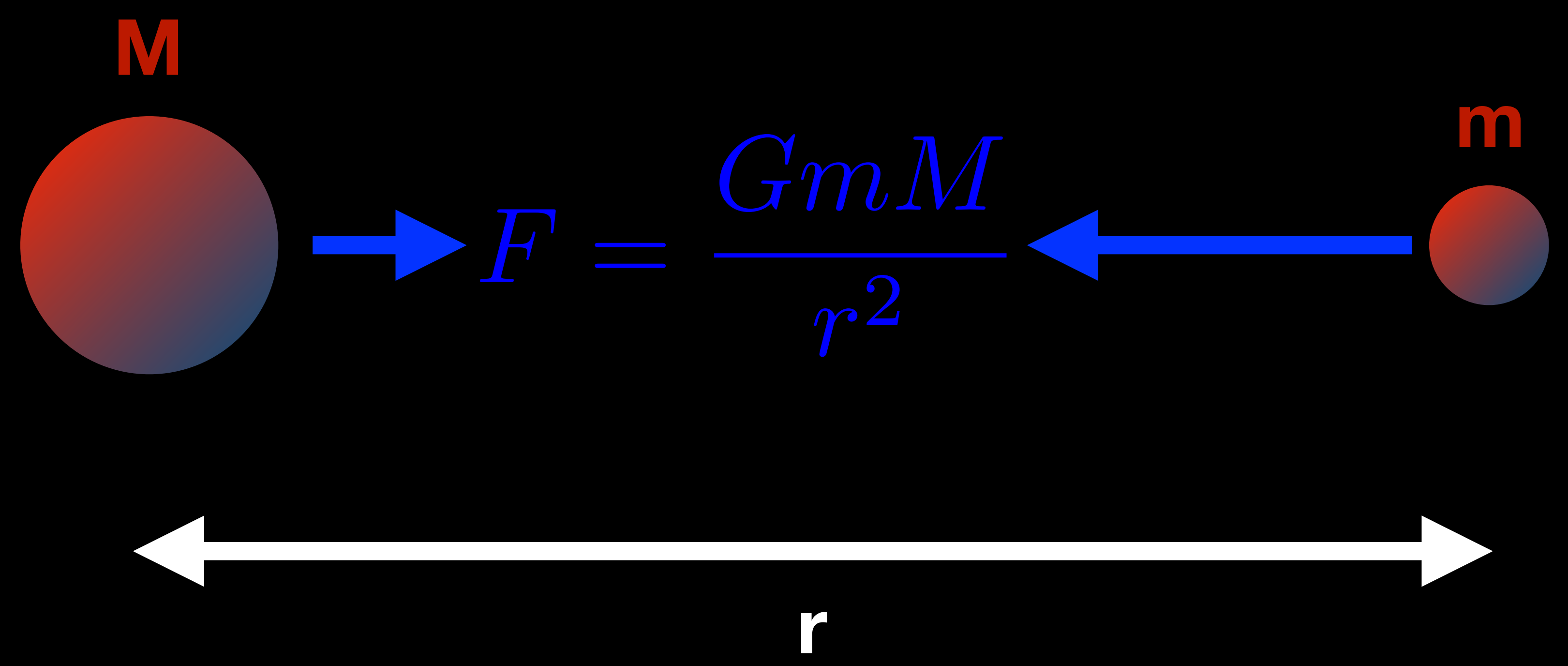

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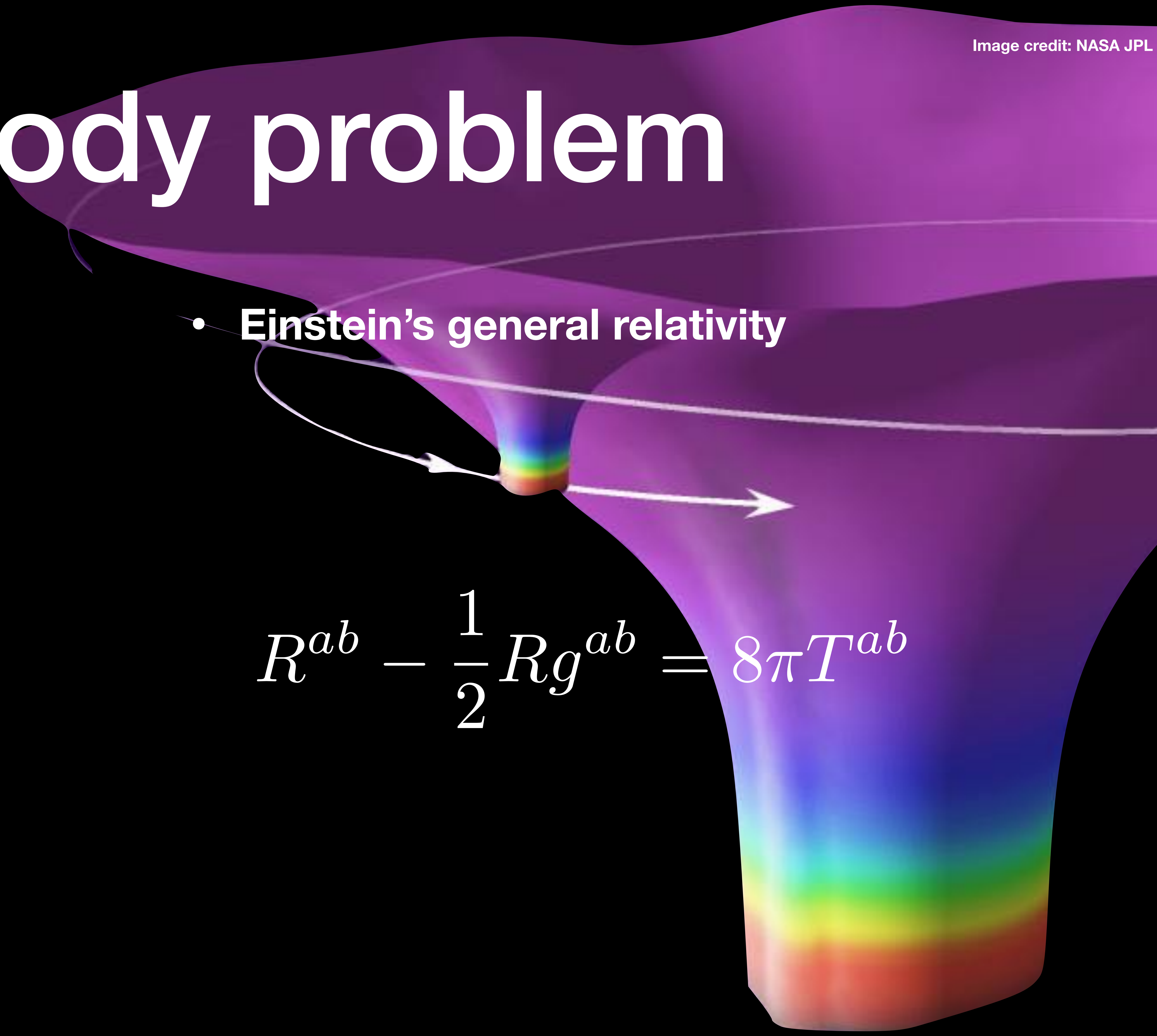
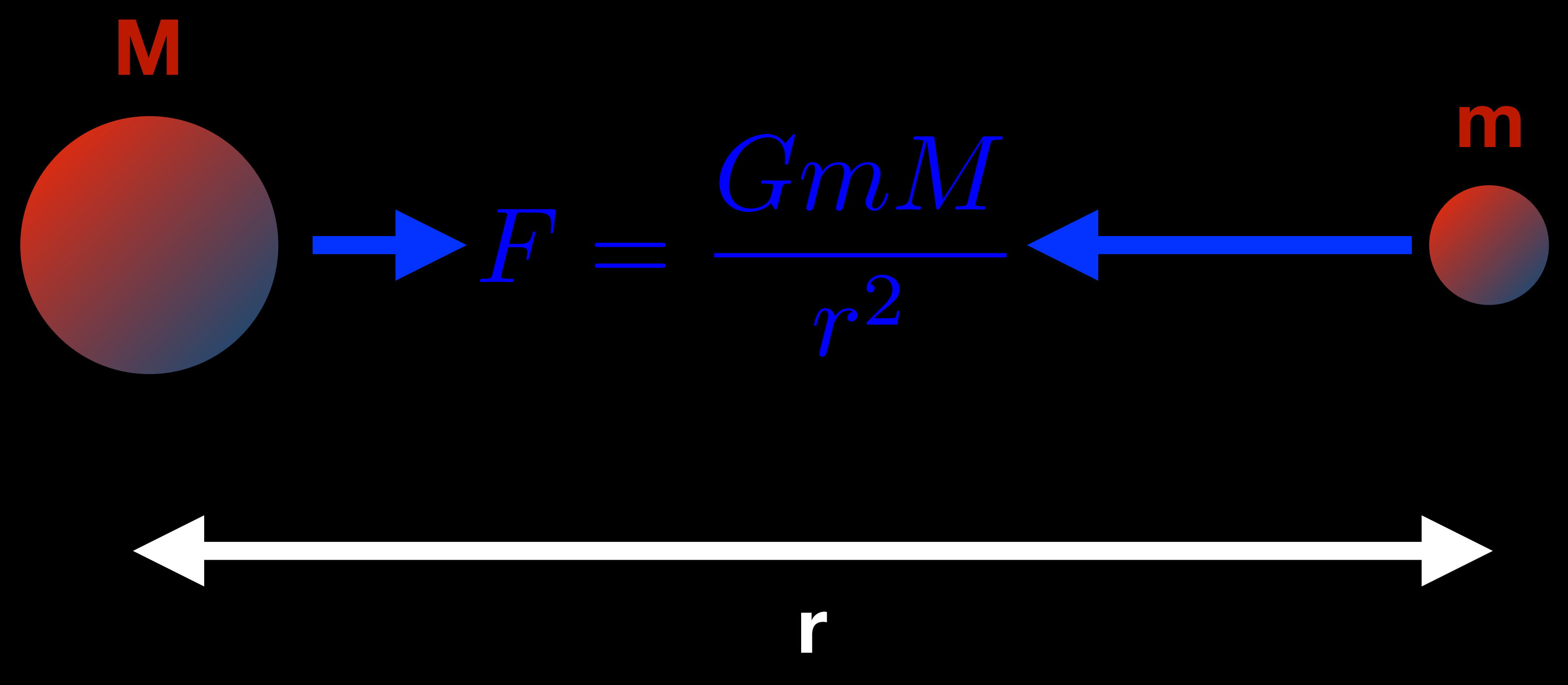


$$R^{ab} - \frac{1}{2} R g^{ab} = 8\pi T^{ab}$$

The two body problem

Image credit: NASA JPL

- **Newton's flat space**



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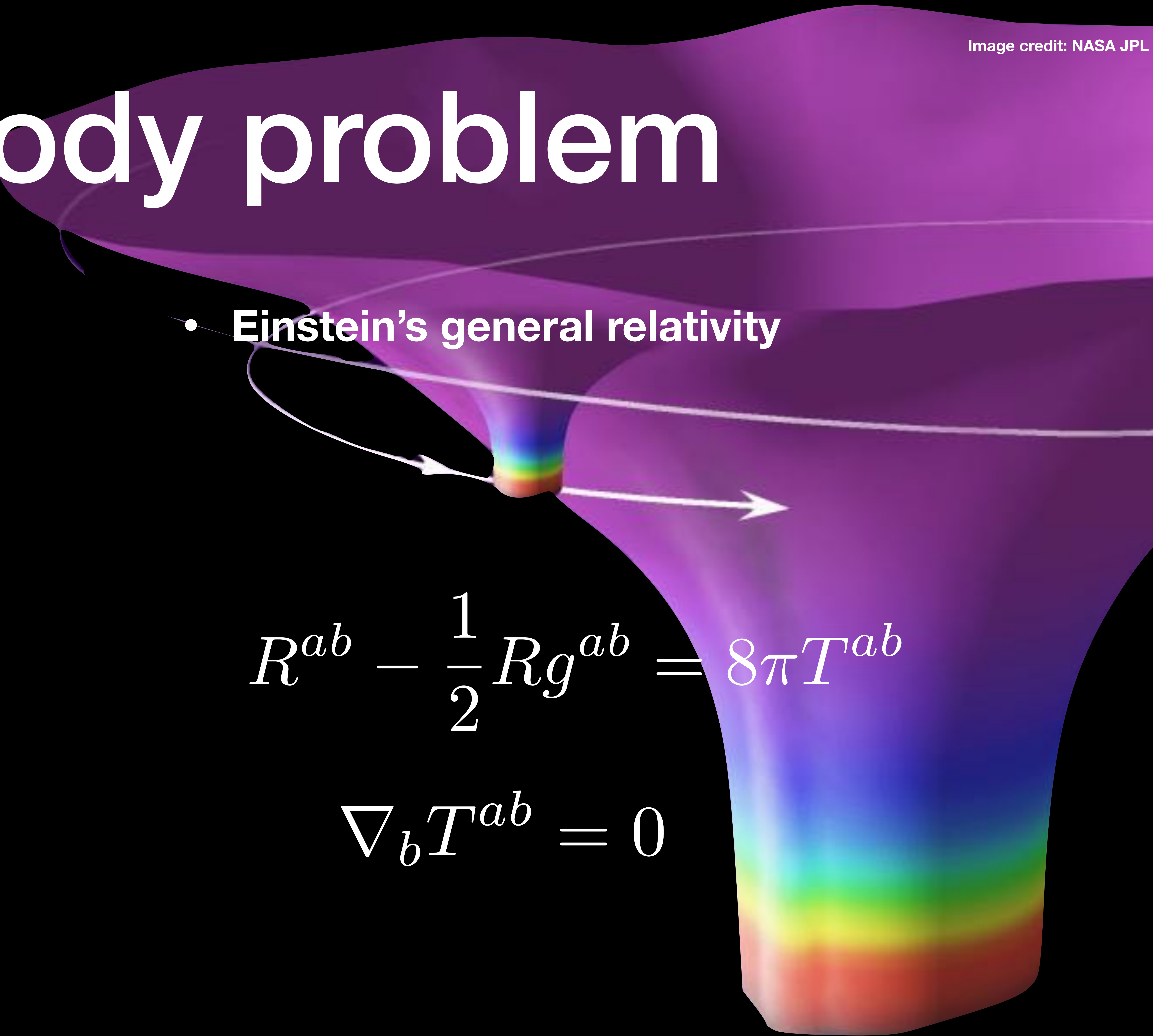
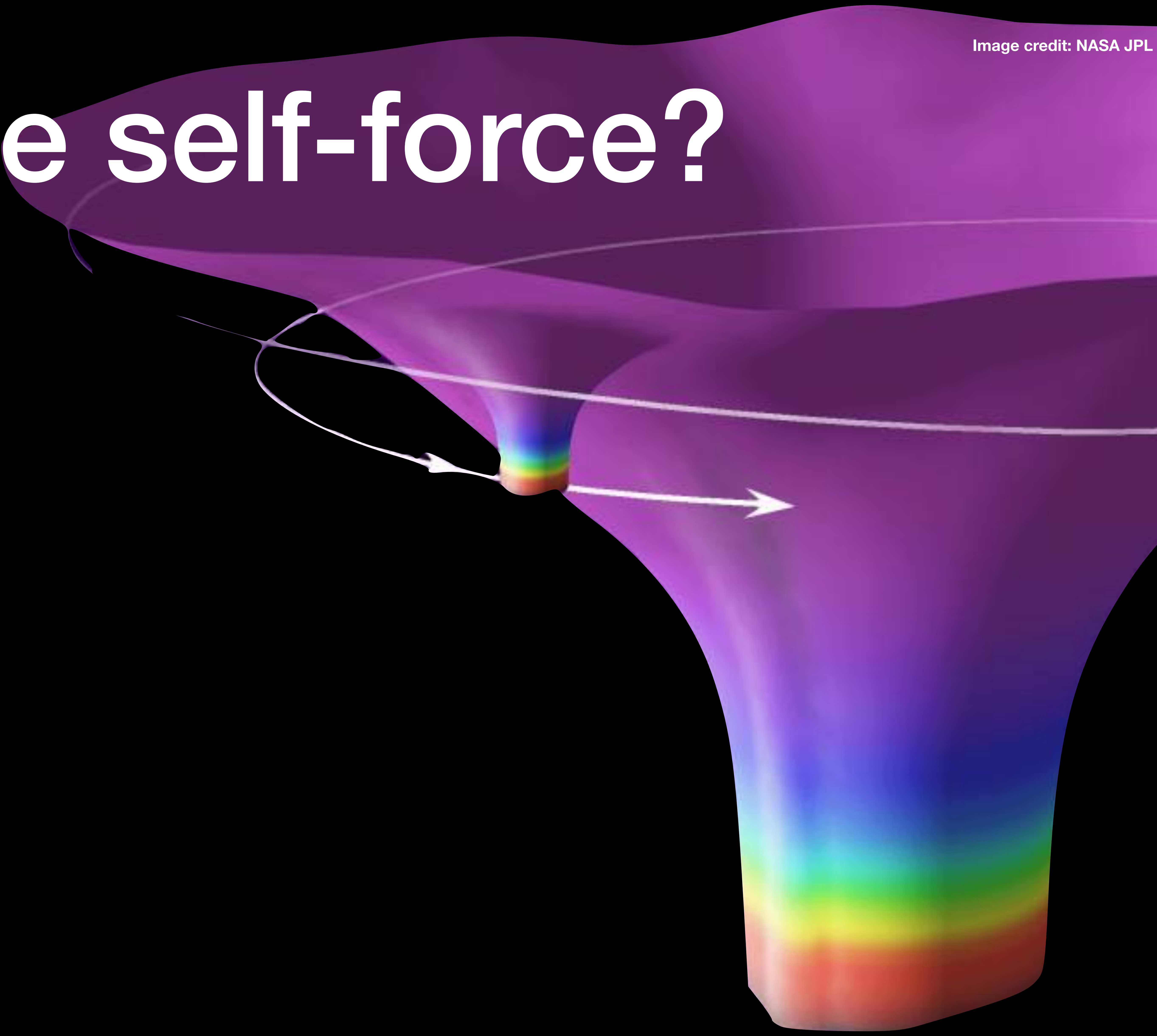


$$R^{ab} - \frac{1}{2} R g^{ab} = 8\pi T^{ab}$$

$$\nabla_b T^{ab} = 0$$

What is the self-force?

Image credit: NASA JPL



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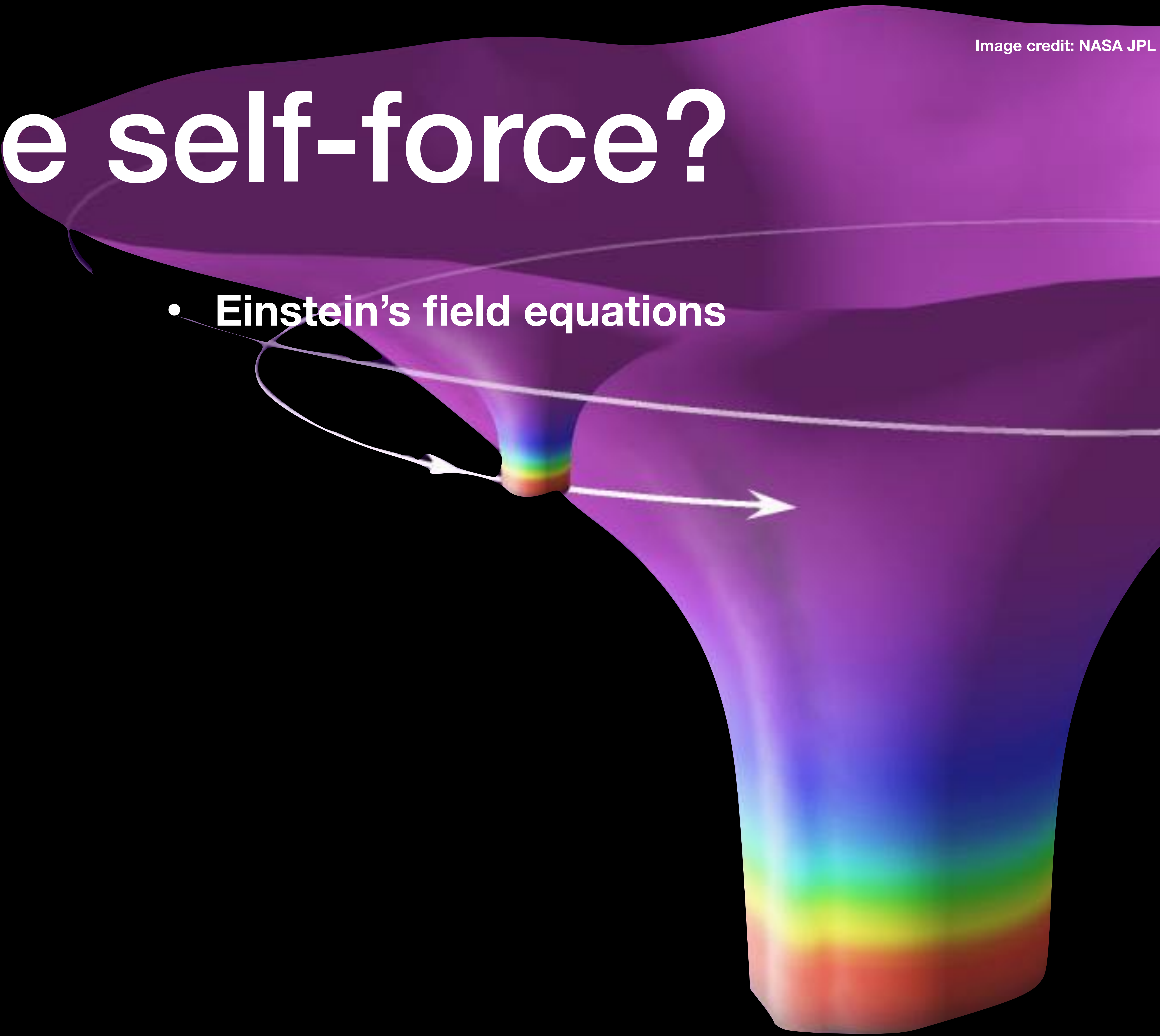

- Einstein's field equations

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What is the self-force?

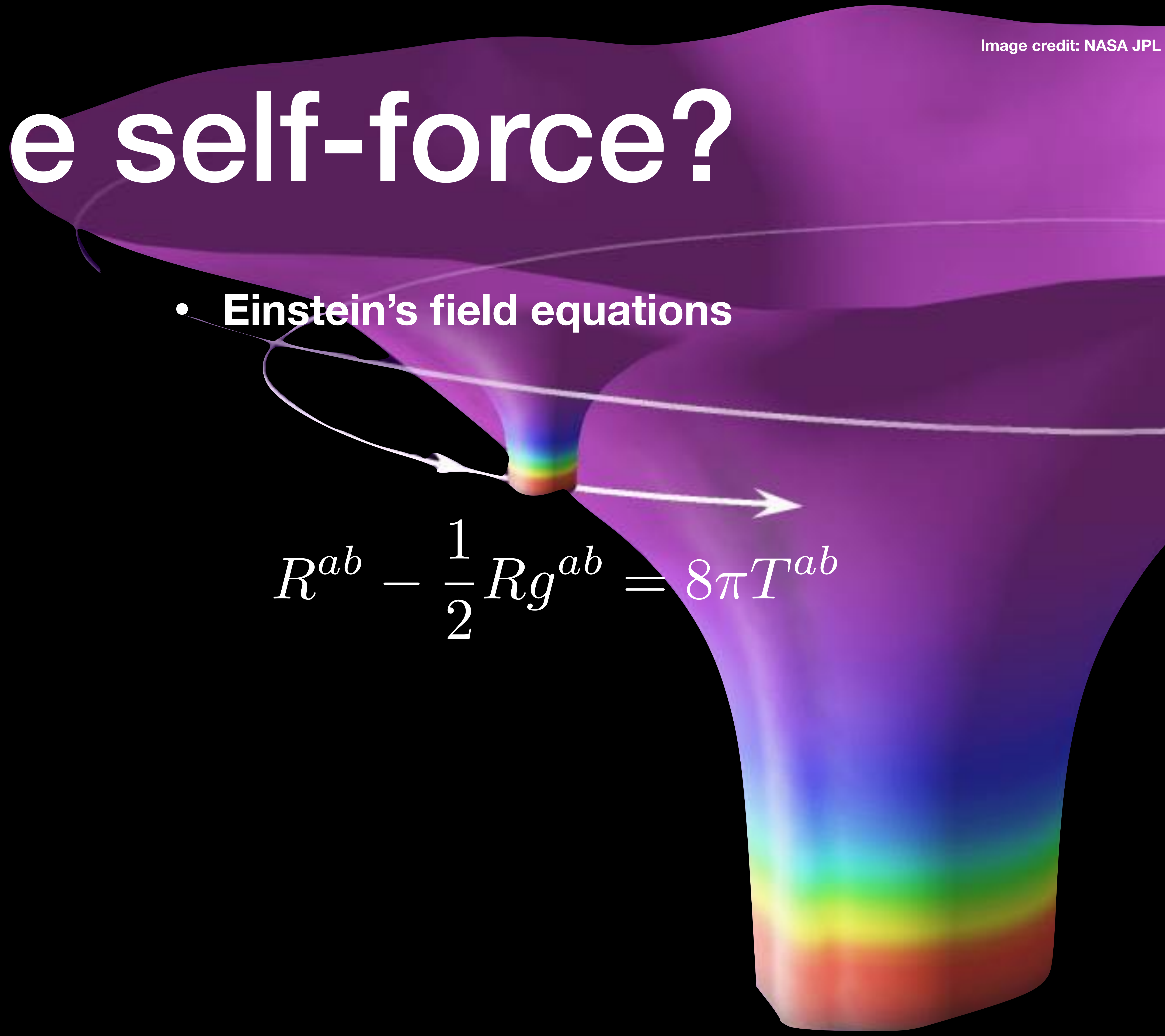

Image credit: NASA JPL

- **Einstein's field equations**

$$R^{ab} - \frac{1}{2} R g^{ab} = 8\pi T^{ab}$$

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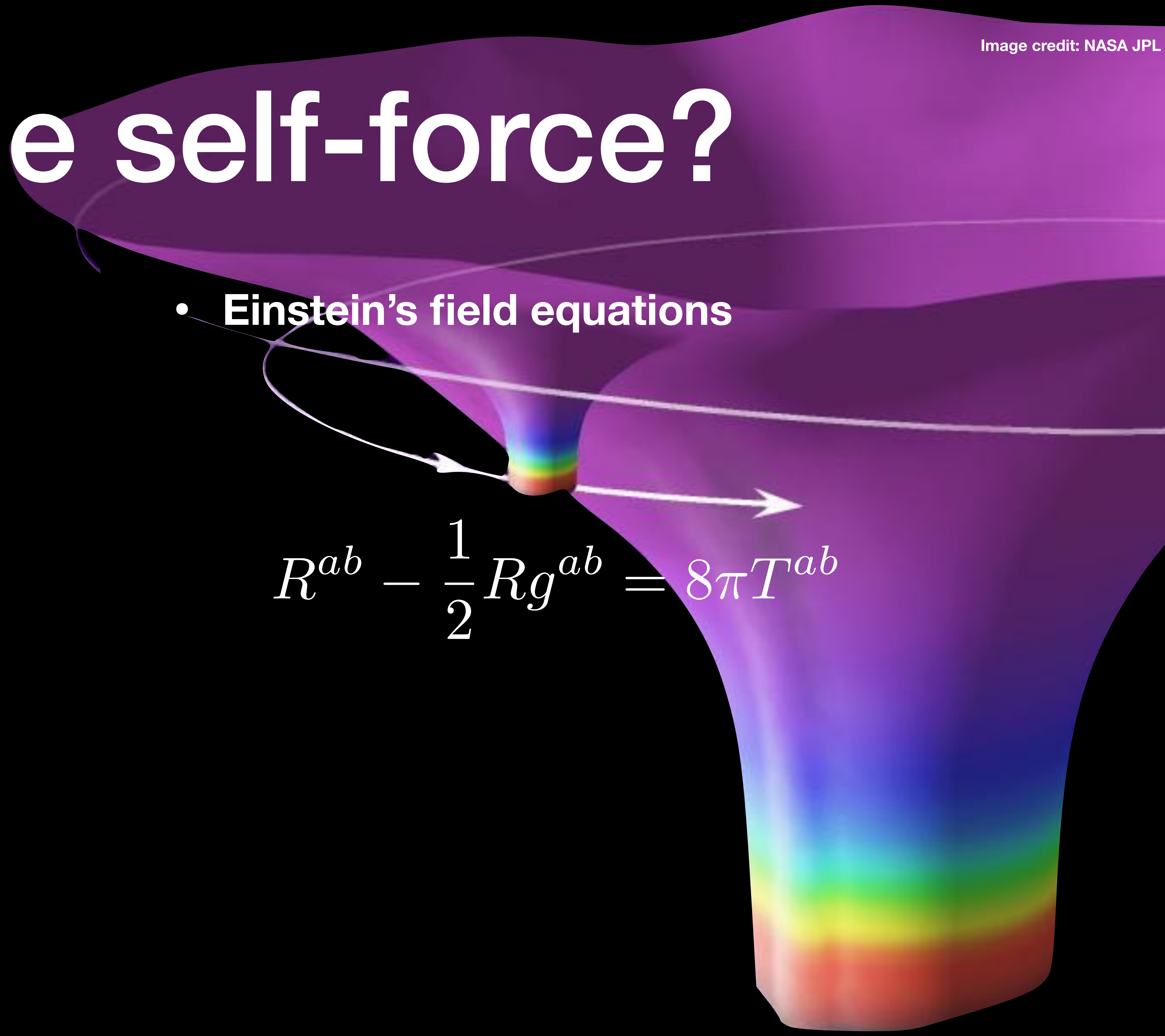
What is the self-force?

- **Perturbing the metric in the mass ratio**
- **Einstein's field equations**



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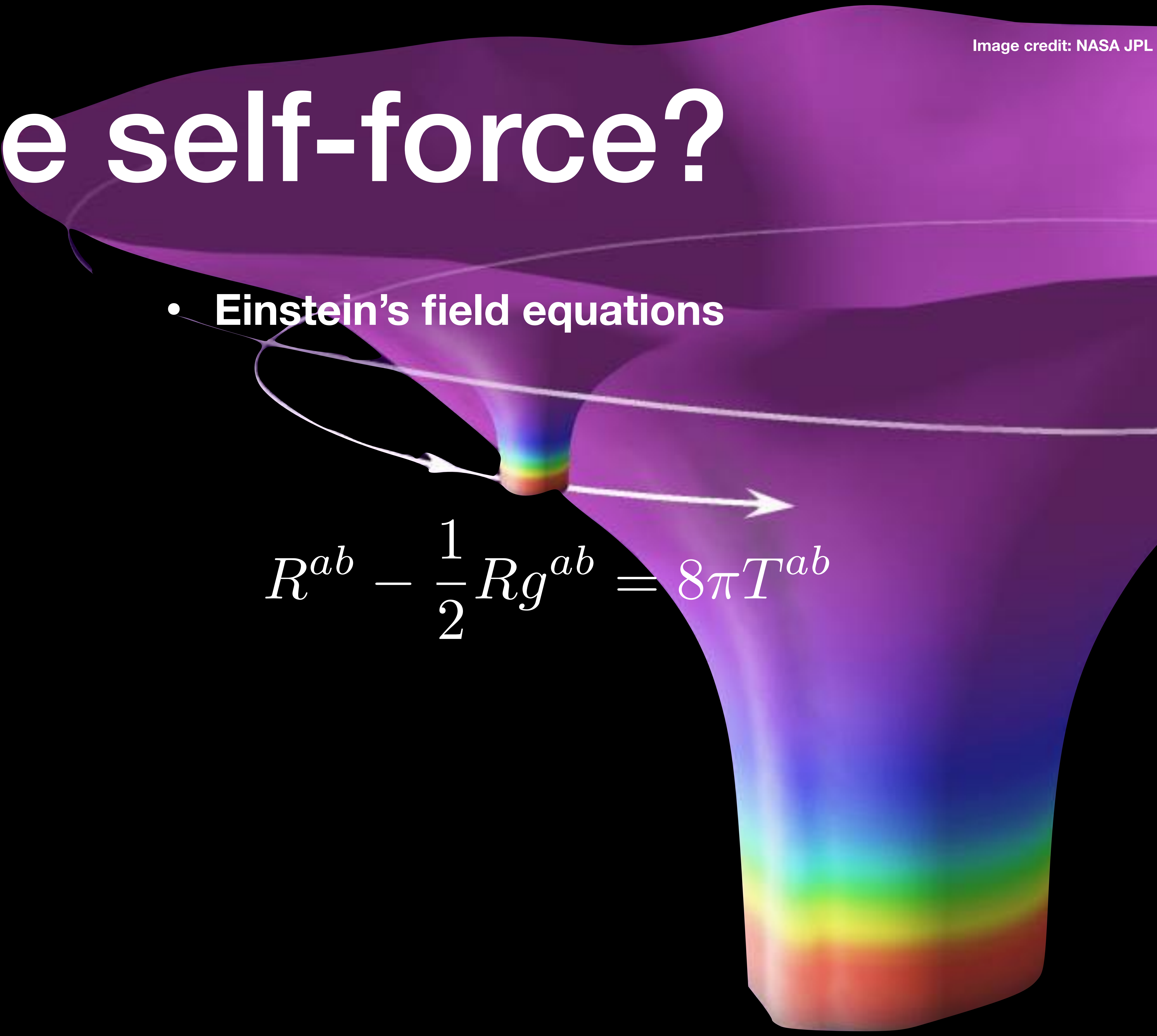

Image credit: NASA JPL

What is the self-force?

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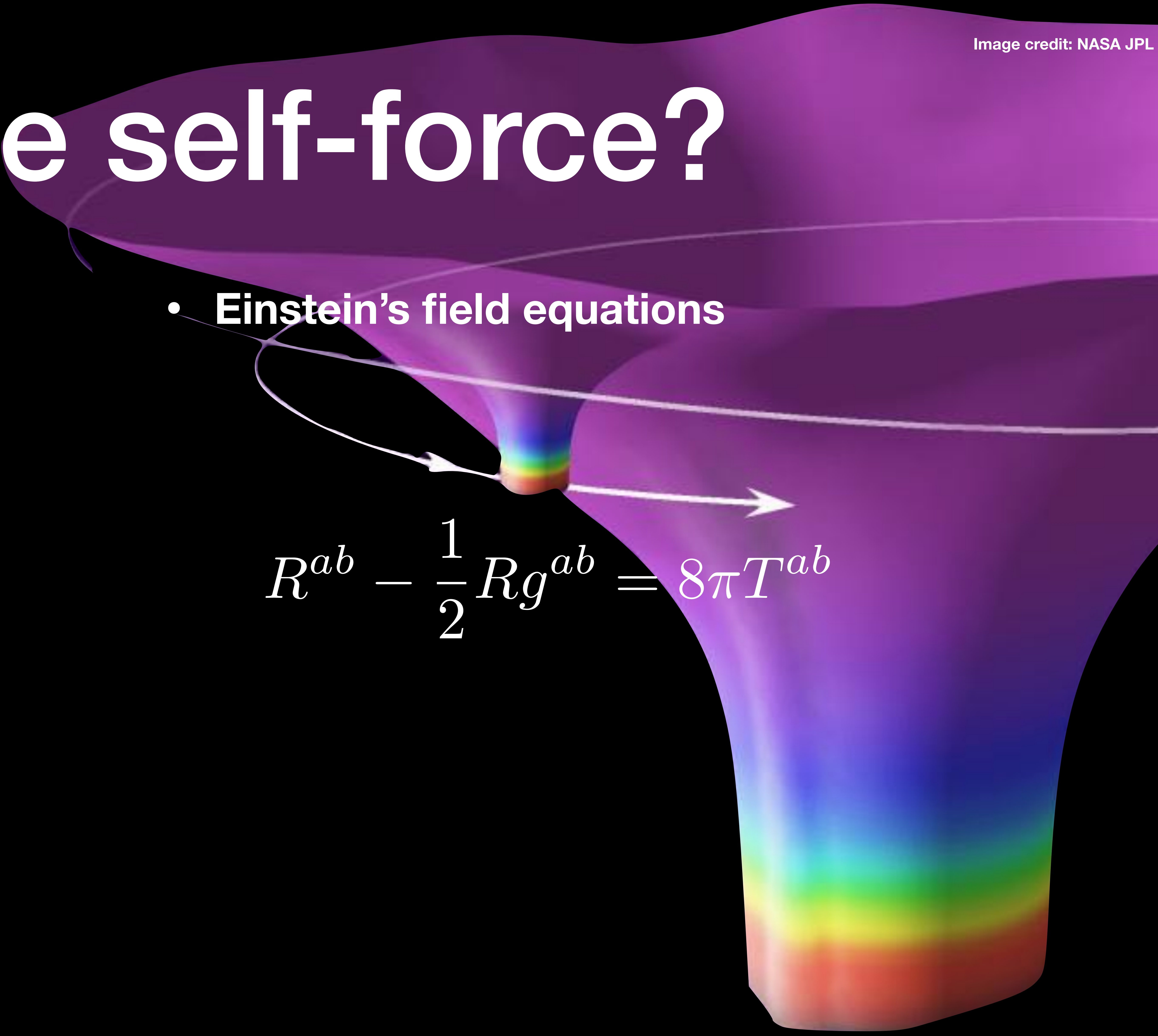
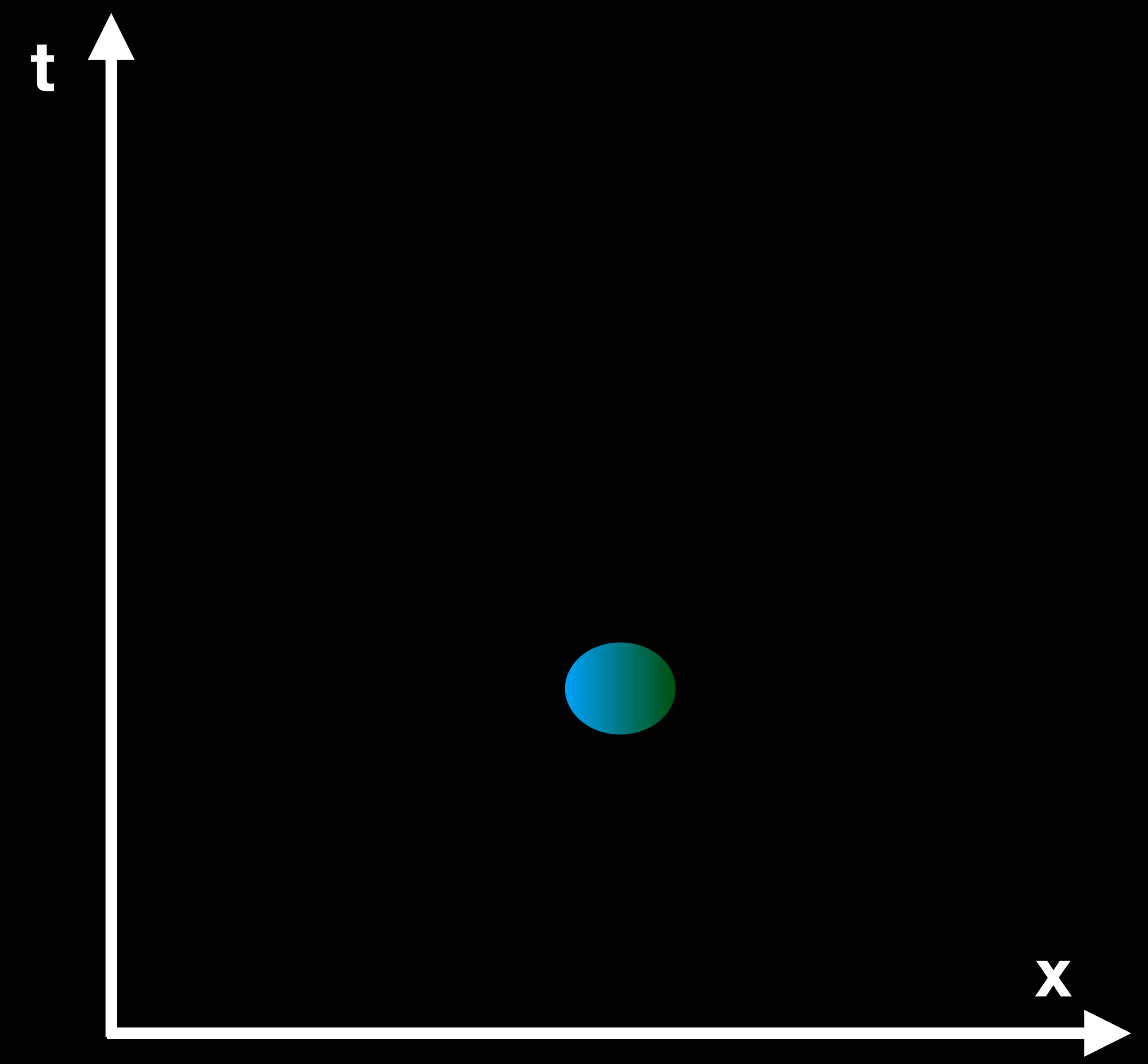


- Perturbing the metric in the mass ratio



- Einstein's field equations

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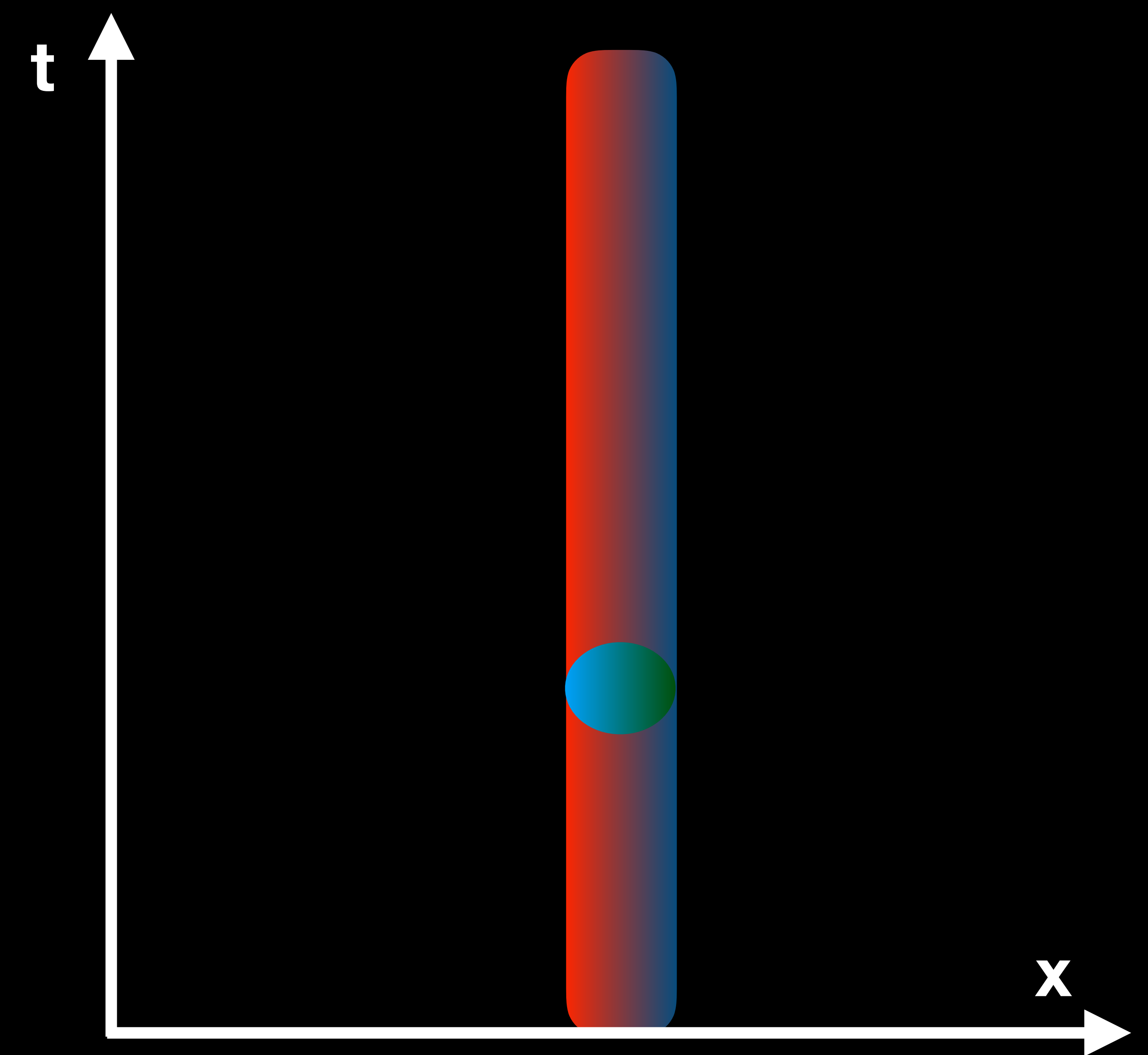

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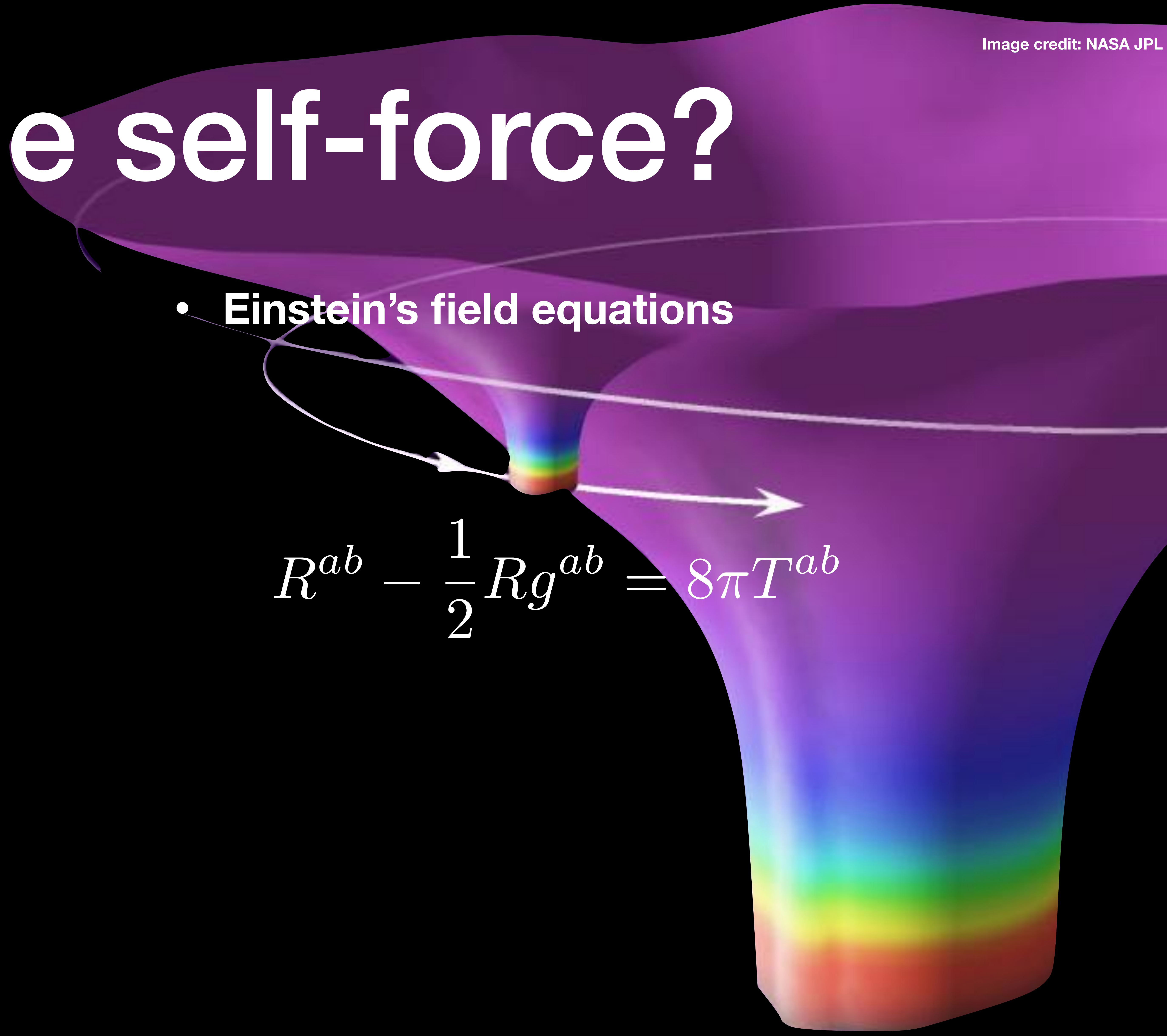

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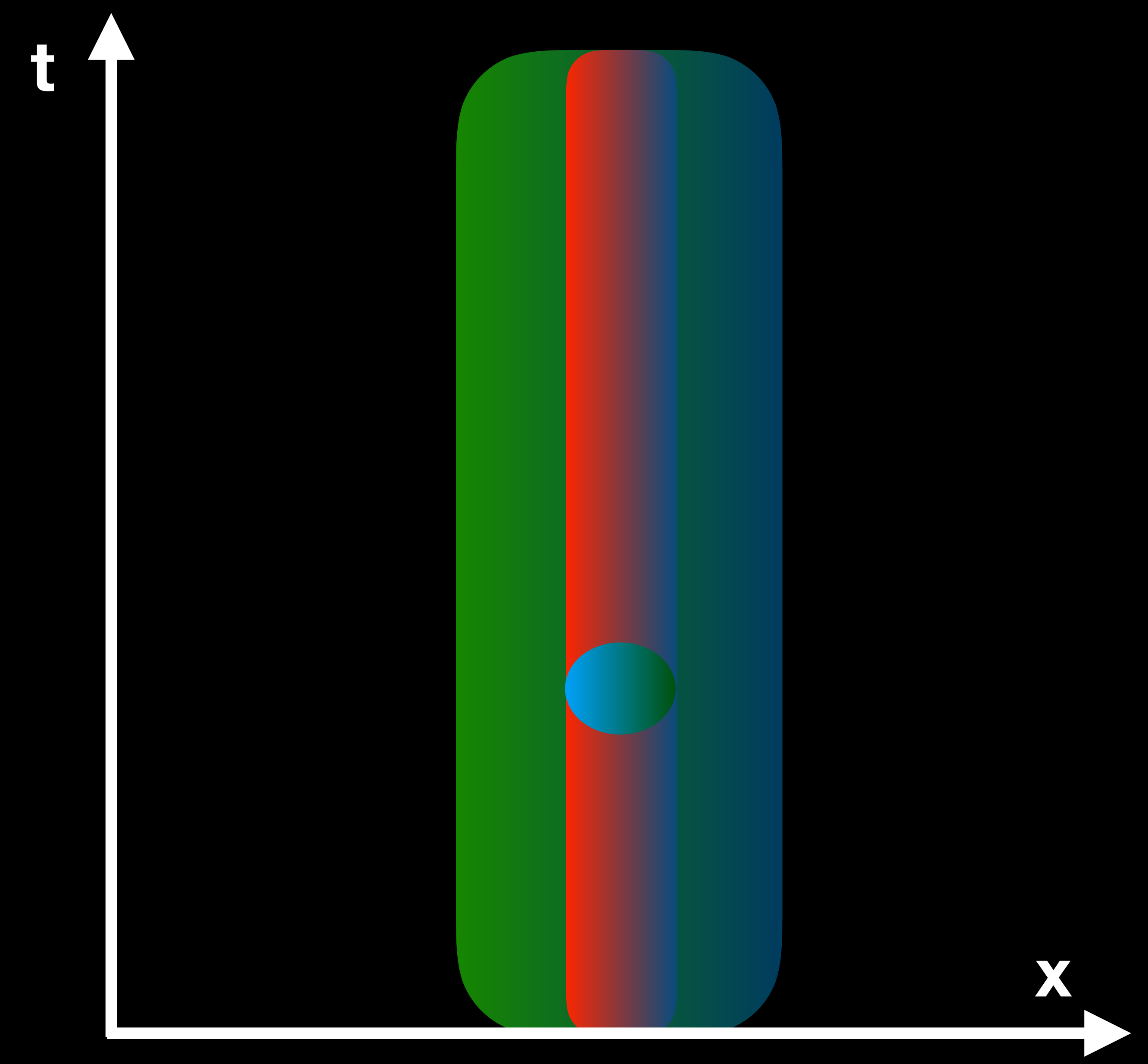

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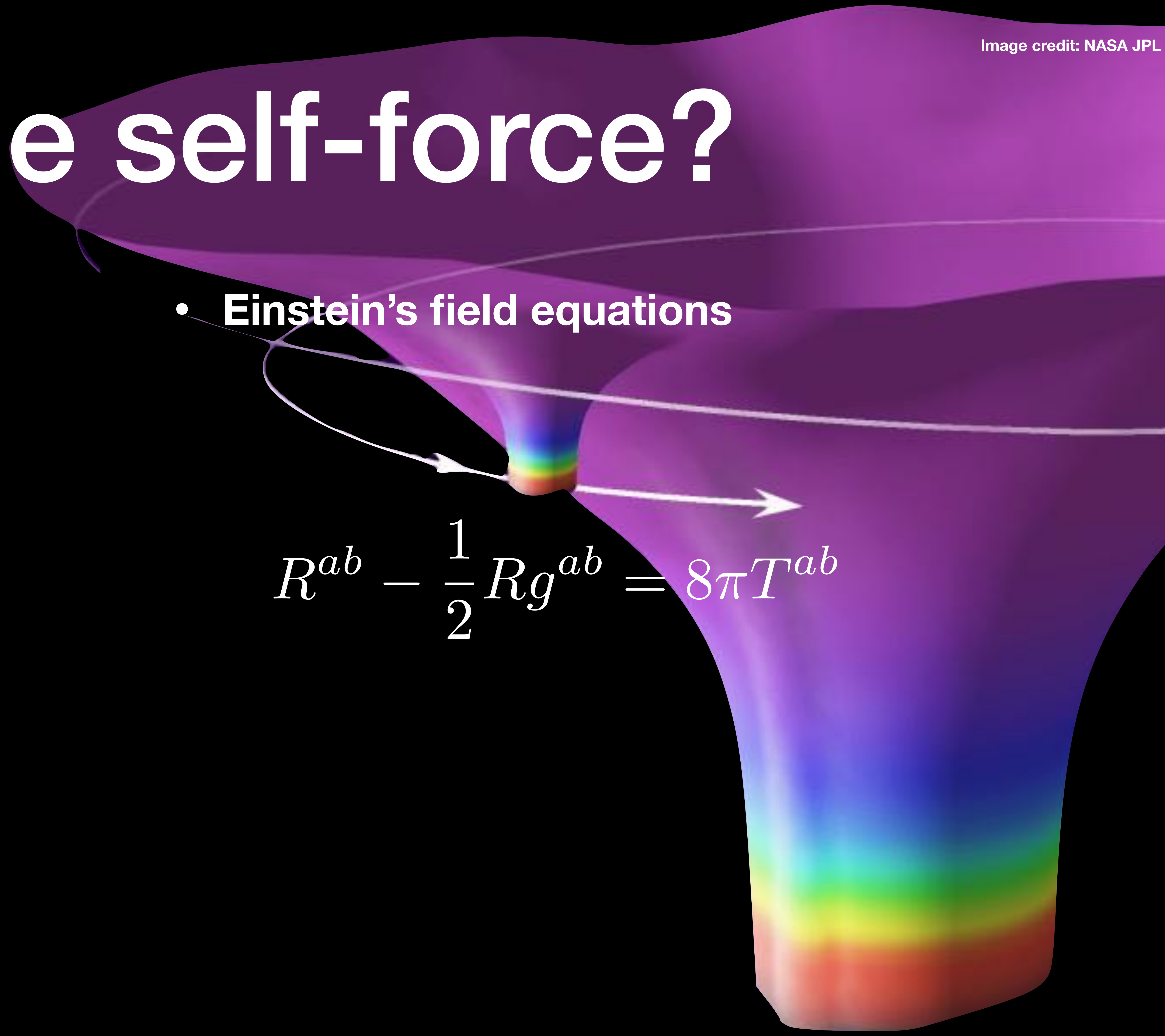

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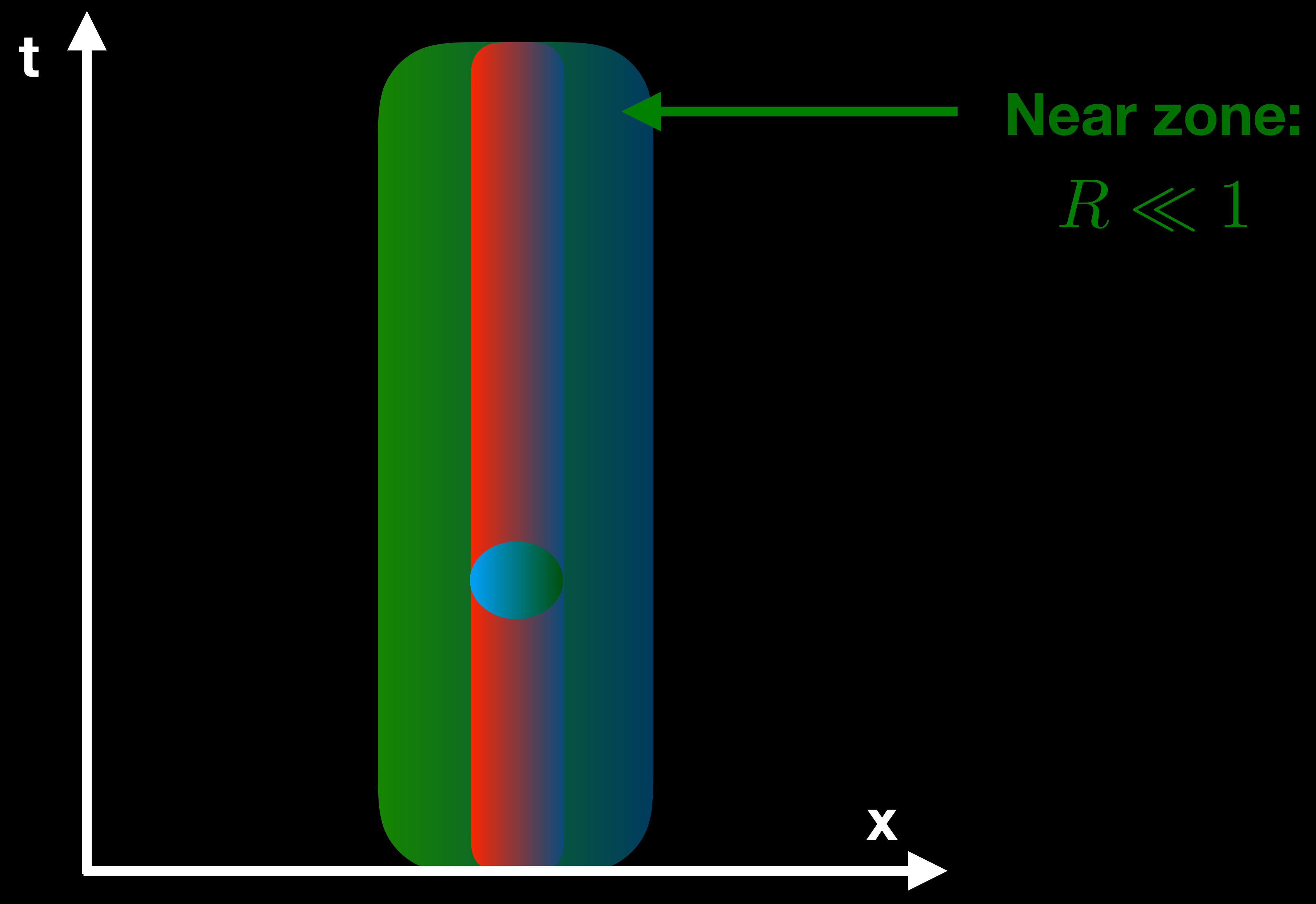

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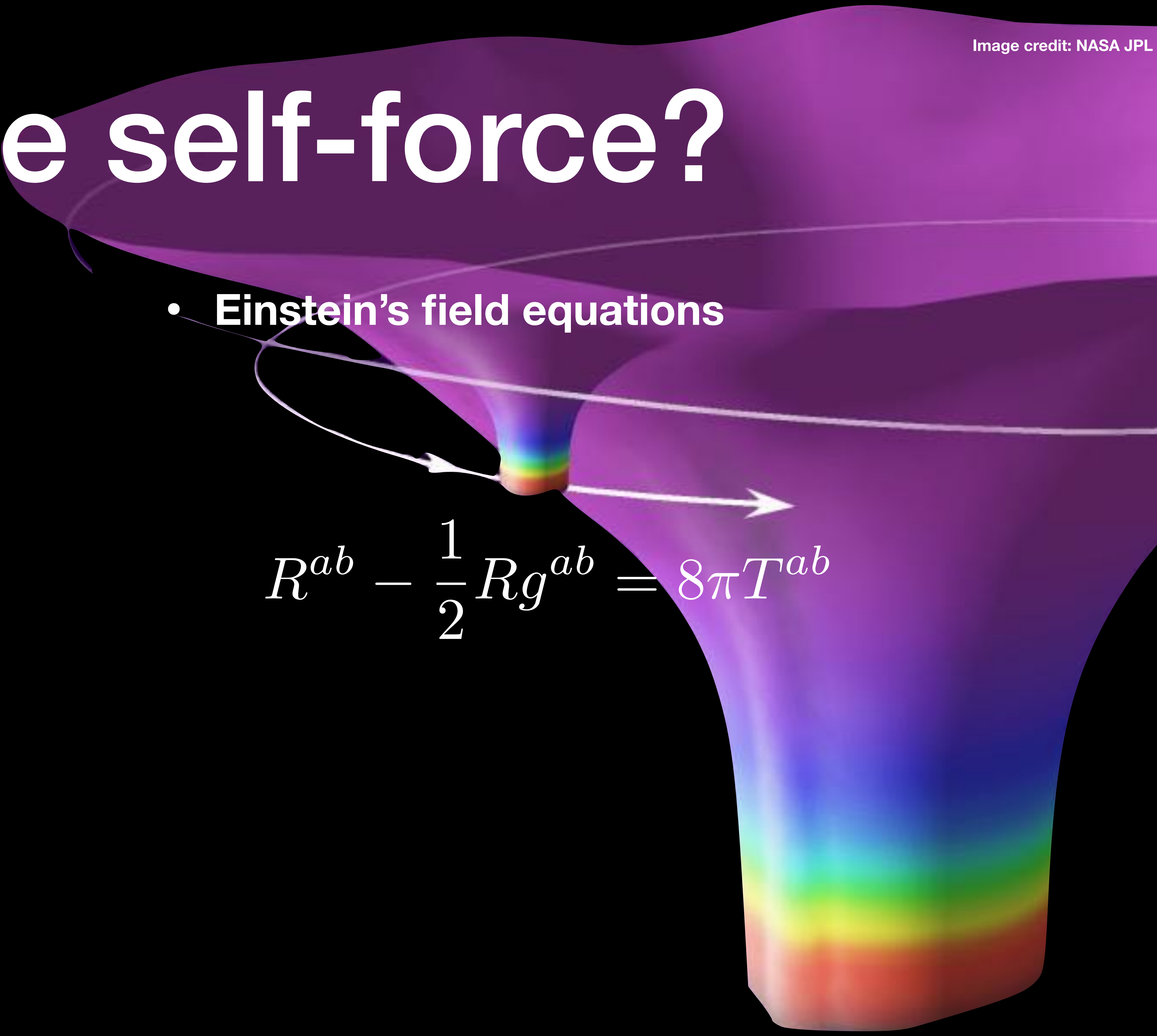

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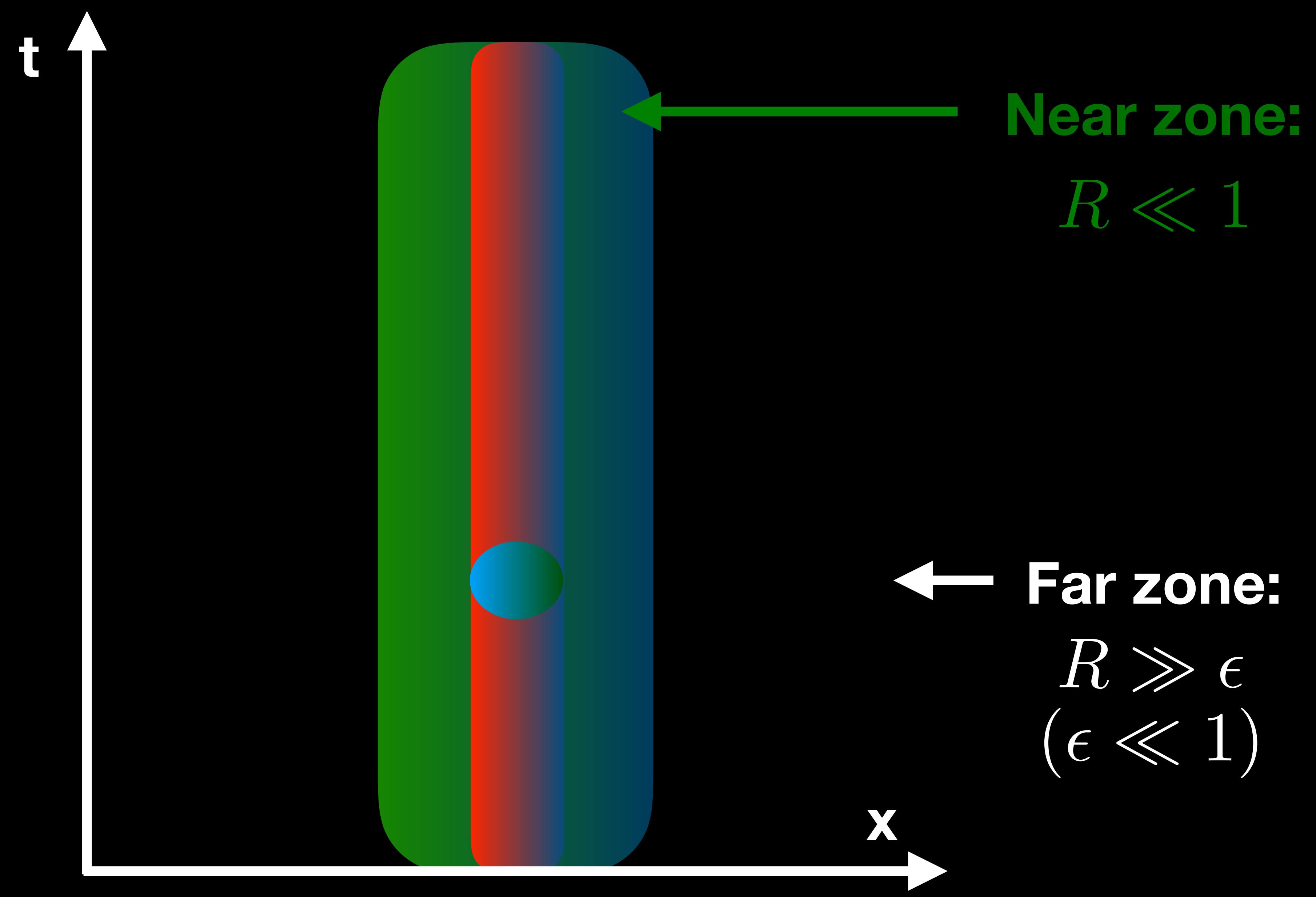

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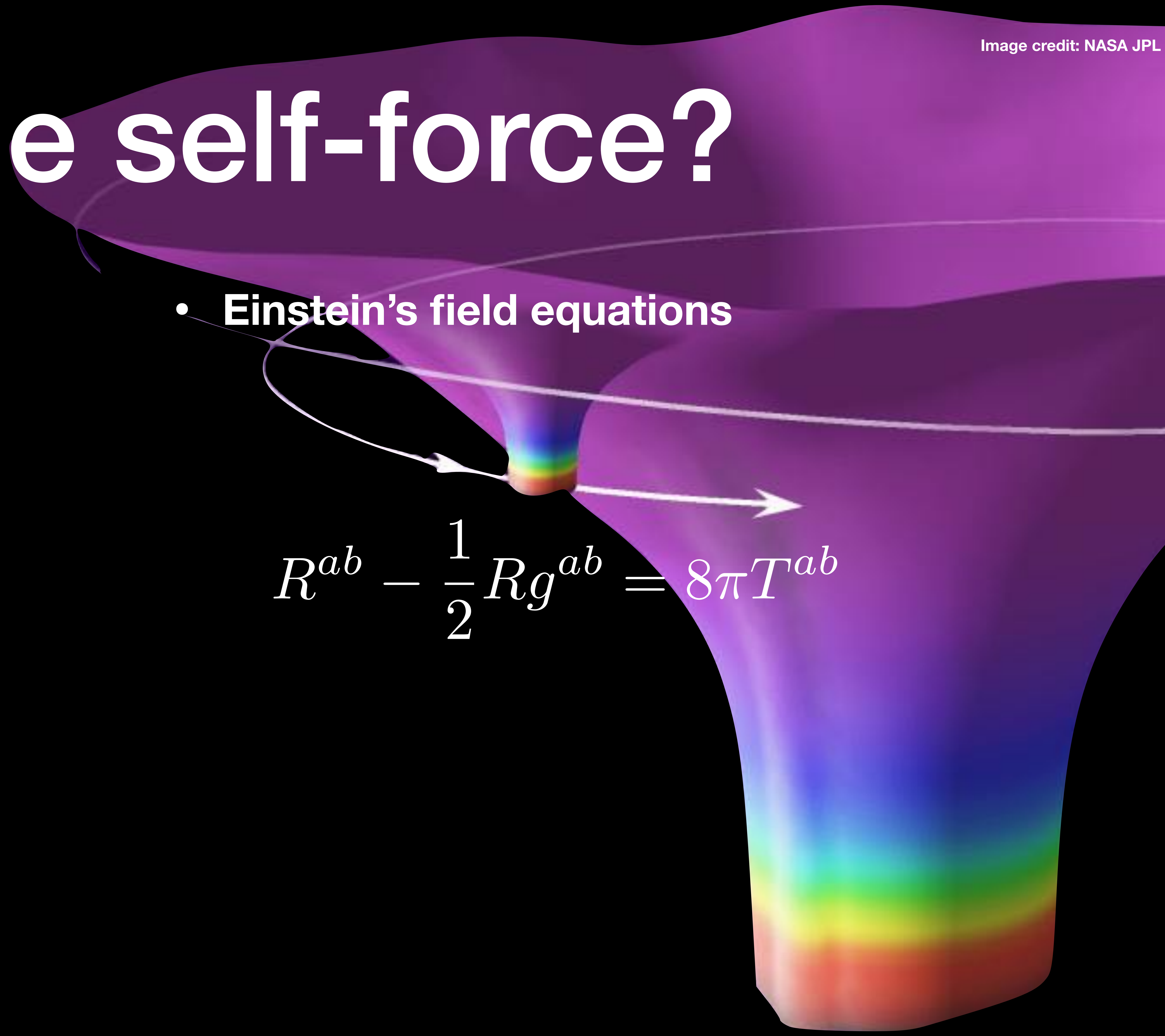

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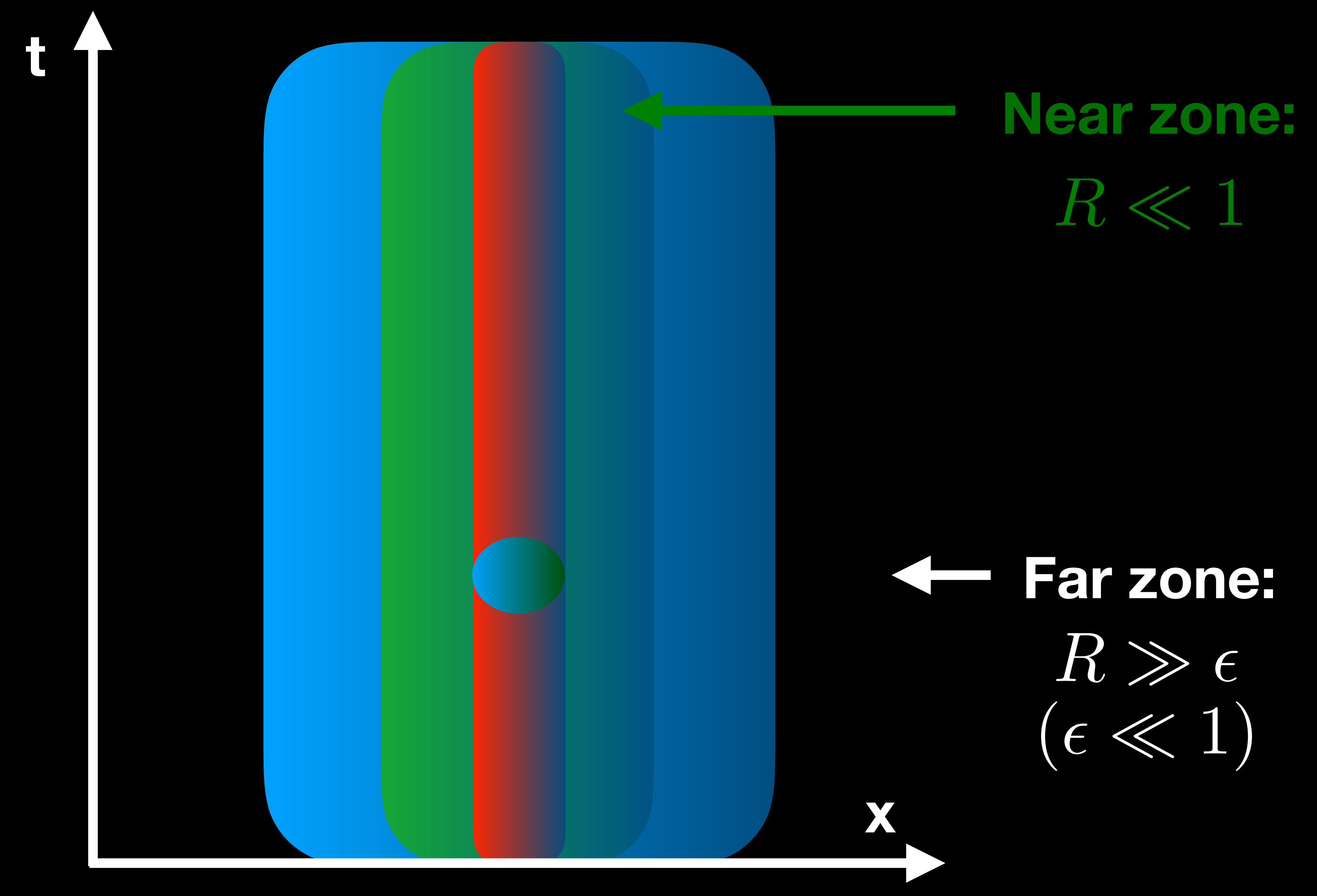

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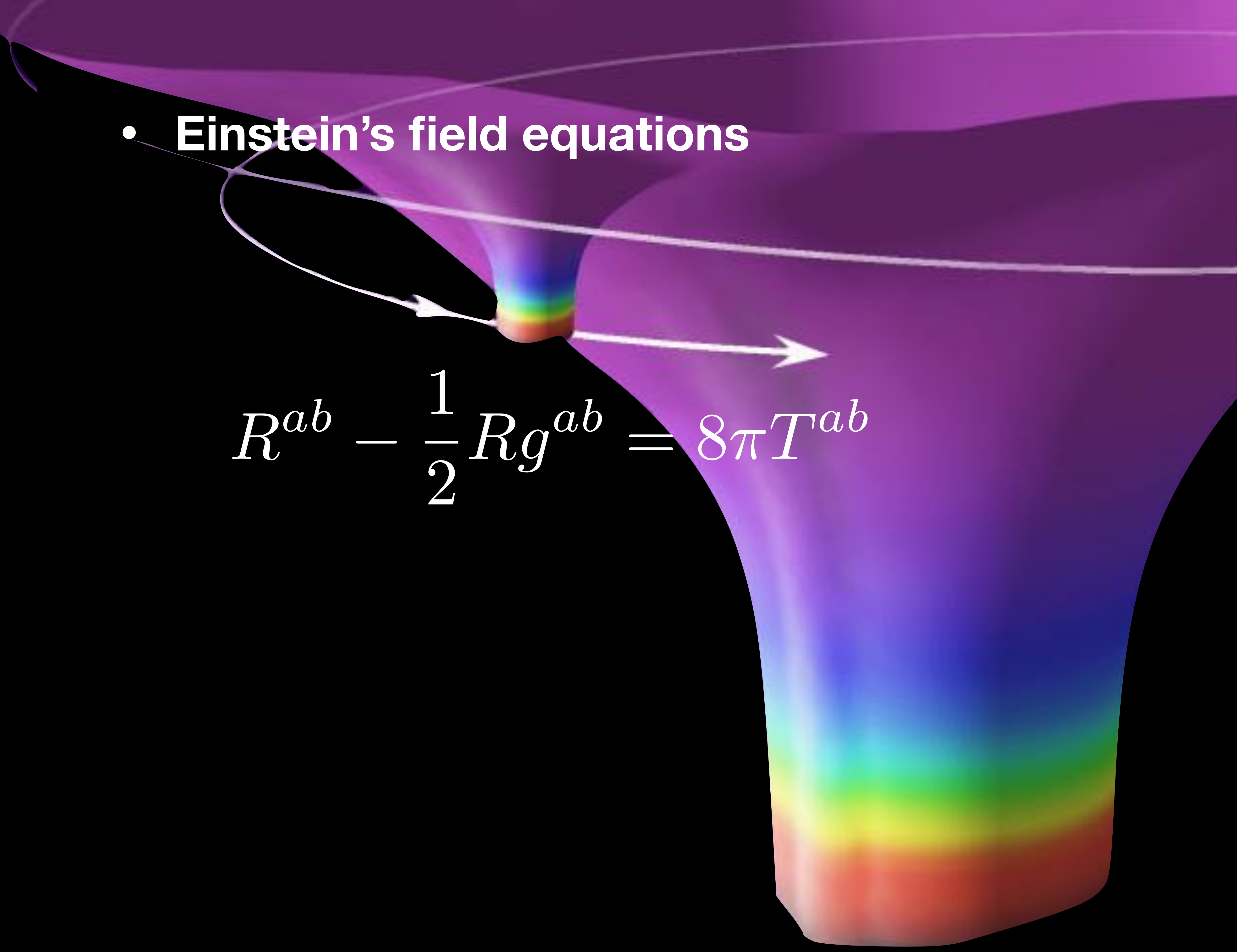

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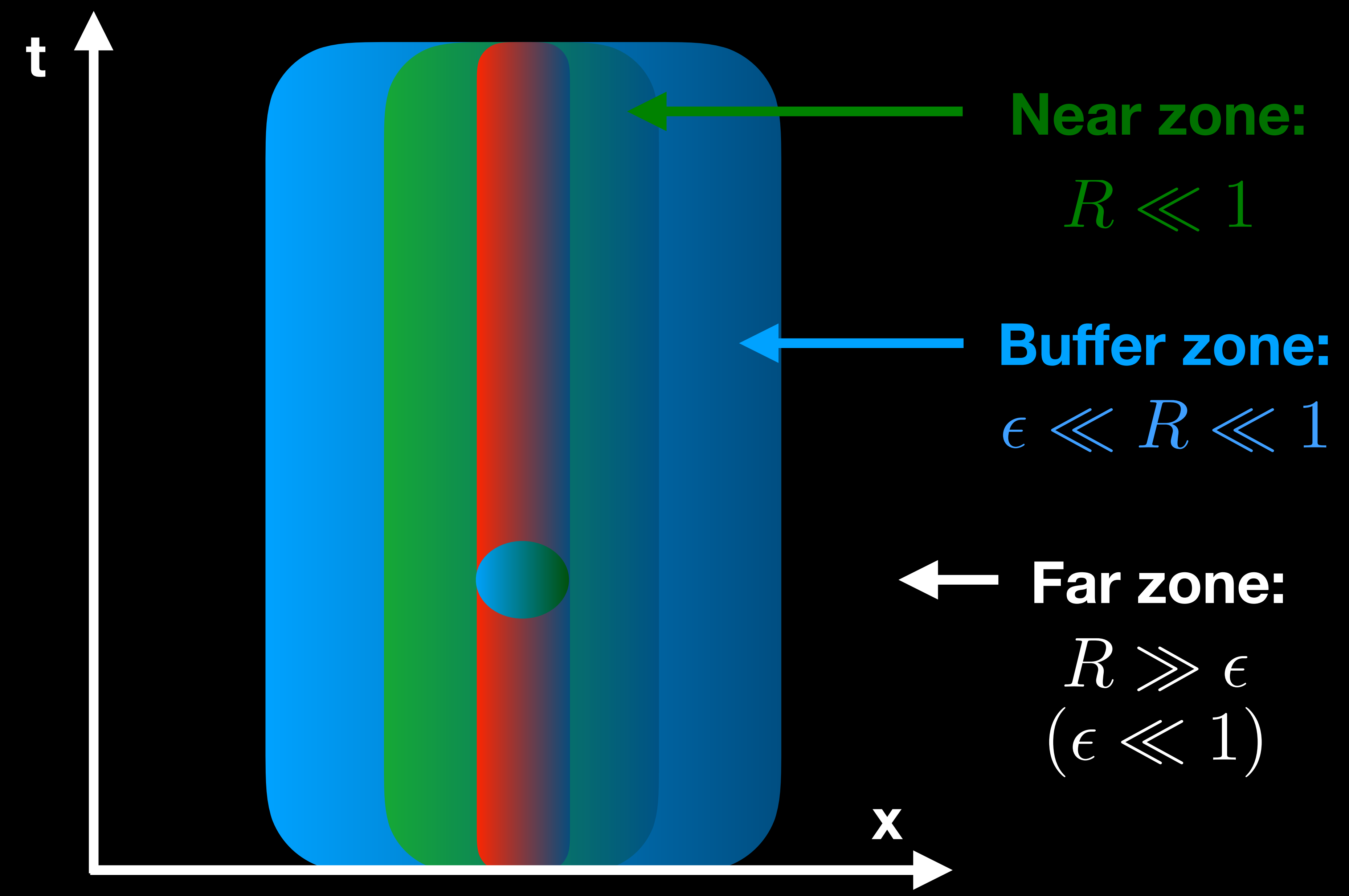

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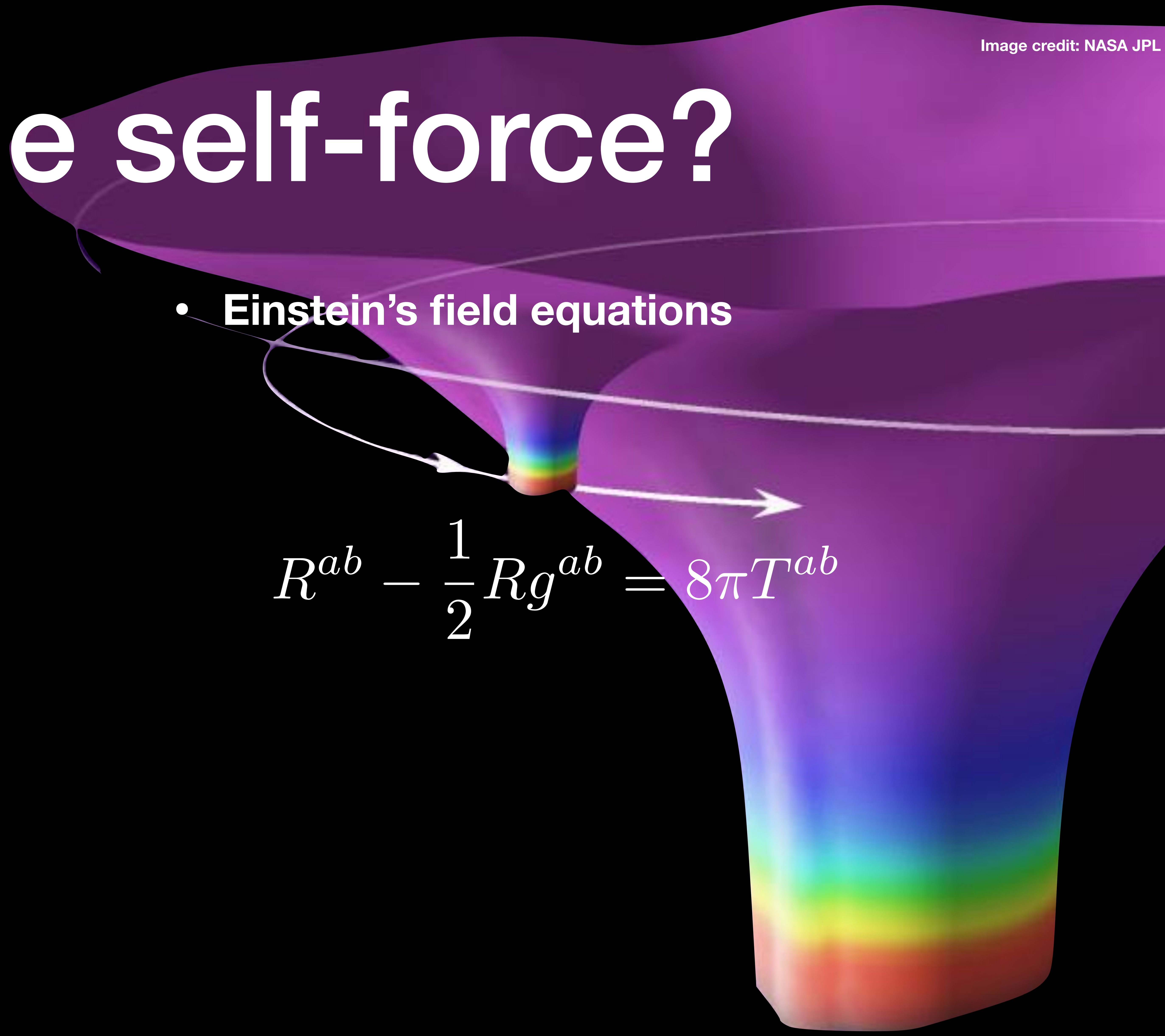

$$R^{ab} - \frac{1}{2} R g^{ab} = 8\pi T^{ab}$$

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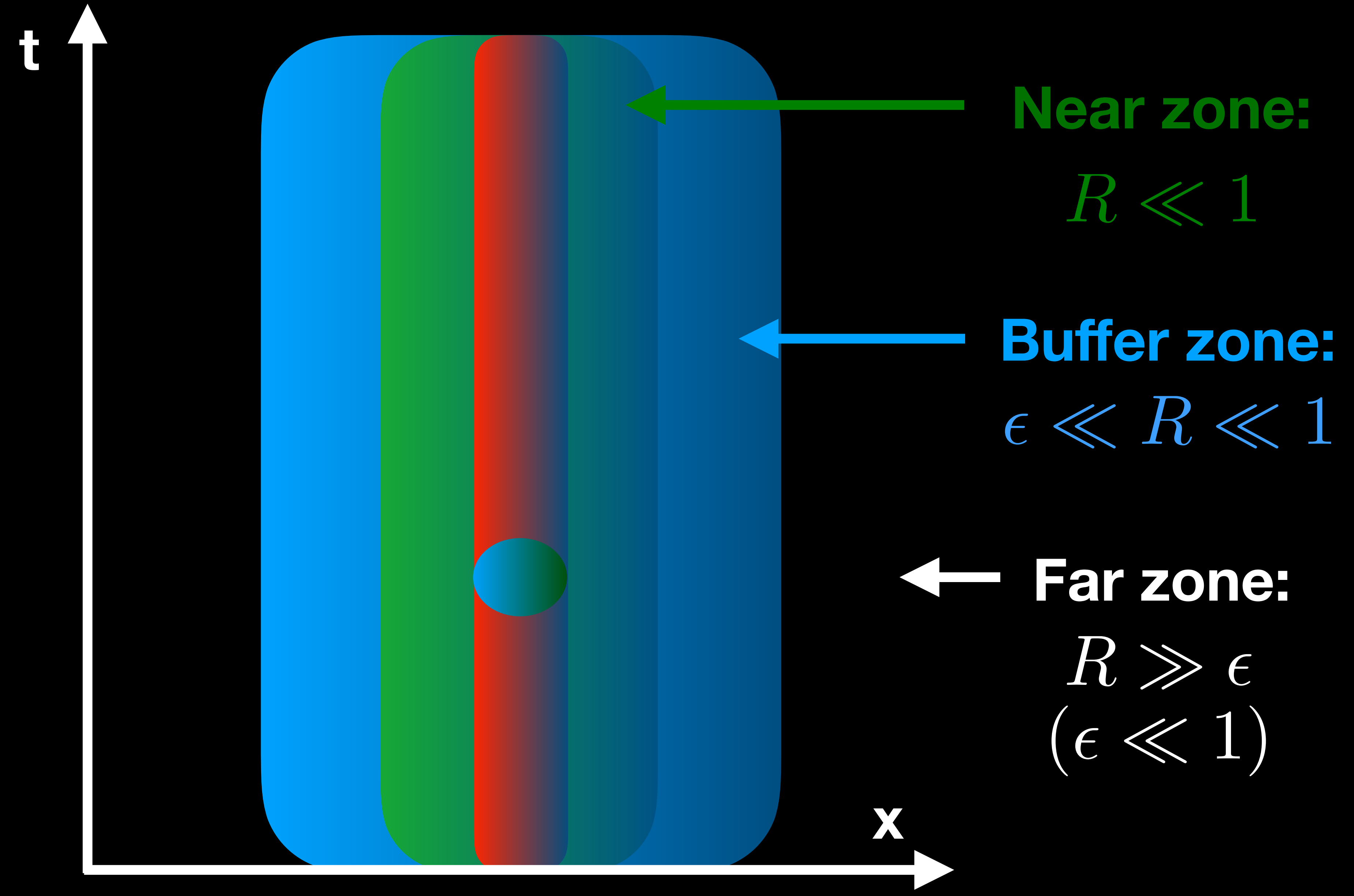
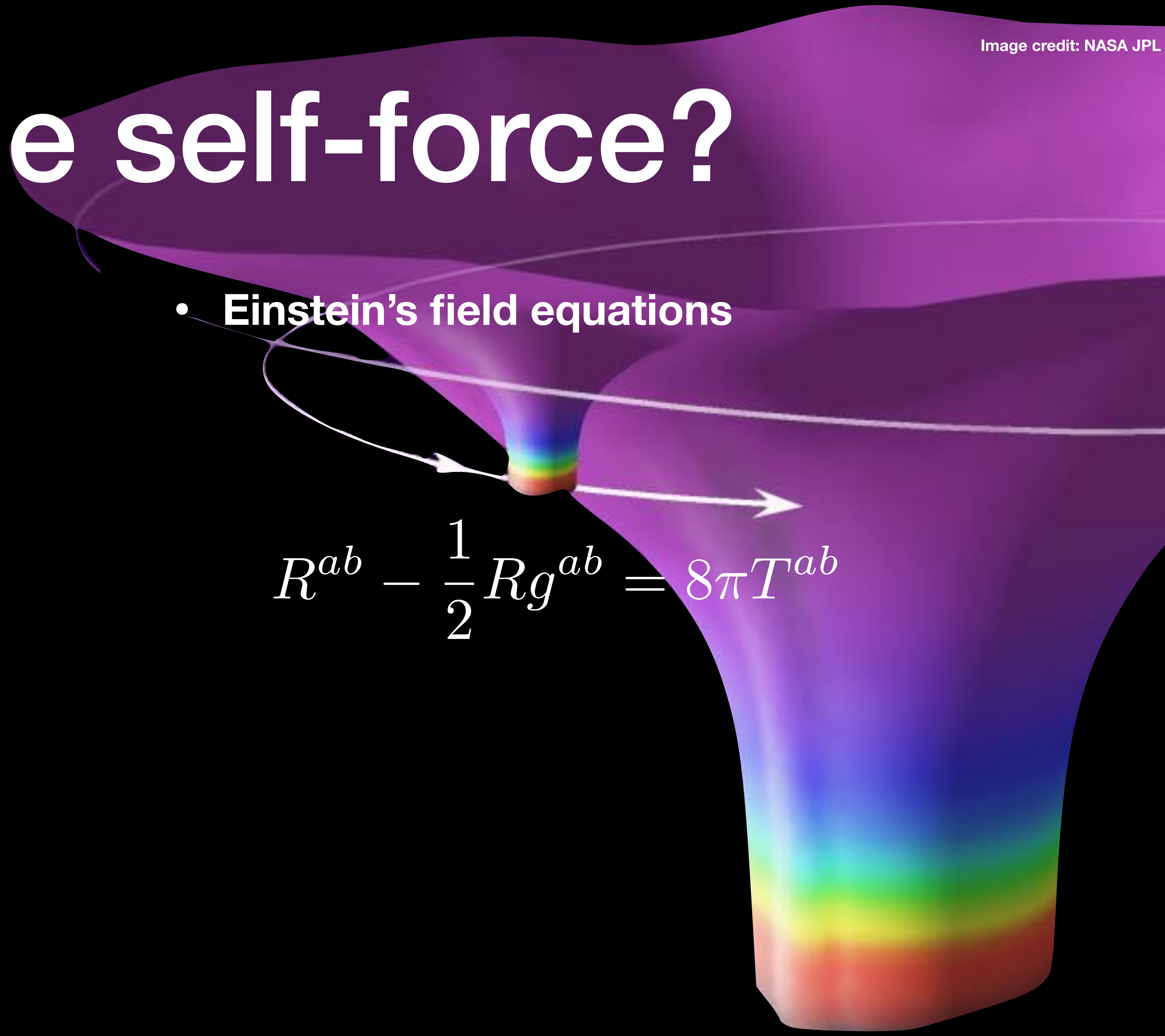
What is the self-force?

- **Perturbing the metric in the mass ratio**



Near zone: $\mathrm{g}_{\mu\nu} = g^{(m)}_{\mu\nu} + \epsilon H_{\mu\nu} + \mathcal{O}(\epsilon^2)$

- **Einstein's field equations**



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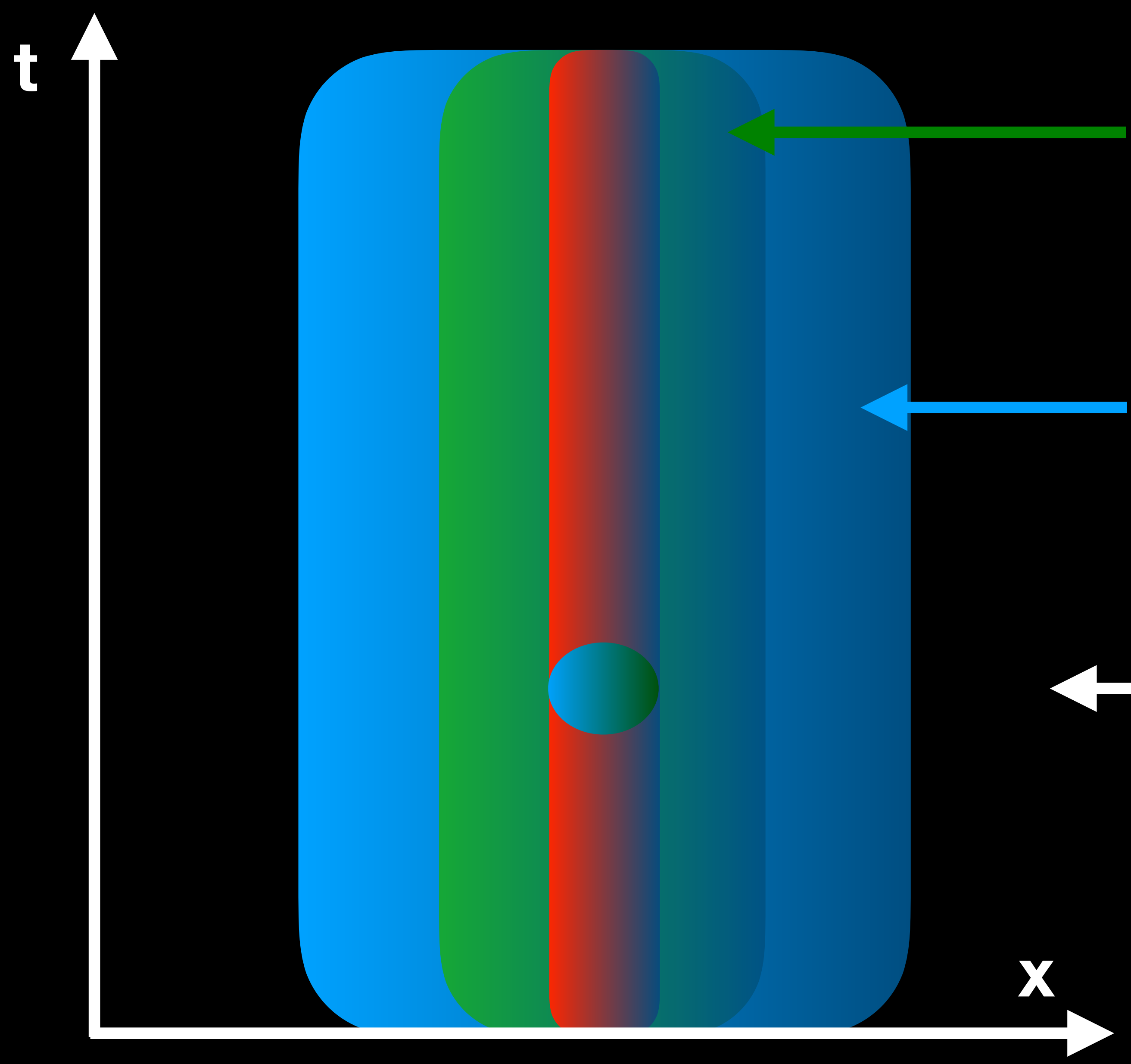


Near zone:
$R \ll 1$

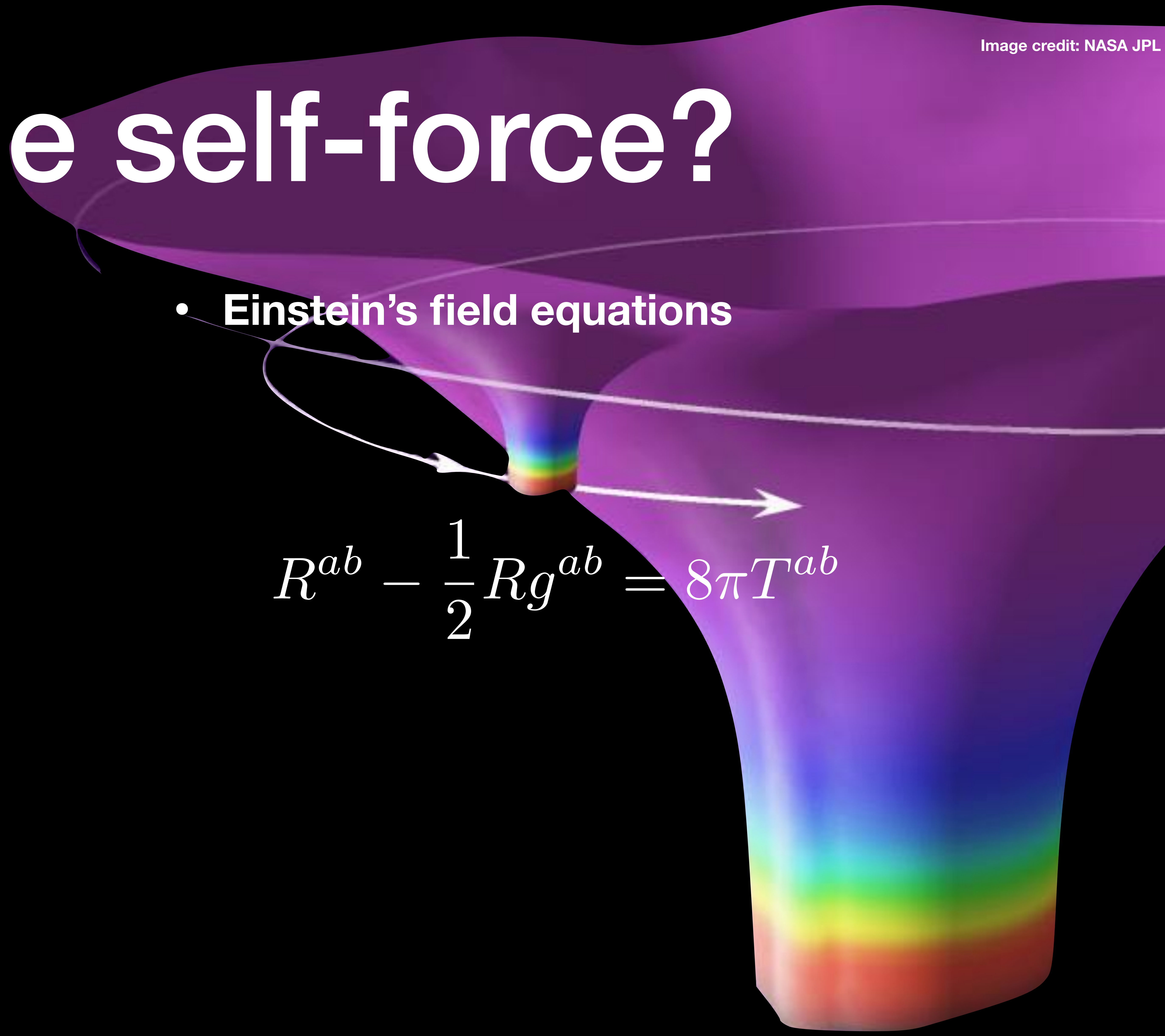
Buffer zone:
$\epsilon \ll R \ll 1$

Far zone:
$R \gg \epsilon$
$(\epsilon \ll 1)$

Near zone: $\mathrm{g}_{\mu\nu} = g^{(m)}_{\mu\nu} + \epsilon H_{\mu\nu} + \mathcal{O}(\epsilon^2)$

Far zone: $\mathrm{g}_{\mu\nu} = g^{(M)}_{\mu\nu} + \epsilon h_{\mu\nu} + \mathcal{O}(\epsilon^2)$

- Einstein's field equations



$$R^{ab} - \frac{1}{2} R g^{ab} = 8\pi T^{ab}$$

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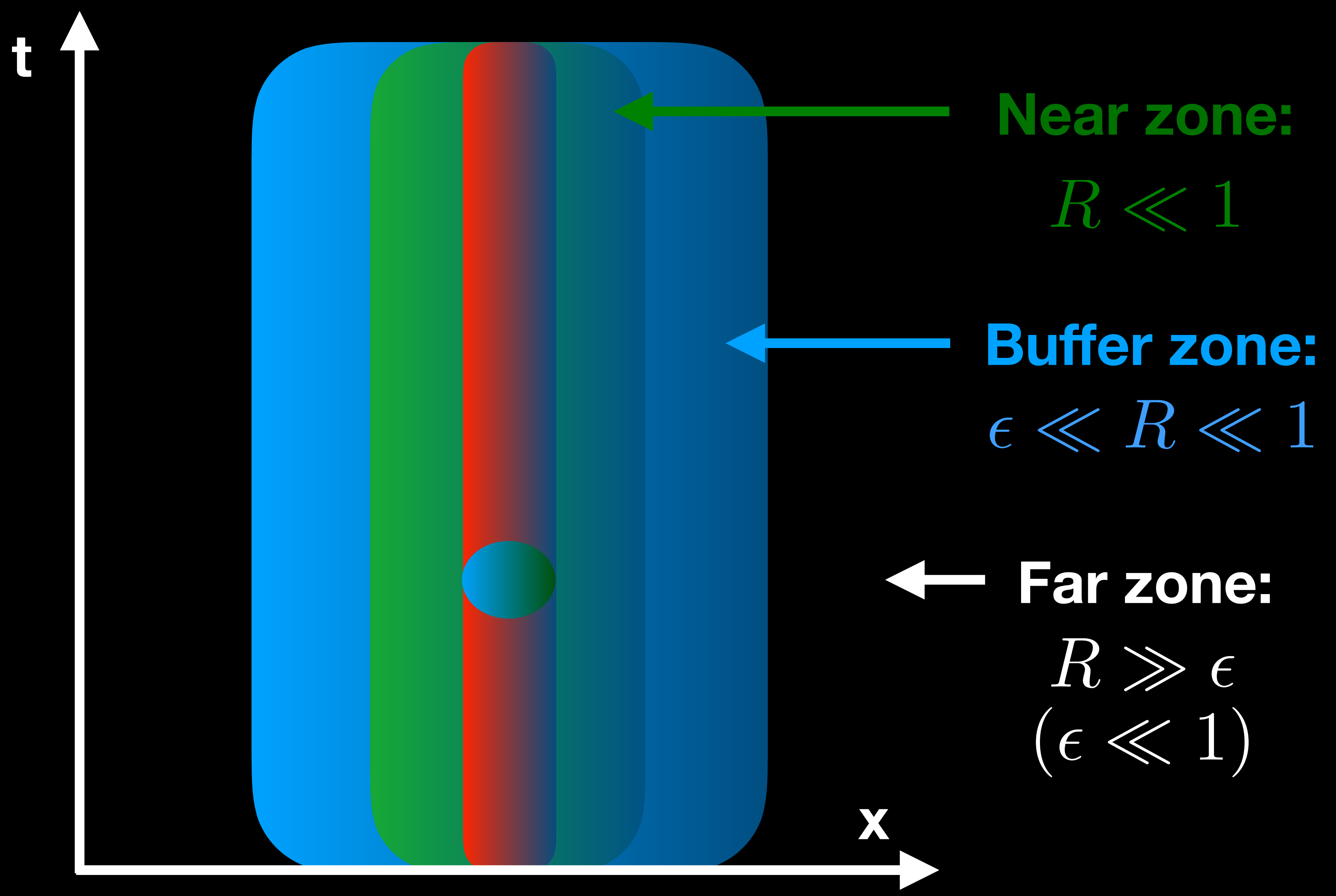
- Perturbing the metric in the mass ratio

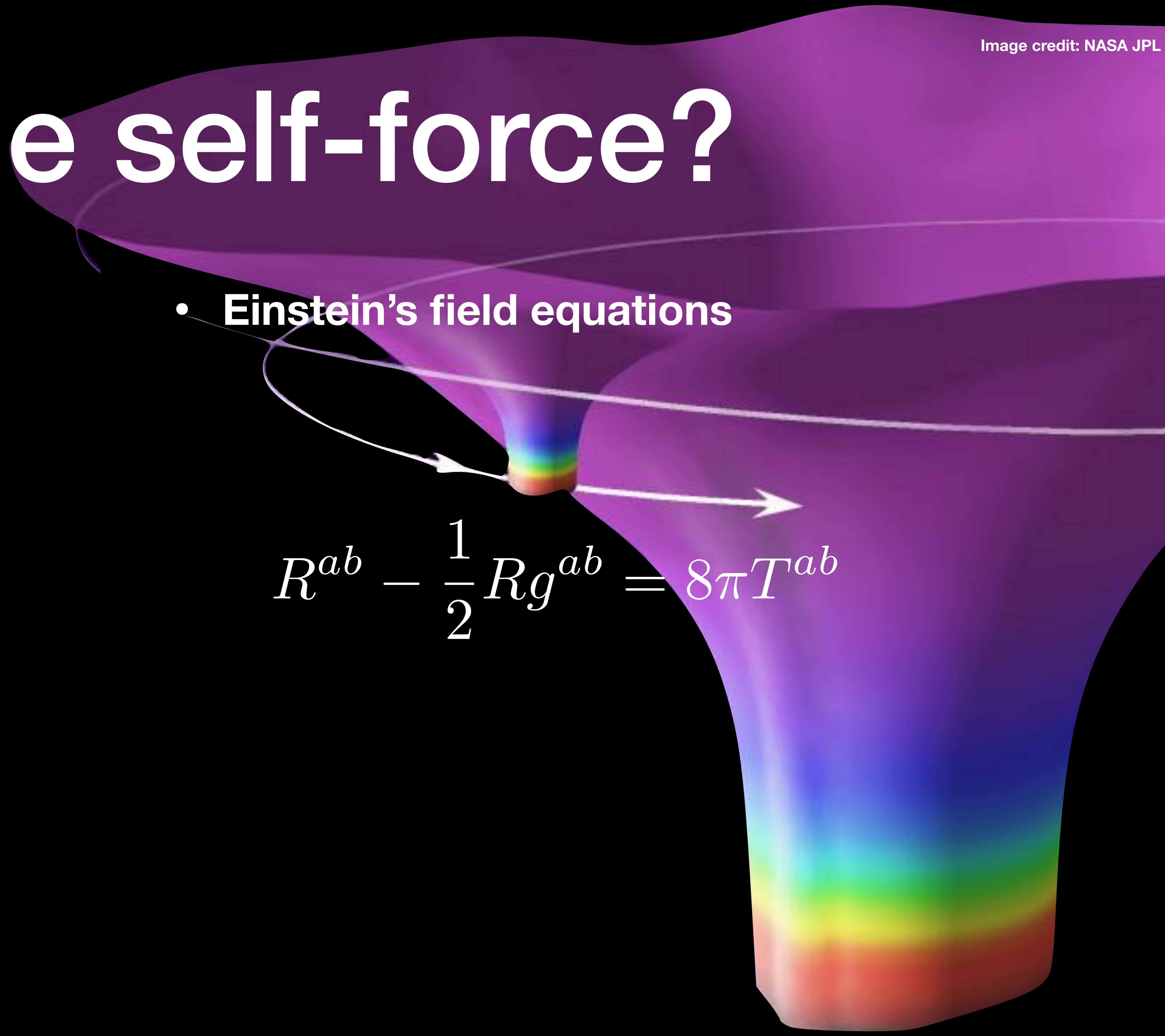


Near zone: $g_{\mu\nu} = g^{(m)}_{\mu\nu} + \epsilon H_{\mu\nu} + \mathcal{O}(\epsilon^2)$

Far zone: $g_{\mu\nu} = g^{(M)}_{\mu\nu} + \epsilon h_{\mu\nu} + \mathcal{O}(\epsilon^2)$

Solve in the buffer zone!

- Einstein's field equations



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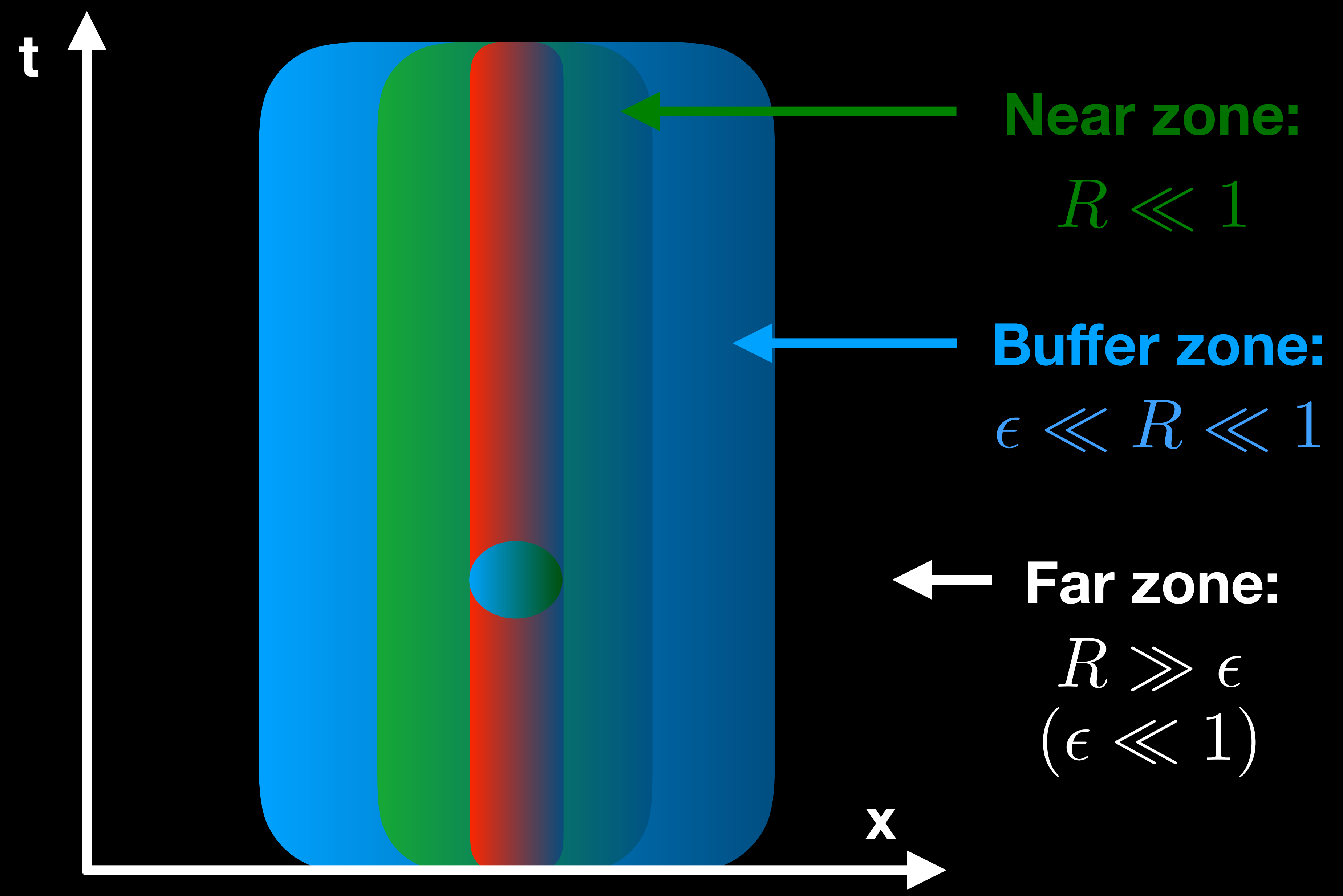

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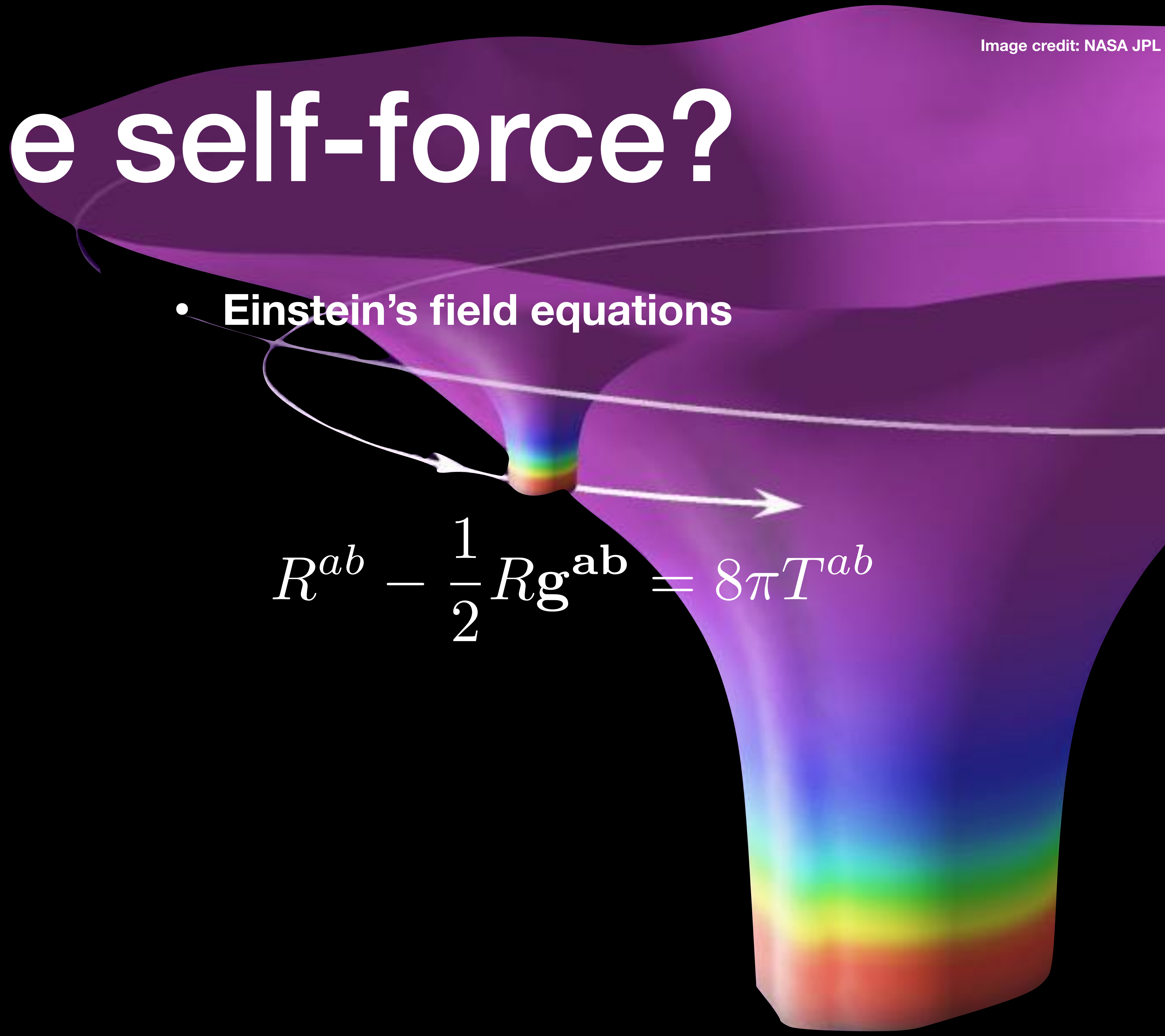
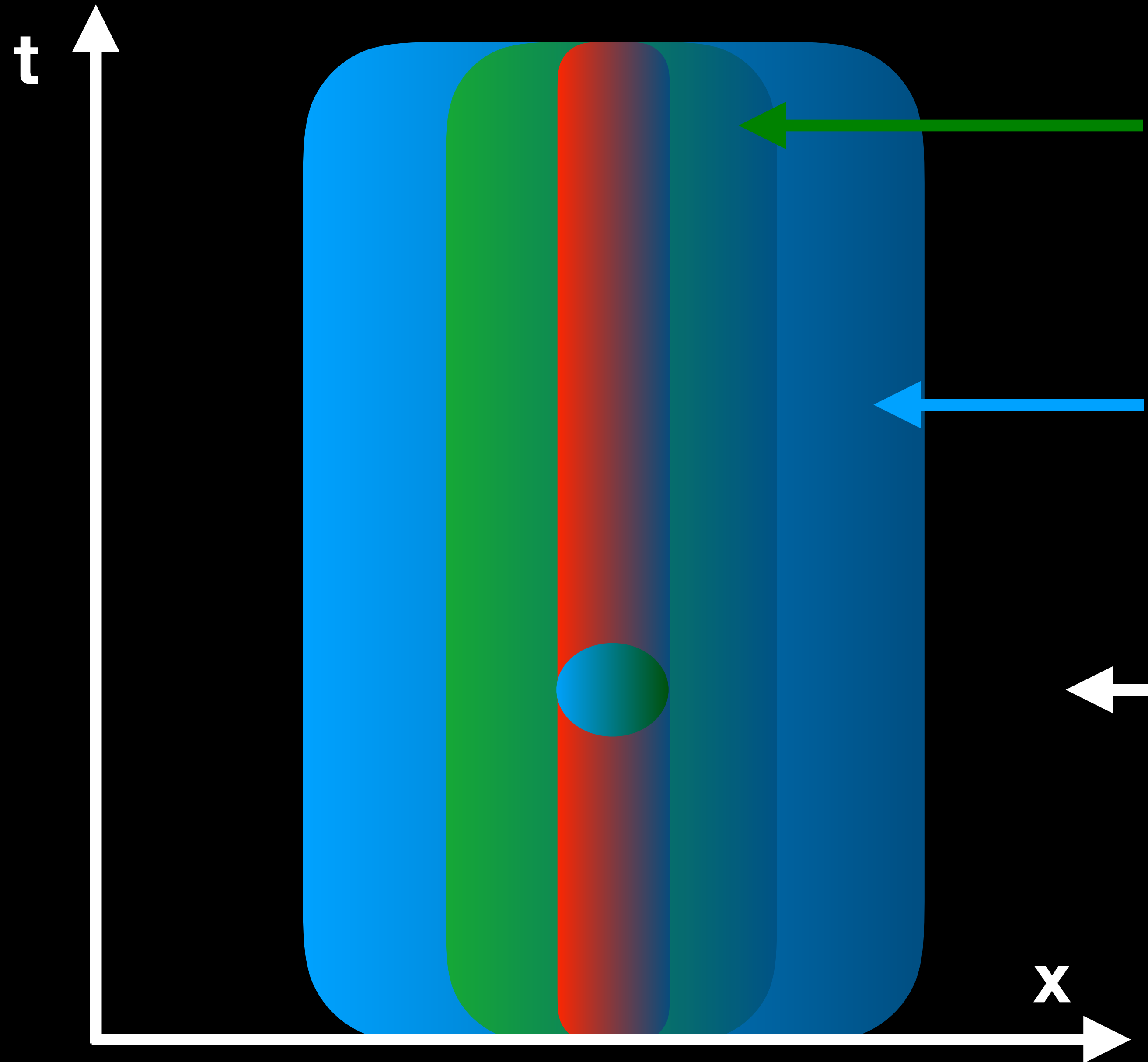


$$R^{ab} - \frac{1}{2} R \mathbf{g}^{\mathbf{ab}} = 8\pi T^{ab}$$

Image credit: NASA JPL

What is the self-force?

- **Perturbing the metric in the mass ratio**



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Near zone: $\mathrm{g}_{\mu\nu} = g^{(m)}_{\mu\nu} + \epsilon H_{\mu\nu} + \mathcal{O}(\epsilon^2)$

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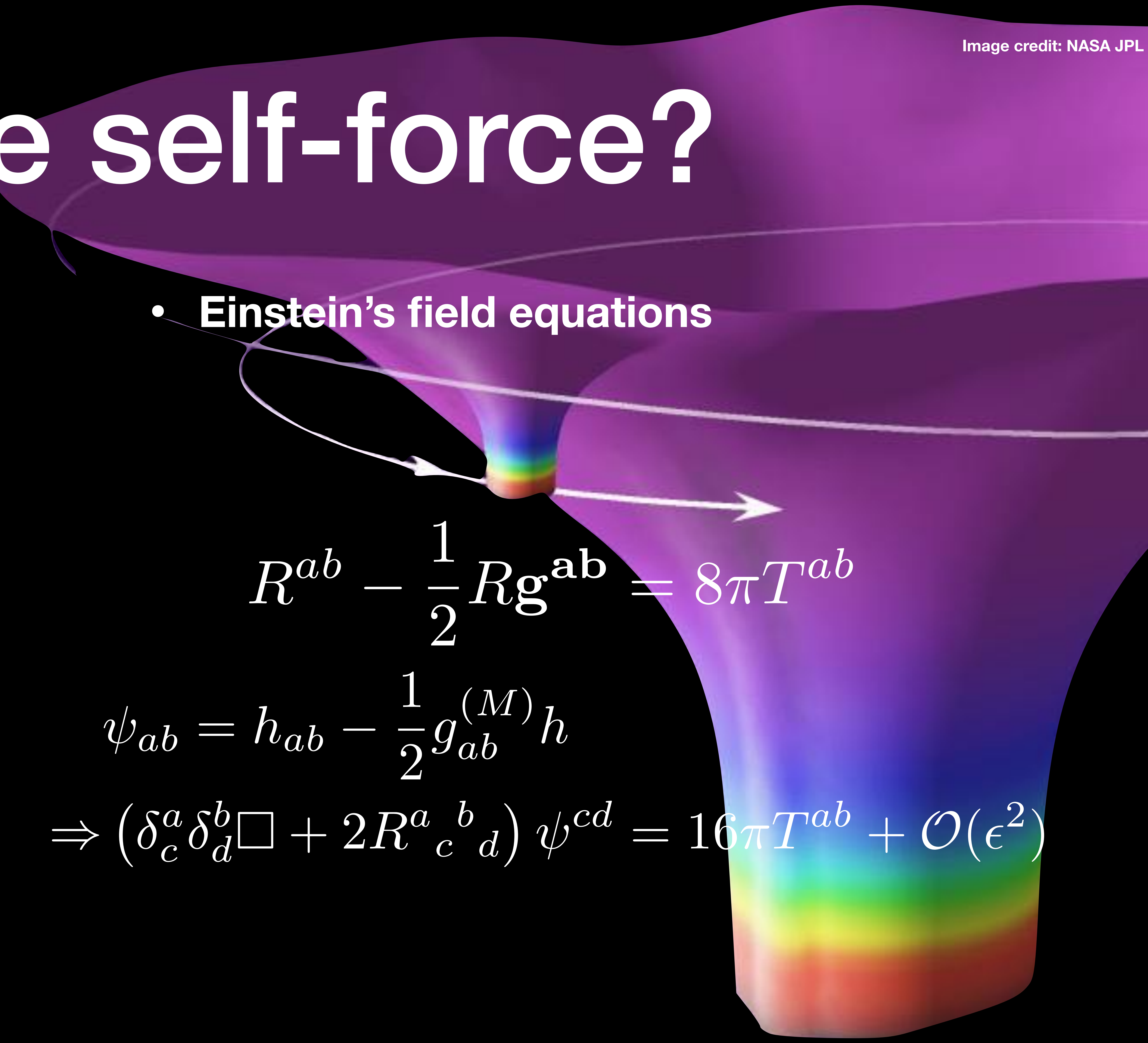
Solve in the buffer zone!

- **Einstein's field equations**



$$R^{ab} - \frac{1}{2} R \mathbf{g}^{\mathbf{ab}} = 8\pi T^{ab}$$

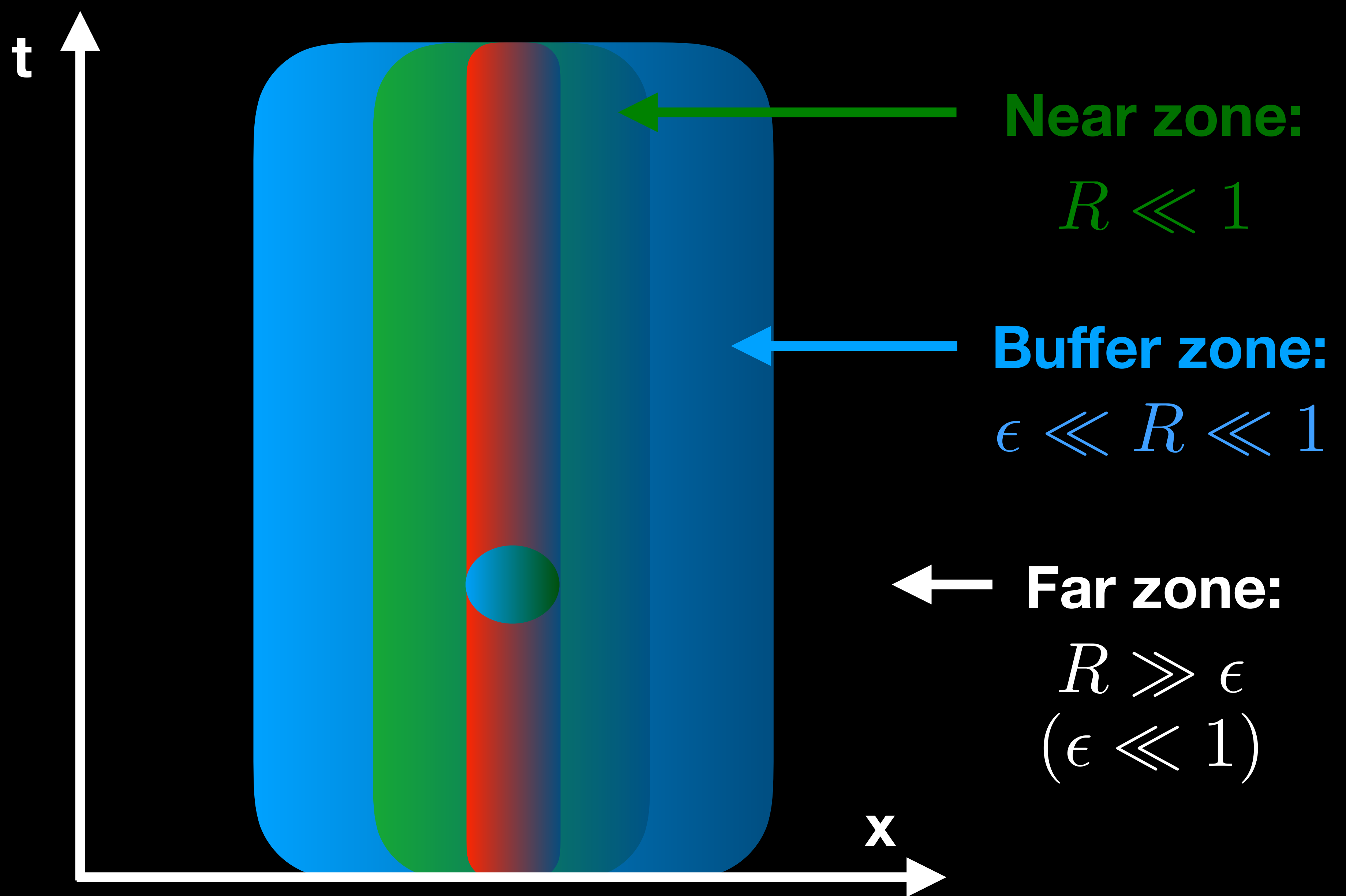
$$\psi_{ab} = h_{ab} - \frac{1}{2} g^{(M)}_{ab} h$$

$$\Rightarrow \left(\delta^a_c \delta^b_d \Box + 2 R^a{}_c{}^b{}_d\right) \psi^{cd} = 16\pi T^{ab} + \mathcal{O}(\epsilon^2)$$

What is the self-force?

Image credit: NASA JPL

- **Perturbing the metric in the mass ratio**



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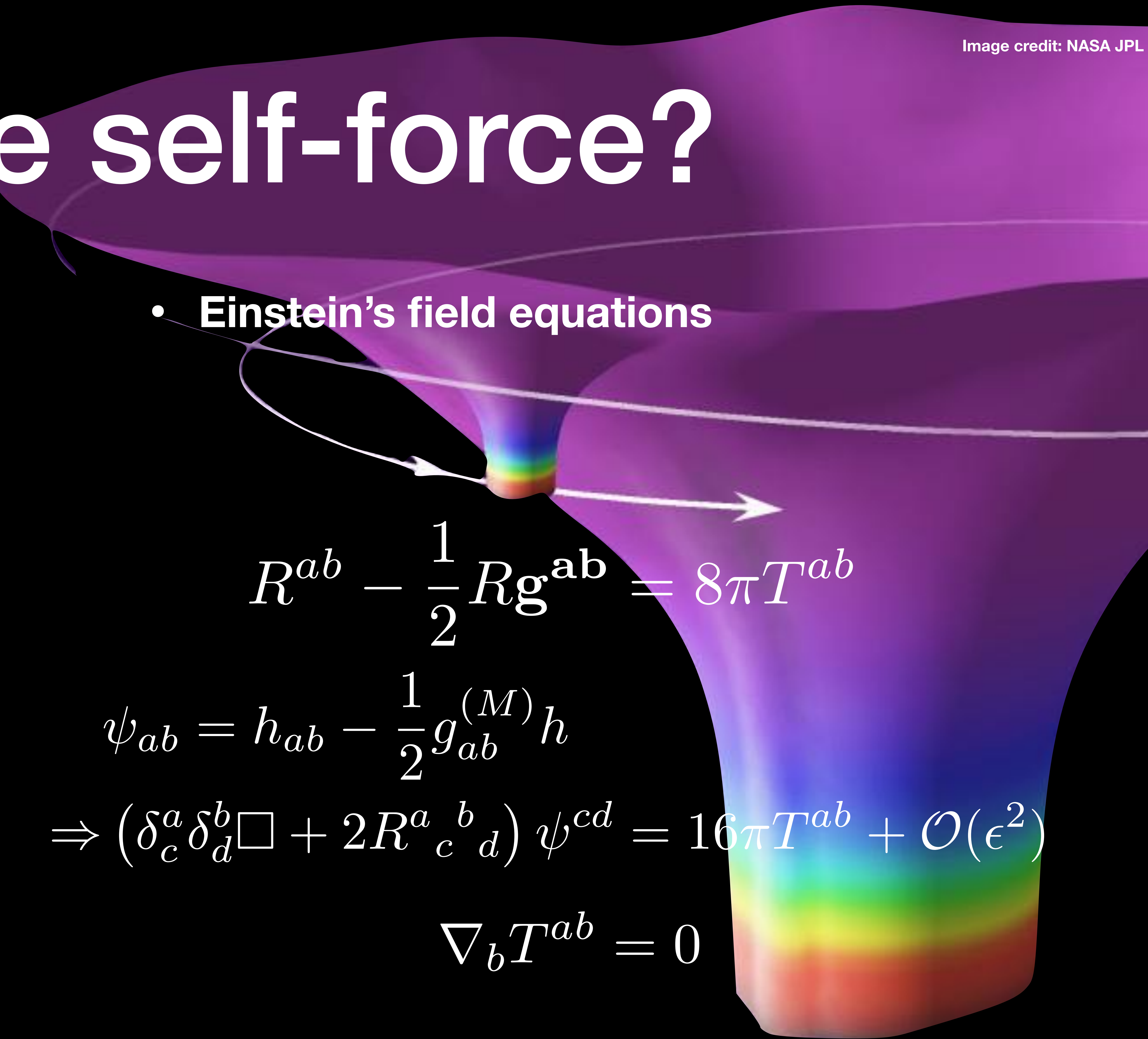
Solve in the buffer zone!

- **Einstein's field equations**



$$R^{ab} - \frac{1}{2} R \mathbf{g}^{\mathbf{ab}} = 8\pi T^{ab}$$

$$\psi_{ab} = h_{ab} - \frac{1}{2} g^{(M)}_{ab} h$$

$$\Rightarrow \left(\delta^a_c \delta^b_d \Box + 2R^a{}_c{}^b{}_d\right) \psi^{cd} = 16\pi T^{ab} + \mathcal{O}(\epsilon^2)$$

$$\nabla_b T^{ab} = 0$$

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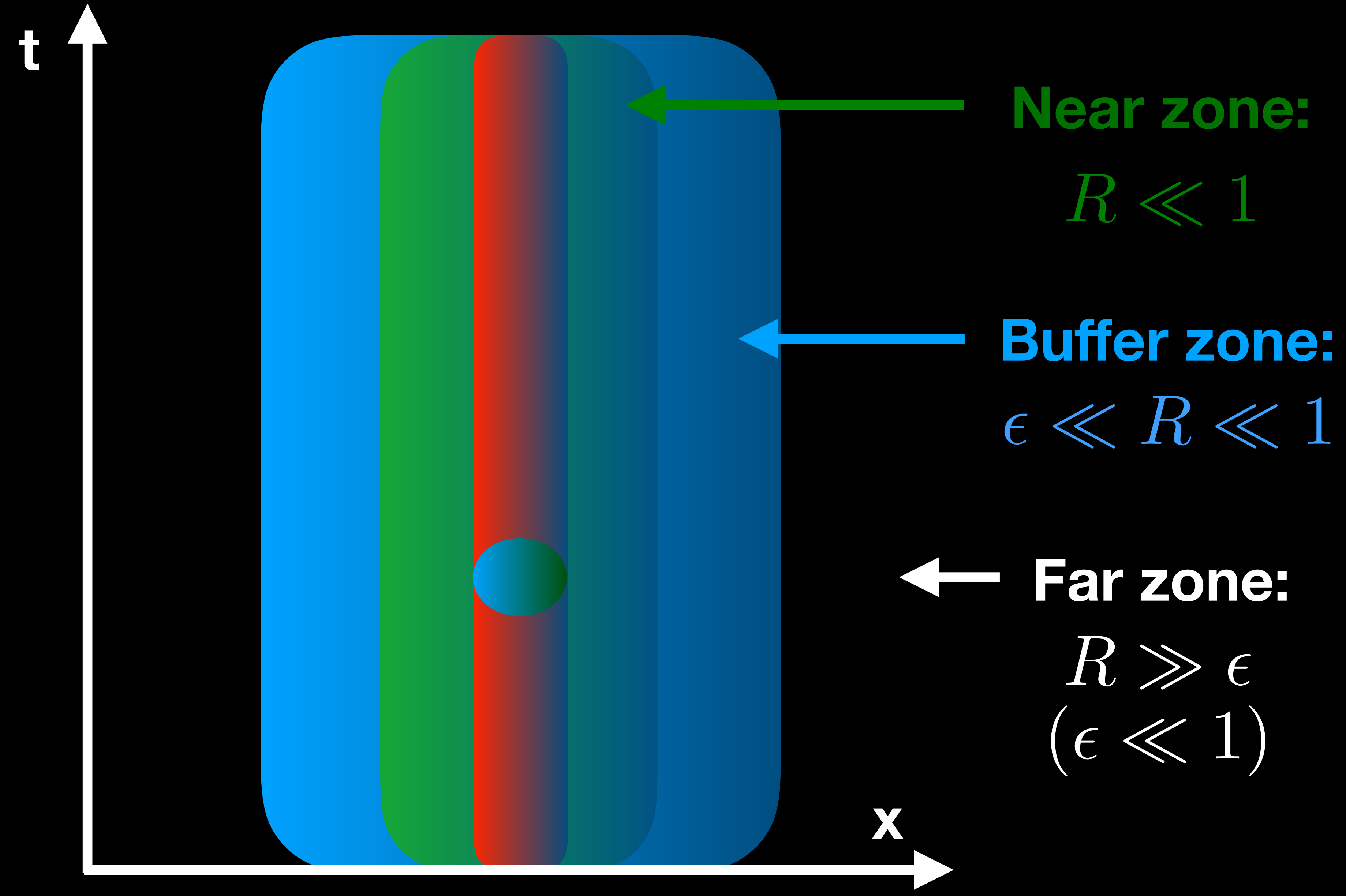
What is the self-force?

Image credit: NASA JPL

- Perturbing the metric in the mass ratio



Near zone: $\mathrm{g}_{\mu\nu} = g^{(m)}_{\mu\nu} + \epsilon H_{\mu\nu} + \mathcal{O}(\epsilon^2)$

Far zone: $\mathrm{g}_{\mu\nu} = g^{(M)}_{\mu\nu} + \epsilon h_{\mu\nu} + \mathcal{O}(\epsilon^2)$

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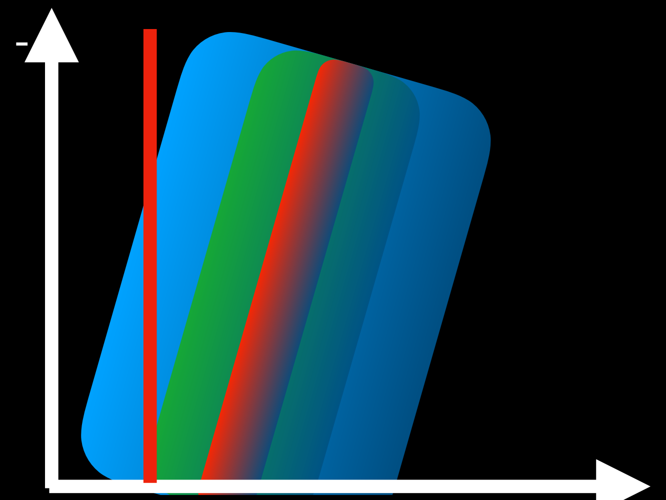

Self consistent

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What is the self-force?

Image credit: NASA JPL

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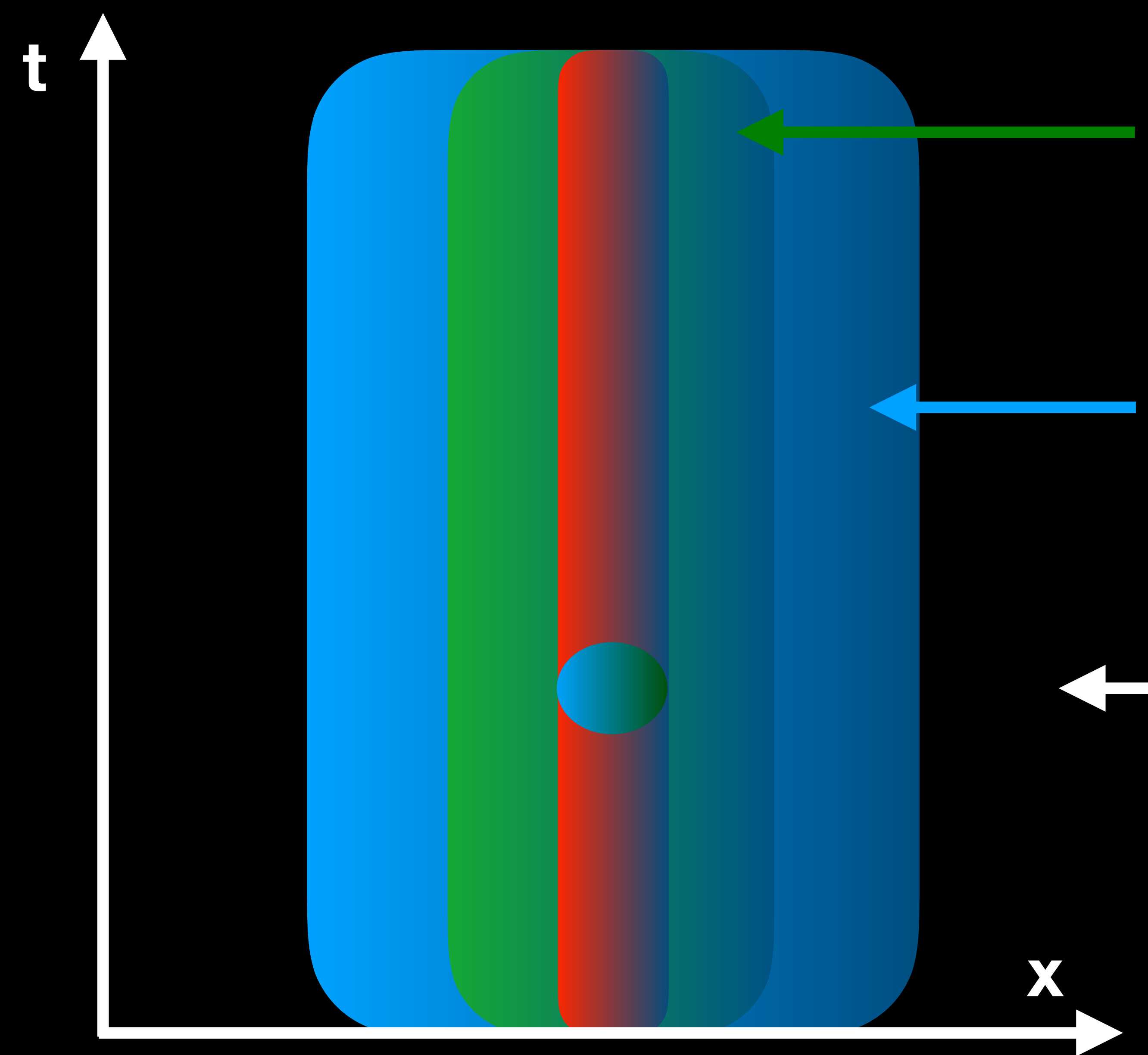

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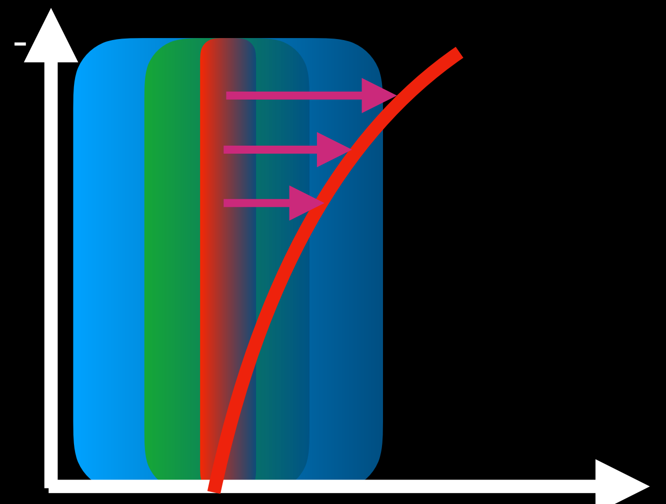
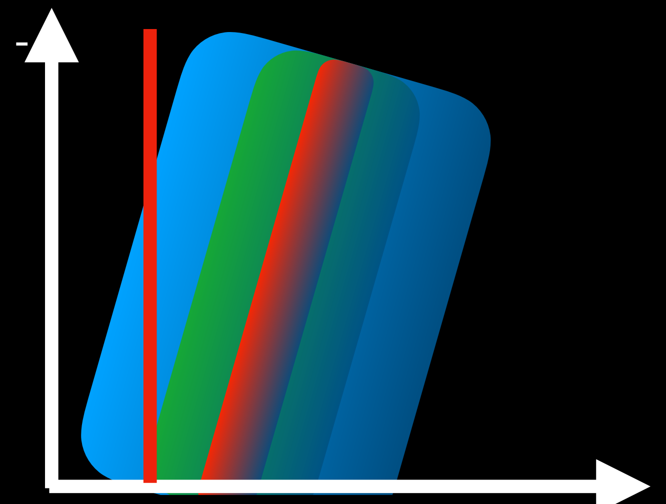


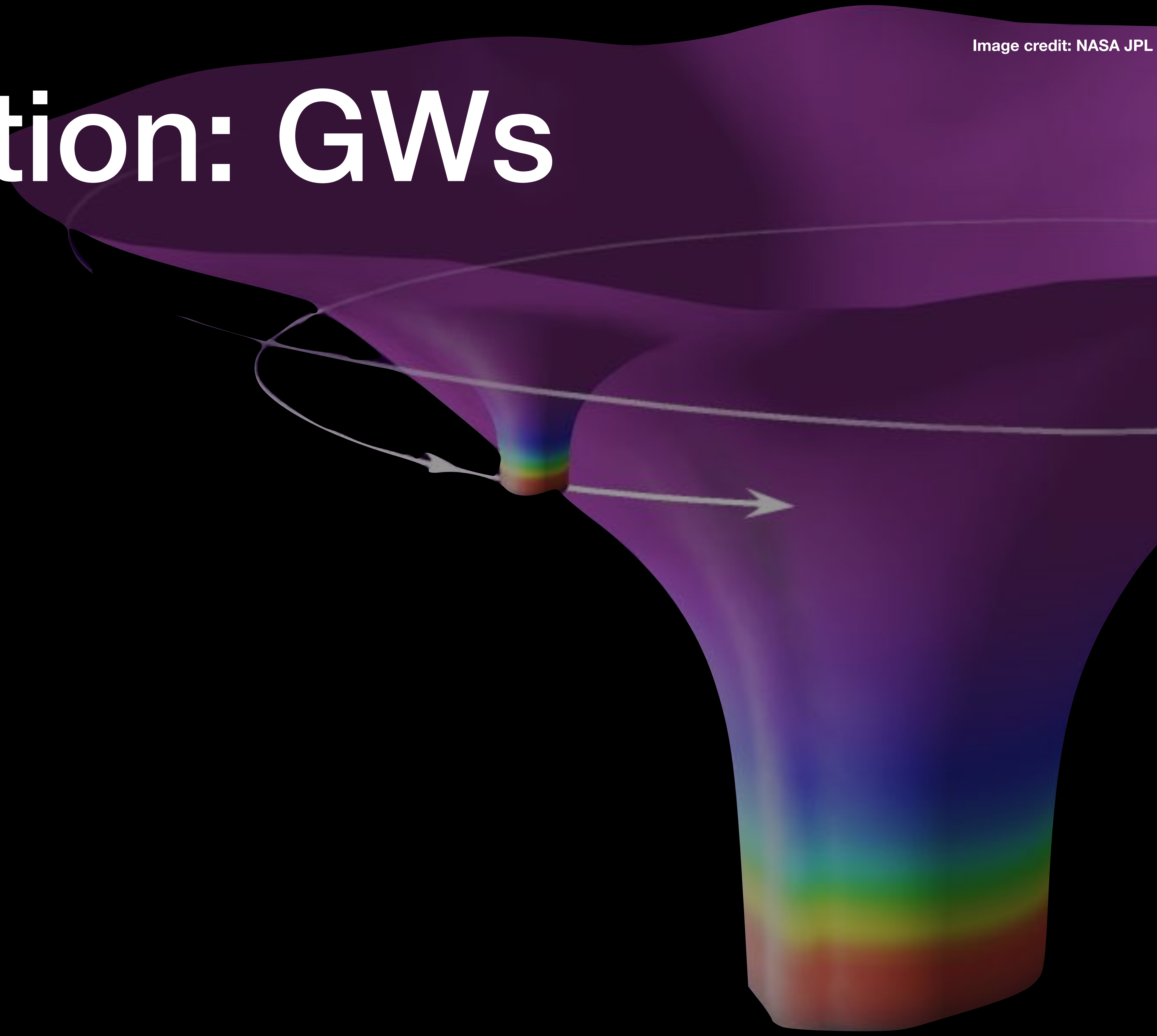
Gralla Wald



Self consistent

Motivation: GWs

Image credit: NASA JPL



Motivation: GWs

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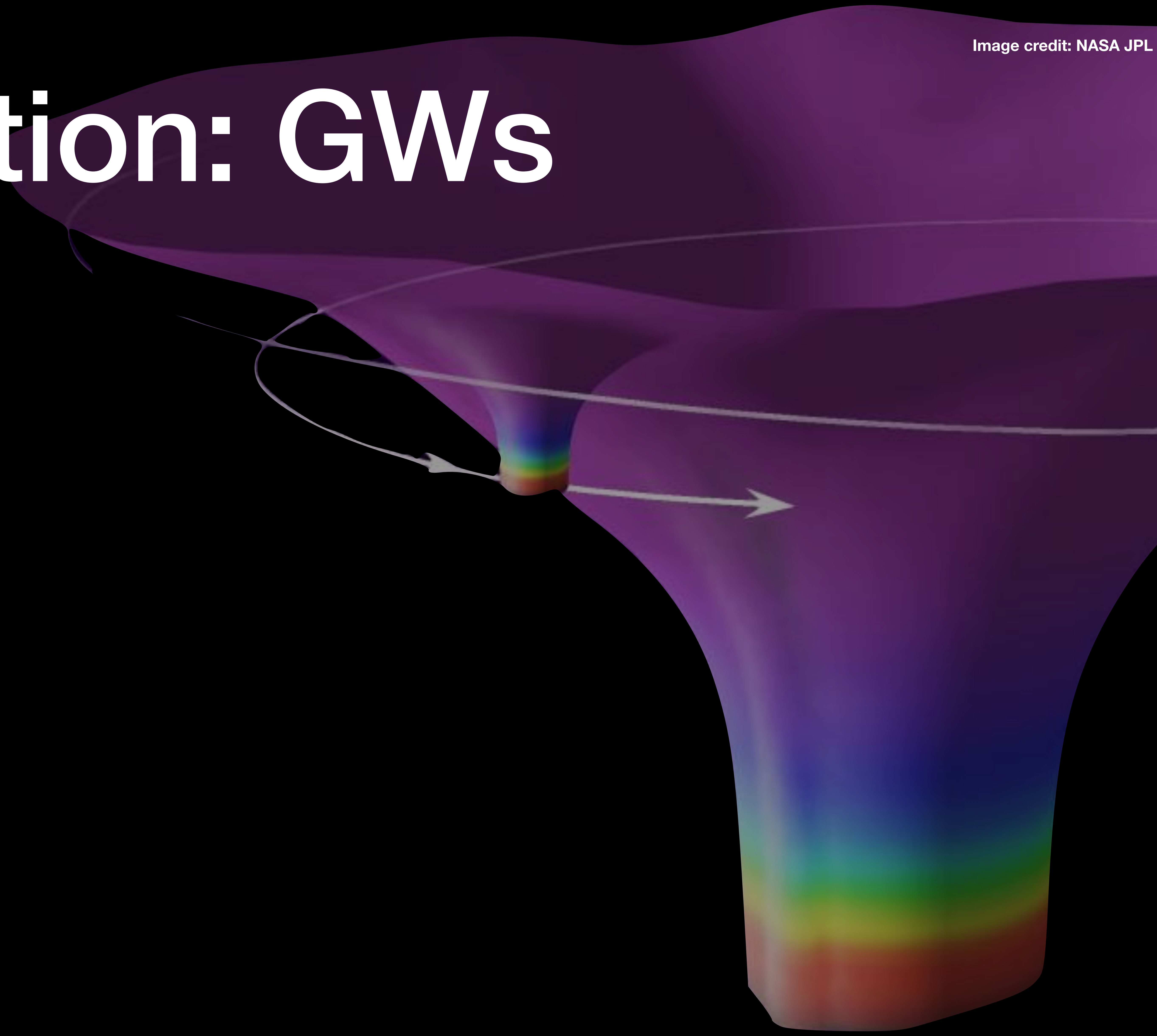
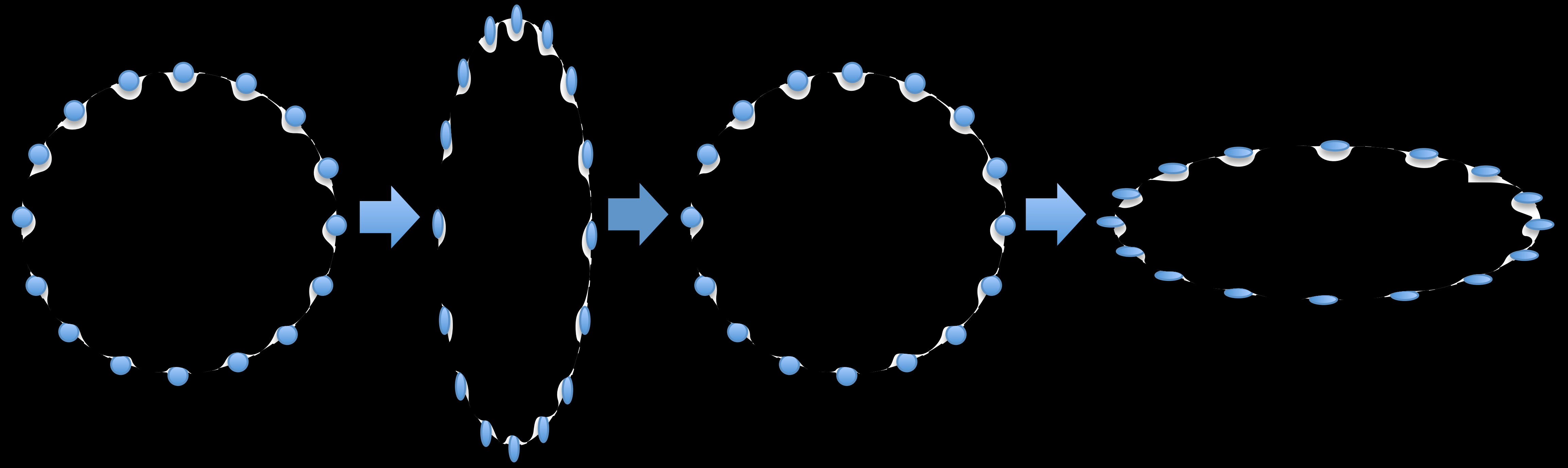


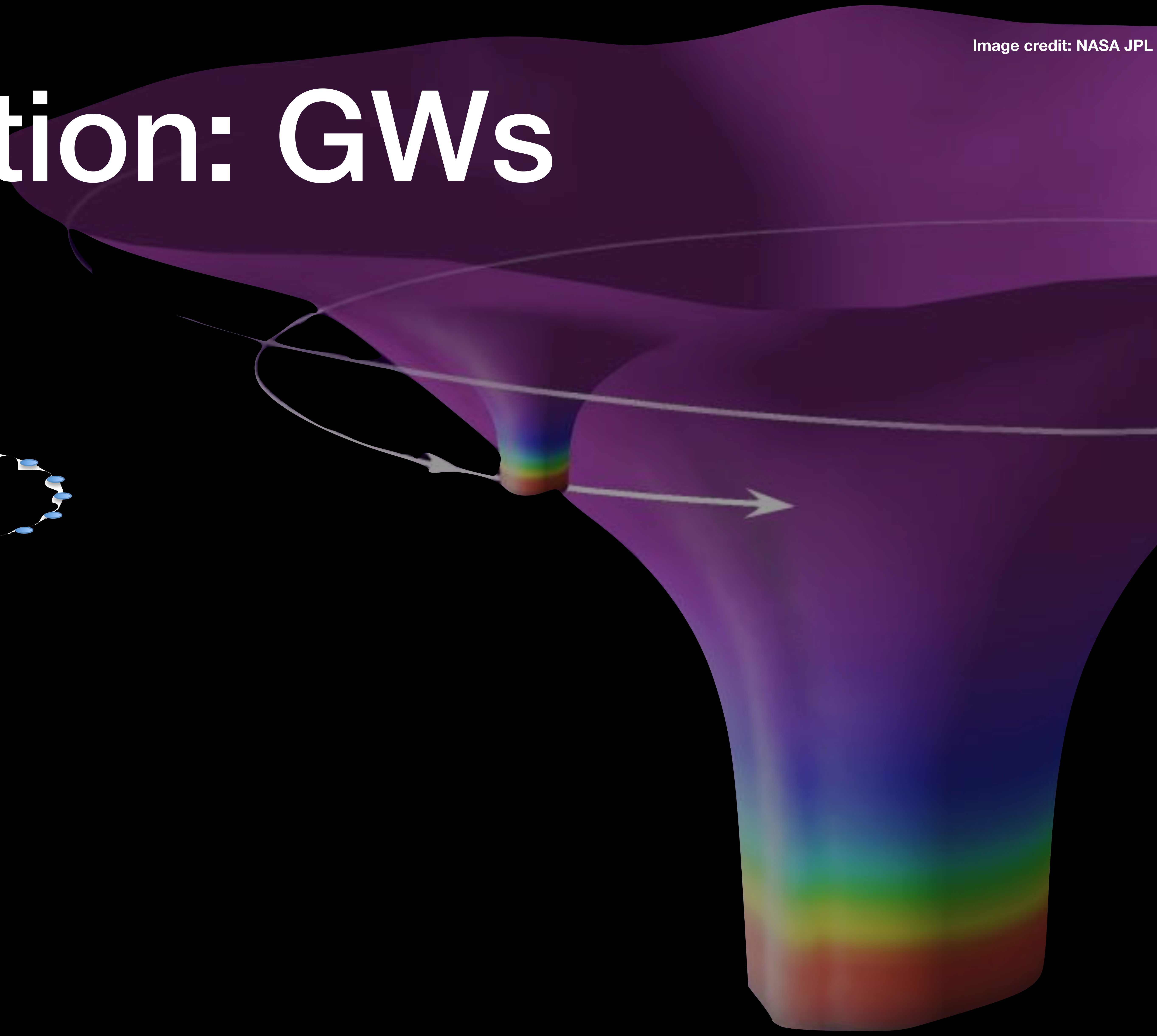
- **Gravitational waves**

Image credit: NASA JPL

Motivation: GWs

- **Gravitational waves**





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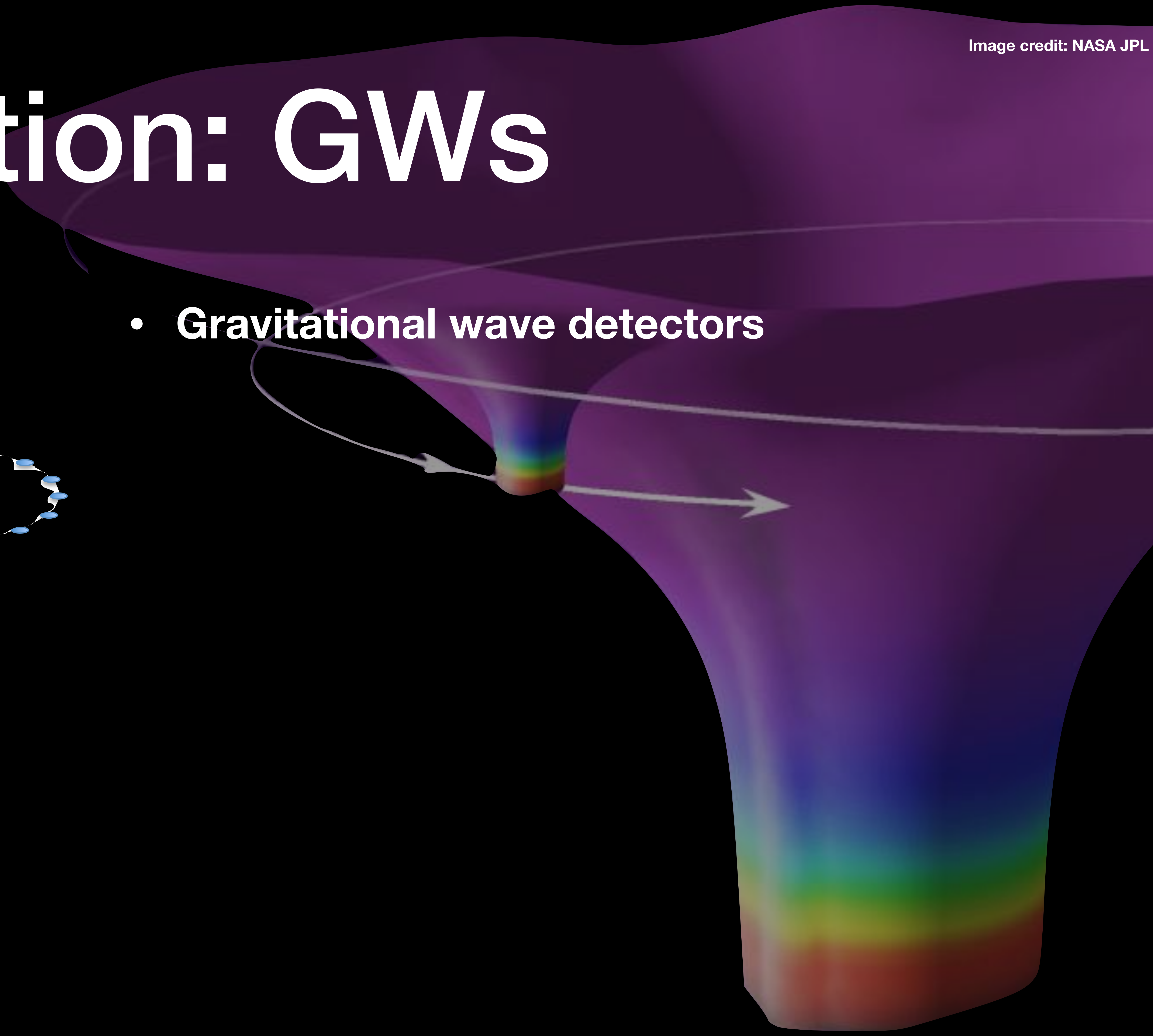

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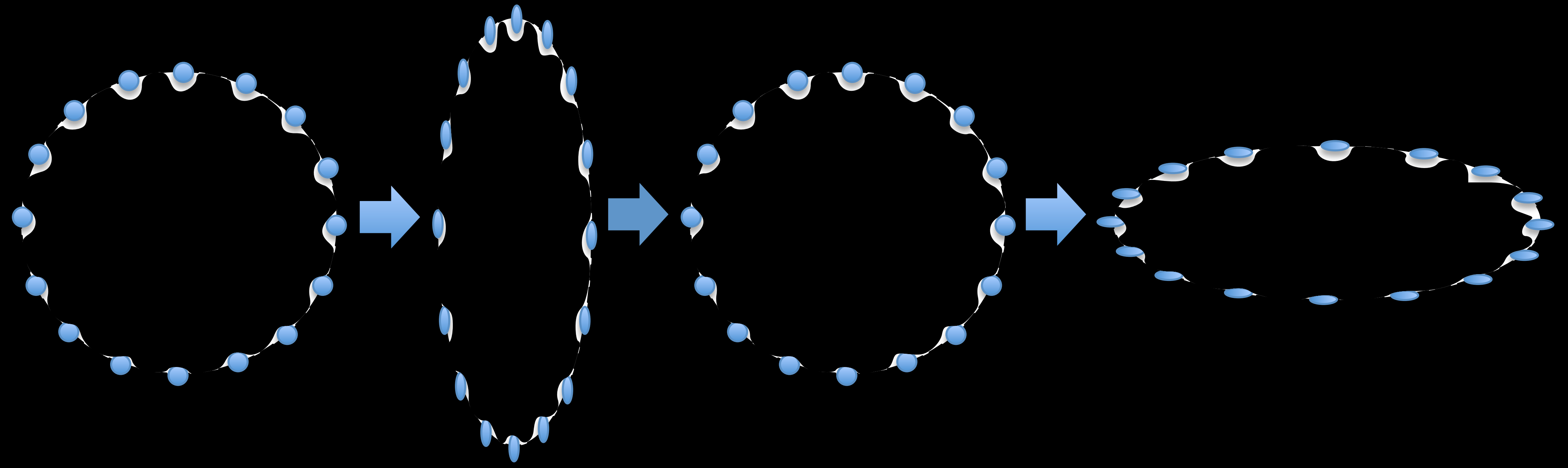

- **Gravitational wave detectors**

Motivation: GWs

Image credit: NASA JPL

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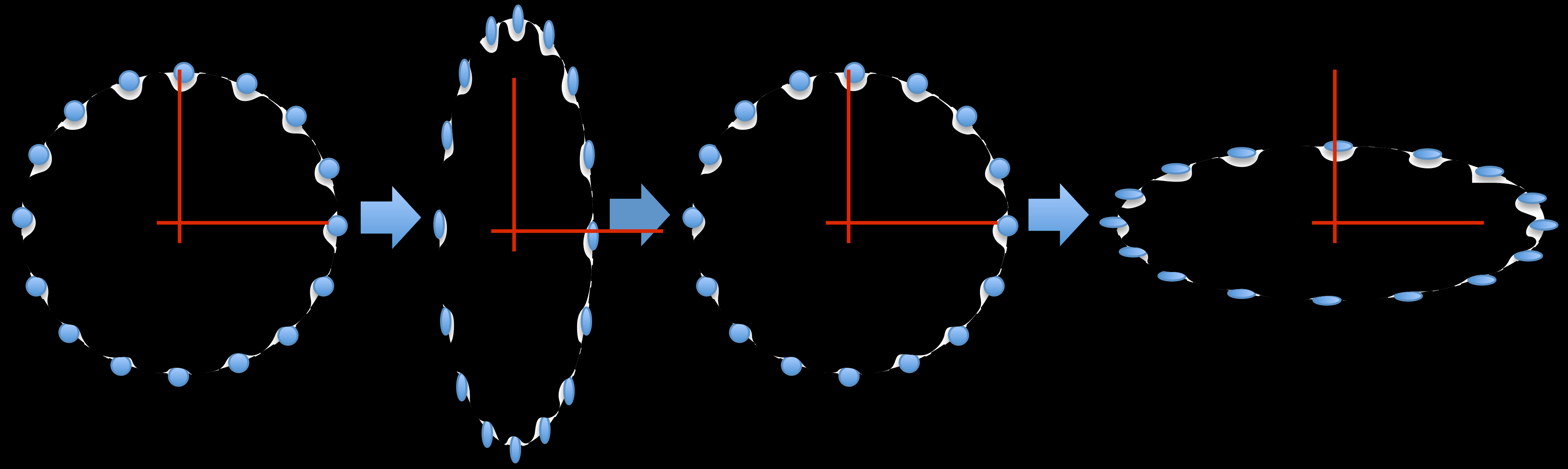

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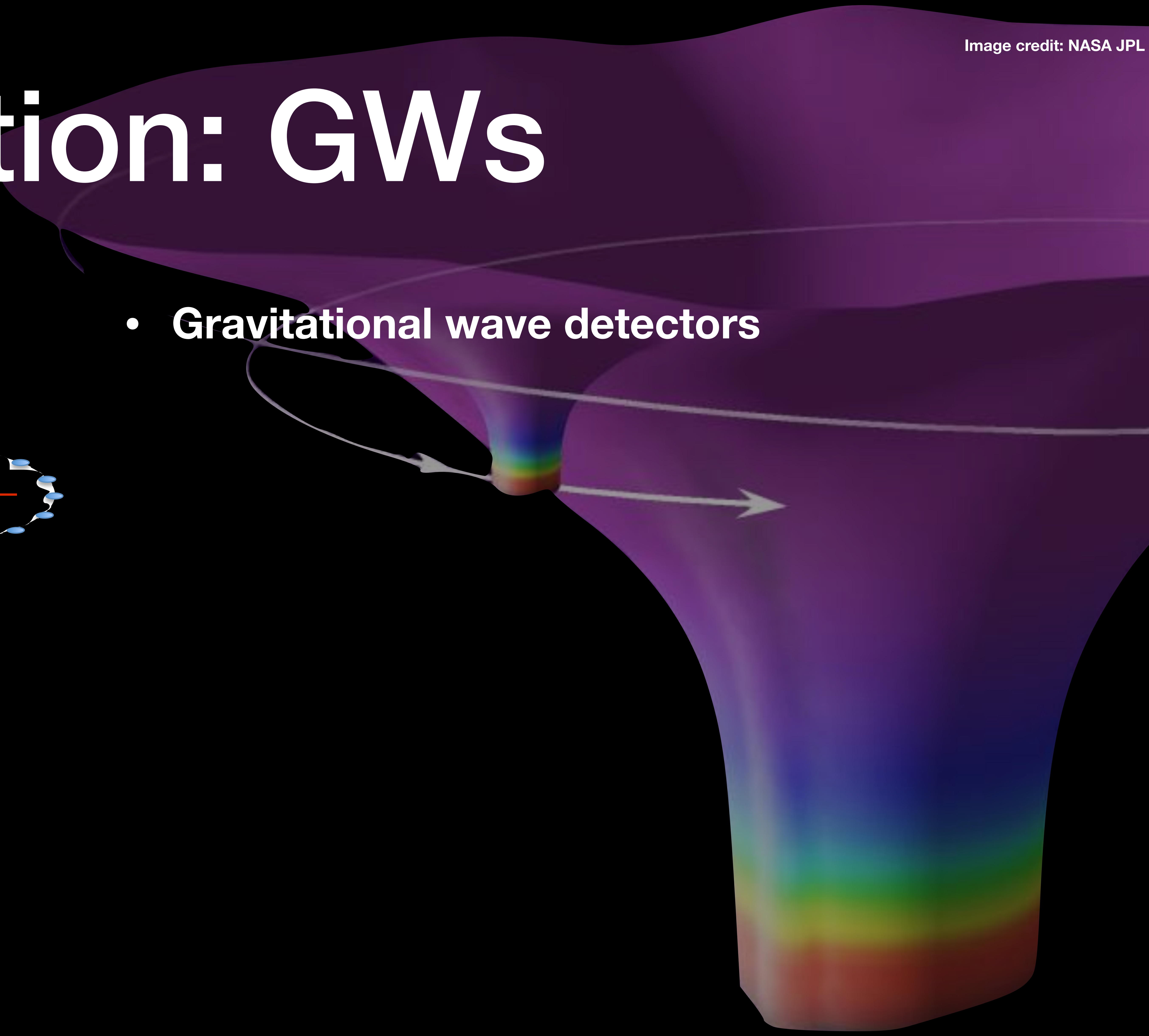

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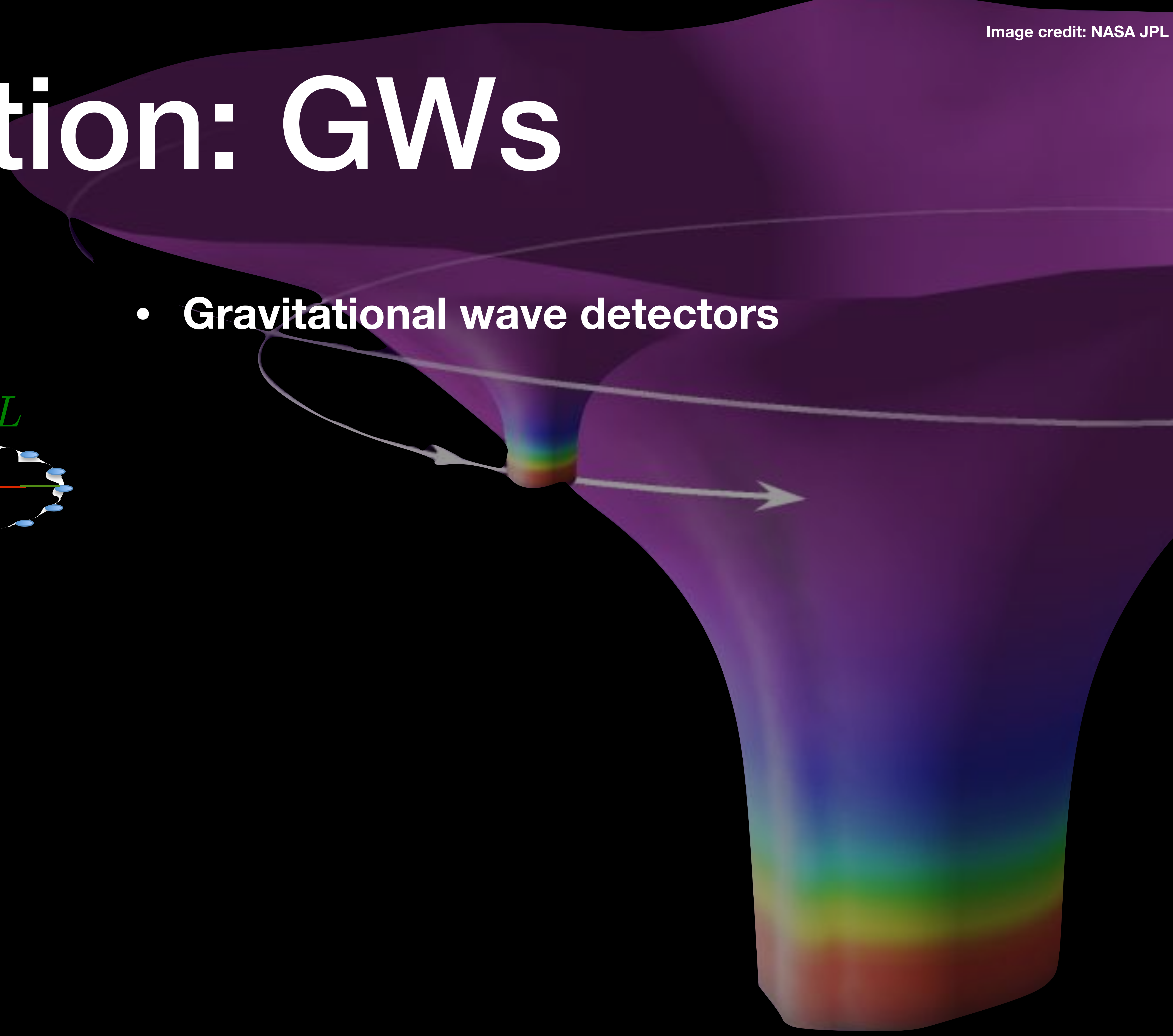

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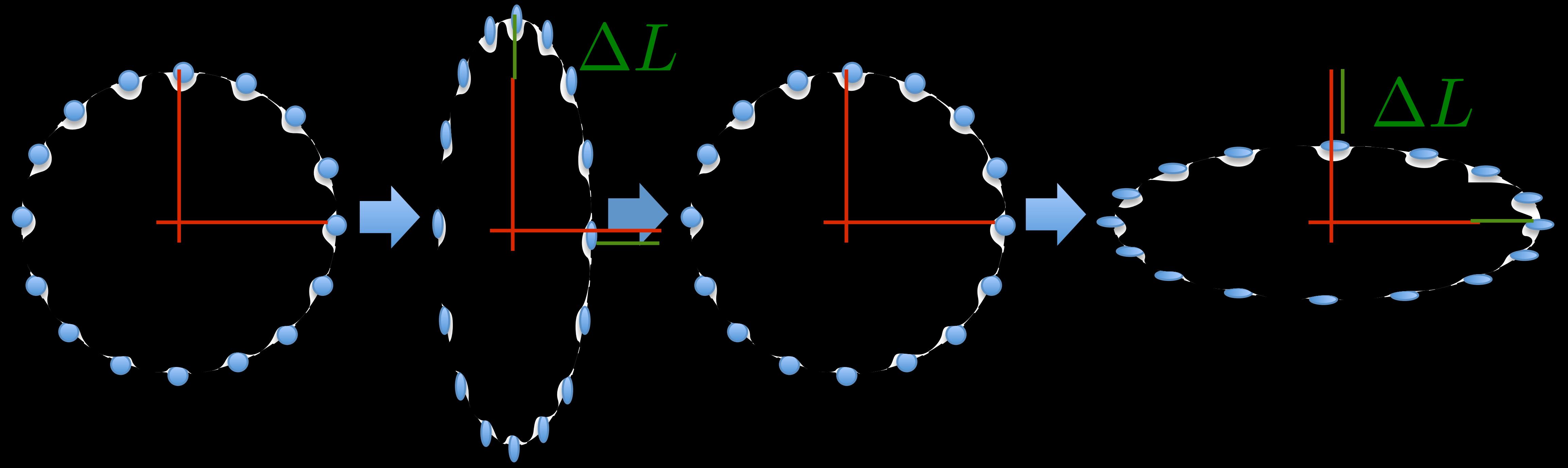

- **Gravitational wave detectors**

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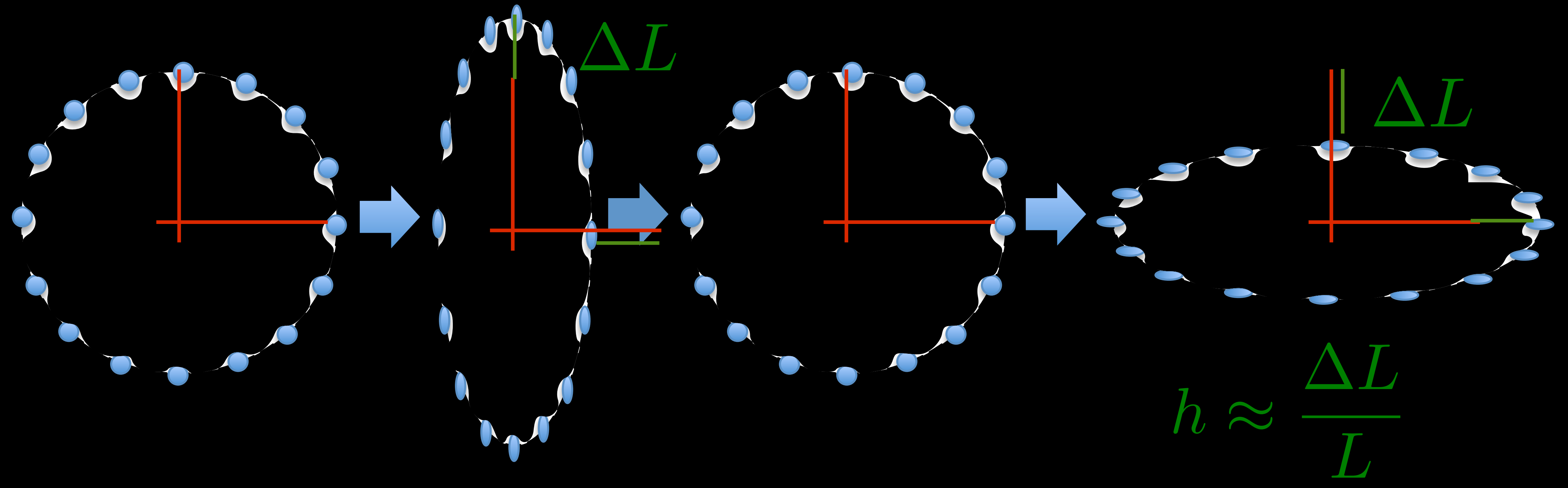

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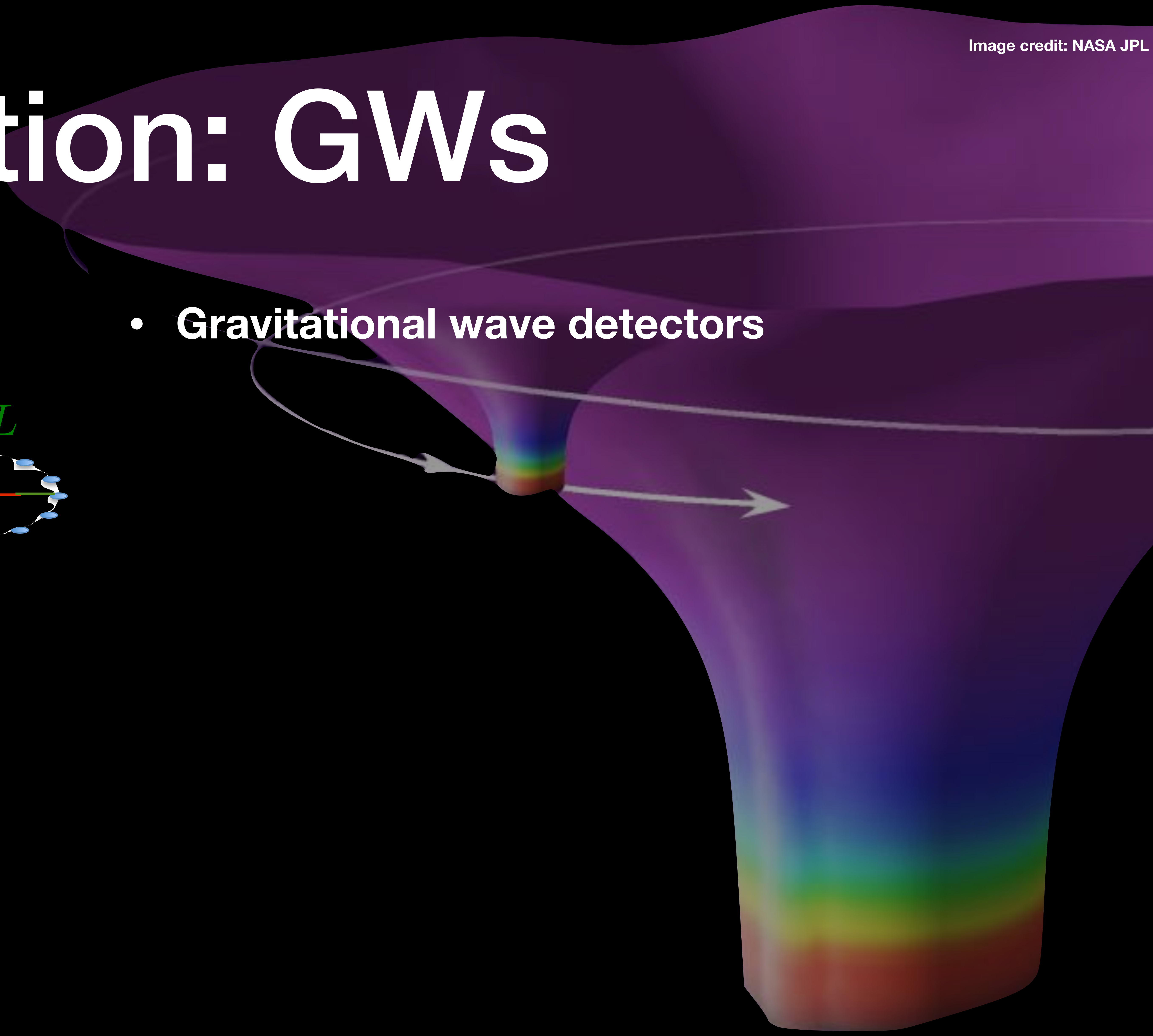

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Image credit: NASA JPL

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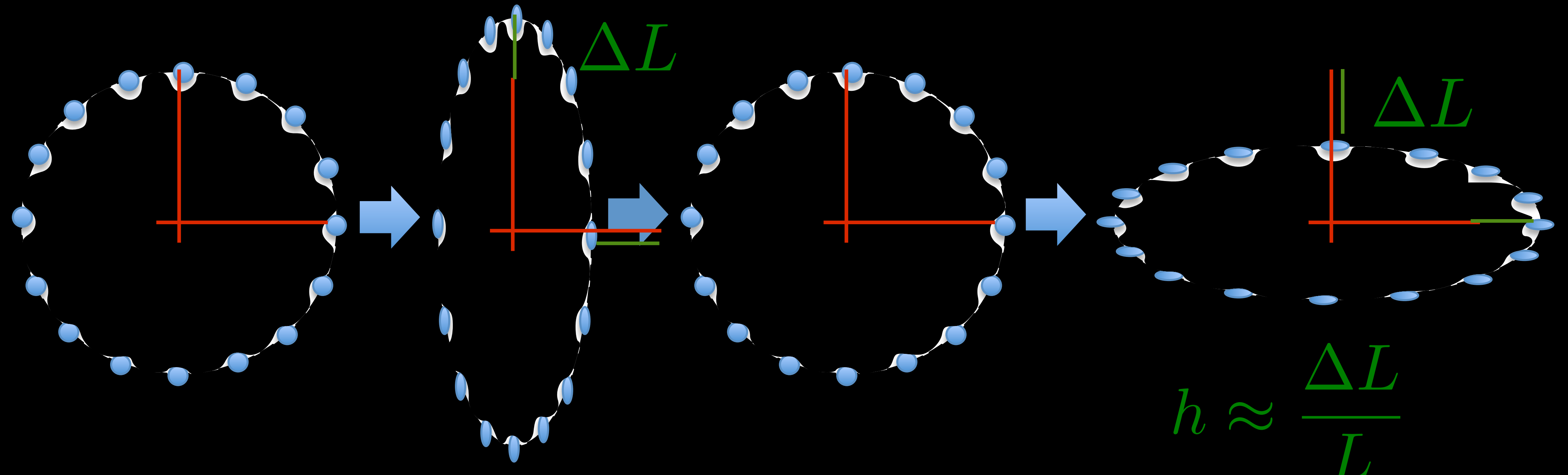

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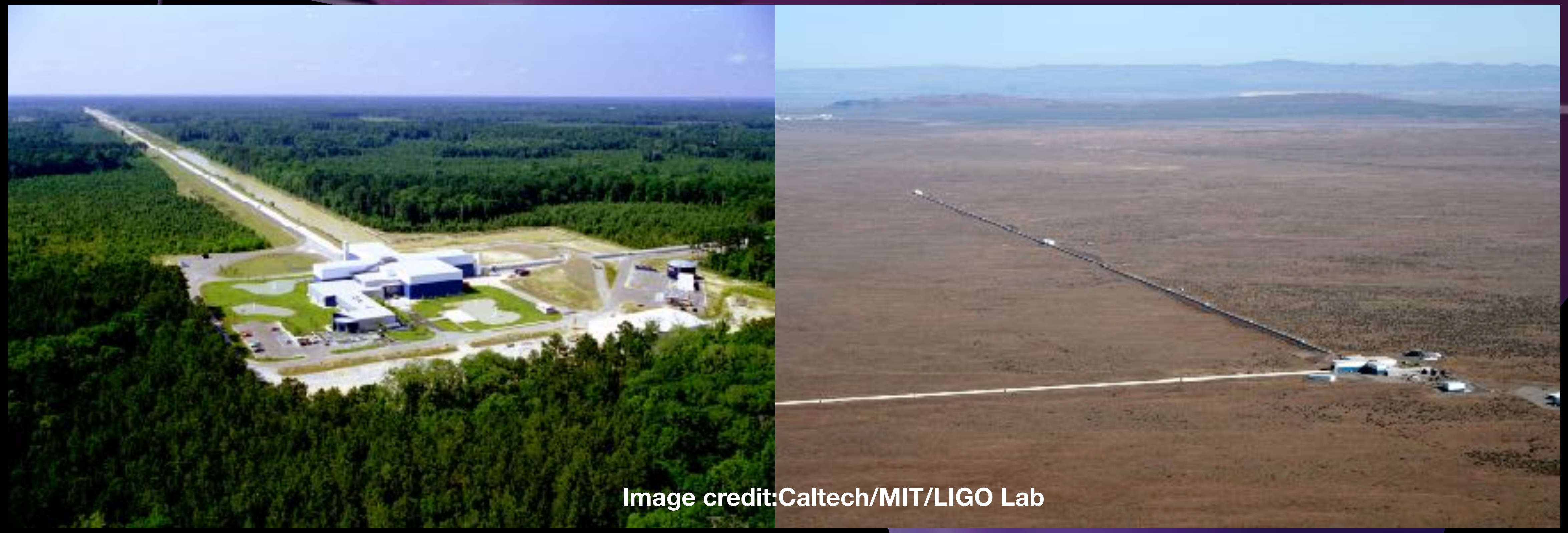

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Image credit: NASA JPL

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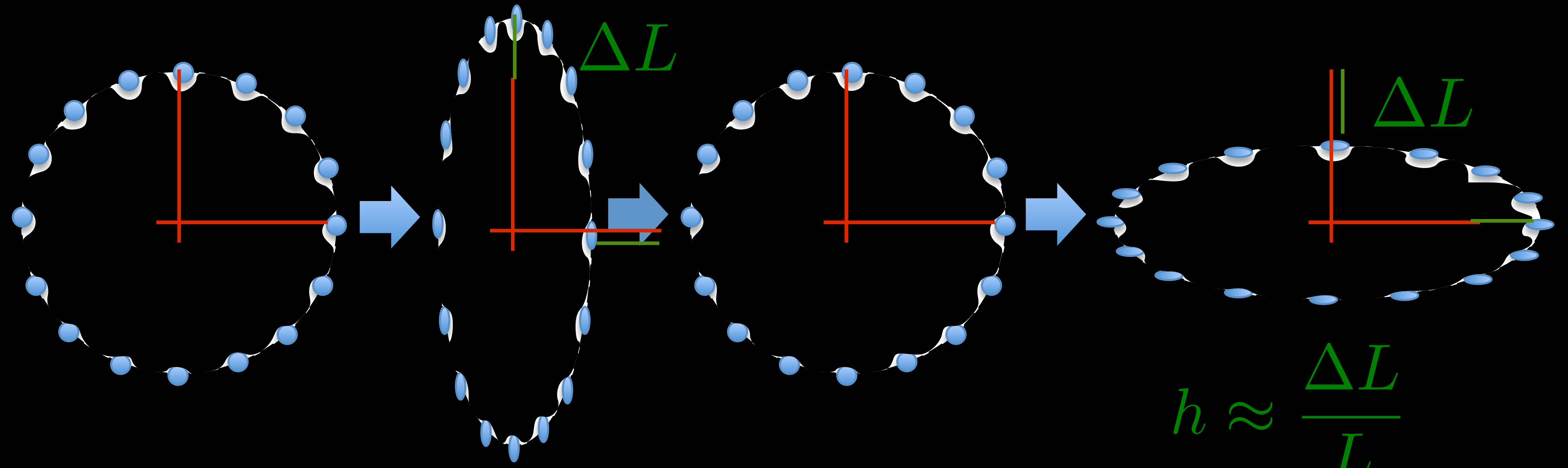

$$h \approx \frac{\Delta L}{L}$$

- **Gravitational wave detectors**

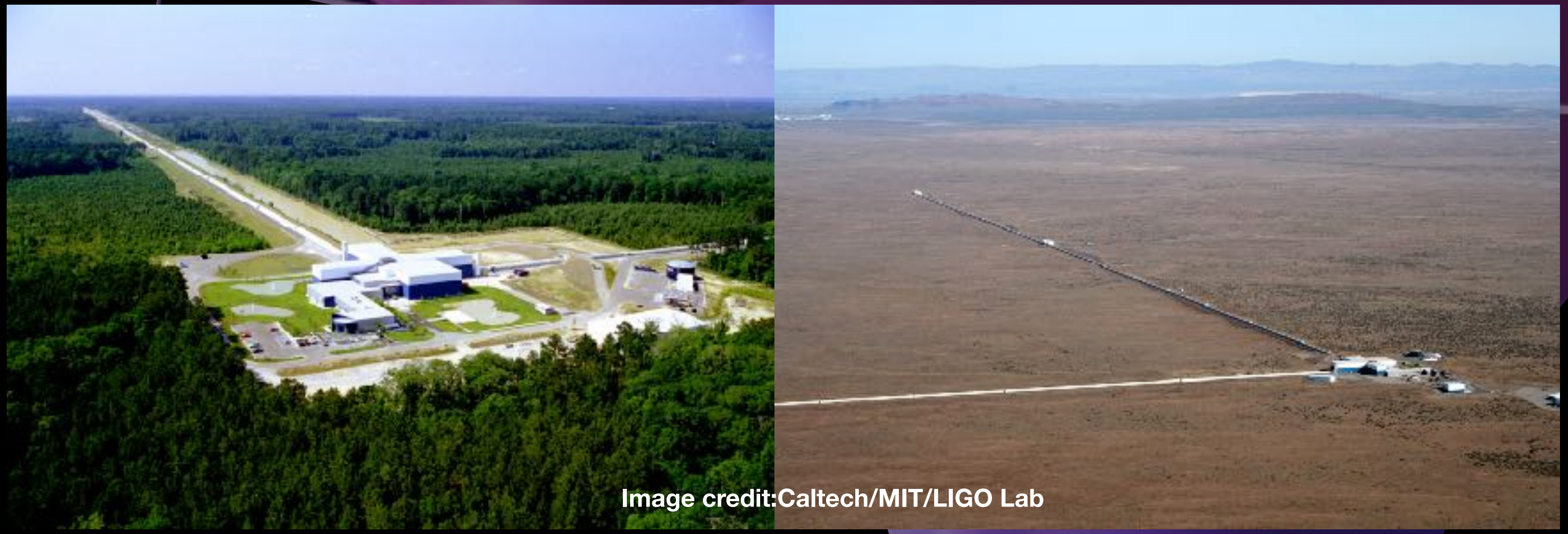

Image credit:Caltech/MIT/LIGO Lab

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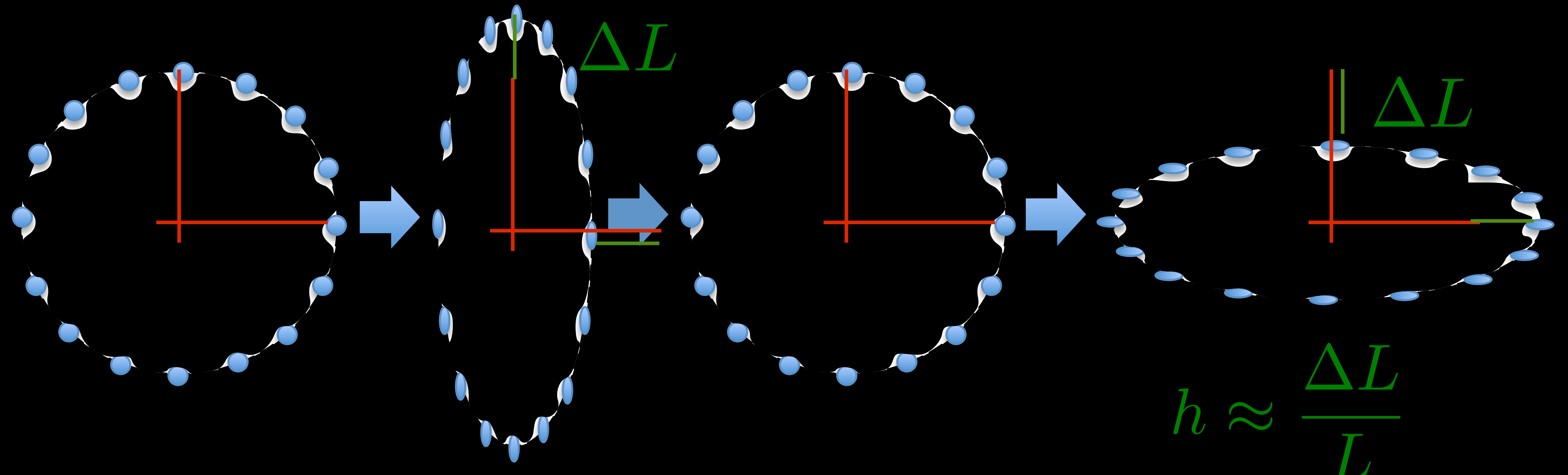

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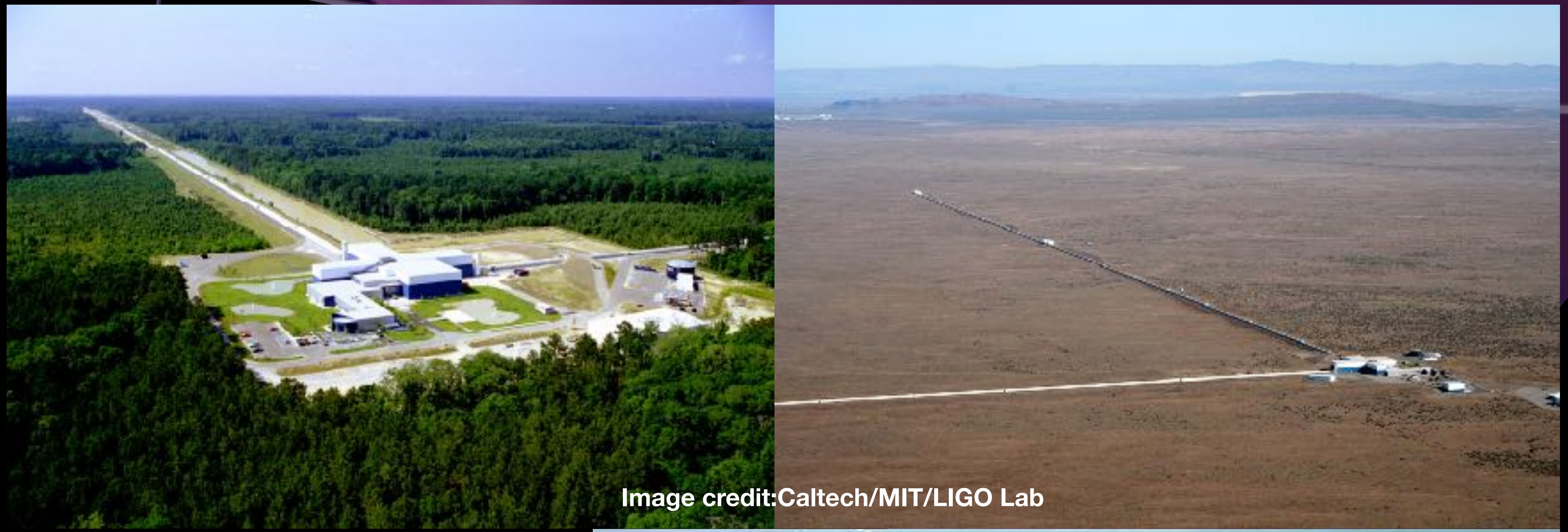
Image credit:Caltech/MIT/LIGO Lab


Image credit:The Virgo collaboration/CCO 1.0

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Image credit: NASA JPL

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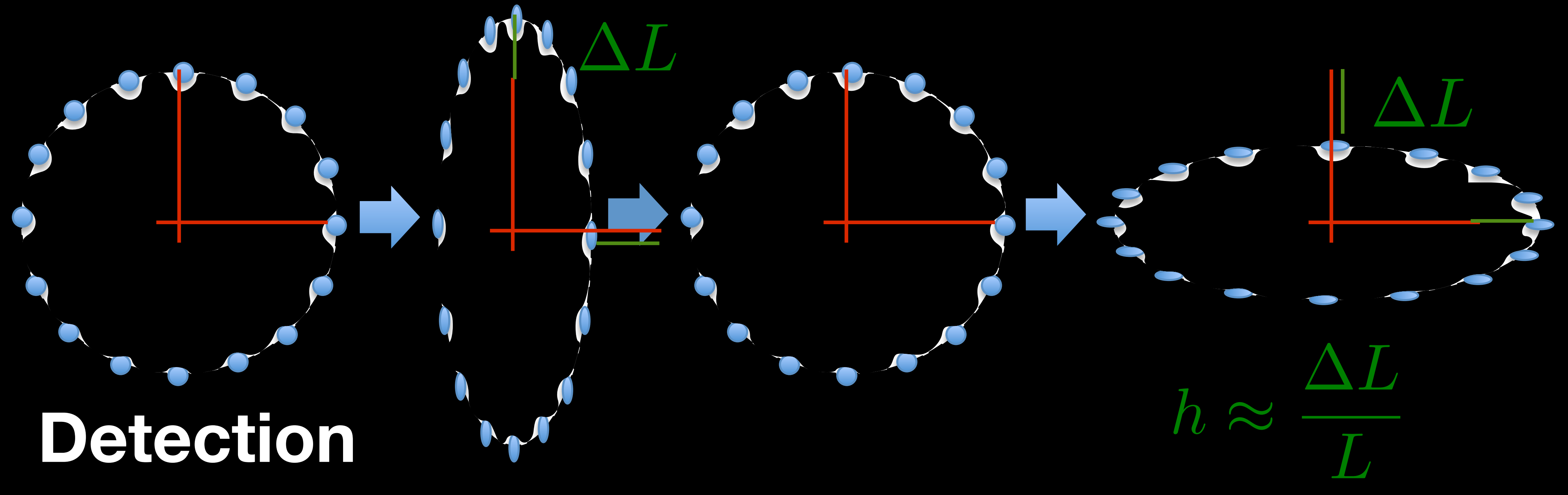
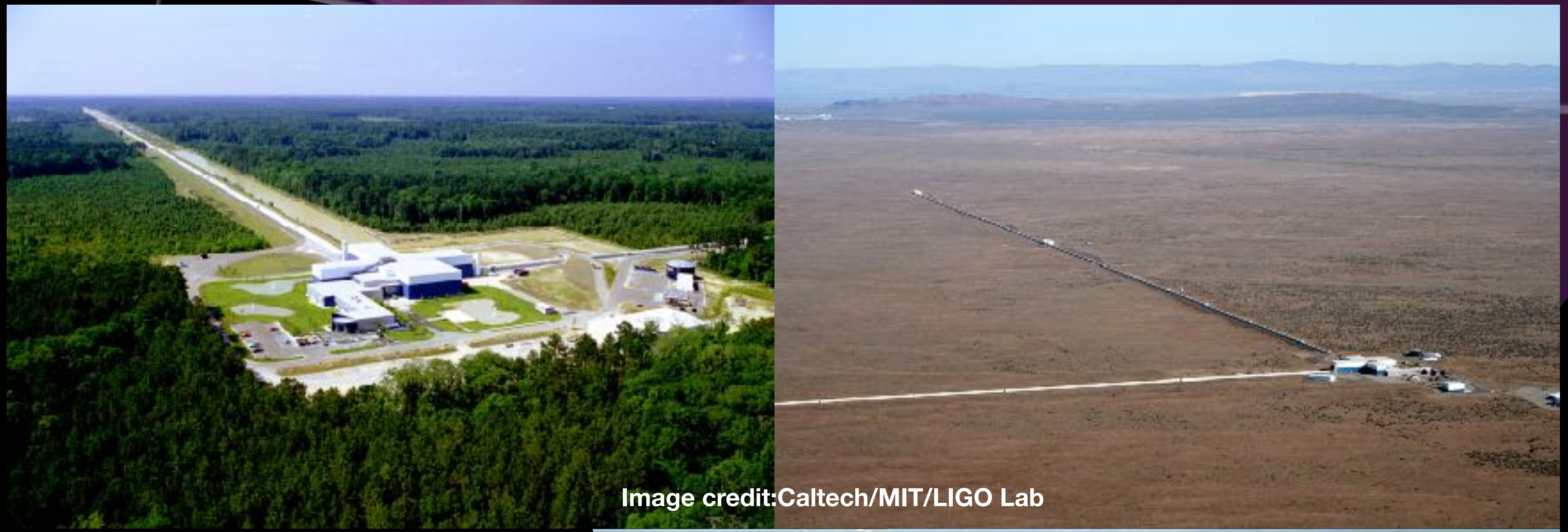


- **Detection**

- **Gravitational wave detectors**



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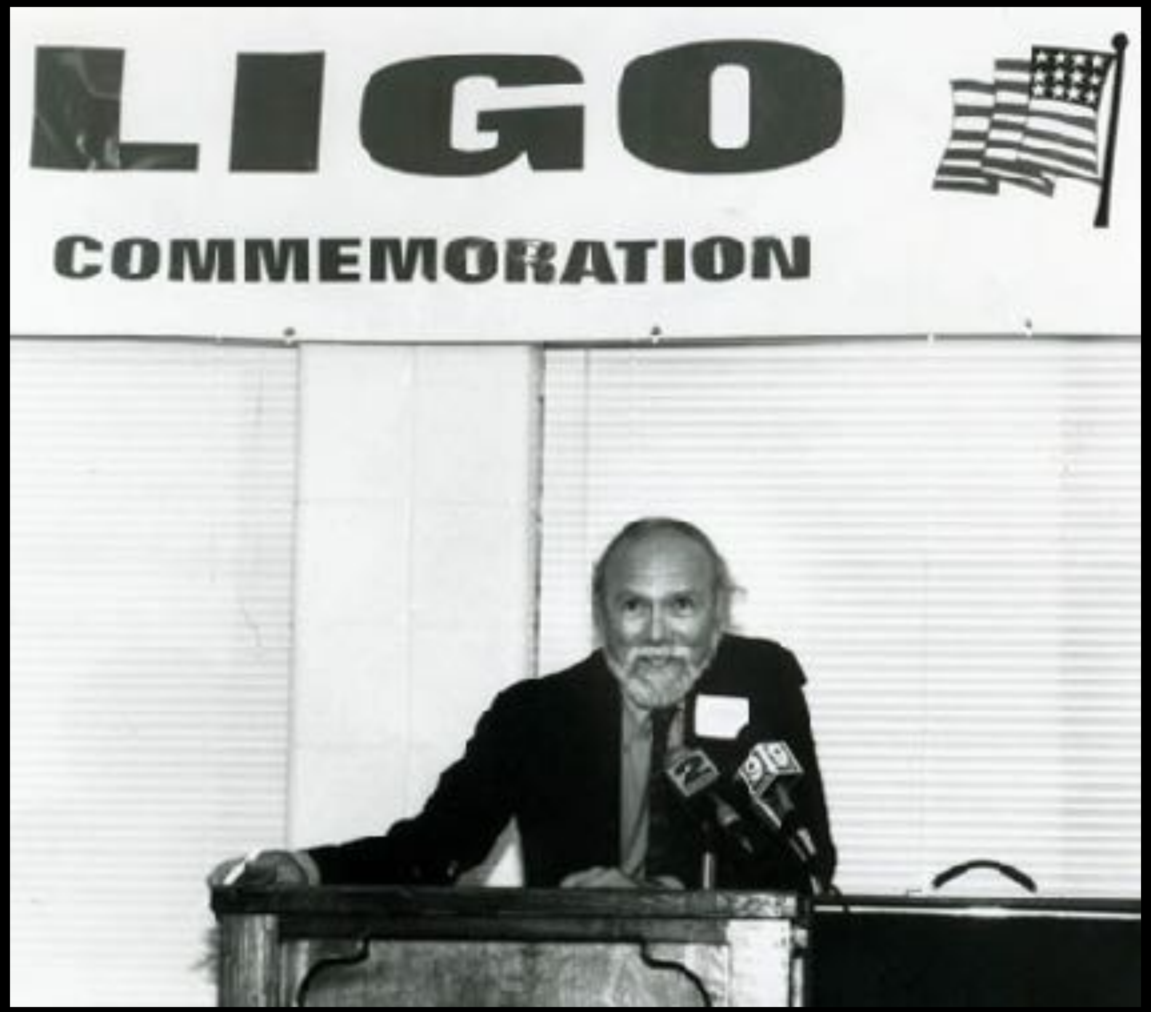

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Image credit: NASA JPL

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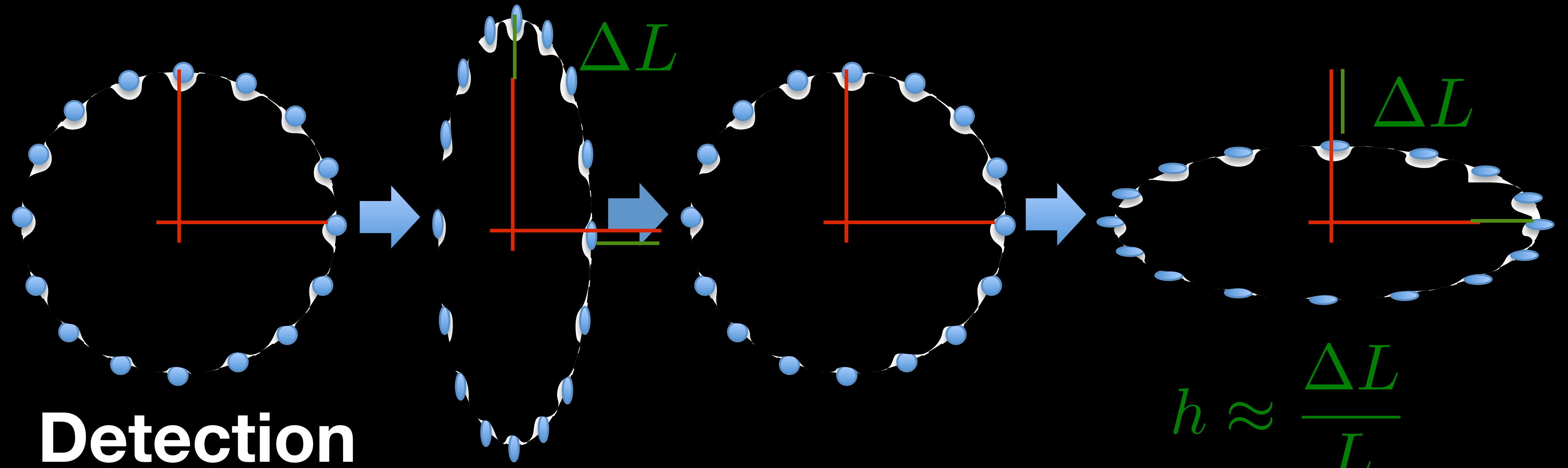

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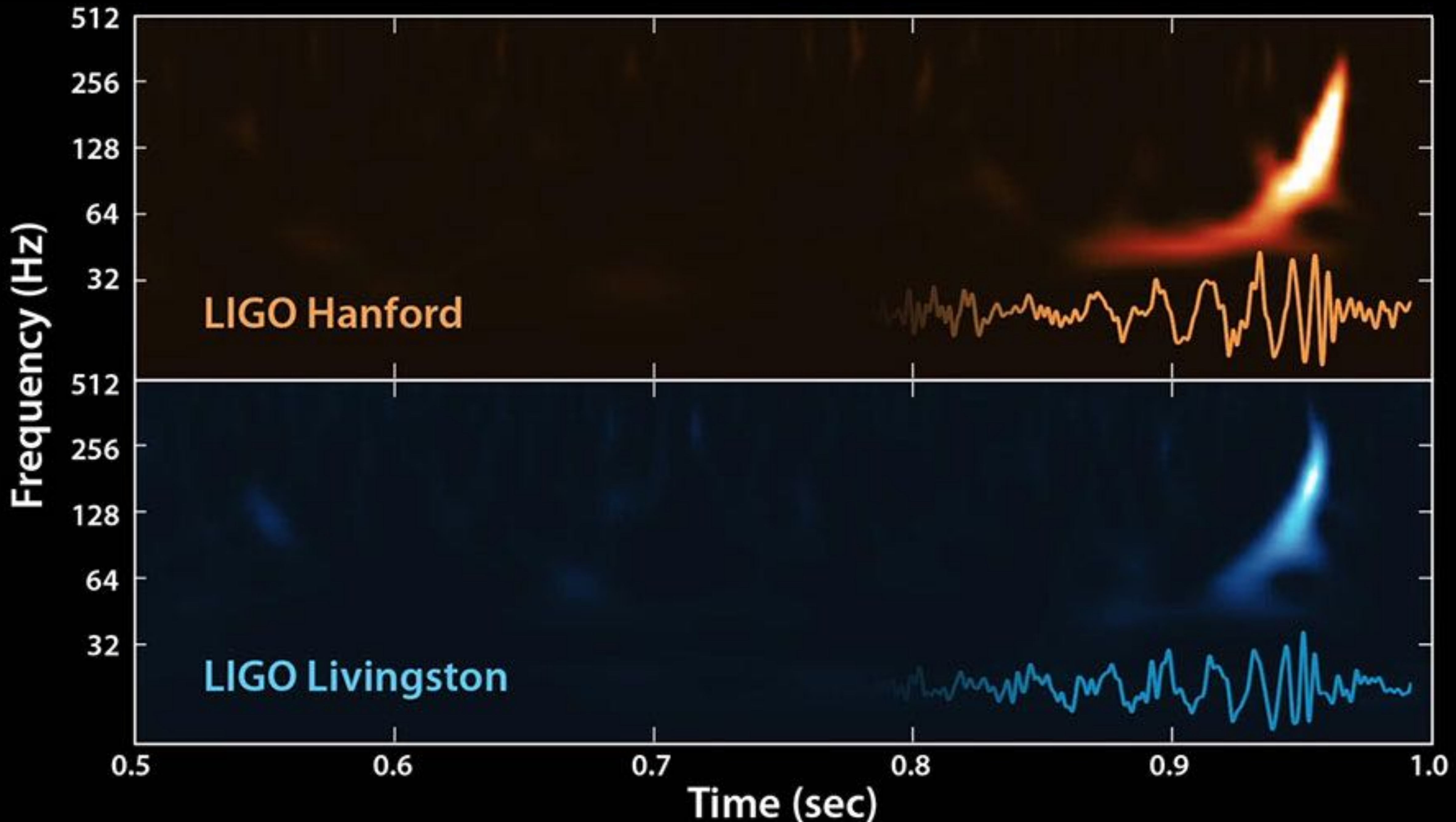

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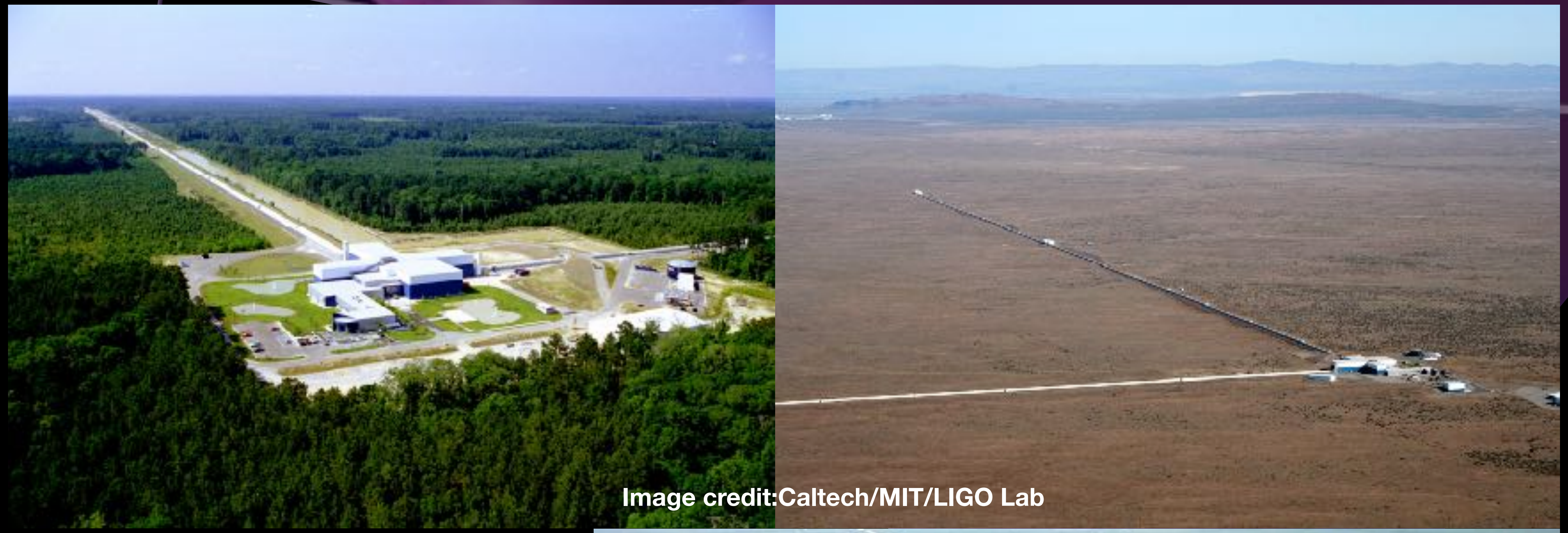

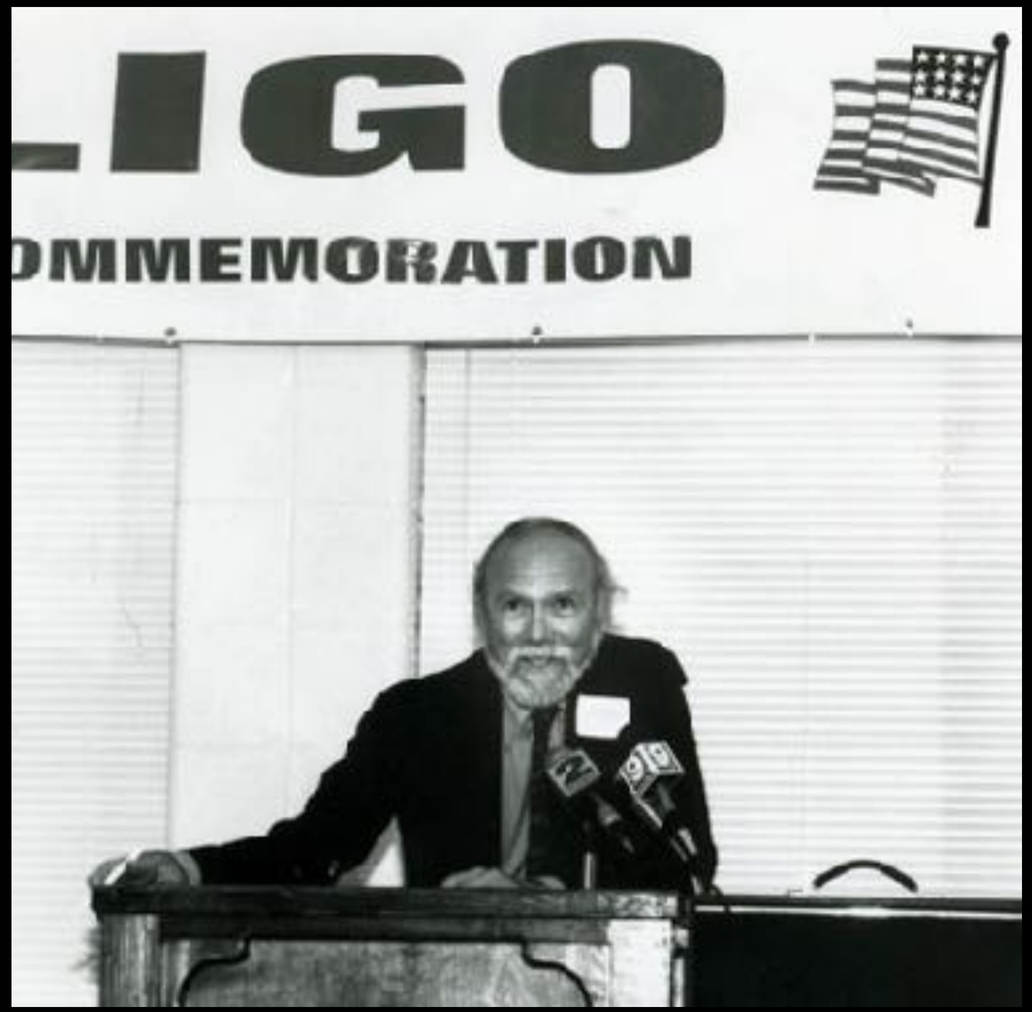

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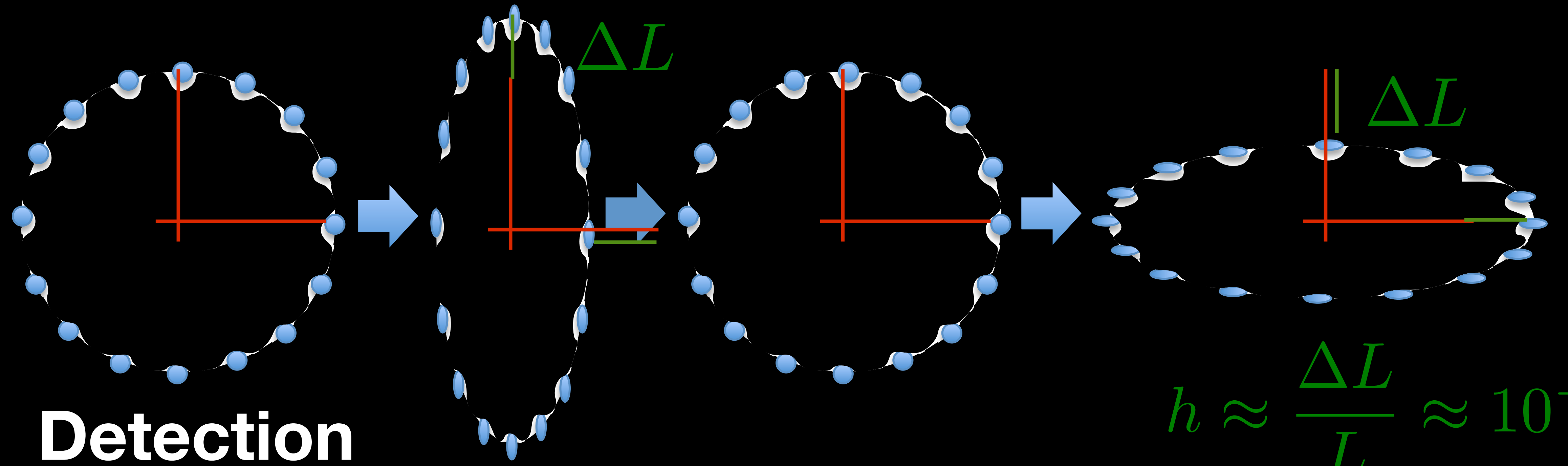

$$h \approx \frac{\Delta L}{L} \approx 10^{-21}$$

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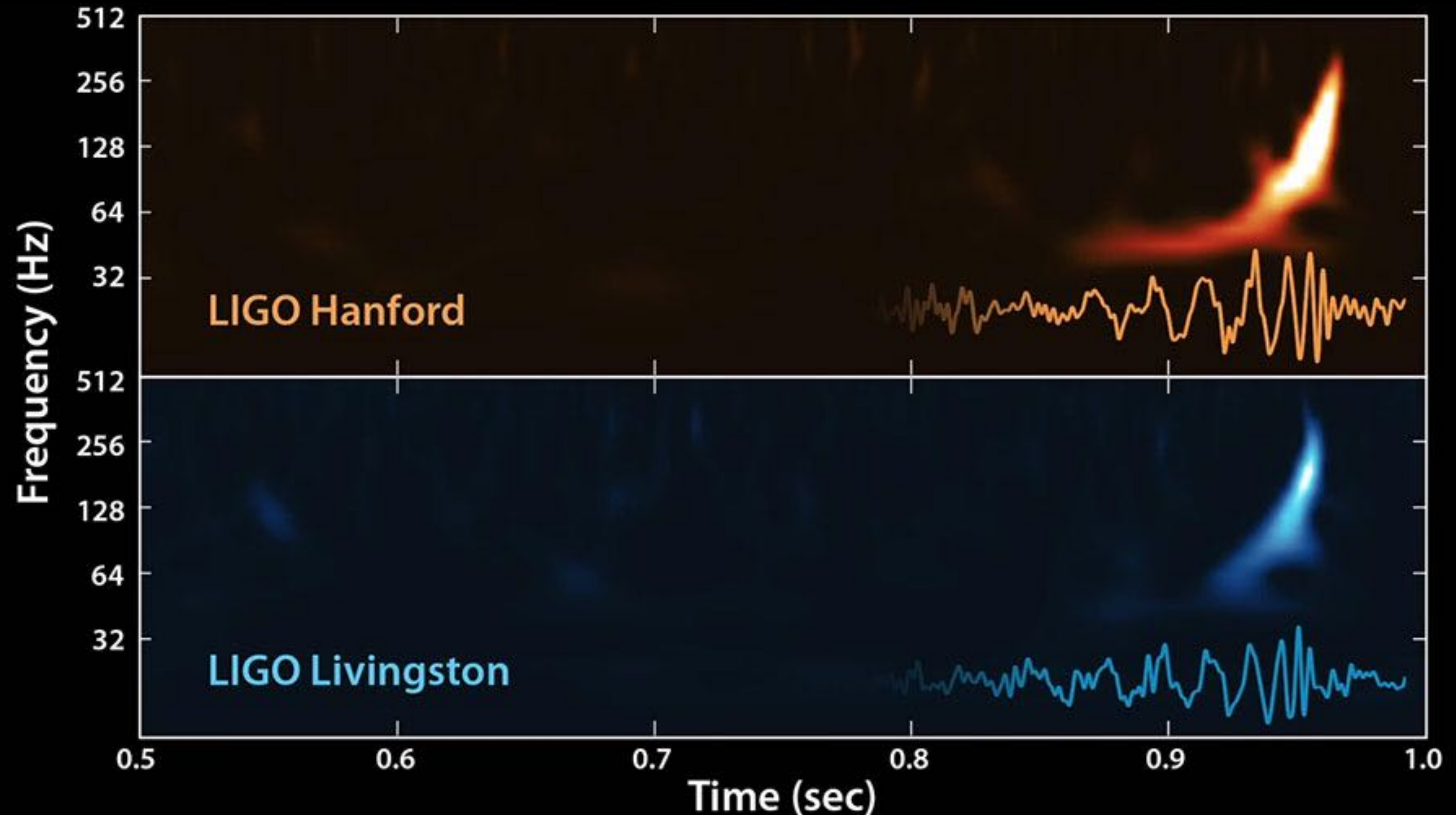

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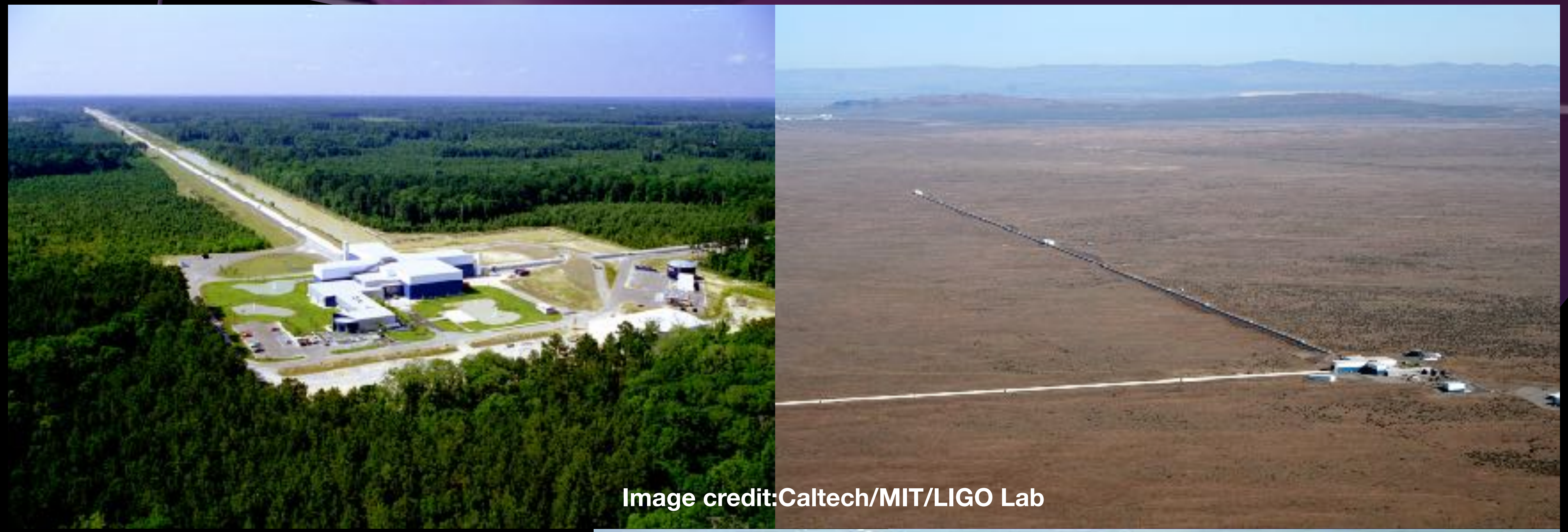
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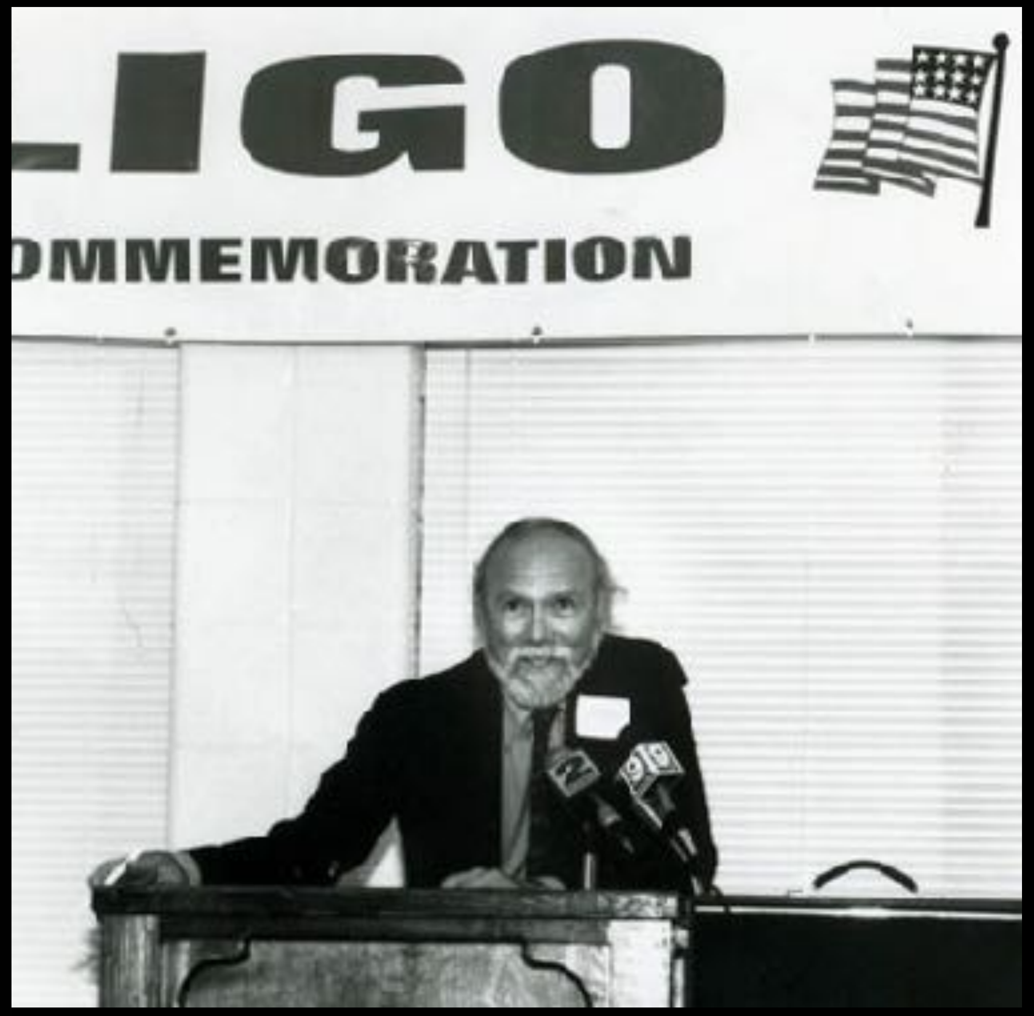
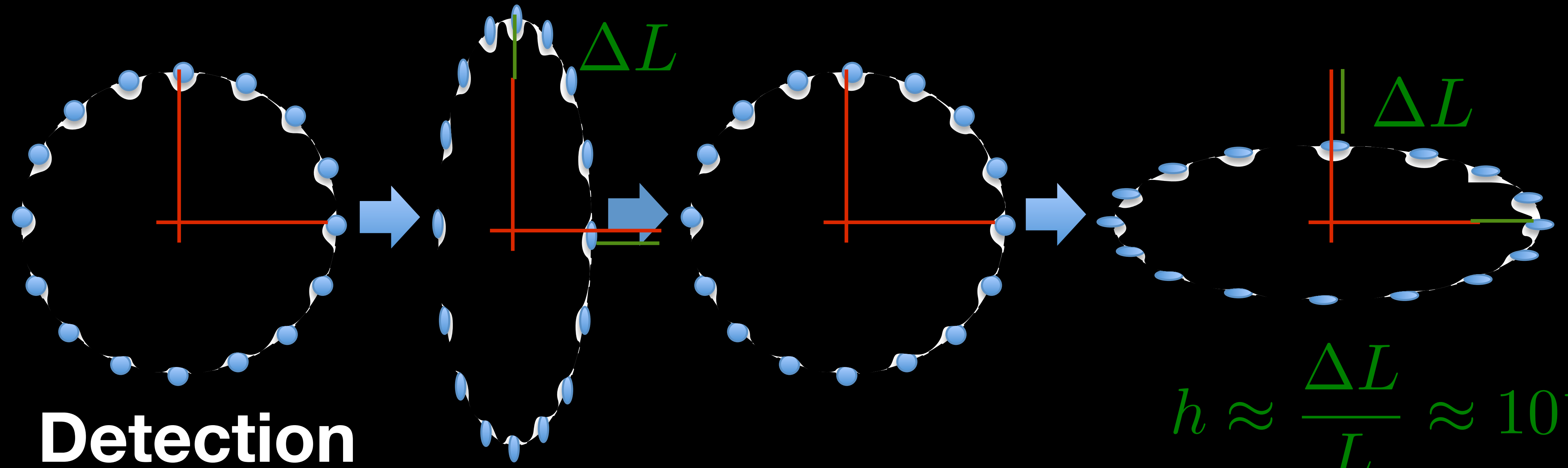



Image credit:The Virgo collaboration/CCO 1.0

Image credit: NASA JPL

Motivation: GWs

- **Gravitational waves** $\Delta L = 4 \times 10^{-18} m$



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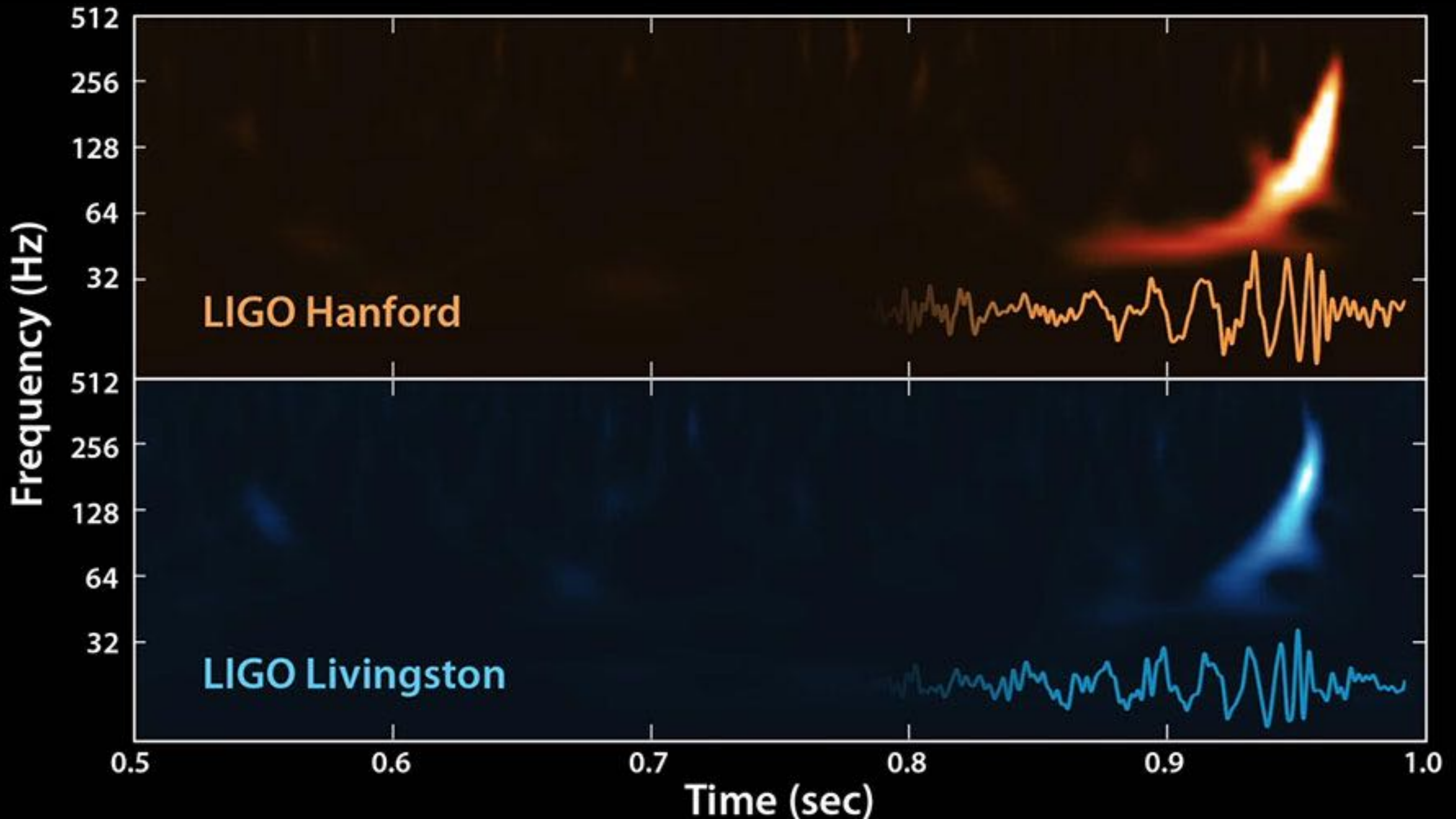

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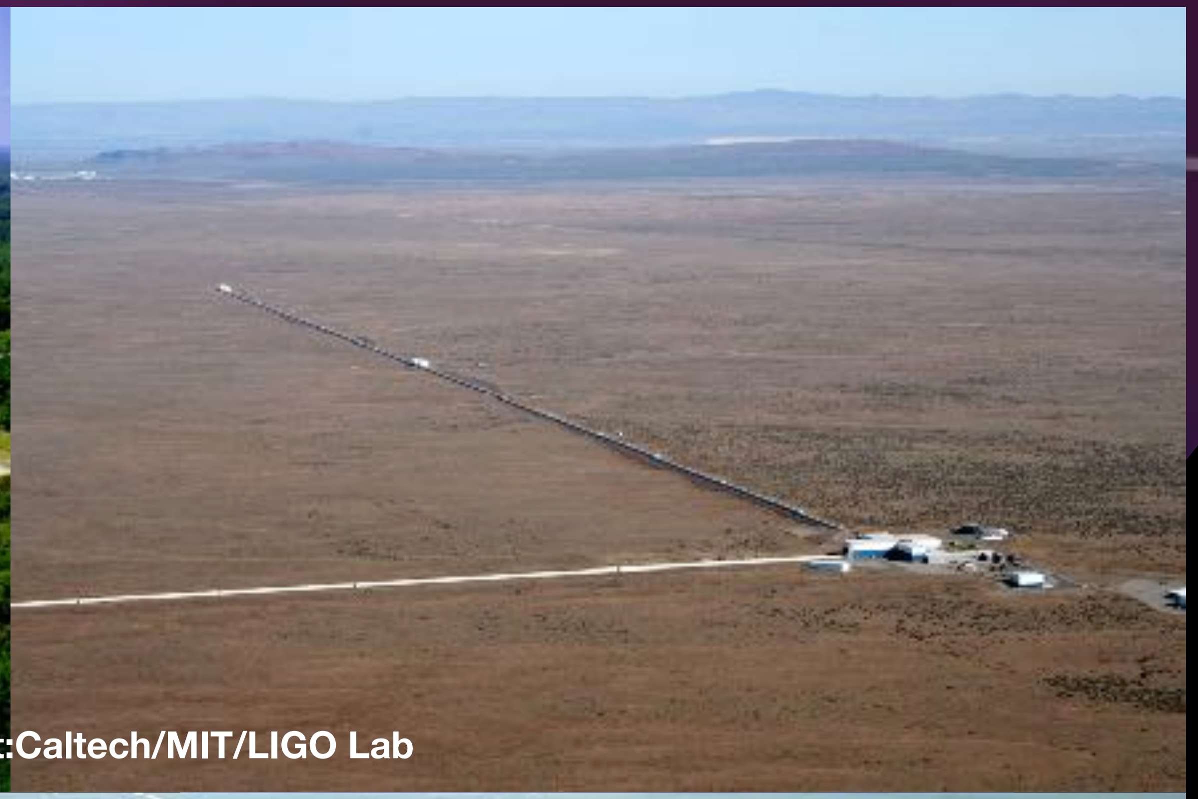

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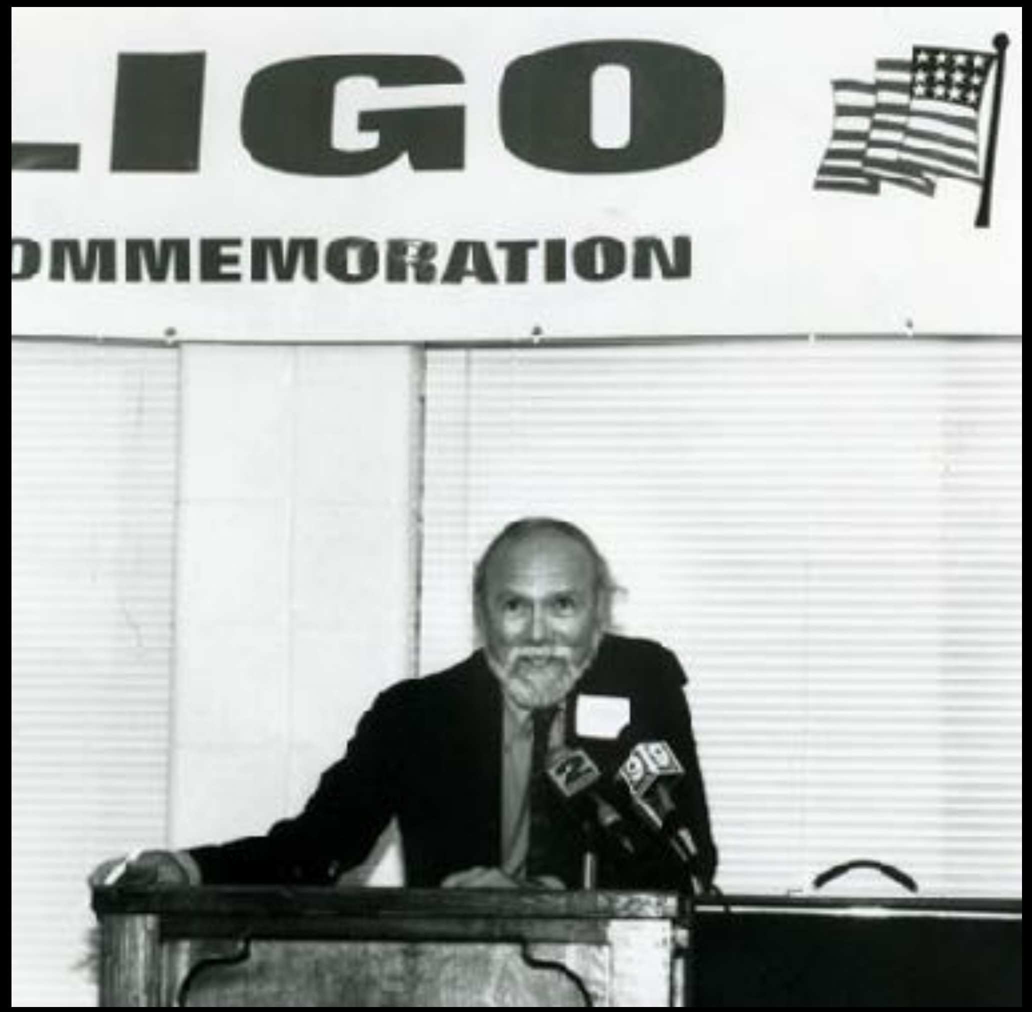

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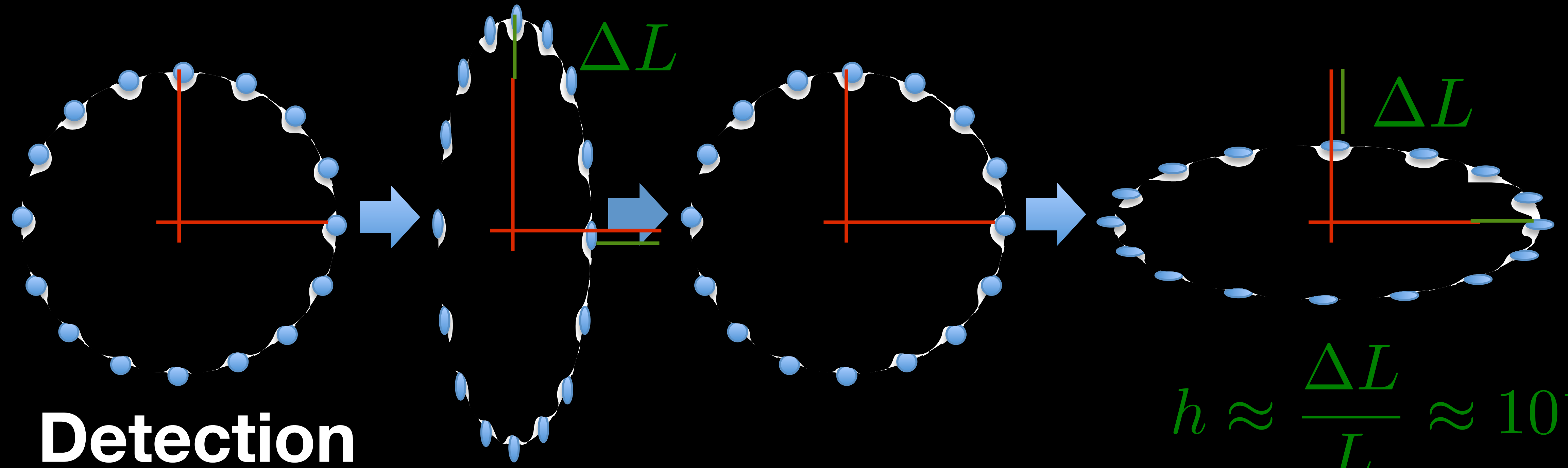

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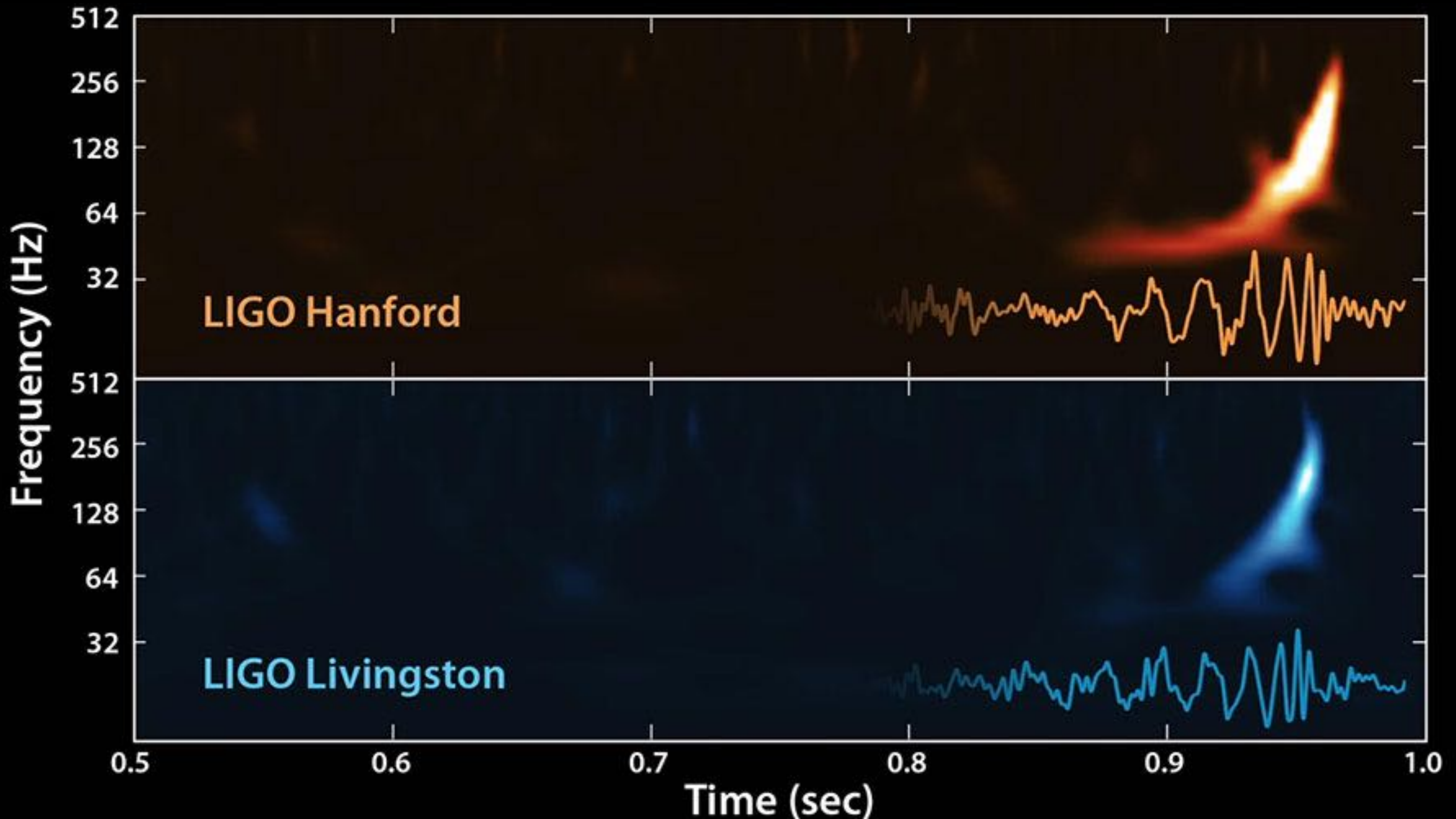

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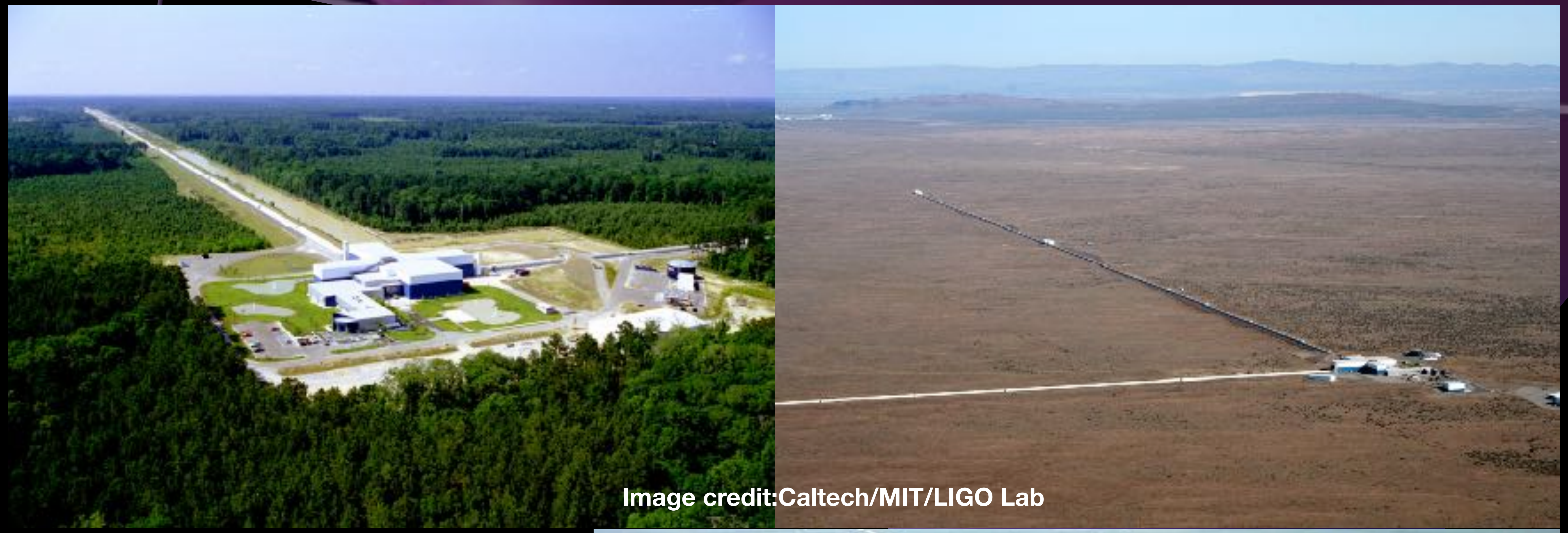

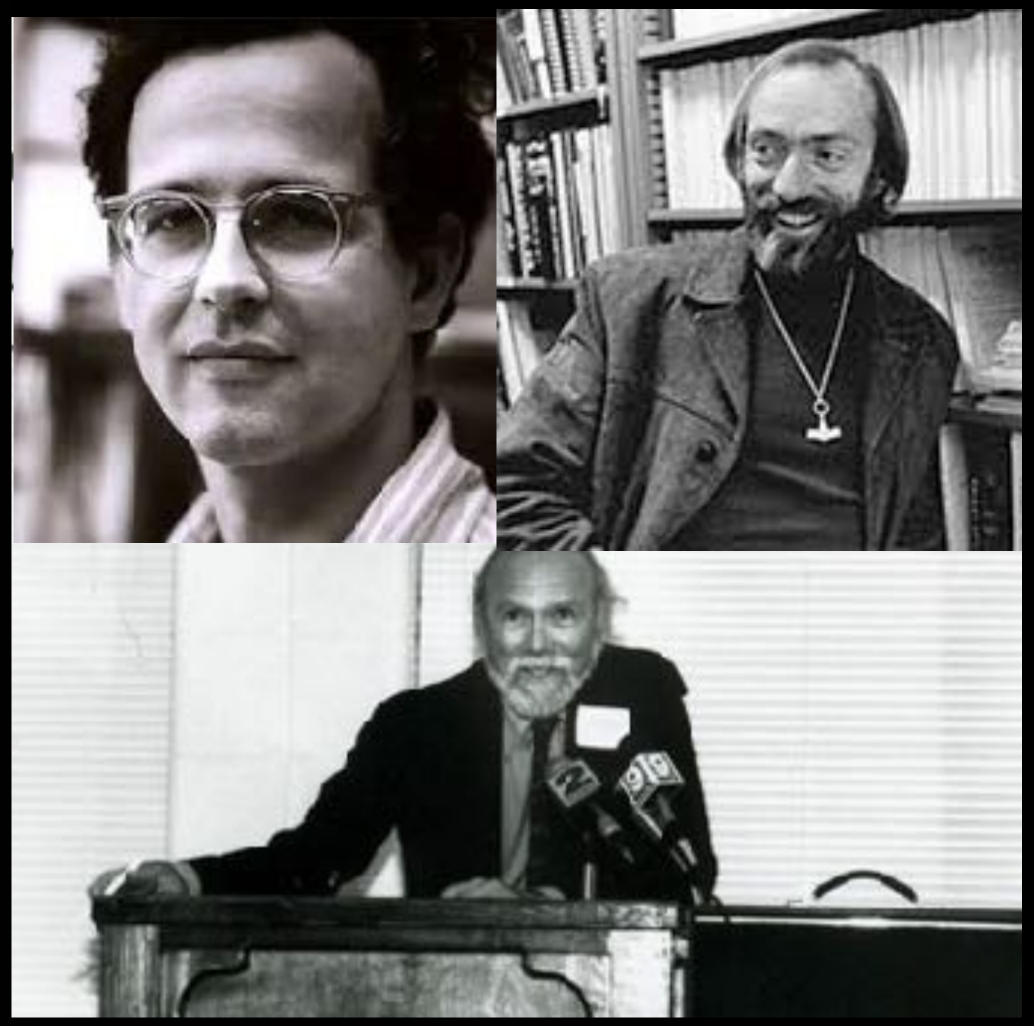

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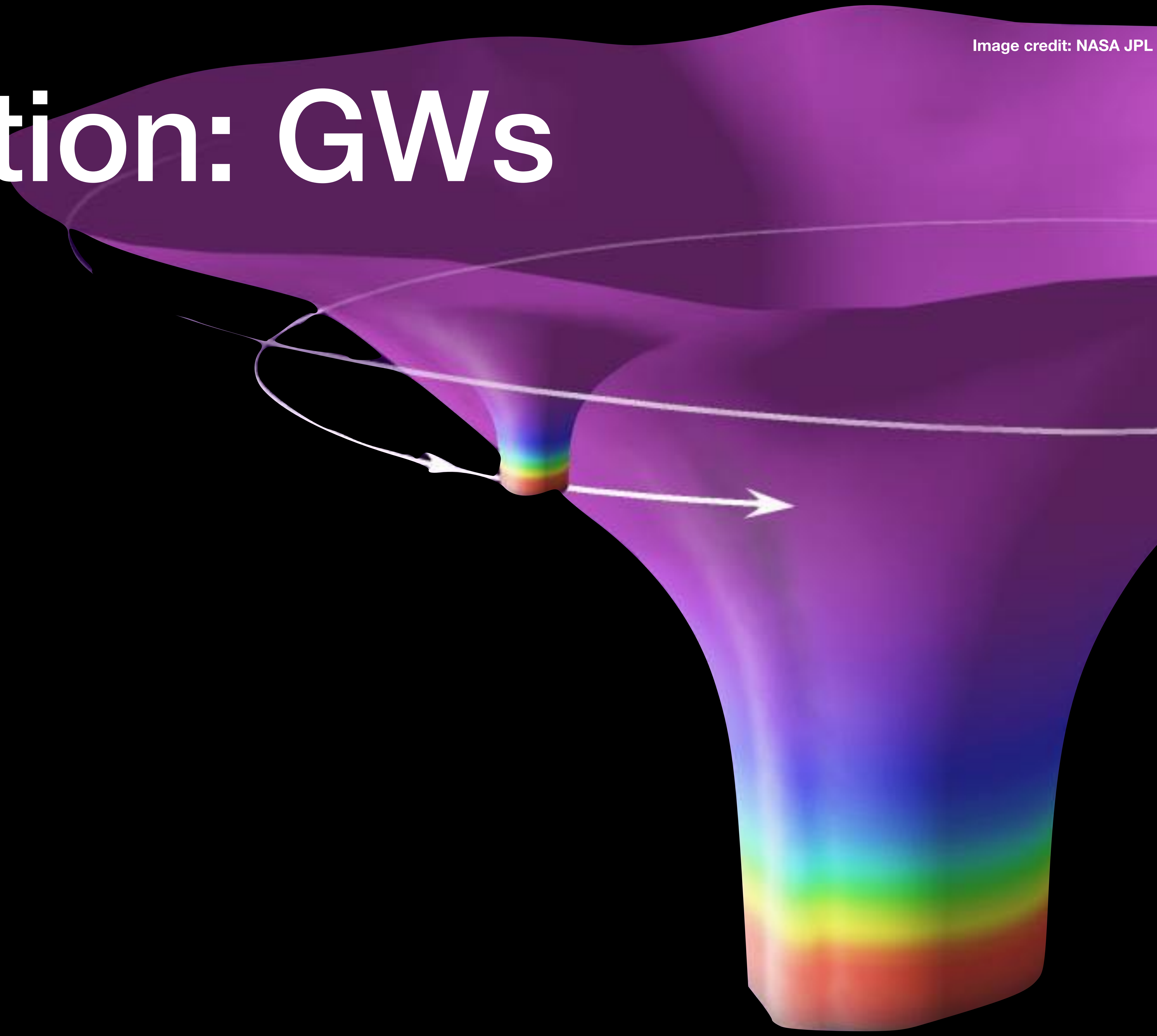
Motivation: GWs

Image credit: NASA JPL



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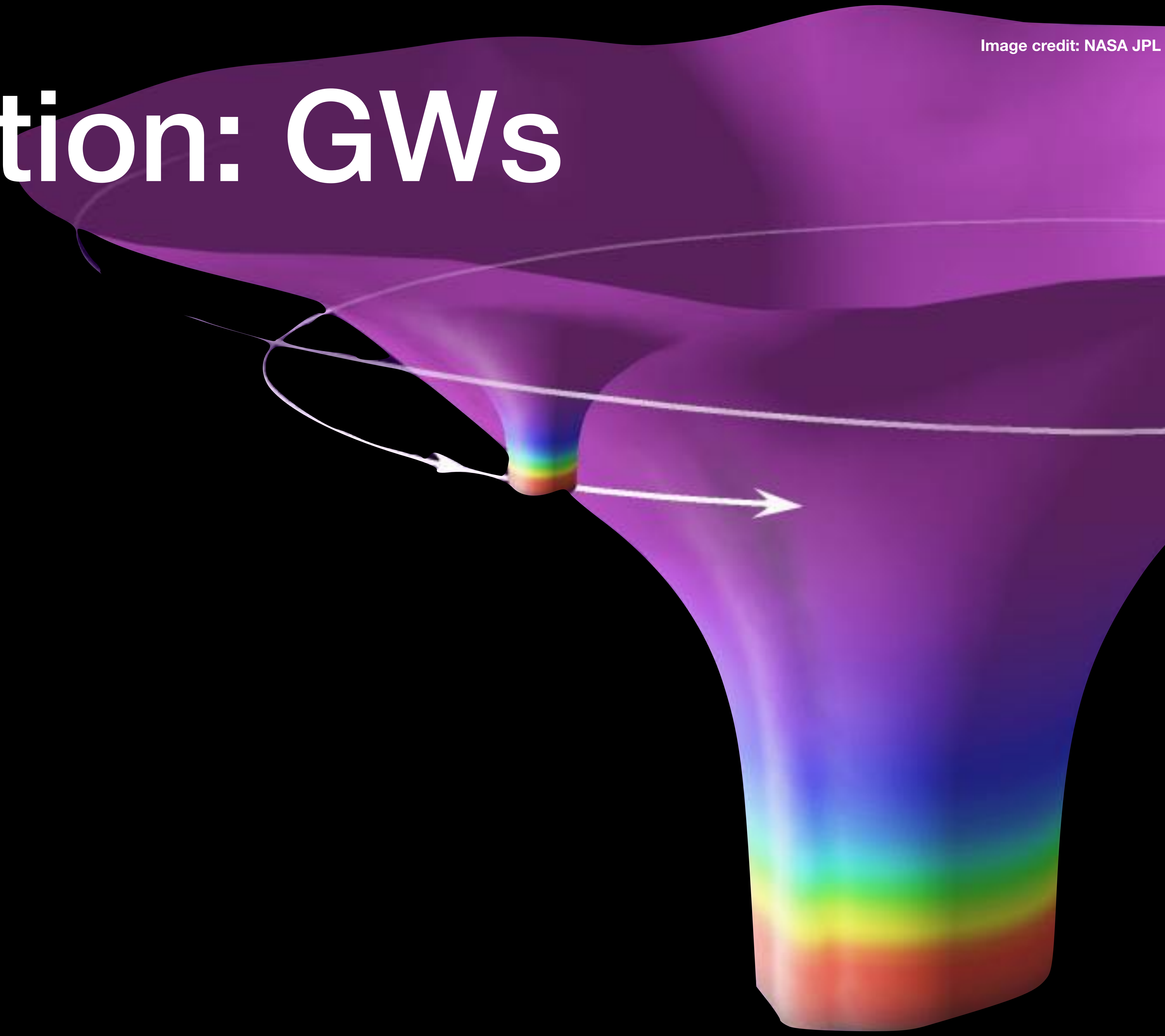
Image credit: NASA JPL

- Gravitational wave spectroscopy

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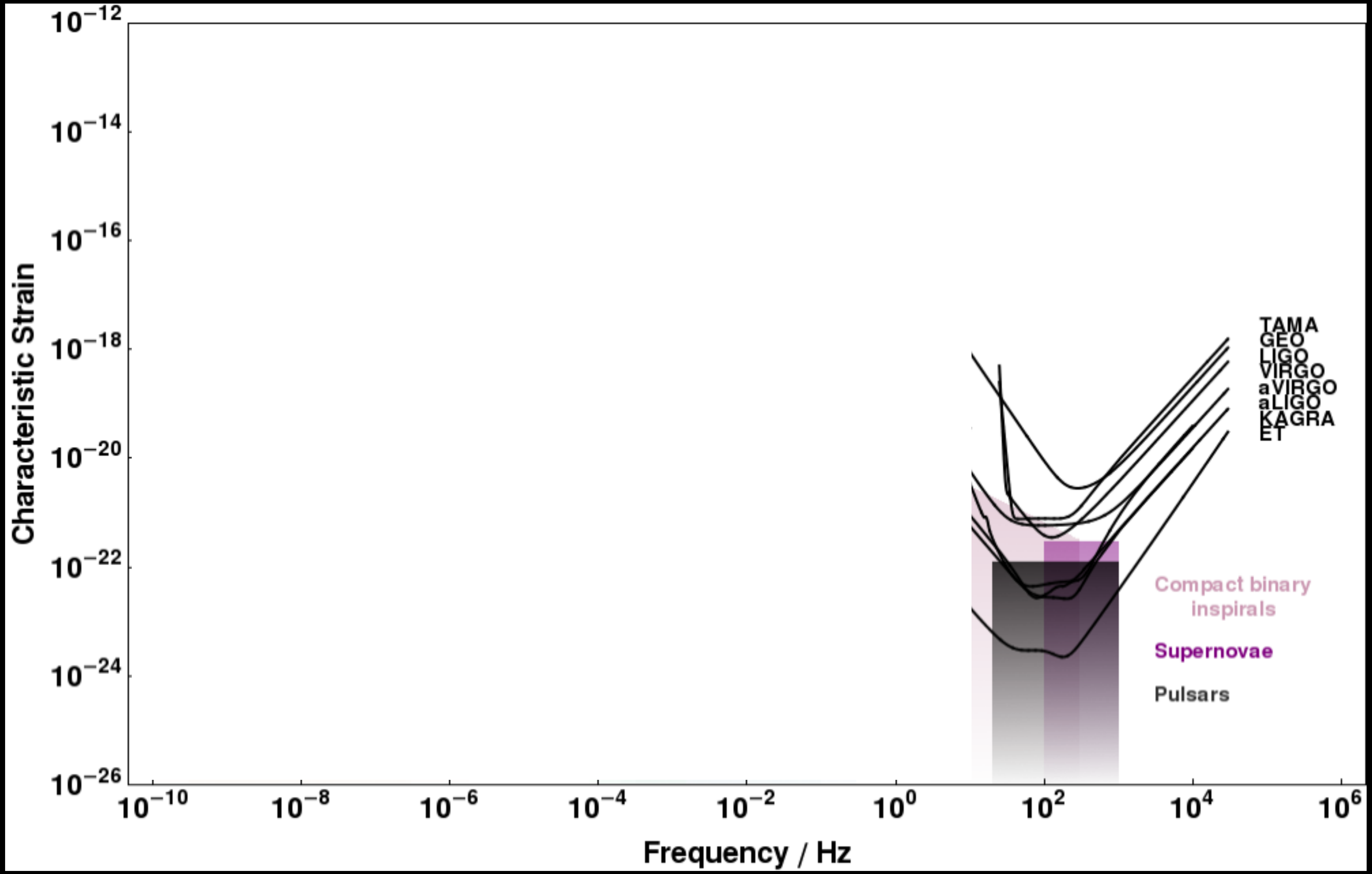
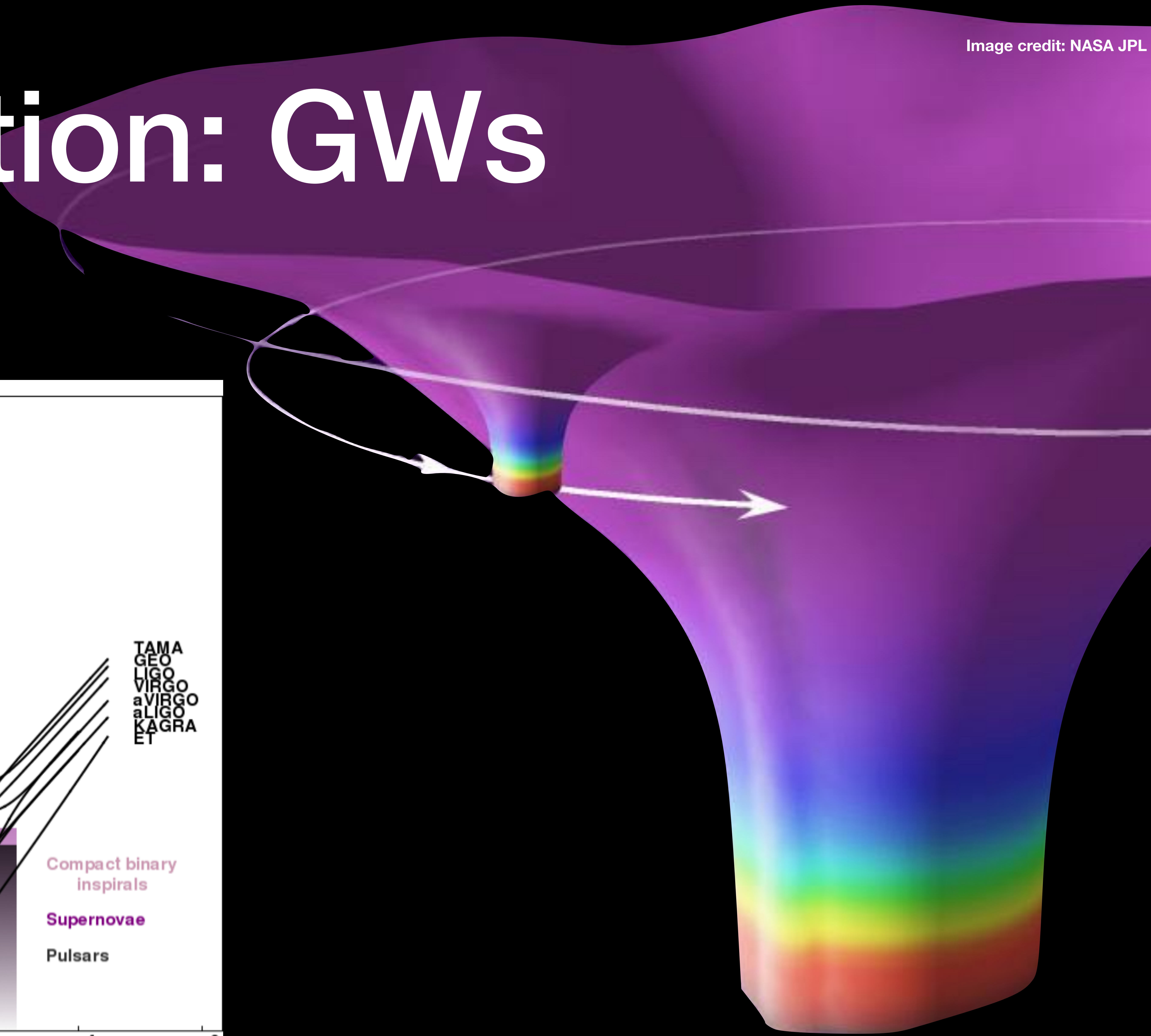
- Gravitational wave spectroscopy



C.J. Moore et al., Class. Quantum Grav 32, 015014 (2015)



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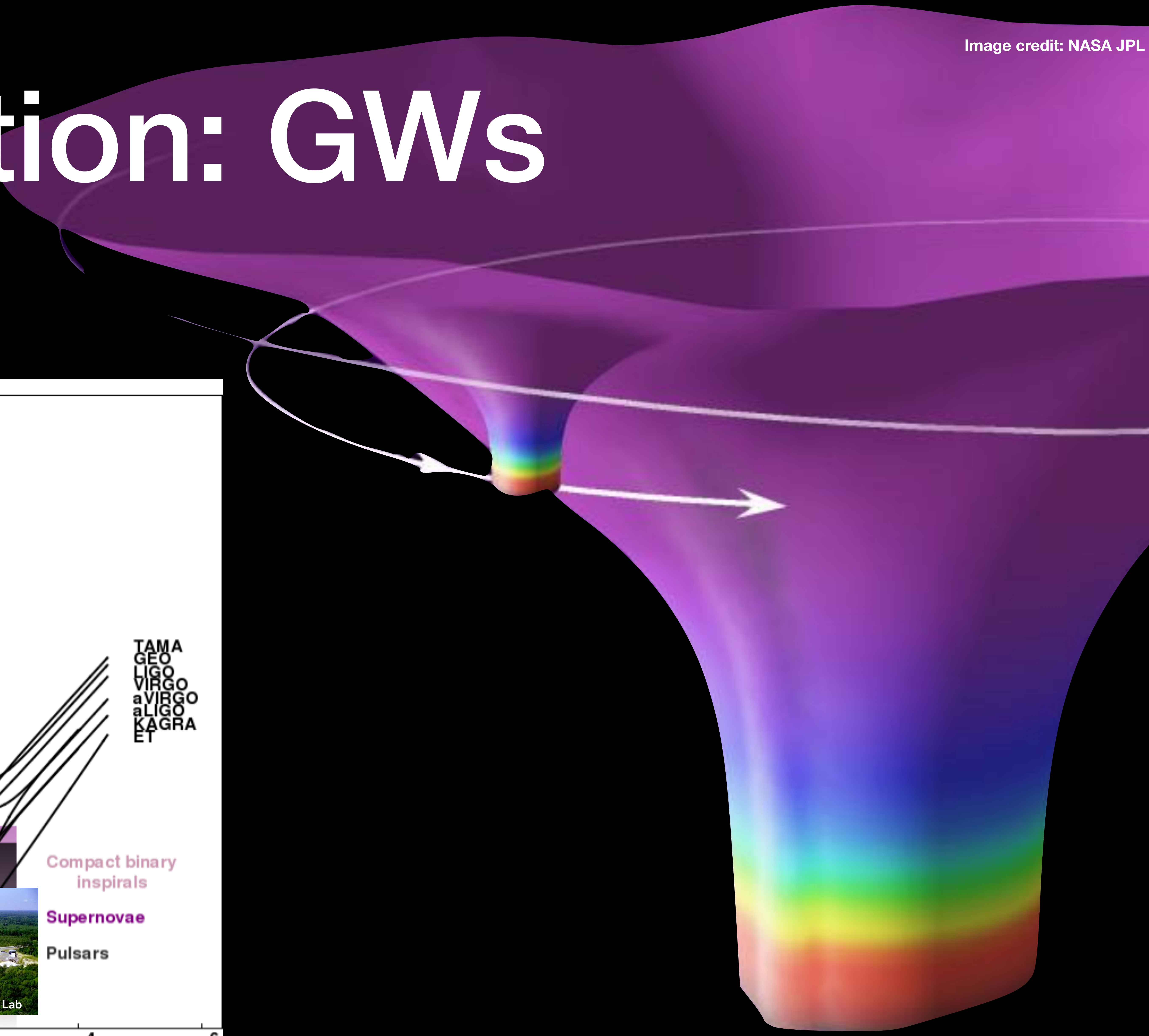

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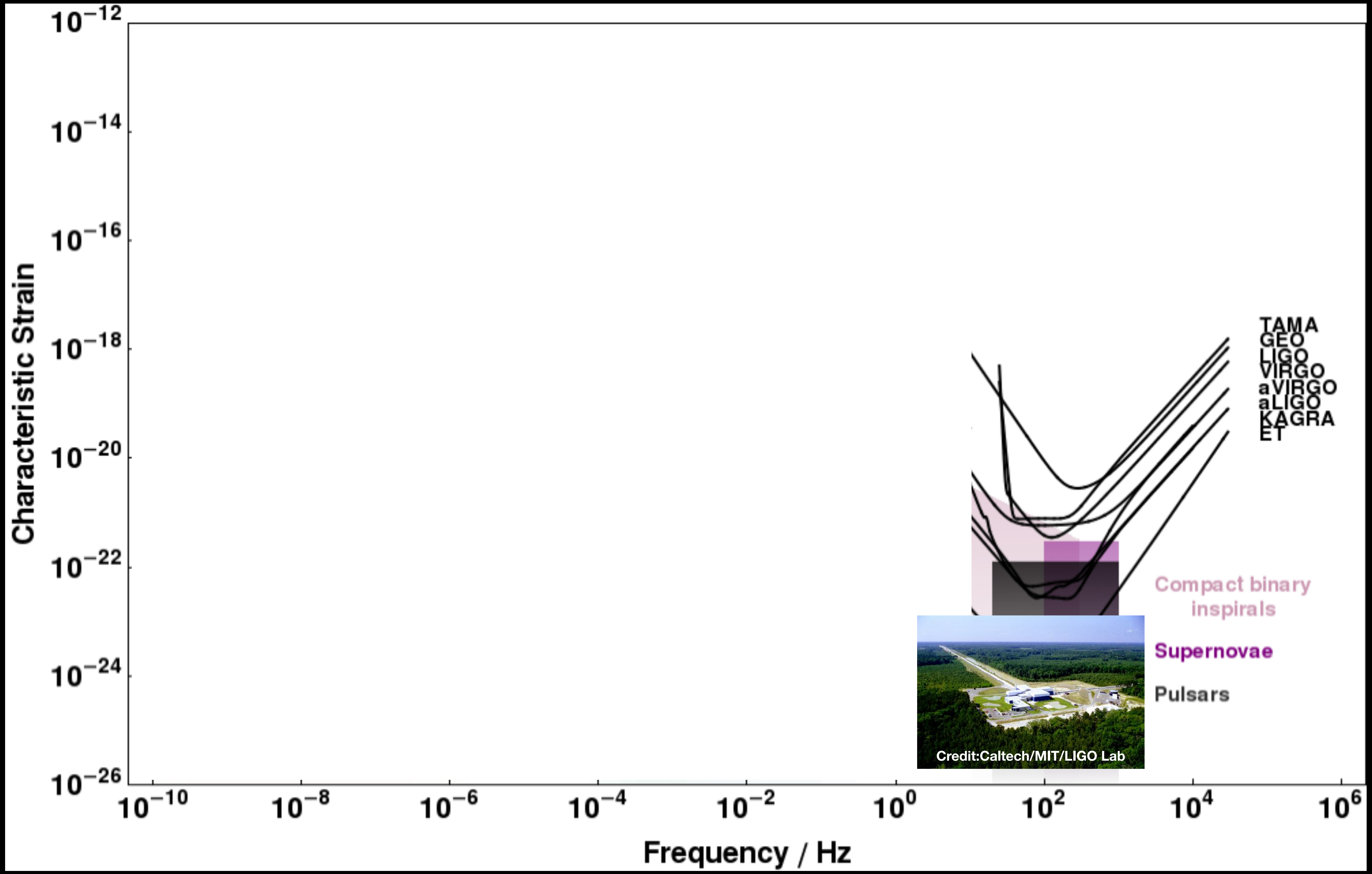

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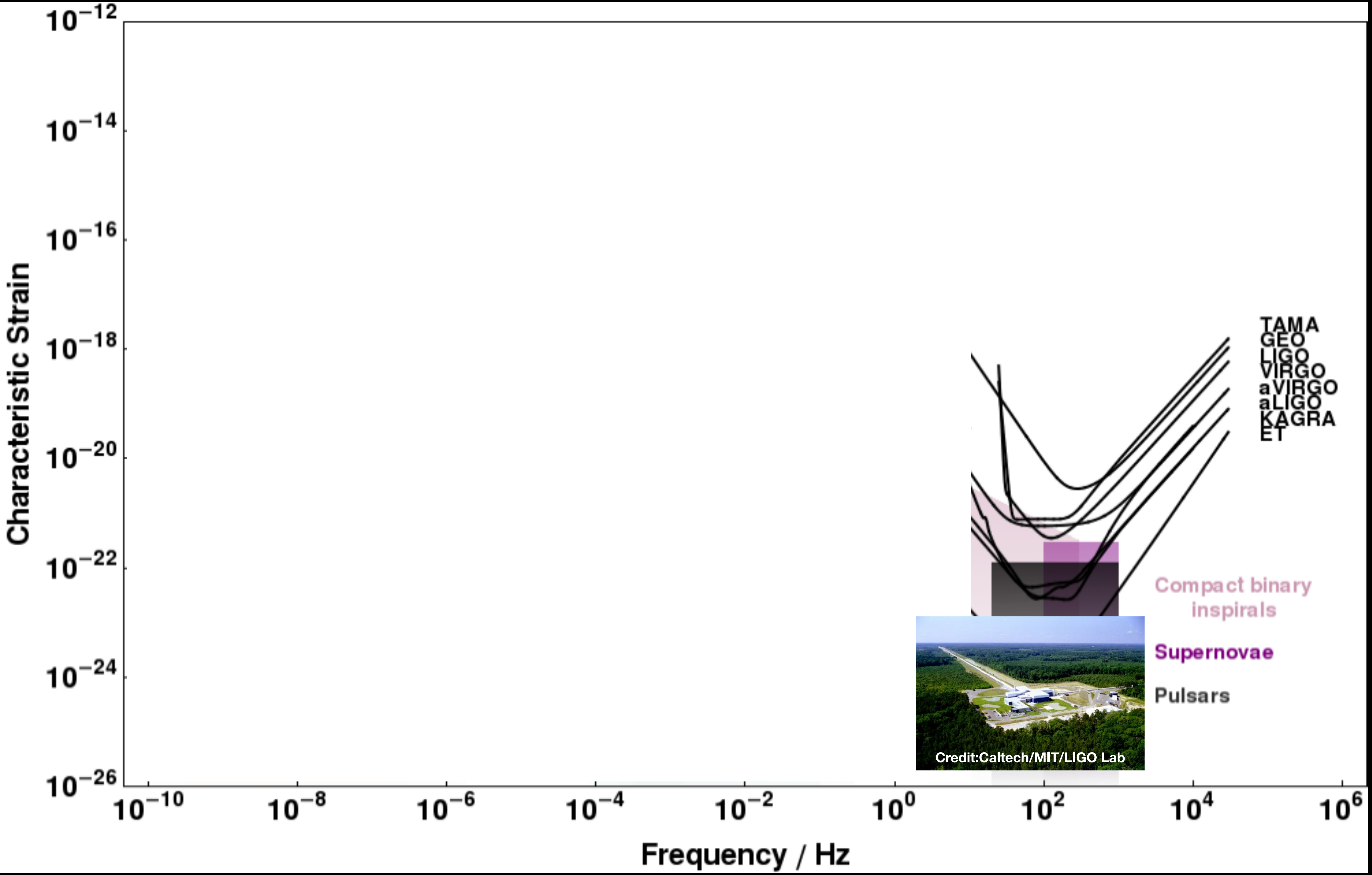

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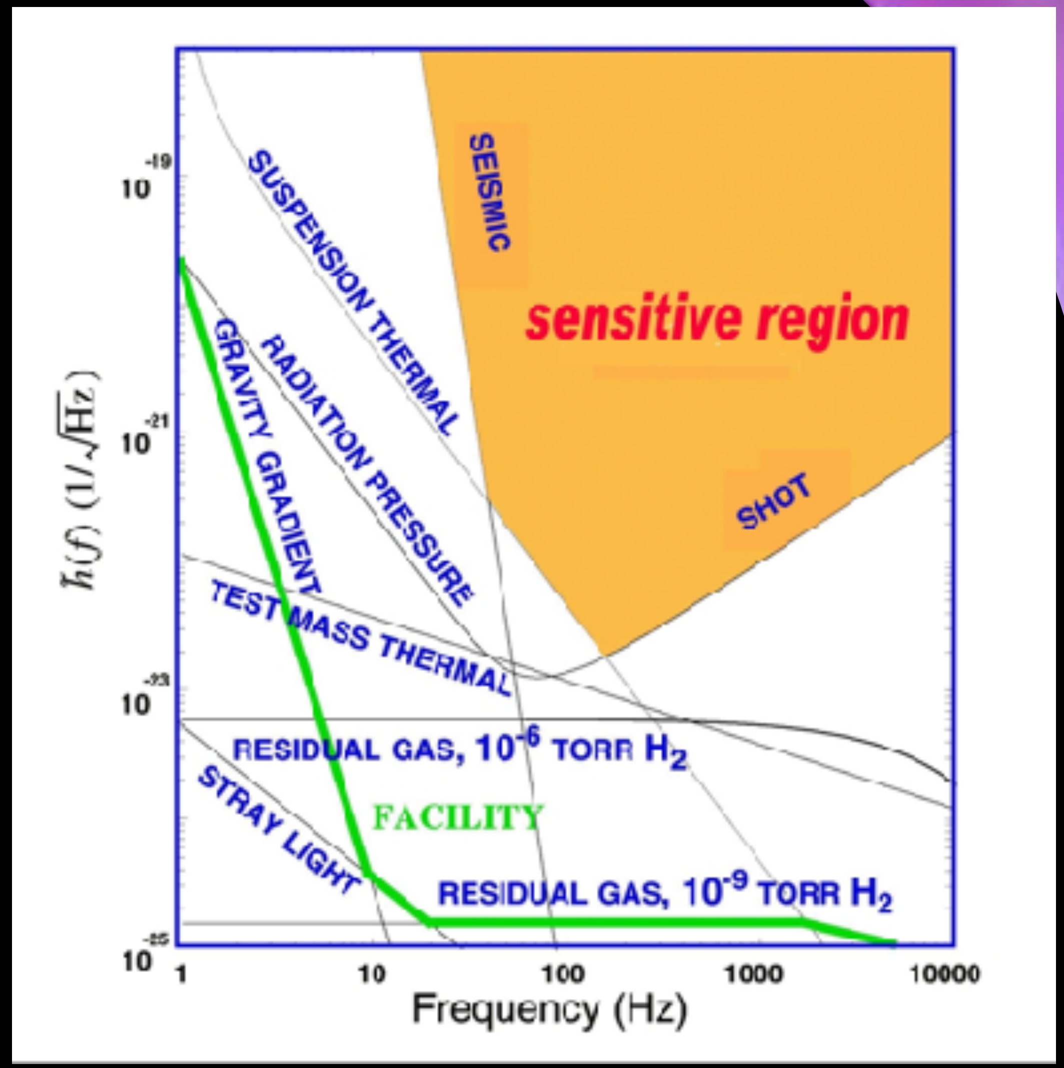
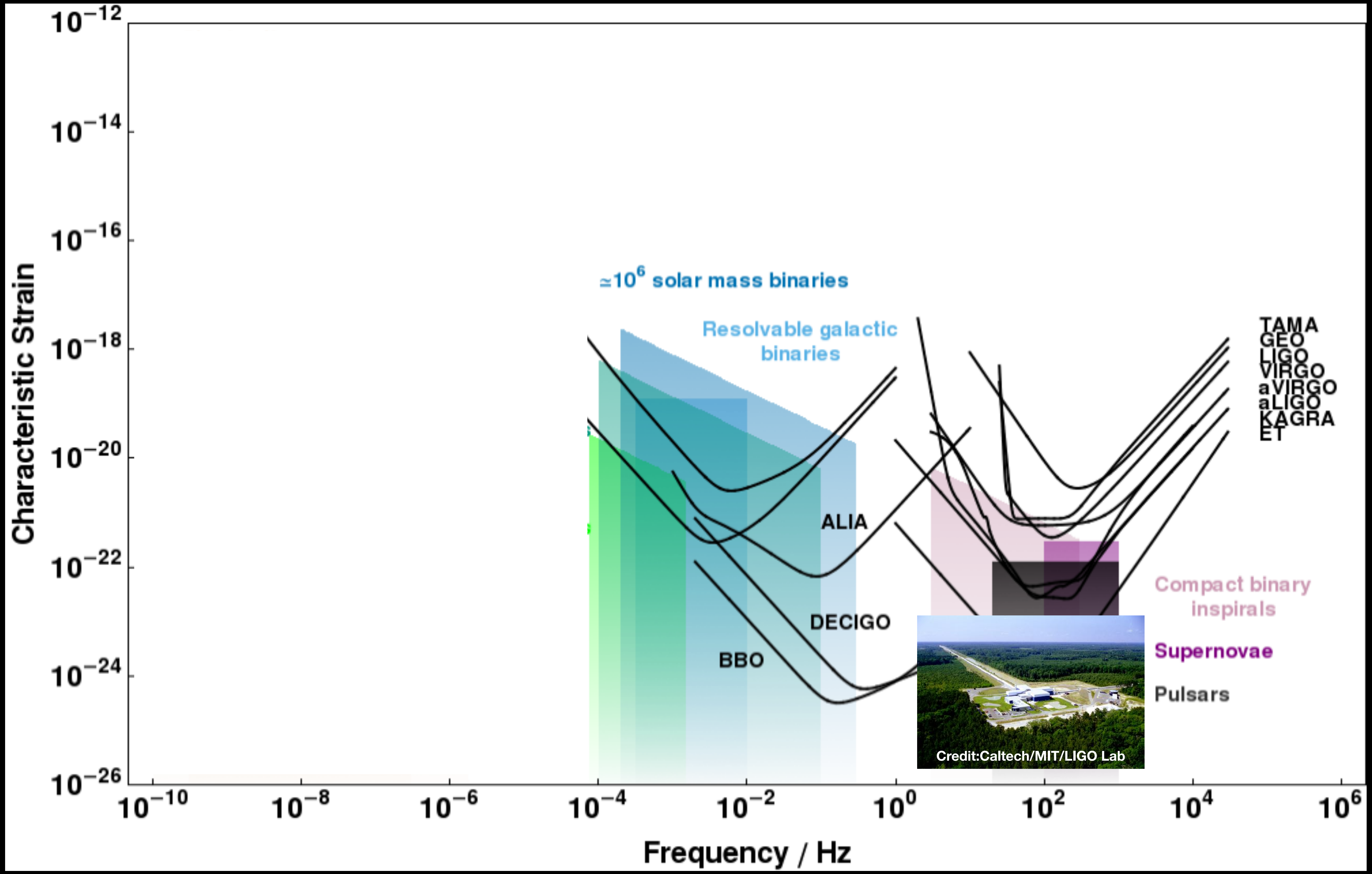


LIGO sensitivity / noise floor

Image credit: NASA JPL

Motivation: GWs

- Gravitational wave spectroscopy



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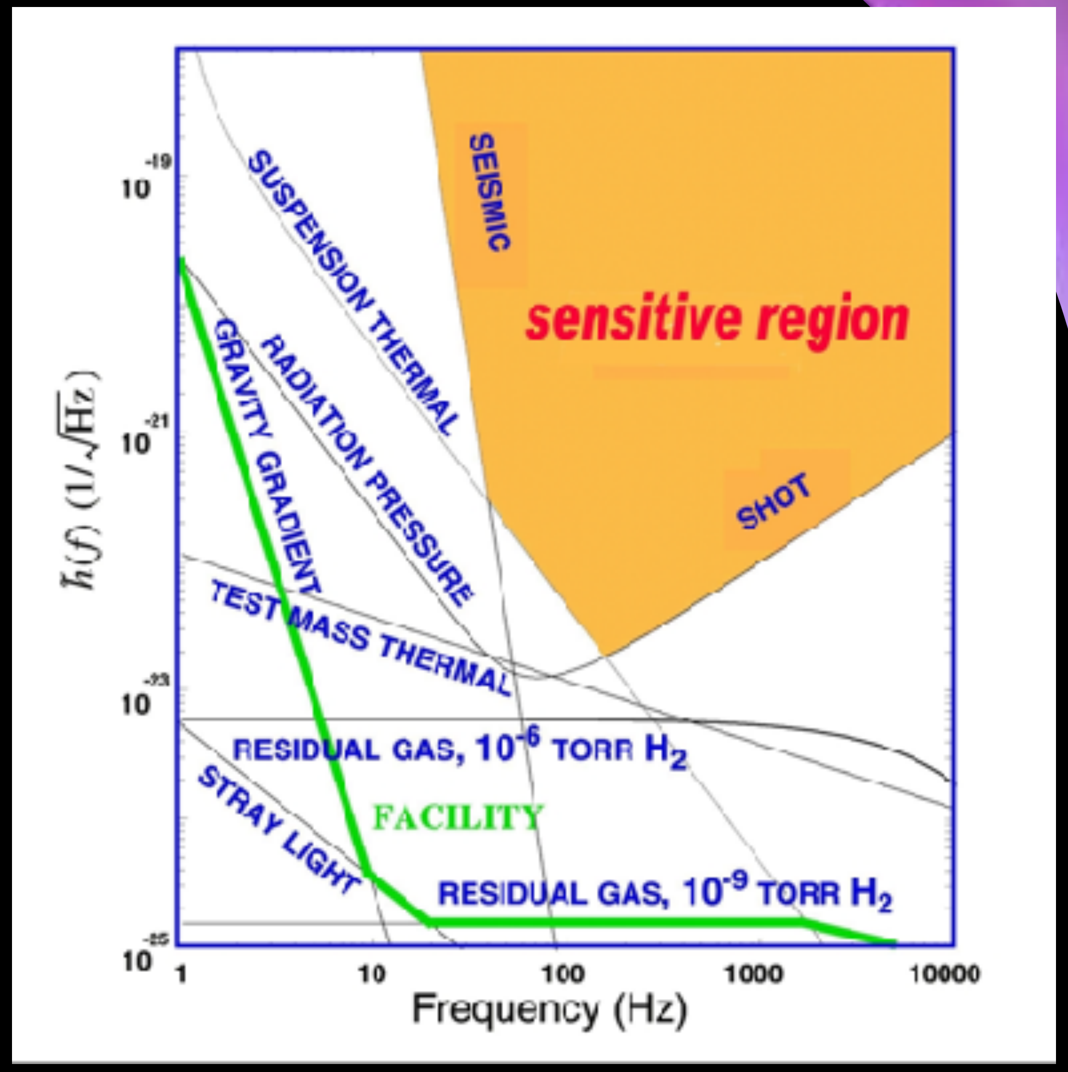

LIGO sensitivity / noise floor

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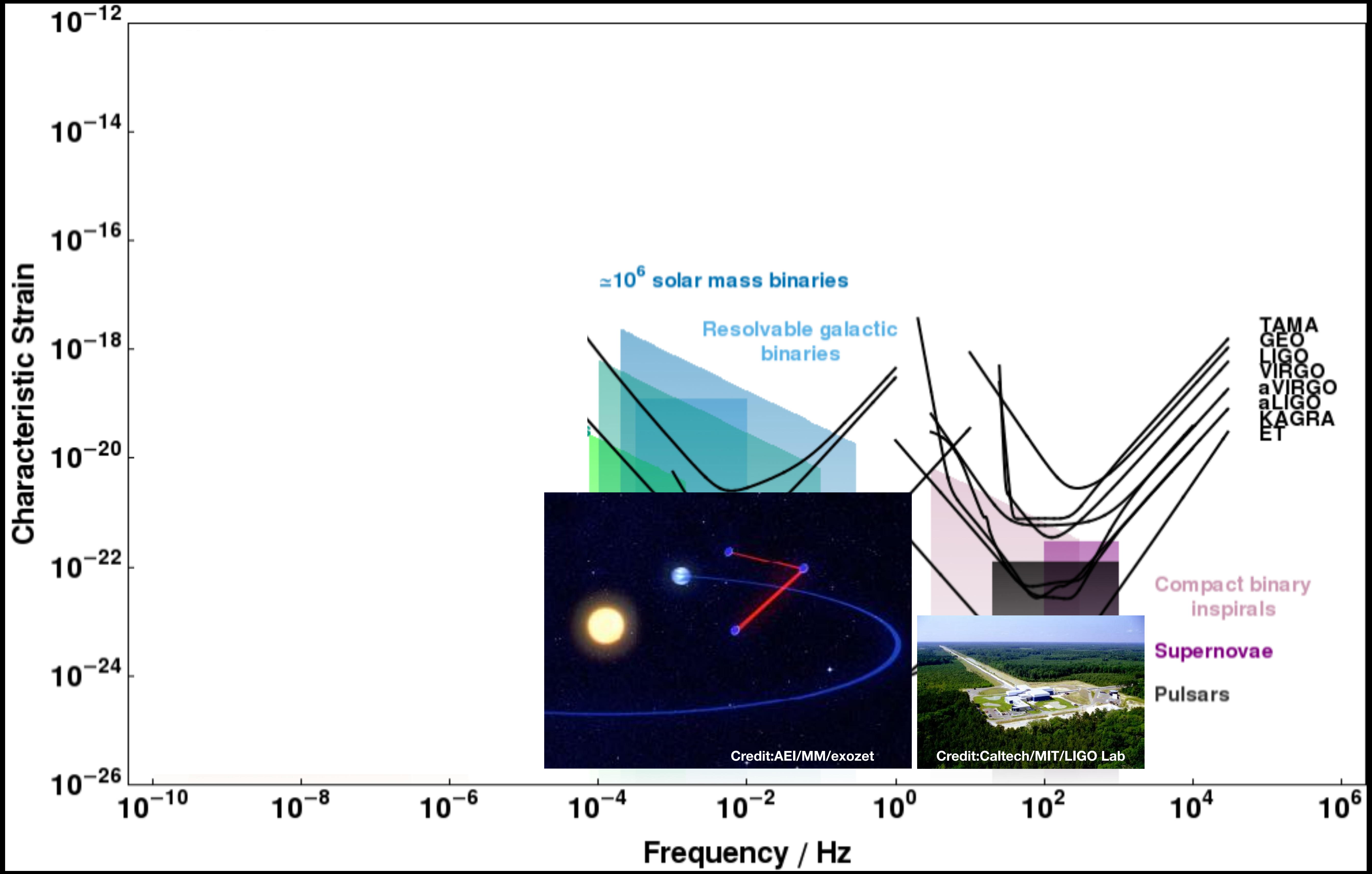

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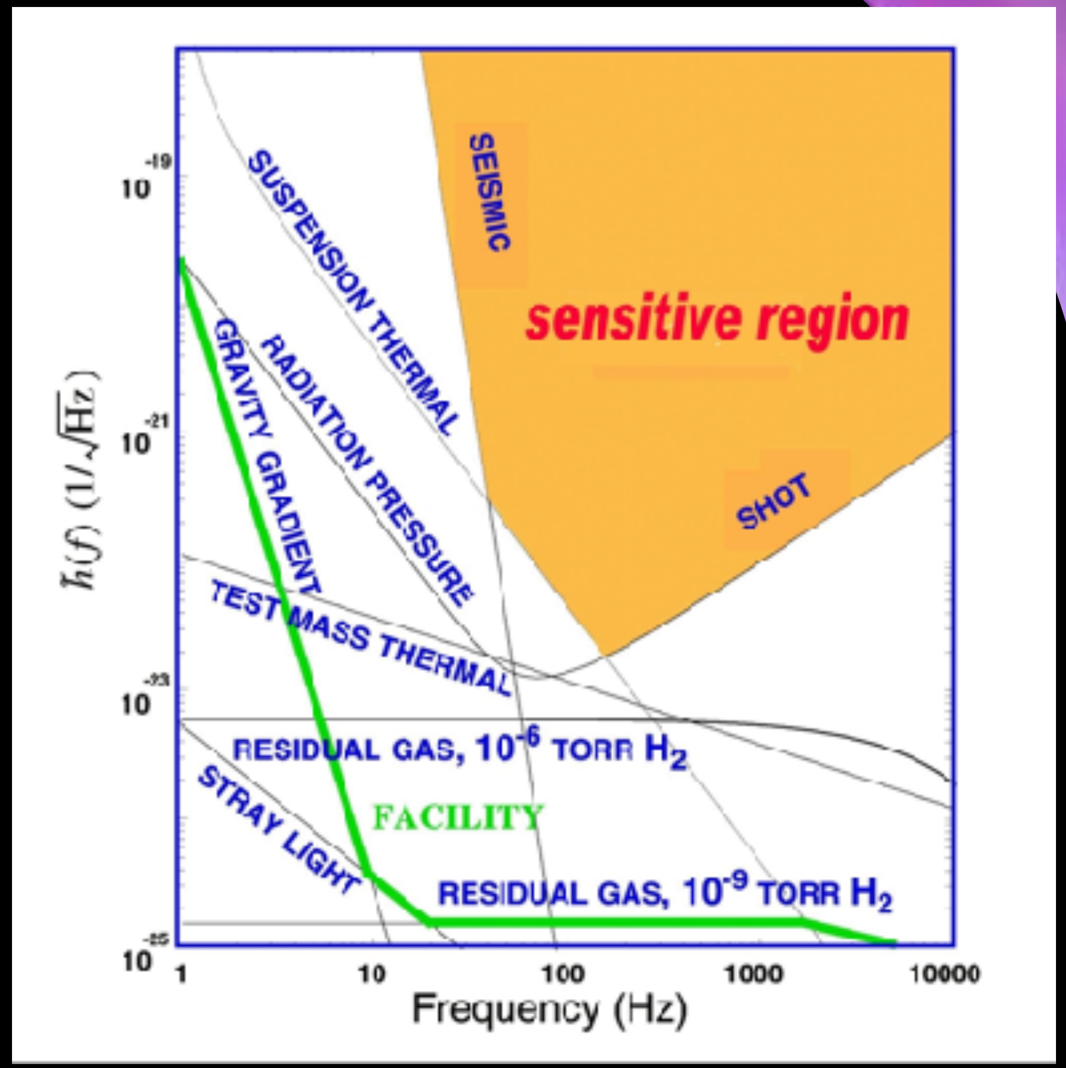

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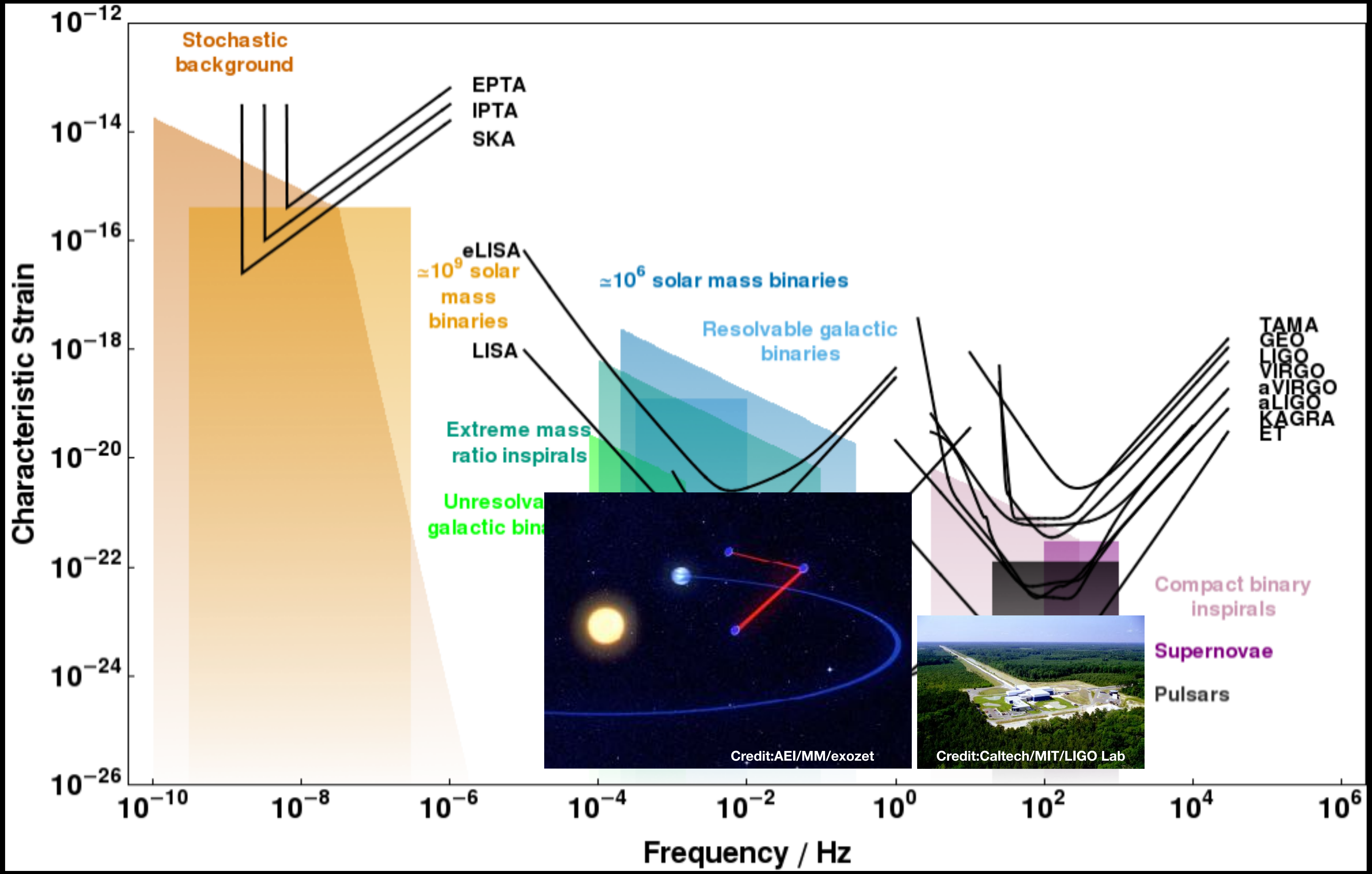

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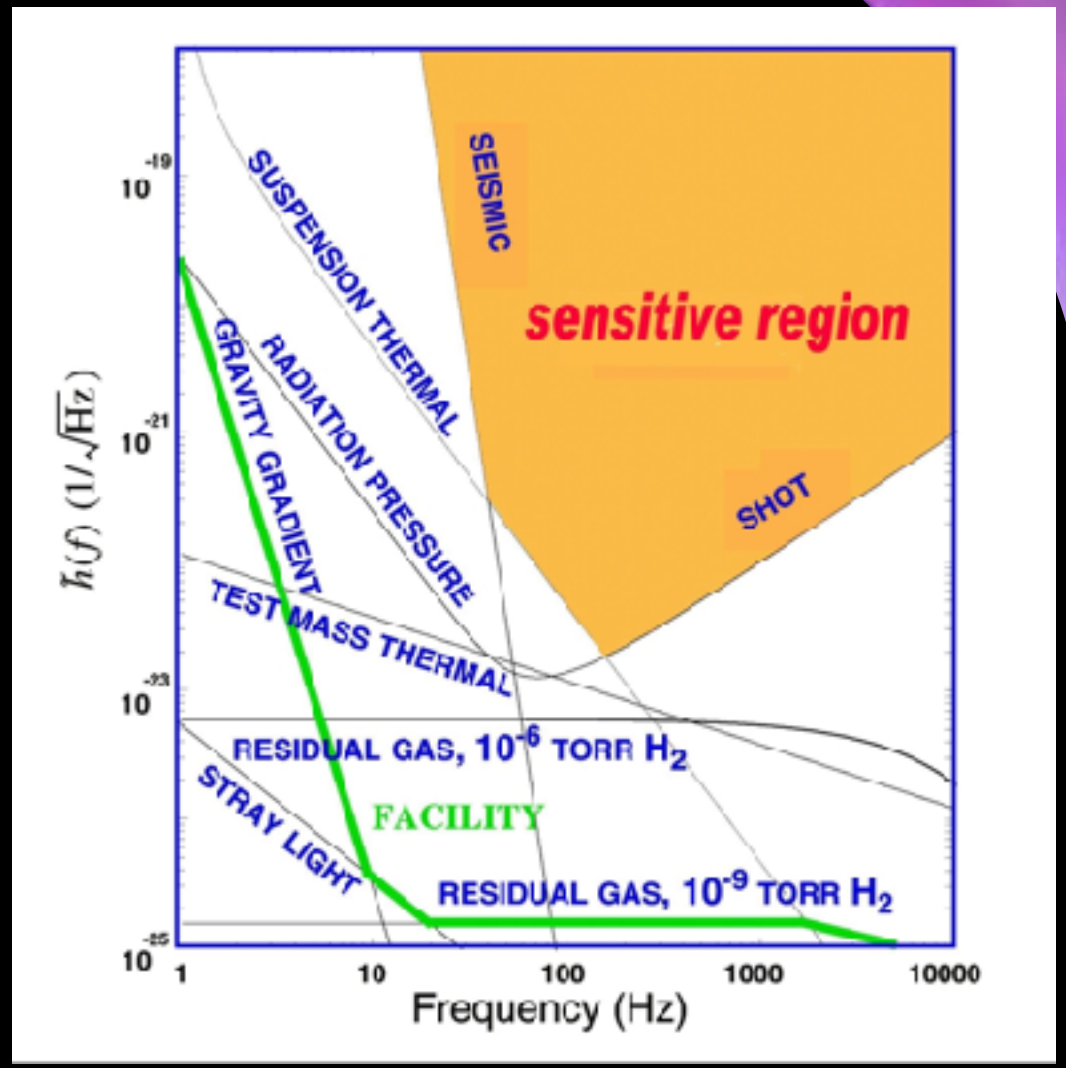

LIGO sensitivity / noise floor

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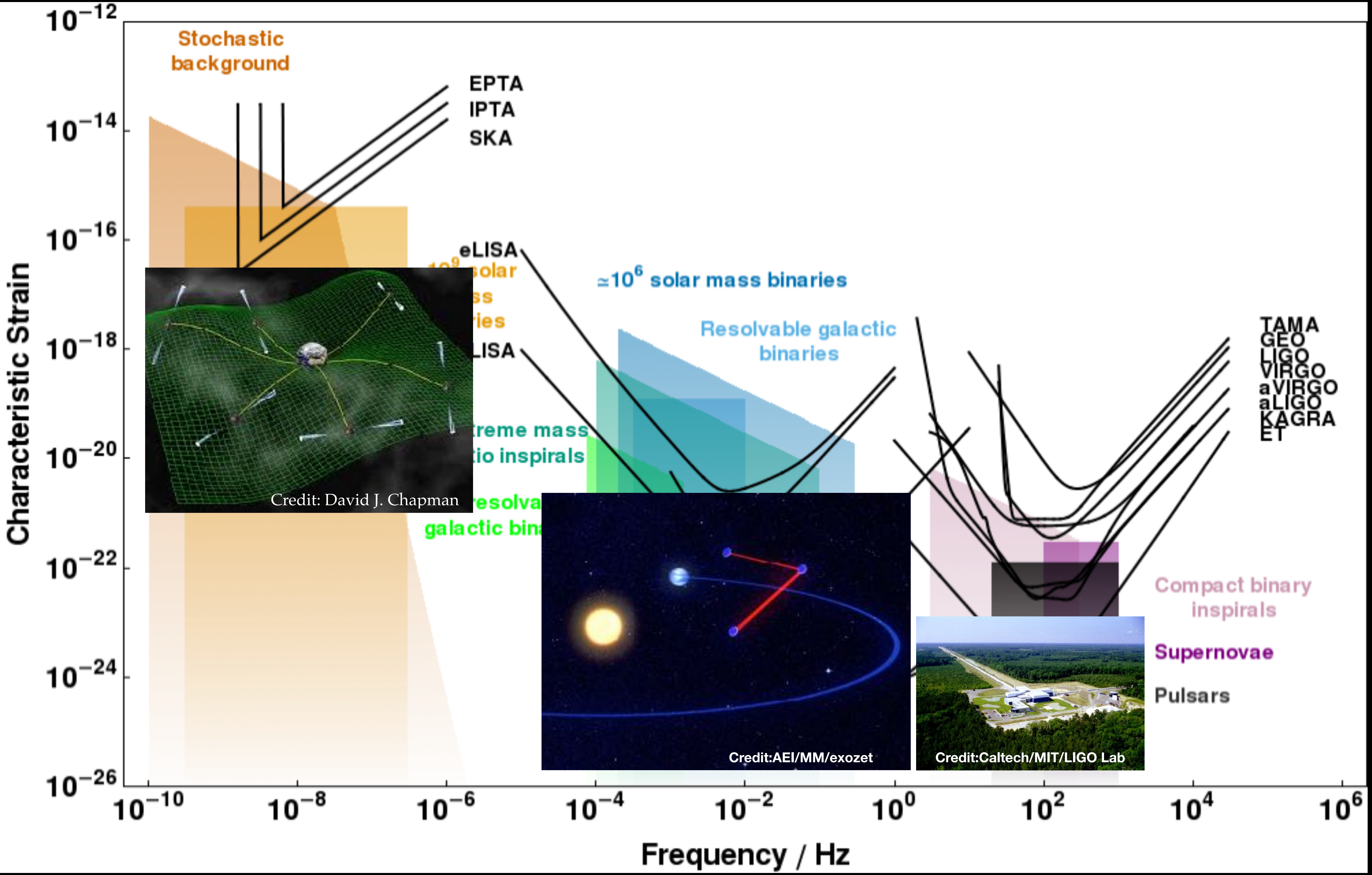
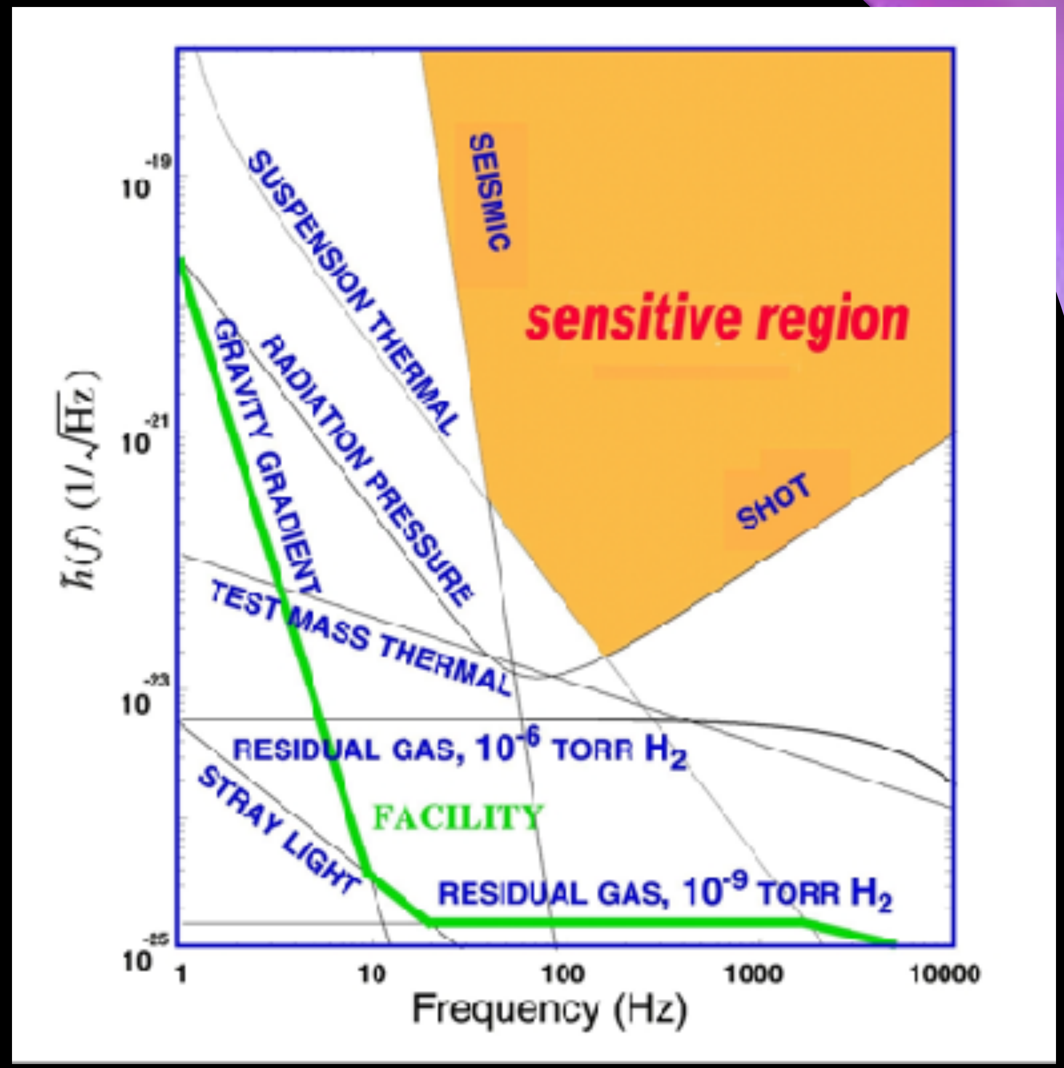


C.J. Moore et al., Class. Quantum Grav 32, 015014 (2015)

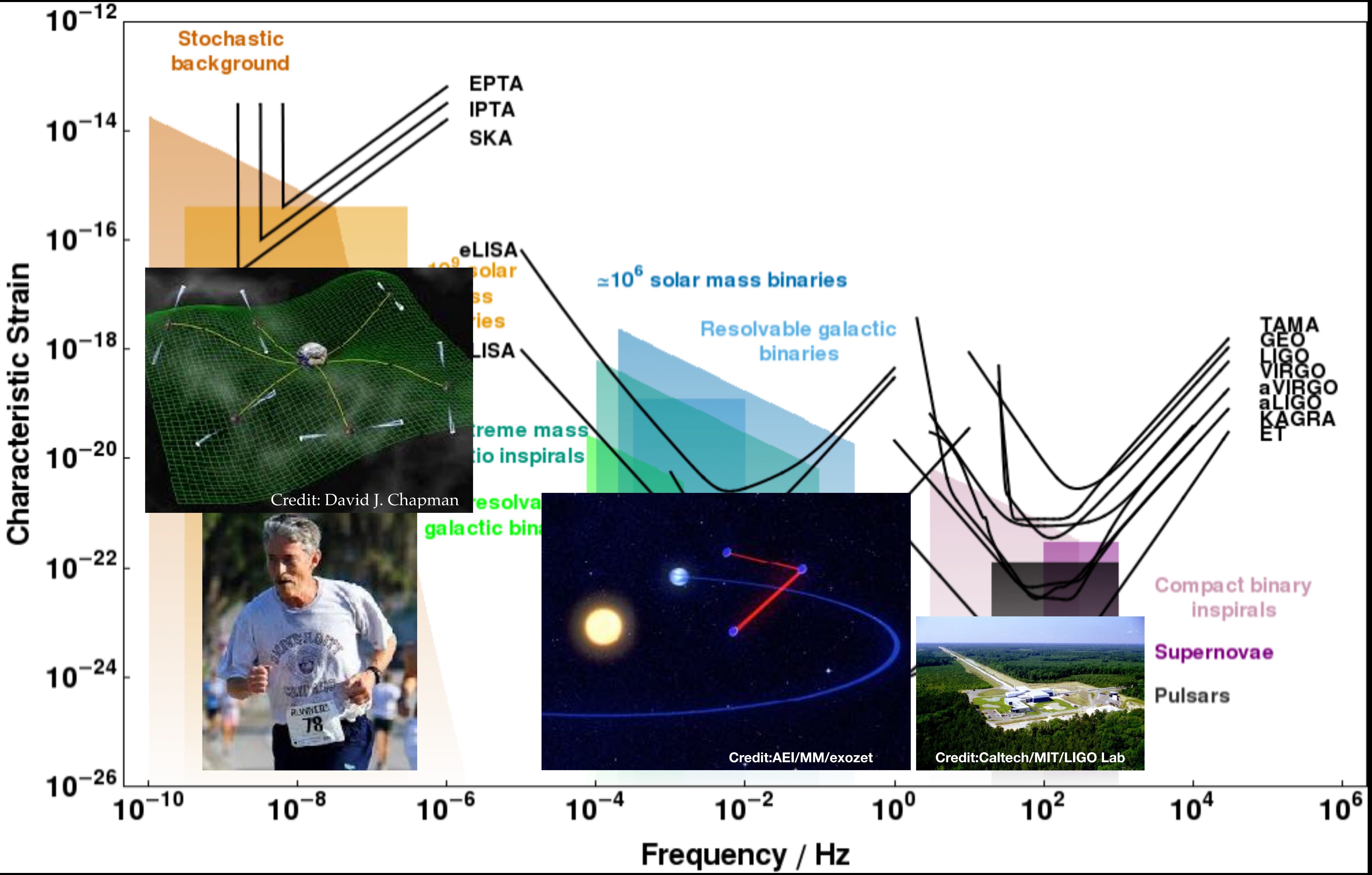


LIGO sensitivity / noise floor

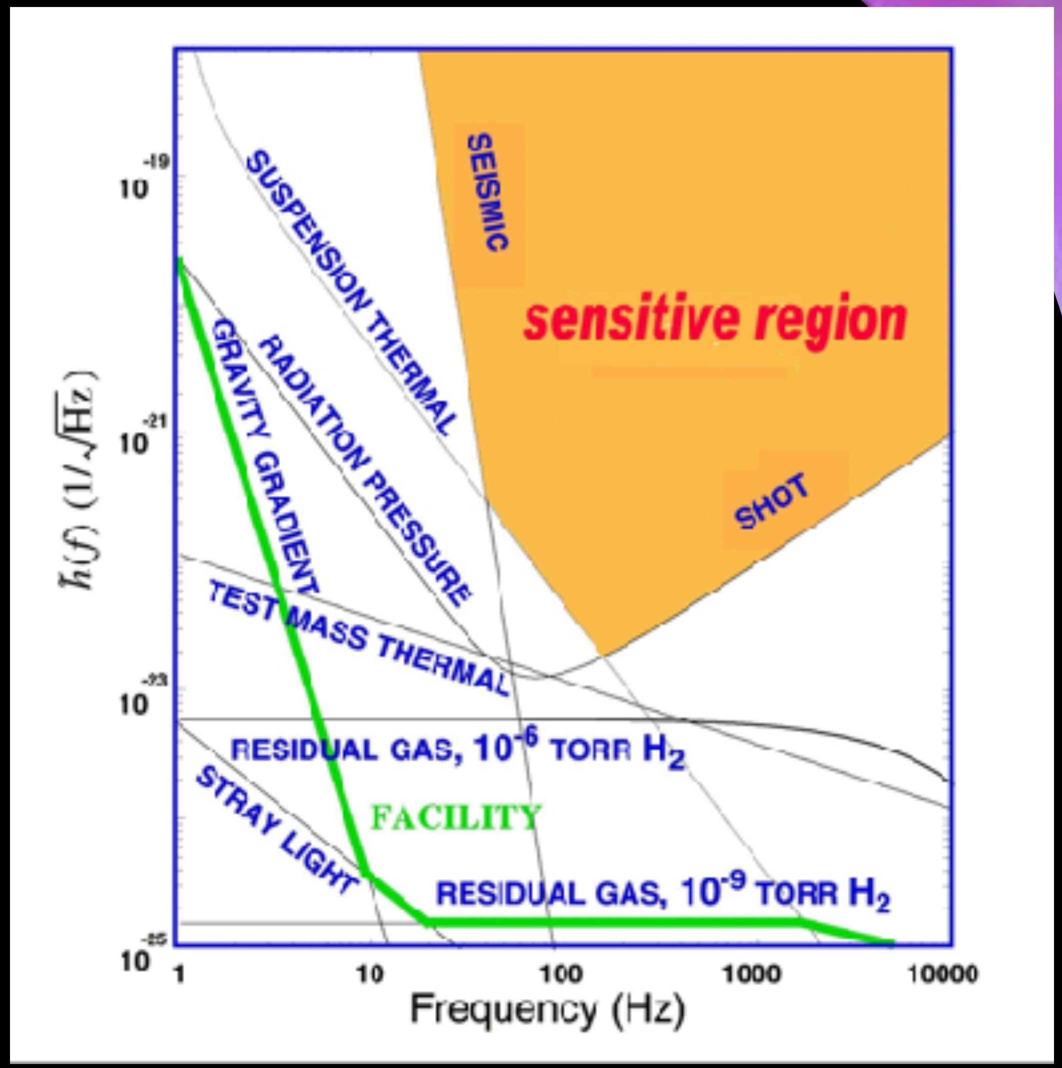
Motivation: GWs

Image credit: NASA JPL

- Gravitational wave spectroscopy



C.J. Moore et al., Class. Quantum Grav 32, 015014 (2015)



LIGO sensitivity / noise floor

Motivation:EMRIs

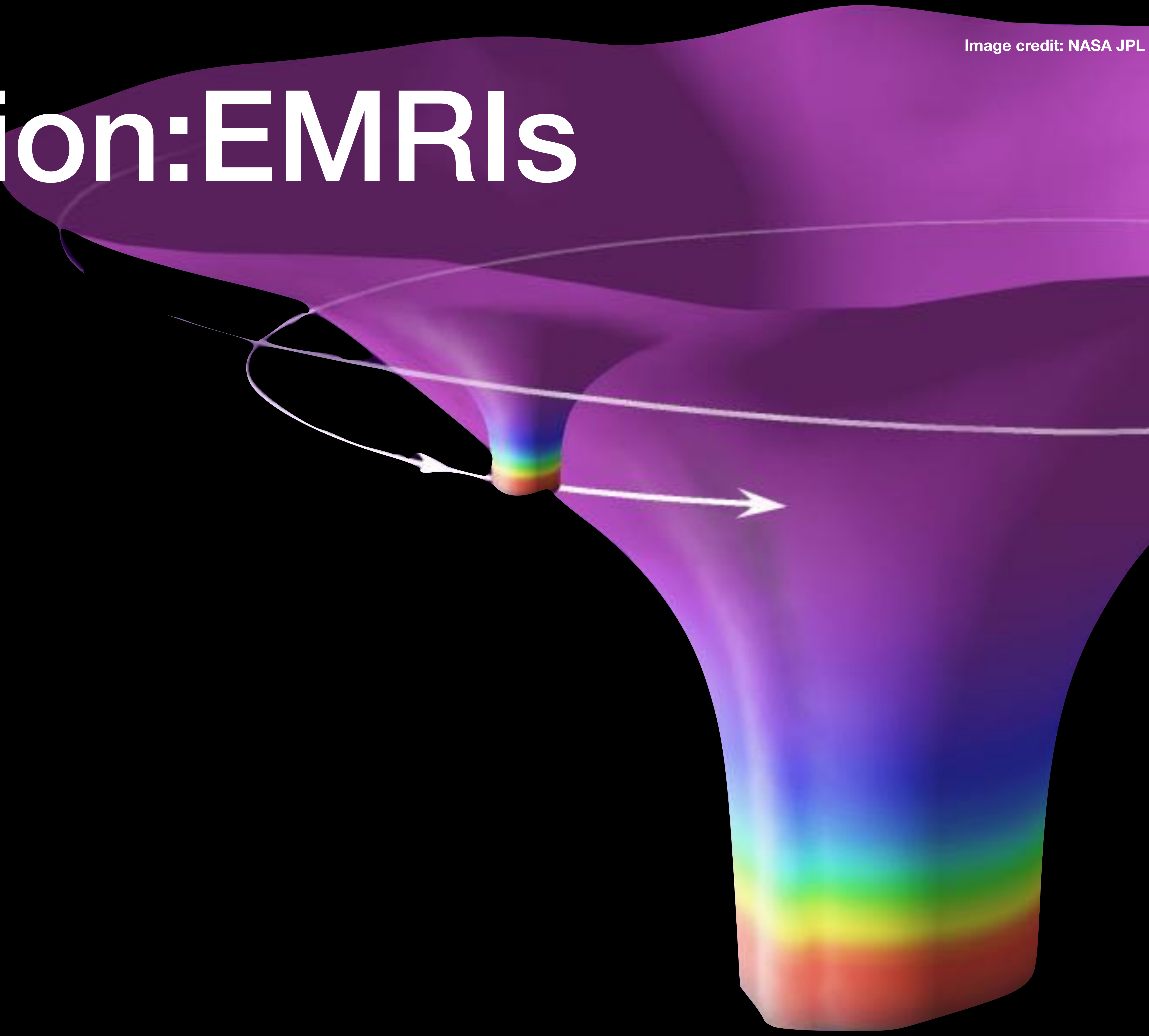


Image credit: NASA JPL

Motivation:EMRIs

- LISA

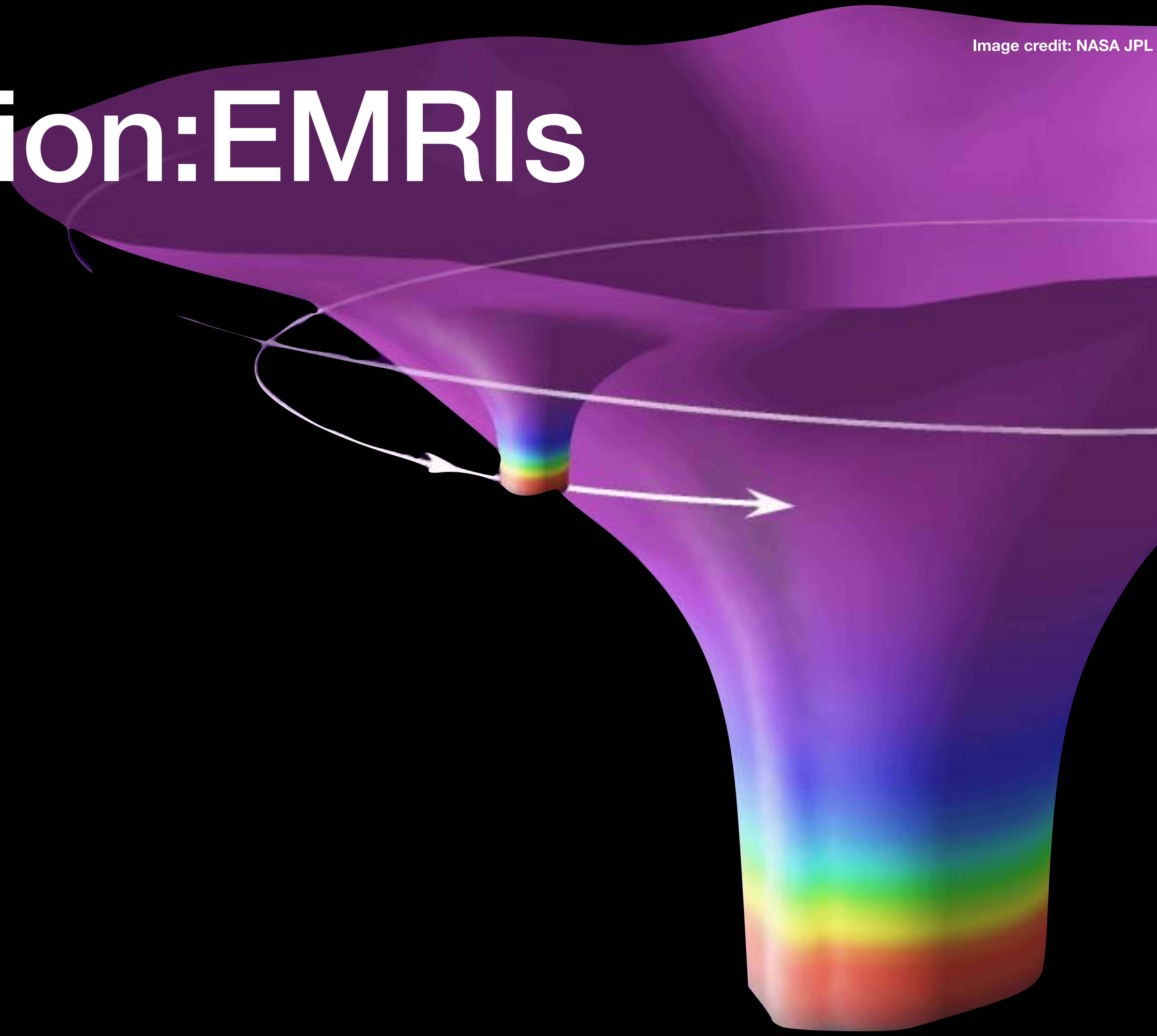
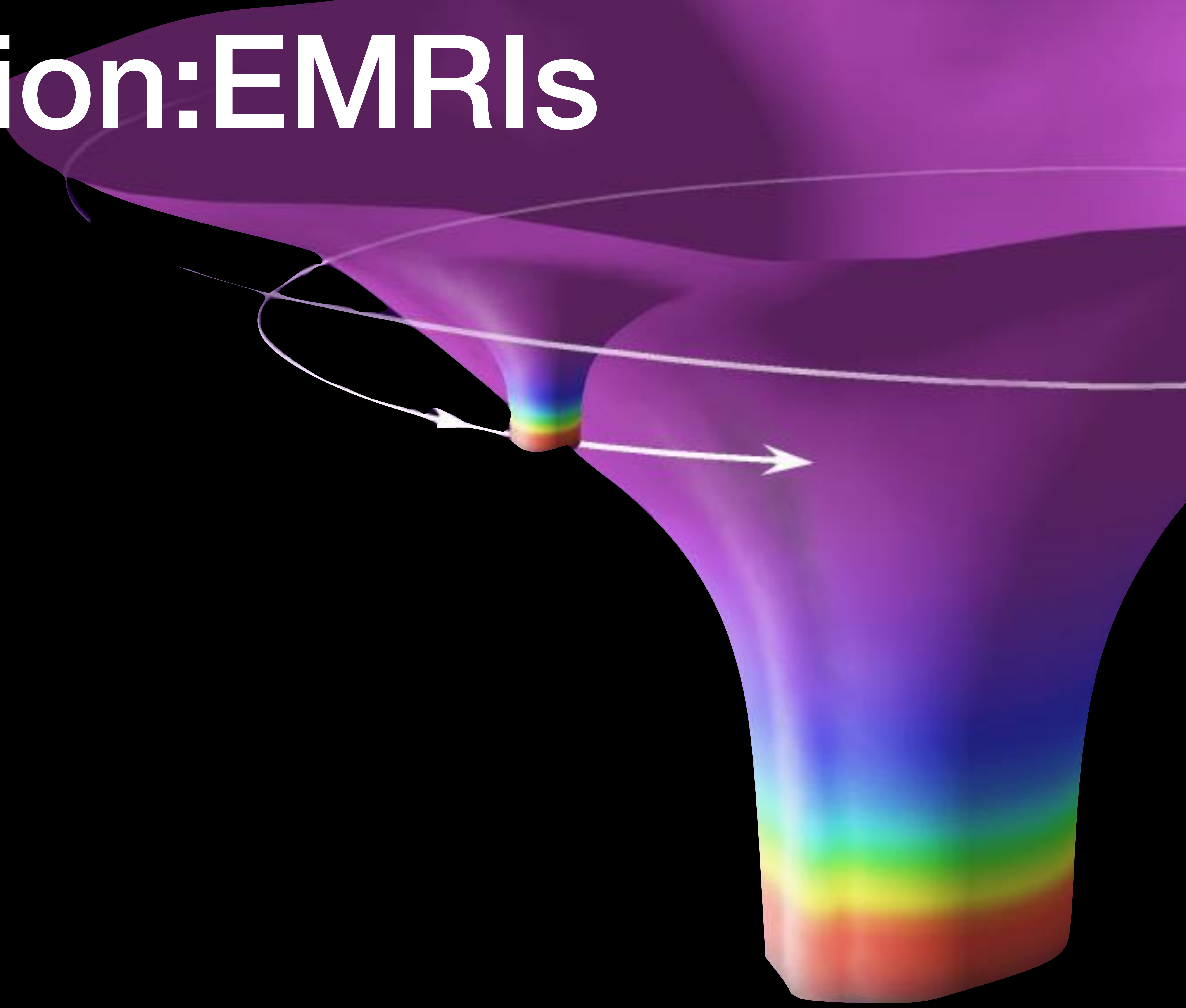


Image credit: NASA JPL

Image credit: NASA JPL

Motivation:EMRIs

- LISA



Motivation:EMRIs

- LISA

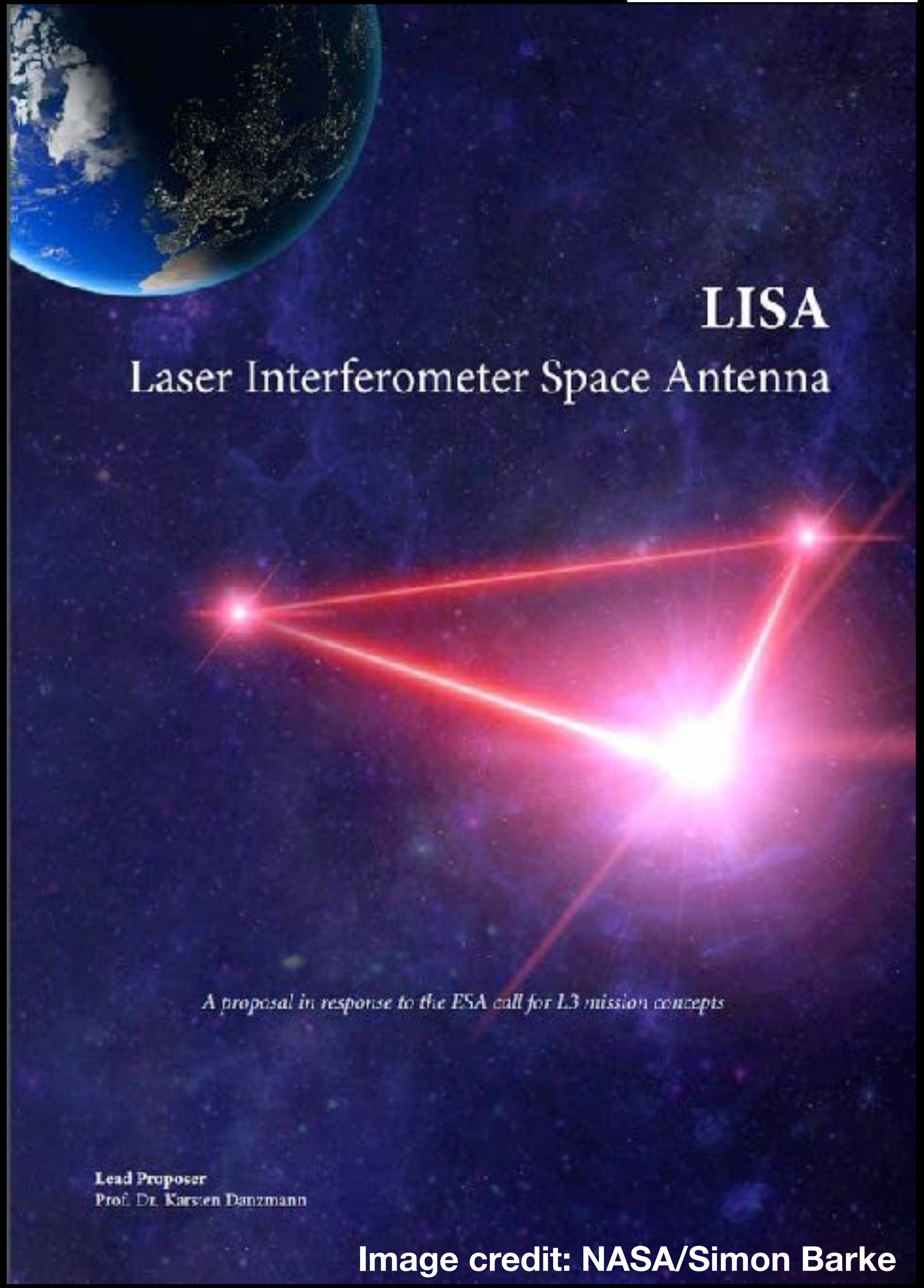
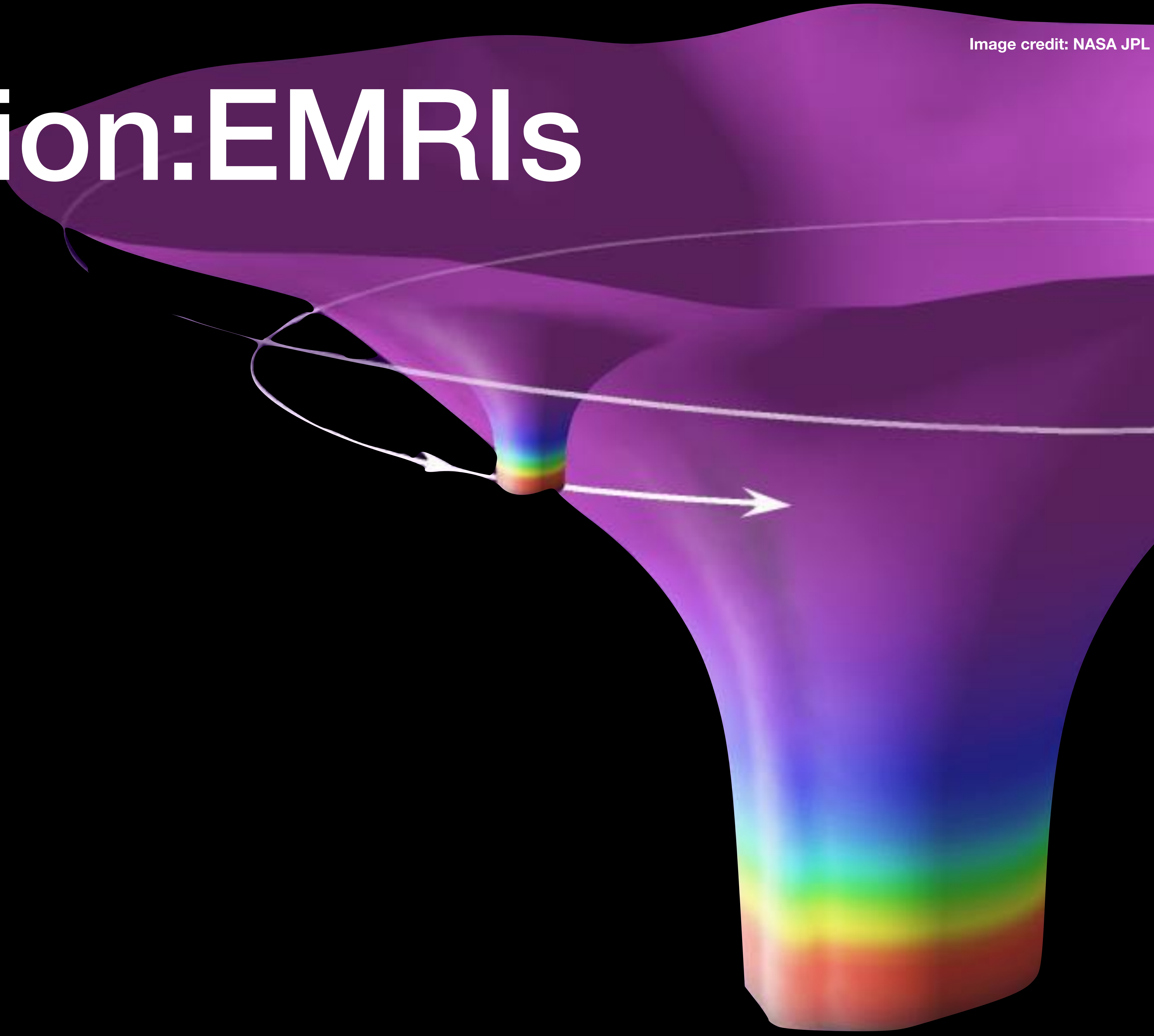


Image credit: NASA/Simon Barke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Motivation:EMRIs

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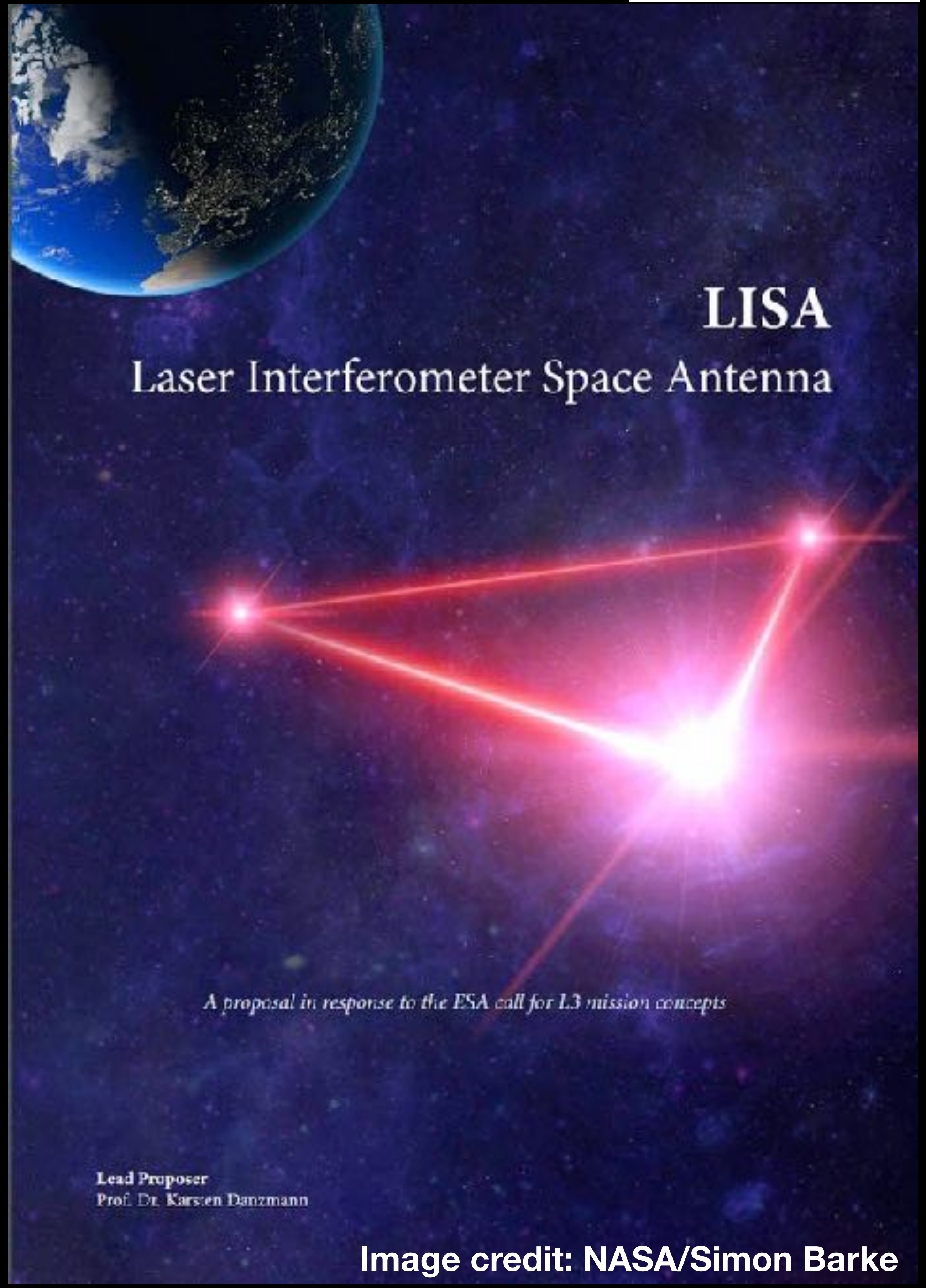
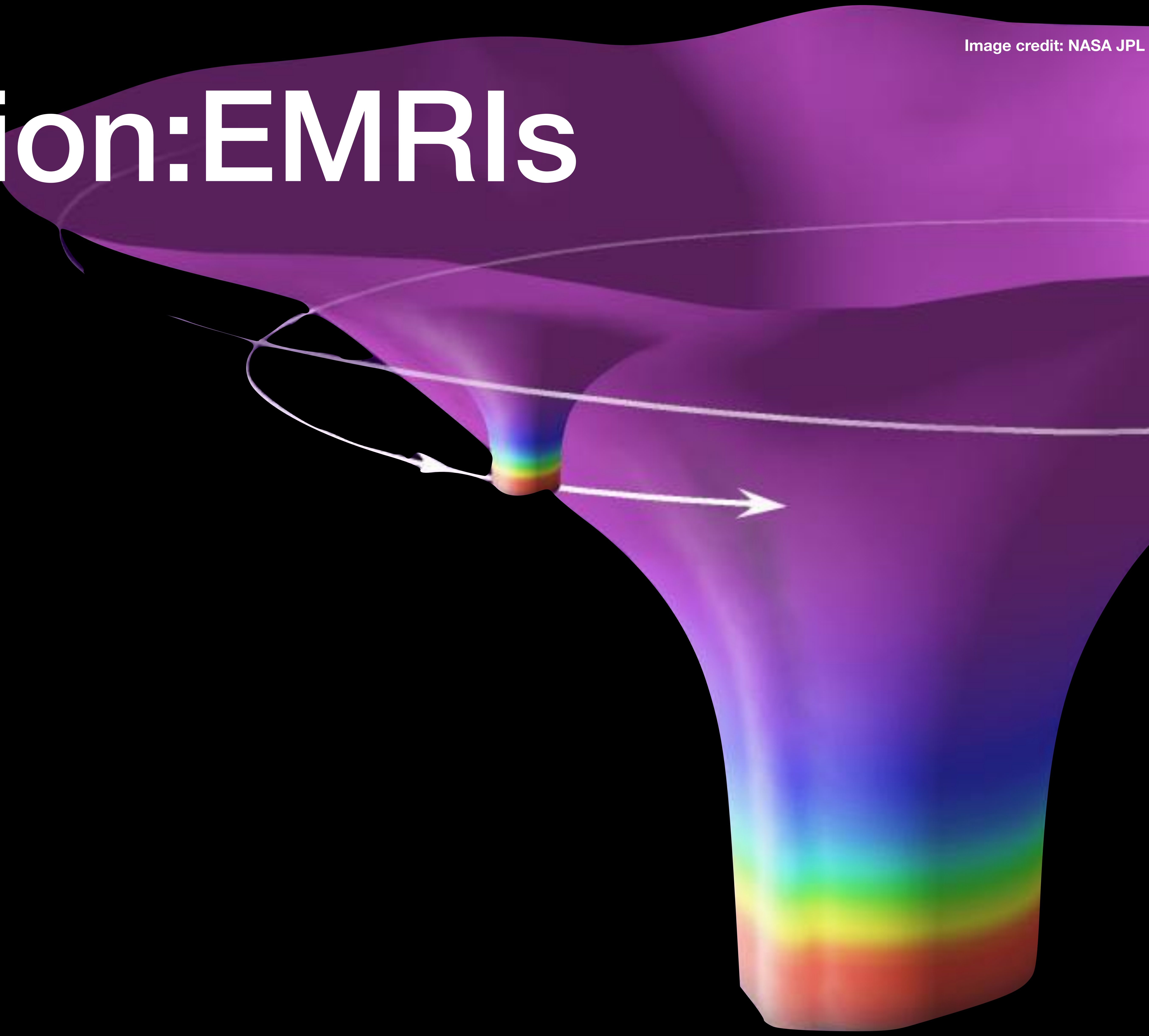


Image credit: NASA/Simon Barke

Launch date: 2034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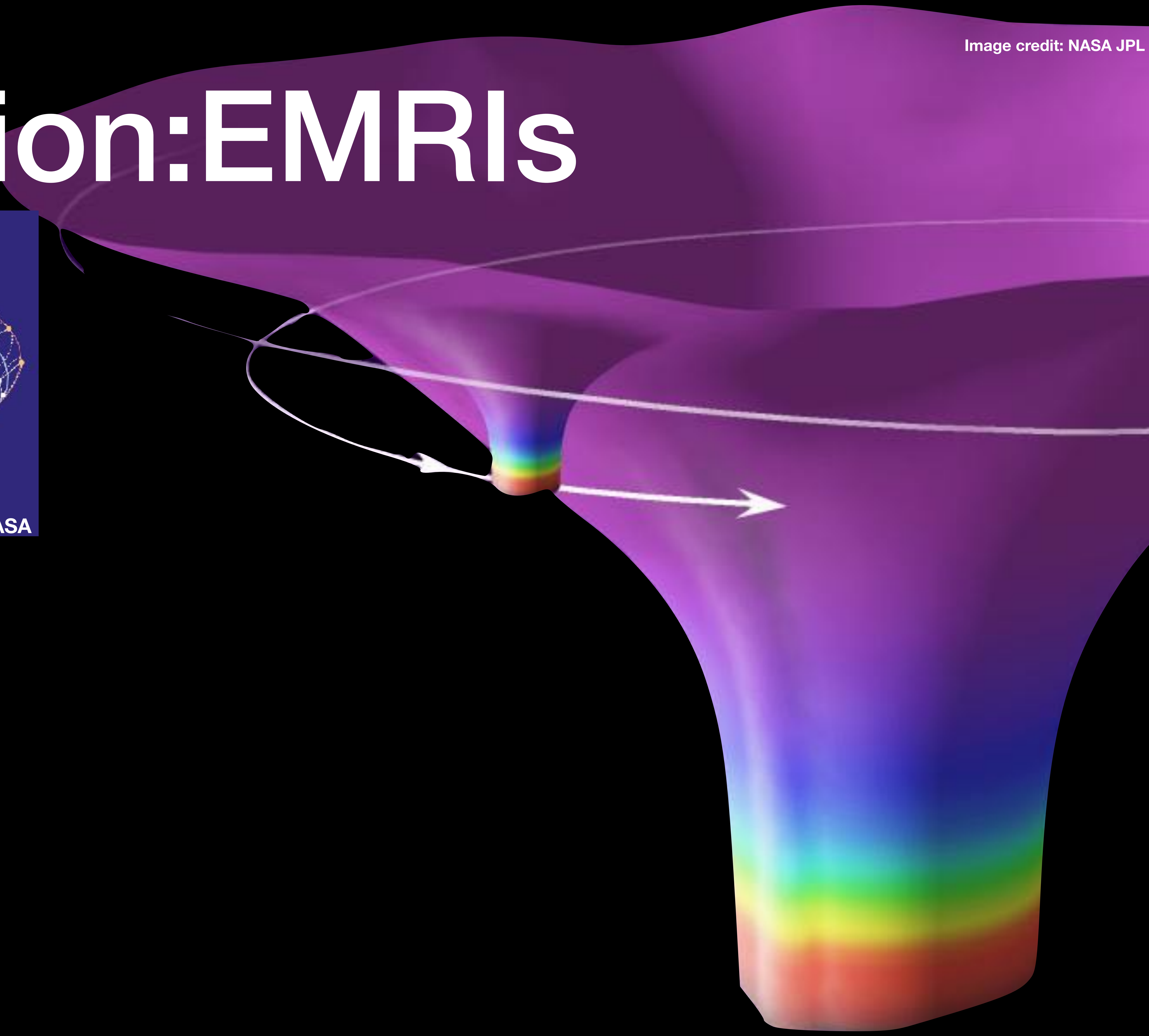
Motivation:EMRIs



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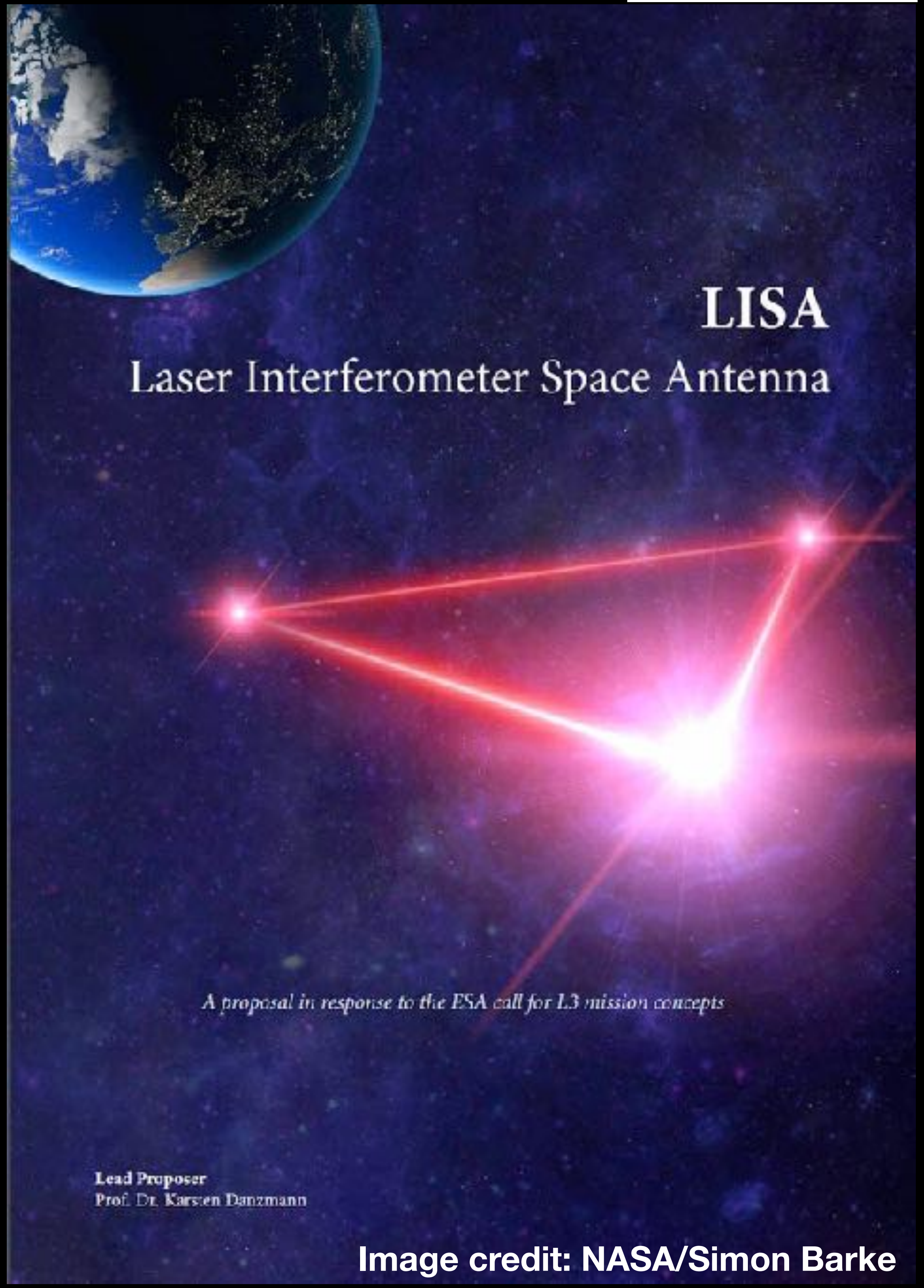

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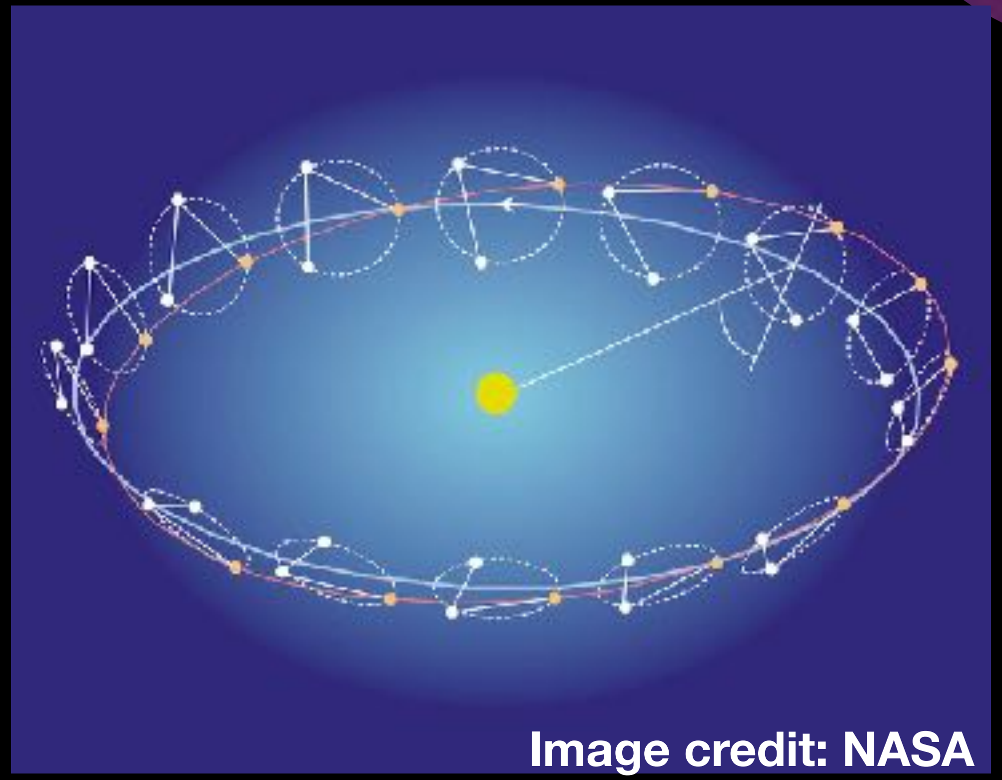

Image credit: NASA

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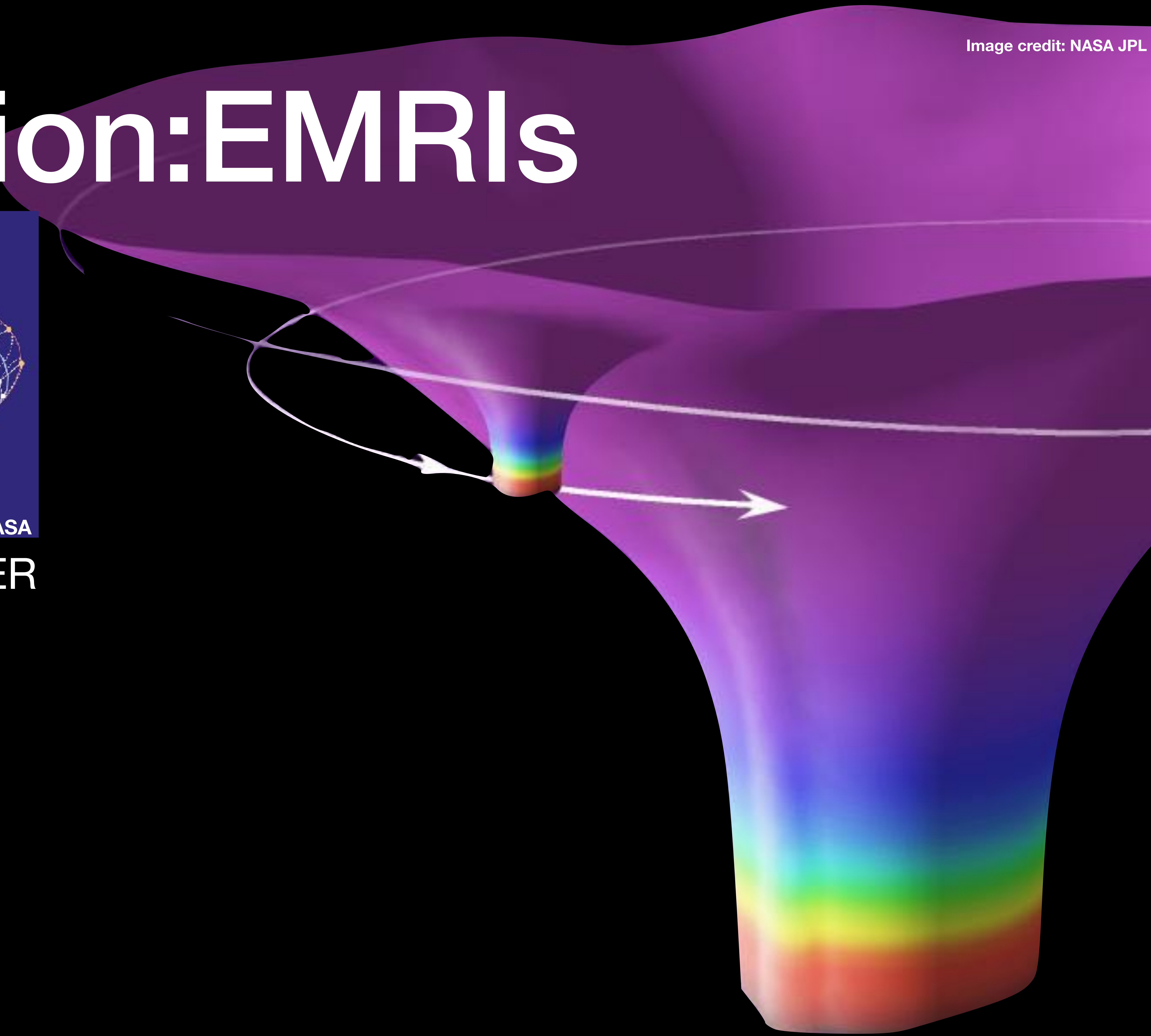

Image credit: NASA JPL

- LISA

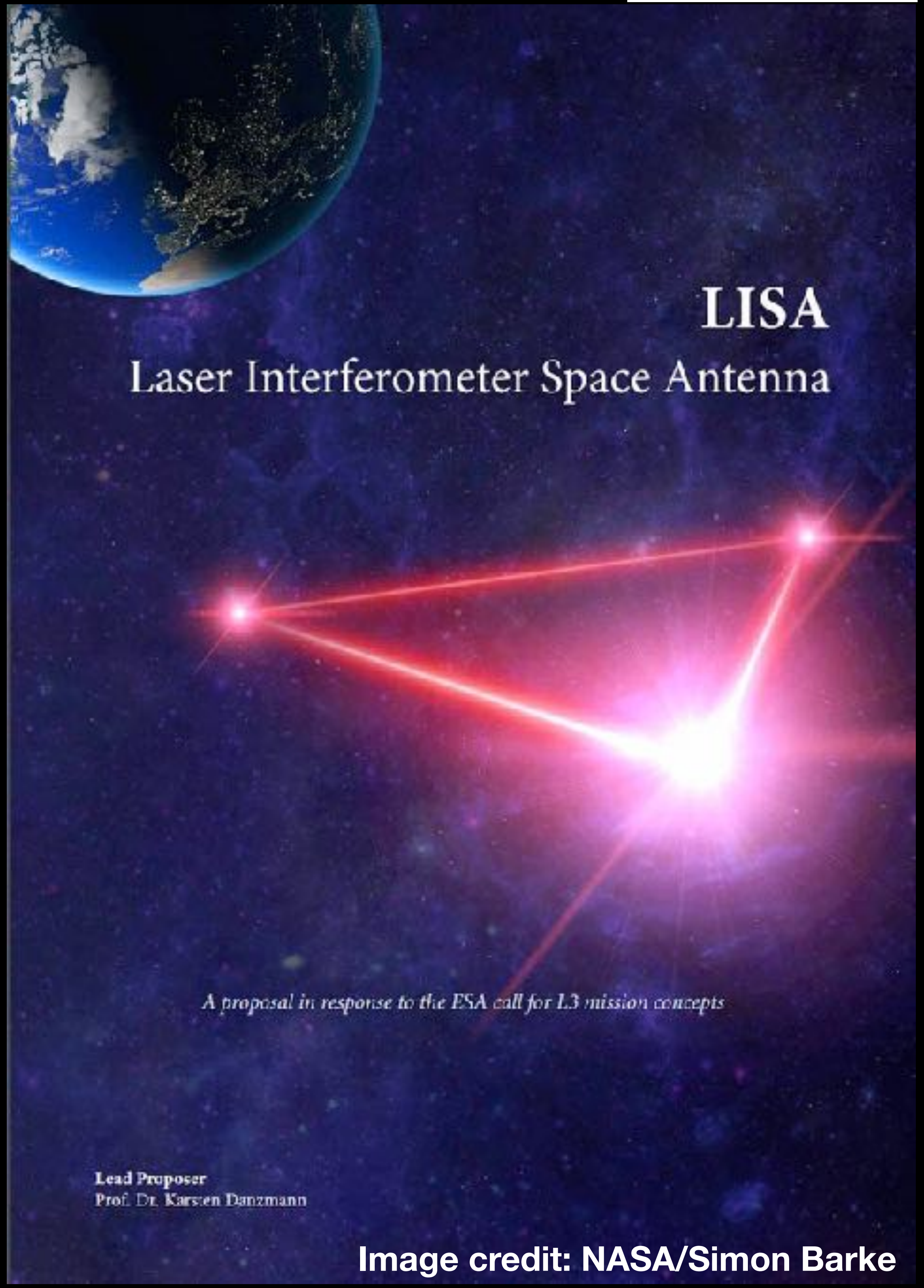

Image credit: NASA/Simon Barke

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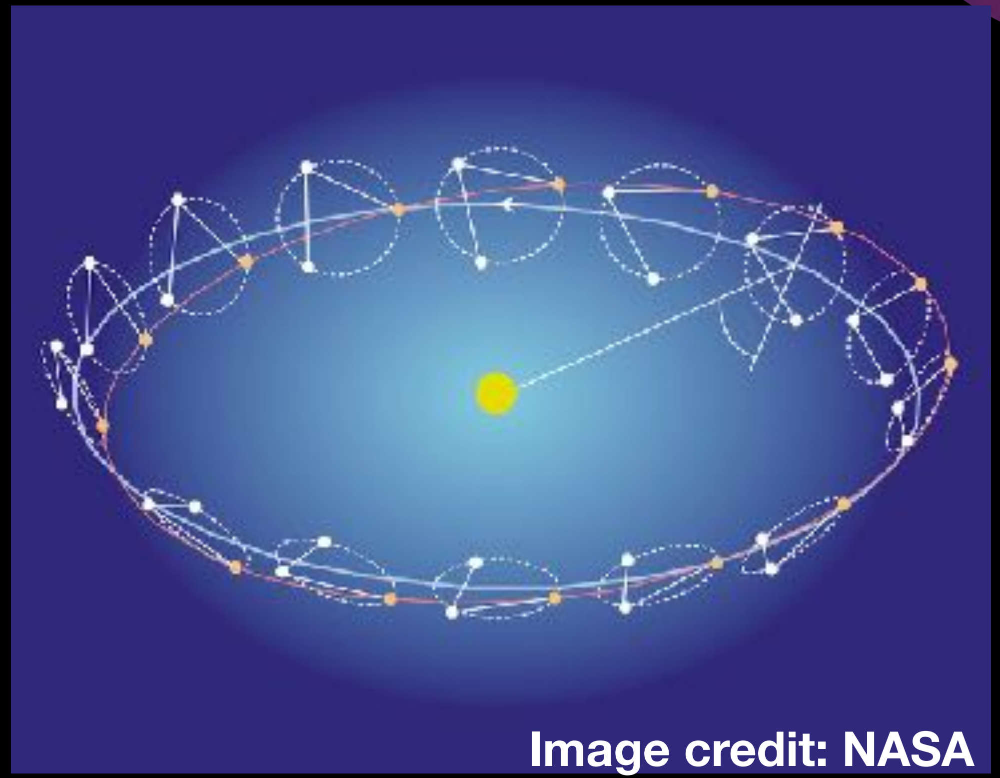
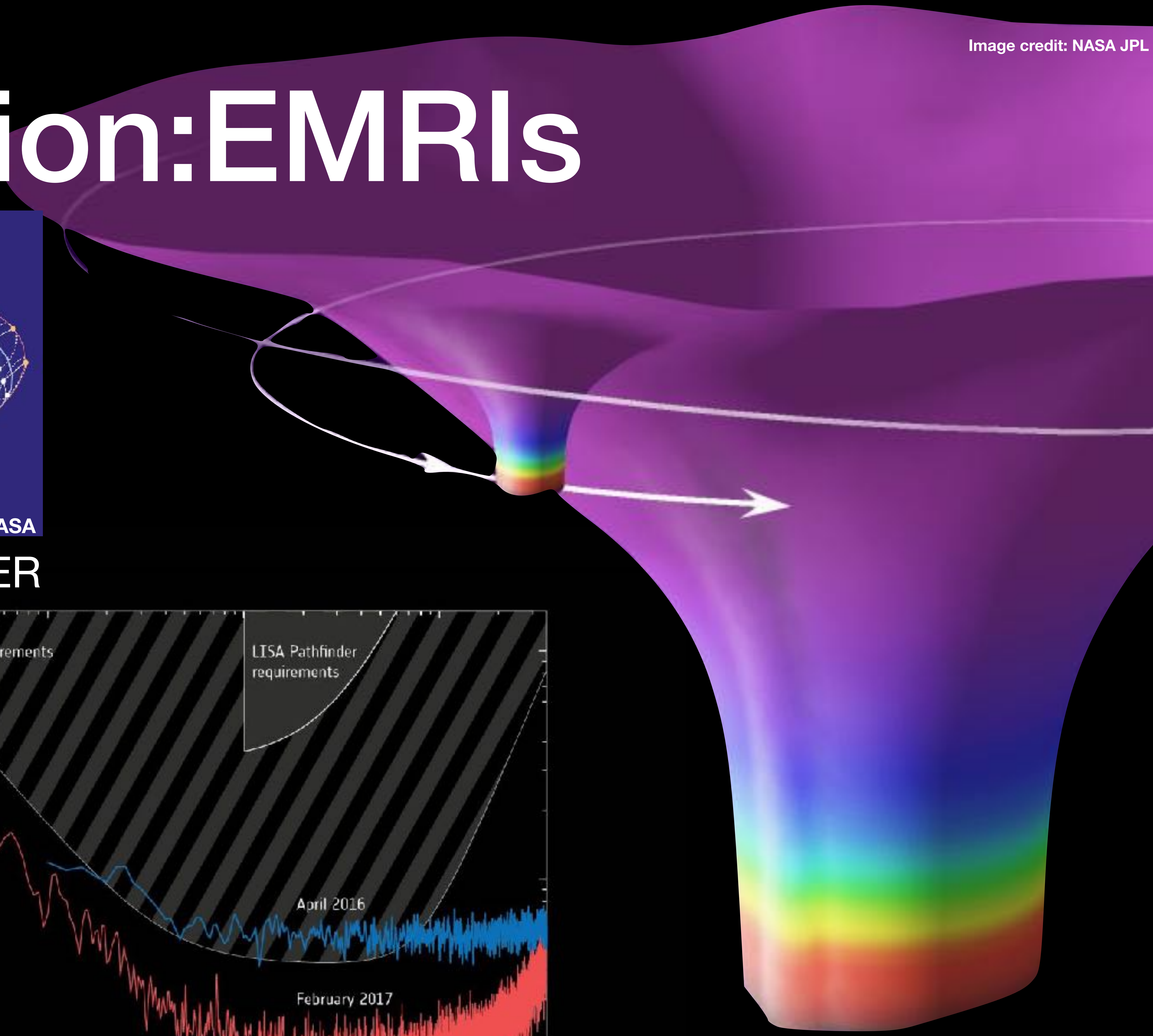


Image credit: NASA

- LISA PATHFINDER

Motivation:EMRIs



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- LISA

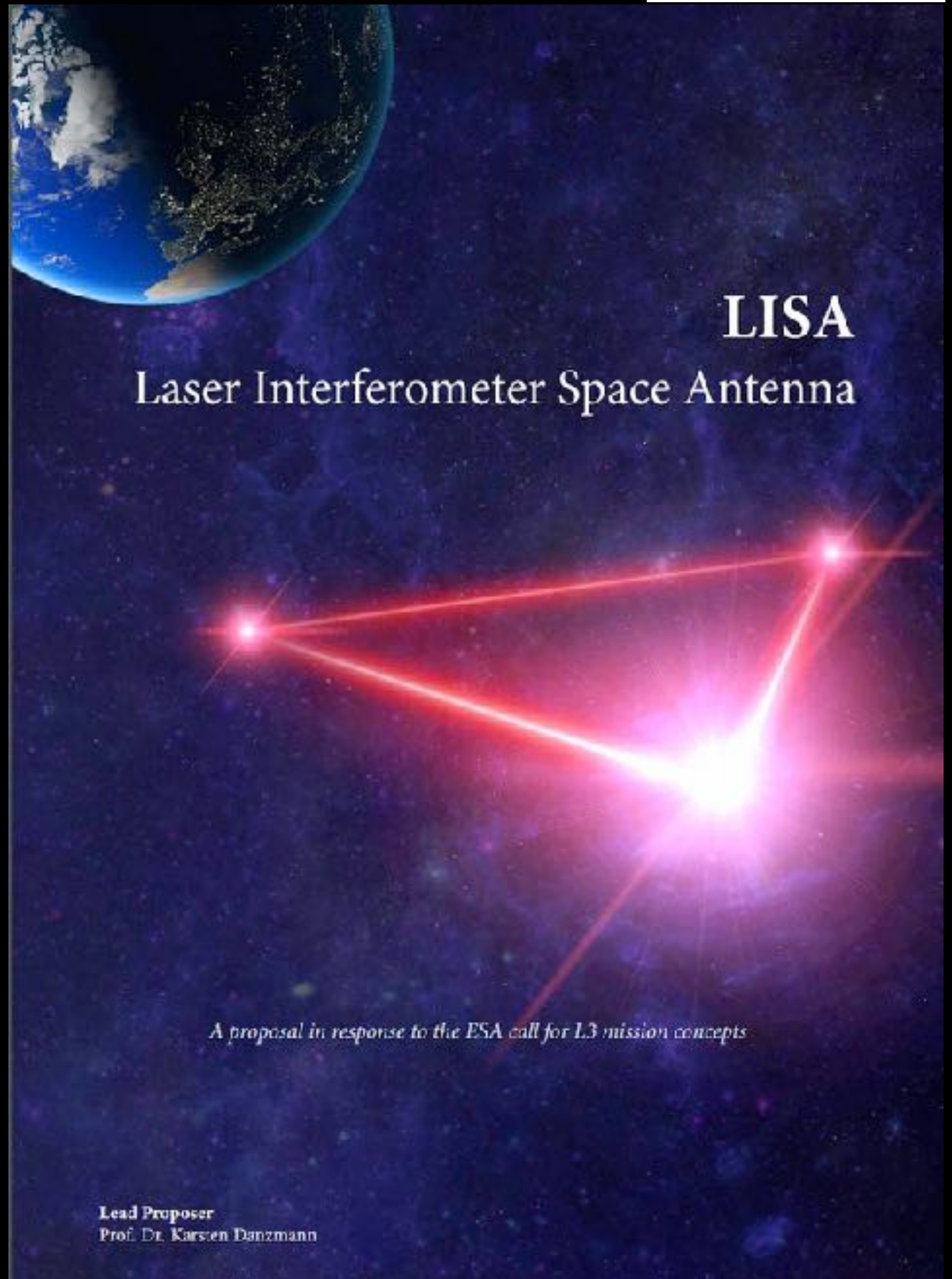


Image credit: NASA/Simon Barke

Launch date: 2034

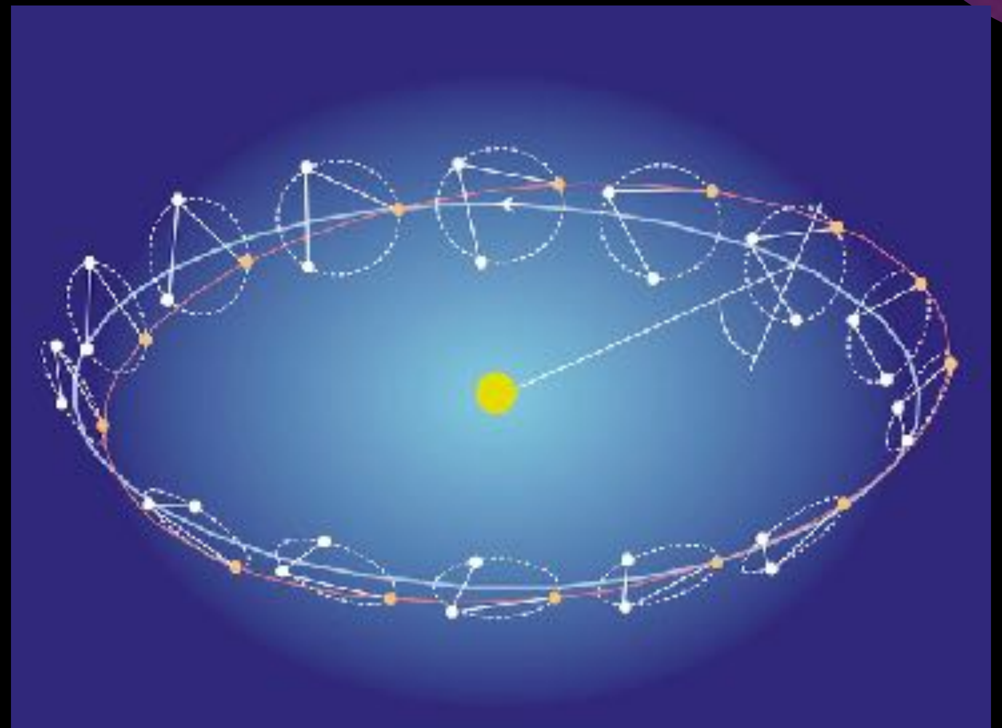
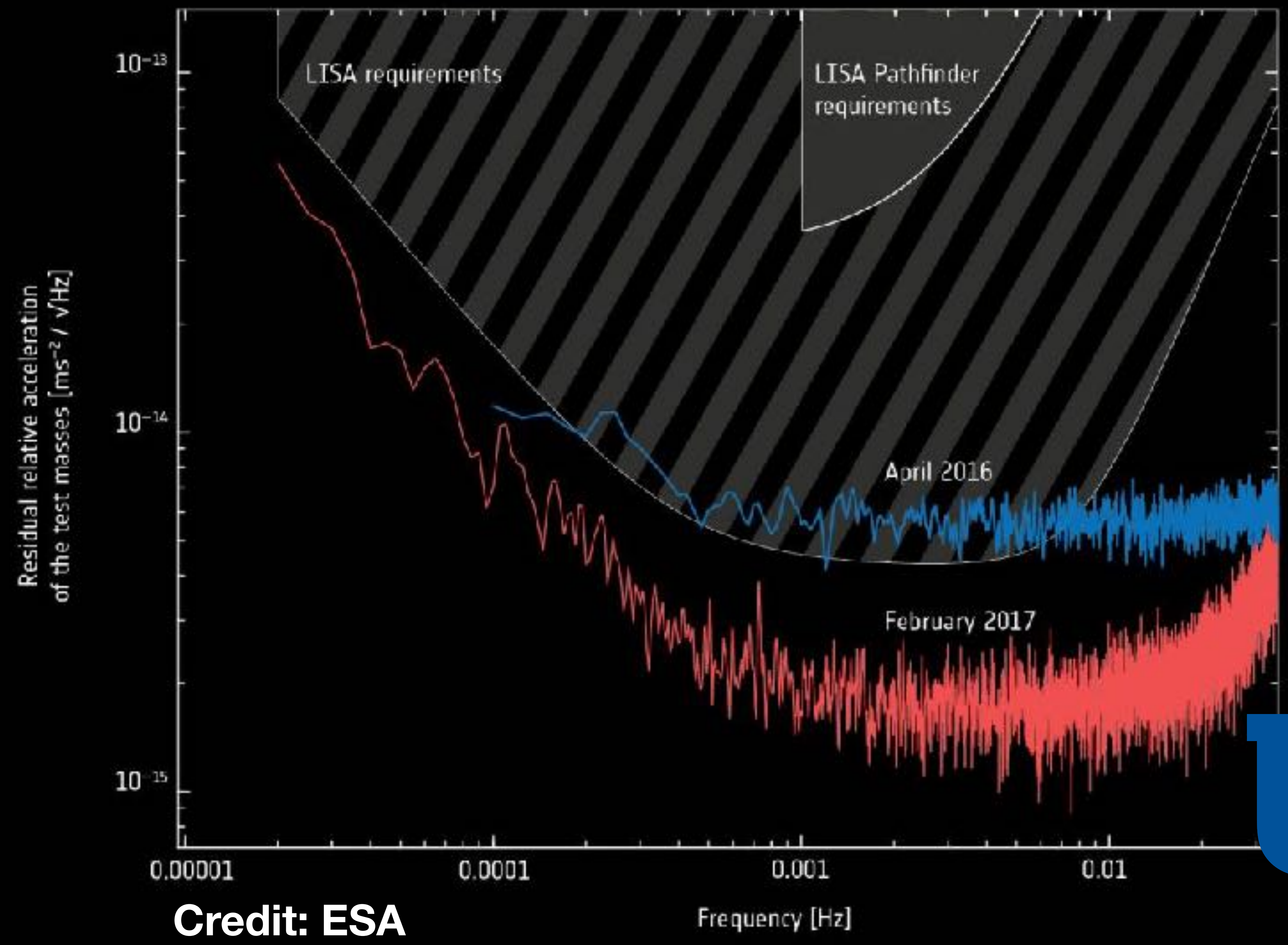


Image credit: NASA

- LISA PATHFINDER



Credit: ESA

Motivation: EMRIs

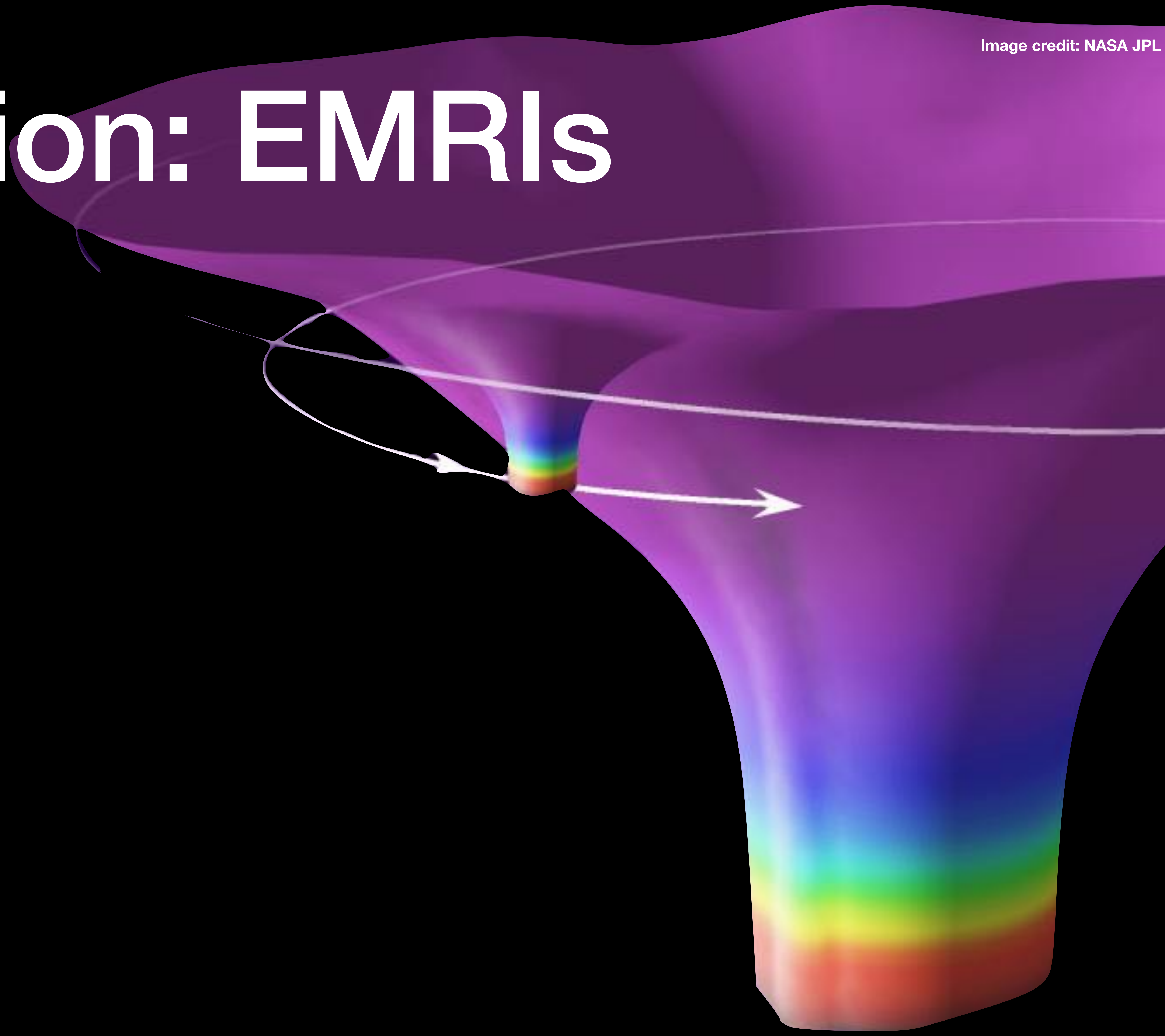


Image credit: NASA JPL

Motivation: EMRIs

- Deepest view of galactic nuclei

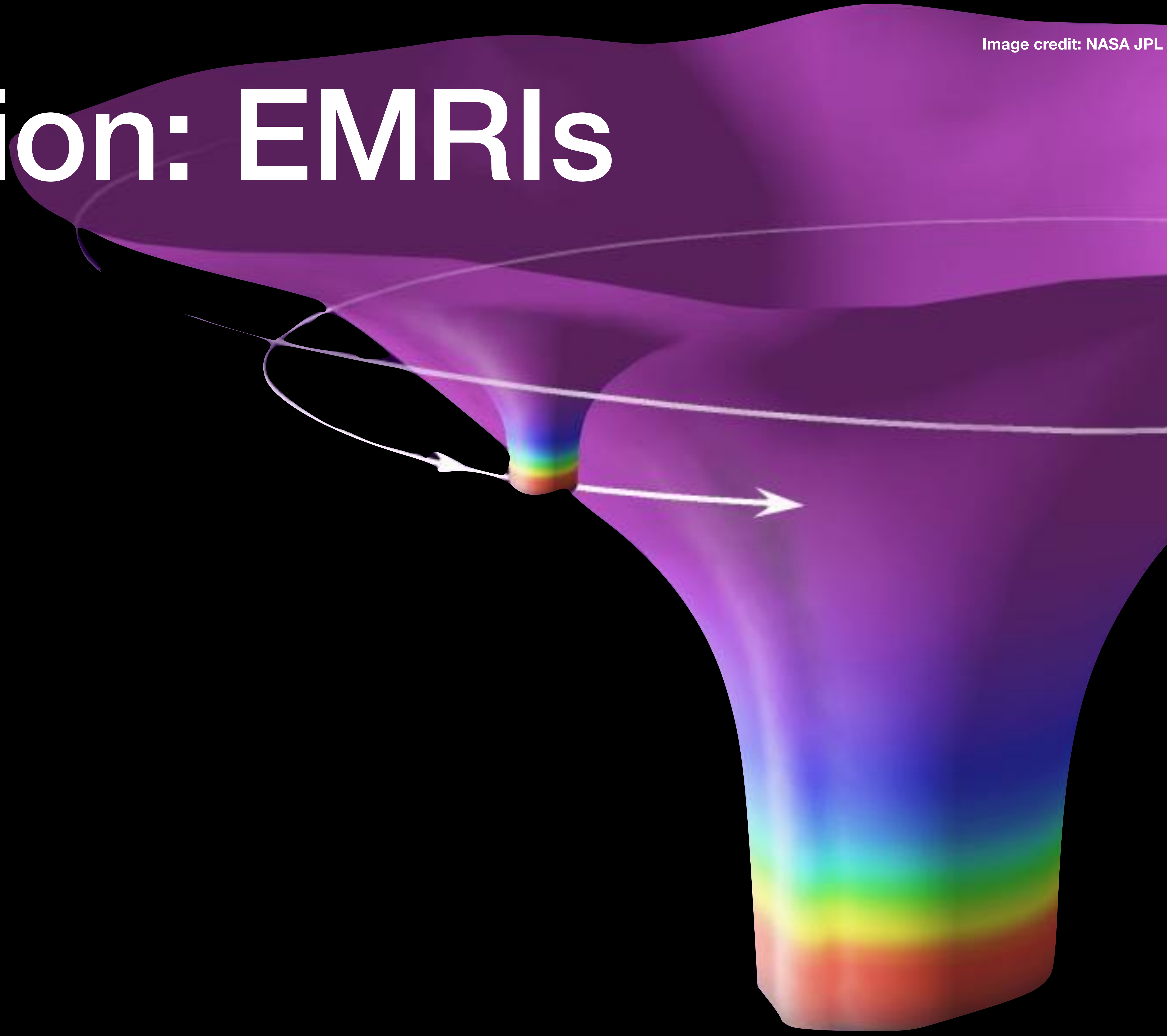


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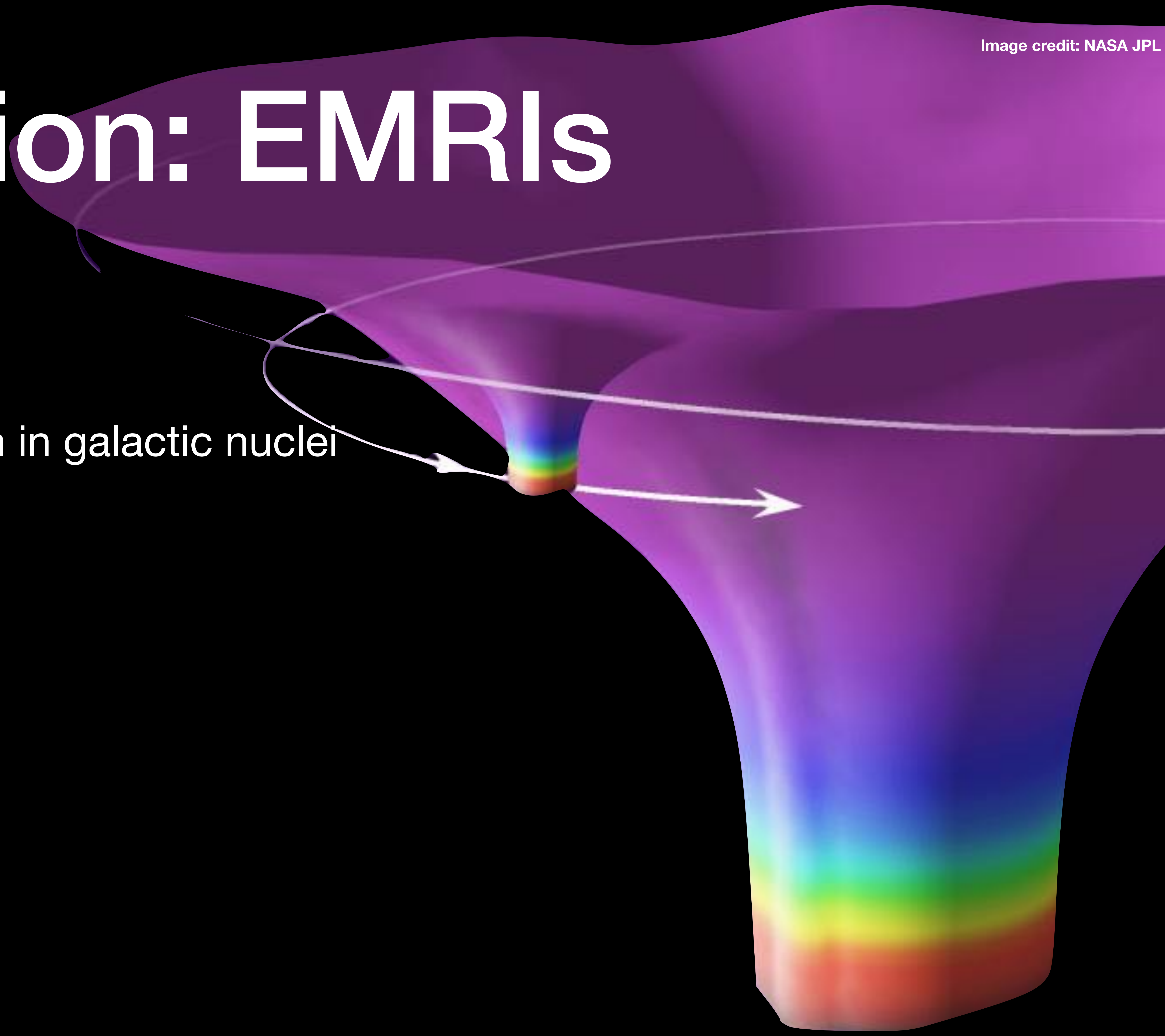
Image credit: NASA JPL

Motivation: EMRIs

- Deepest view of galactic nuclei
    - stellar mass compact body distribution in galactic nuclei



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Motivation: EMRIs

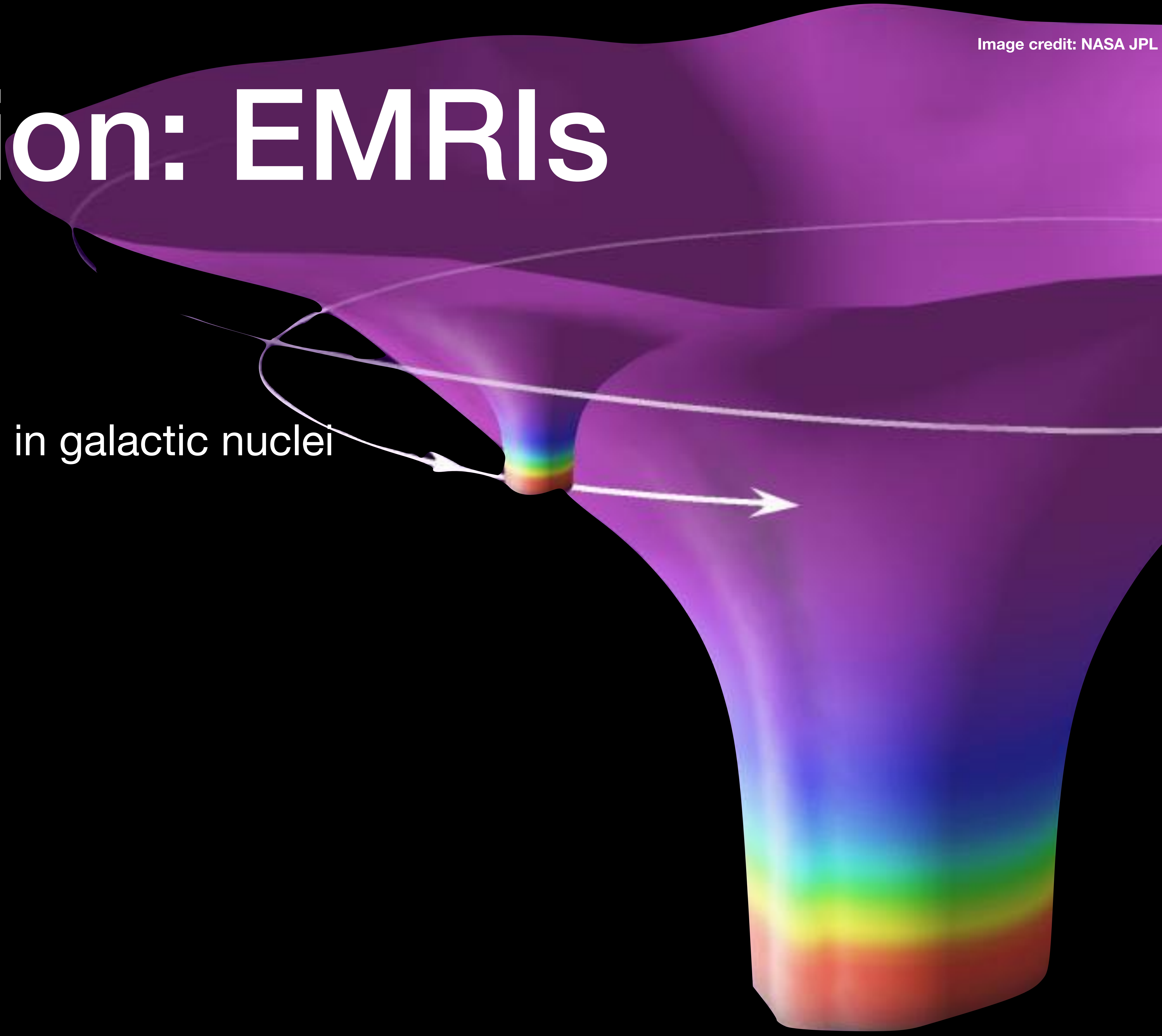
Image credit: NASA JPL

- Deepest view of galactic nuclei
  - stellar mass compact body distribution in galactic nuclei
- Intermediate mass black holes - IMRI’s?

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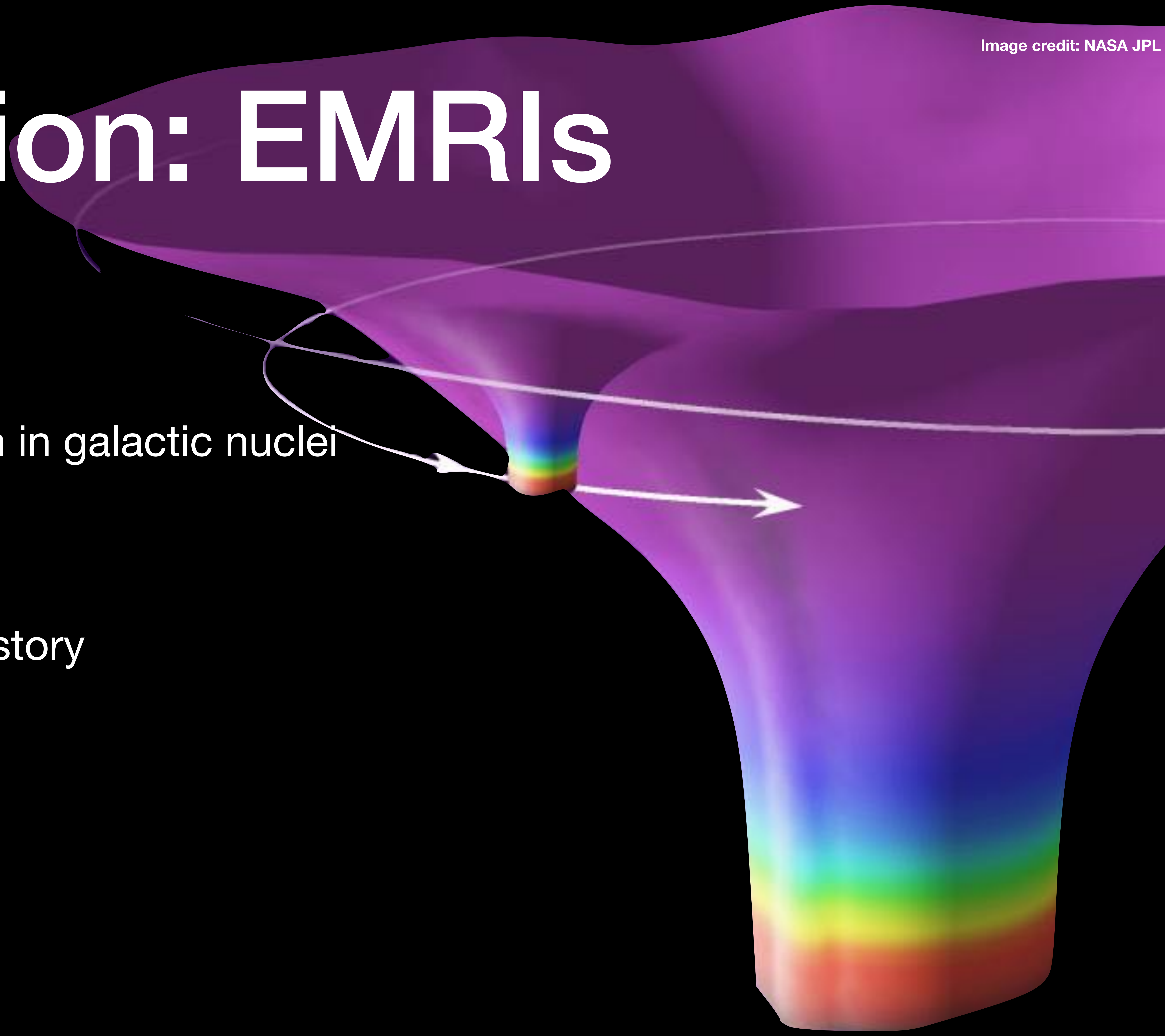
Image credit: NASA JPL

Motivation: EMRIs



- Deepest view of galactic nuclei
  - stellar mass compact body distribution in galactic nuclei
- Intermediate mass black holes - IMRI’s?
- EMRI’s as sirens, we gain acceleration history

Motivation: EMRIs

- Deepest view of galactic nuclei
  - stellar mass compact body distribution in galactic nuclei
- Intermediate mass black holes - IMRI’s?
- EMRI’s as sirens, we gain acceleration history
  - Combined with galaxy redshift curves constrains the hubble constant

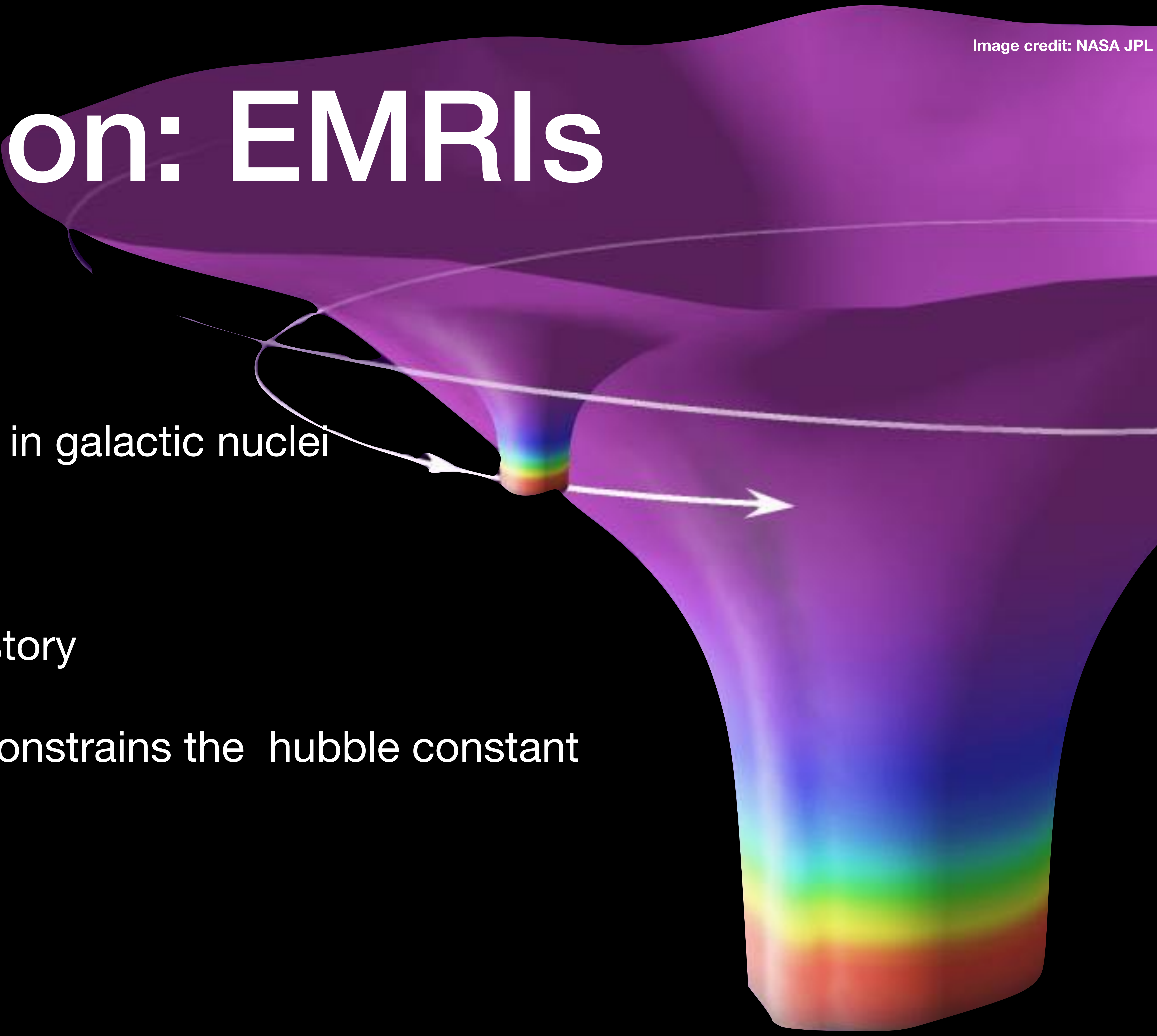


Image credit: NASA JPL

Motivation: EMRIs

- Deepest view of galactic nuclei
    - stellar mass compact body distribution in galactic nuclei
- Intermediate mass black holes - IMRI’s?
- EMRI’s as sirens, we gain acceleration history
    - Combined with galaxy redshift curves constrains the hubble constant
- Mapping spacetime geometry

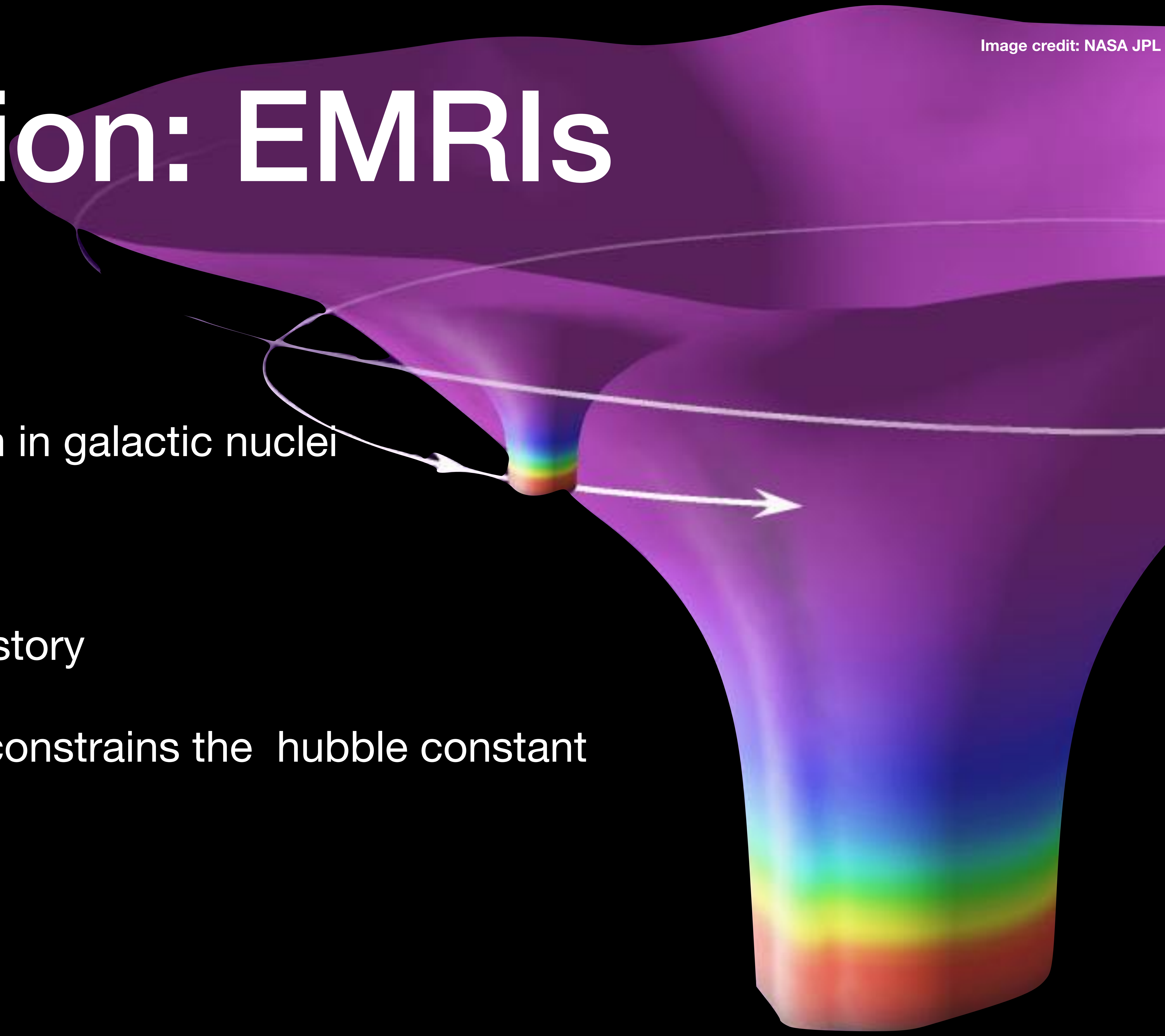


Image credit: NASA JPL

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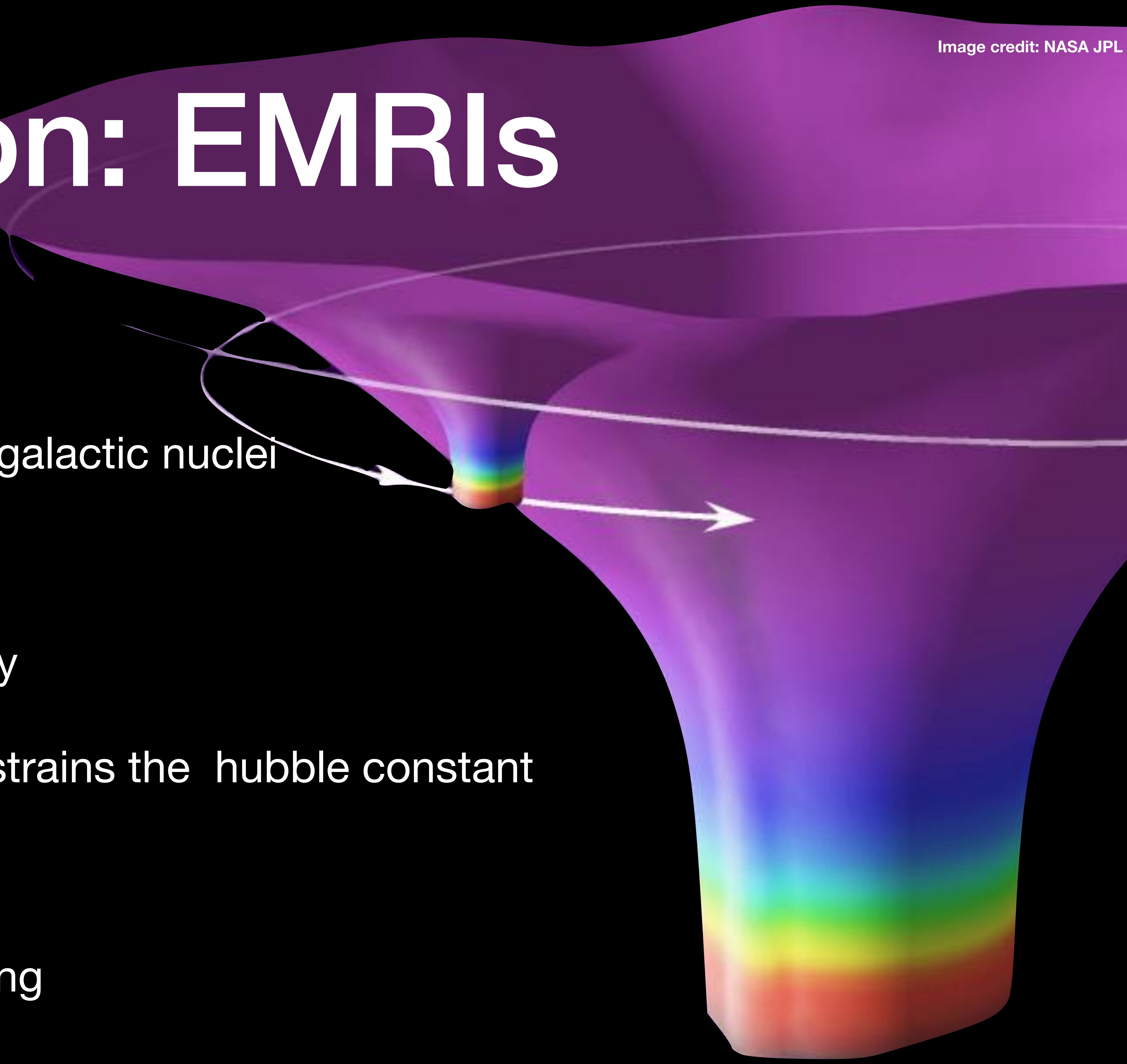
Motivation: EMRIs

- Deepest view of galactic nuclei
    - stellar mass compact body distribution in galactic nuclei
- Intermediate mass black holes - IMRI’s?
- EMRI’s as sirens, we gain acceleration history
    - Combined with galaxy redshift curves constrains the hubble constant
- Mapping spacetime geometry
- Deviations from GR - more orbits, better testing



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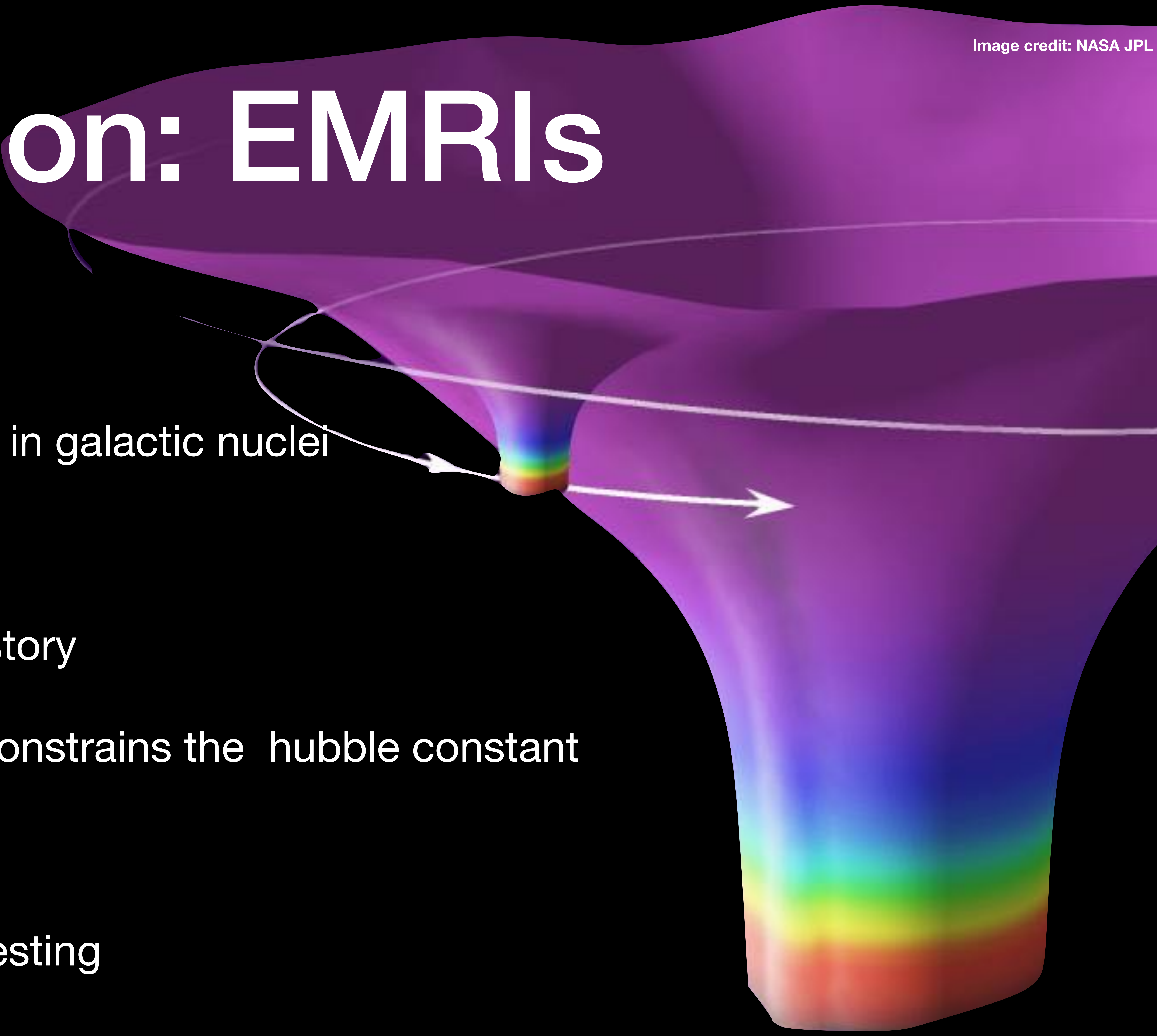
Motivation: EMRIs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- Deepest view of galactic nuclei
  - stellar mass compact body distribution in galactic nuclei
- Intermediate mass black holes - IMRI's?
- EMRI's as sirens, we gain acceleration history
  - Combined with galaxy redshift curves constrains the hubble constant
- Mapping spacetime geometry
- Deviations from GR - more orbits, better testing
  - Tighter constraints on alternative theories of gravity

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Motivation: EMRIs

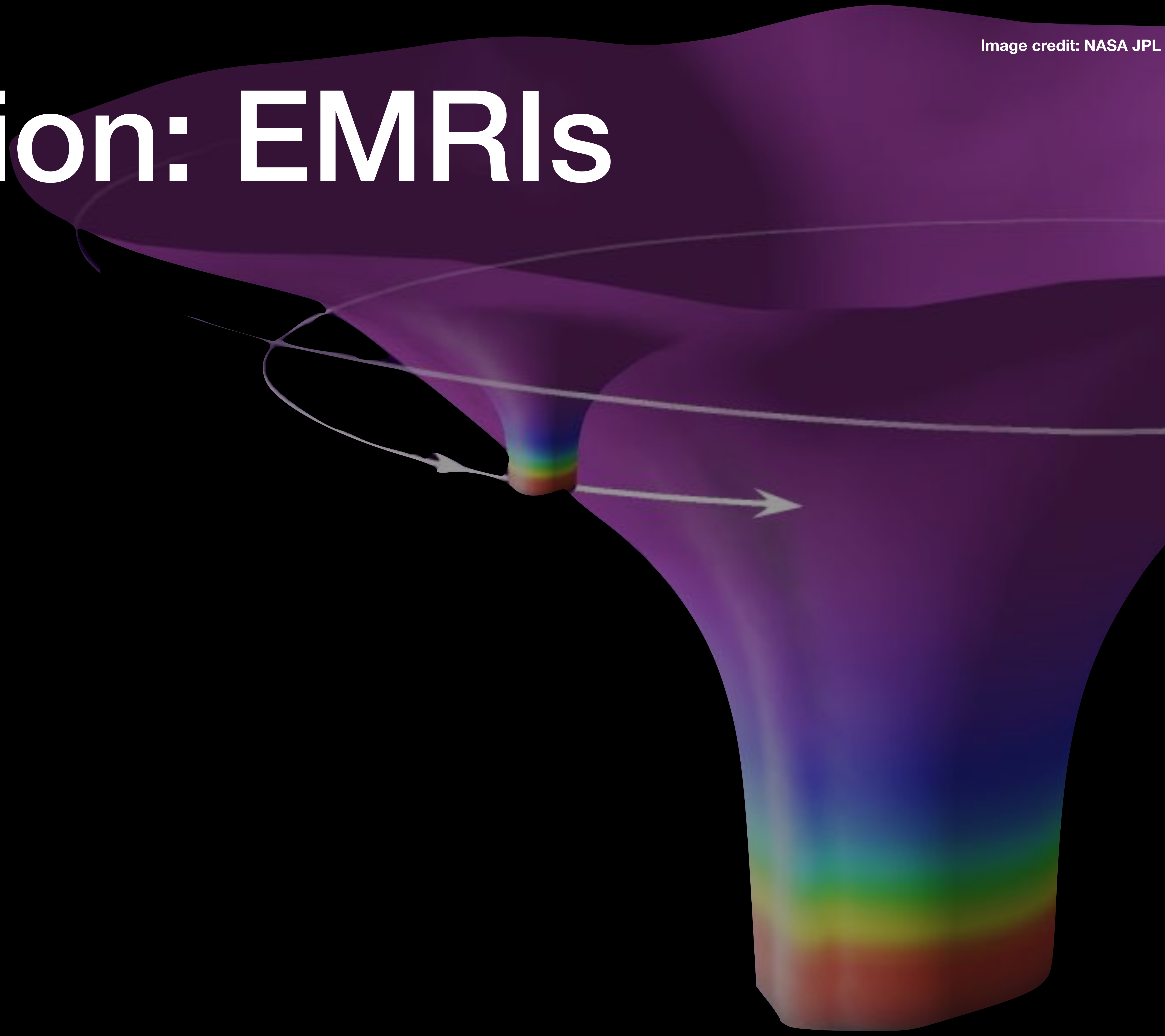

Image credit: NASA JPL

Motivation: EMRIs

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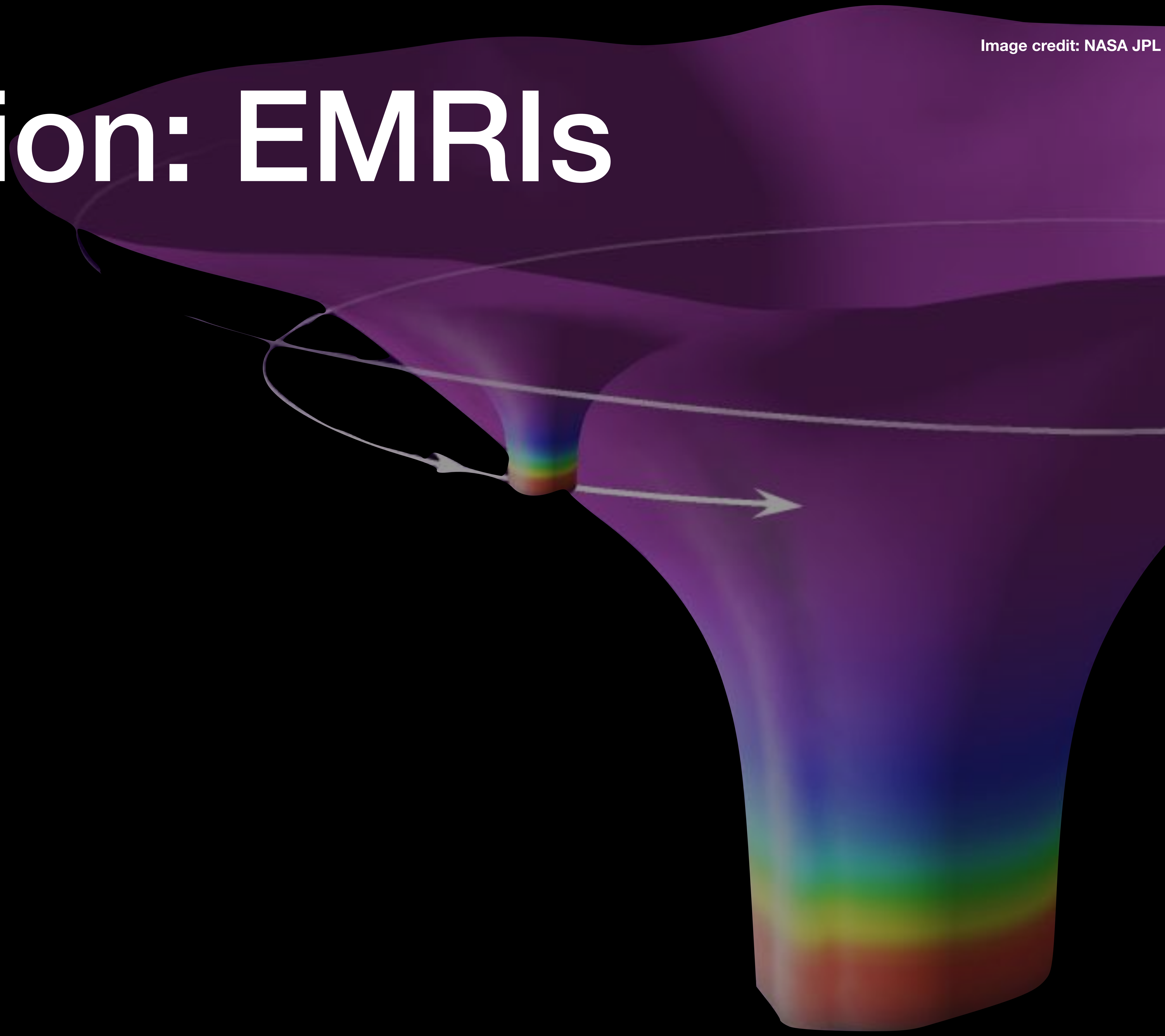


- Parameter Space

Motivation: EMRIs

Image credit: NASA JPL

- Parameter Space

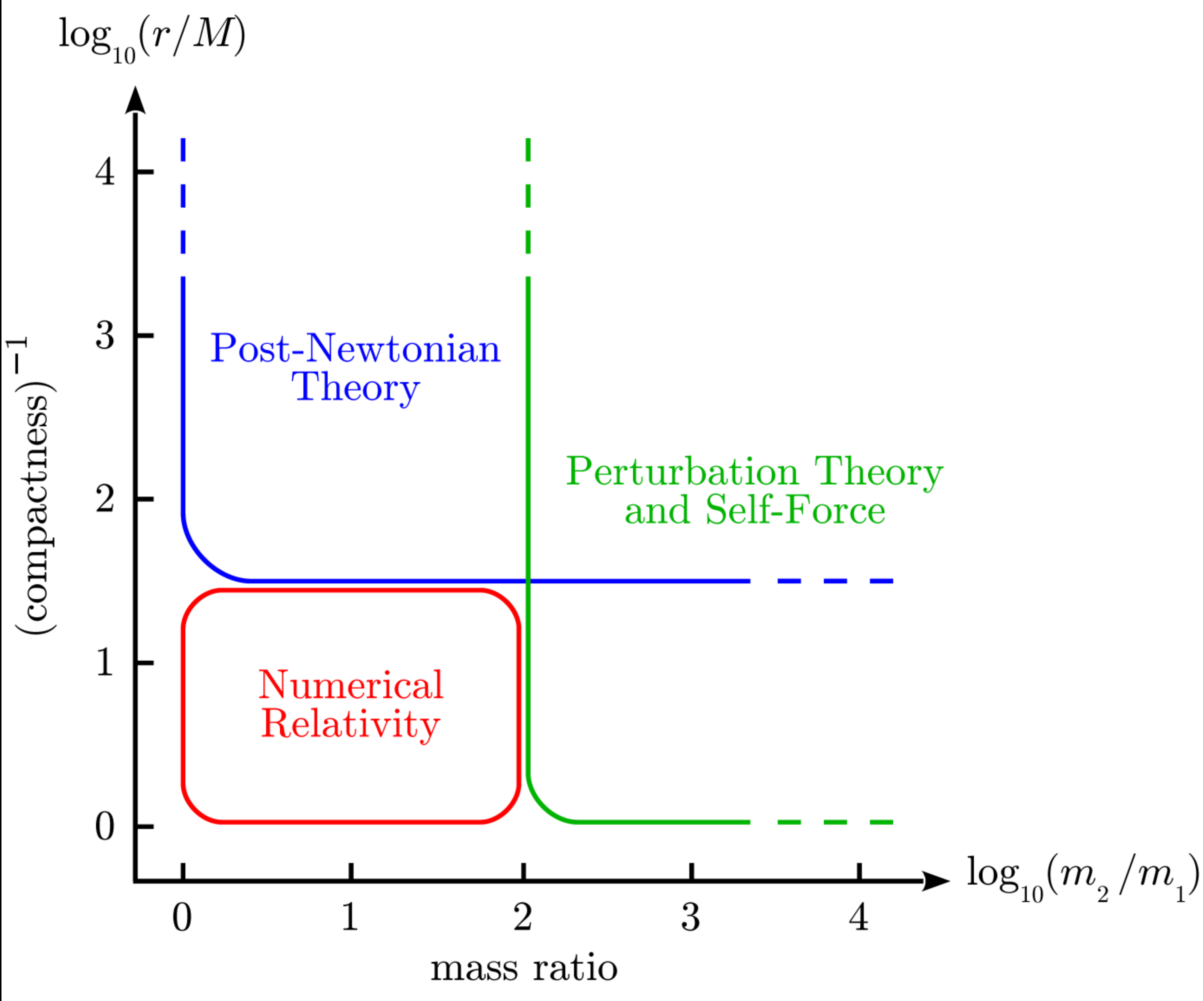
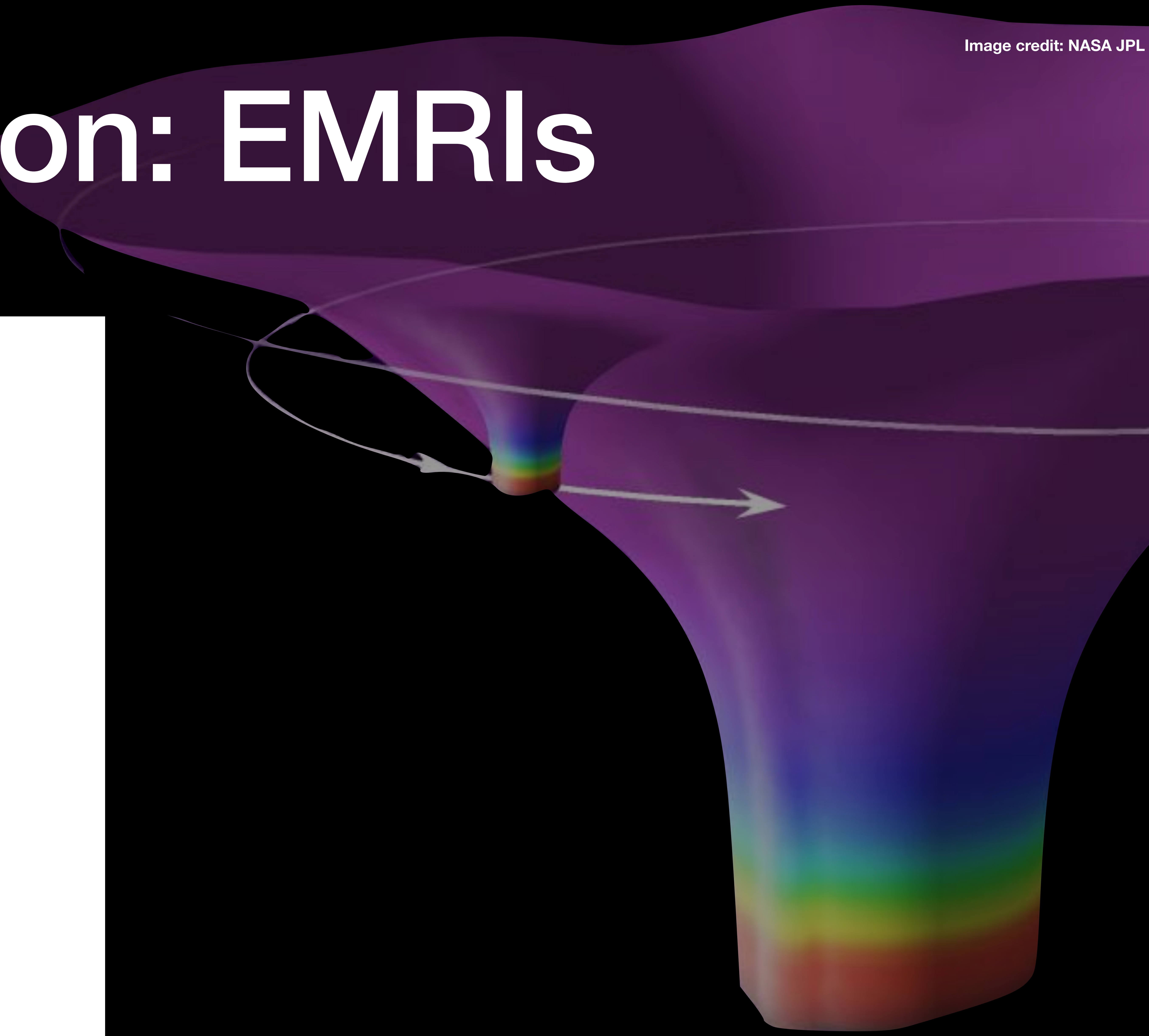


Diagram: Alex Le Tiec, Int. J. Mod. Phys. D. 23:1430022, 2014



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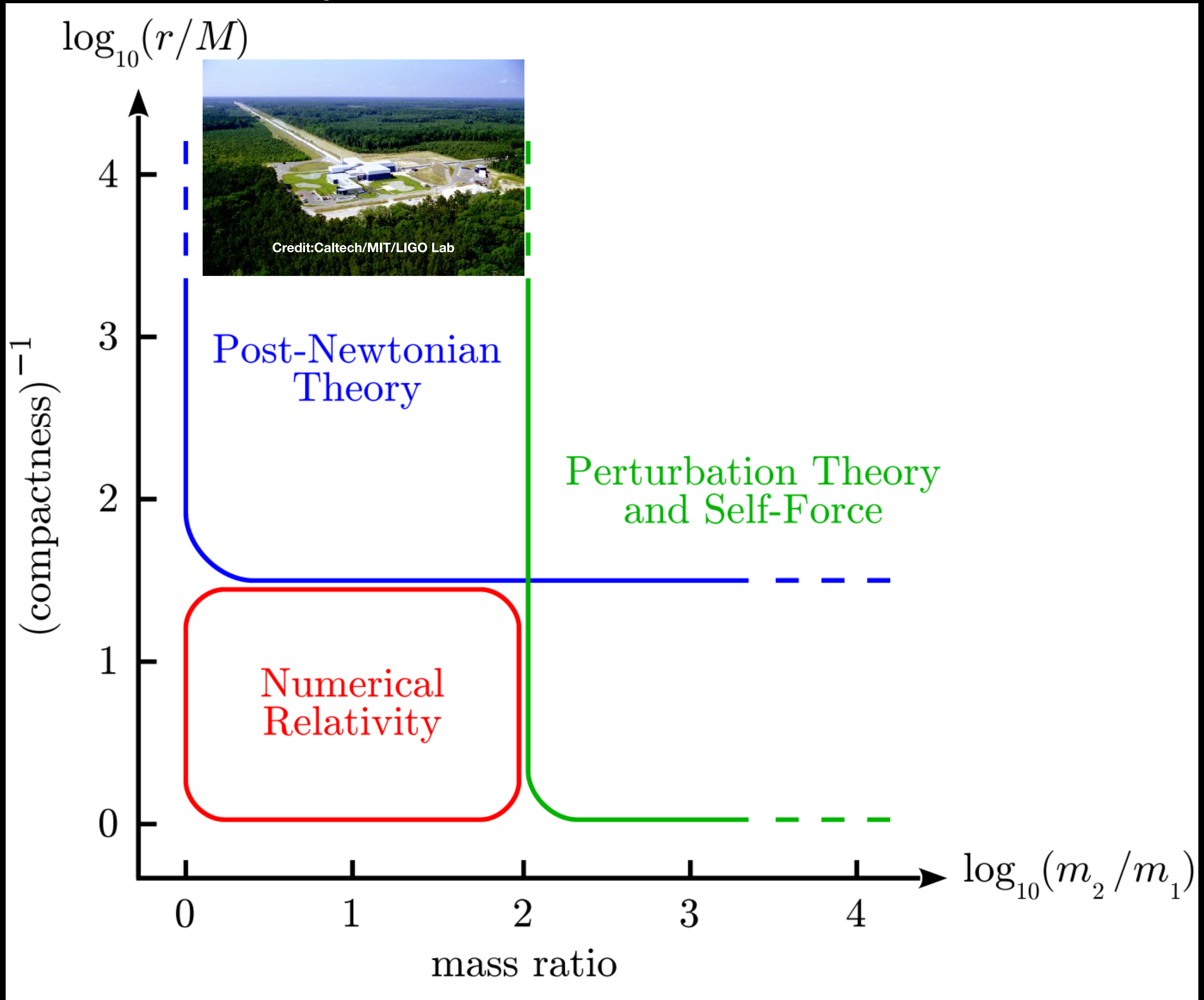
- Parameter Space



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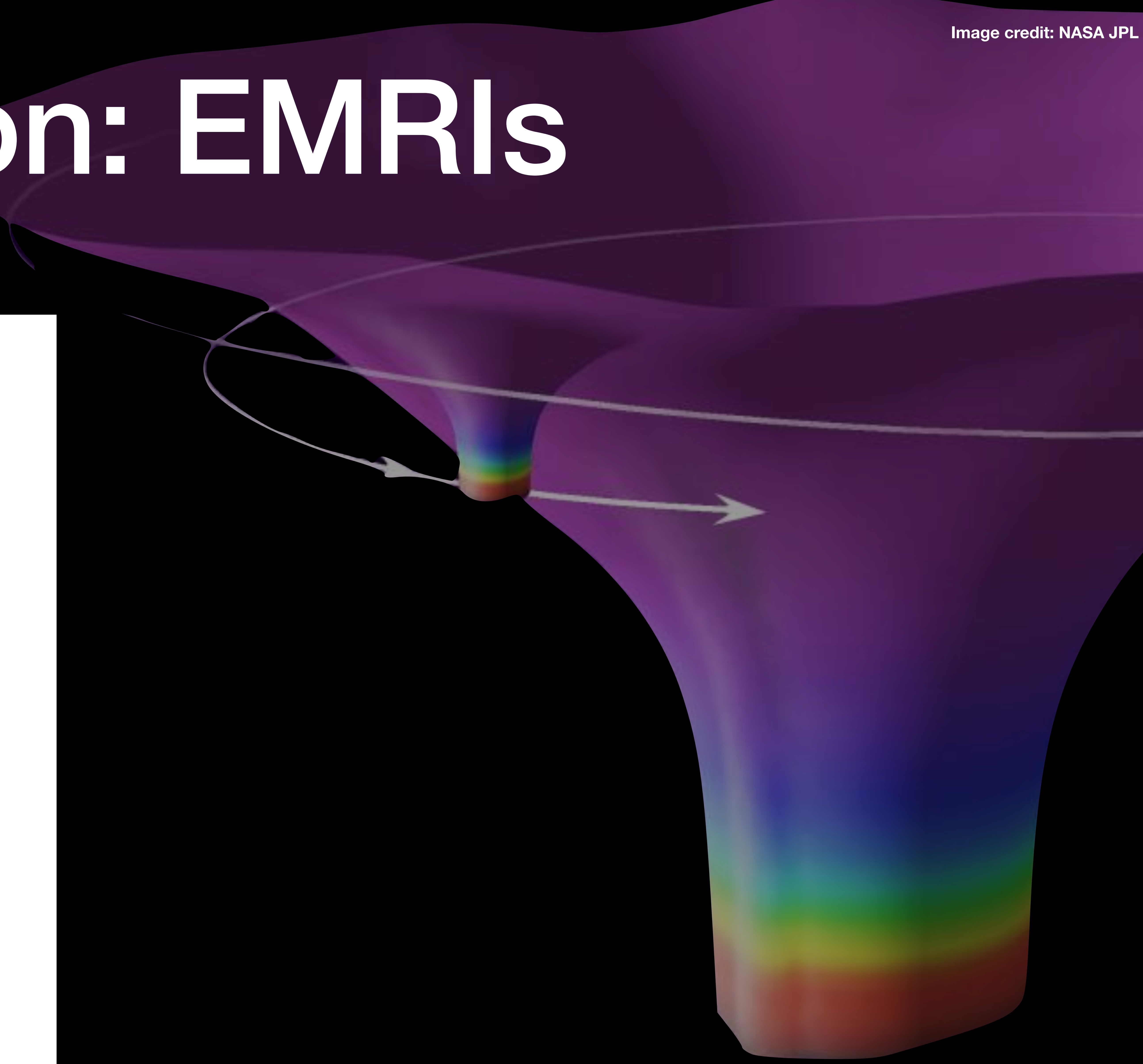

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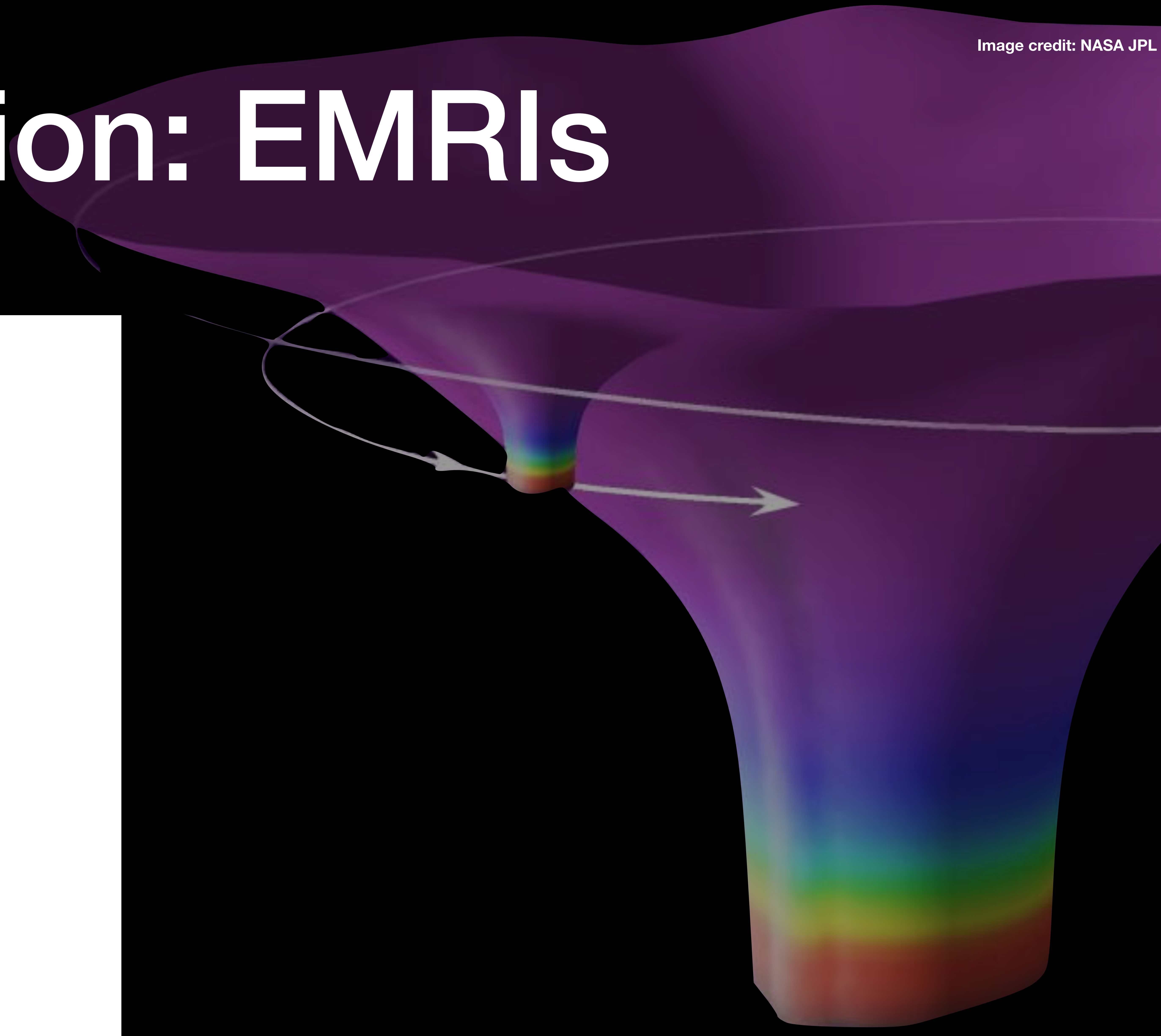
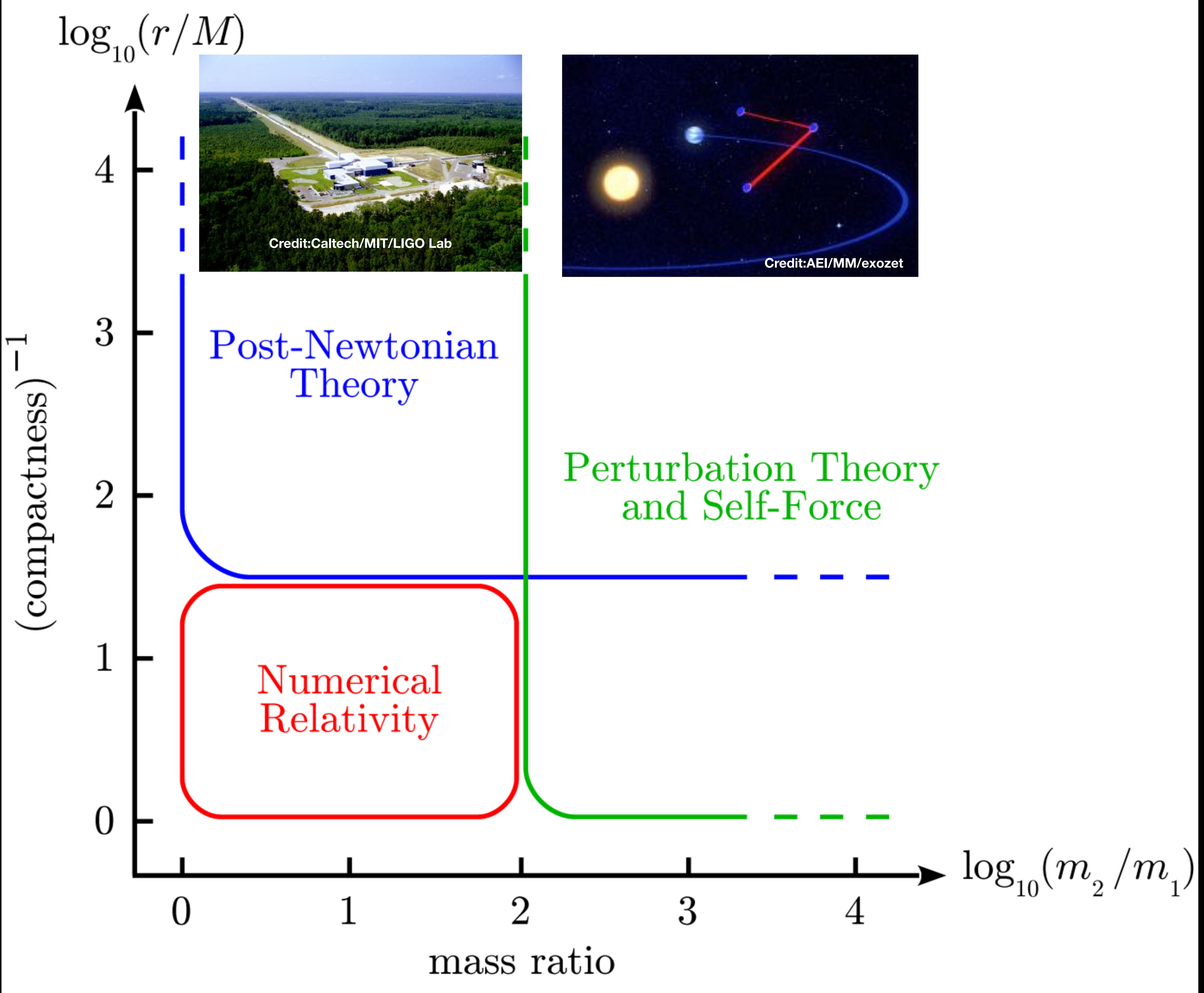
Image credit: NASA JPL



- Parameter Space



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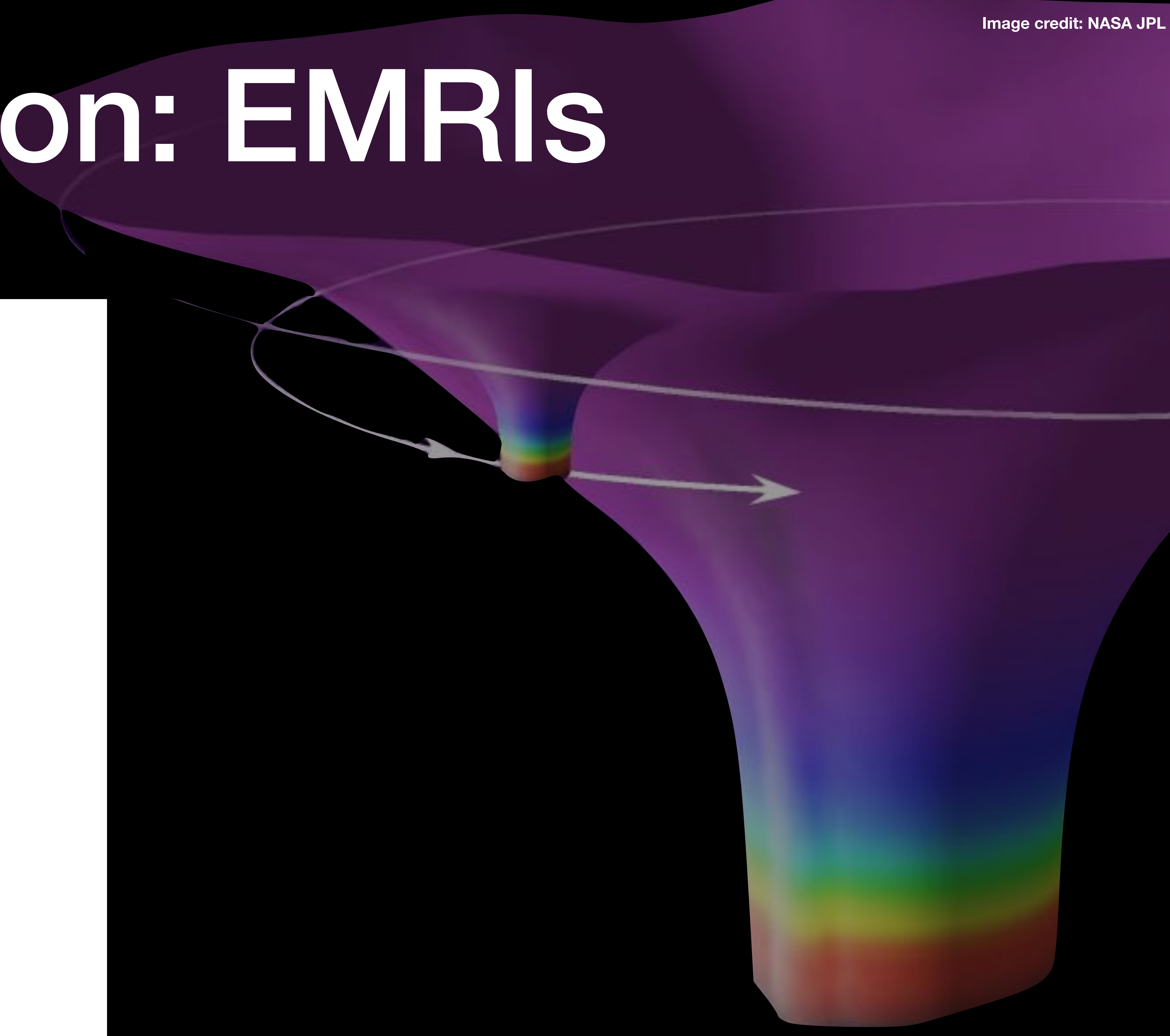

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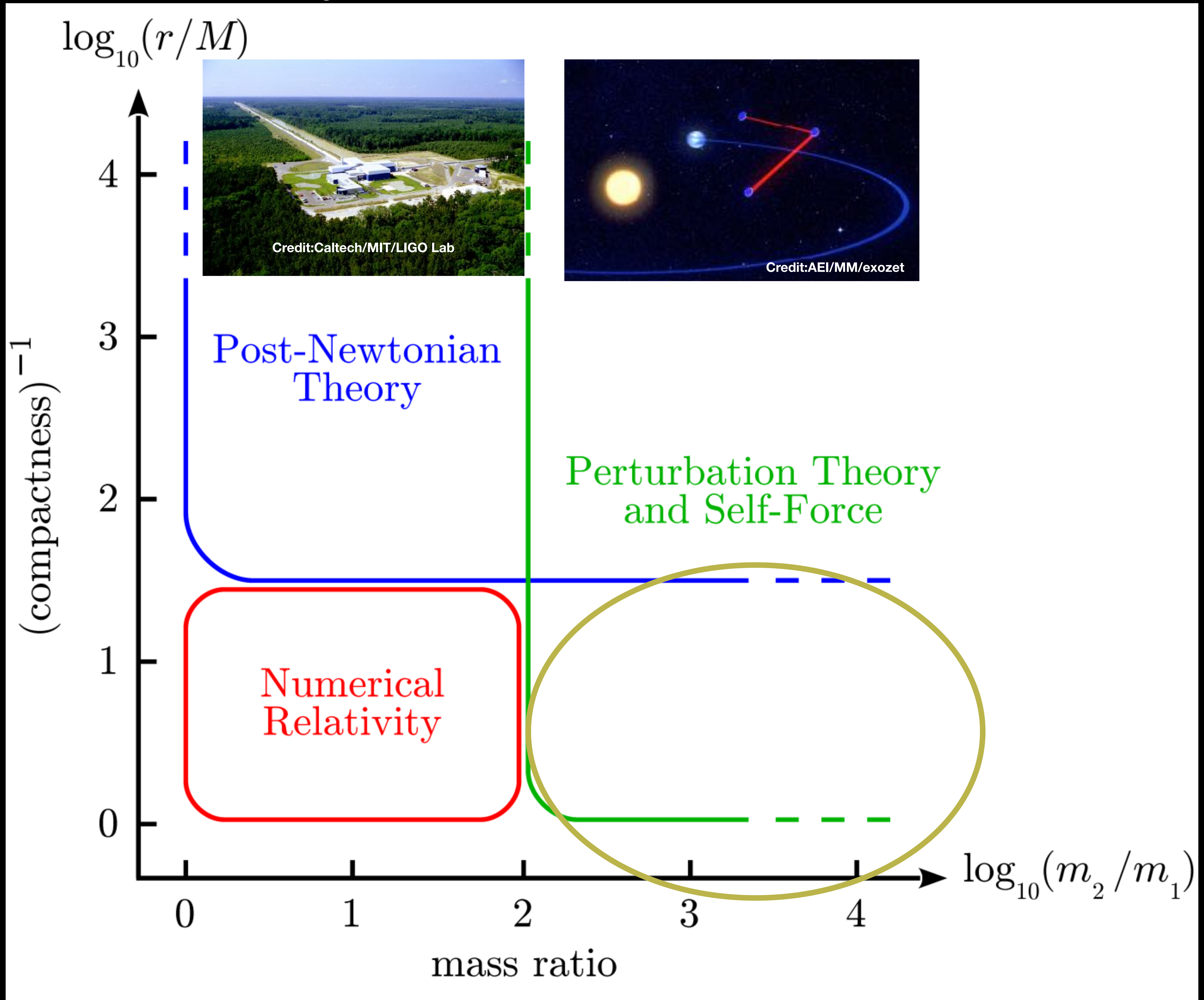

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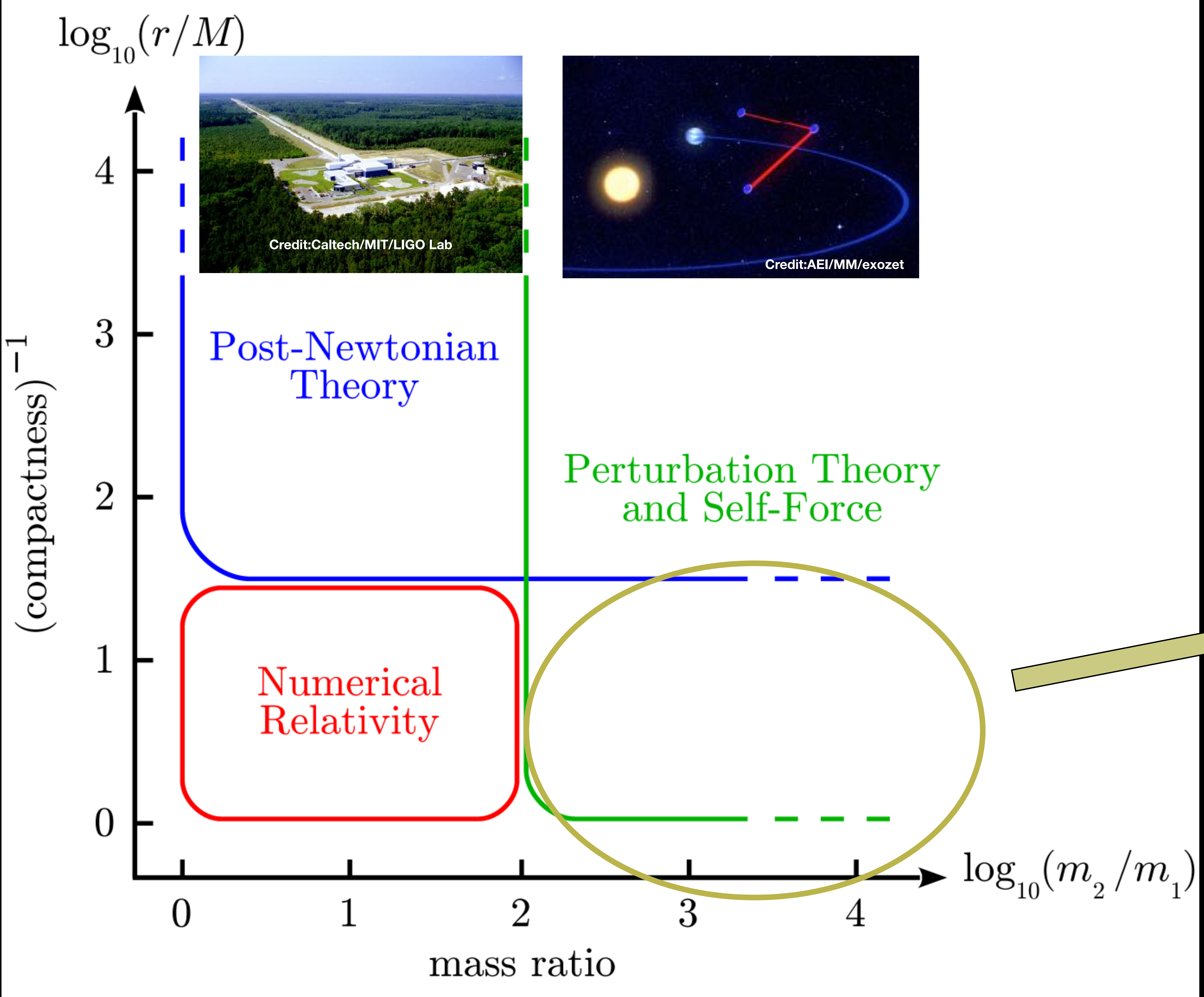
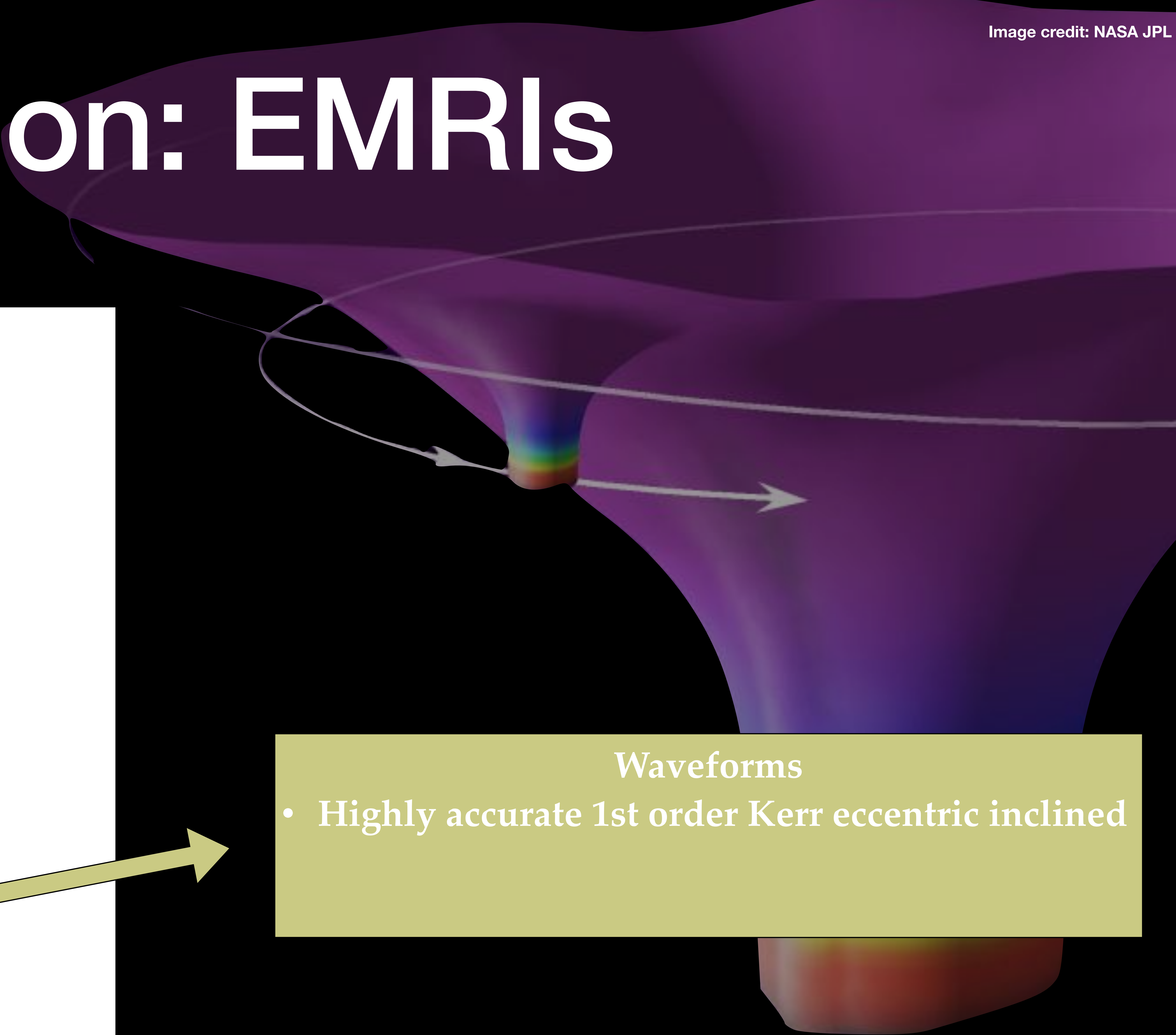


Diagram: Alex Le Tiec, Int. J. Mod. Phys. D. 23:1430022, 2014

Waveforms

Motivation: EMRIs



- Parameter Space

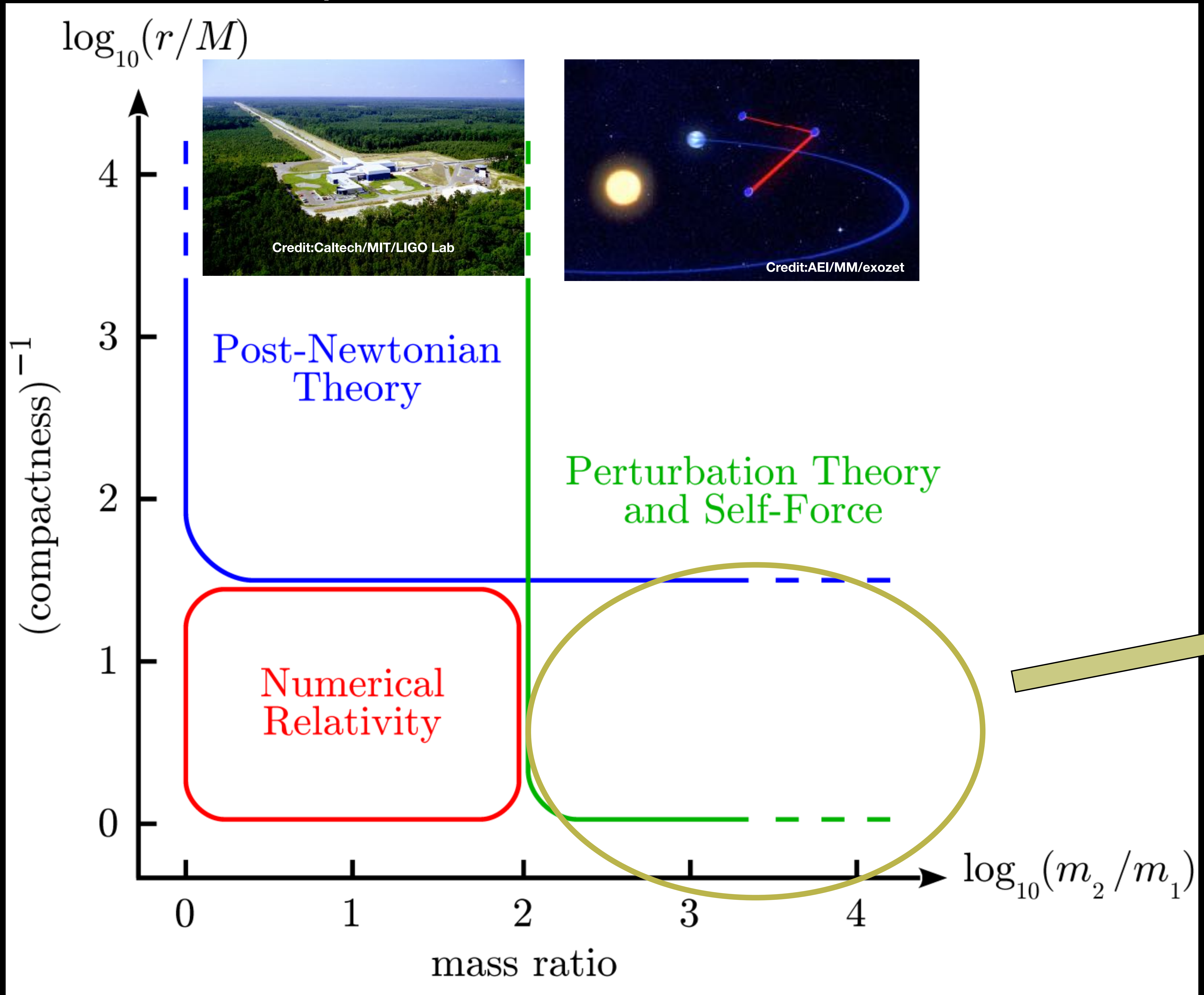


Diagram: Alex Le Tiec, Int. J. Mod. Phys. D. 23:1430022, 2014

Waveforms

- Highly accurate 1st order Kerr eccentric inclined

Image credit: NASA JPL

Motivation: EMRIs

- Parameter Space

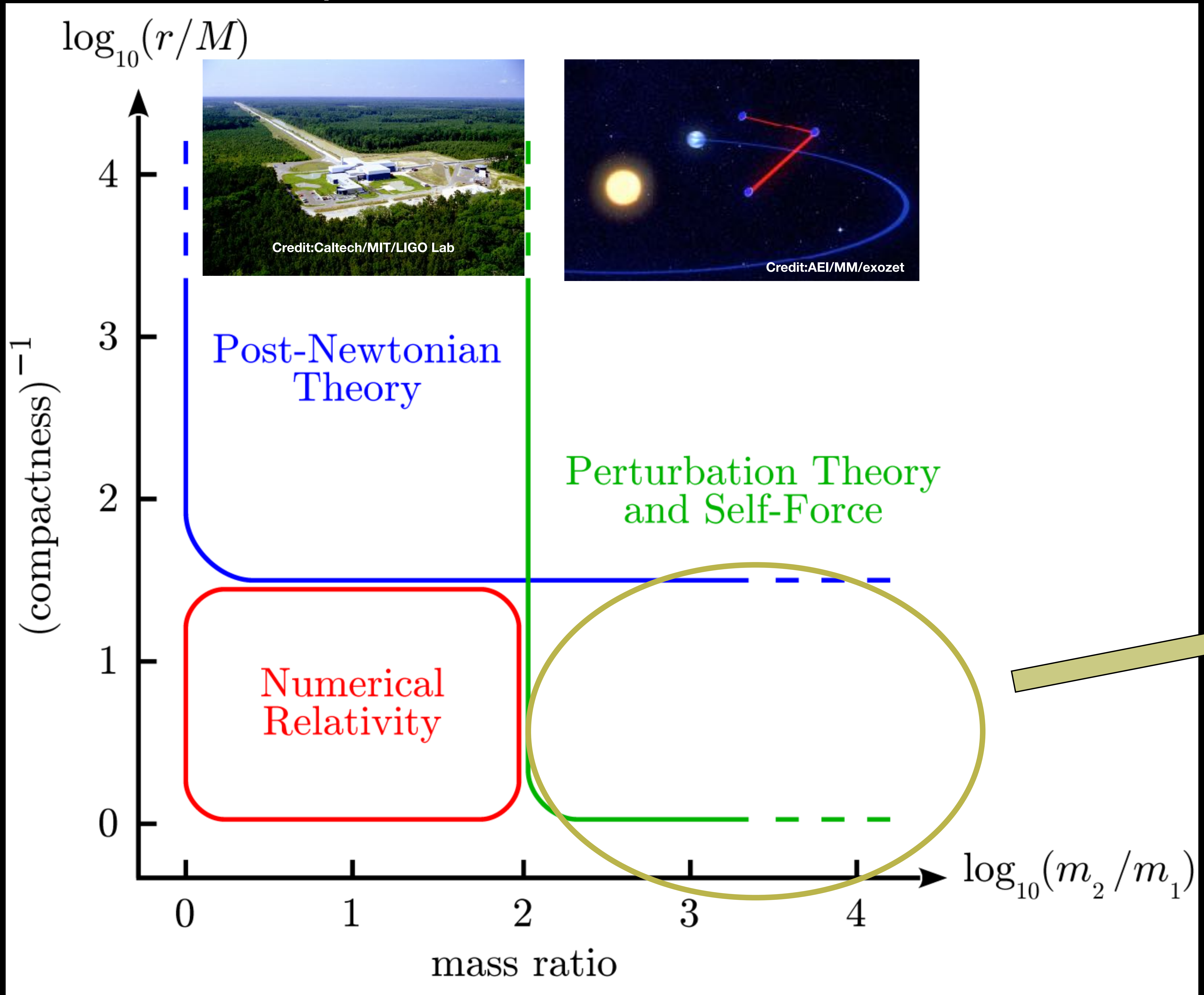


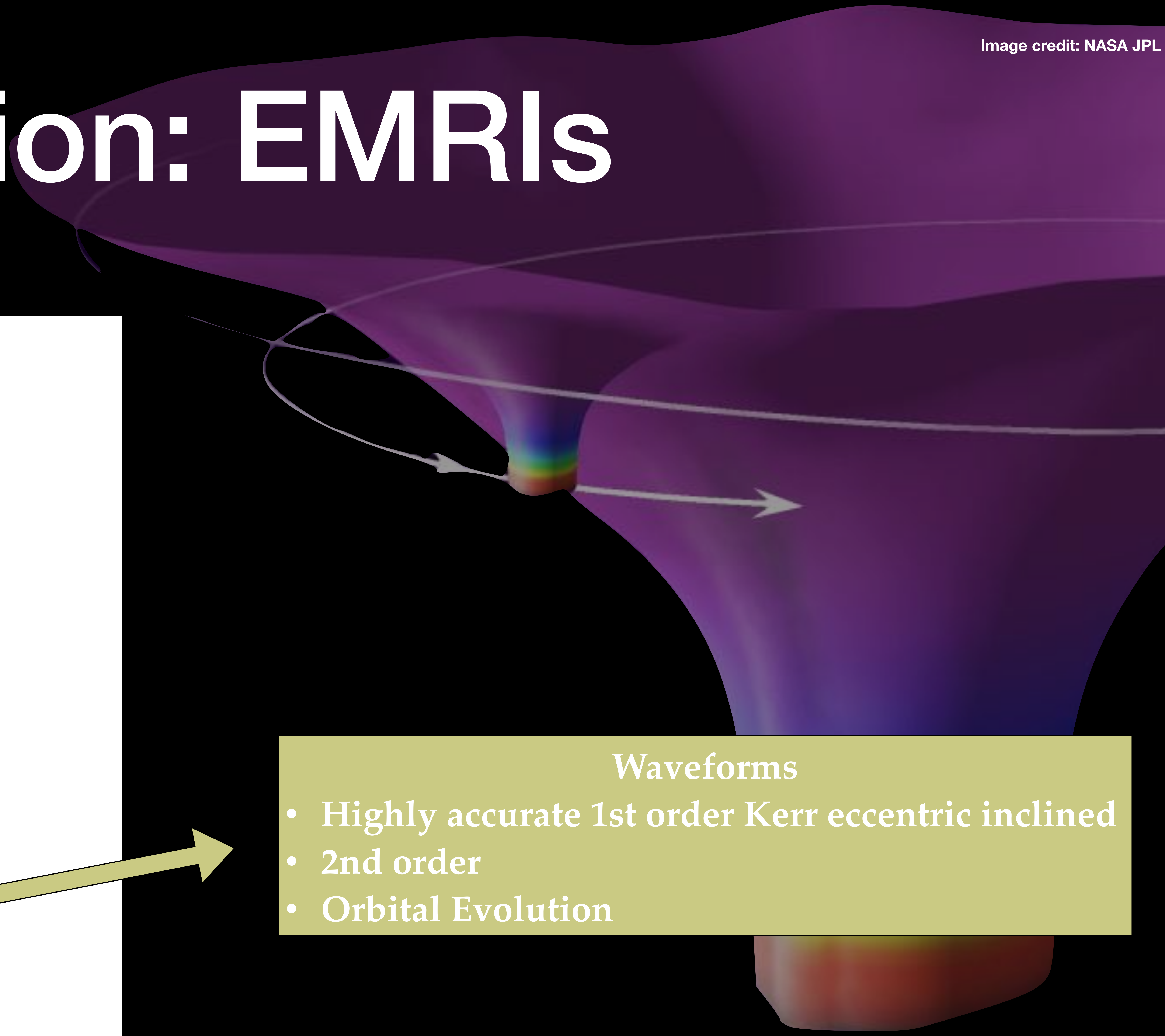
Diagram: Alex Le Tiec, Int. J. Mod. Phys. D. 23:1430022, 2014

Waveforms

- Highly accurate 1st order Kerr eccentric inclined
- 2nd order

Motivation: EMRIs

Image credit: NASA JPL



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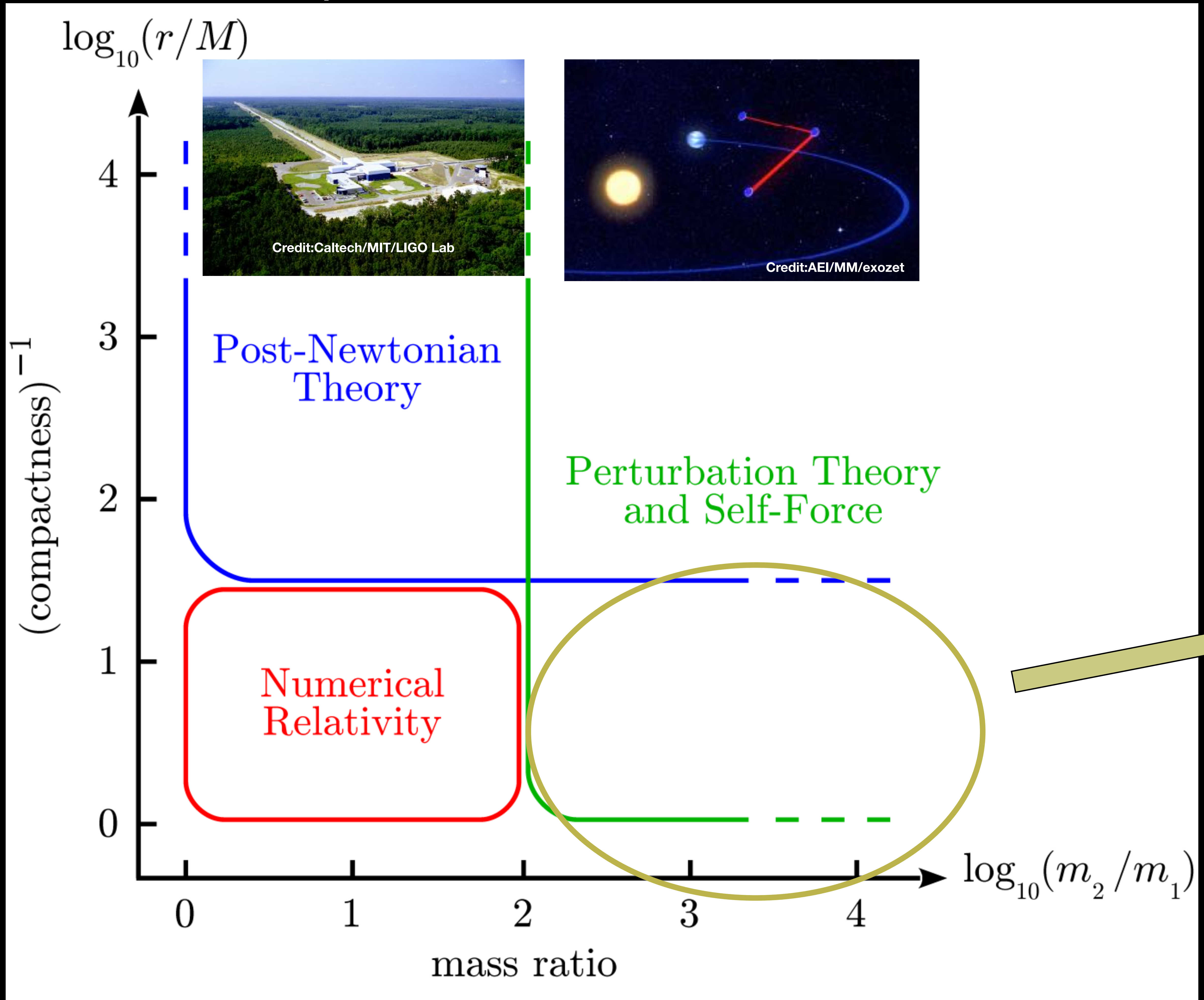

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Waveforms

- Highly accurate 1st order Kerr eccentric inclined
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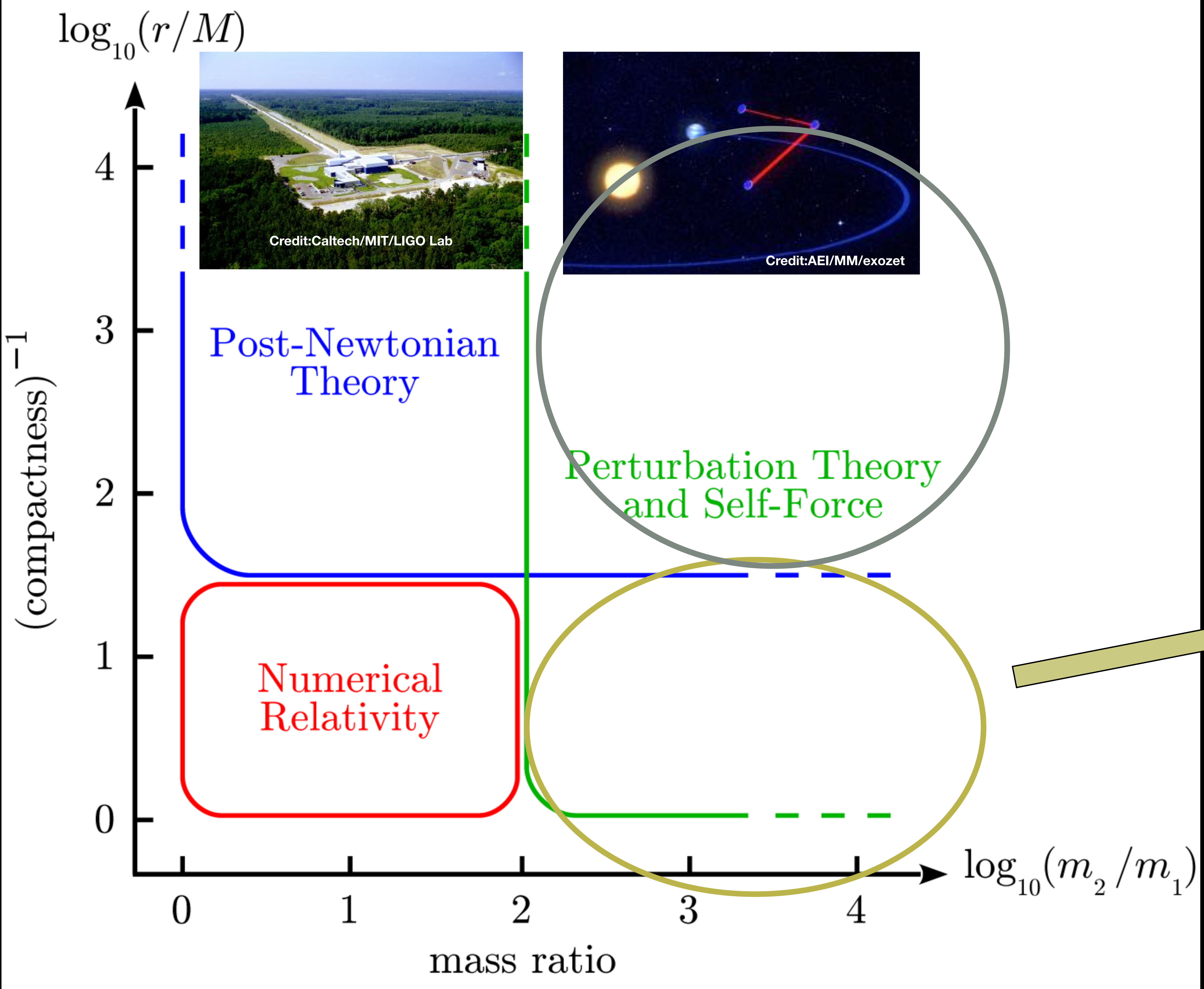
- Orbital Evolution

Image credit: NASA JPL

Motivation: EMRIs

- Parameter Space



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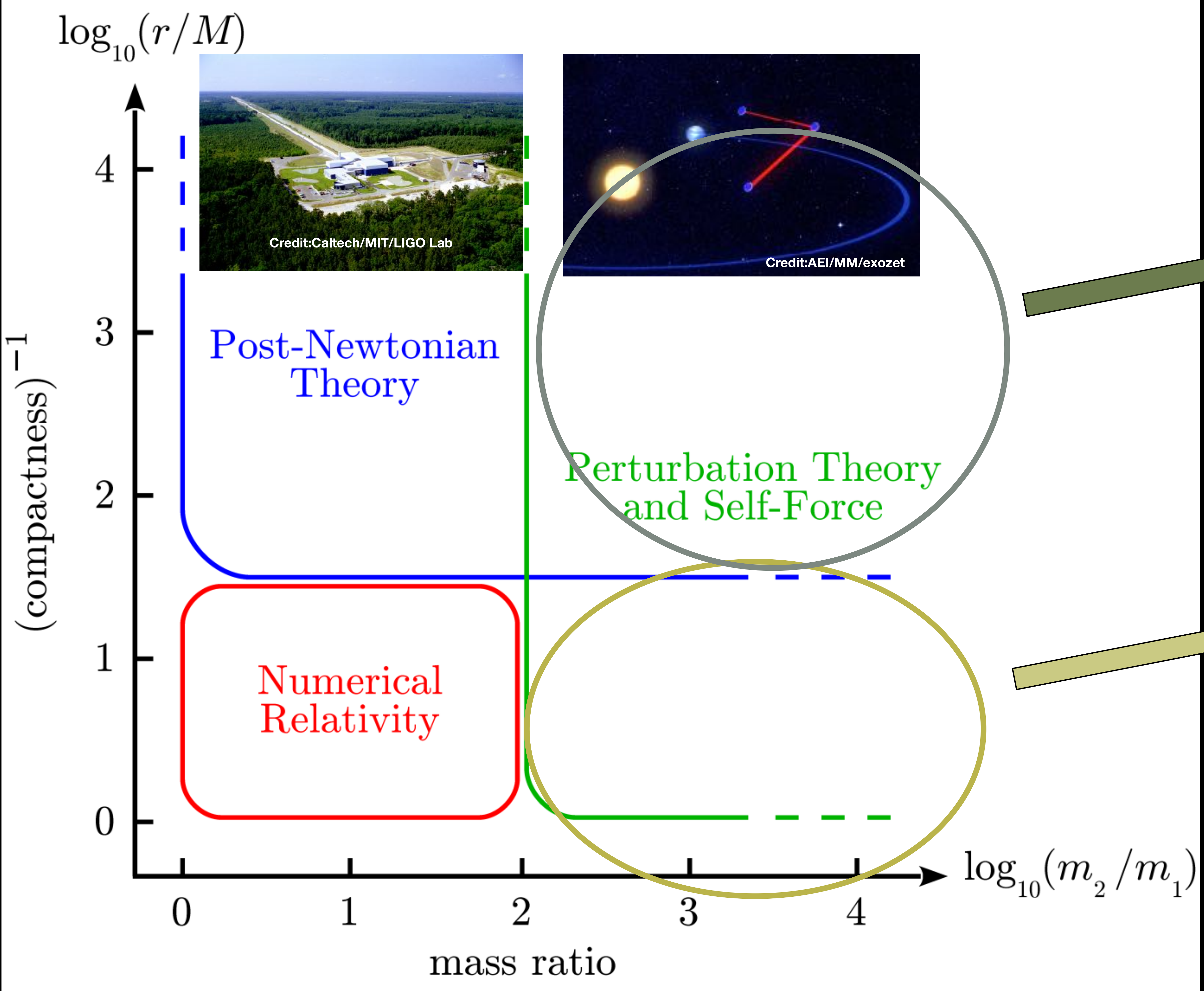

Diagram: Alex Le Tiec, Int. J. Mod. Phys. D. 23:1430022, 2014

Comparison with PN

Waveforms

- Highly accurate 1st order Kerr eccentric inclined
- 2nd order
- Orbital Evolution

Motivation: EMRIs

- Parameter Space

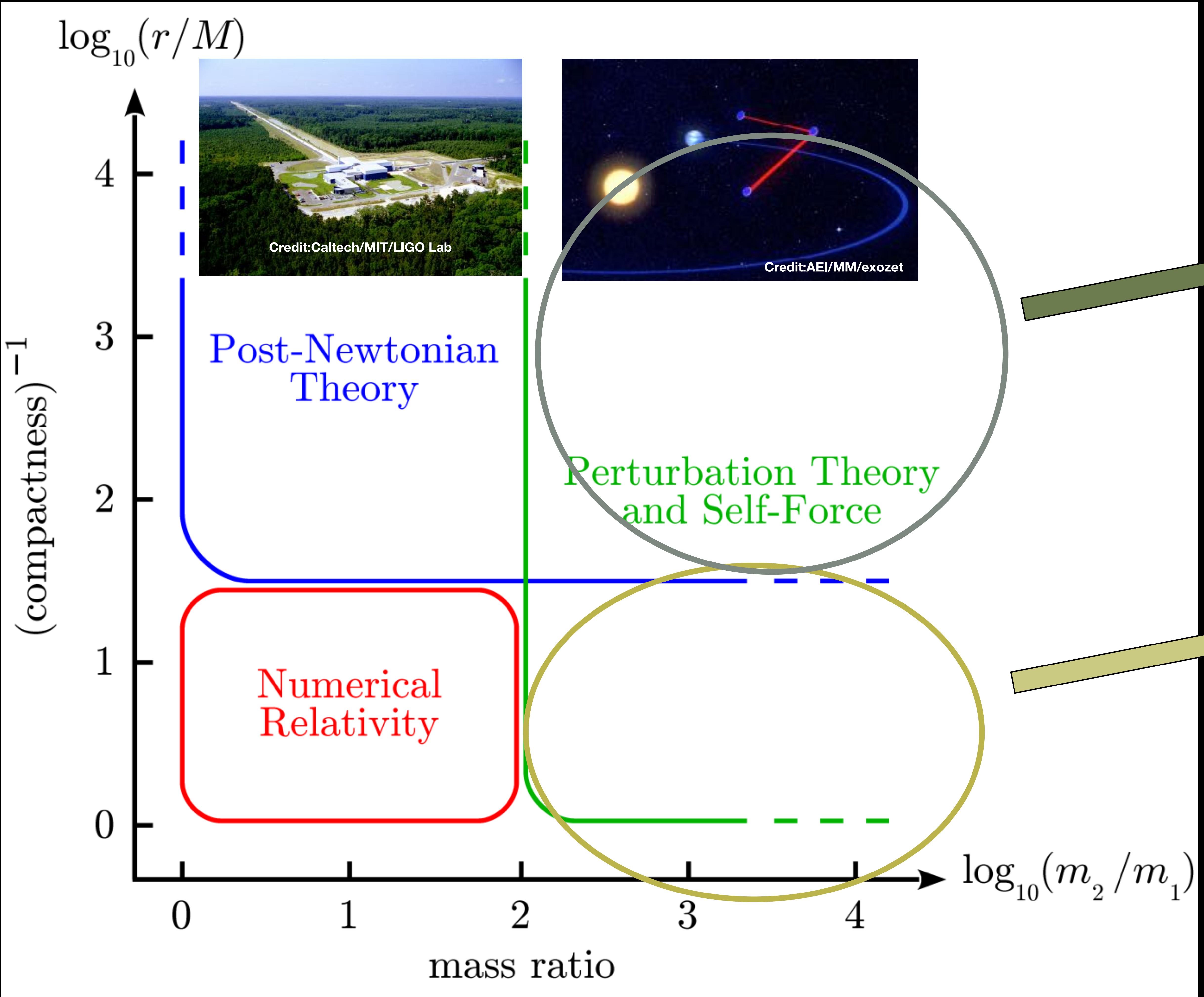

Diagram: Alex Le Tiec, Int. J. Mod. Phys. D. 23:1430022, 2014

Comparison with PN
- Gauge invariant quantities

Waveforms
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- Orbital Evolution

Motivation: EMRIs

- Parameter Space

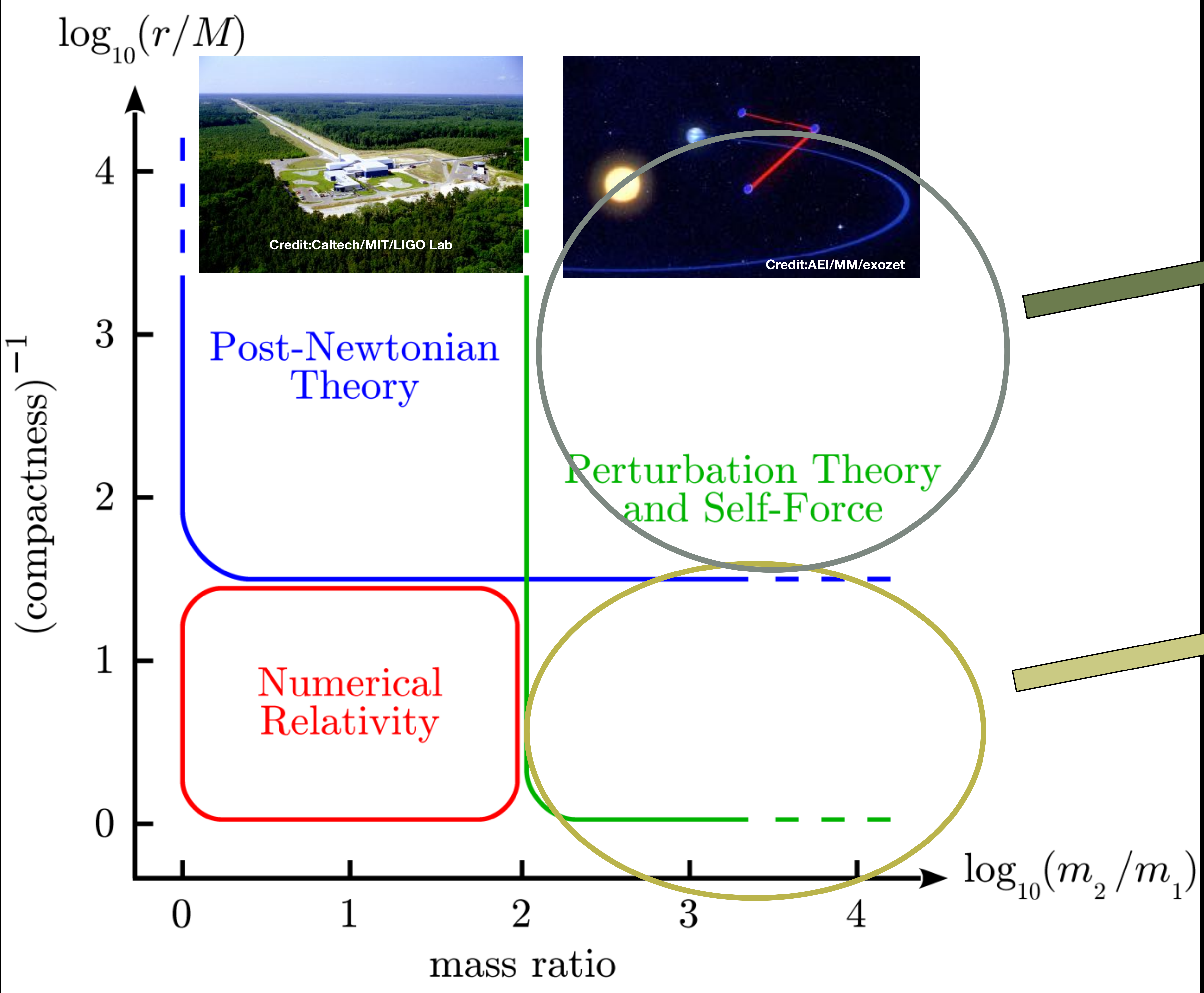

Diagram: Alex Le Tiec, Int. J. Mod. Phys. D. 23:1430022, 2014

Comparison with PN
- Gauge invariant quantities
- Read off unknown PN coefficients

Waveforms
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- Orbital Evolution

Motivation: EMRIs

- Parameter Space

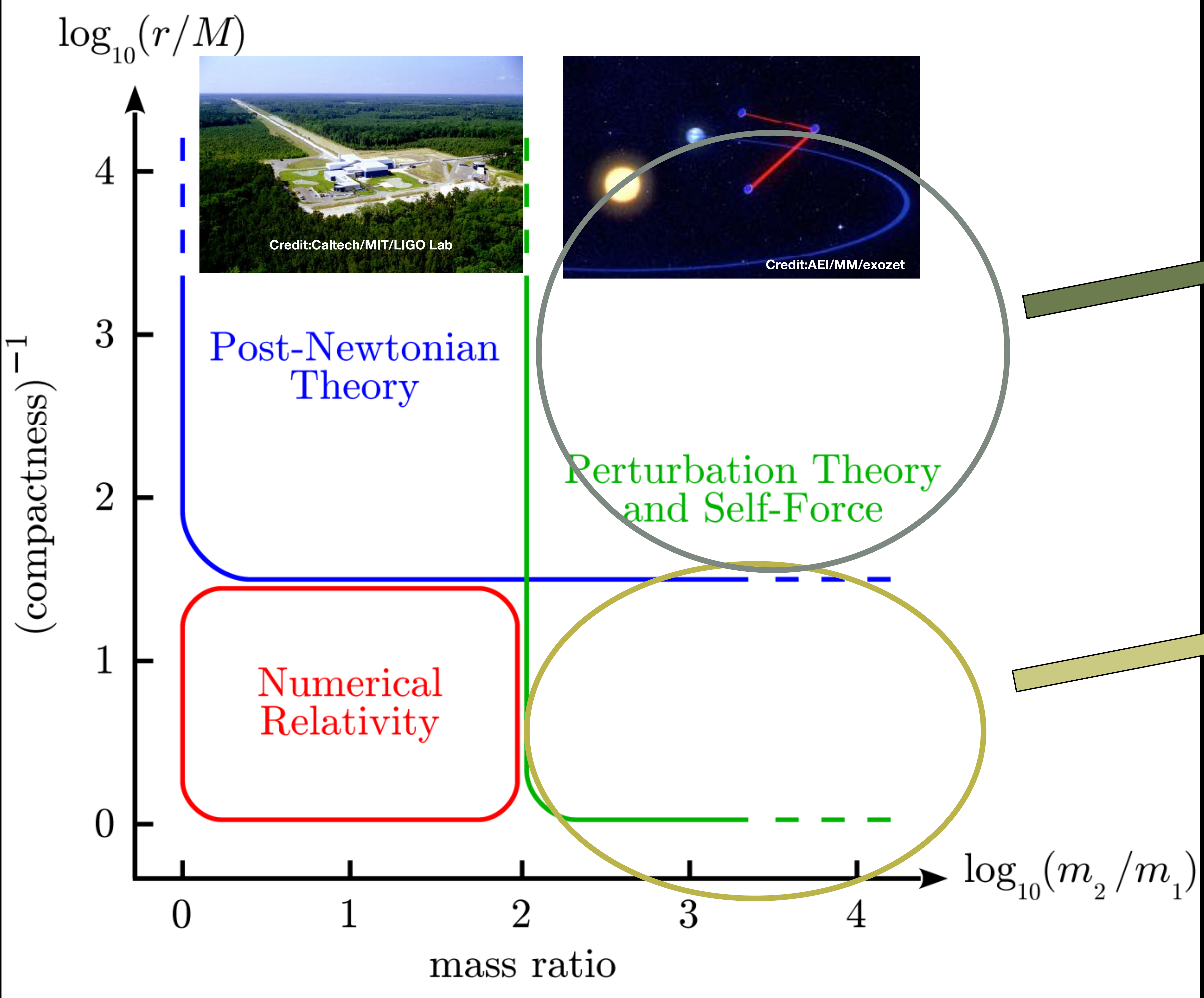


Diagram: Alex Le Tiec, Int. J. Mod. Phys. D. 23:1430022, 2014

Comparison with PN
- Gauge invariant quantities
- Read off unknown PN coefficients
- Calibrate EOB

Waveforms
- Highly accurate 1st order Kerr eccentric inclined
- 2nd order
- Orbital Evolution

Regularisation: Flat space

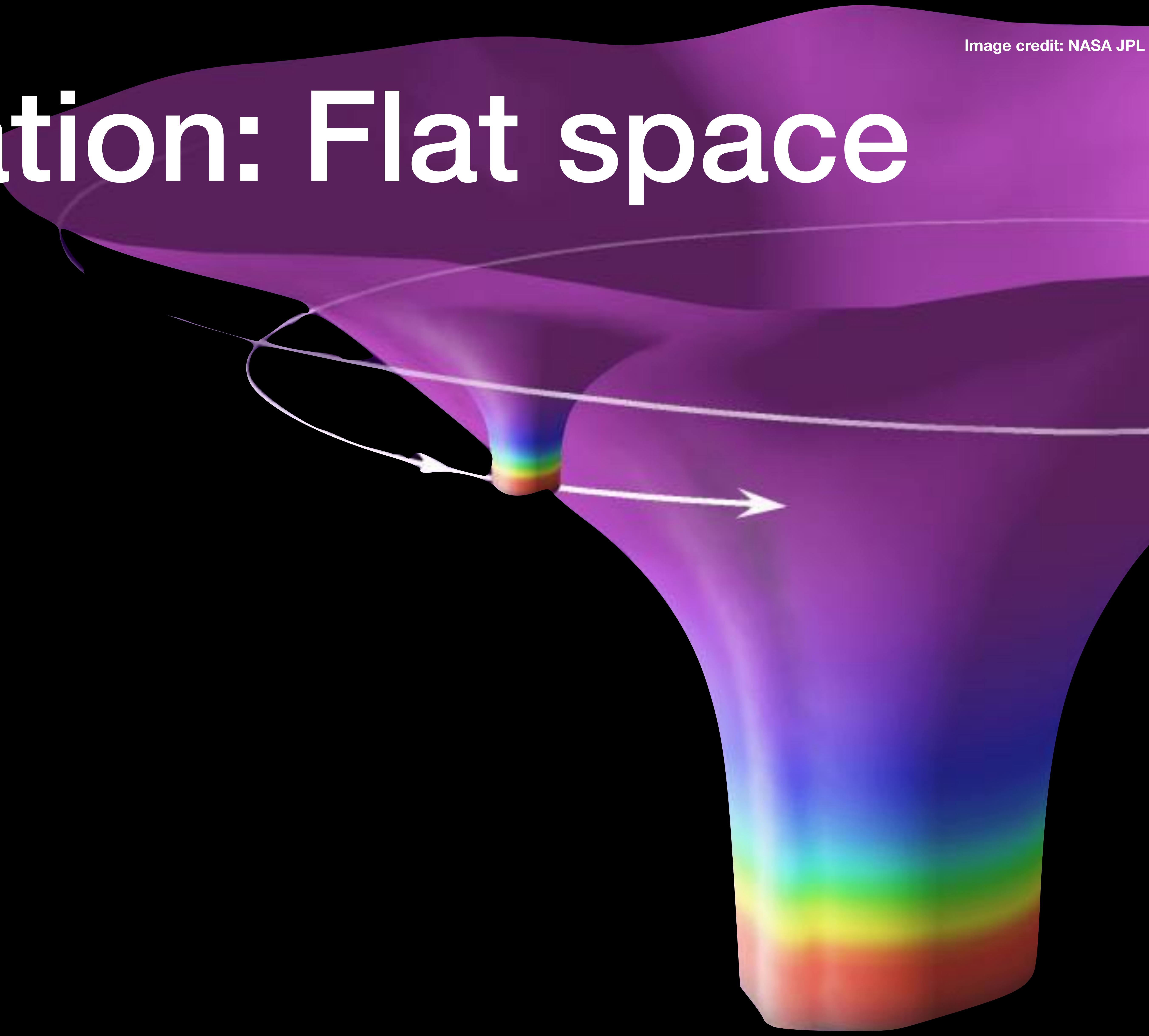
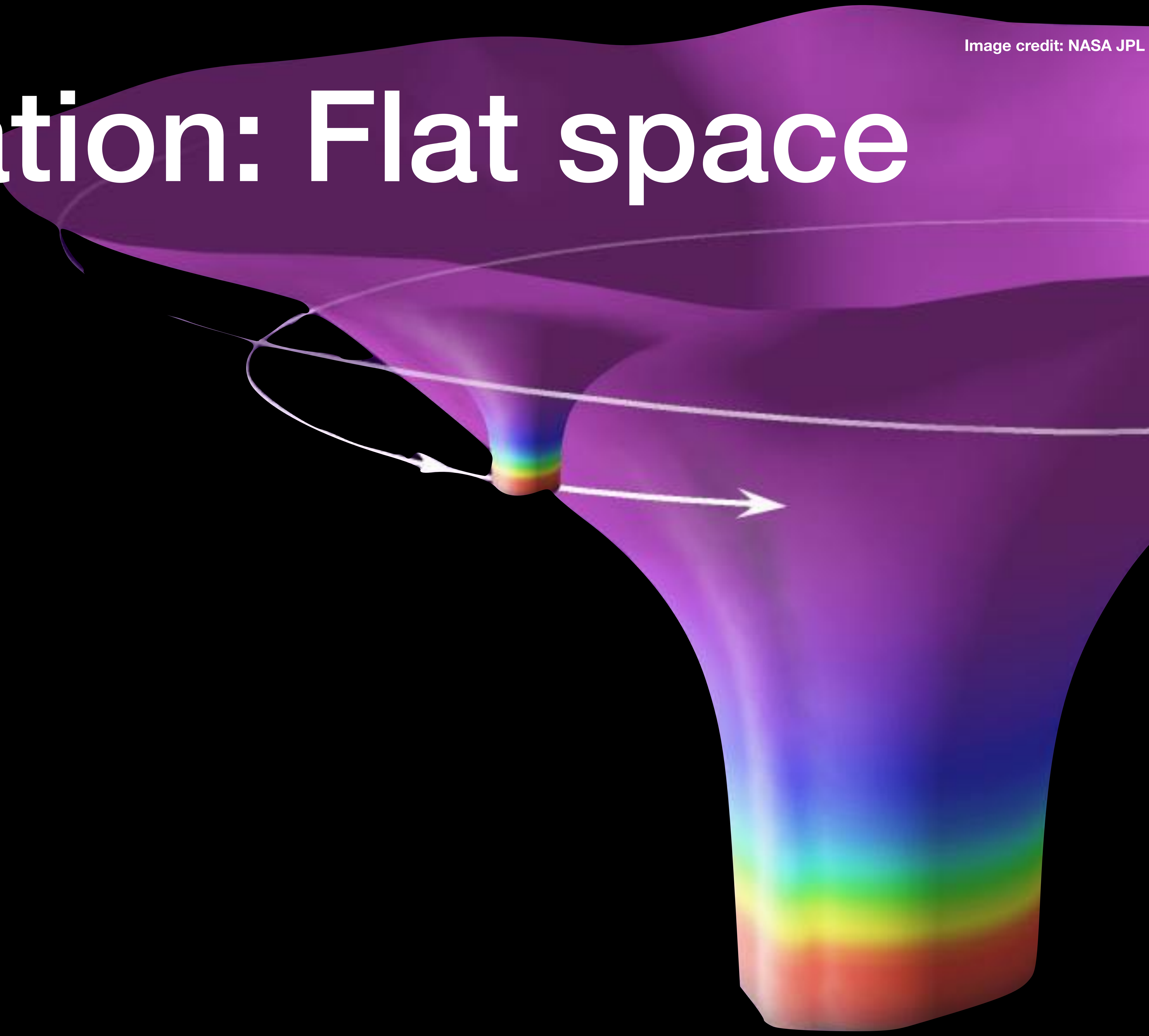


Image credit: NASA JPL

Regularisation: Flat space

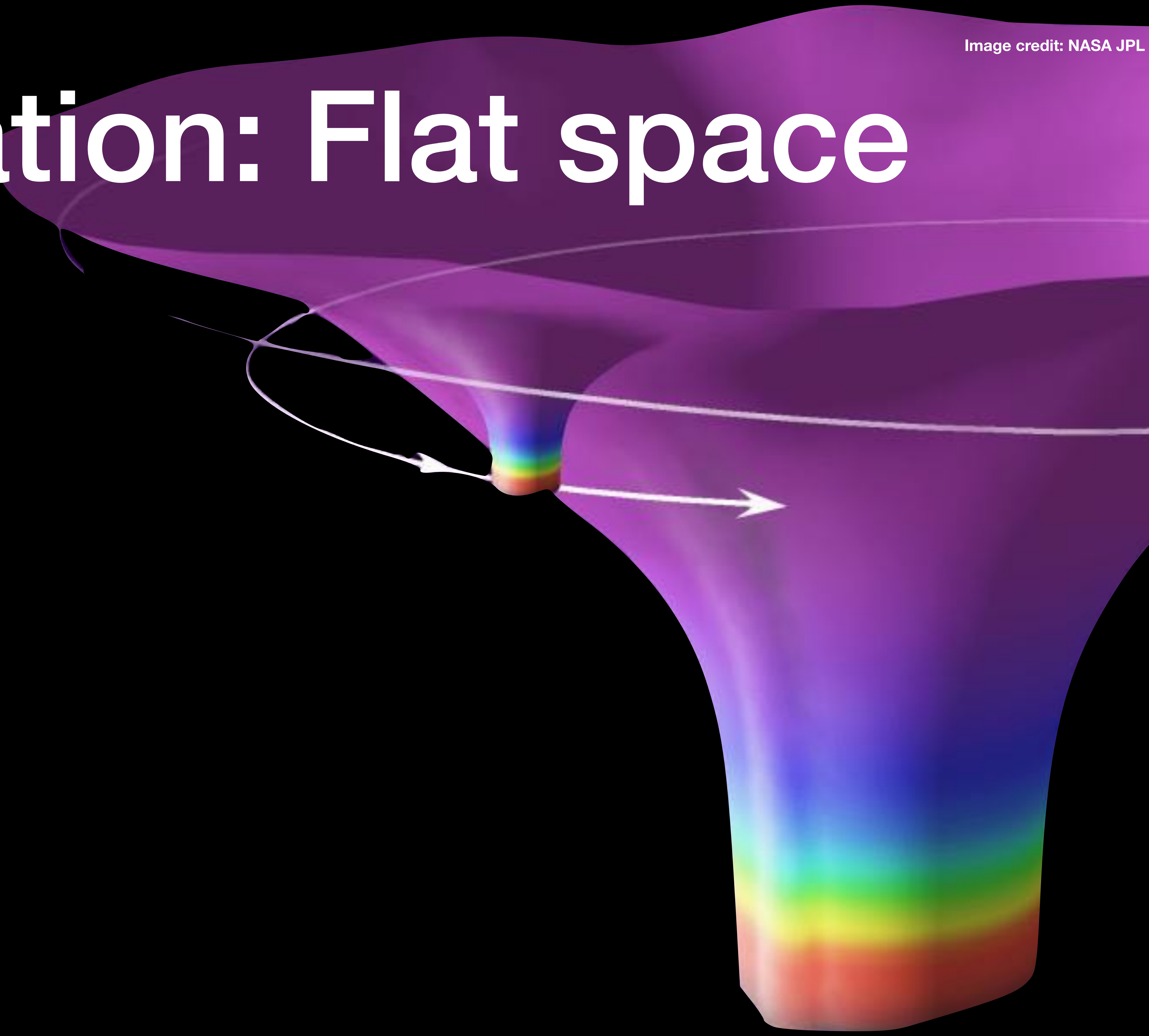
Image credit: NASA JPL

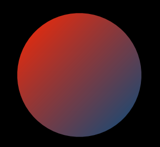


- Flat space

Regularisation: Flat space

Image credit: NASA JPL





- Flat space

Image credit: NASA JPL

Regularisation: Flat space

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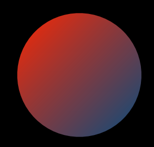
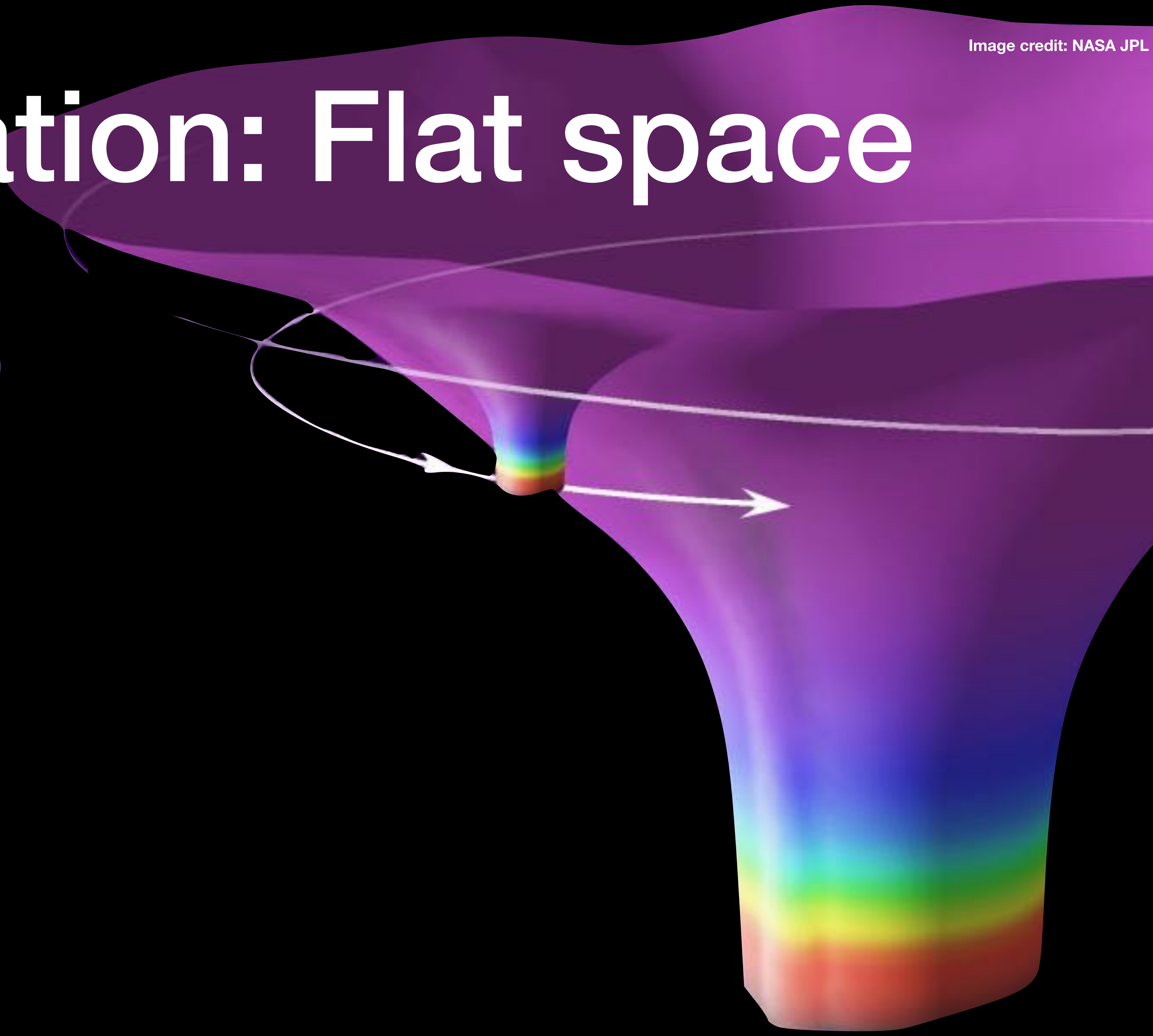
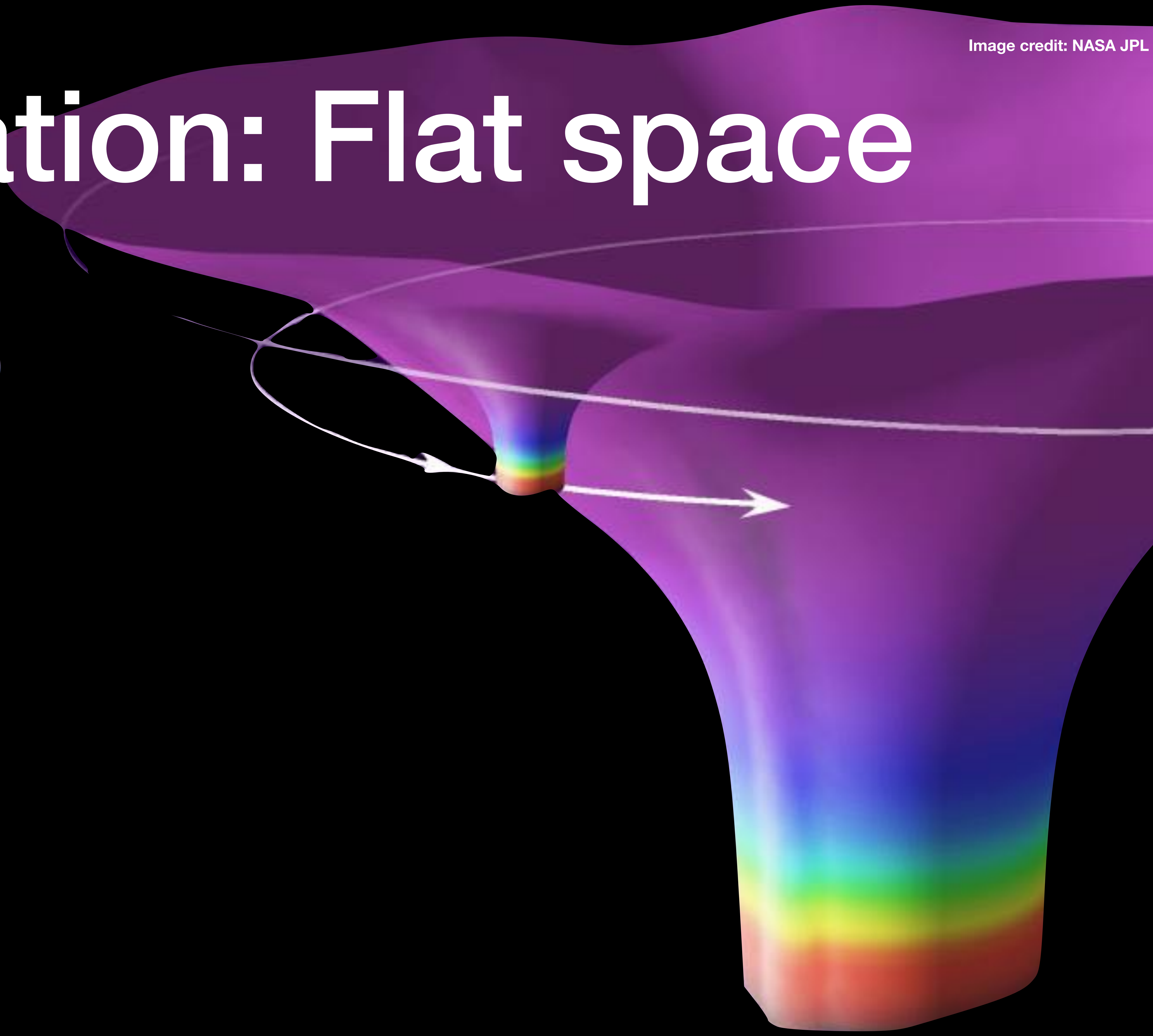


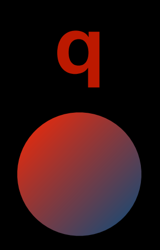


- Flat space
  - Electromagnetism $\Box A^{\mu} = -4\pi j^{\mu}$

Regularisation: Flat space

Image credit: NASA JPL

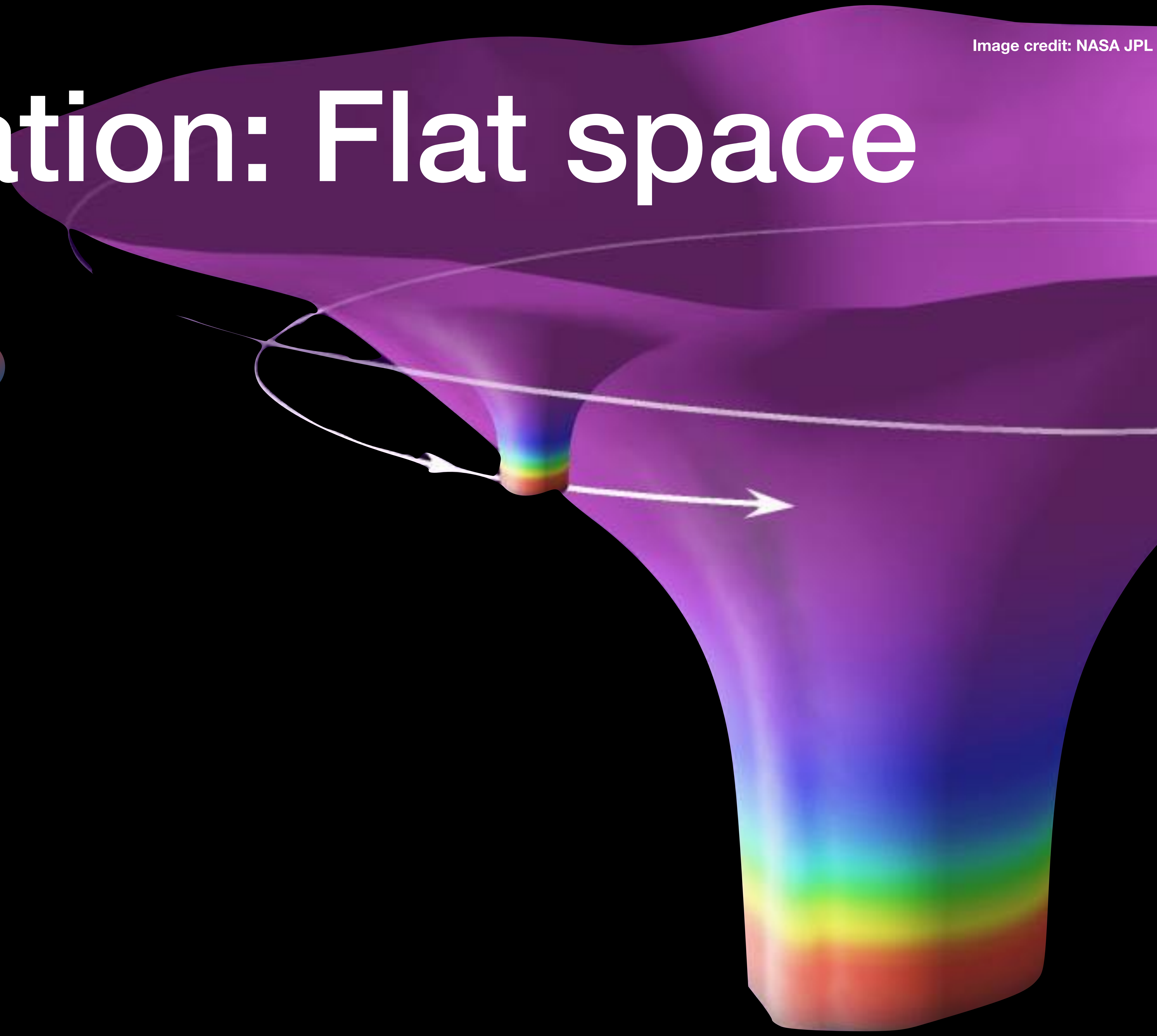




- Flat space
  - Electromagnetism $\Box A^\mu = -4\pi j^\mu$
  - 2 Solutions: A^μ_{ret}, A^μ_{adv}

Image credit: NASA JPL

Regularisation: Flat space



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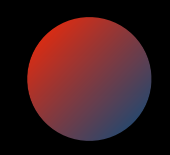

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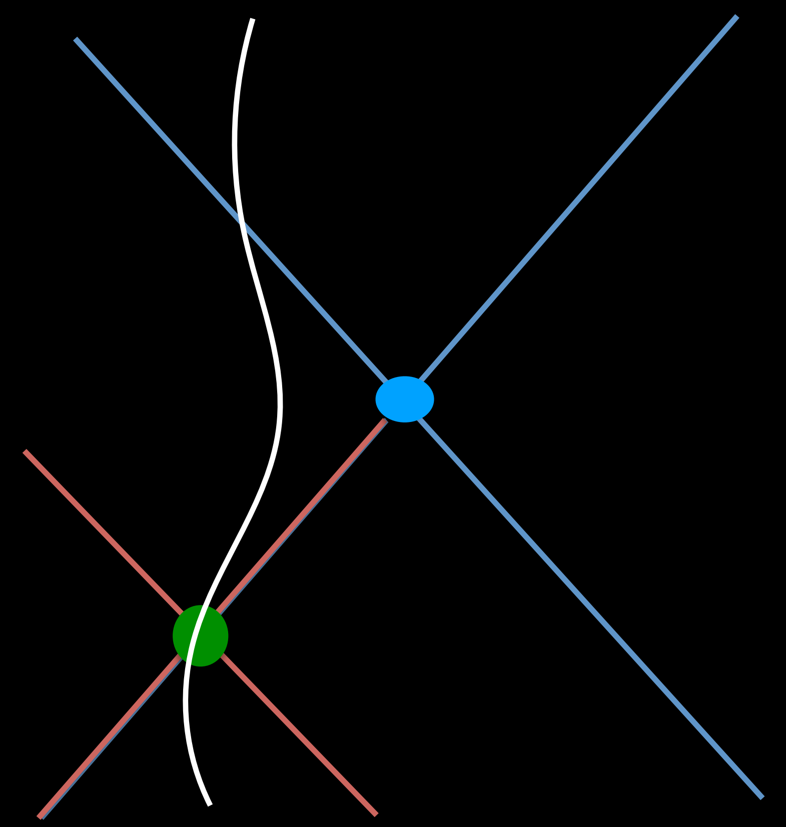
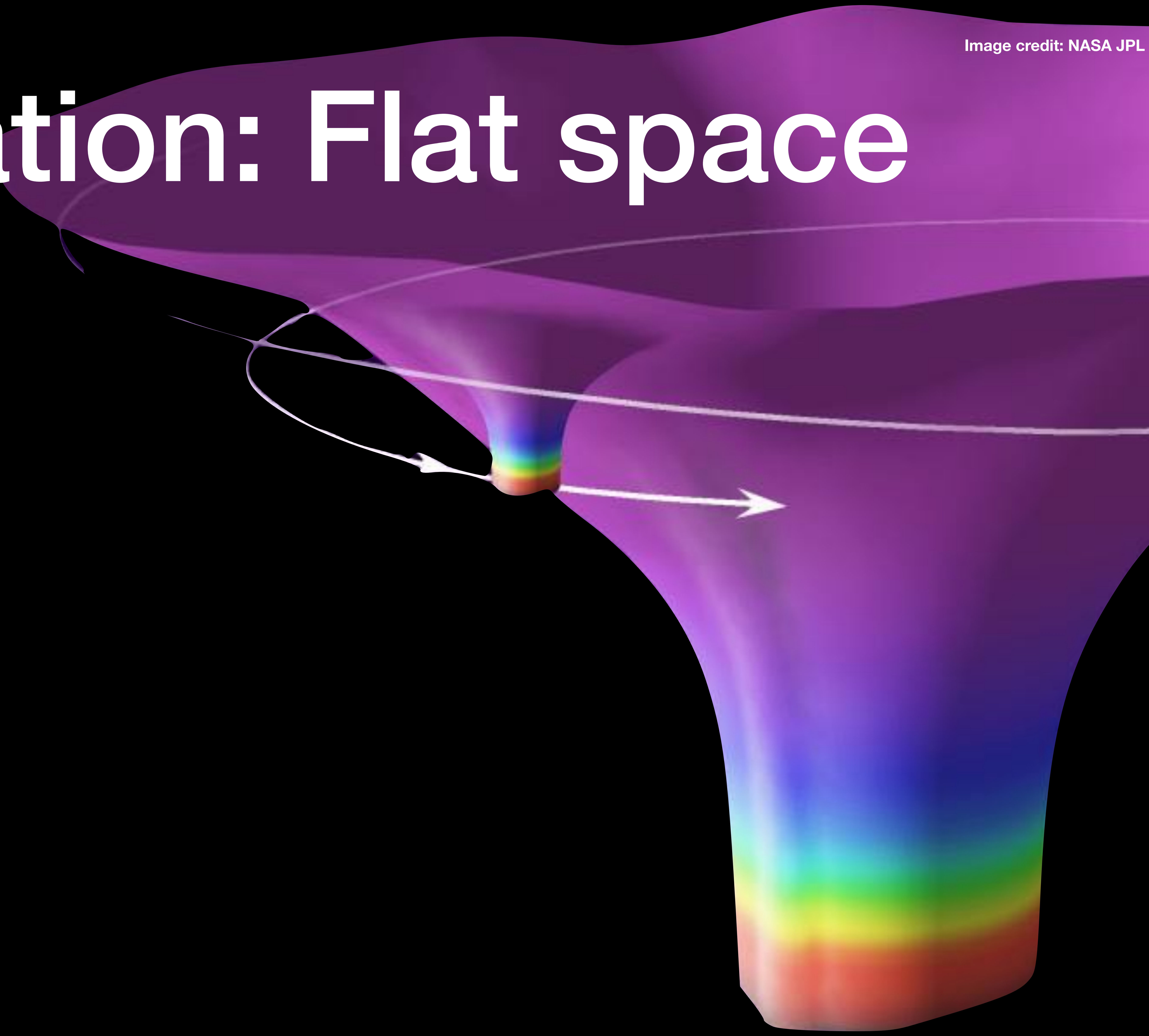
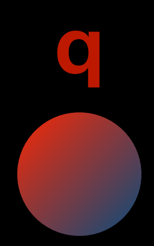


Retarded solution

Regularisation: Flat space





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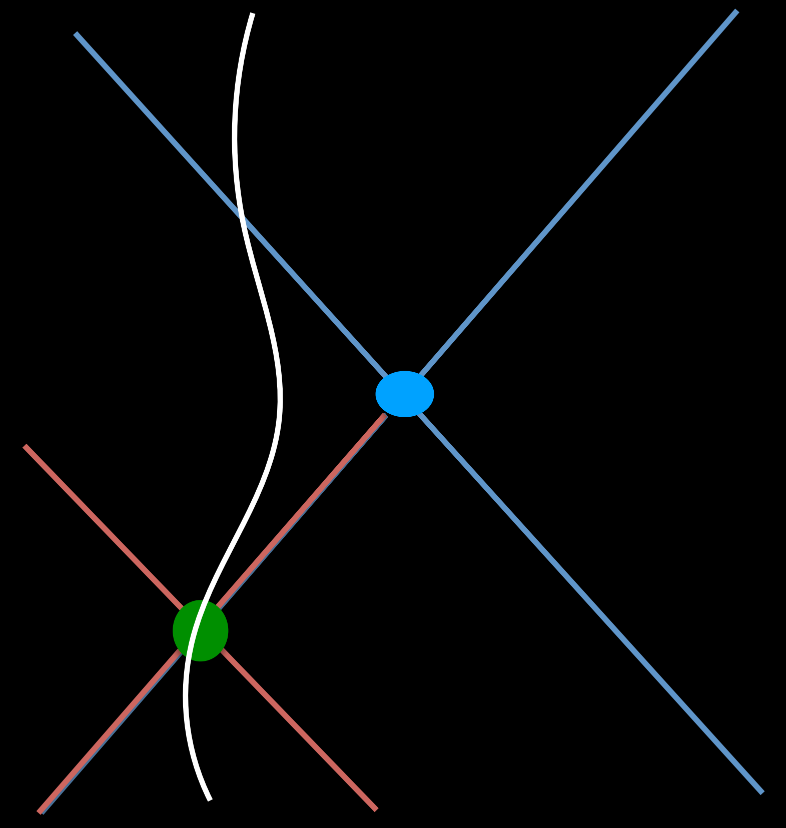

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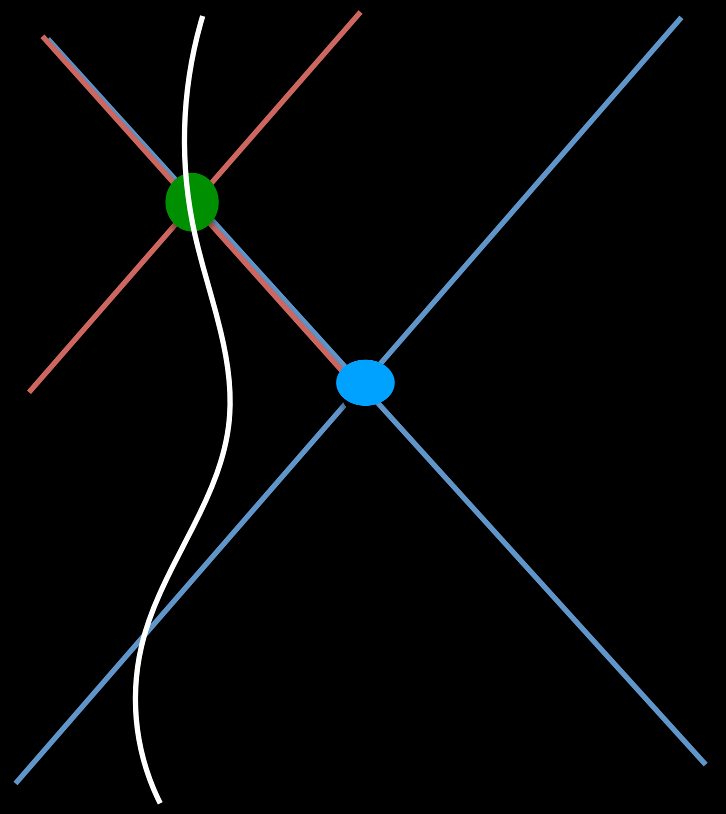
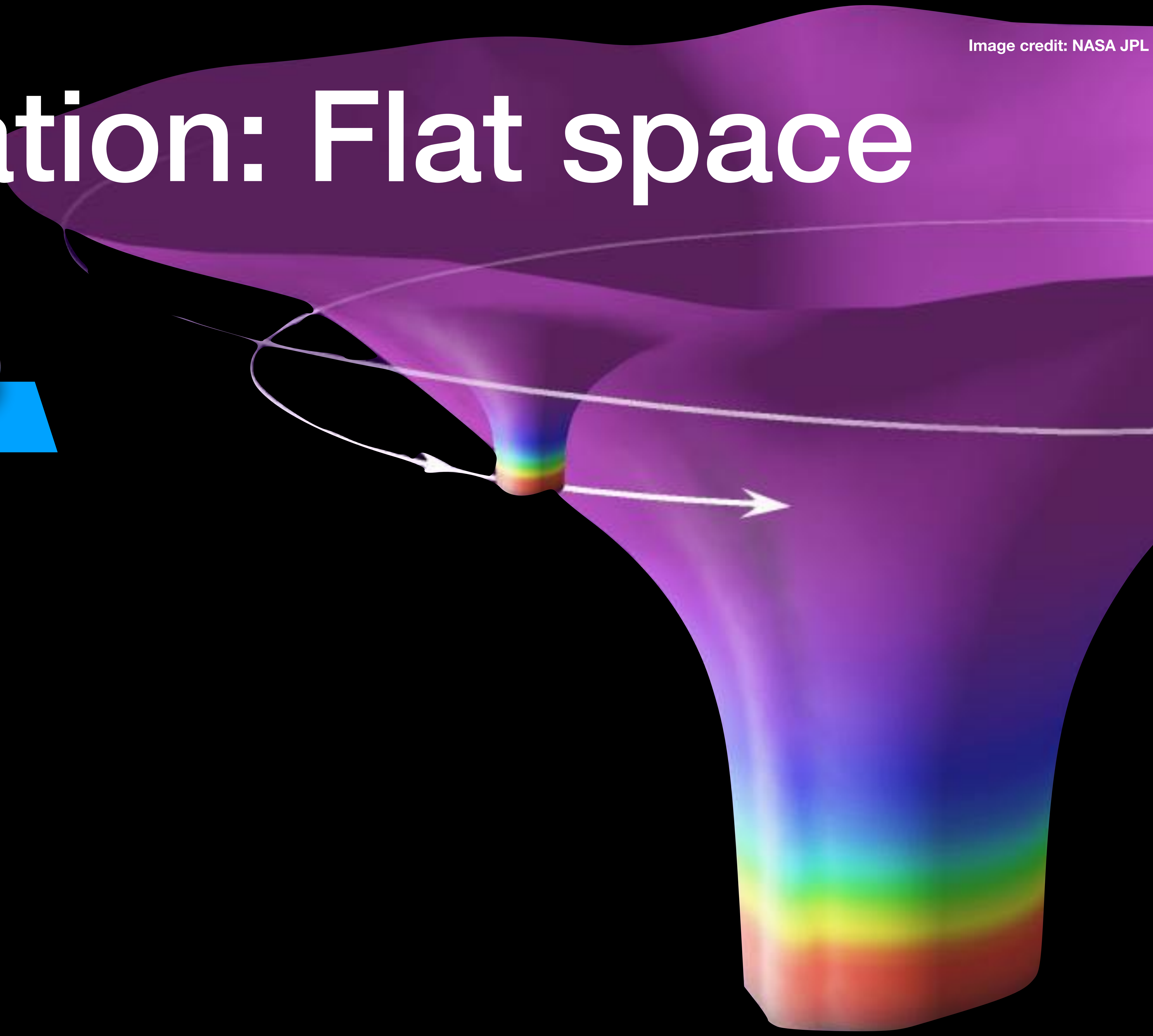
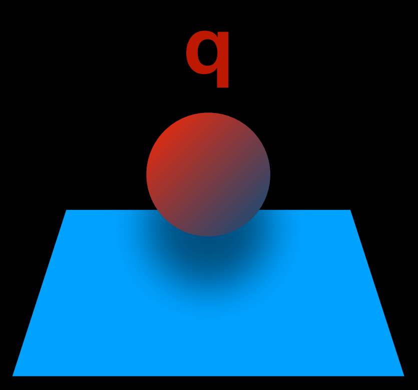


Advanced solution

Regularisation: Flat space





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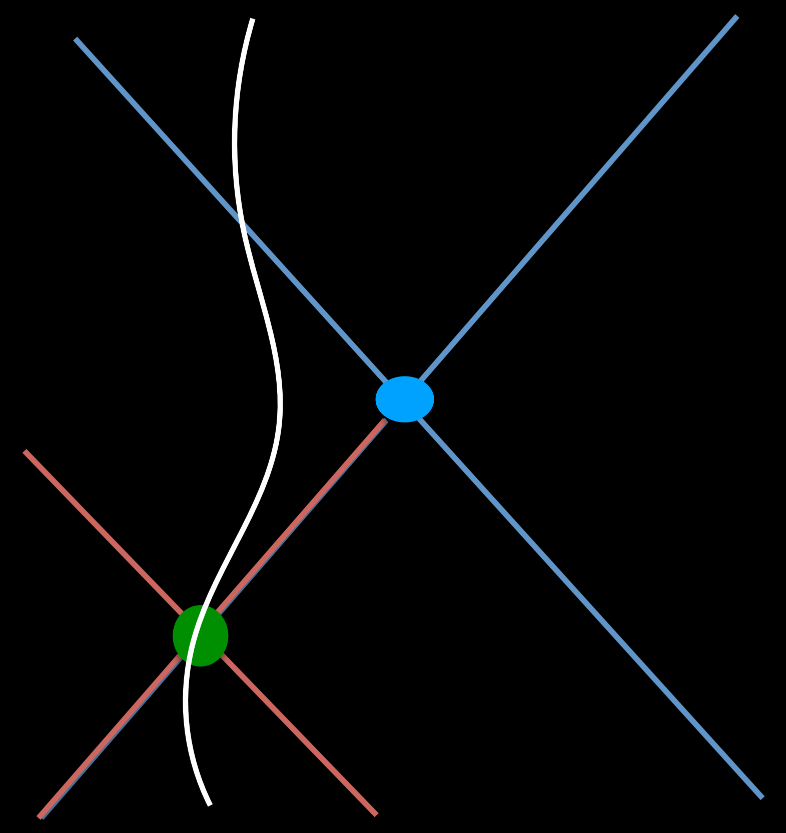

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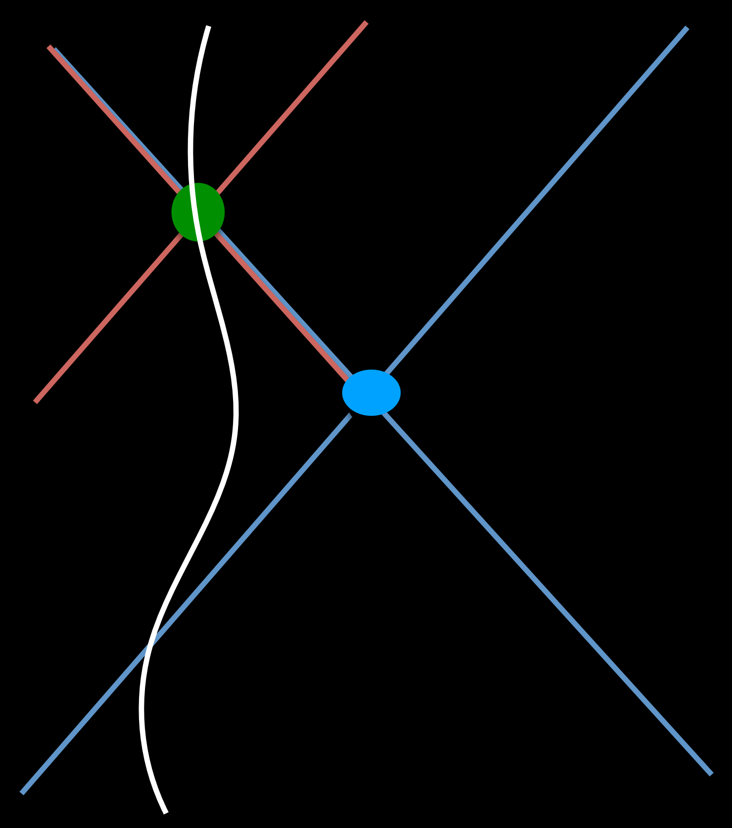

Advanced solution

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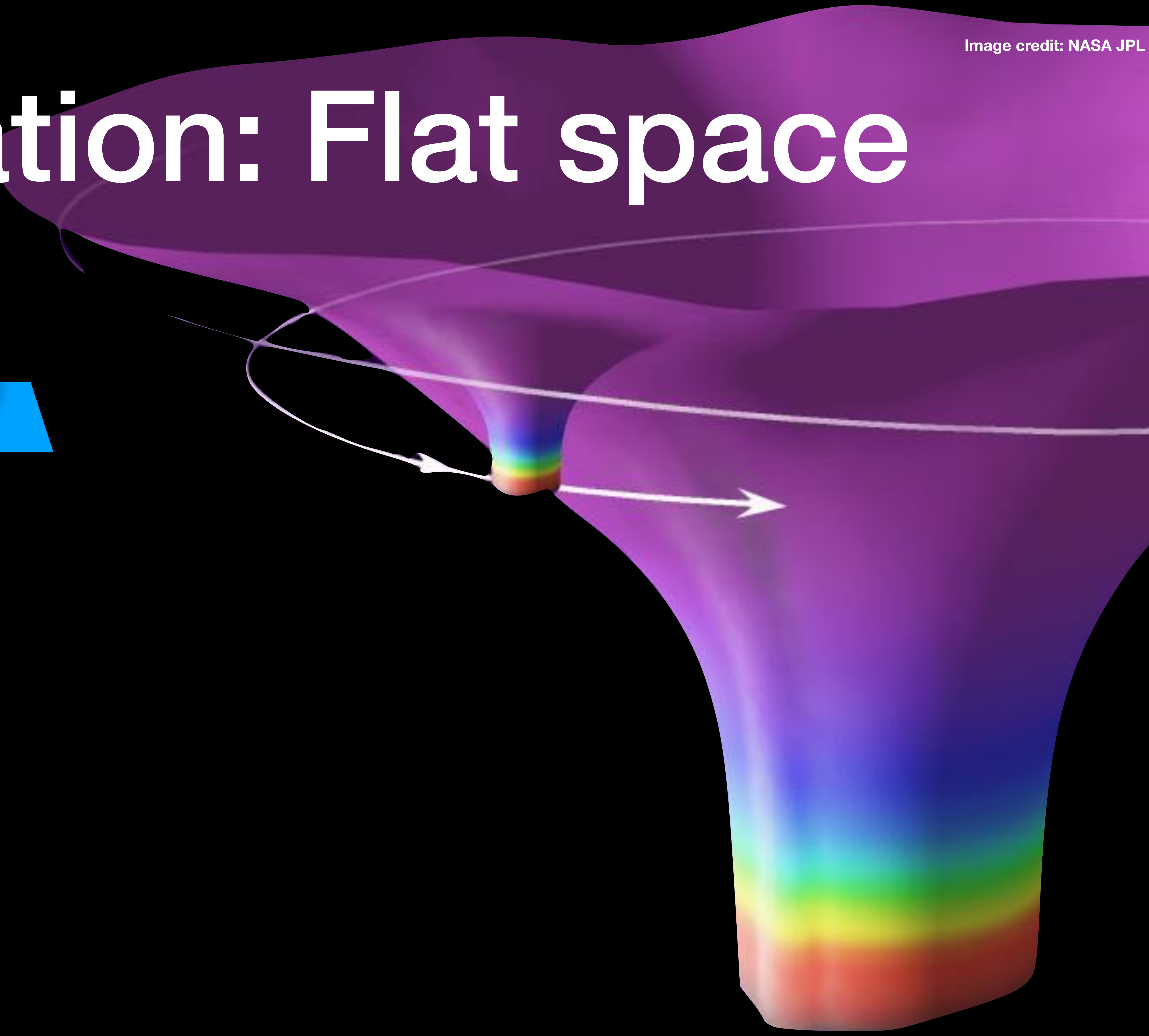

Image credit: NASA JPL

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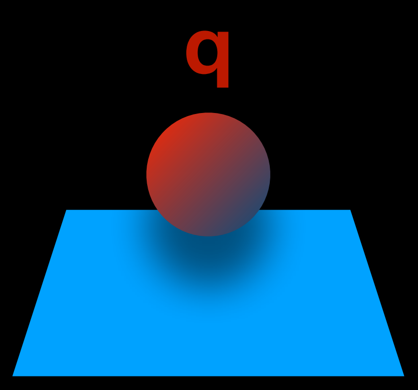

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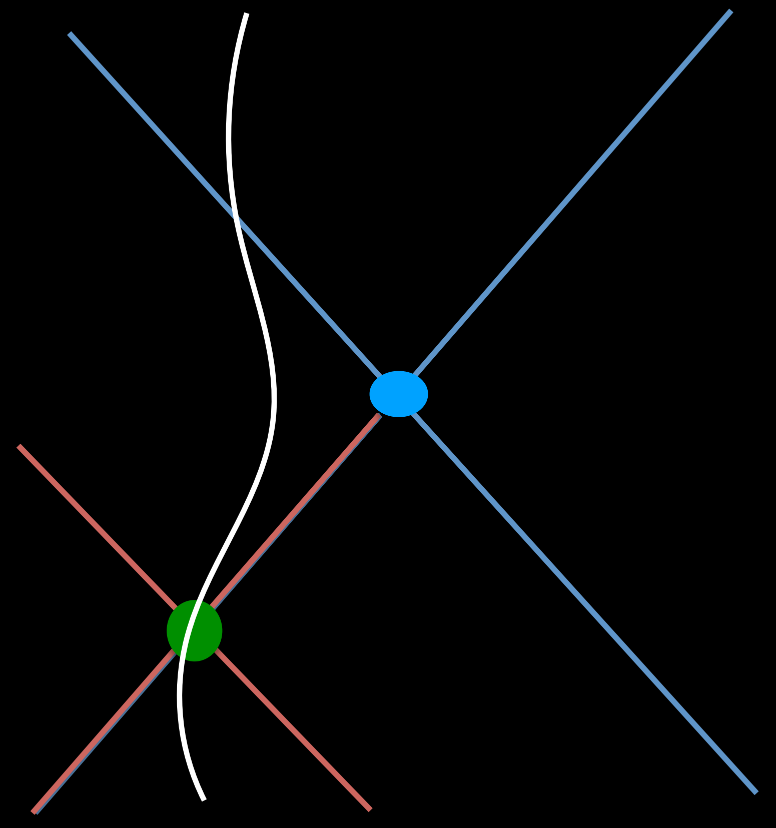

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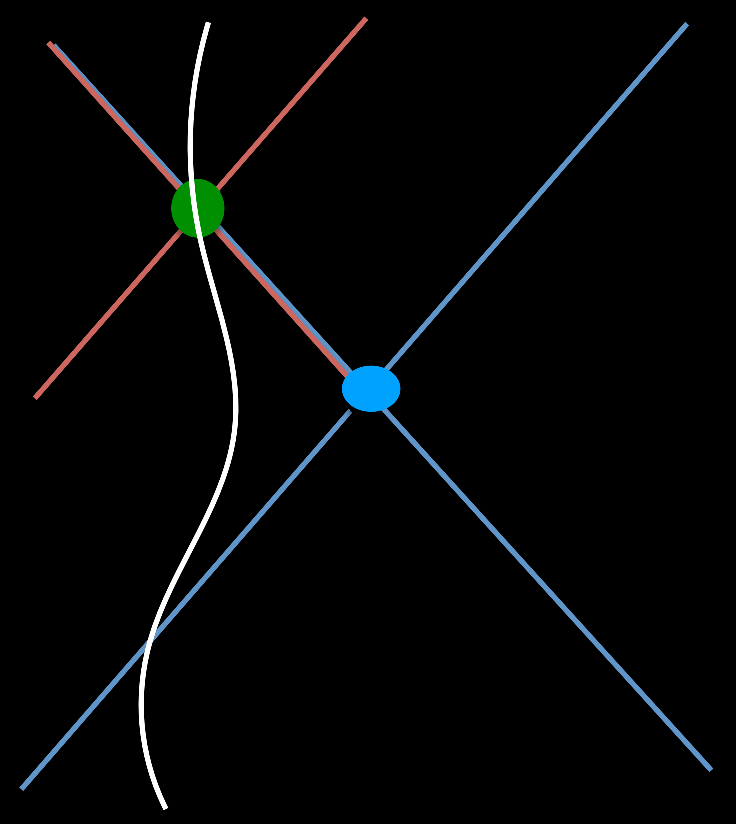

Advanced solution

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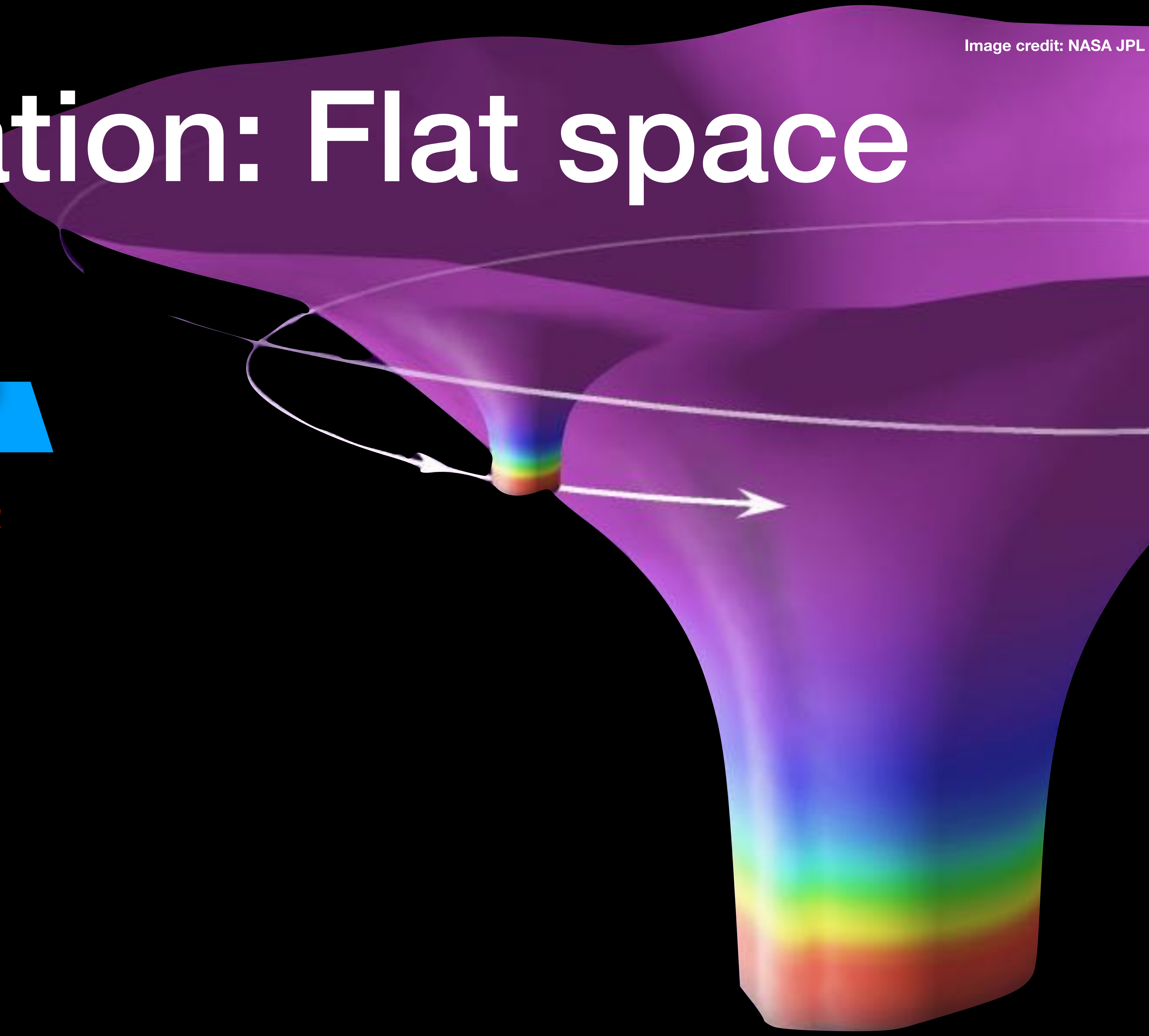
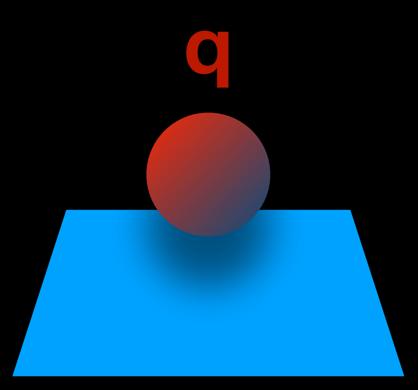


Image credit: NASA JPL



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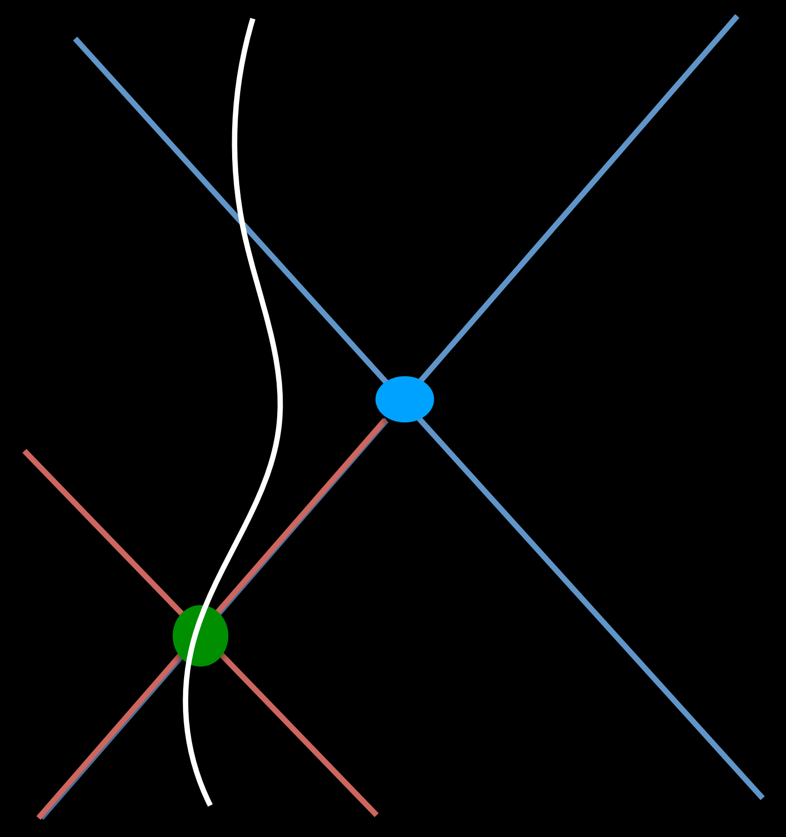
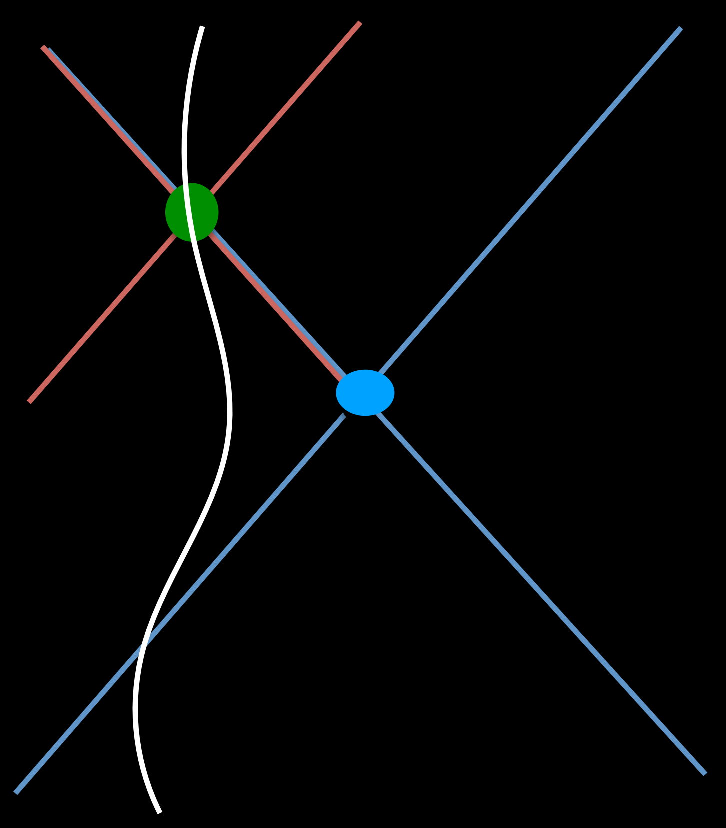

Retarded solution


Advanced solution

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Regularisation: Flat space

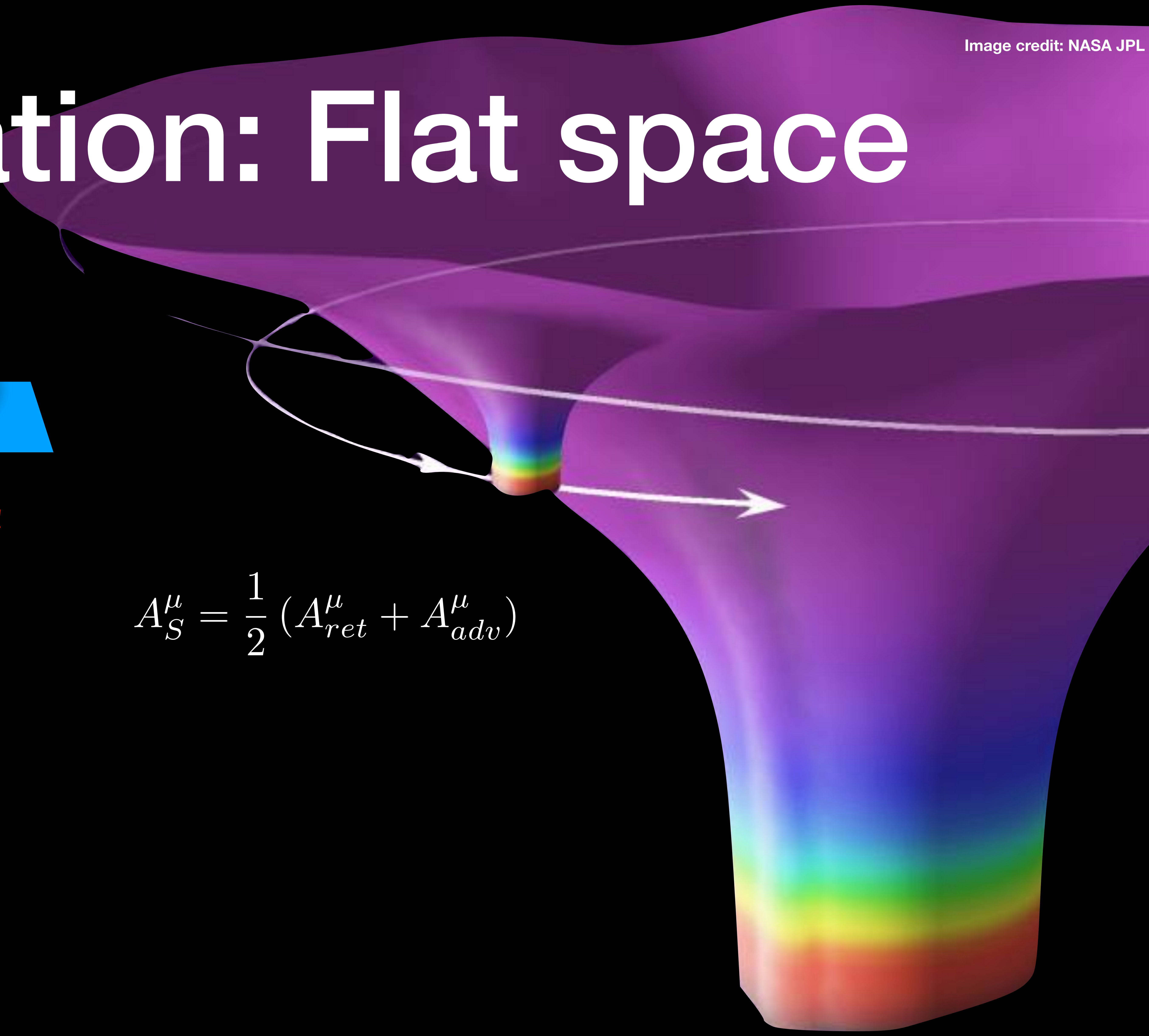
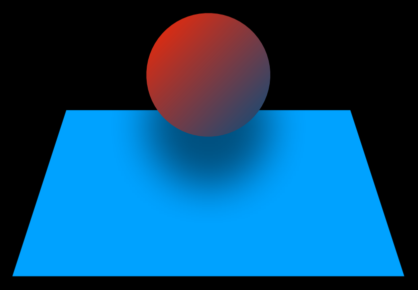


Image credit: NASA JPL



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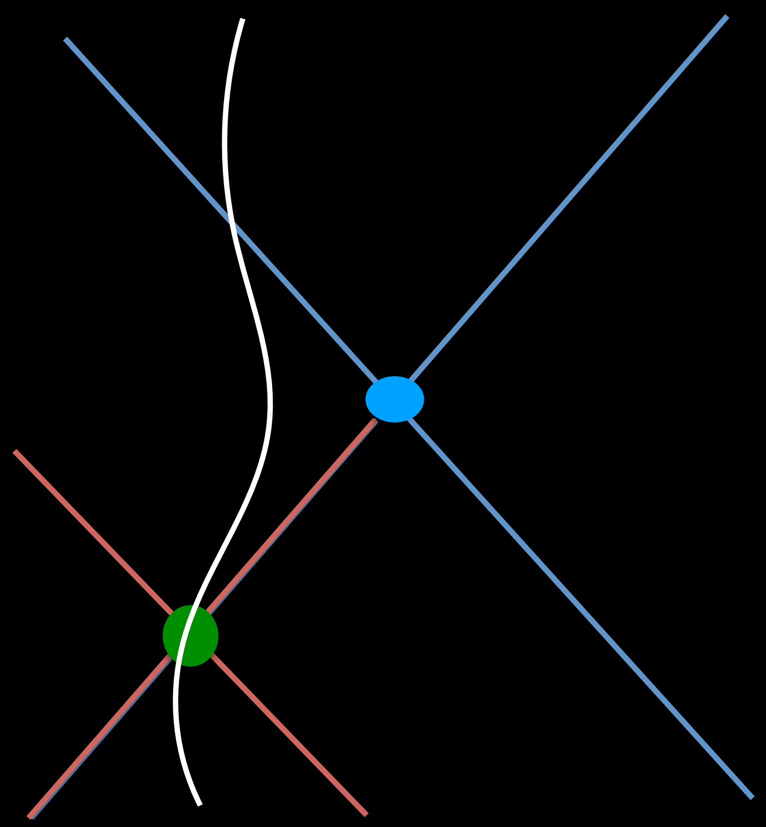
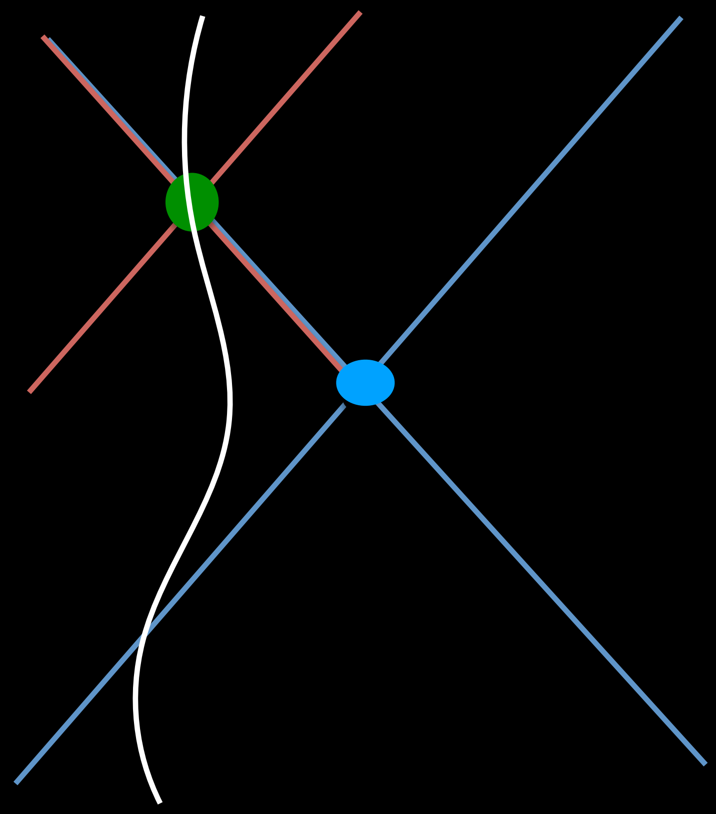


Retarded solution



Advanced solution

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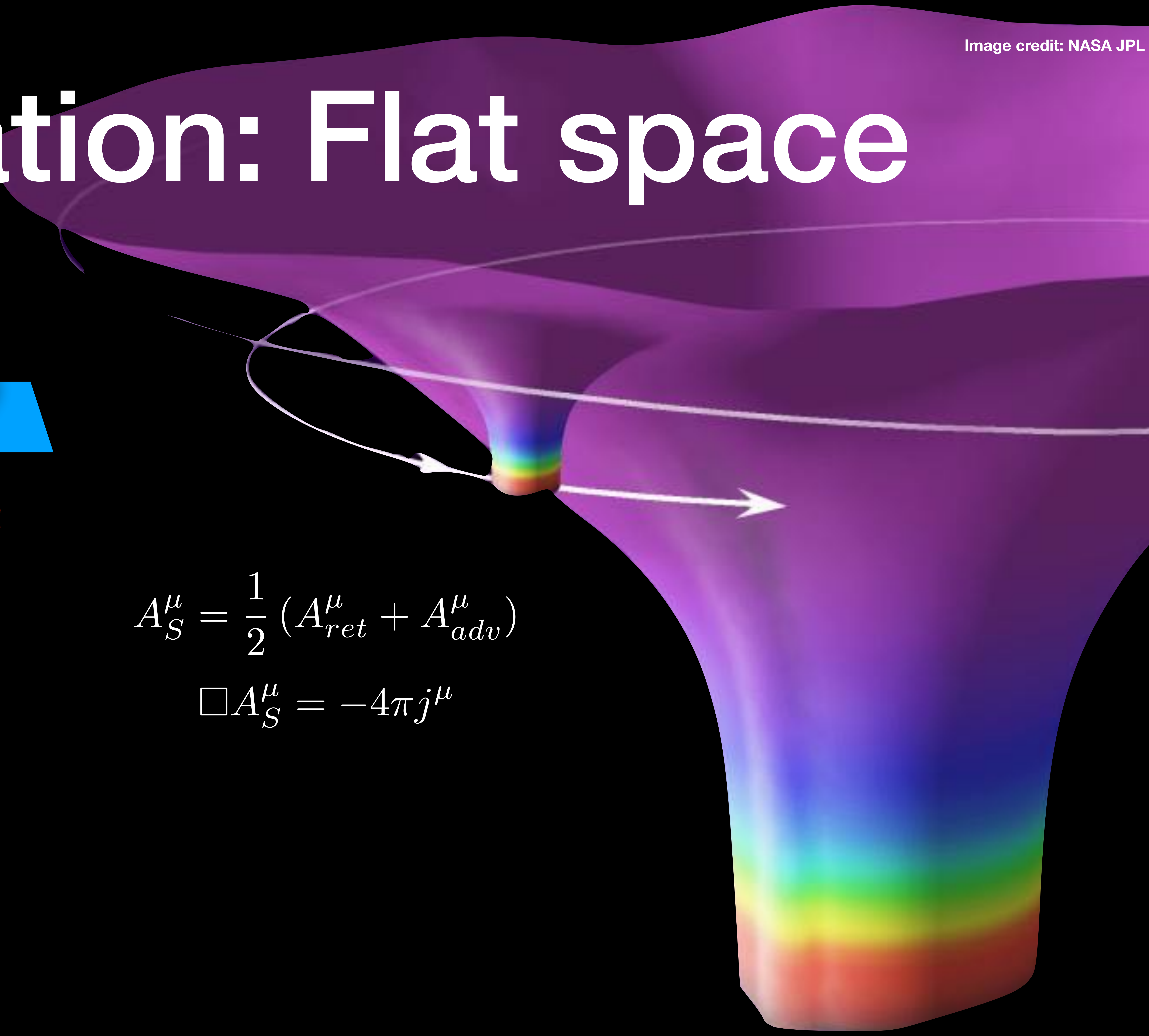
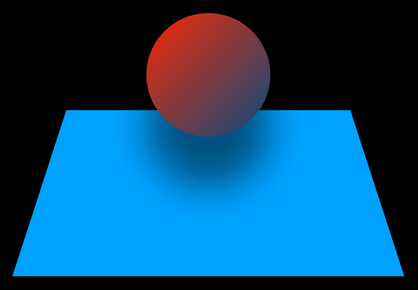
Regularisation: Flat space





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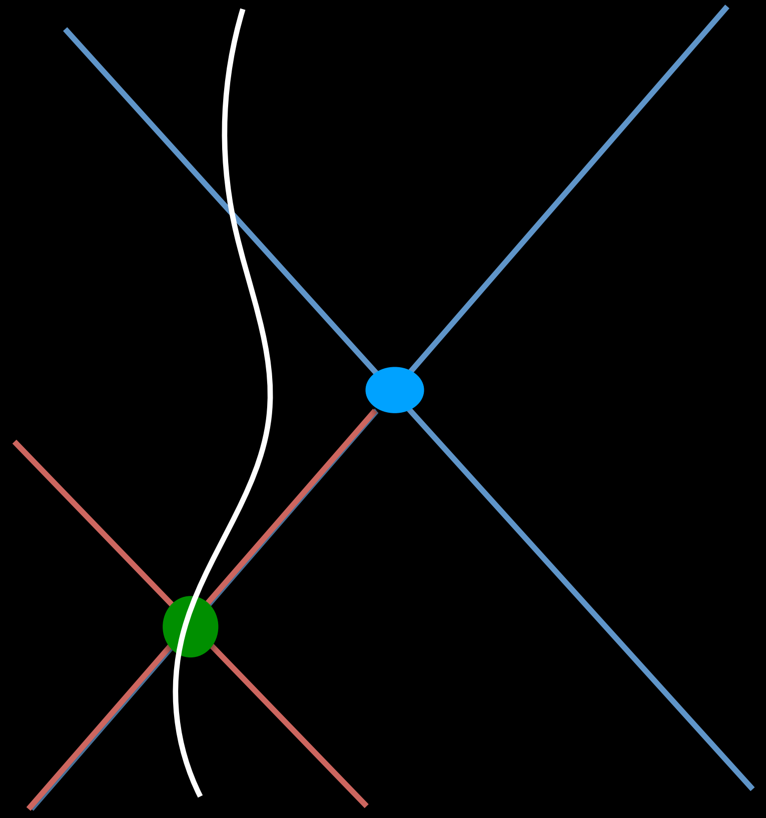

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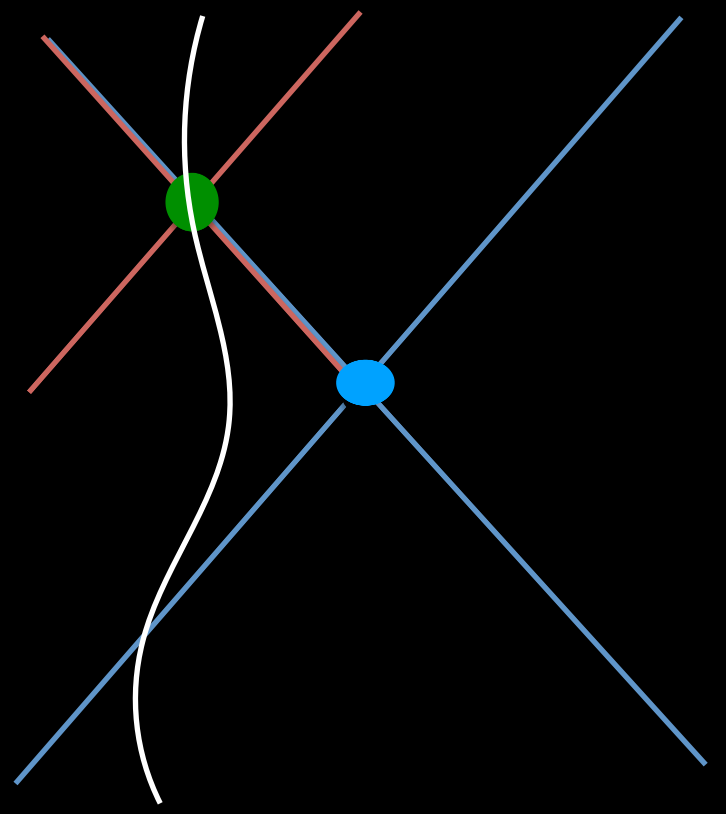

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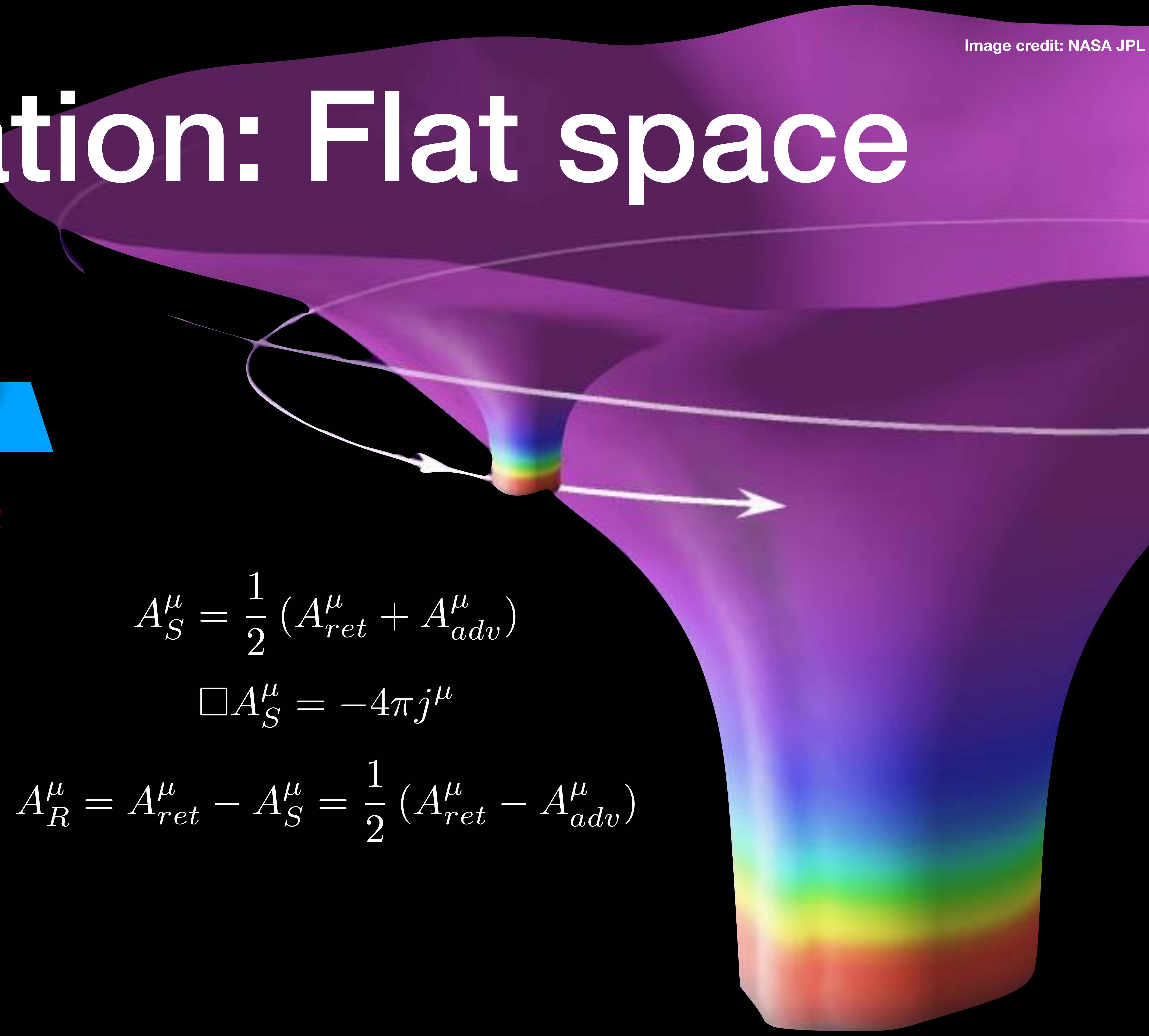

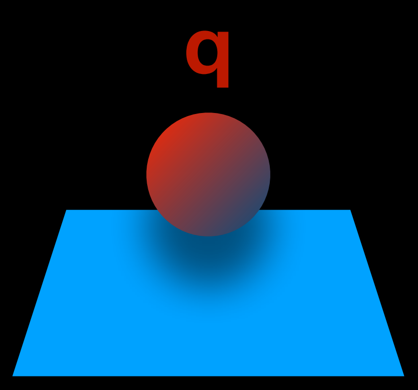

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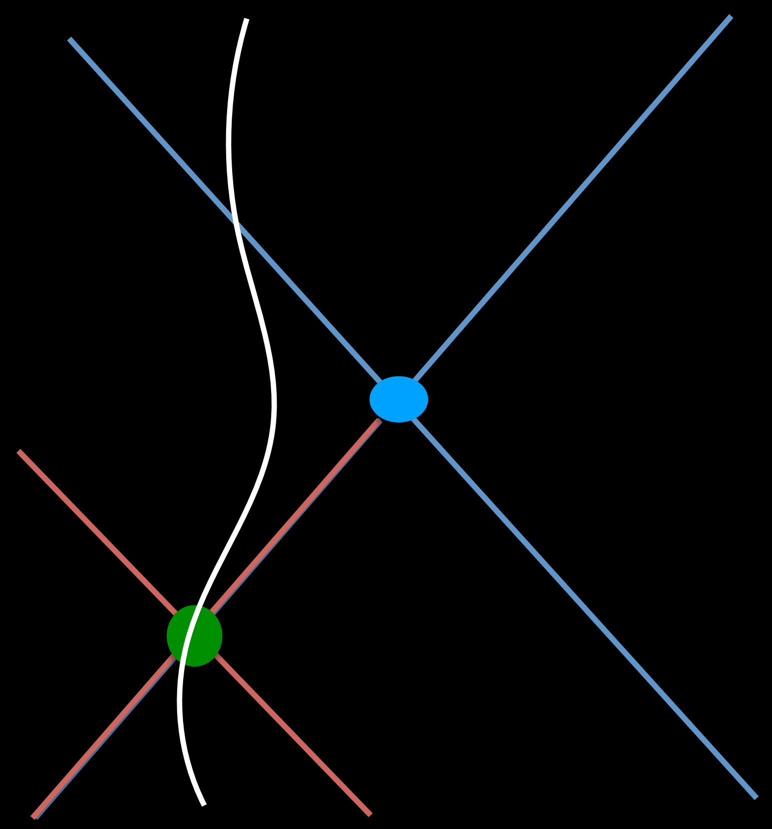

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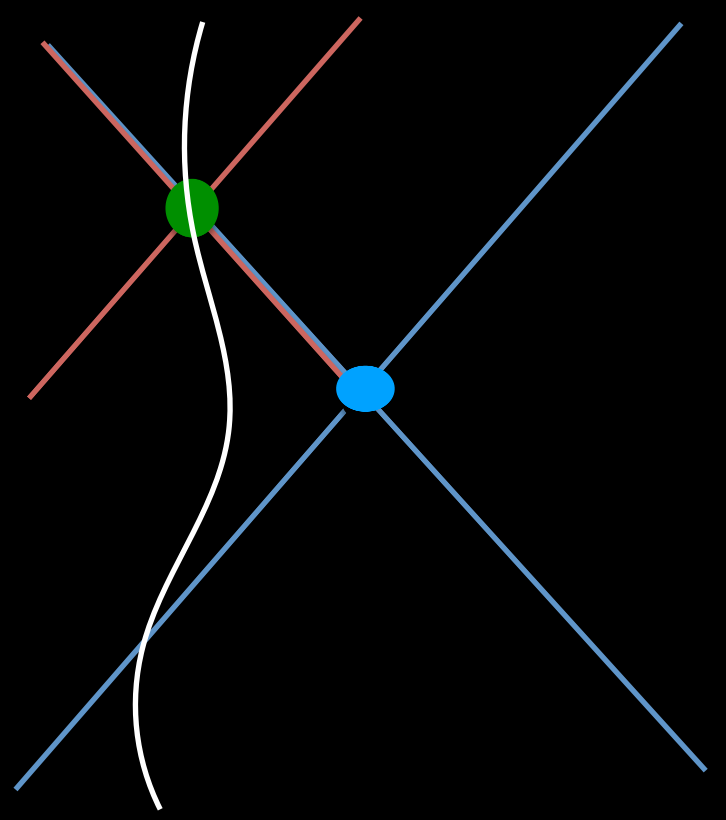

Advanced solution

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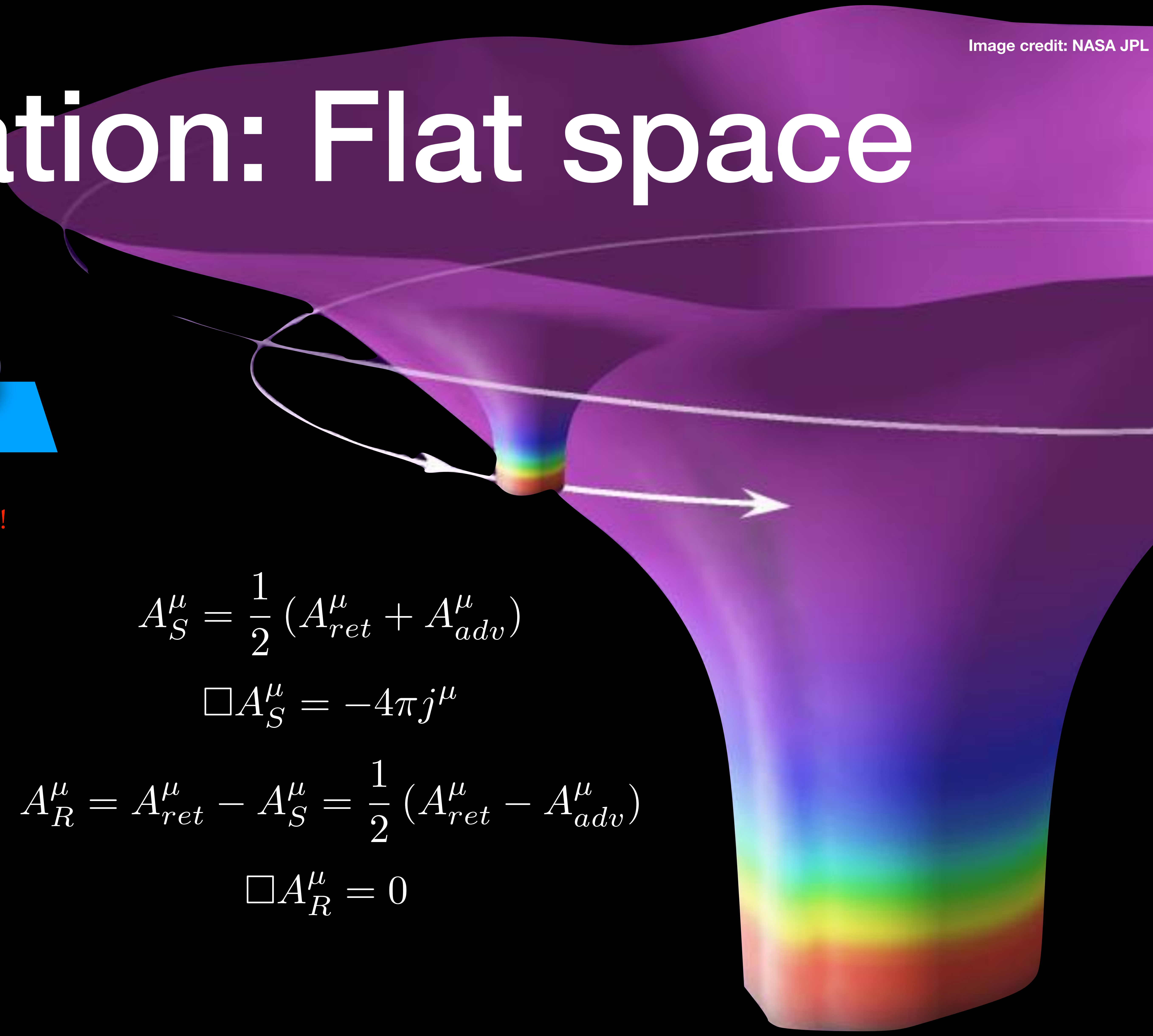
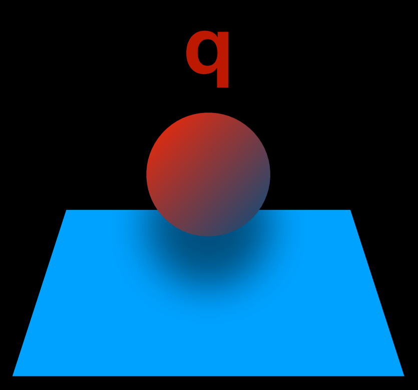
Regularisation: Flat space





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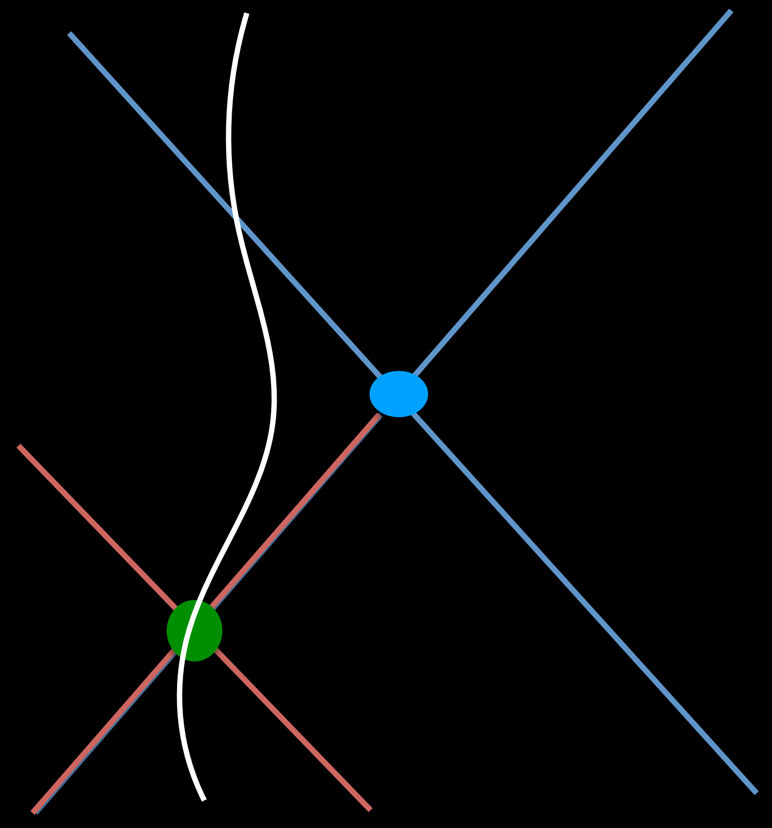

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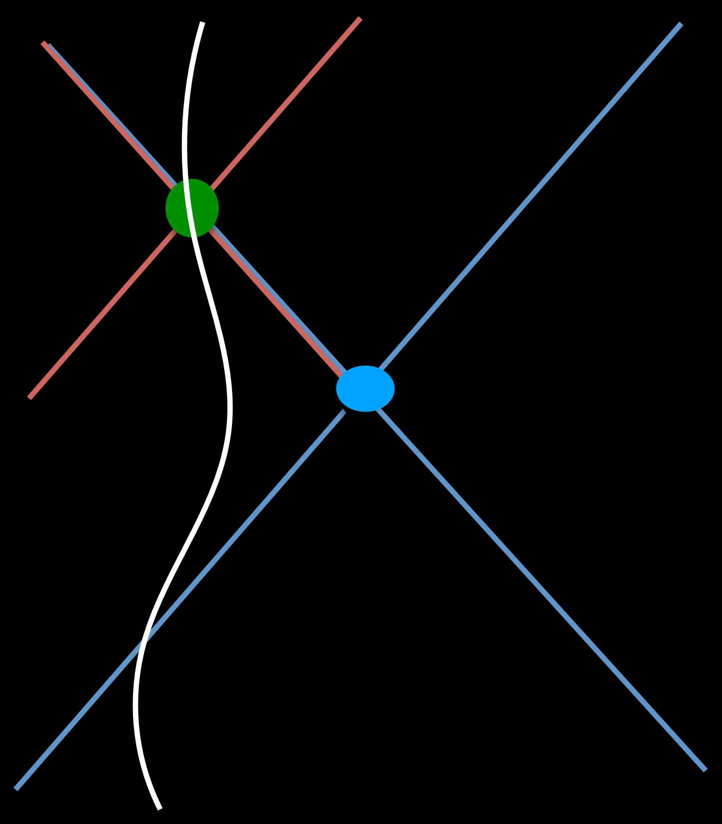
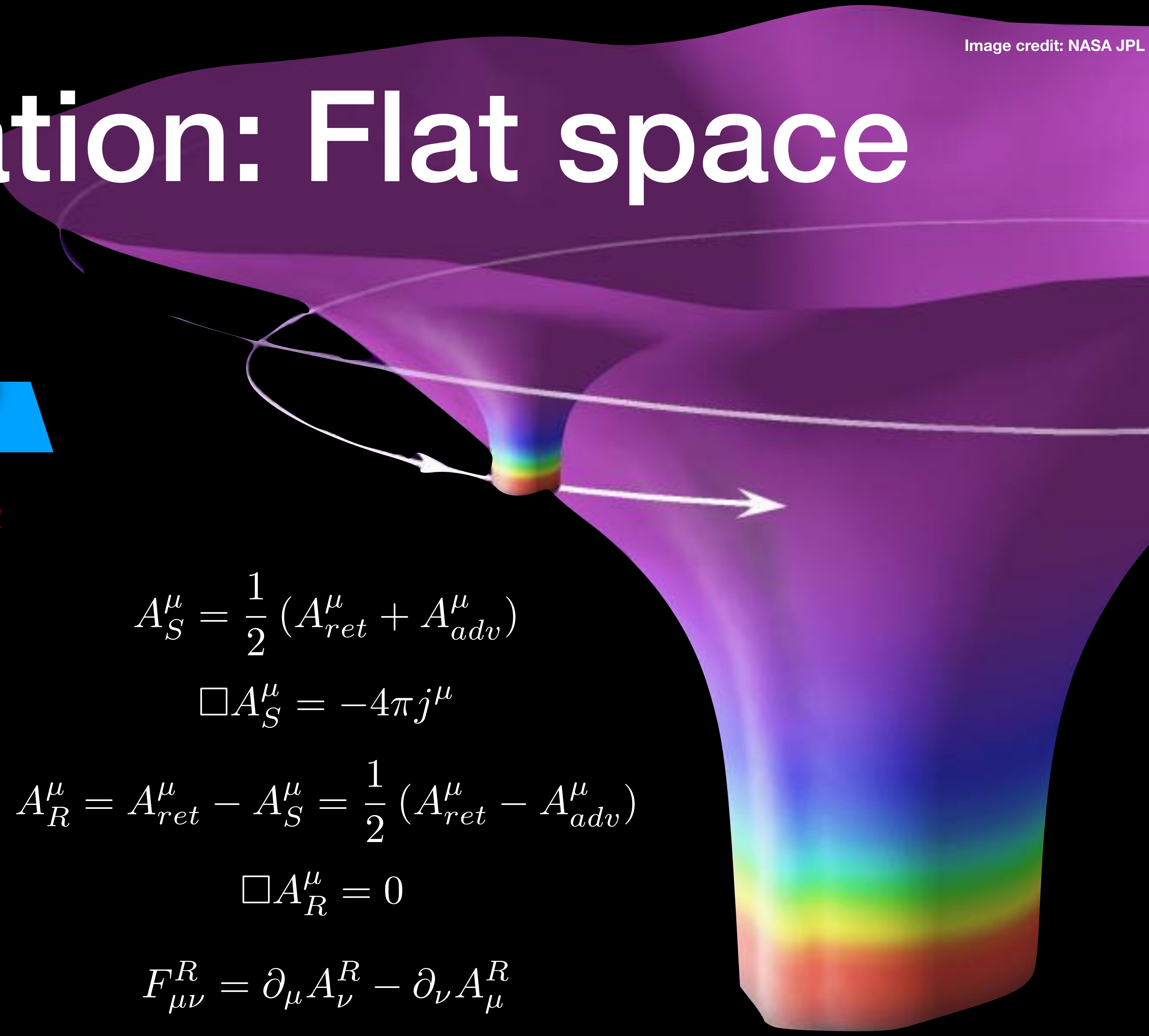
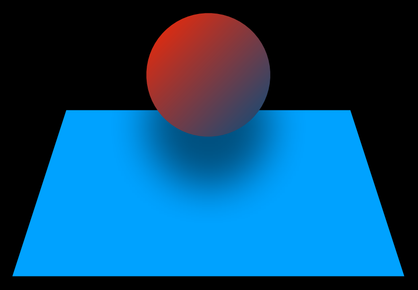


Advanced solution



Regularisation: Flat space

q



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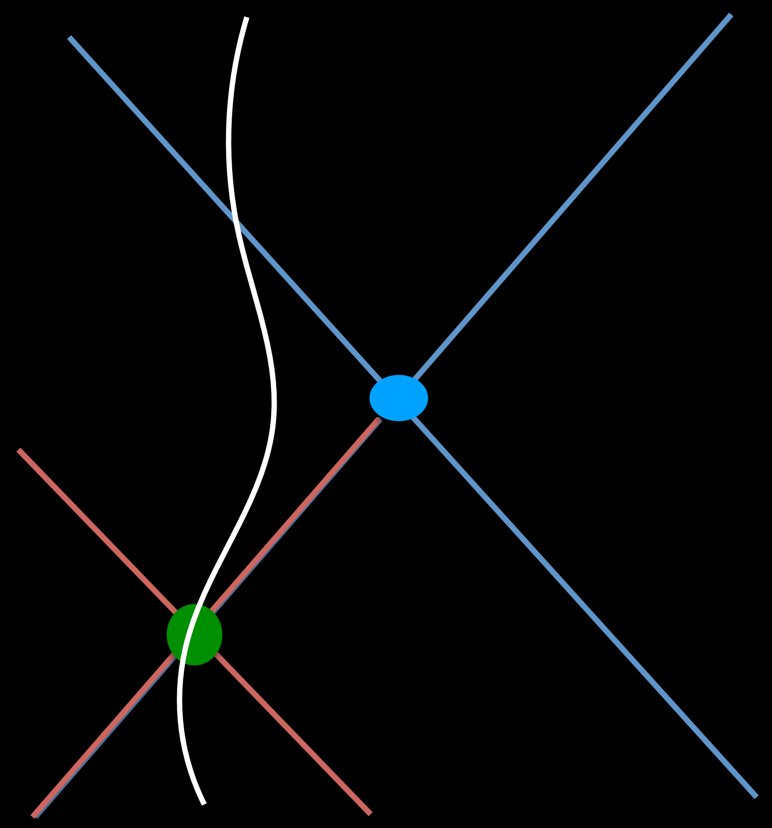
$$F^R_{\mu\nu} = \partial_\mu A^R_\nu - \partial_\nu A^R_\mu$$



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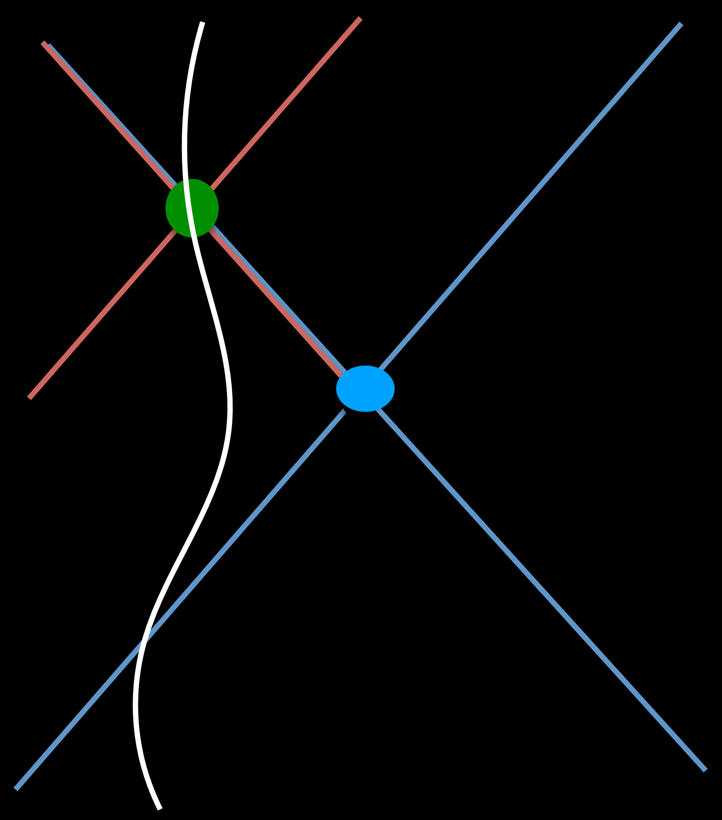

Advanced solution

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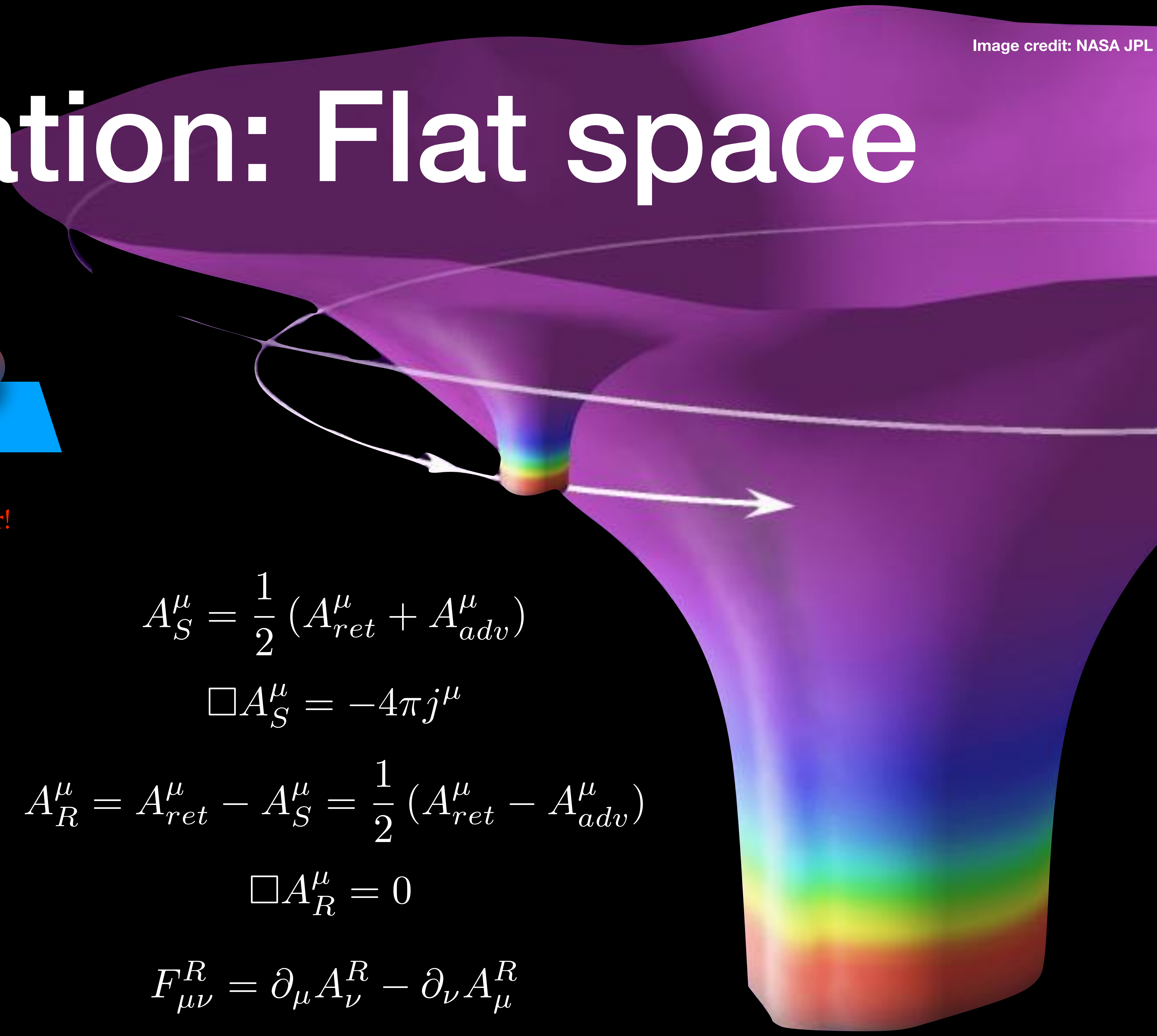
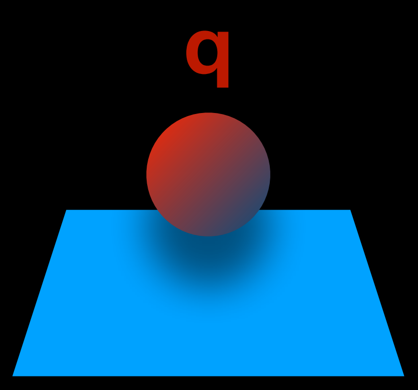


Image credit: NASA JPL



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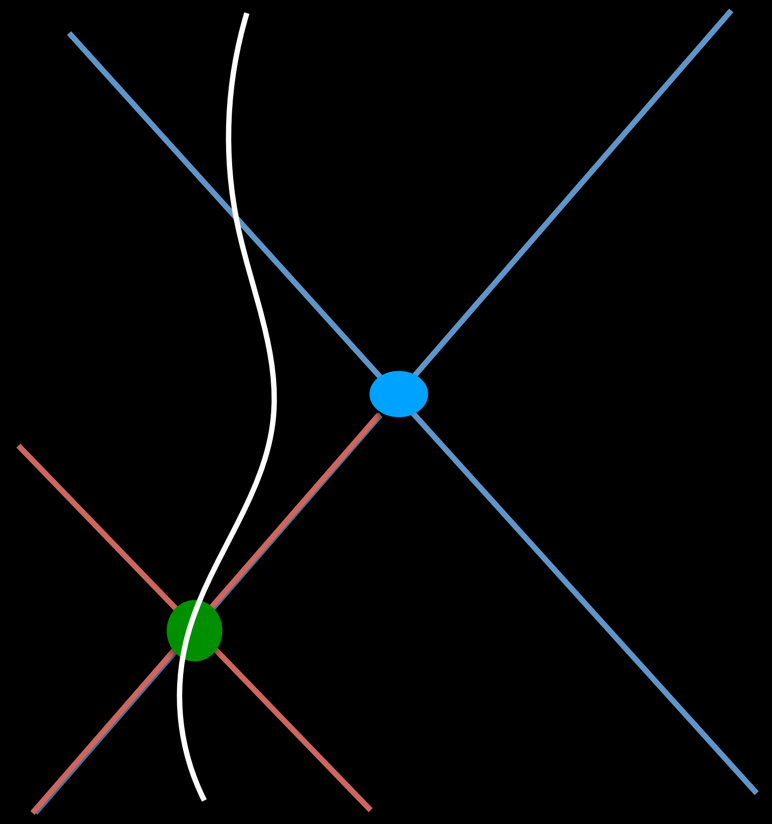
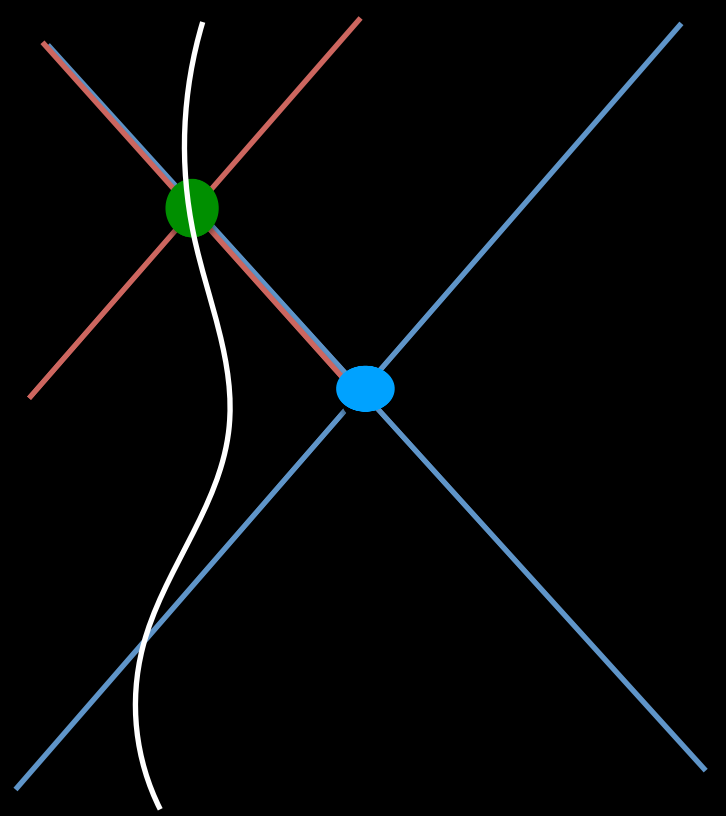
$$F^{R}_{\mu\nu} = \partial_{\mu} A^{R}_{\nu} - \partial_{\nu} A^{R}_{\mu}$$

$$\rightarrow \quad m a_{\mu} = f^{ext}_{\mu} + e F^{R}_{\mu\nu} u^{\nu}$$


Retarded solution


Advanced solution

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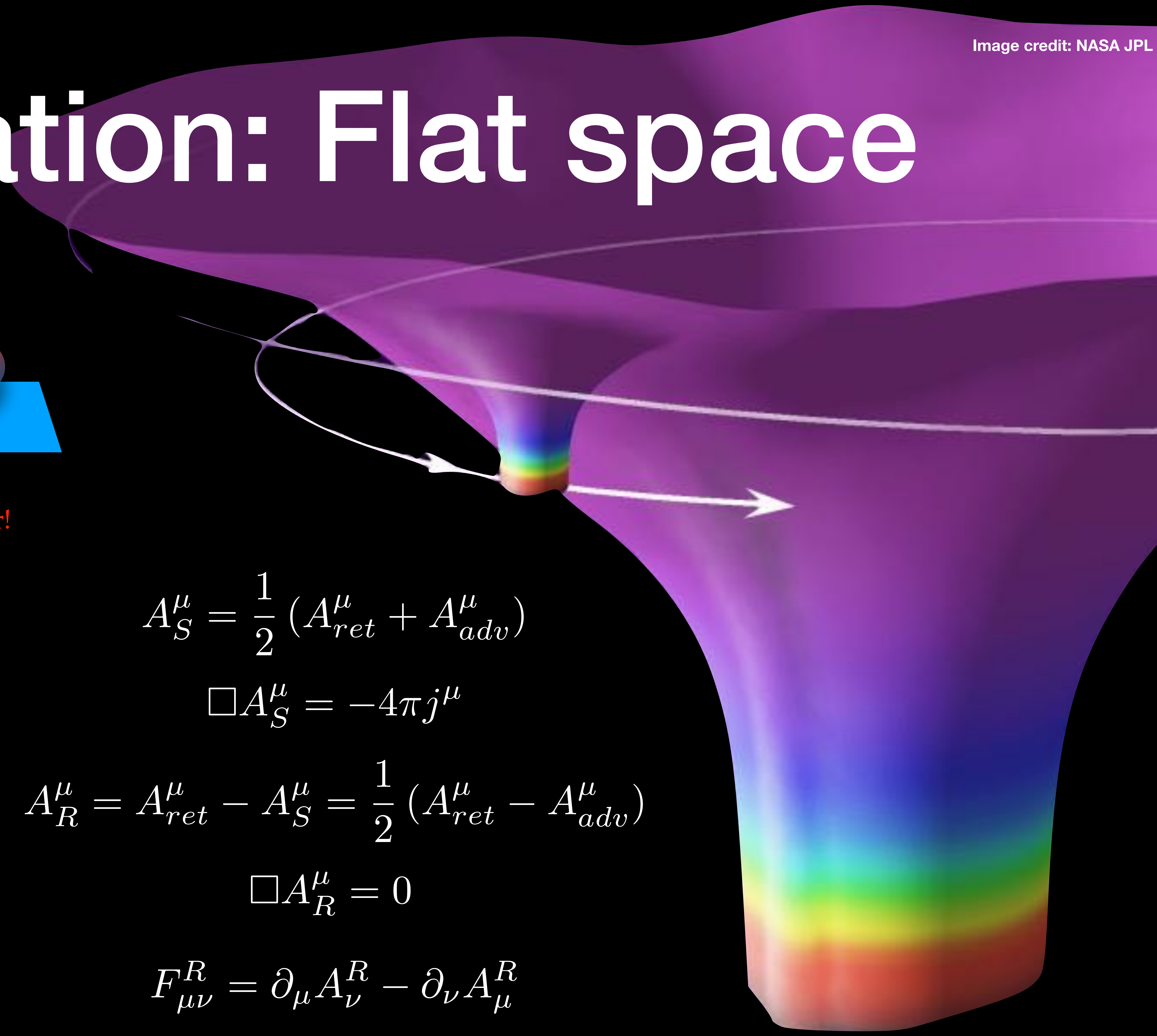

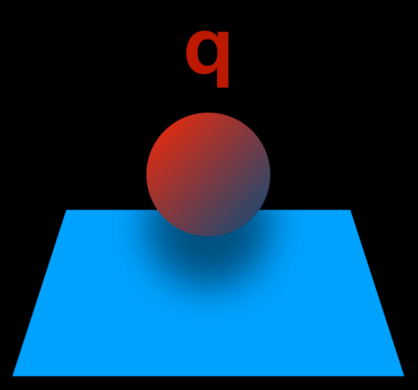

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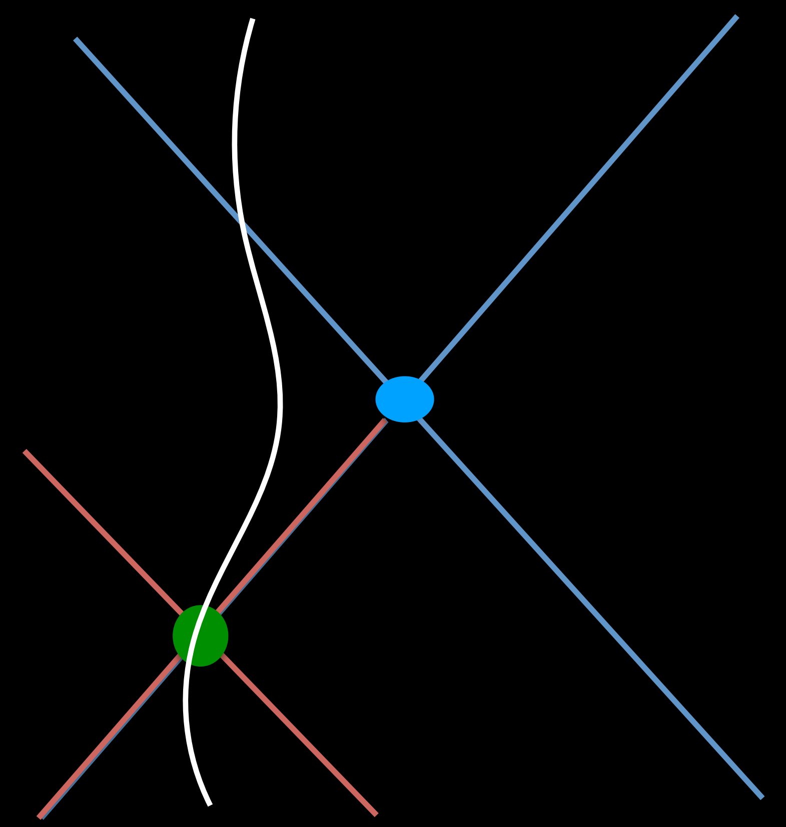
Dirac, 1938



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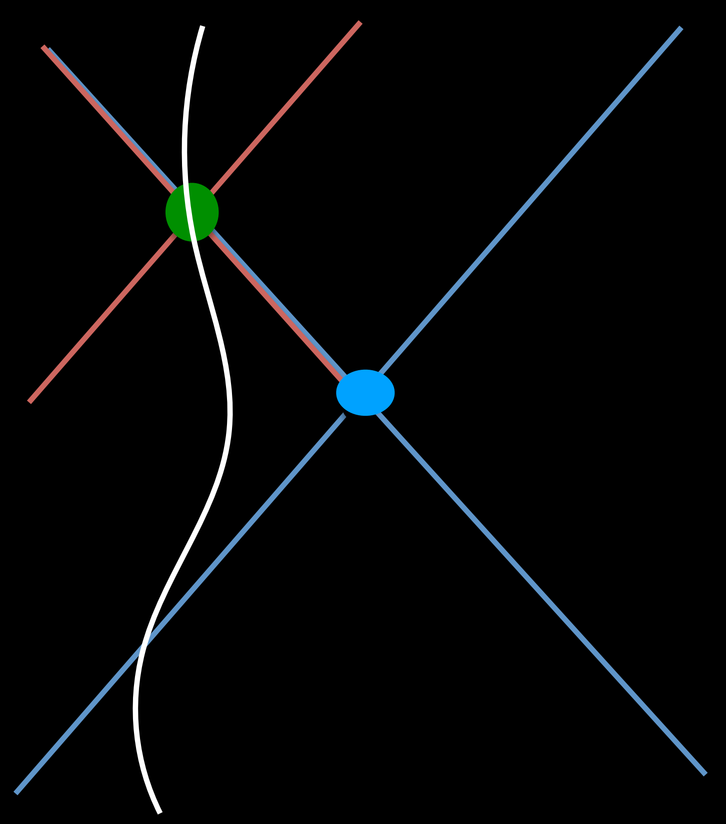
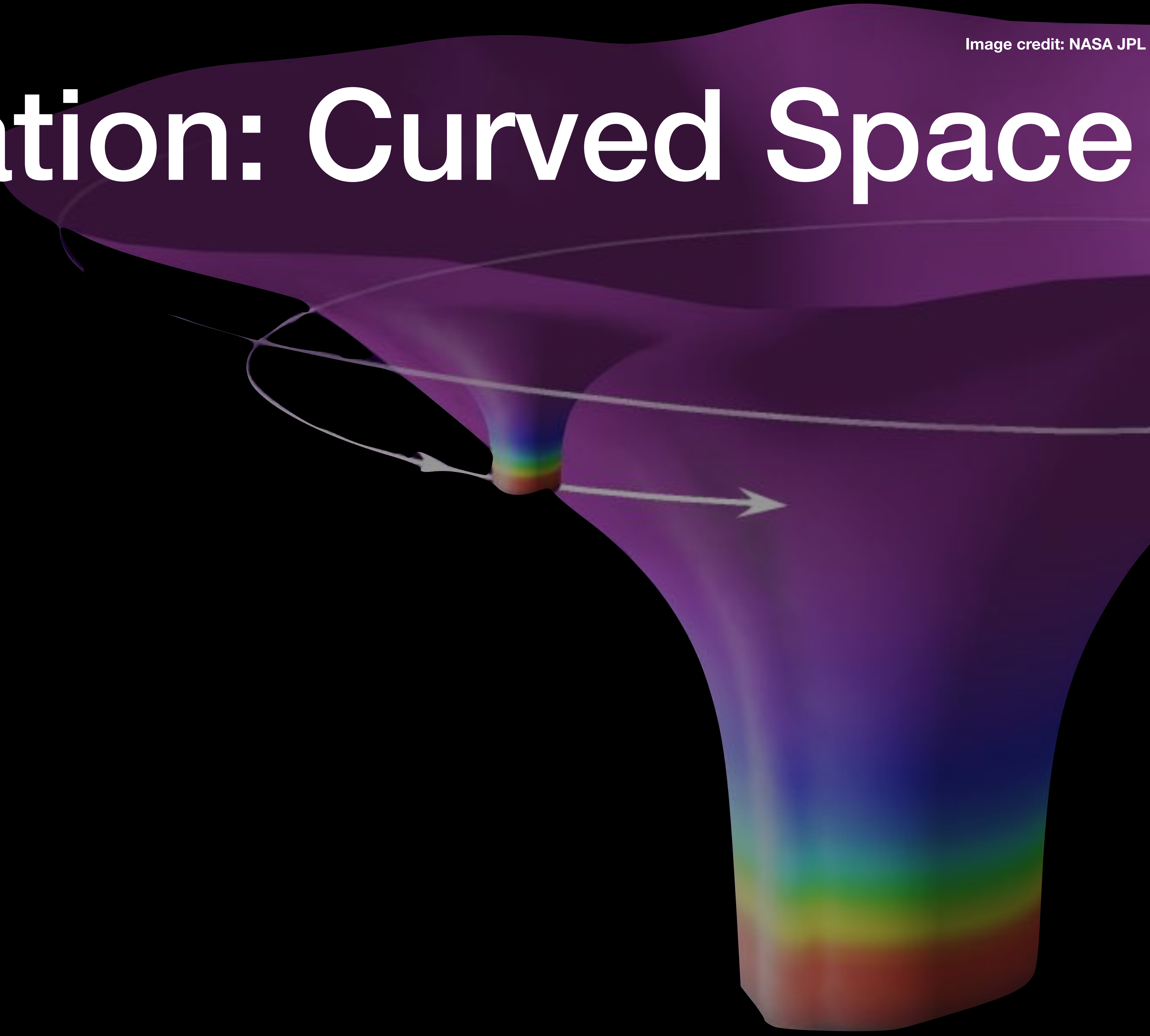


Advanced solution

Regularisation: Curved Space

Image credit: NASA JPL



Regularisation: Curved Space

- Curved spacetime

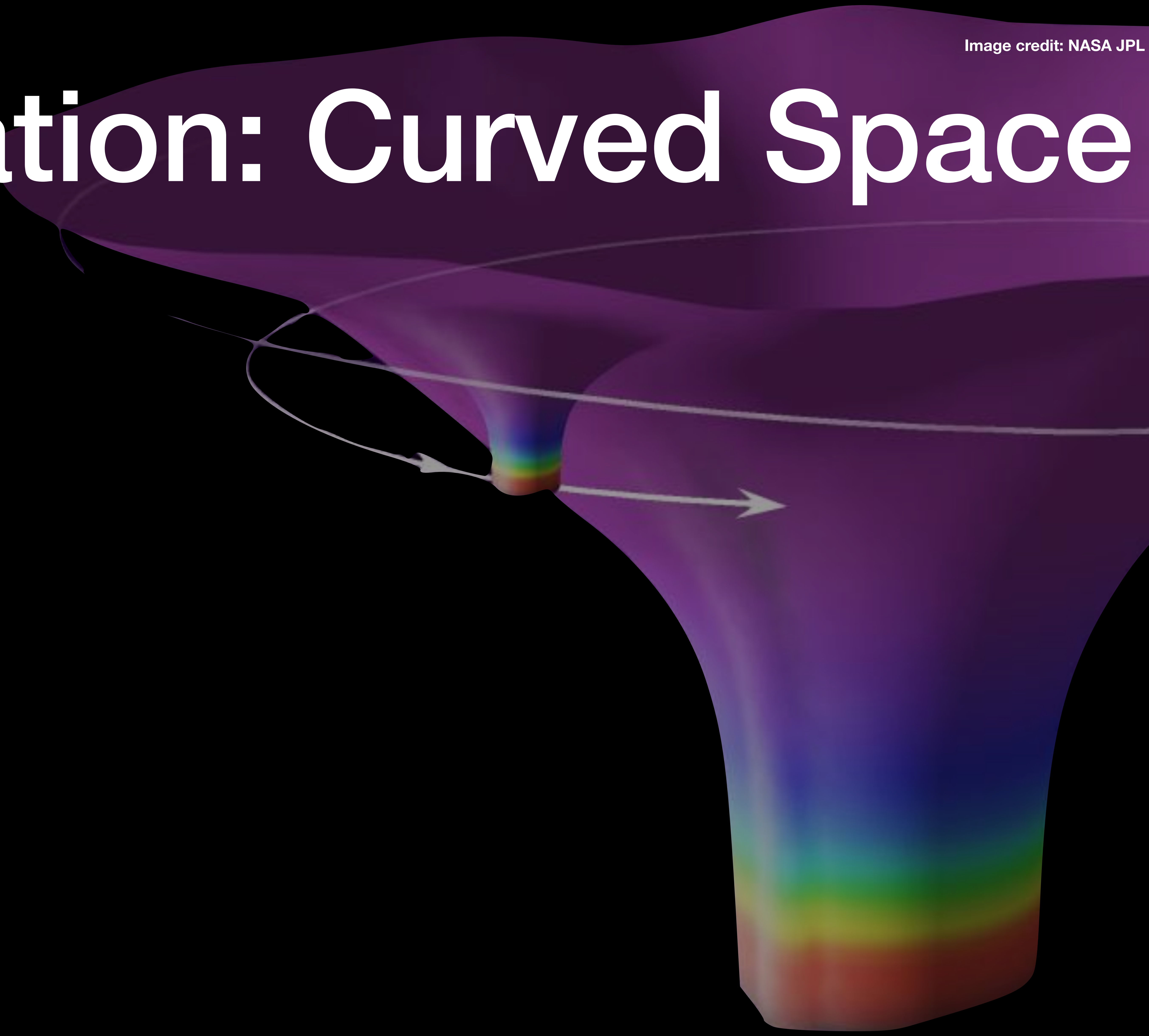


Image credit: NASA JPL

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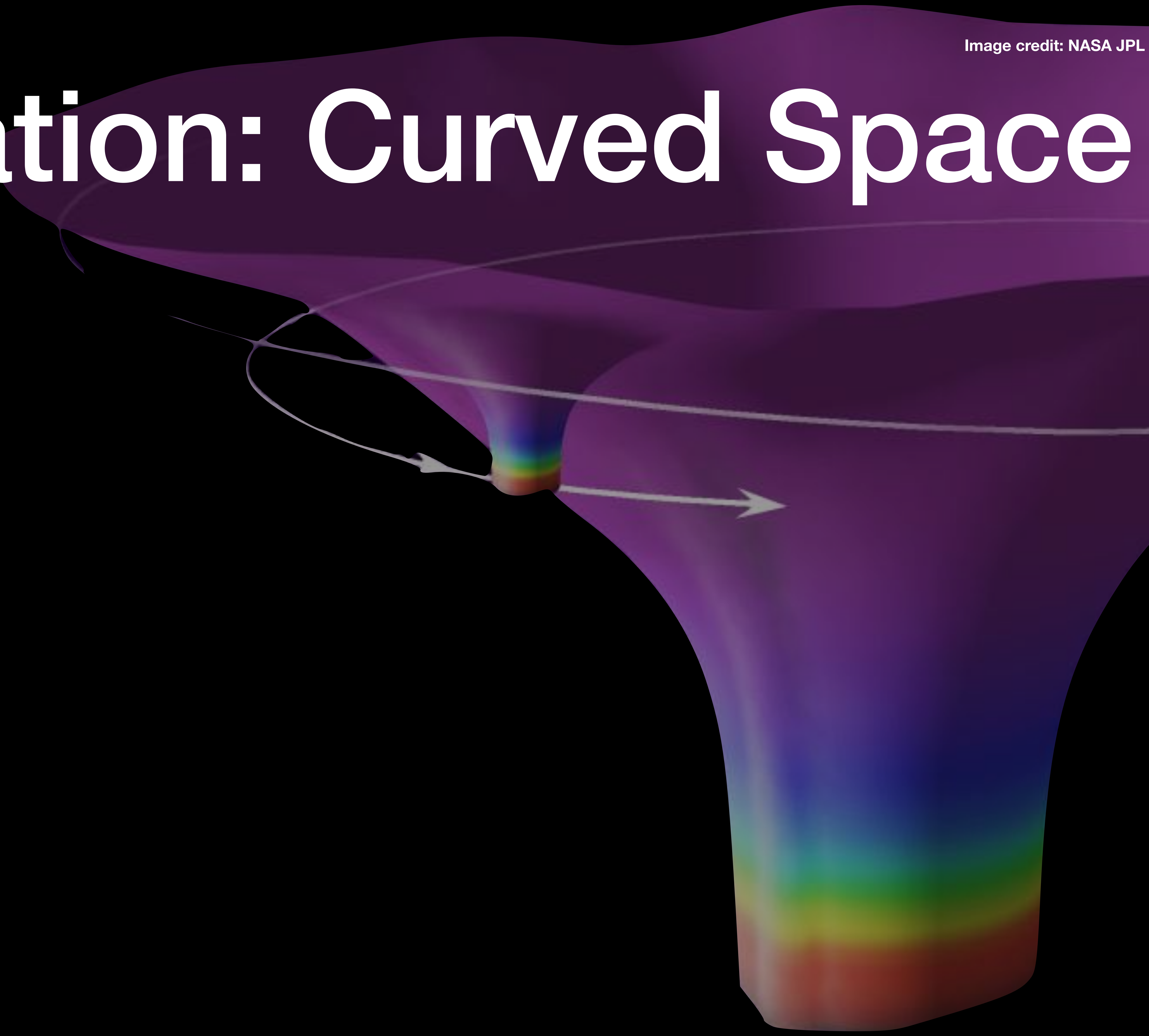

Image credit: NASA JPL

- Curved spacetime
  - Scalar case:

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Regularisation: Curved Space

- Curved spacetime
  - Scalar case:

$$(\Box - \zeta R)\,\Phi = -4\pi\mu + \mathcal{O}(\epsilon^2)$$

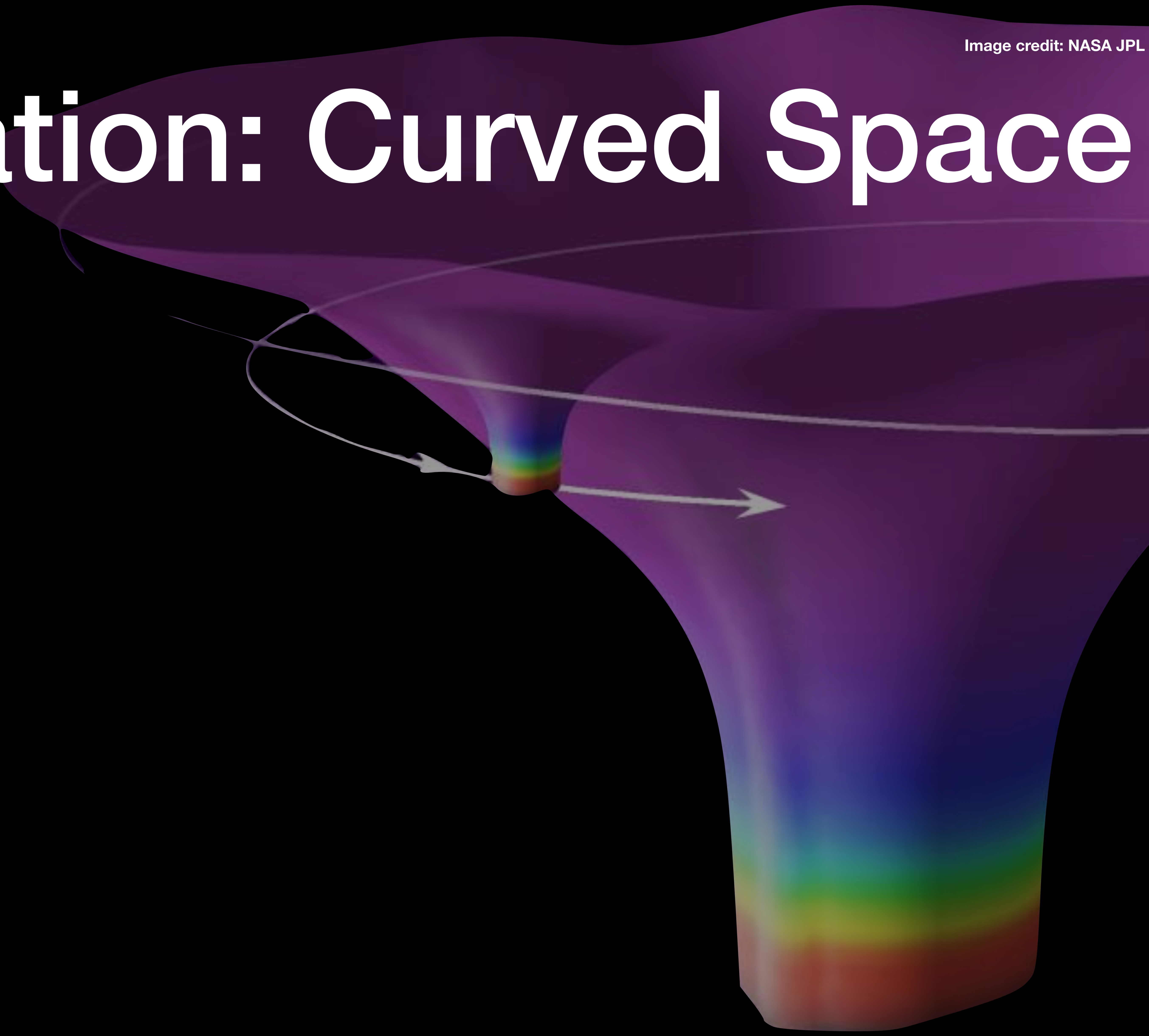

Image credit: NASA JPL

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Regularisation: Curved Space

- Curved spacetime
  - Scalar case:
    $$(\Box - \zeta R)\,\Phi = -4\pi\mu + \mathcal{O}(\epsilon^2)$$
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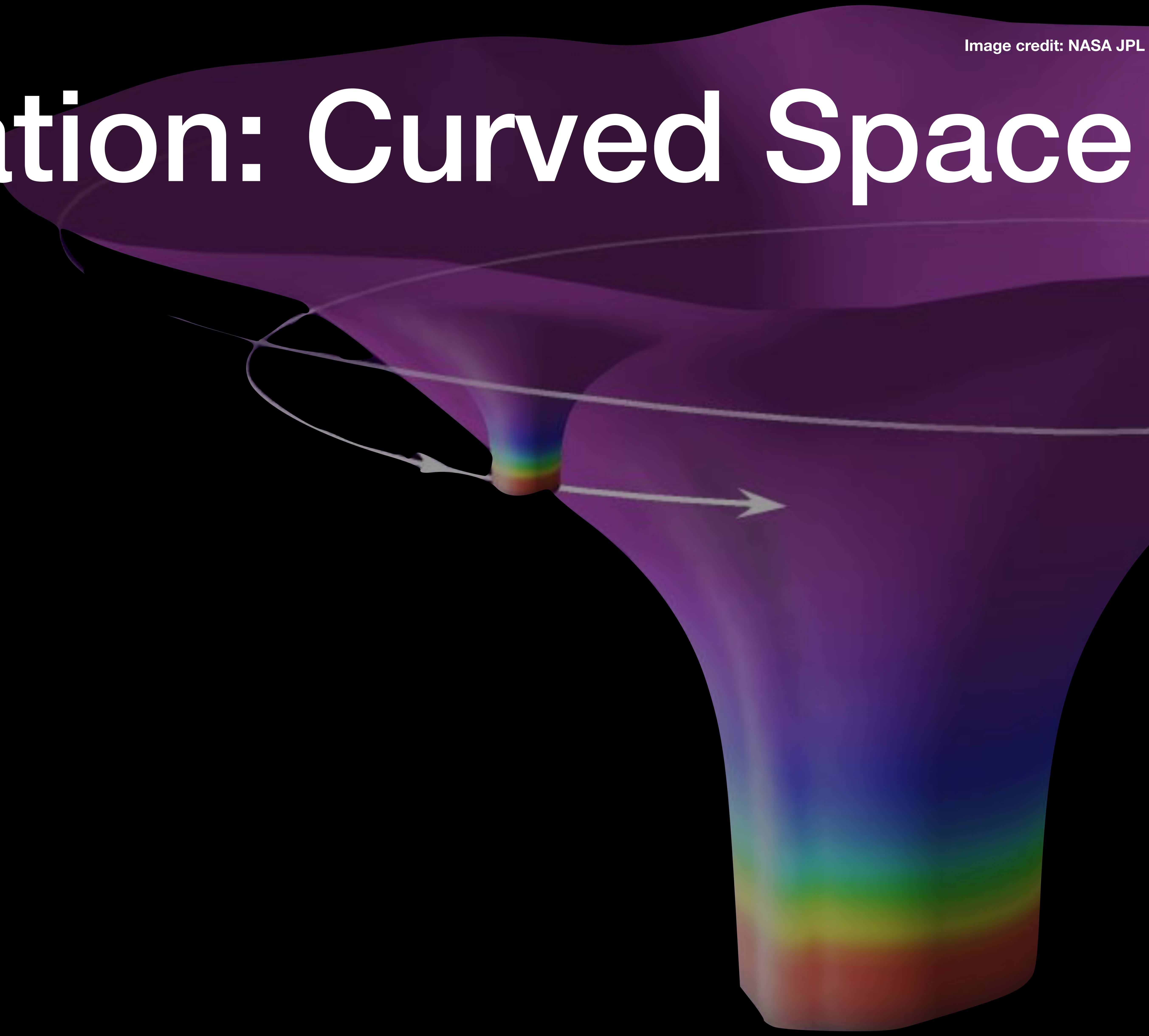

Image credit: NASA JPL

Regularisation: Curved Space

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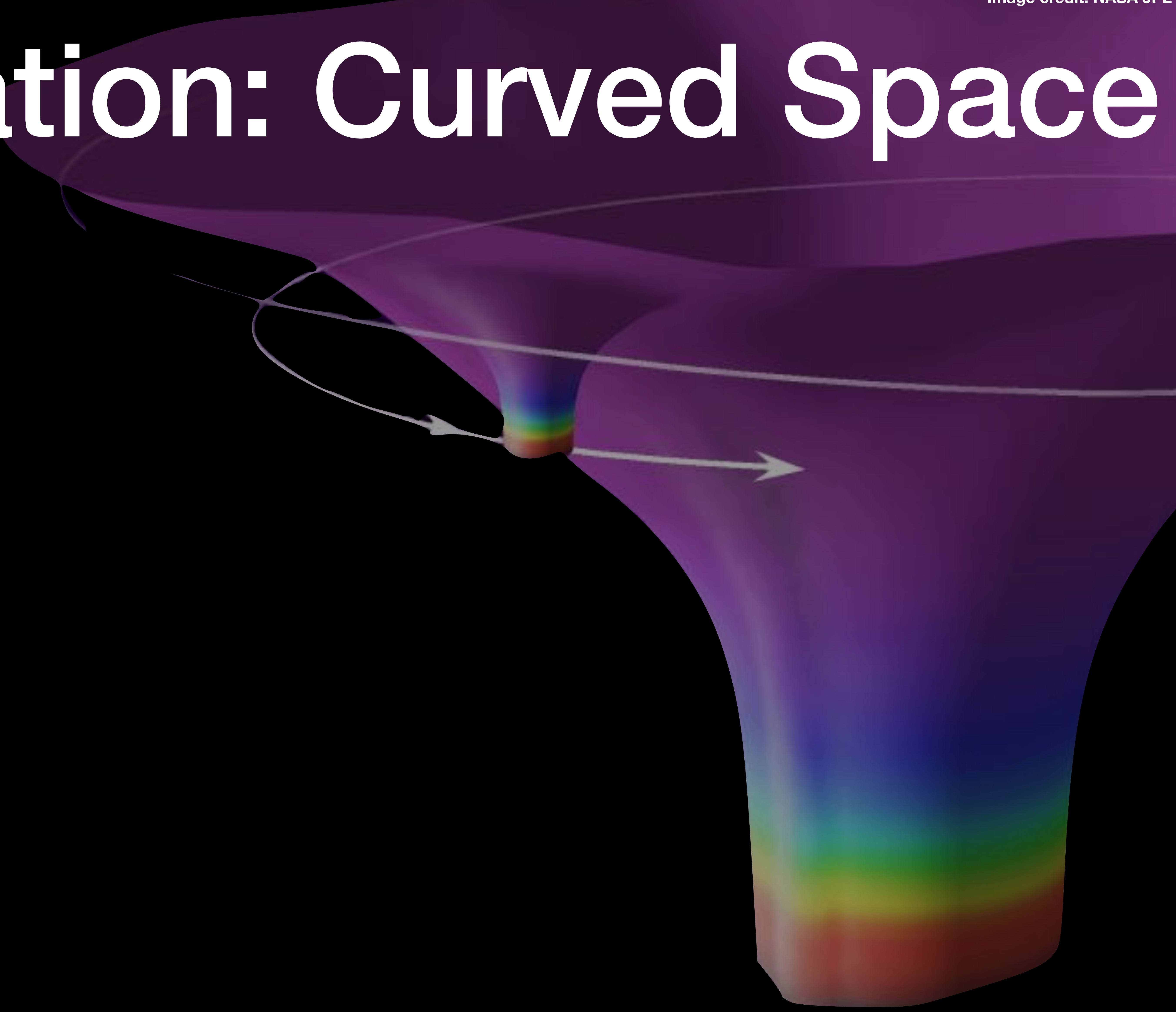
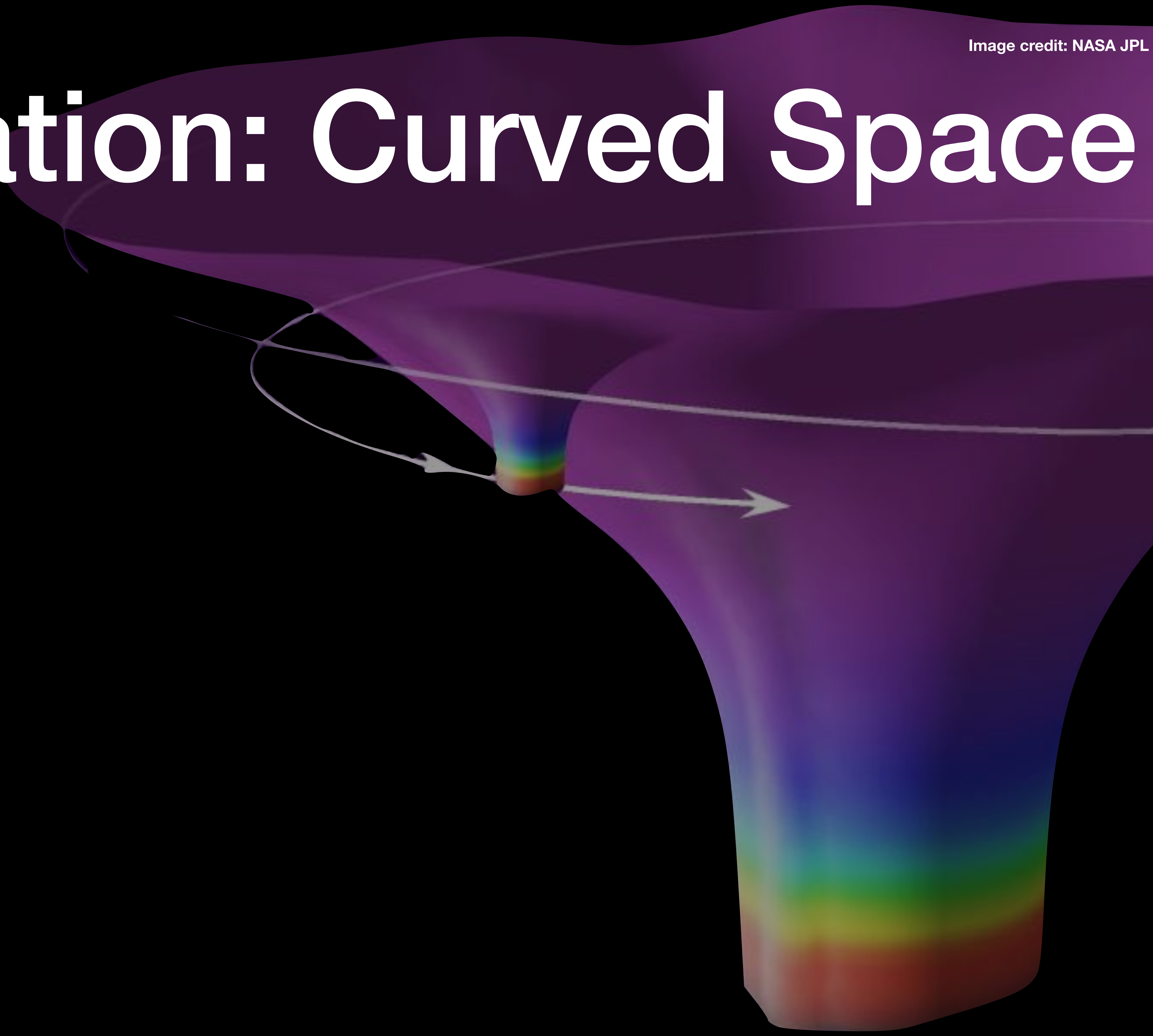


- Curved spacetime
  - Scalar case:
    $$(\Box - \zeta R)\,\Phi = -4\pi\mu + \mathcal{O}(\epsilon^2)$$
  - Electromagnetic case:
    $$(\delta^a{}_b\Box - R^a_b)\,A^b = -4\pi j^a + \mathcal{O}(\epsilon^2)$$

Regularisation: Curved Space

Image credit: NASA JPL



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  - Gravitational case:

Regularisation: Curved Space

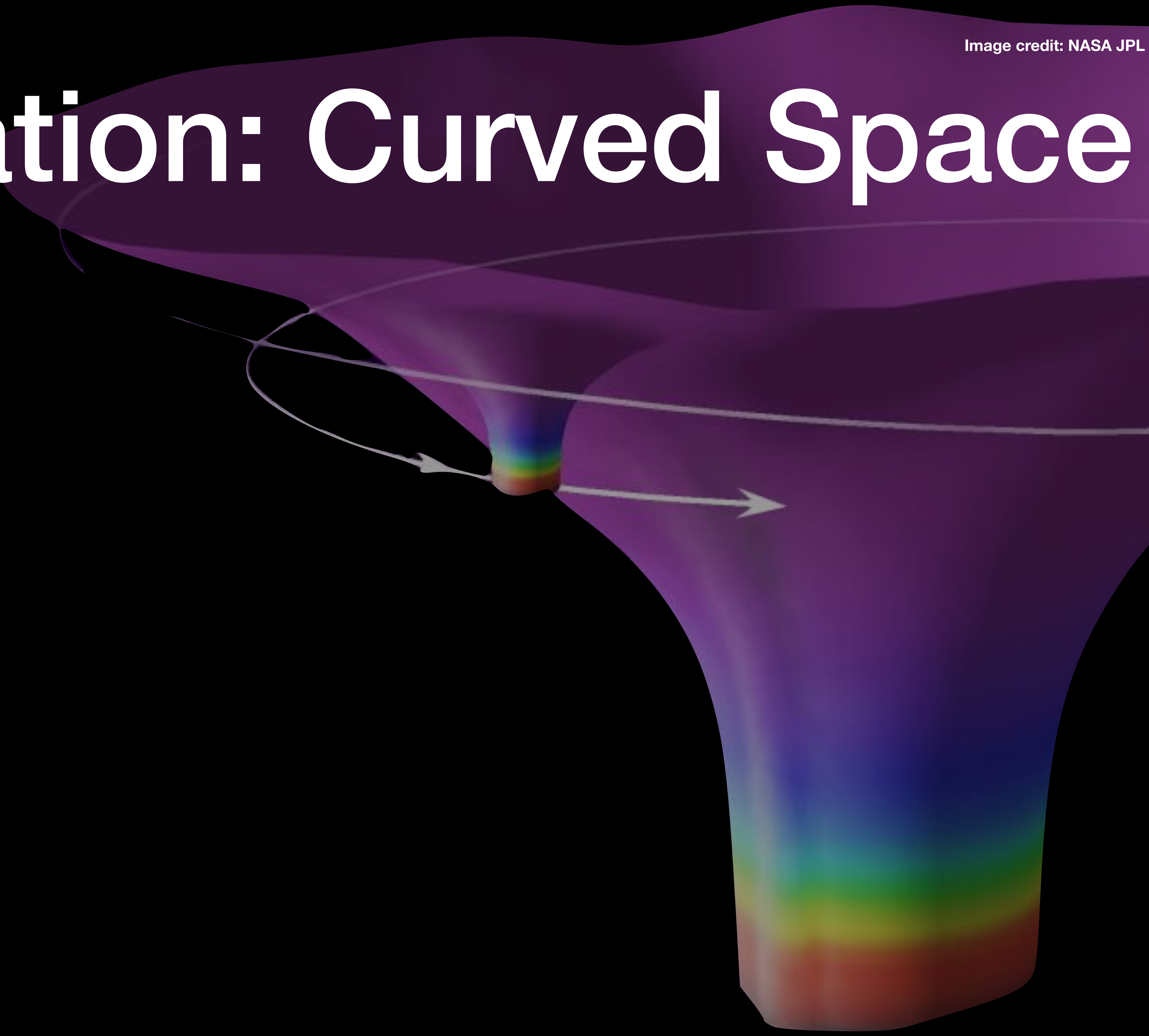

Image credit: NASA JPL

- Curved spacetime
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  $$(\delta_{ac}\delta_{bd}\Box + 2R_{cadb})\,\psi^{cd} = -16\pi T^{ab}_{(eff)} + \mathcal{O}(\epsilon^2)$$

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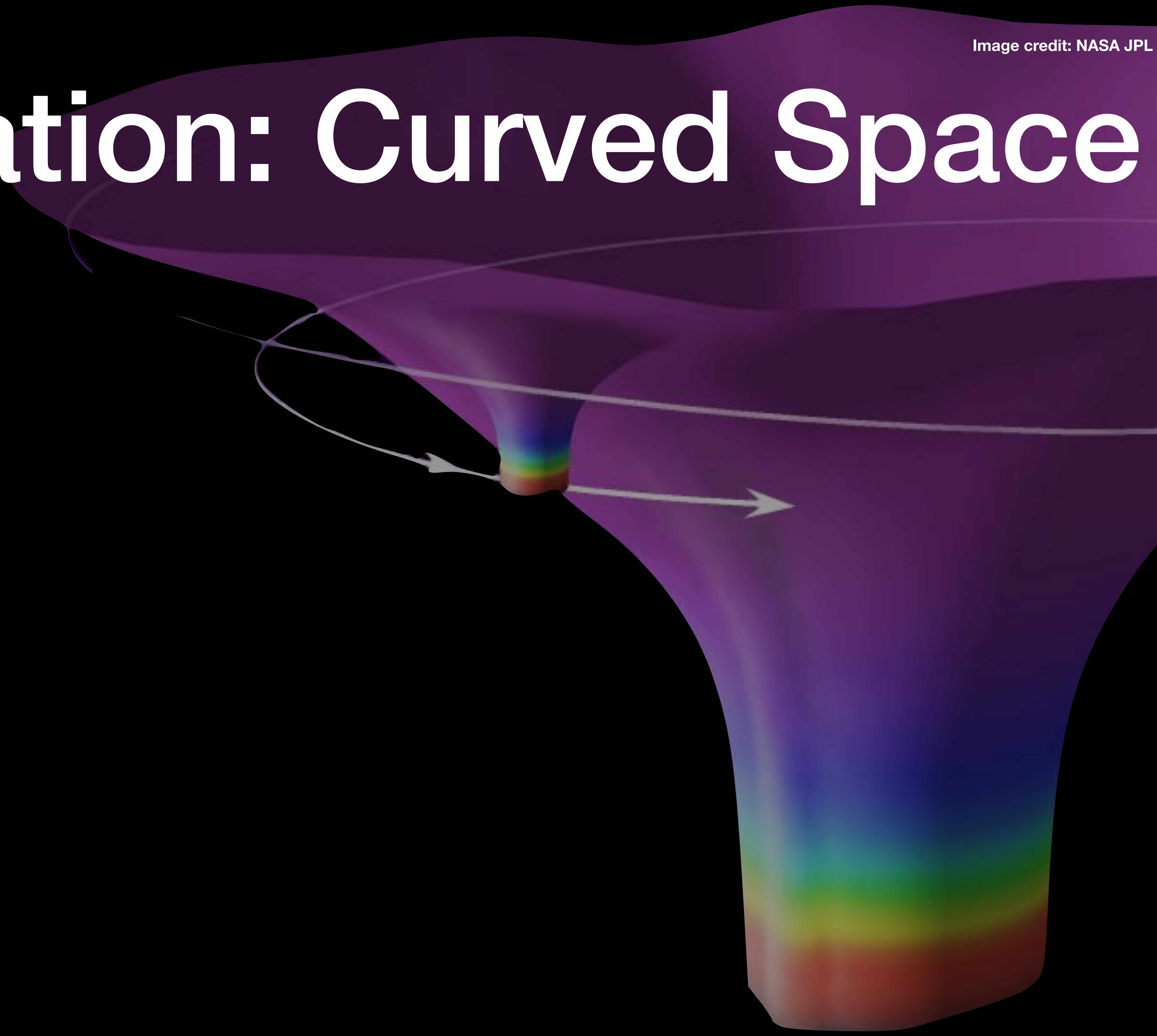
Regularisation: Curved Space



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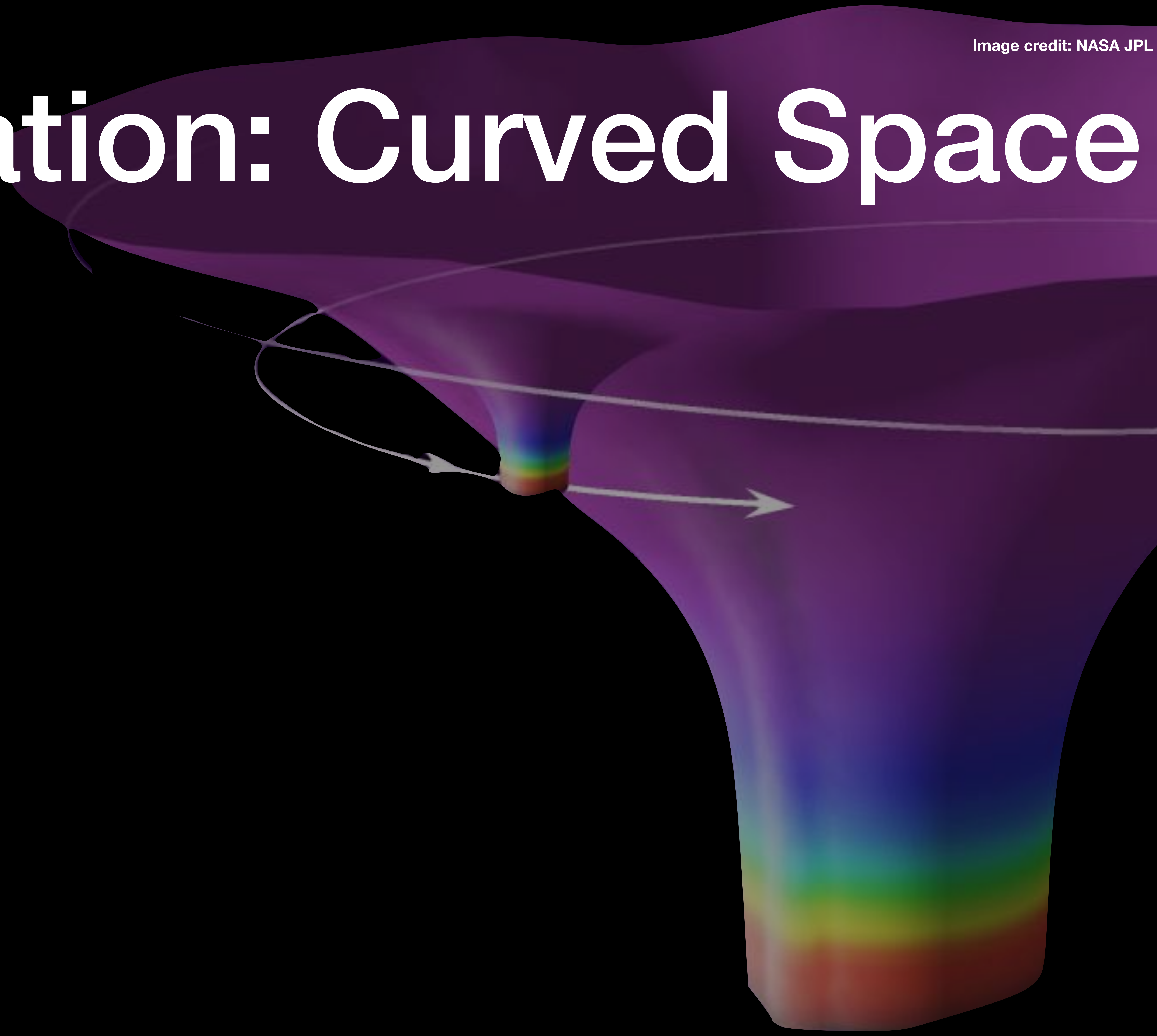
    $$(\Box - \zeta R)\,\Phi = -4\pi\mu + \mathcal{O}(\epsilon^2)$$
  - Electromagnetic case:
    $$(\delta^a{}_b \Box - R^a_b)\,A^b = -4\pi j^a + \mathcal{O}(\epsilon^2)$$
  - Gravitational case:
    $$(\delta_{ac}\delta_{bd}\Box + 2R_{cadb})\,\psi^{cd} = -16\pi T^{ab}_{(eff)} + \mathcal{O}(\epsilon^2)$$
  - General case:

Regularisation: Curved Space

Image credit: NASA JPL



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  - General case:
    $$(\delta^A{}_B\Box - P^A{}_B)\,\Psi^B{}_{(ret)/(adv)} = -4\pi\mathcal{M}^A + \mathcal{O}(\epsilon^2)$$

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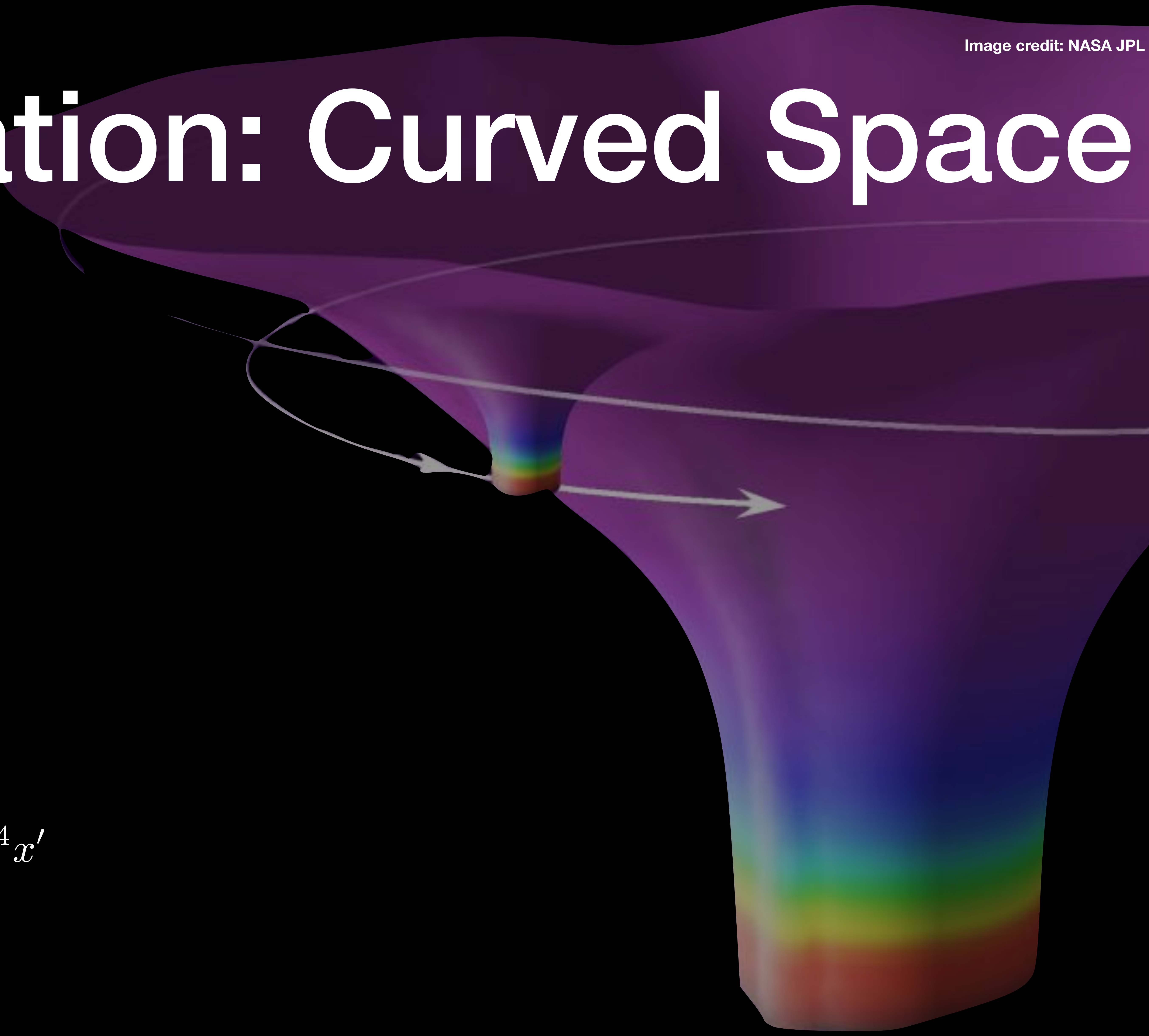
Image credit: NASA JPL

Regularisation: Curved Space



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$$\Psi^A_{(ret)/(adv)} = \int G^A{}_{B'(ret)/(adv)}(x,x')\mathcal{M}^{B'}(x')\sqrt{-g'}d^4x'$$

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Regularisation: Curved Space

Image credit: NASA JPL

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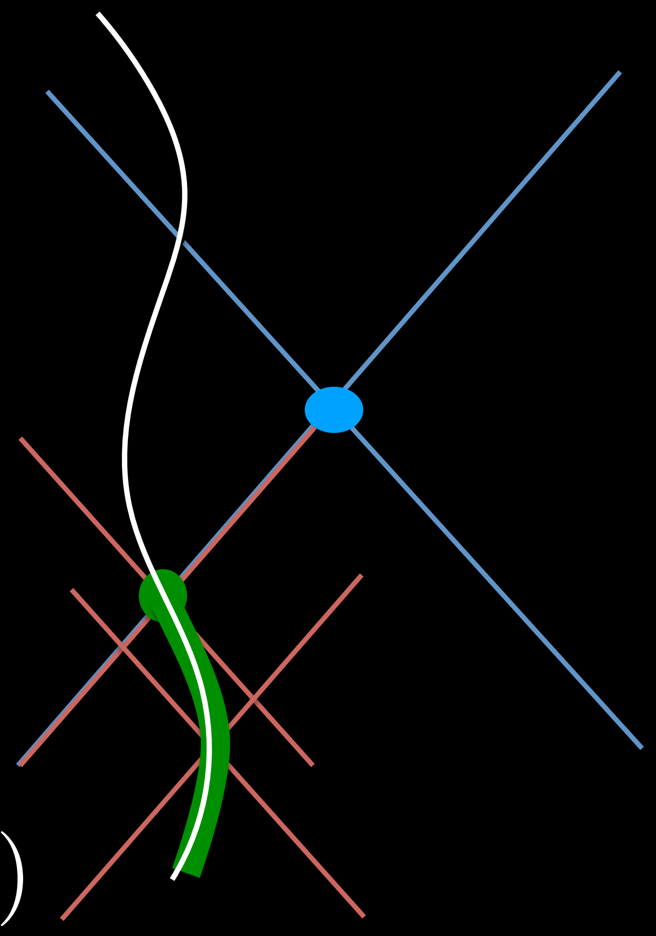
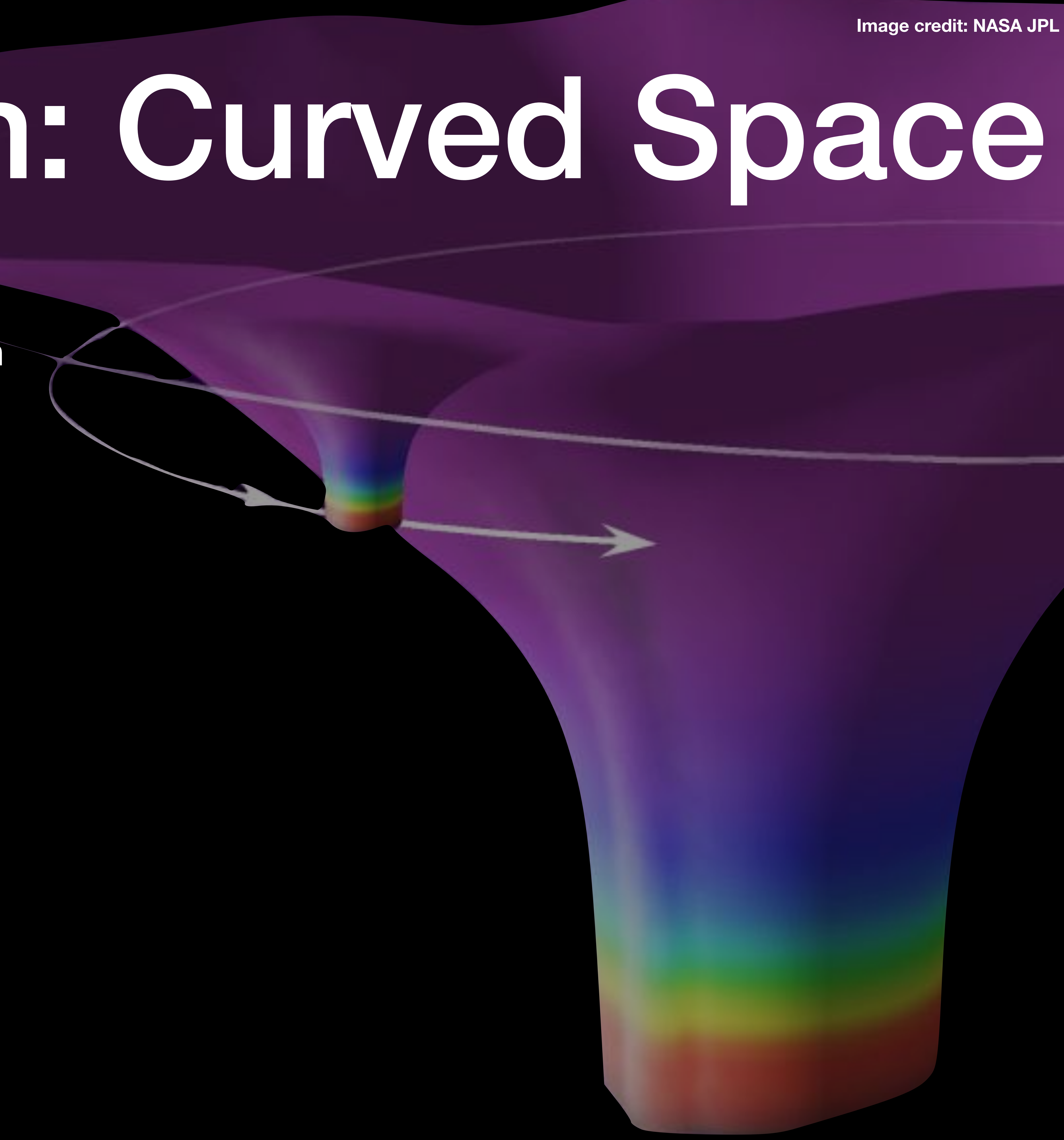
Retarded solution





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Regularisation: Curved Space

Image credit: NASA JPL

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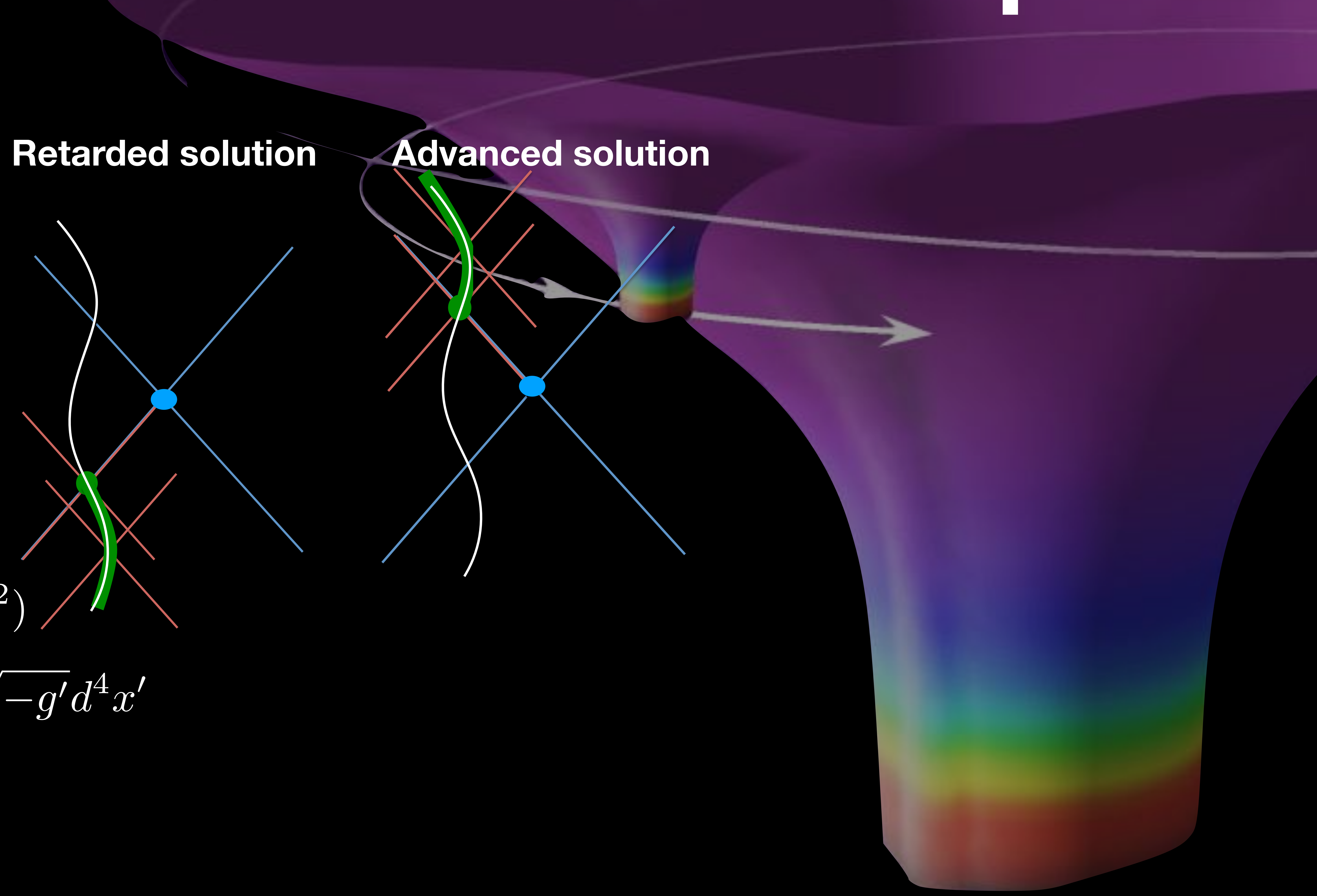

Image credit: NASA JPL

Regularisation: Curved Space

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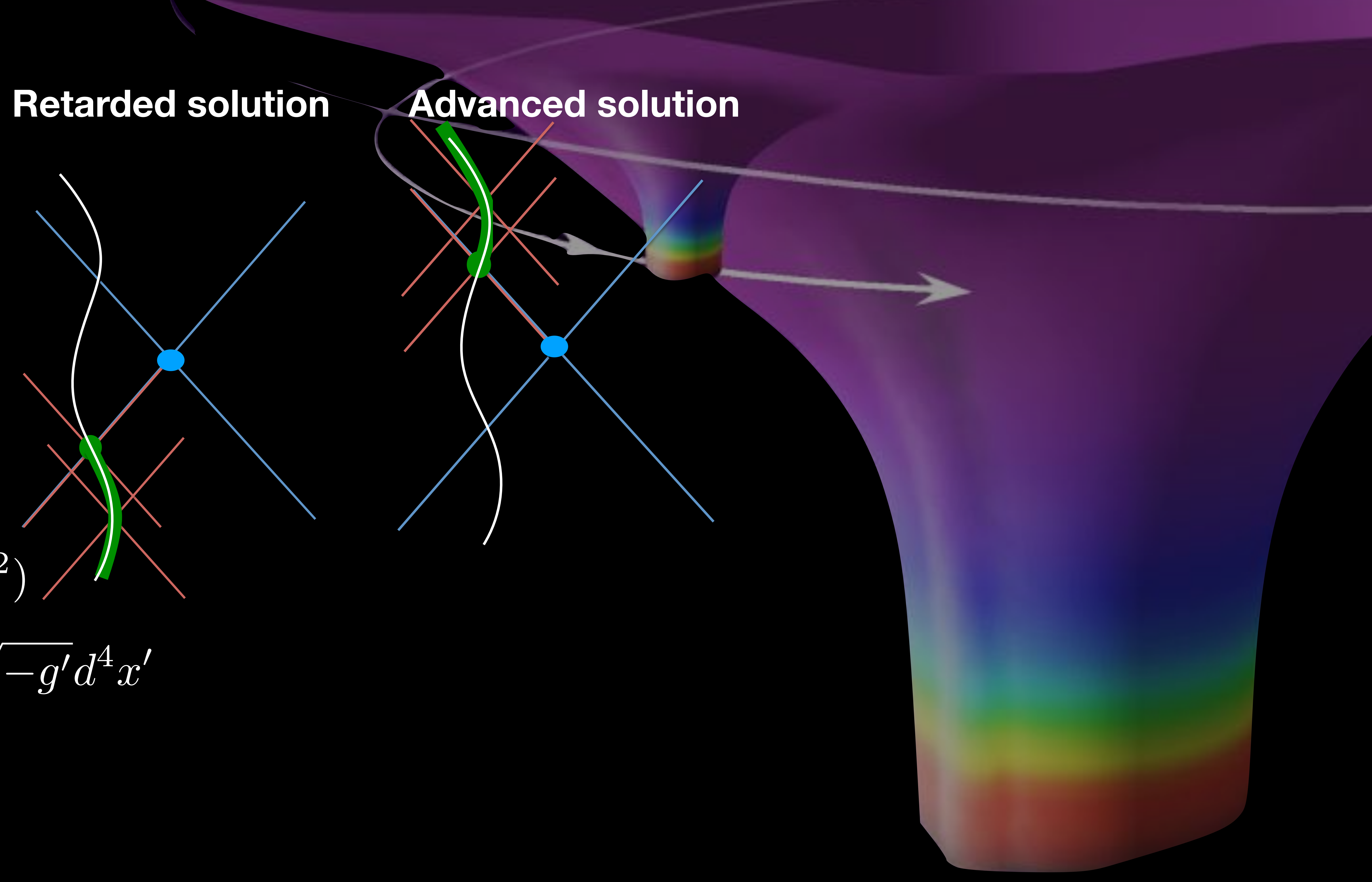
$$\Psi^A_{(ret)/(adv)} = \int {G^A}_{B'(ret)/(adv)}(x,x')\mathcal{M}^{B'}(x')\sqrt{-g'}\,d^4x'$$

- Detweiler-Whiting singular field



Regularisation: Curved Space

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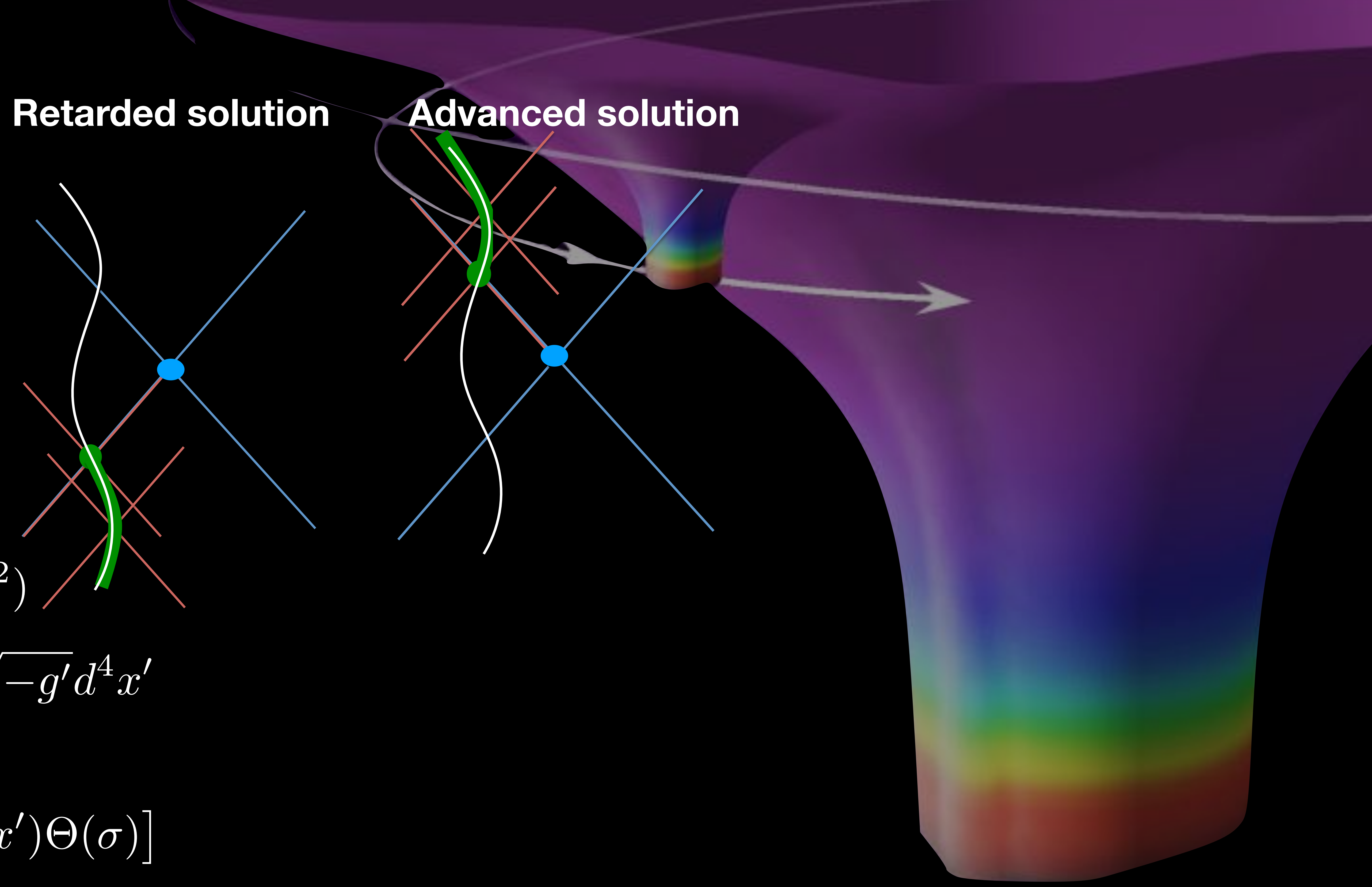
- Detweiler-Whiting singular field
$$ {G^A}_{B(S)} = \frac{1}{2}\left[{U^A}_{B'}(x,x')\delta(\sigma) + {V^A}_{B'}(x,x')\Theta(\sigma)\right]$$



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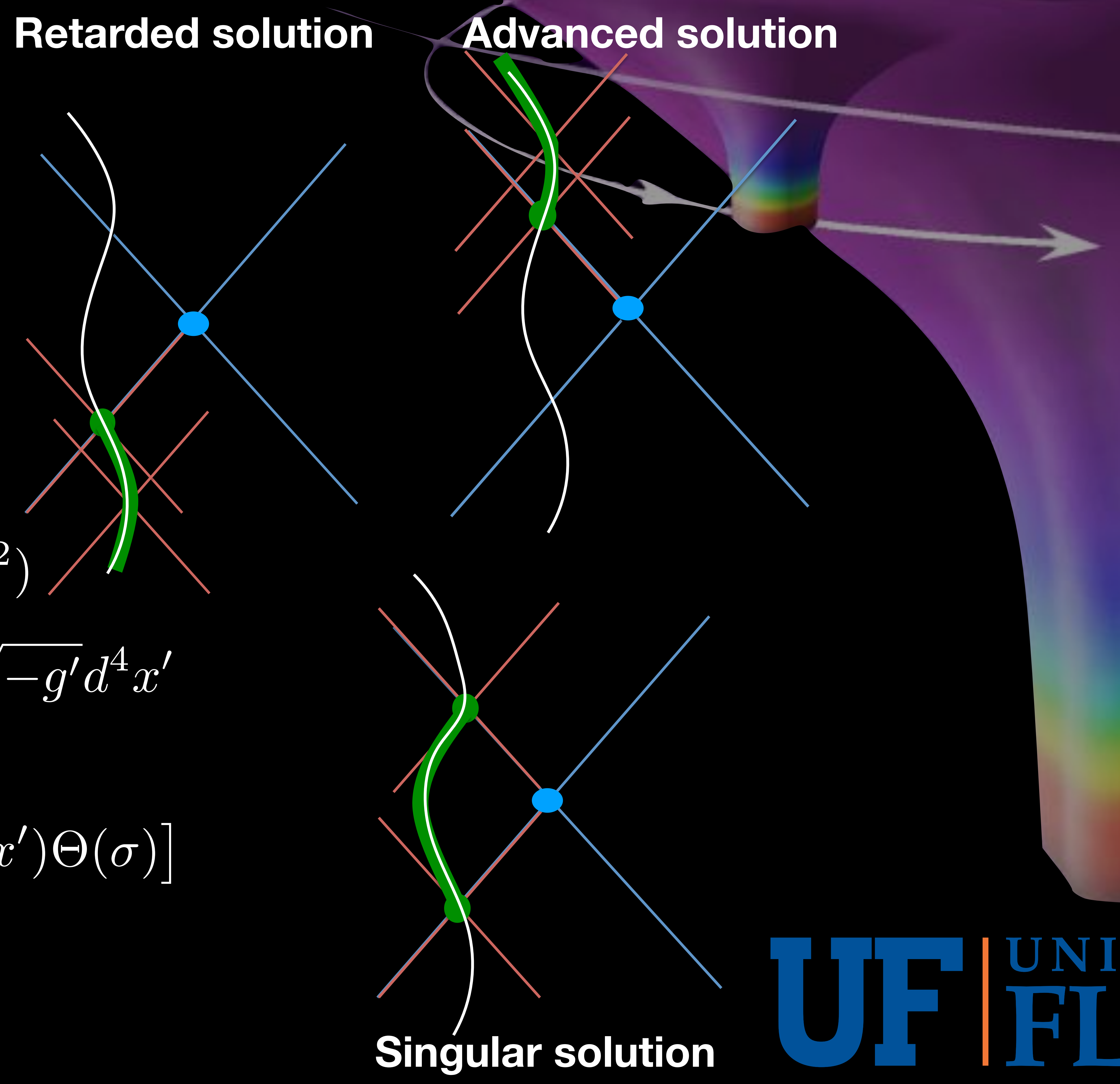

Regularisation: Curved Space

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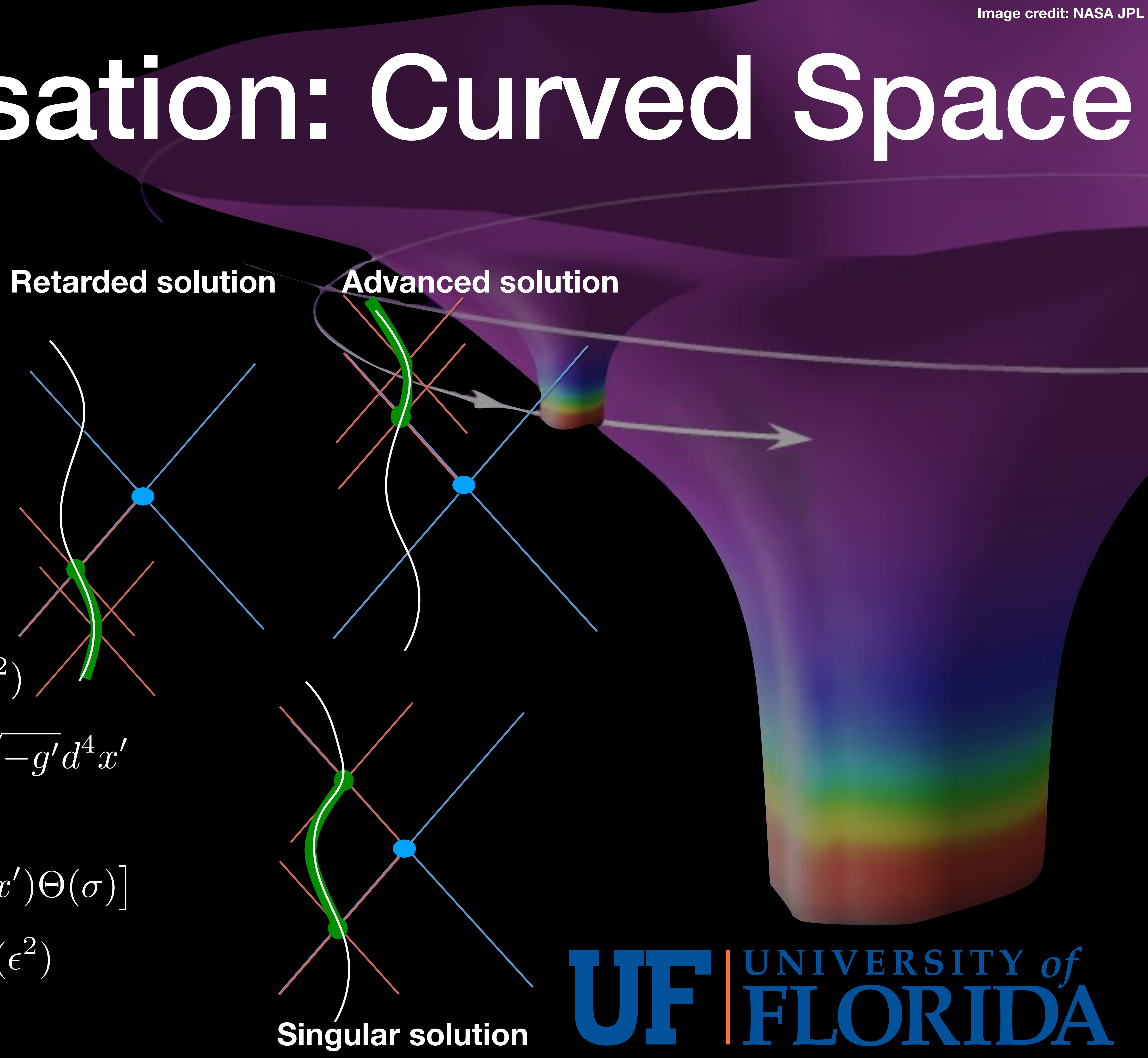

Regularisation: Curved Space

Image credit: NASA JPL

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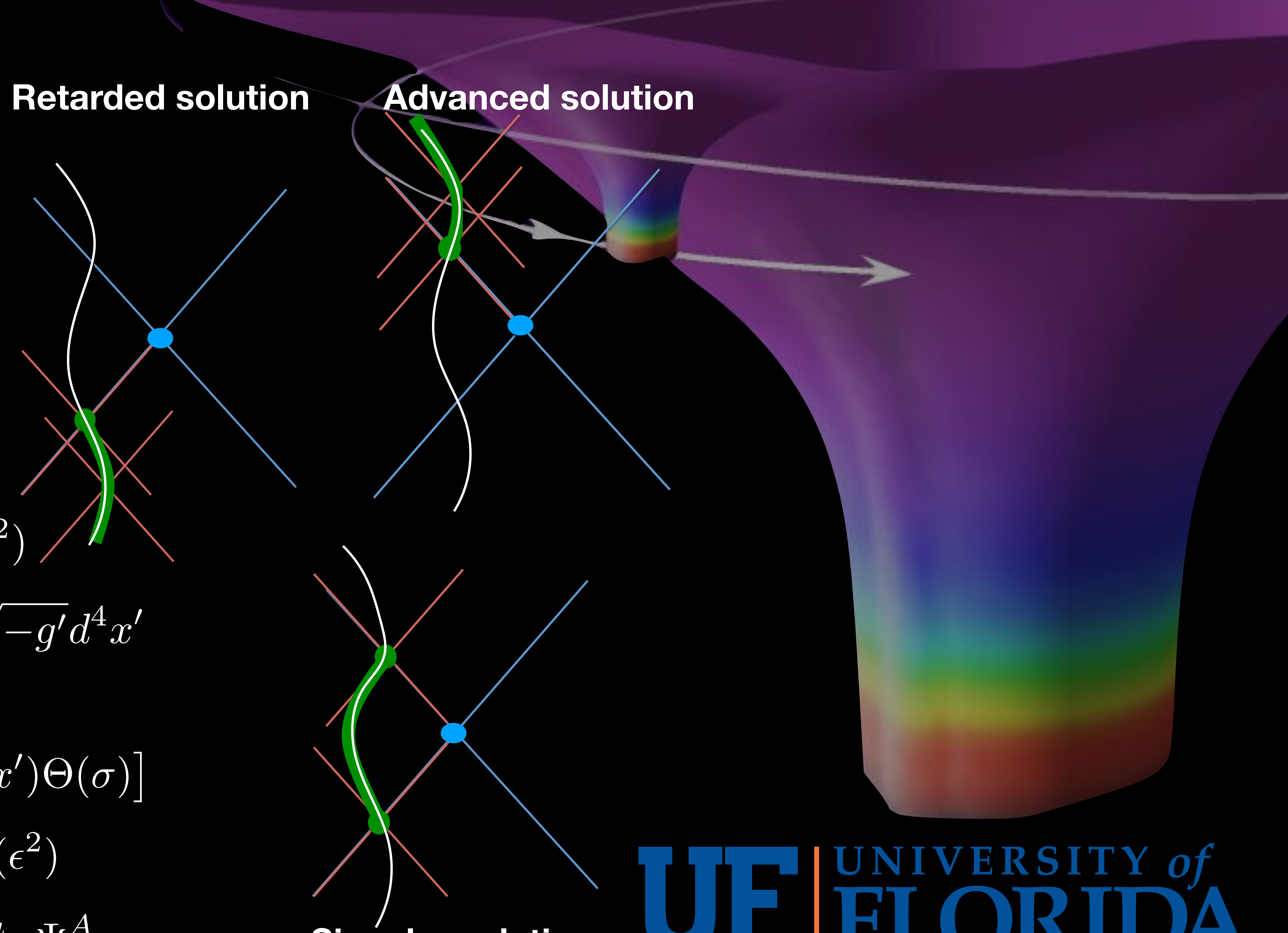
$$\left({\delta^A}_B \Box - {P^A}_B\right){\Psi^B}_S = -4\pi \mathcal{M}^A + \mathcal{O}(\epsilon^2)$$
$$\left({\delta^A}_B \Box - {P^A}_B\right){\Psi^B}_R = 0, \qquad F^a = {p^a}_A \Psi^A_R$$



Methods

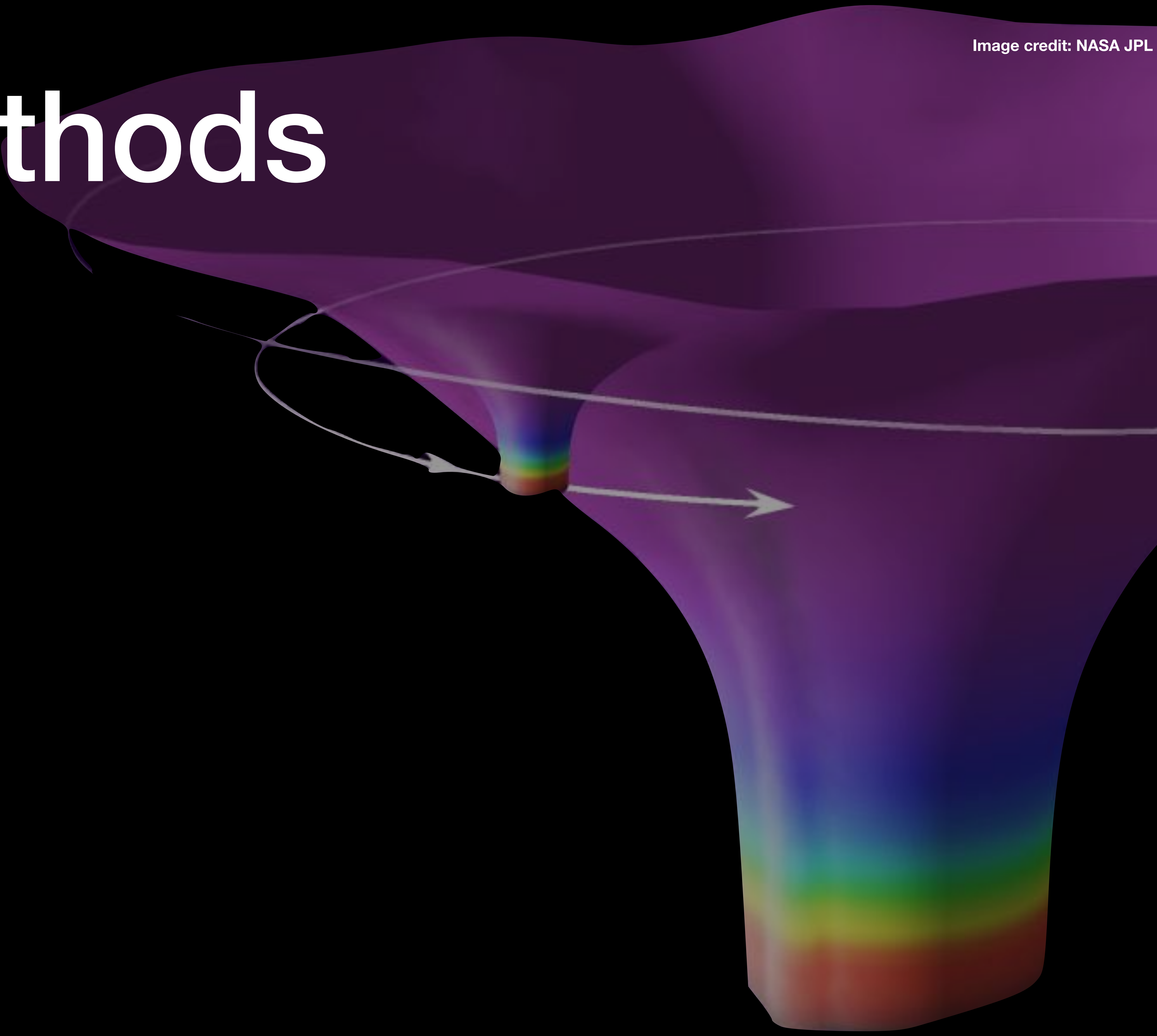


Image credit: NASA JPL

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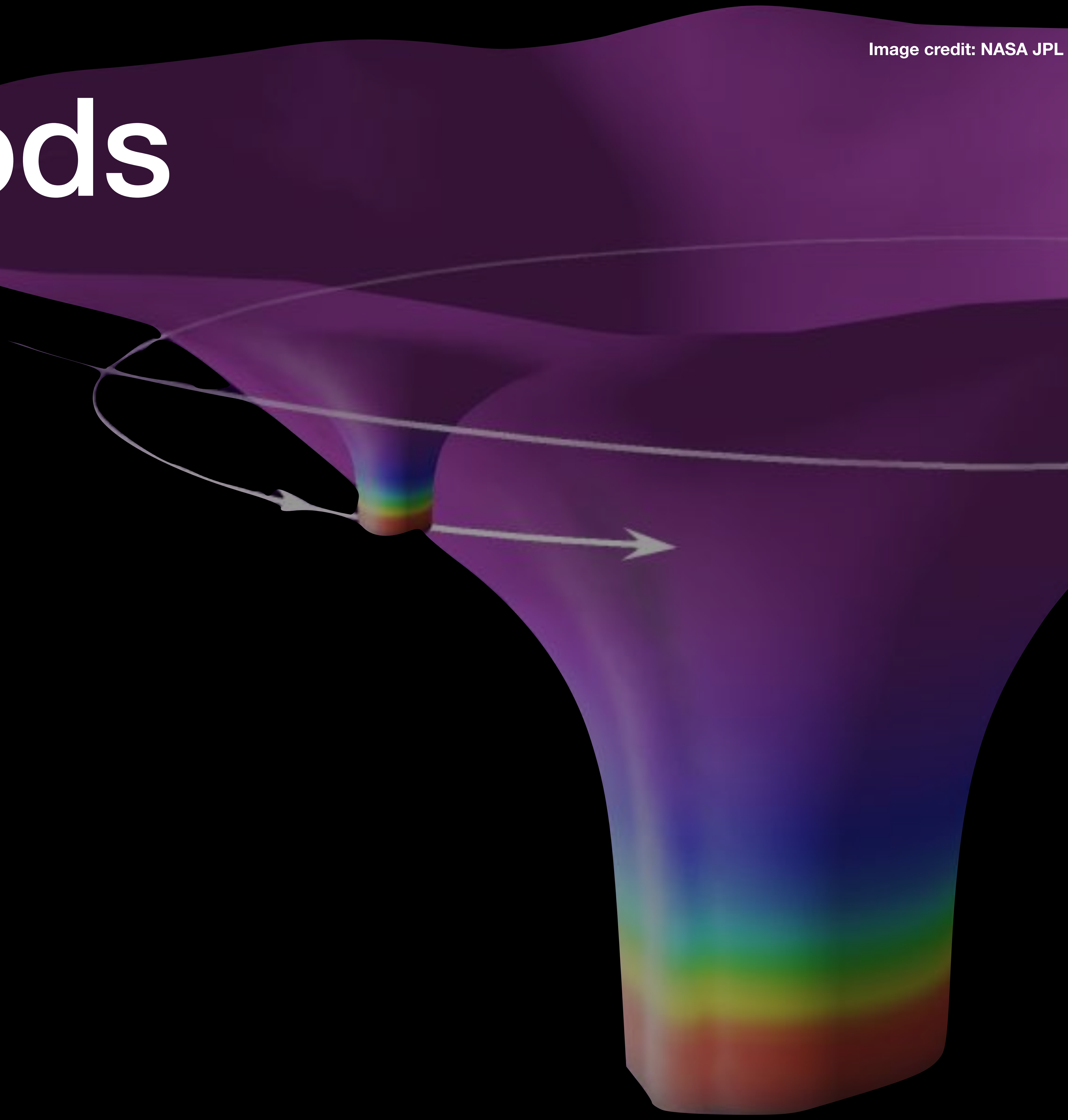
Marie Sklodowska-Curie actions

Image credit: NASA JPL

Methods

$$g_{ab}^{(M)}$$



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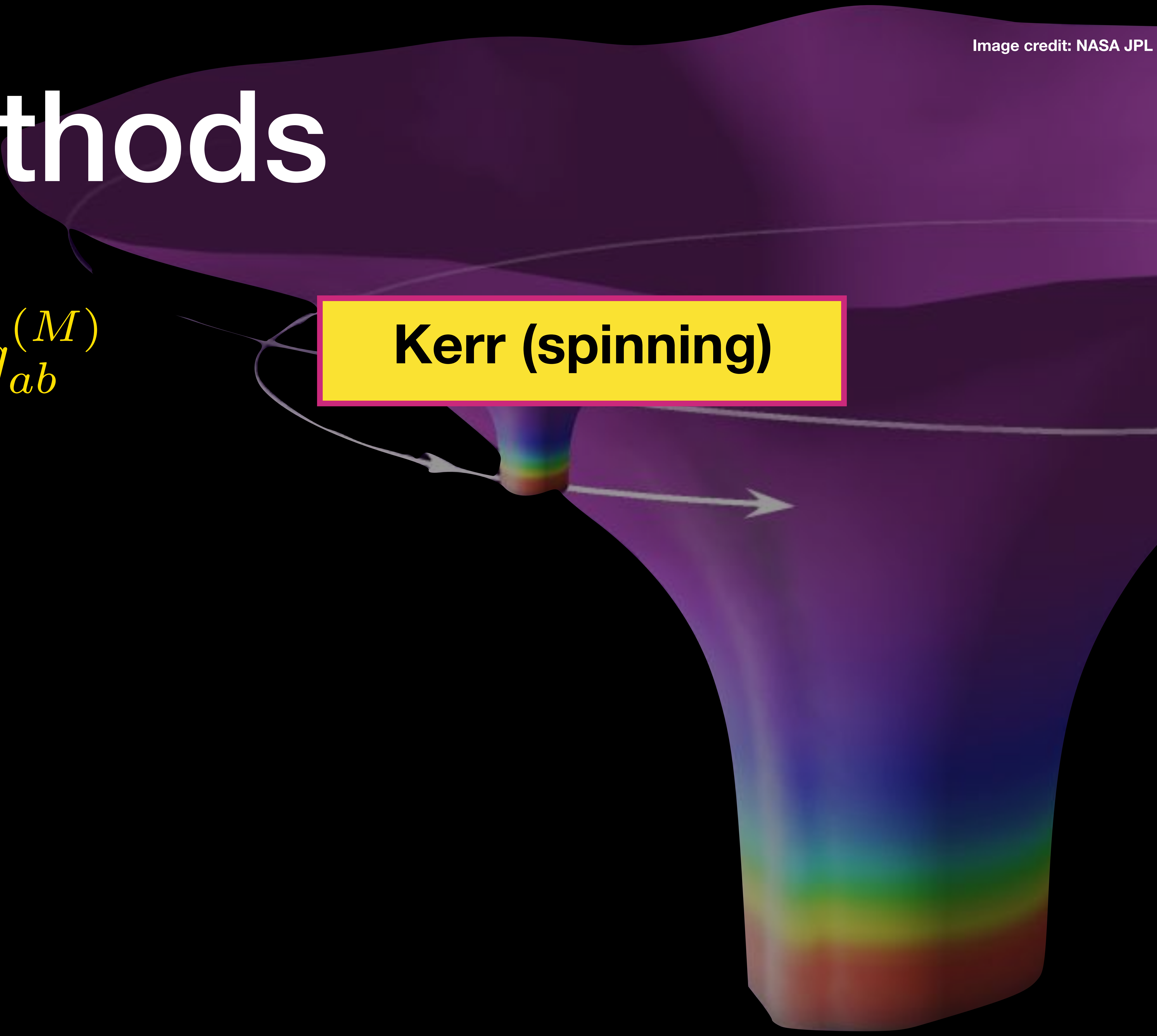
Marie Sklodowska-Curie actions

Image credit: NASA JPL

Methods



Schwarzschild

$g_{ab}^{(M)}$

Kerr (spinning)

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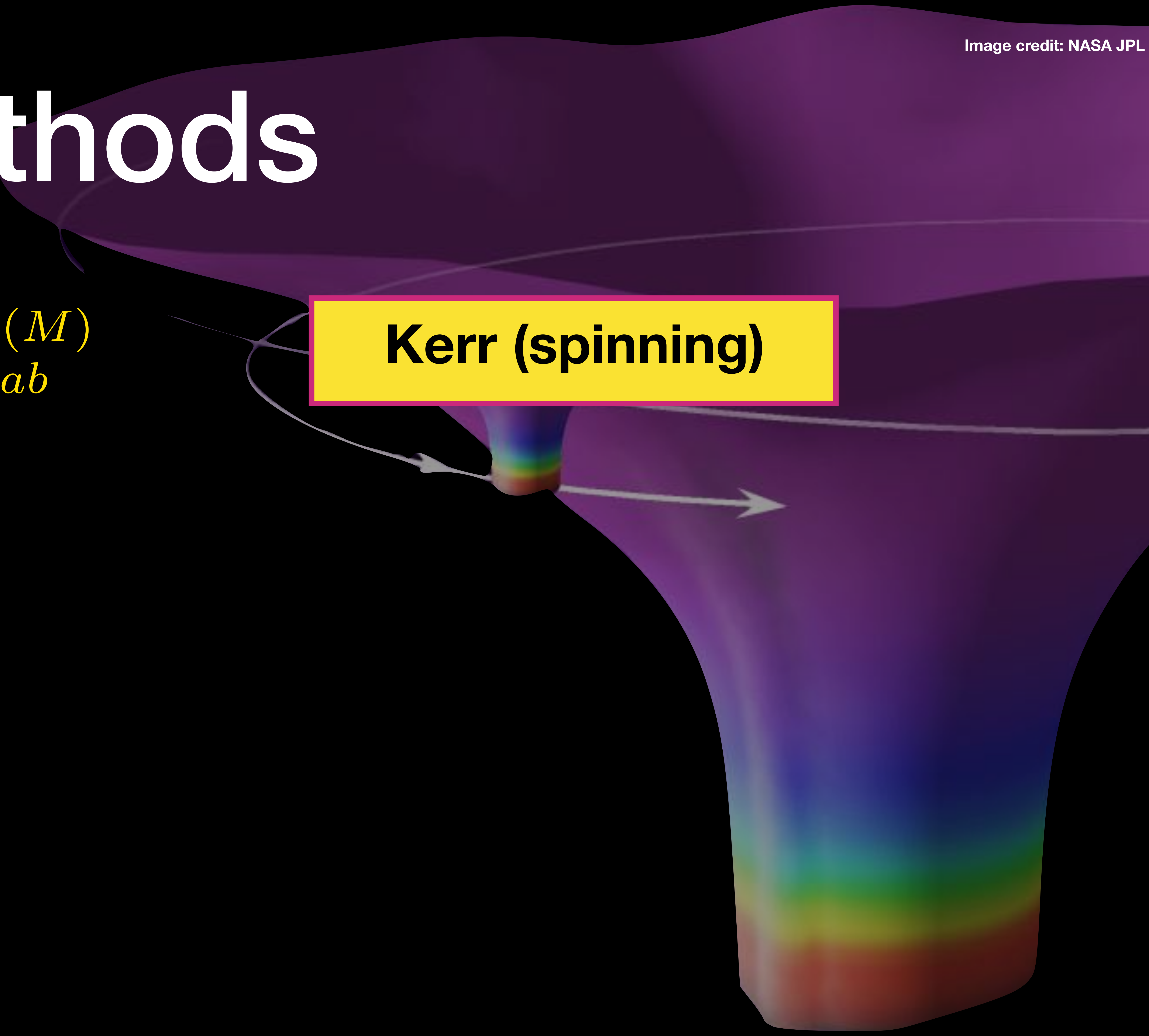
Methods

Image credit: NASA JPL



Schwarzschild

$g_{ab}^{(M)}$

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Spin

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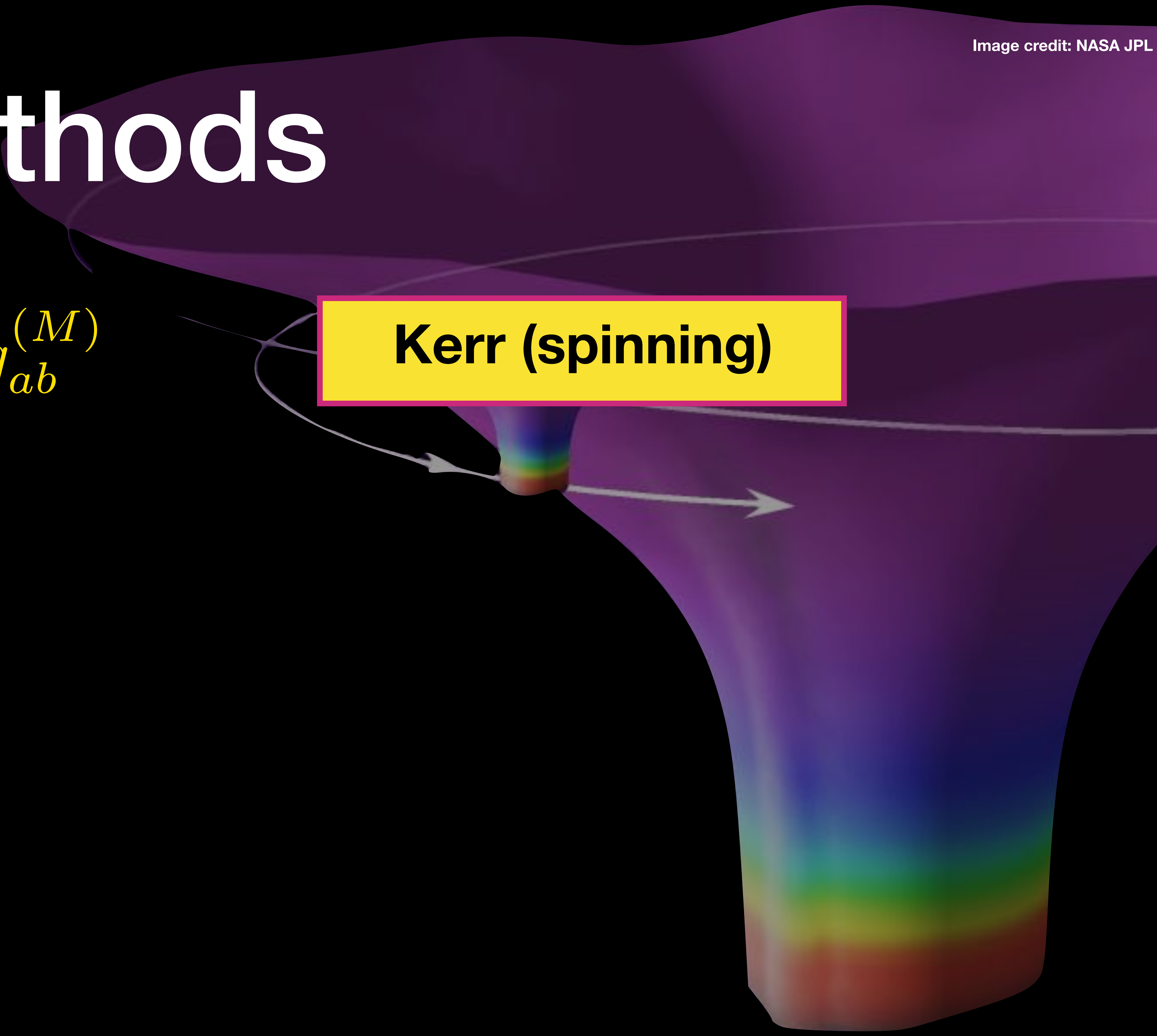
Methods

Image credit: NASA JPL



Schwarzschild

$$g_{ab}^{(M)}$$

Kerr (spinning)

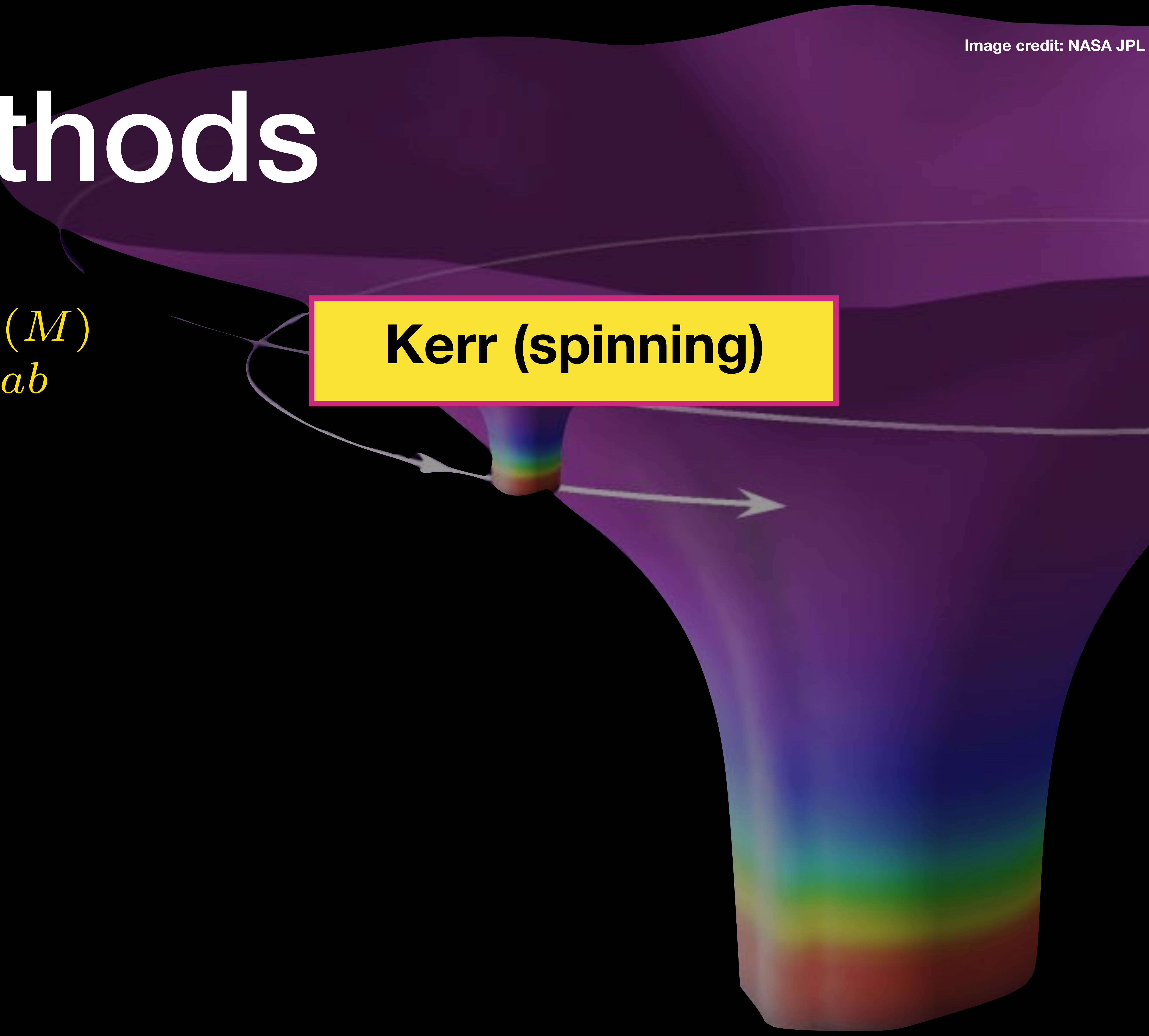
Spin

- Scalar

$$\left(\Box - \zeta R\right)\Phi = -4\pi\mu$$

Methods

Image credit: NASA JPL



Schwarzschild

$g_{ab}^{(M)}$

Kerr (spinning)

Spin

- **Scalar**

$$(\Box - \zeta R)\,\Phi = -4\pi\mu$$

- **Electromagnetism**

$$\left(\delta^{a}{}_{b}\Box - R^{a}_{b}\right) A^{b} = -4\pi j^{a}$$

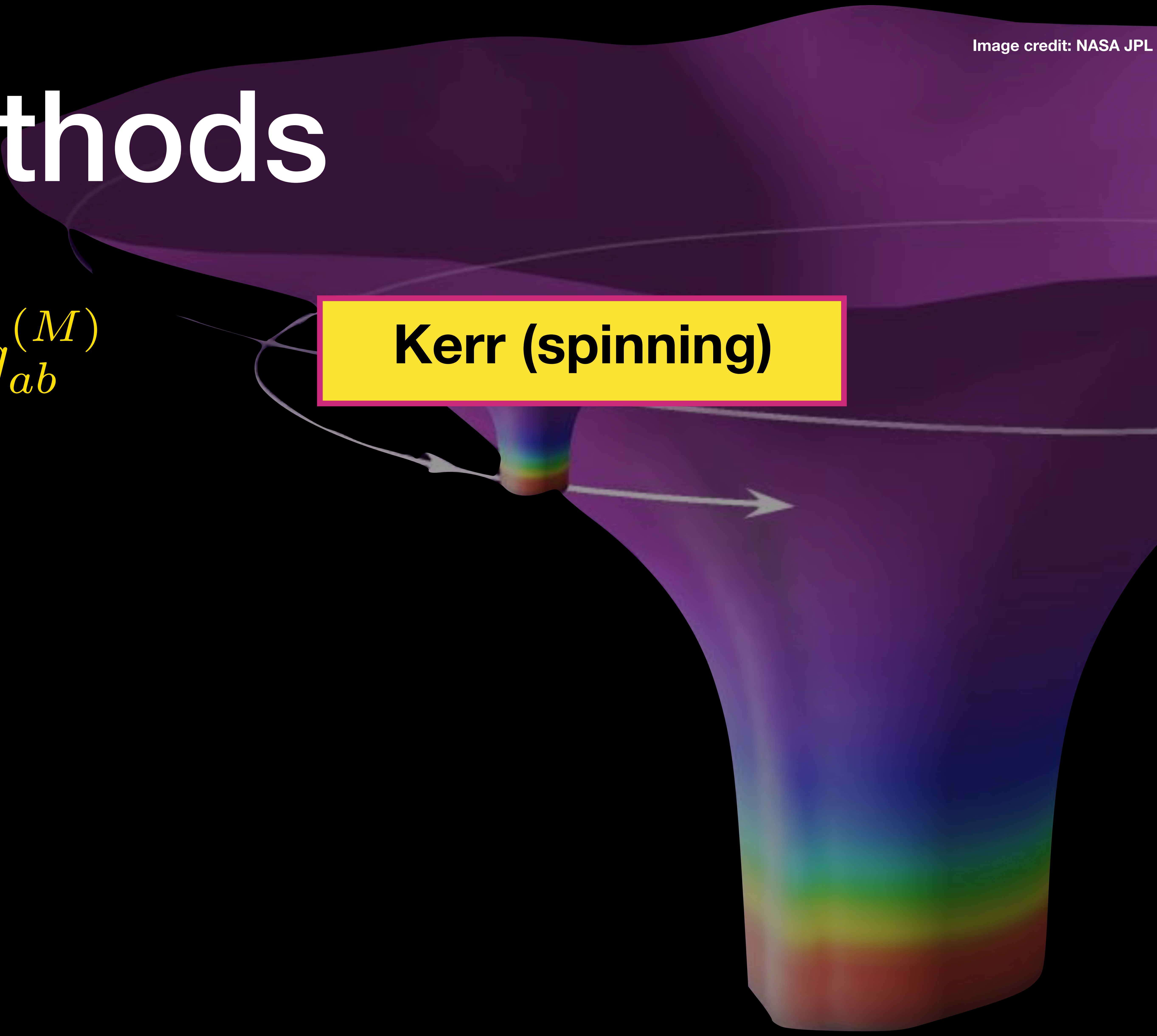


Image credit: NASA JPL

Methods

Schwarzschild

$g_{ab}^{(M)}$

Kerr (spinning)

Spin

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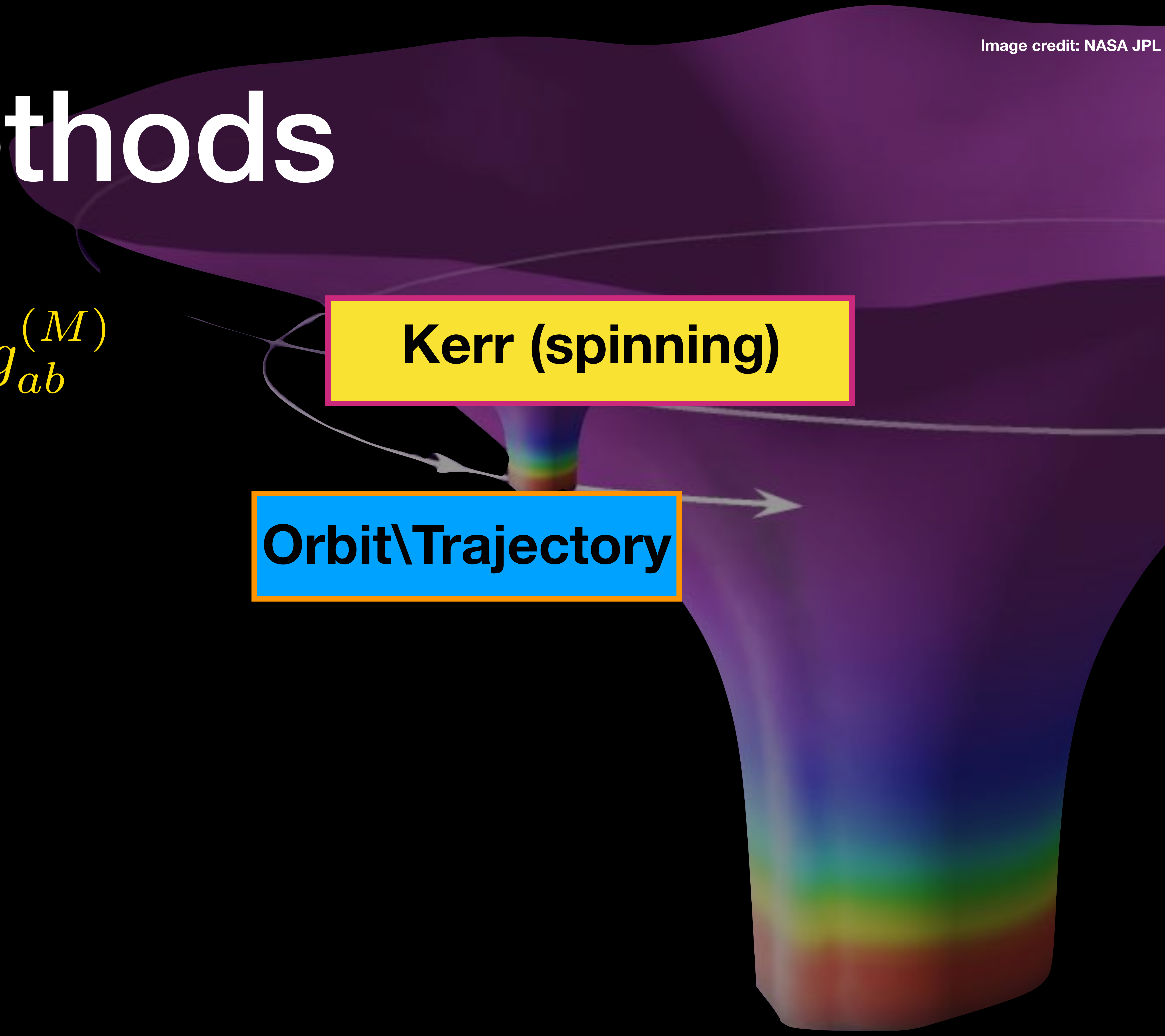
Image credit: NASA JPL

Methods

Schwarzschild

$g_{ab}^{(M)}$

Kerr (spinning)

Spin

Orbit\Trajectory

- **Scalar**

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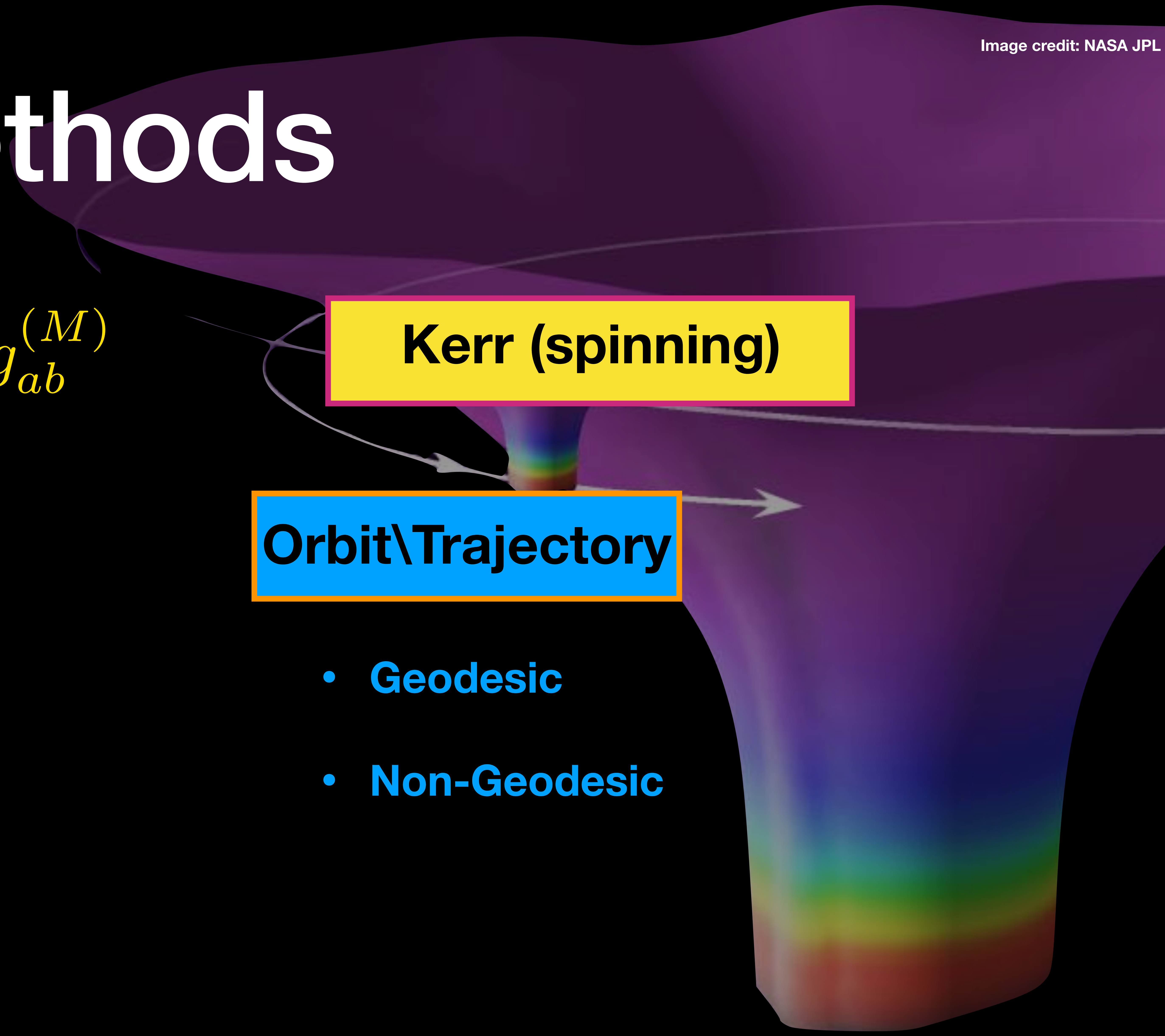
Methods



Schwarzschild

$$g_{ab}^{(M)}$$

Kerr (spinning)

Spin

- **Scalar**

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Orbit\Trajectory

- **Geodesic**
- **Non-Geodesic**

Methods

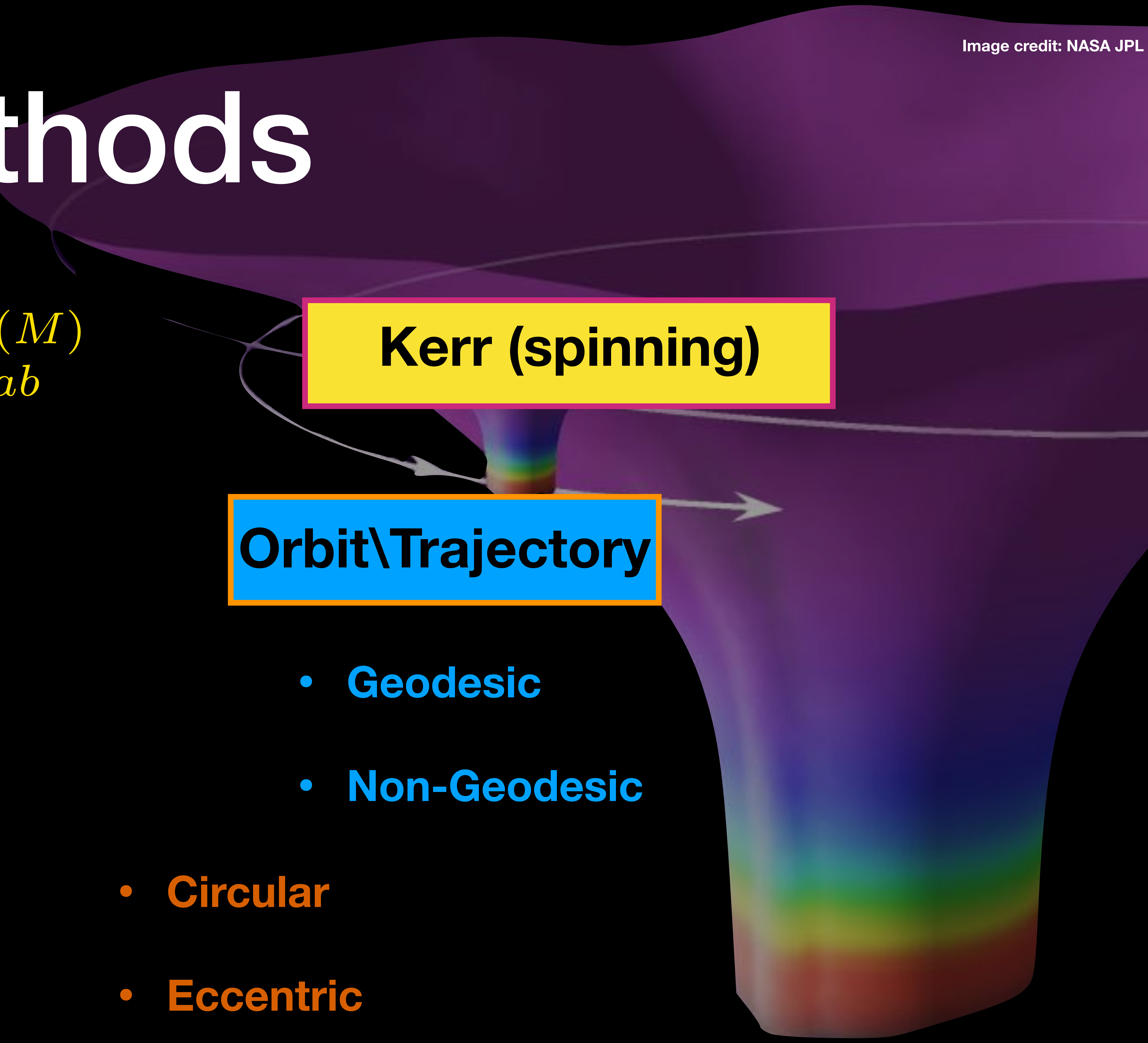
Image credit: NASA JPL

Schwarzschild

$g_{ab}^{(M)}$

Kerr (spinning)

Spin

- Scalar

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$$(\delta^a{}_b\Box - R^a_b)\,A^b = -4\pi j^a$$

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Orbit\Trajectory

- Geodesic
- Non-Geodesic

- Circular
- Eccentric

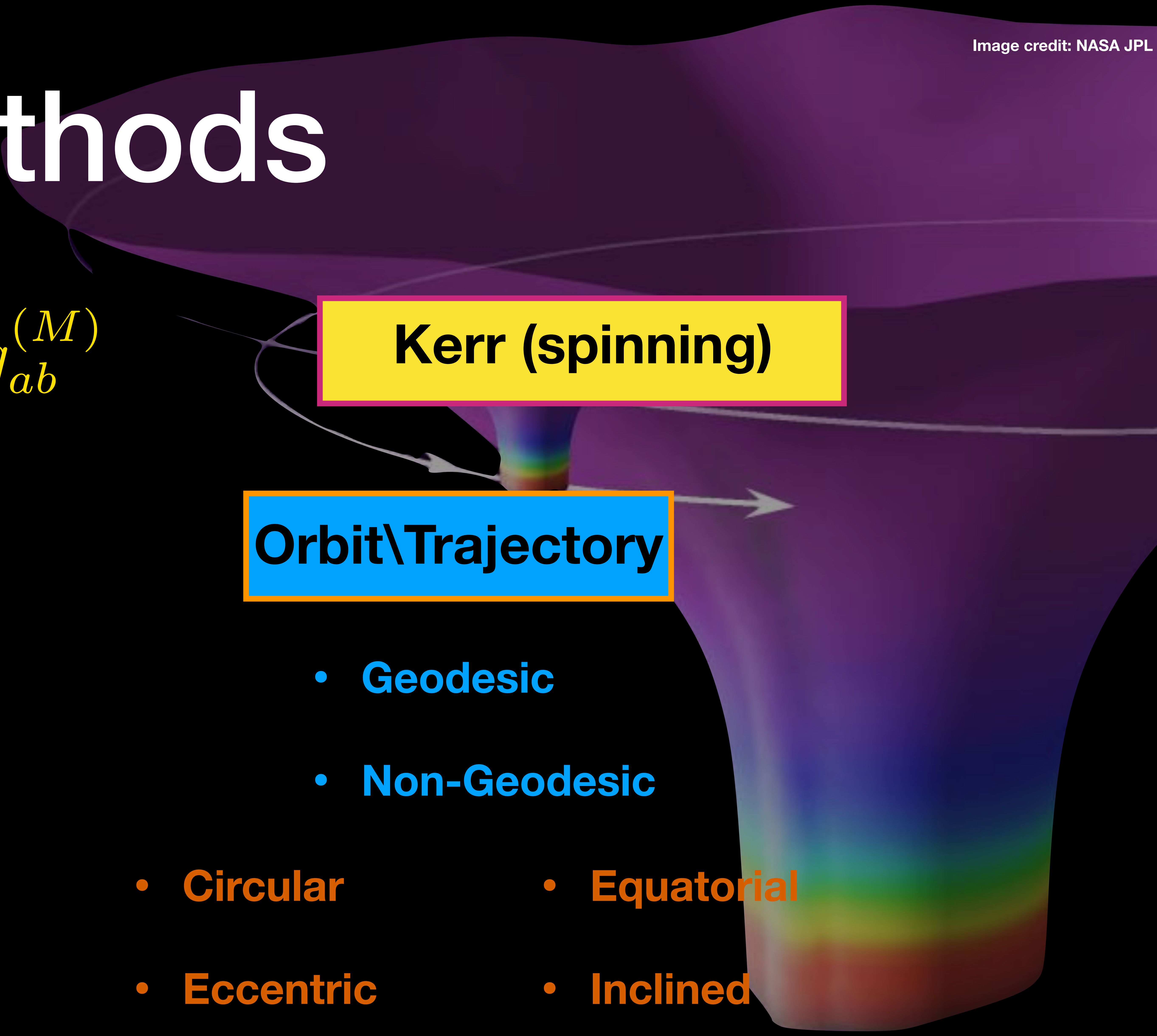
Image credit: NASA JPL

Methods

Schwarzschild $g_{ab}^{(M)}$ **Kerr (spinning)**

Spin

- **Scalar**

$$(\Box - \zeta R)\,\Phi = -4\pi\mu$$

- **Electromagnetism**

$$({\delta^a}_b \Box - R^a_b)\,A^b = -4\pi j^a$$

- **Gravitational**

$$(\delta_{ac}\delta_{bd}\Box + 2R_{cadb})\,\psi^{cd} = -16\pi T^{ab}_{(eff)}$$

Orbit\Trajectory

- **Geodesic**
- **Non-Geodesic**

- **Circular**
- **Eccentric**
- **Equatorial**
- **Inclined**

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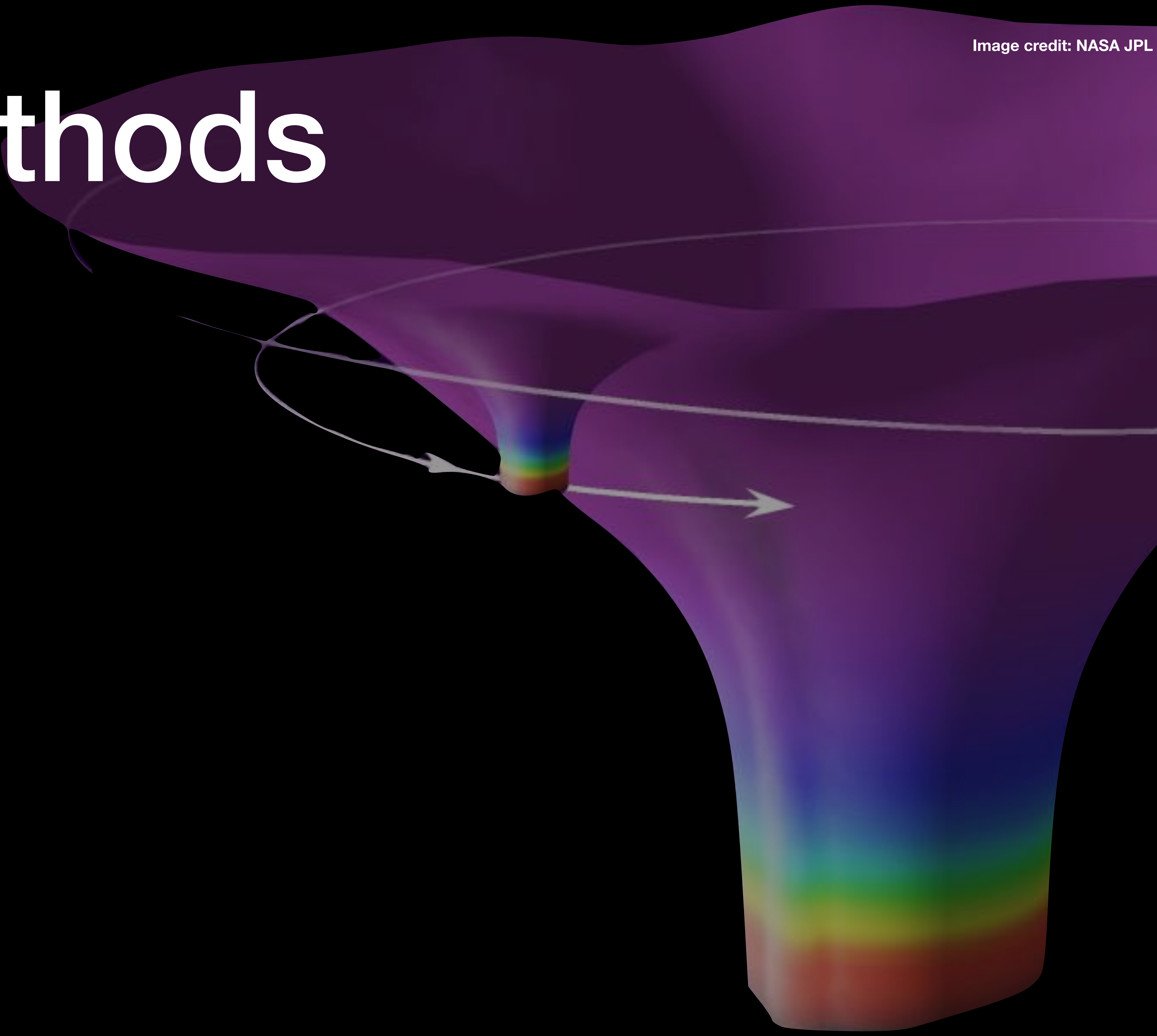
Image credit: NASA JPL

Methods



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Methods

- Matched asymptotic expansions
  - Proposed by Wiseman, Poisson (1998)

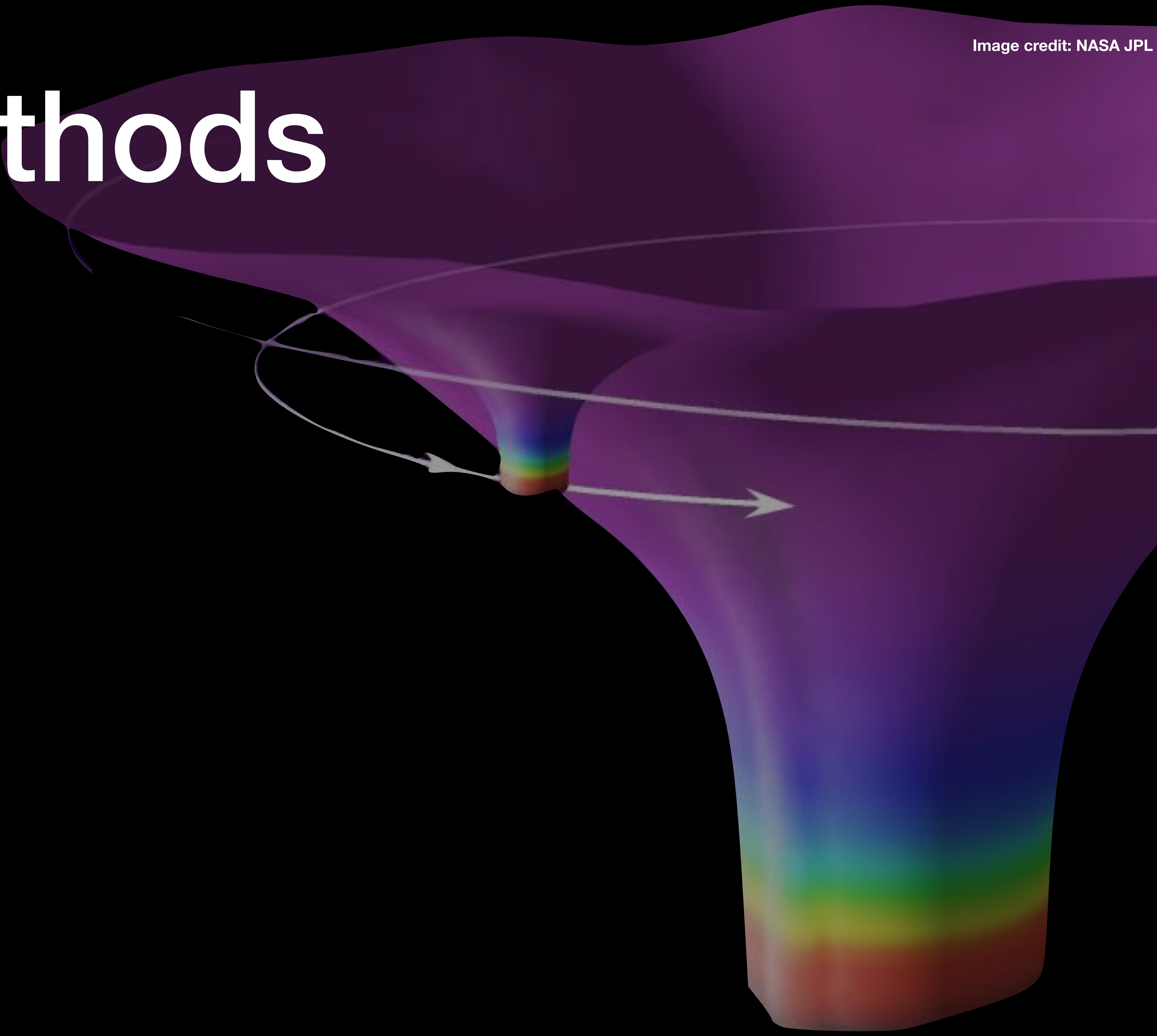

Image credit: NASA JPL

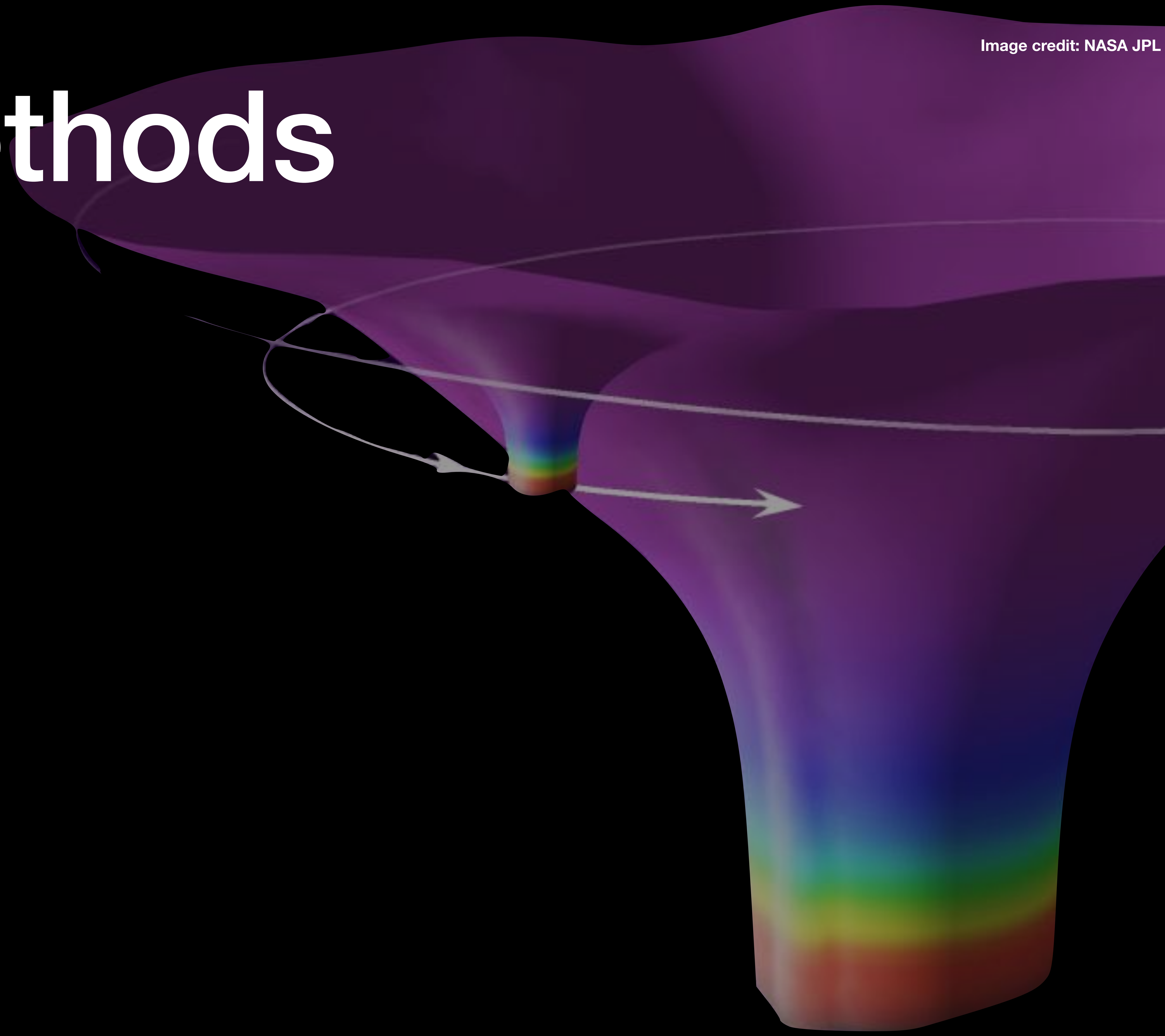
Image credit: NASA JPL

Methods

- Matched asymptotic expansions
  - Proposed by Wiseman, Poisson (1998)

$$
\begin{aligned}
f_a =& q^2 \nabla \Big[\int_{\tau_m}^{\tau^-} d\tau' V(x, x') \\
&+ \int_{\infty}^{\tau^{m}} d\tau' G_{(ret)}(x, x') \Big]
\end{aligned}
$$

Image credit: NASA JPL

Methods

- Matched asymptotic expansions
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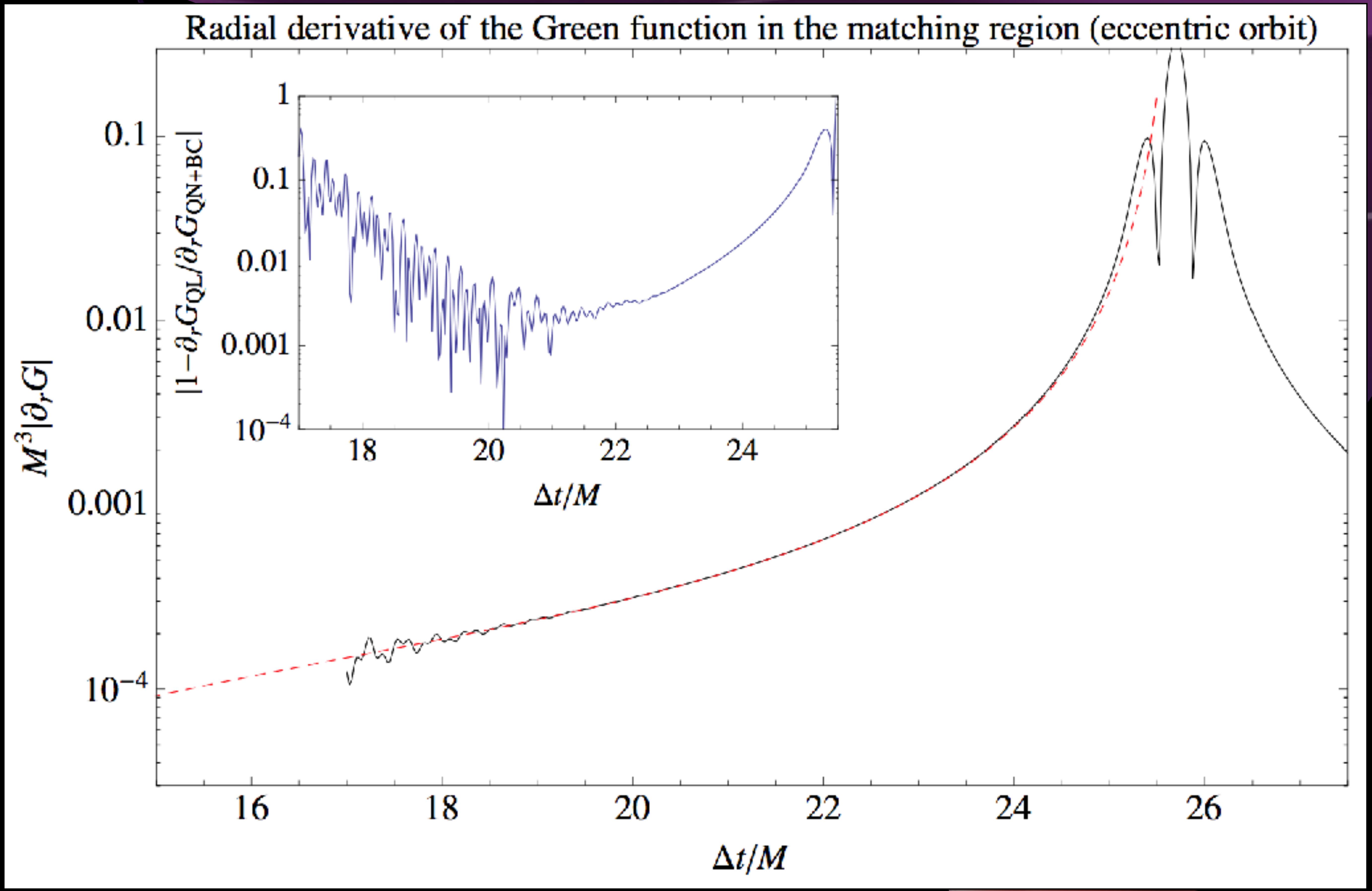
$$f_a = q^2 \nabla \Big[\int_{\tau_m}^{\tau^-} d\tau' V(x, x') + \int_{\infty}^{\tau^m} d\tau' G_{(ret)}(x, x') \Big]$$



M. Casals, S. Dolan, A. Ottewill, B. Wardell, Phys. Rev. D 88, 044022 (2013)

Image credit: NASA JPL

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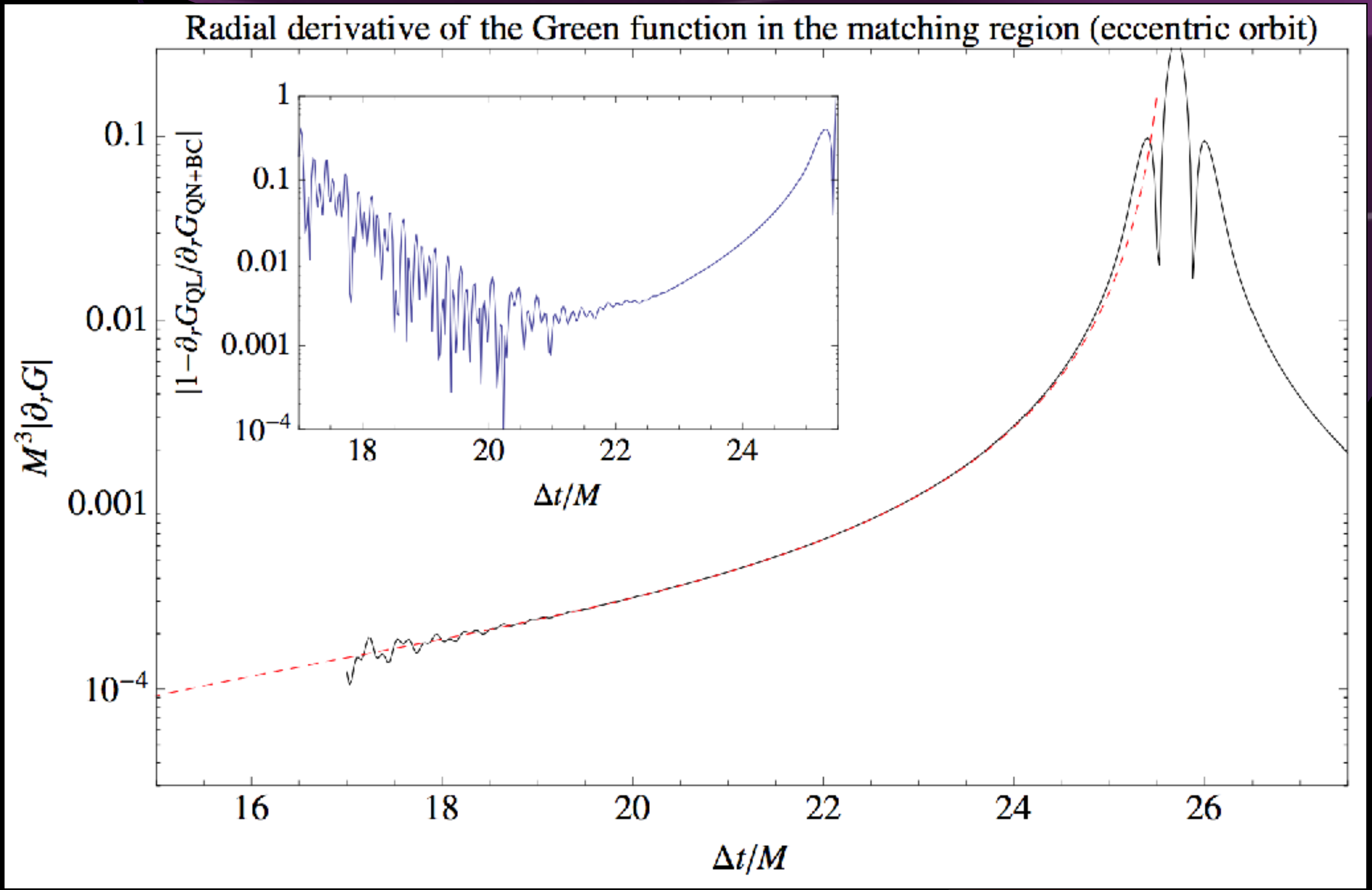

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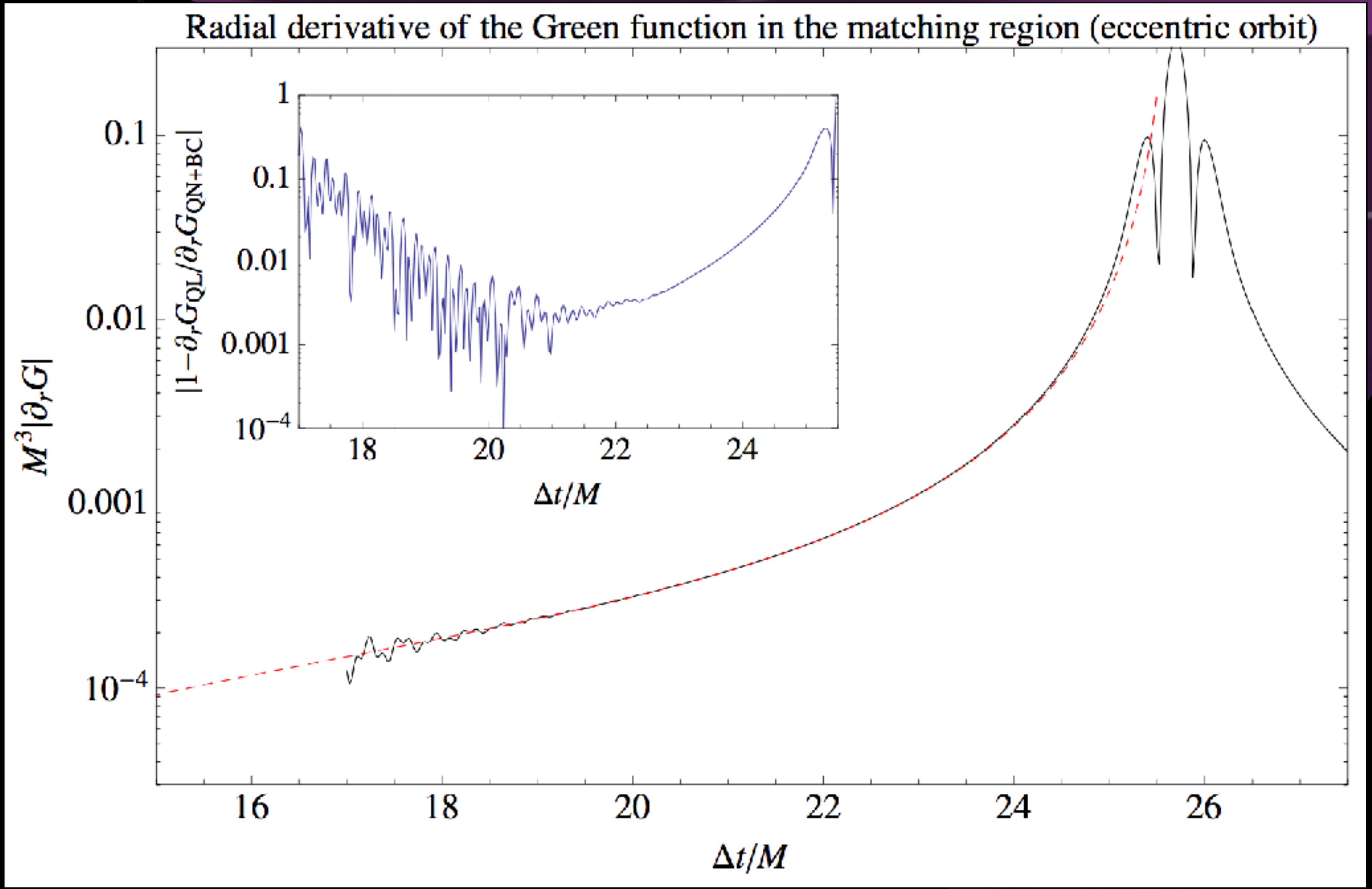

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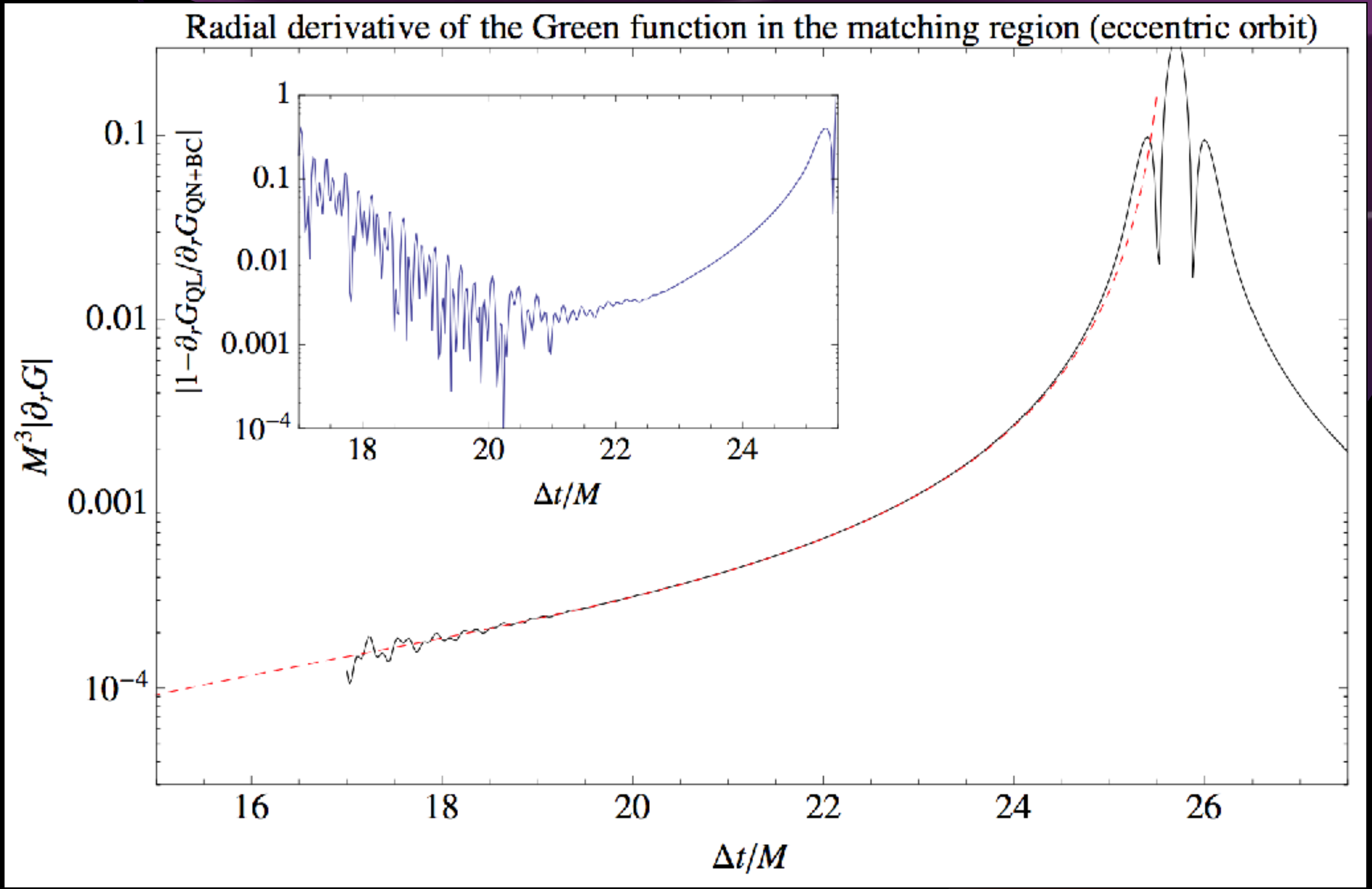
Schwarzschild Scalar



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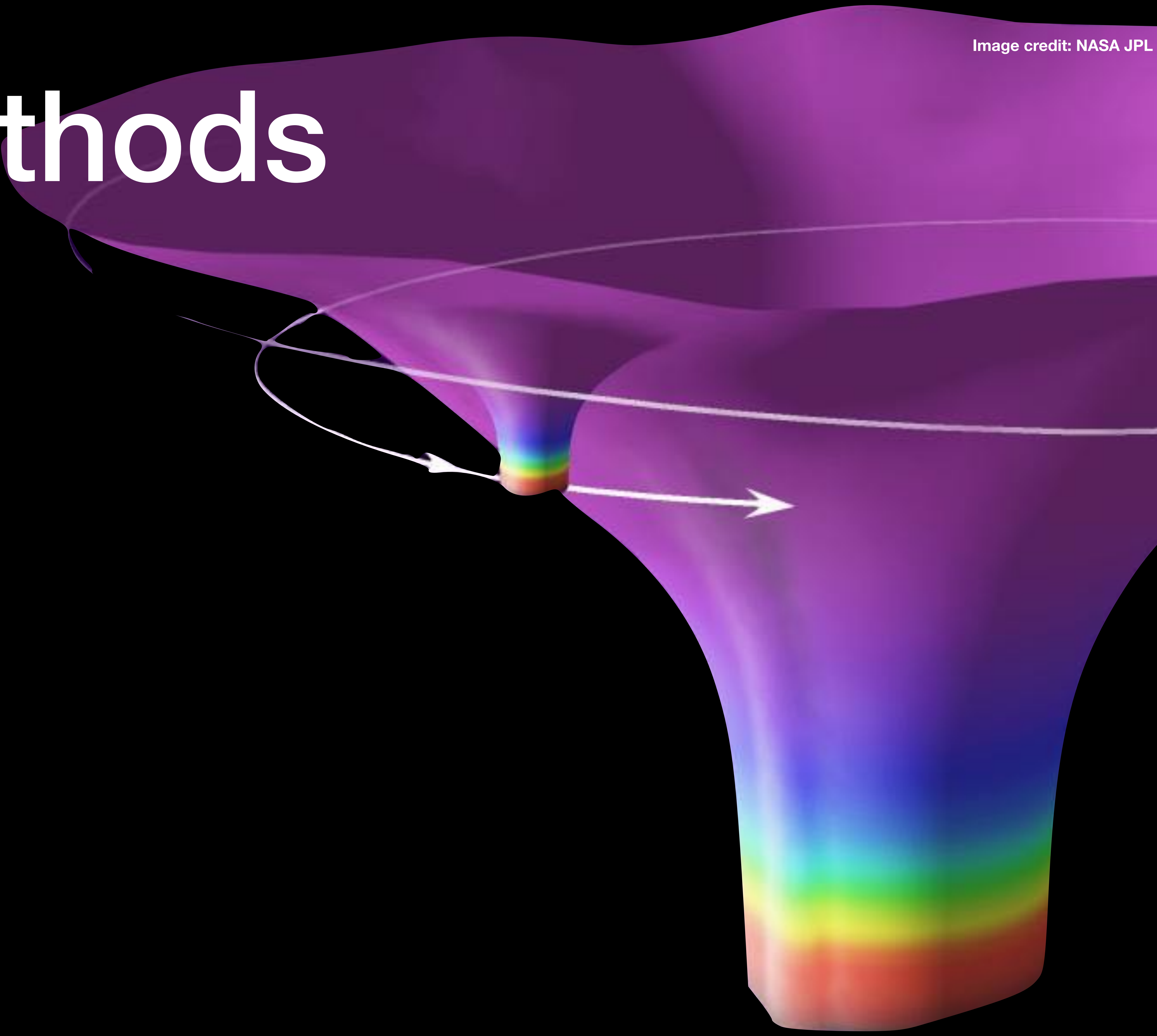
Methods



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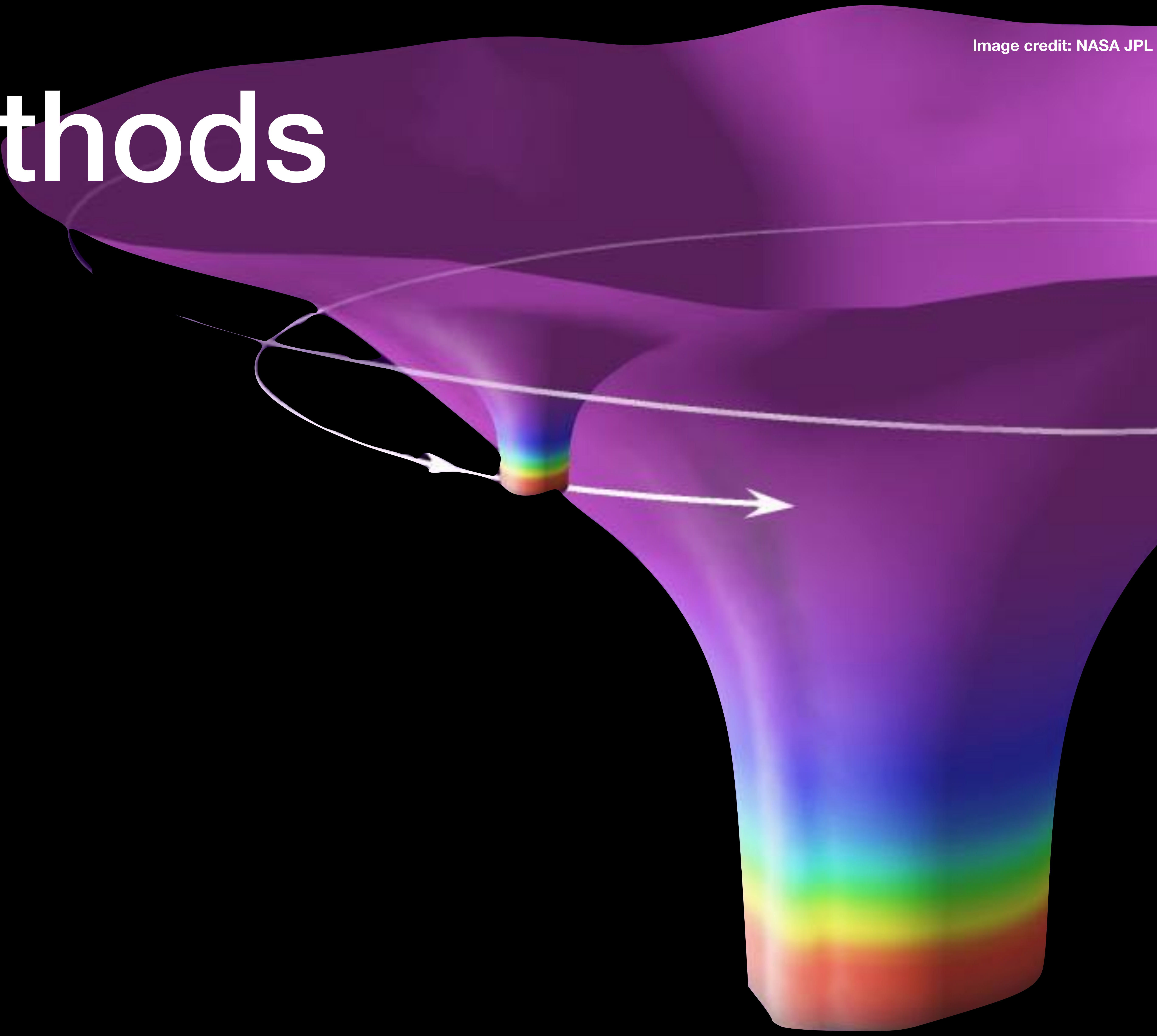
Methods



- Mode-sum regularisation

Methods

- Mode-sum regularisation
  - Barrack, Ori (2001)

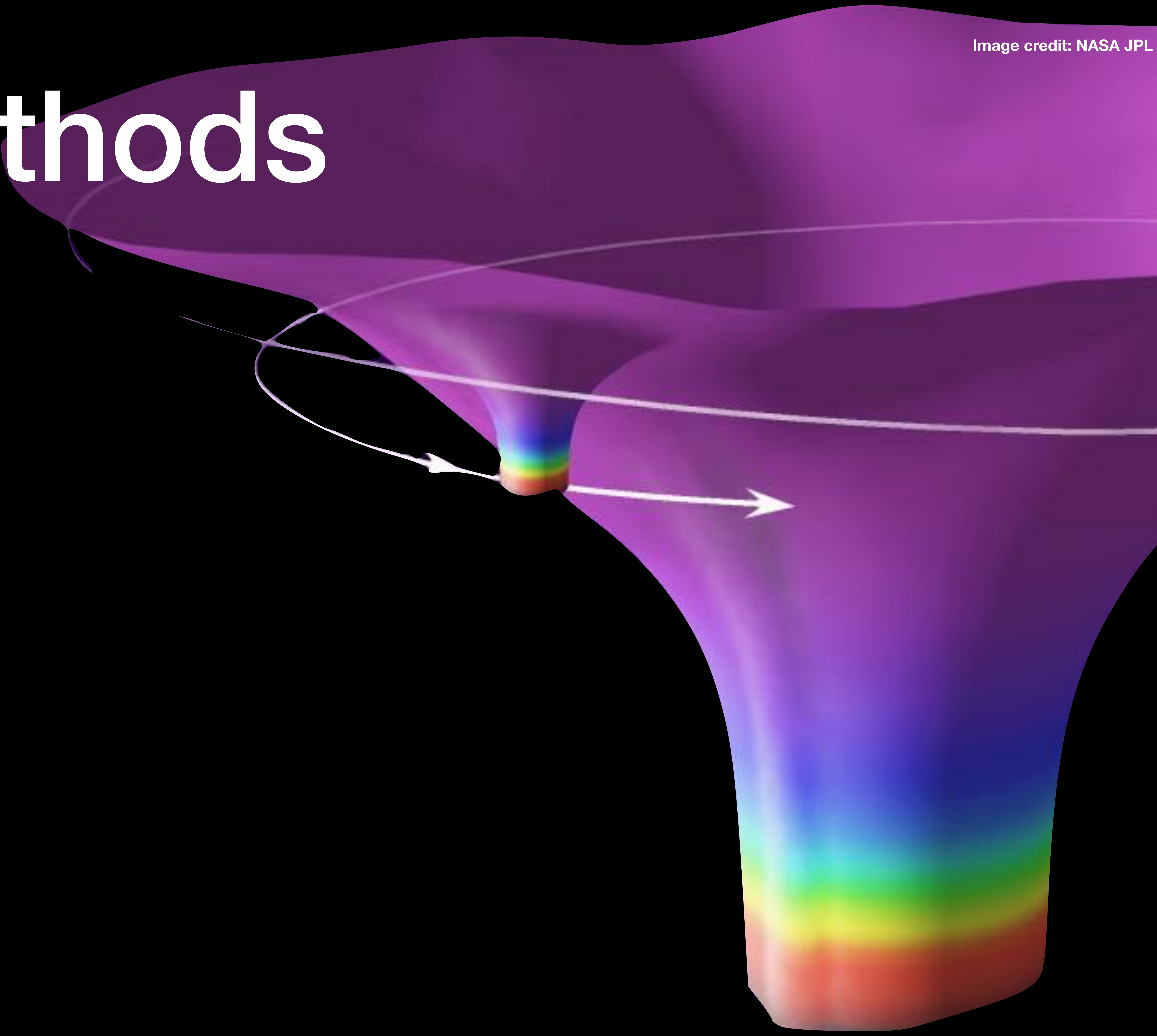


Image credit: NASA JPL

Methods

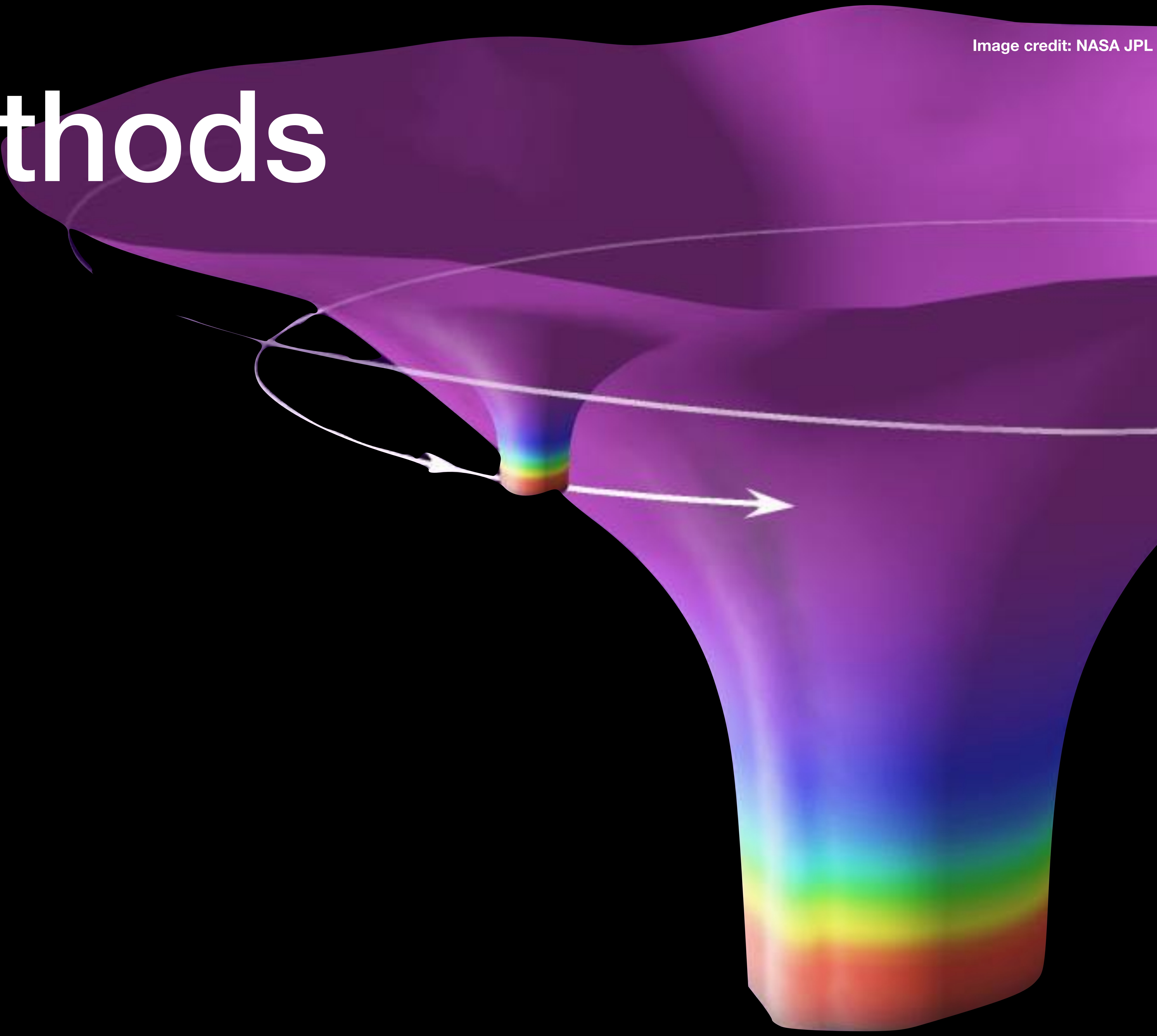


Image credit: NASA JPL

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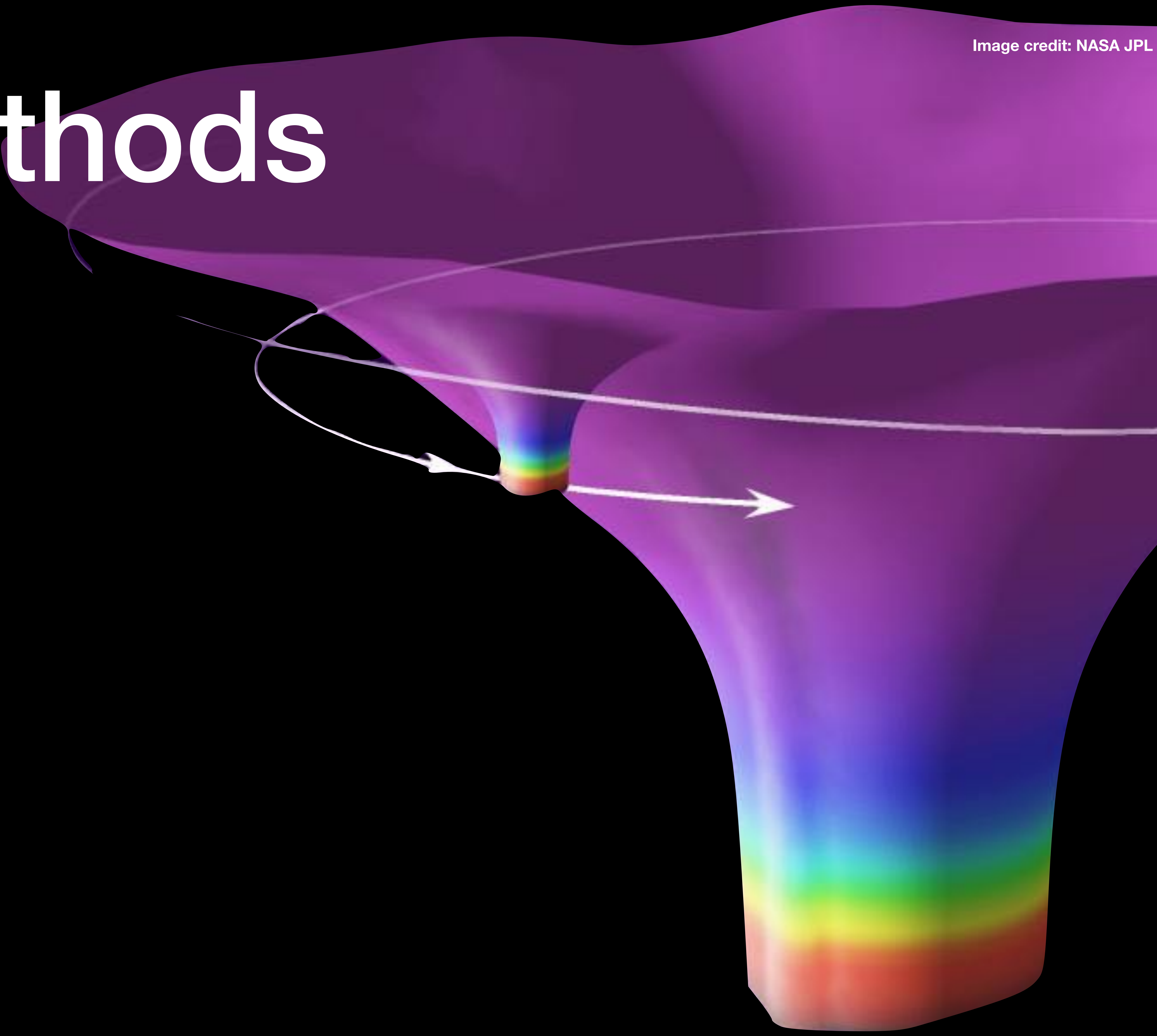

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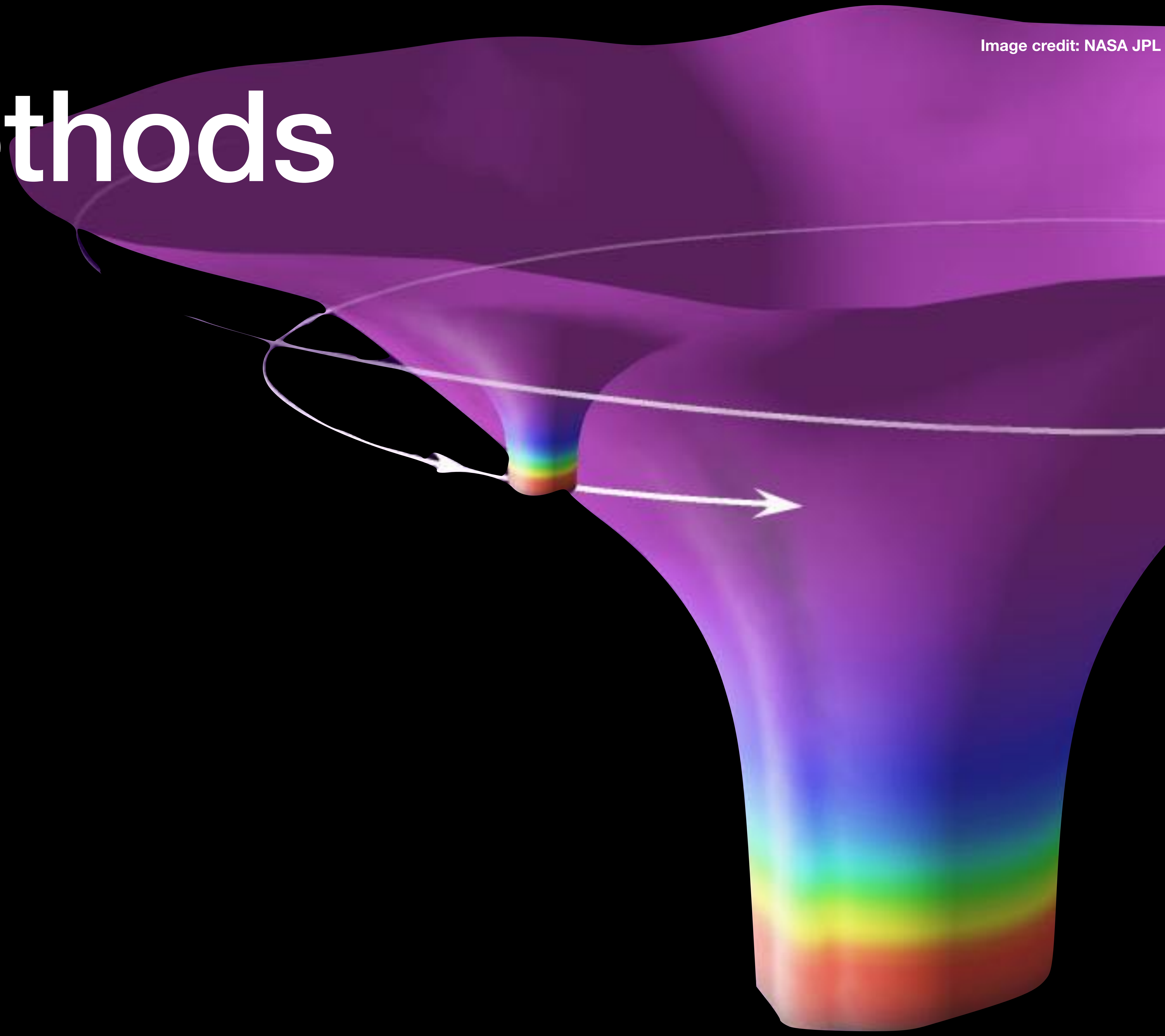

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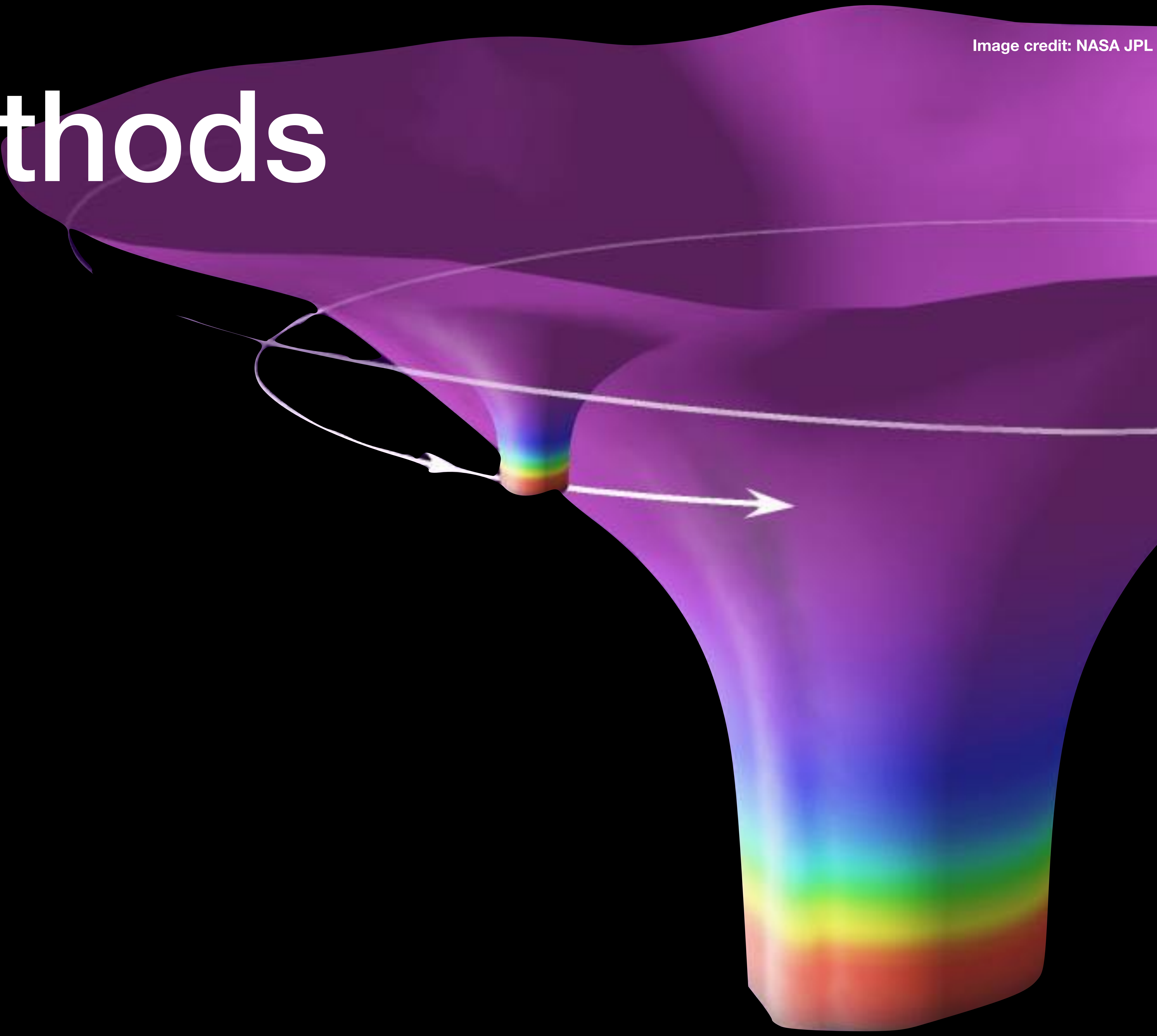
Methods

Image credit: NASA JPL



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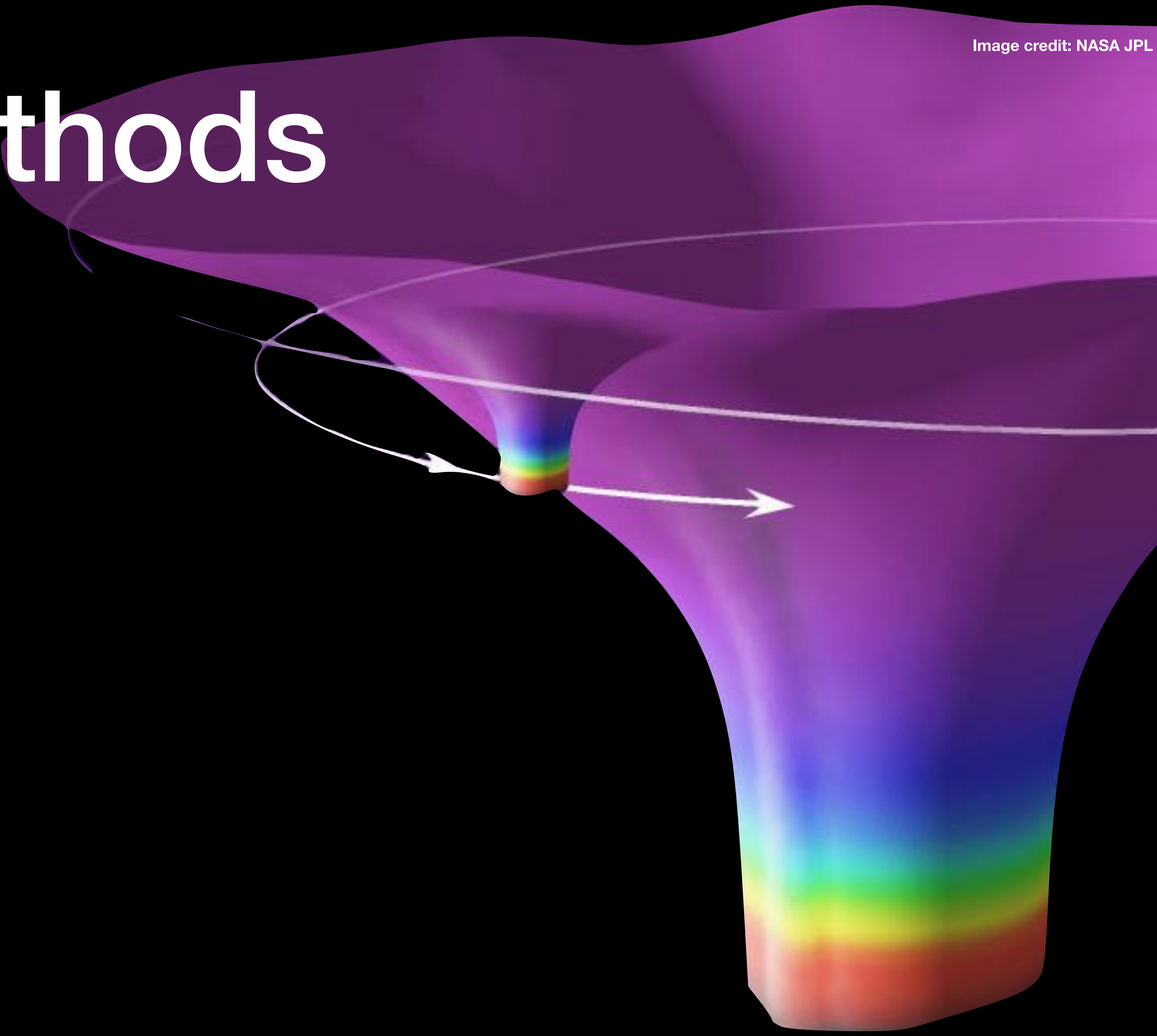

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Methods

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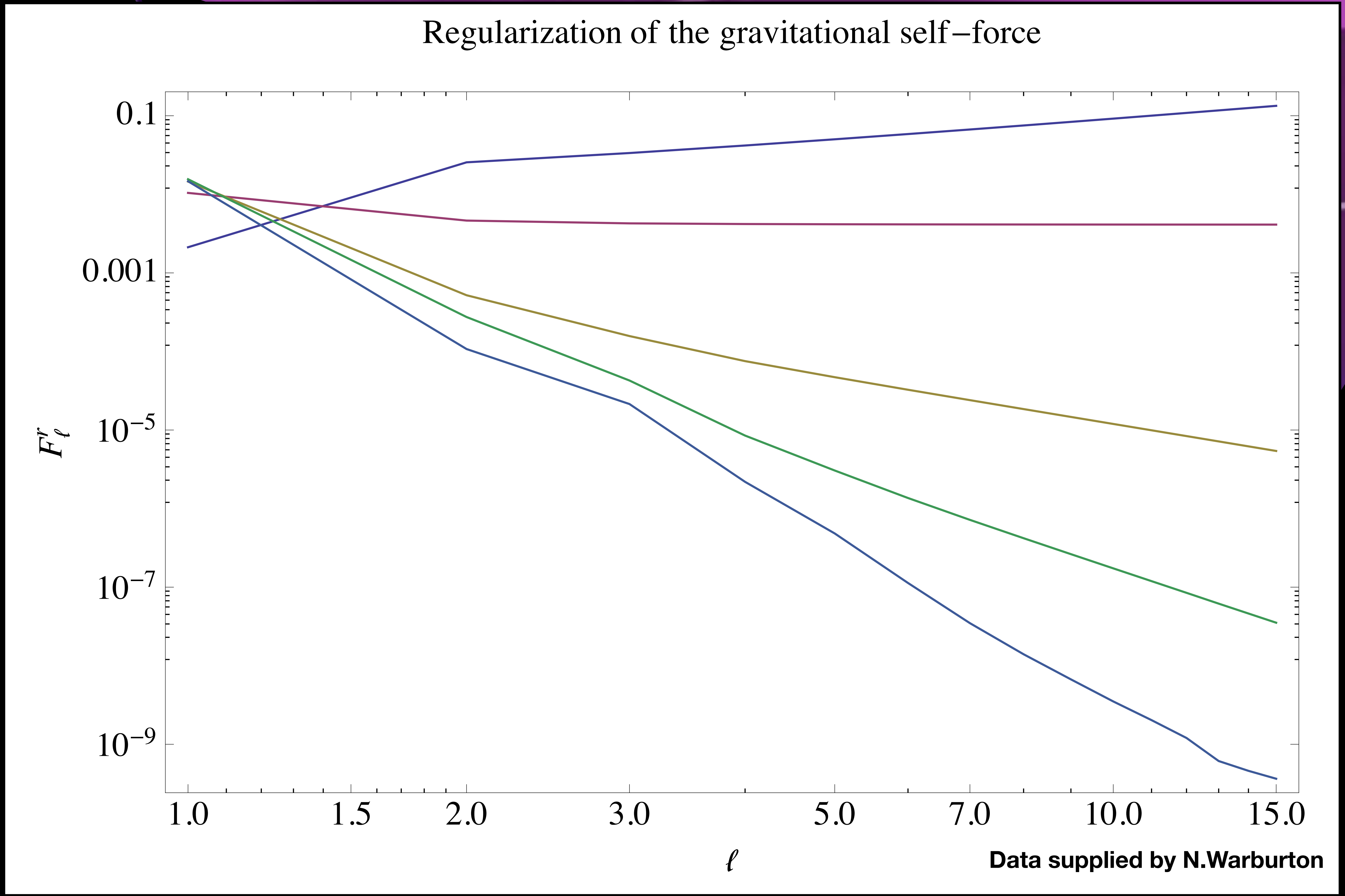
Schwarzschild Gravity



A.Heffernan, A.Ottewill, B.Wardell PRD82, 104023 (2012)

Methods

Image credit: NASA JPL

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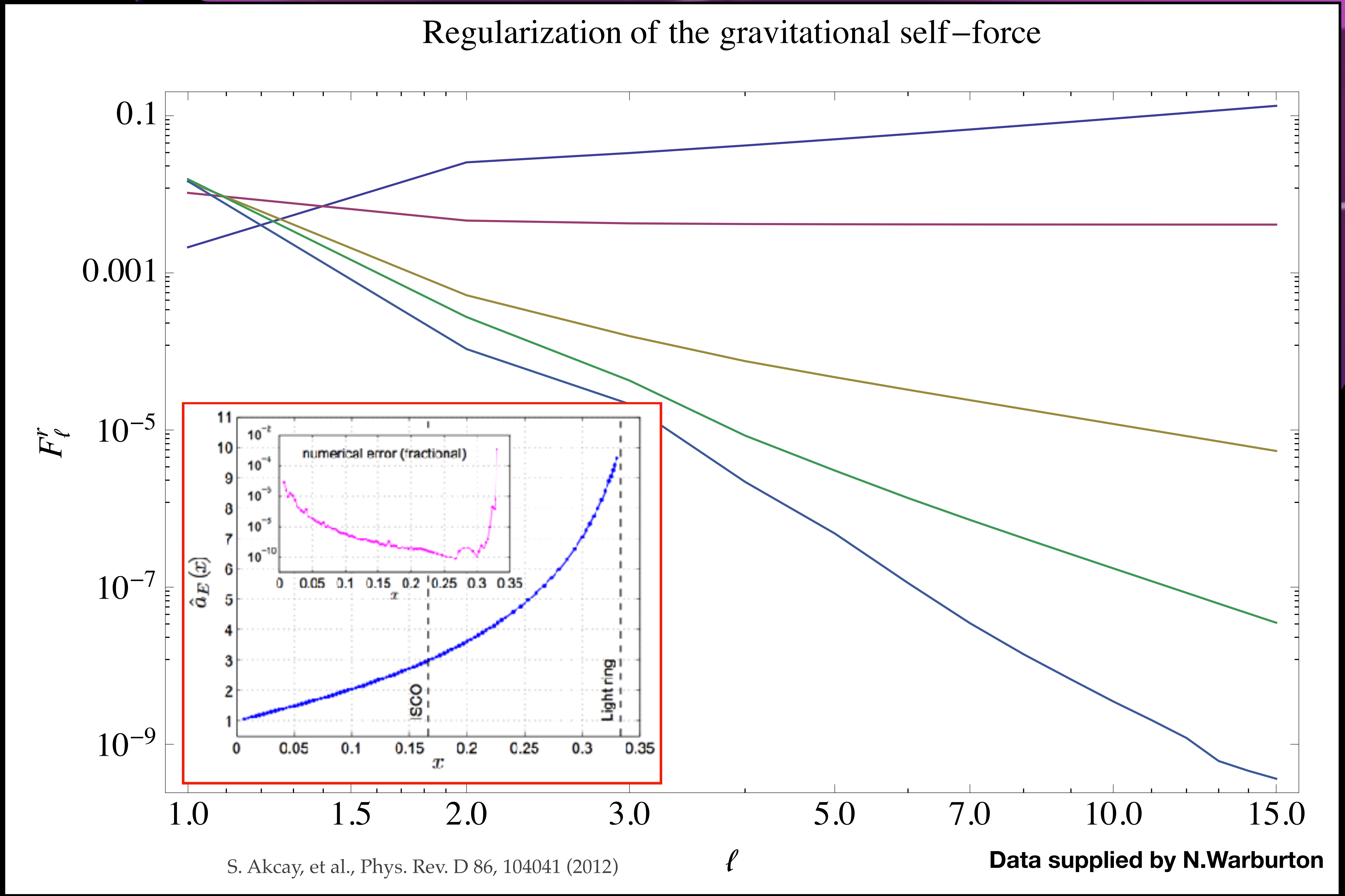

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Image credit: NASA JPL

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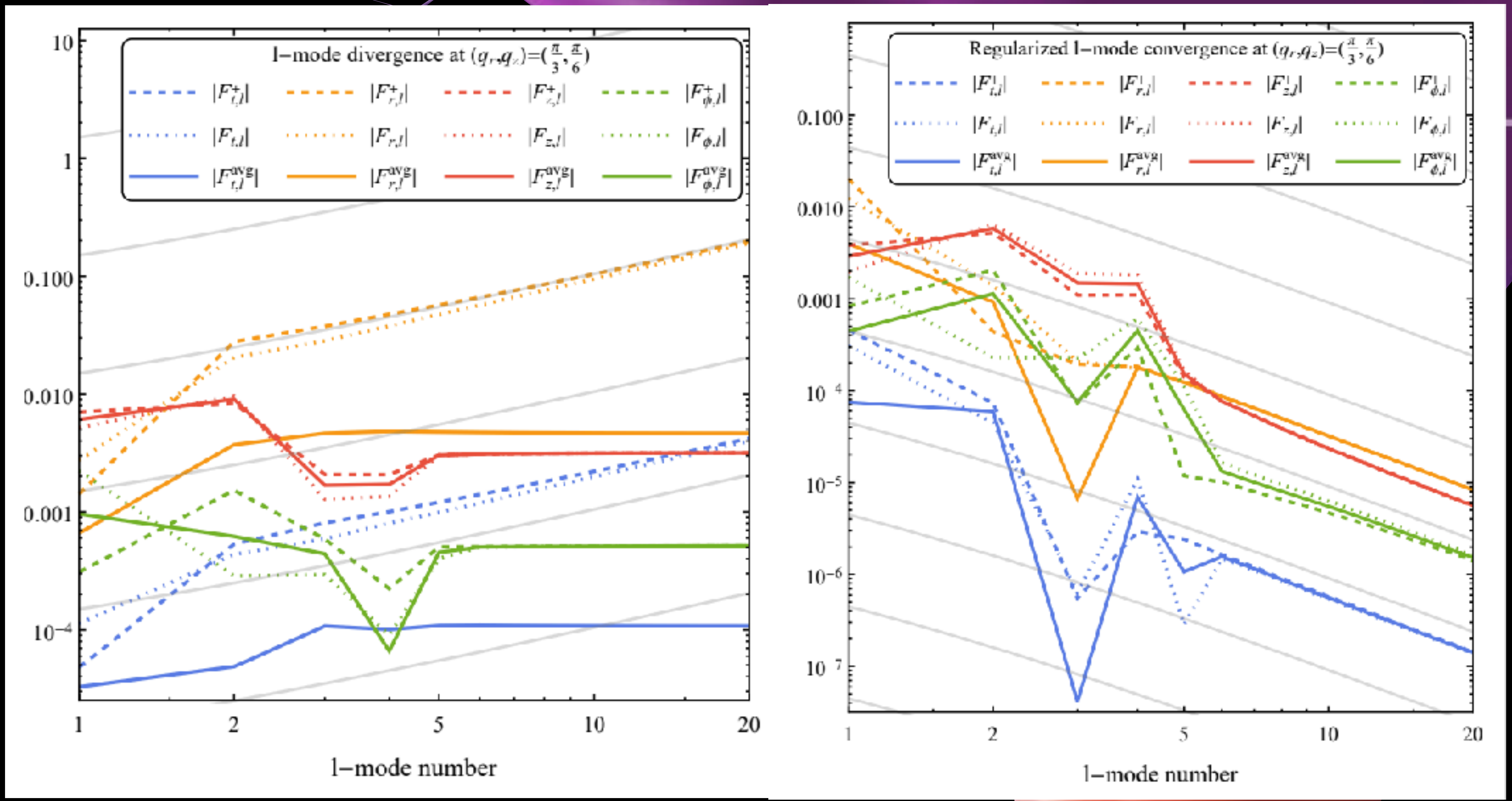
Kerr Gravity Inclined Eccentric



Maarten van de Meent, arXiv:1711.09607

Image credit: NASA JPL

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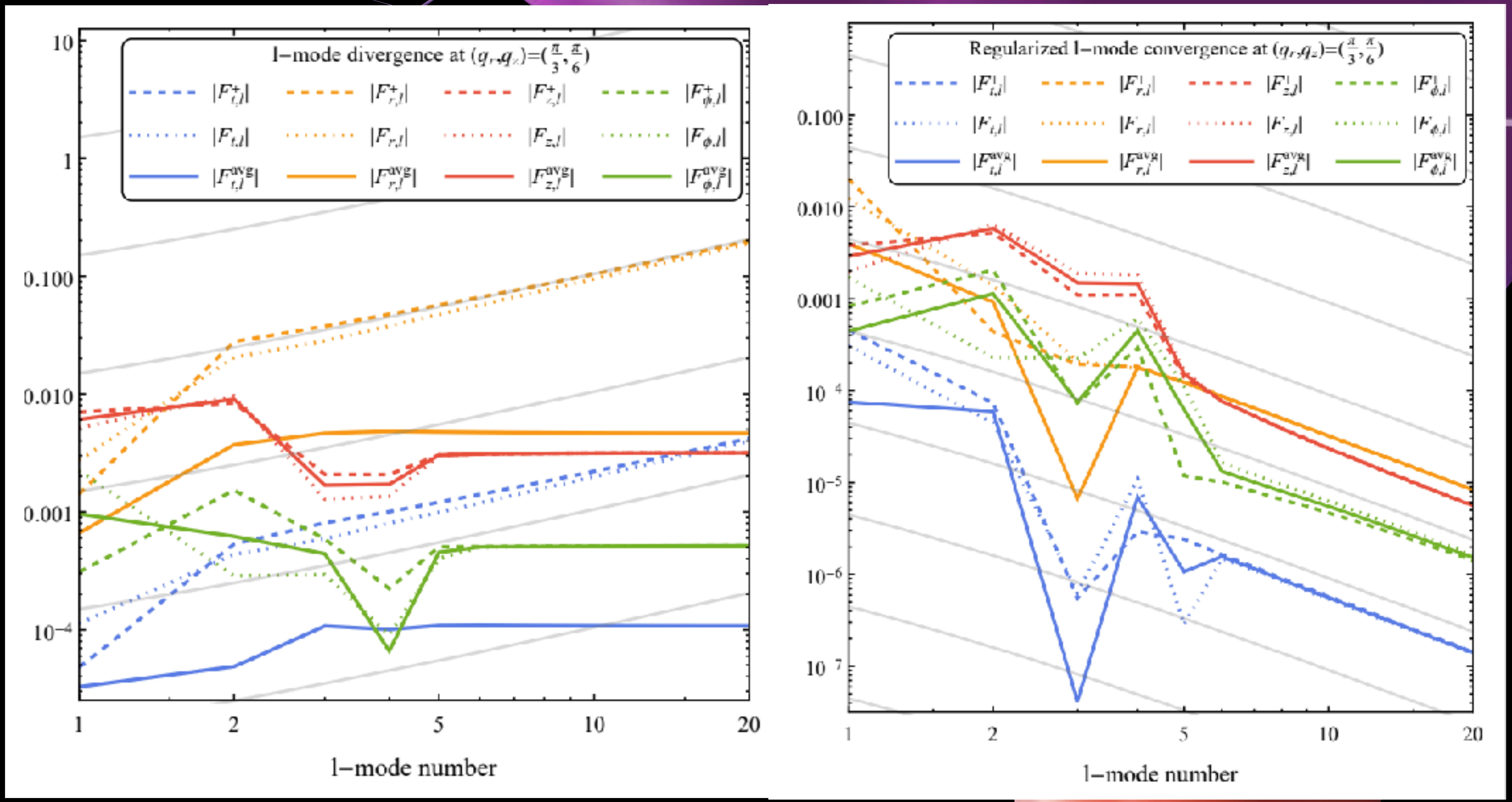

Maarten van de Meent, arXiv:1711.09607

Kerr Gravity Inclined Eccentric

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Methods

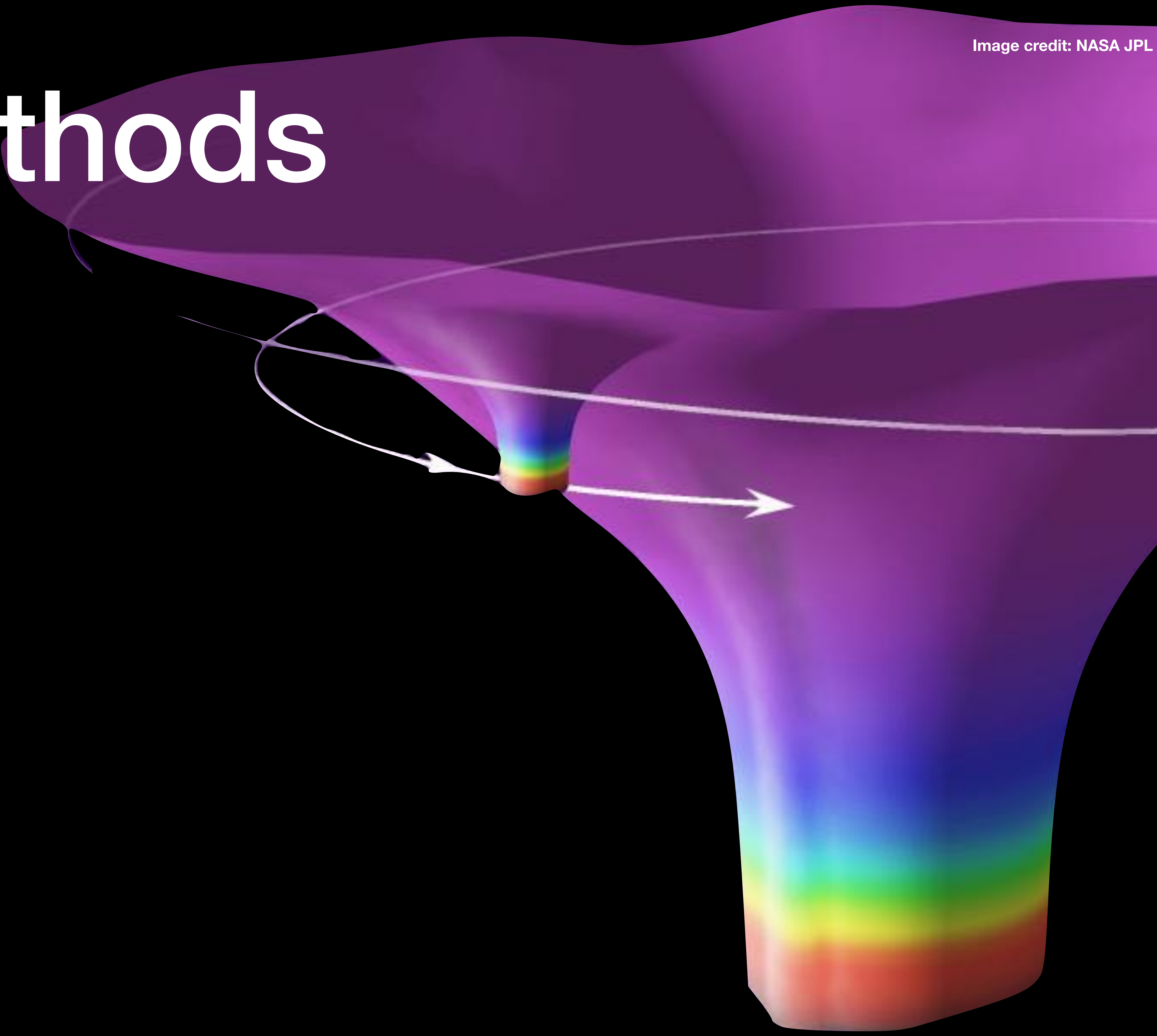

Image credit: NASA JPL

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Methods

- Effective source

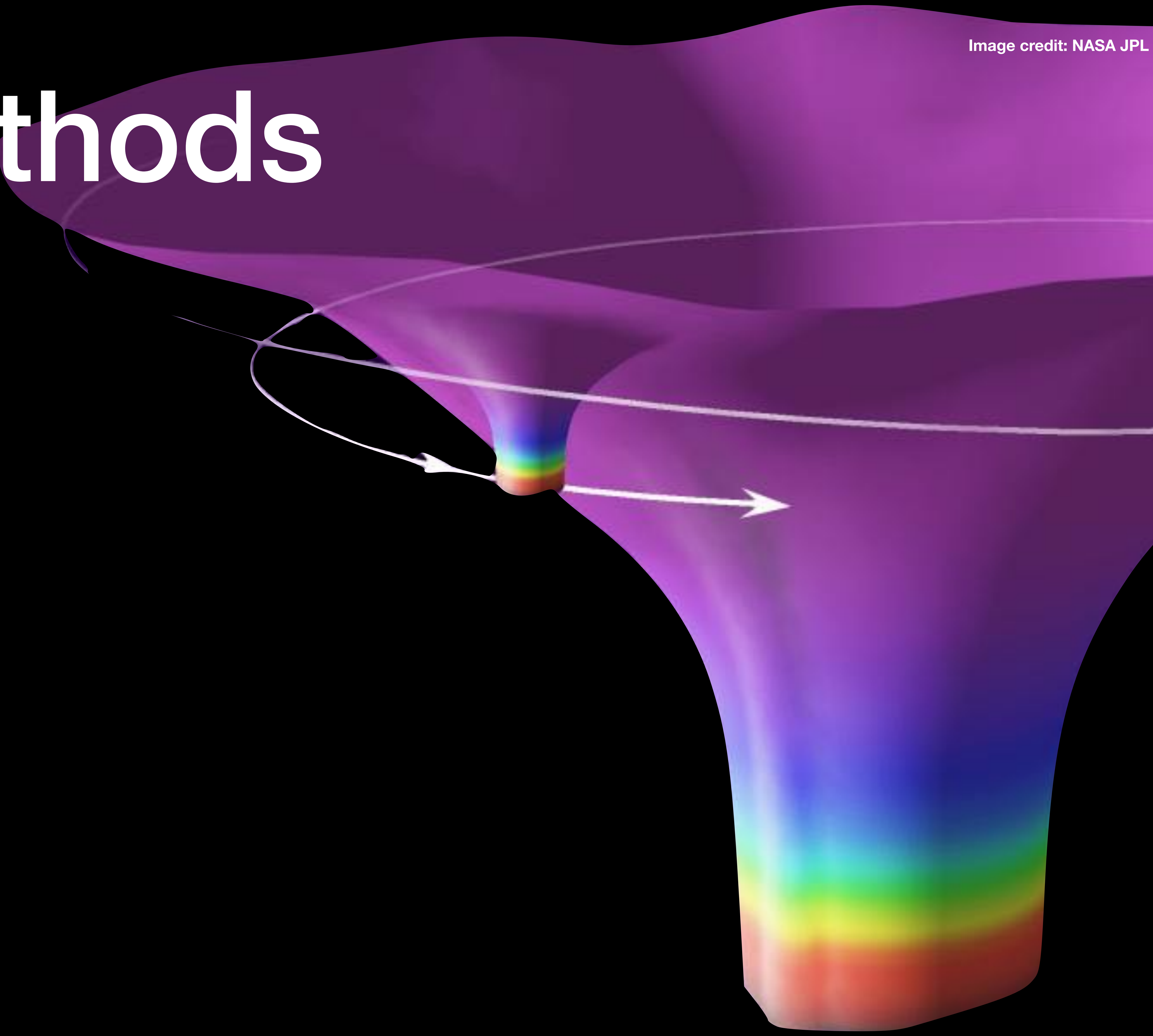

Image credit: NASA JPL

Methods

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    - Vega, Detweiler & Goldburn, Barrack (2007)

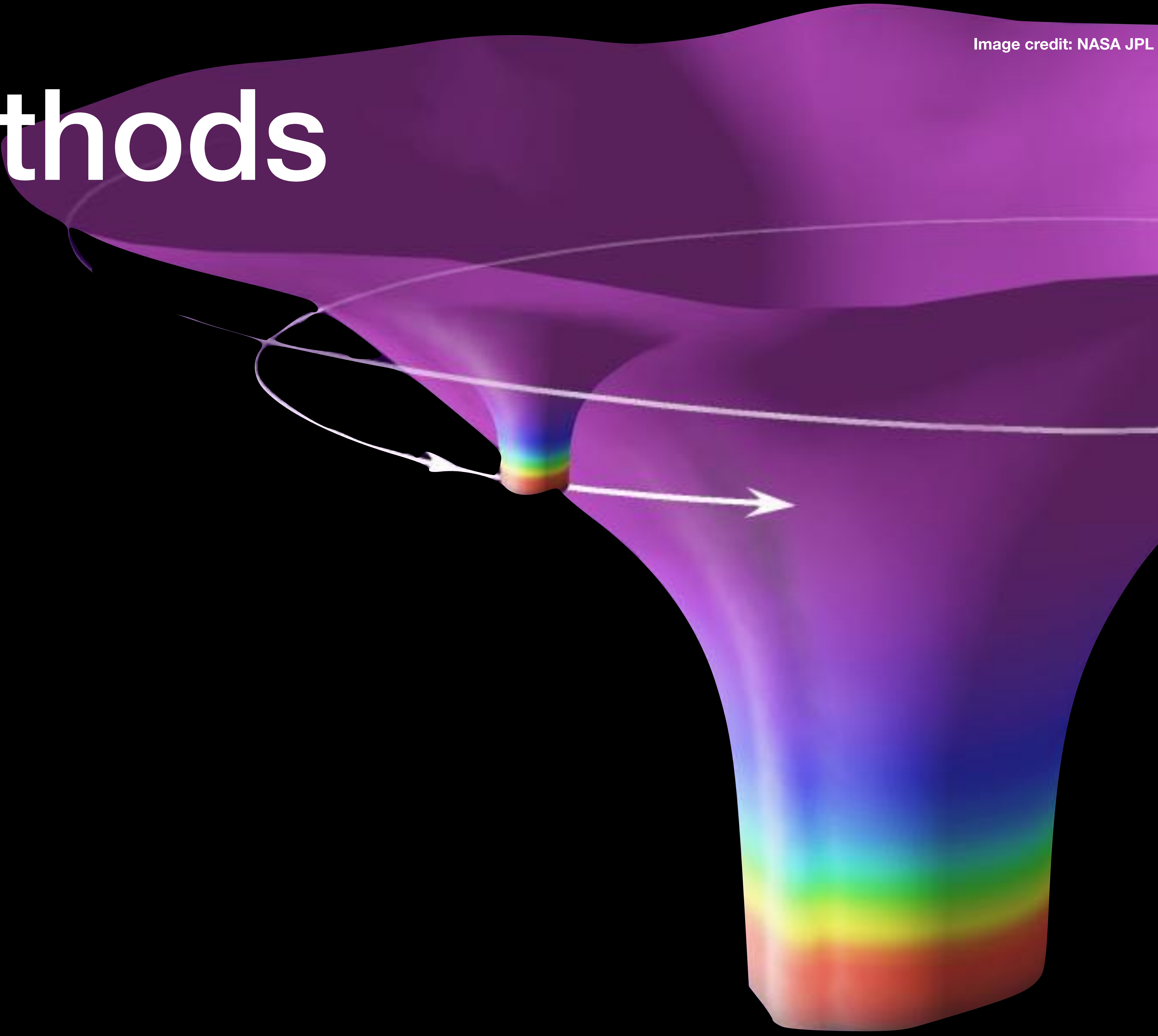

Image credit: NASA JPL

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$$\Psi^{A}_{(ret)} = \tilde{\Psi}^{A}_{(S)} + \tilde{\Psi}^{A}_{(R)}$$

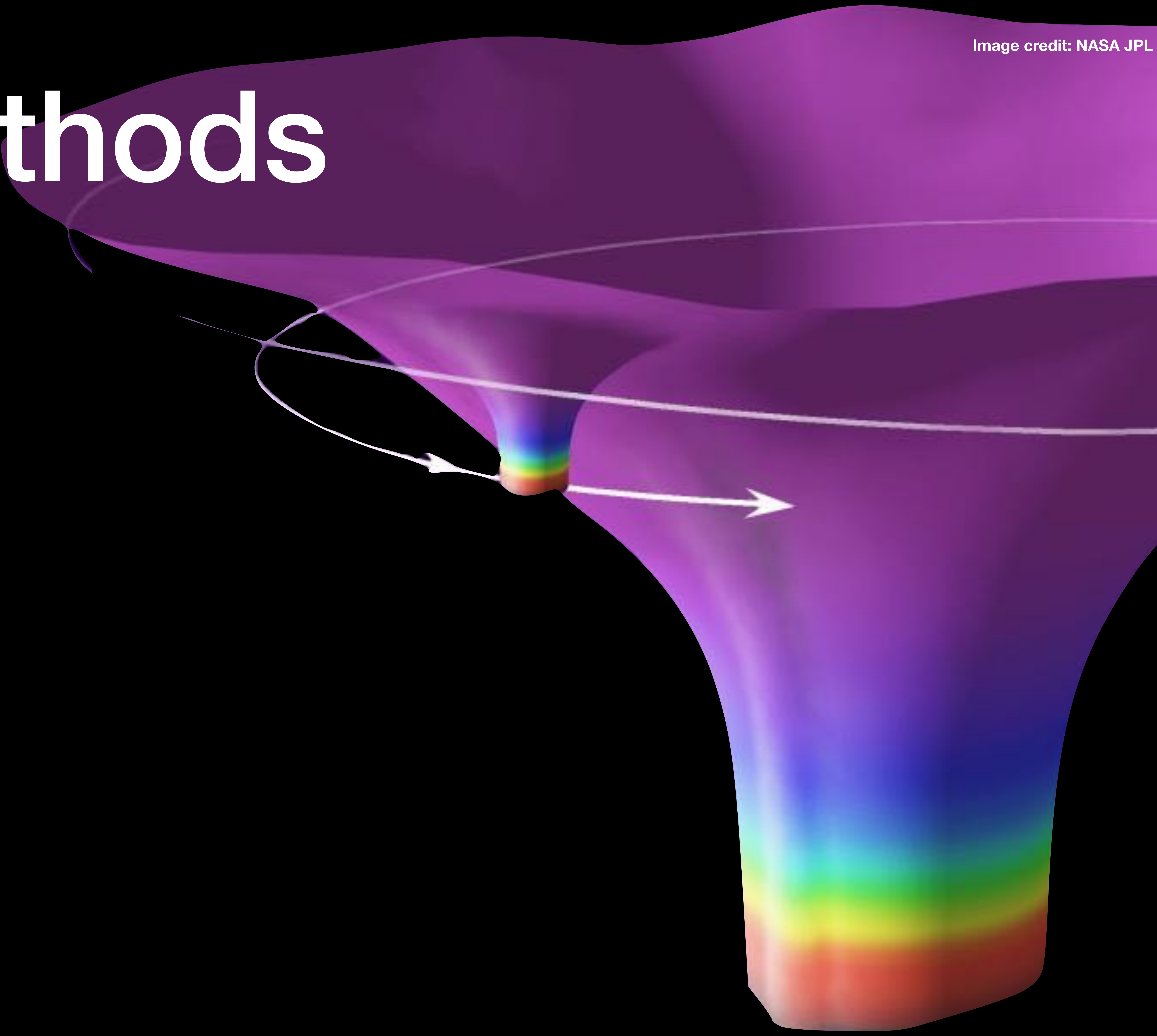

Image credit: NASA JPL

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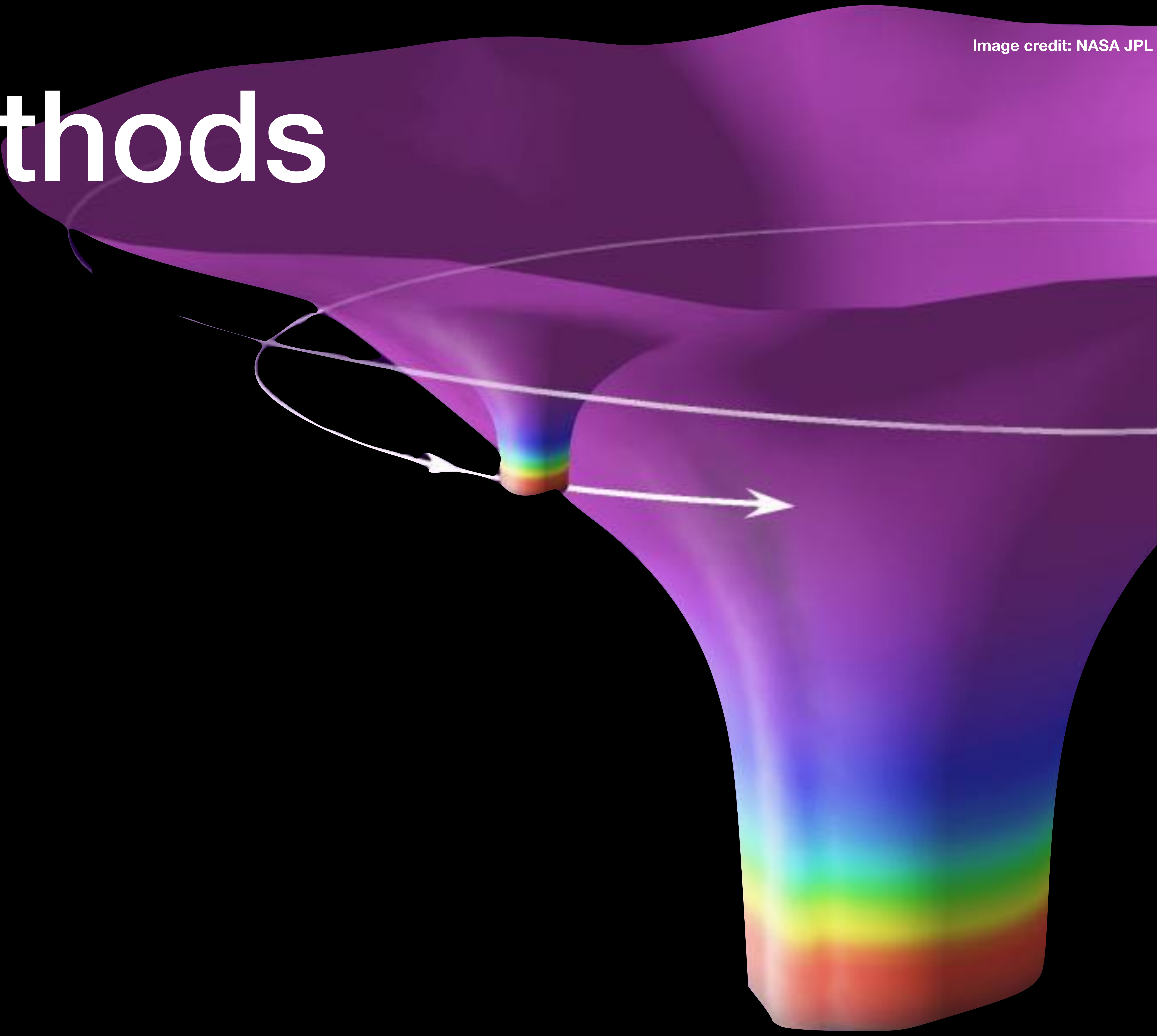
Methods



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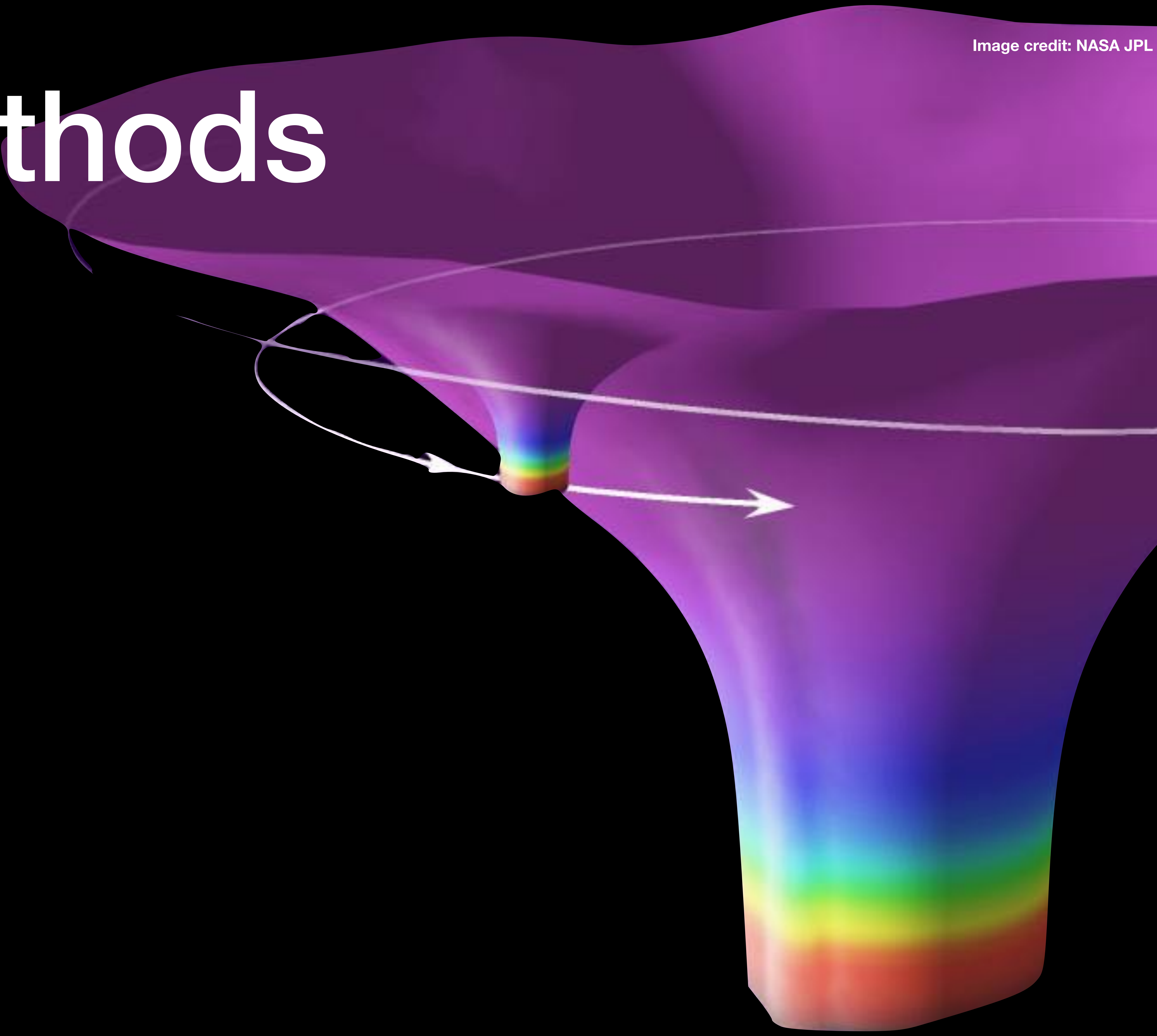
Methods

Image credit: NASA JPL



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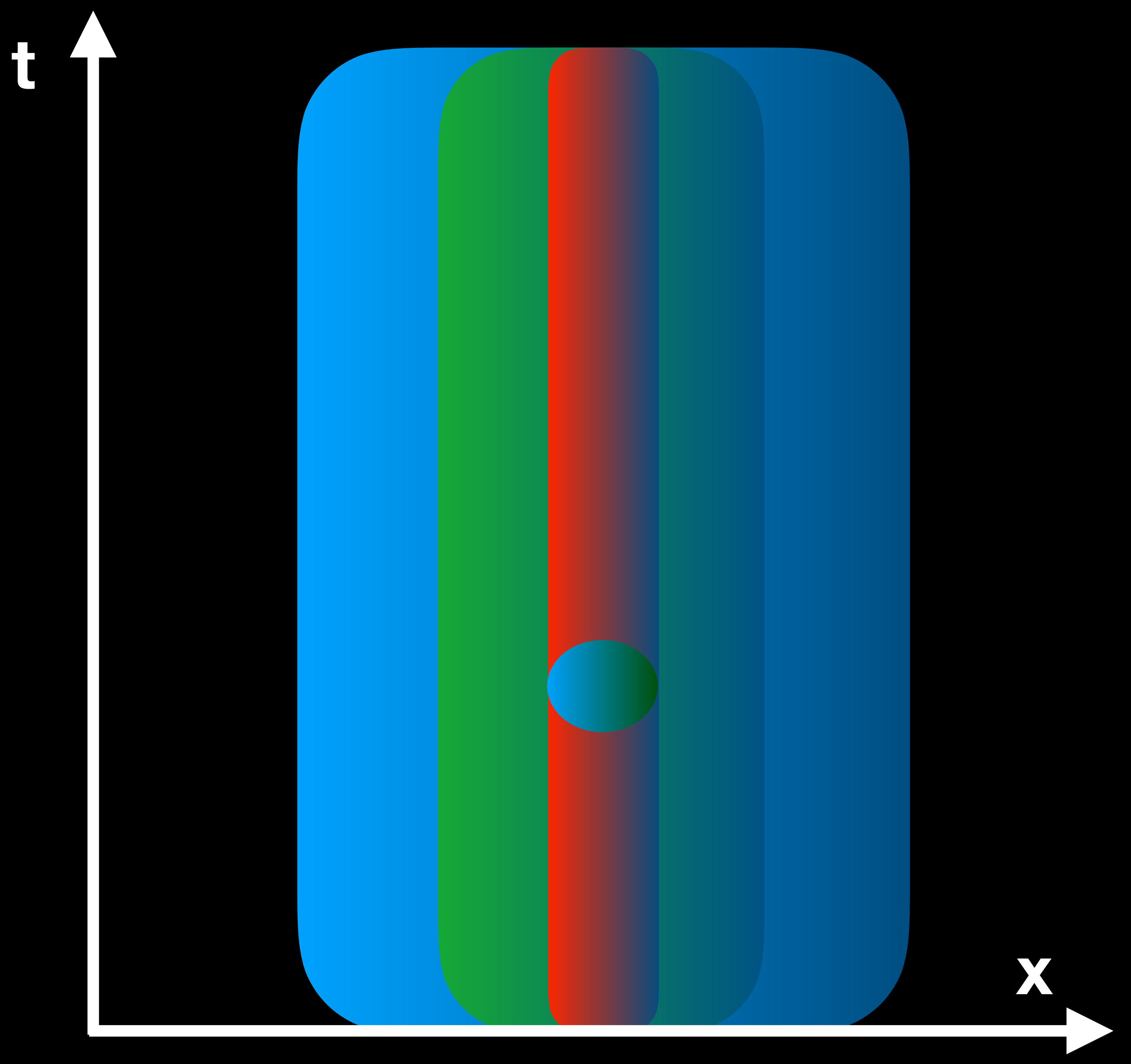

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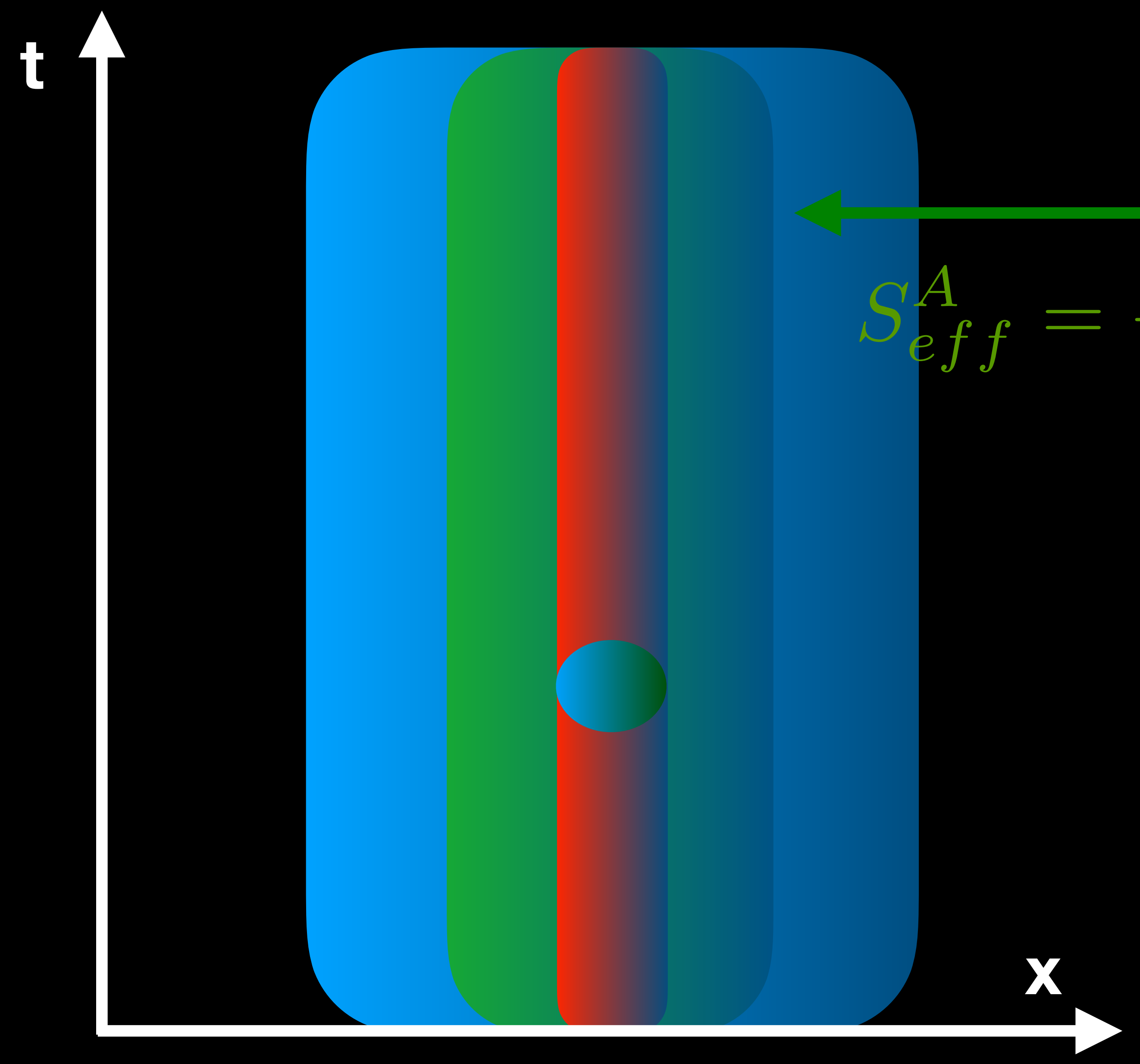

Inside World Tube

$$S^A_{eff} = -4\pi\mathcal{M}^A - (\delta^A_B\Box + P^A_B)\tilde{\Psi}^B_S$$

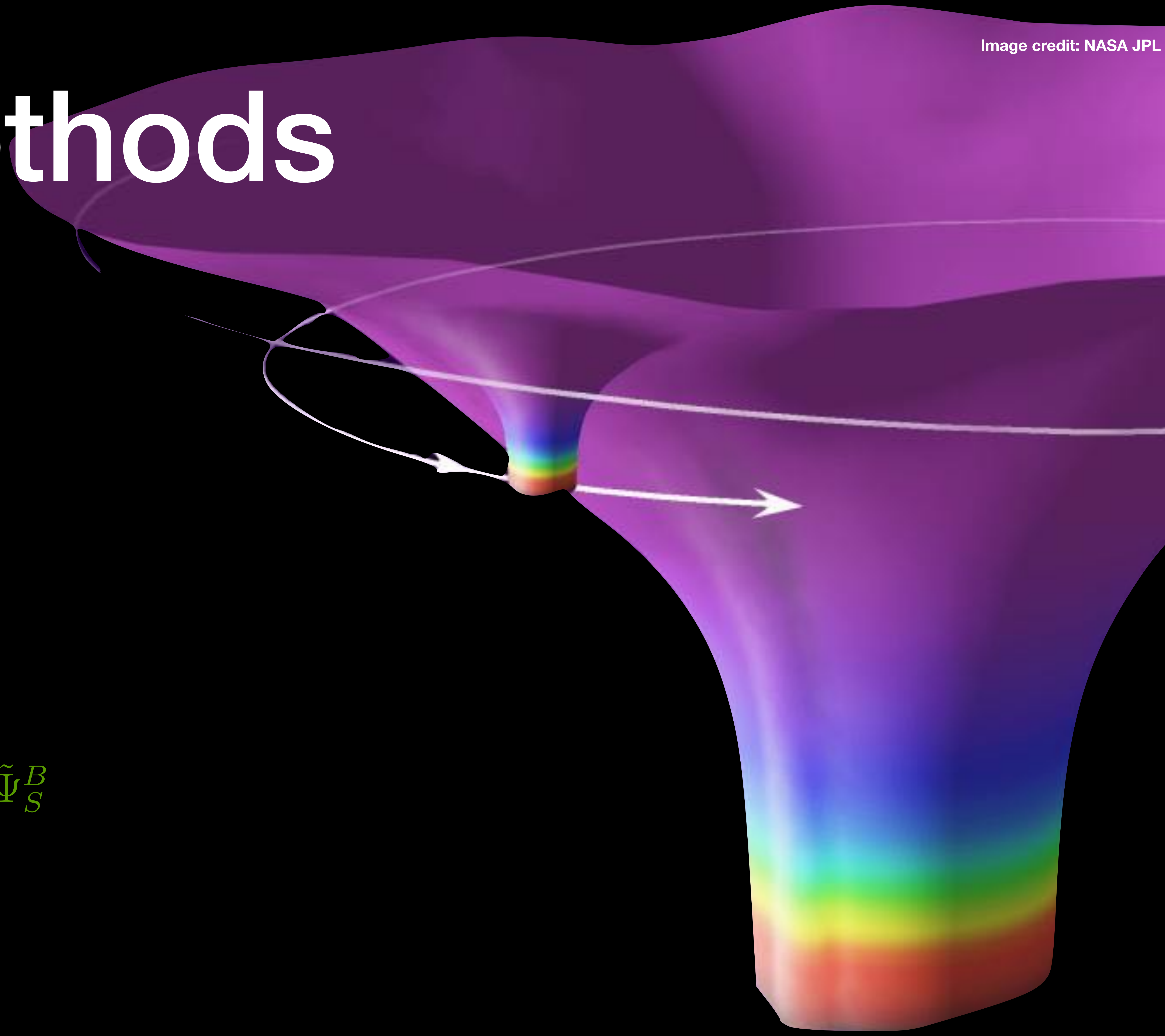

Image credit: NASA JPL

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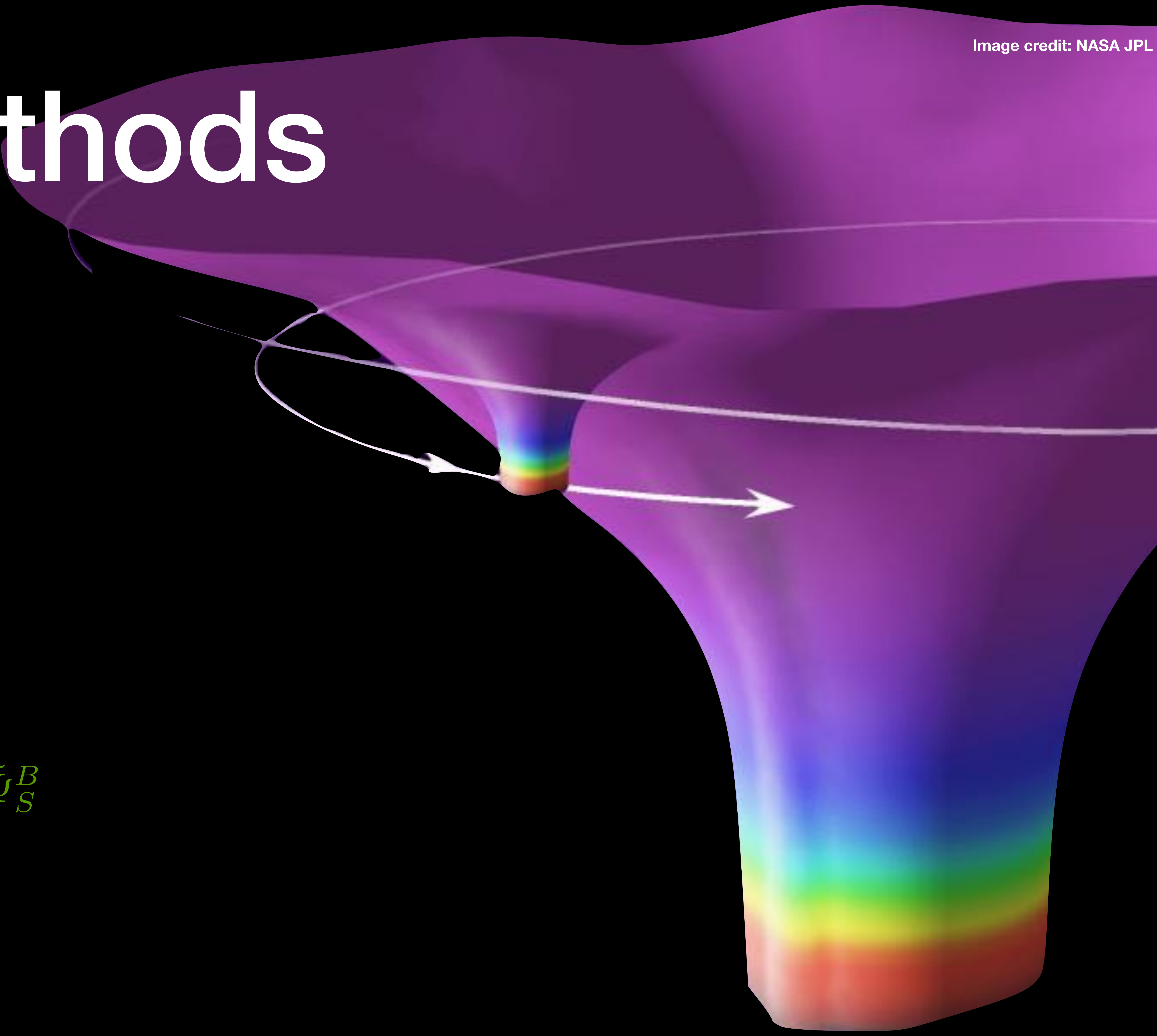

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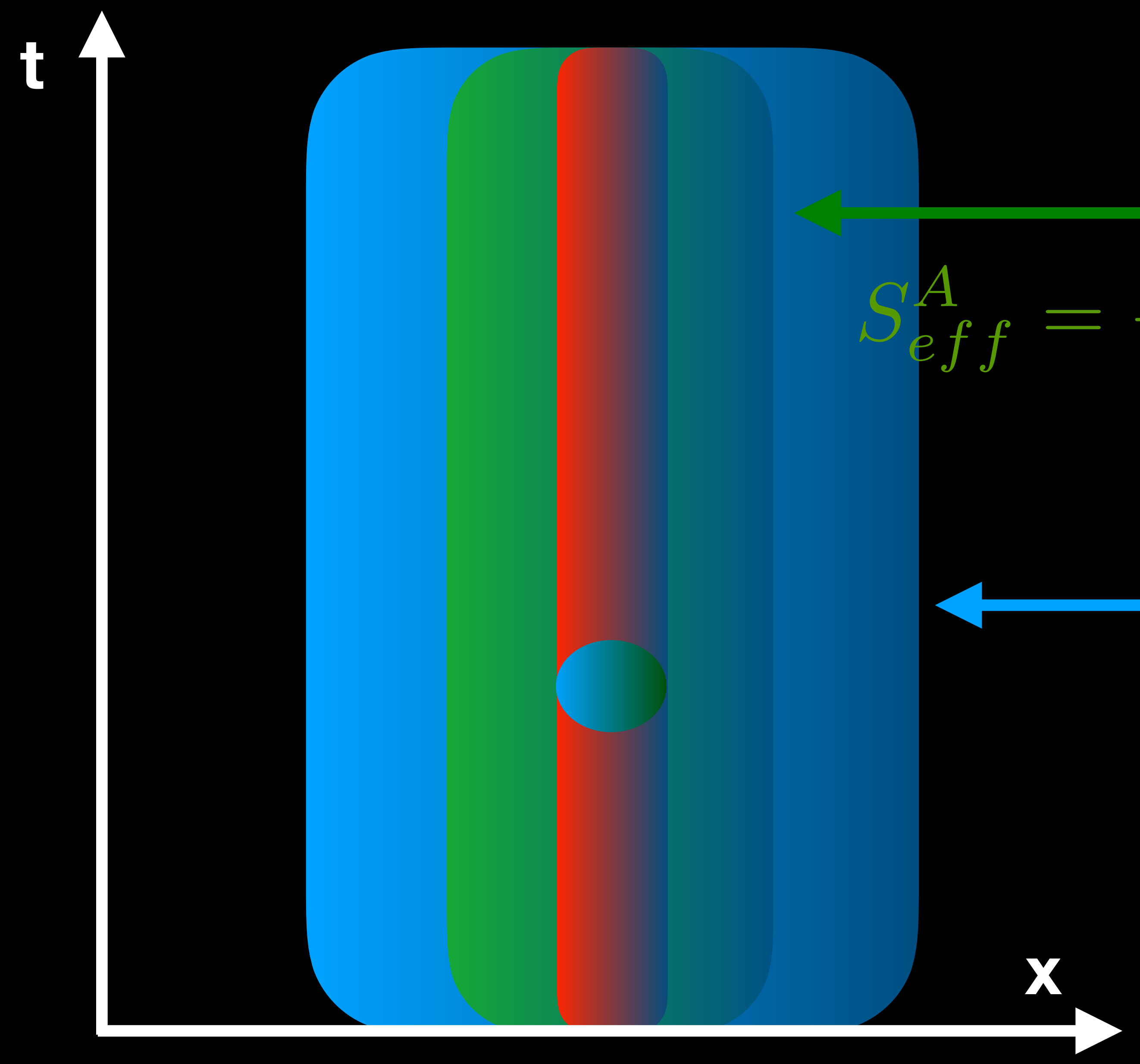

Inside World Tube

$$S^{A}_{eff} = -4\pi\mathcal{M}^{A} - (\delta^{A}_{B}\Box + P^{A}_{B})\tilde{\Psi}^{B}_{S}$$

Outside World Tube

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Methods

Image credit: NASA JPL

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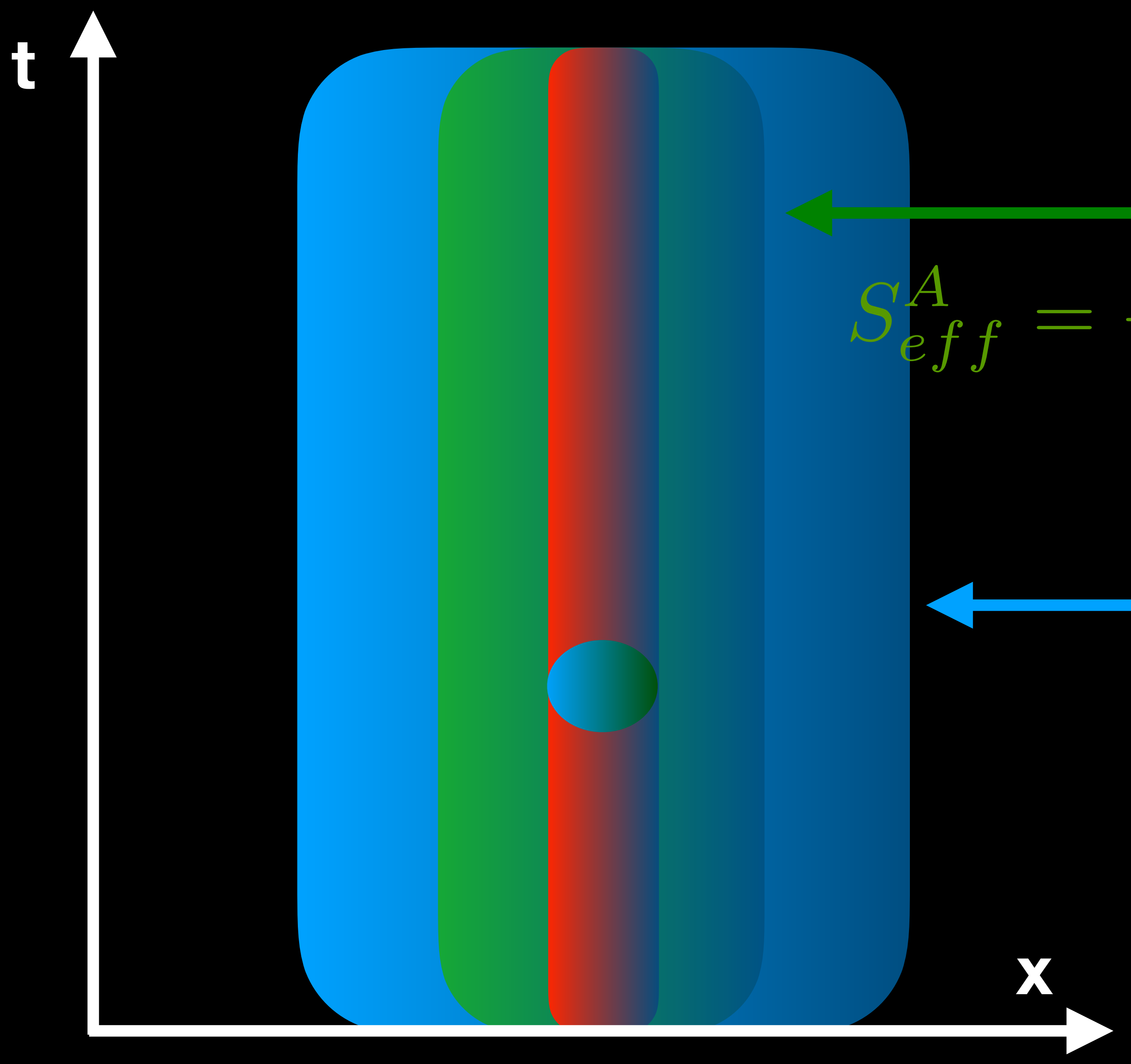

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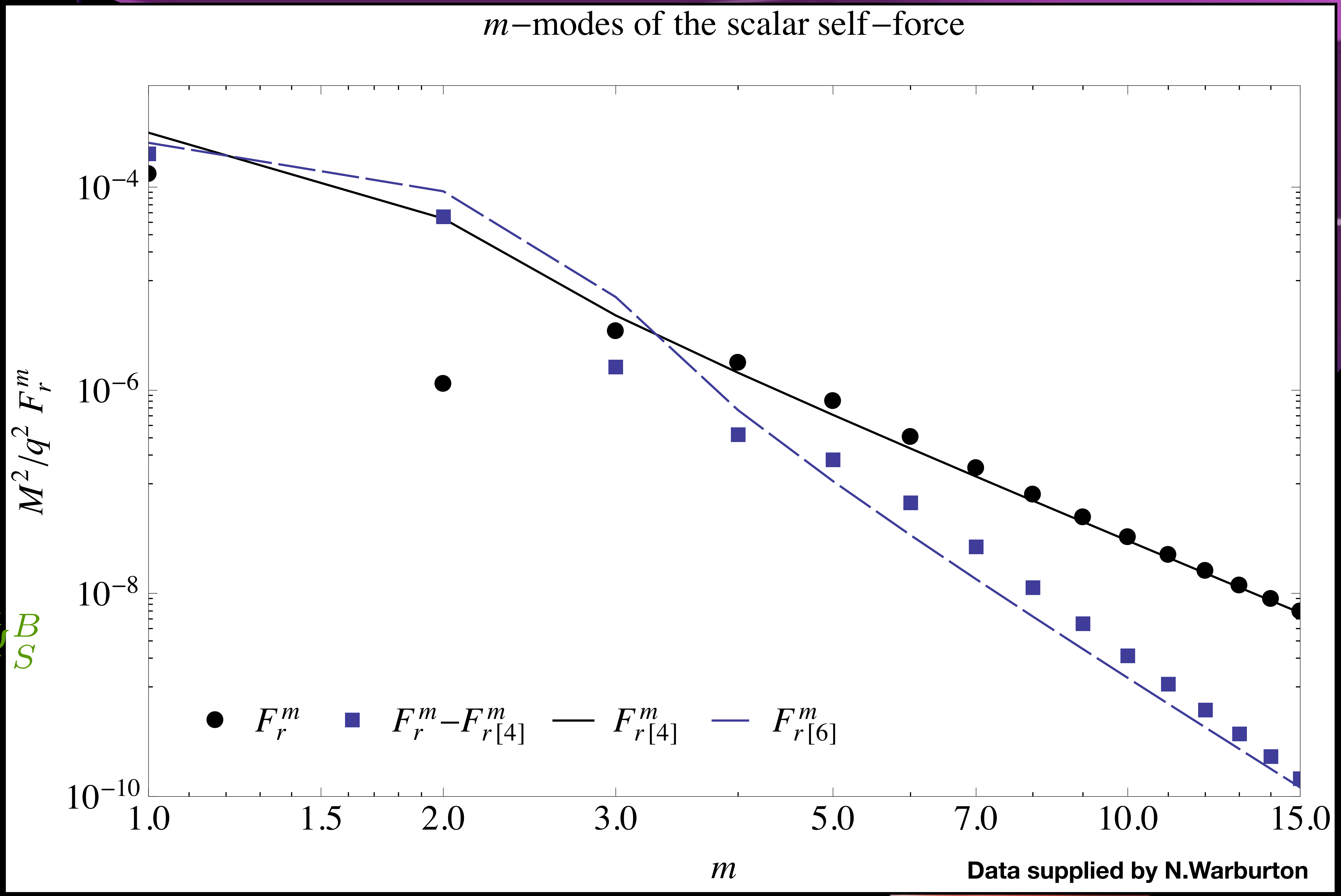
Kerr Scalar



A.Heffernan, A.Ottewill, B.Wardell, PRD89, 024030 (2014)

Methods

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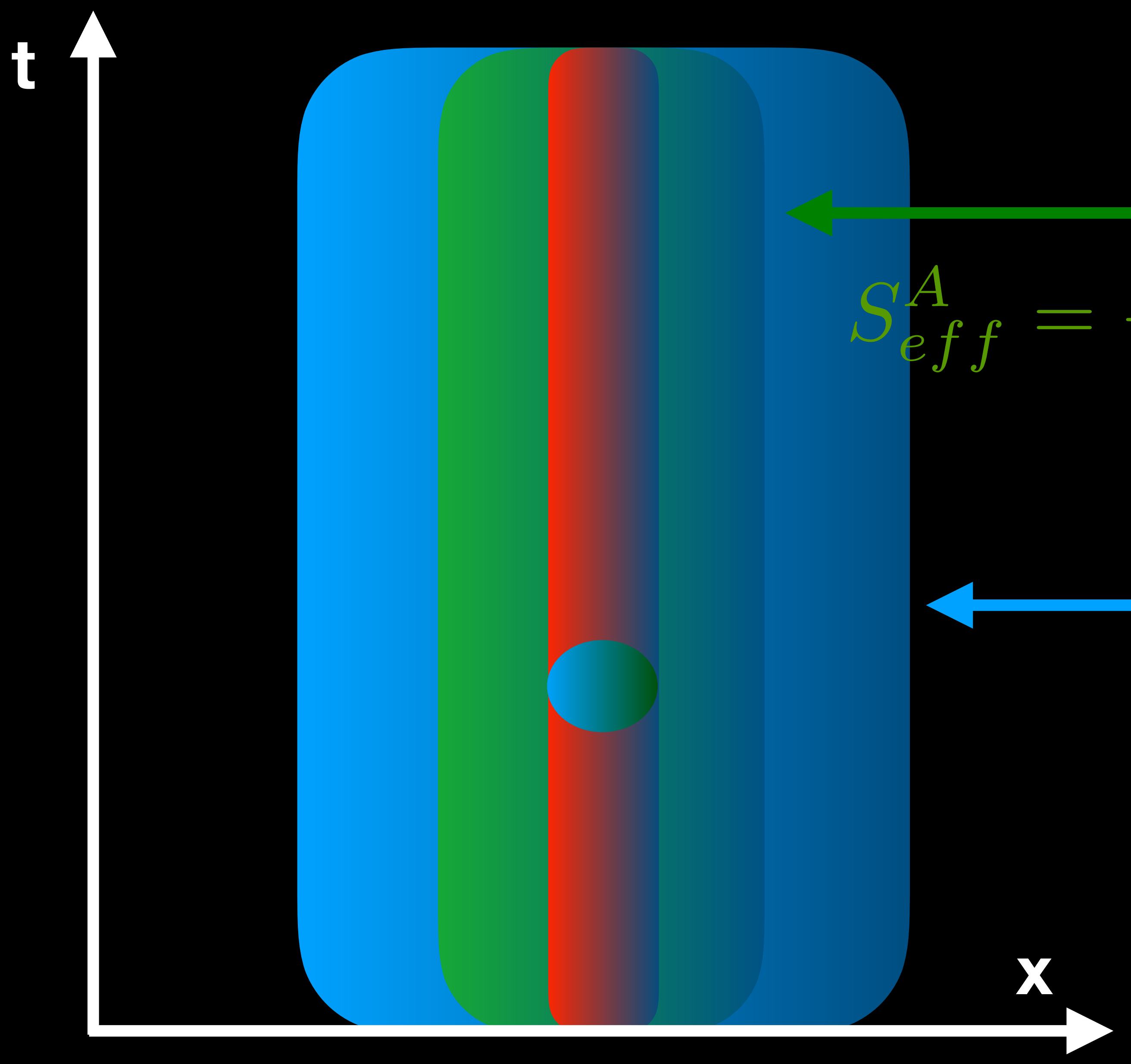

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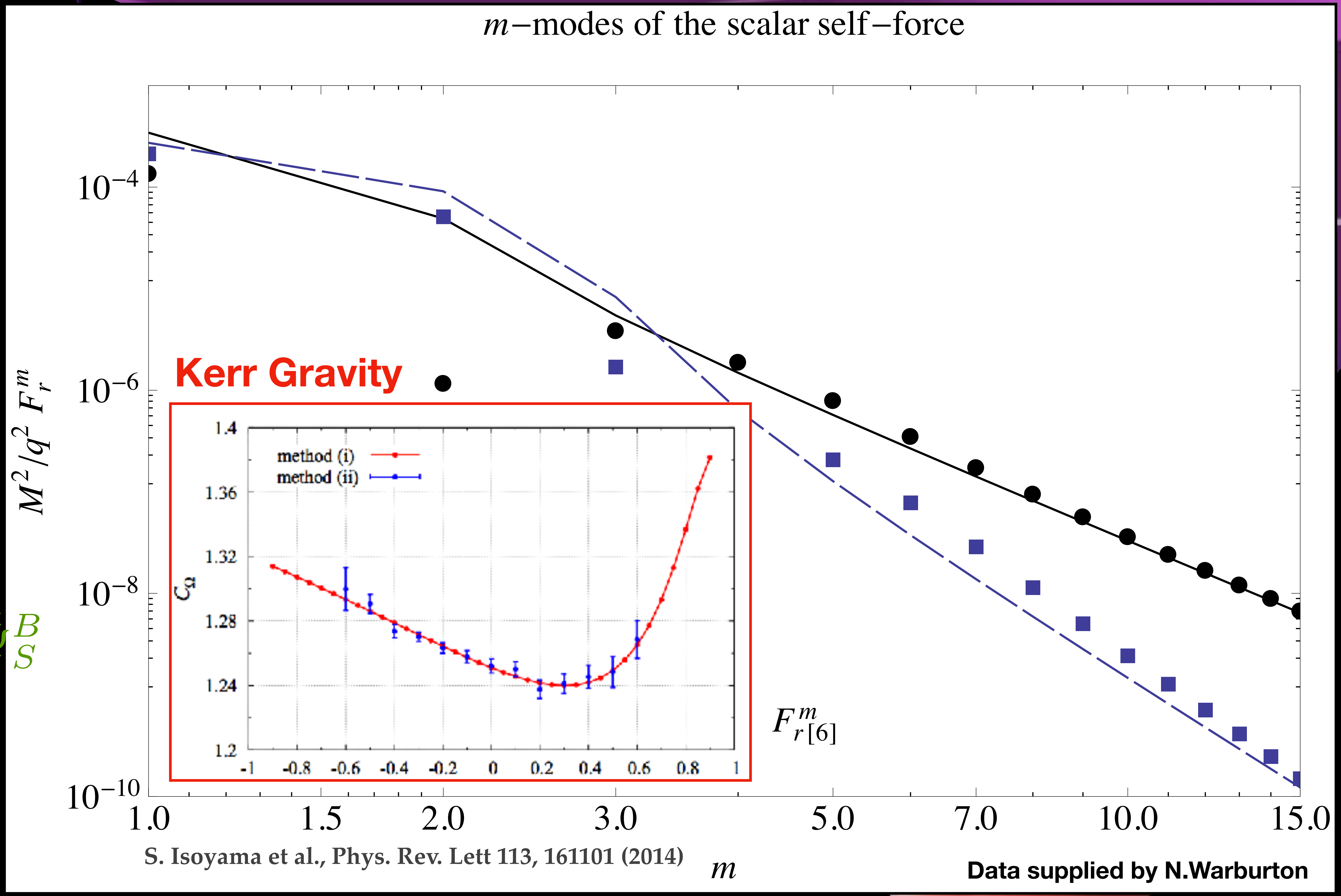

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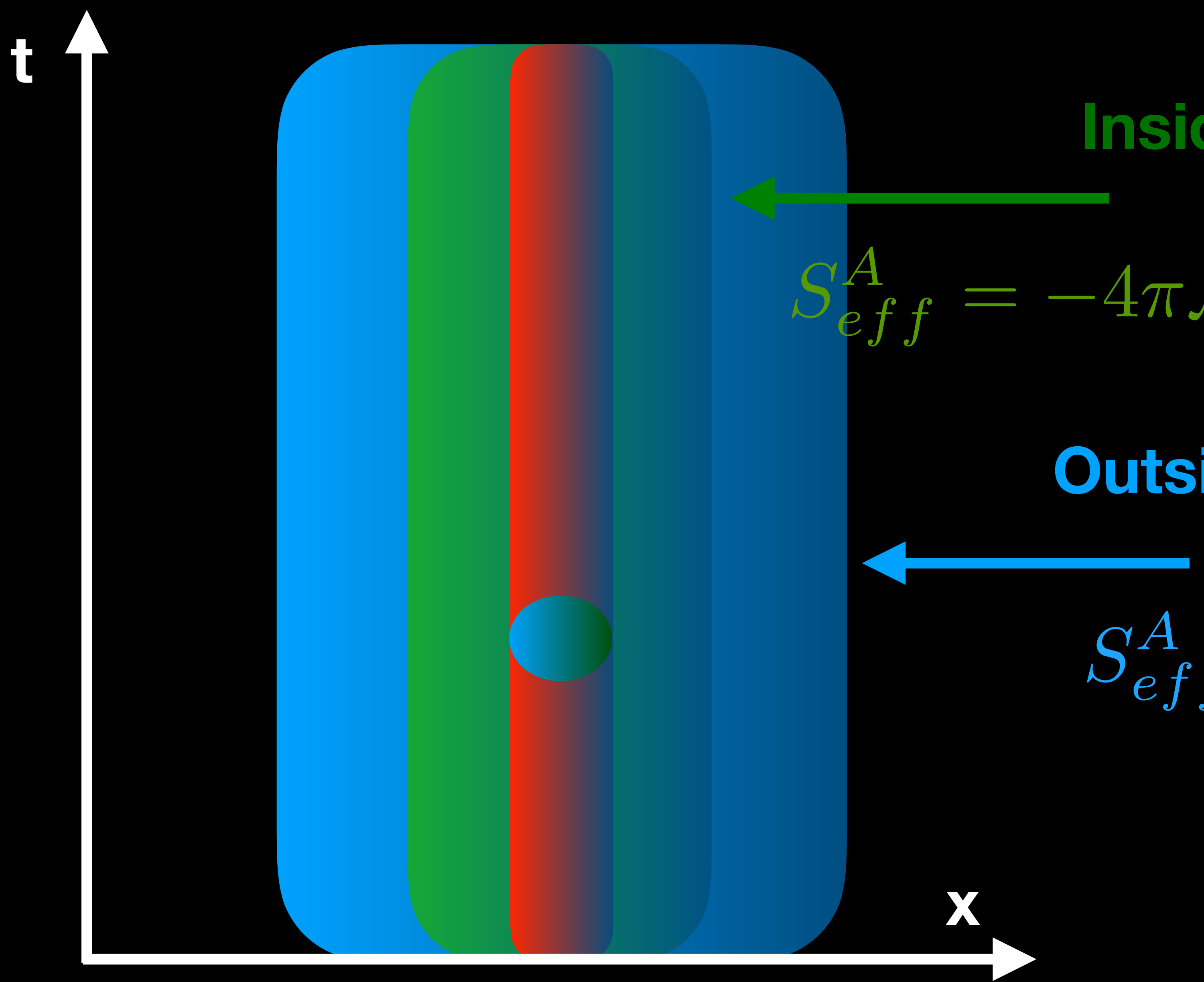

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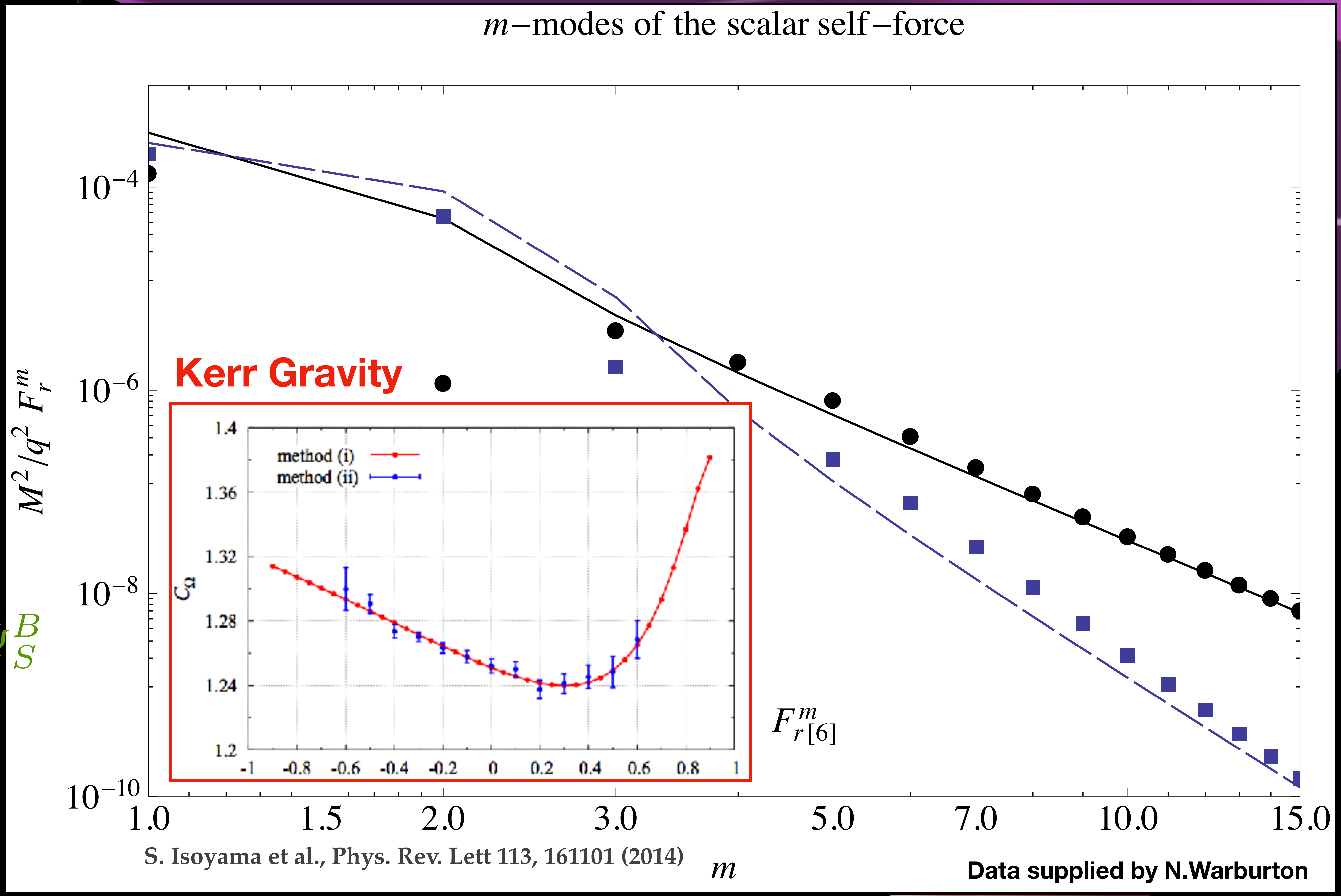

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Kerr Gravity
Circular Equatorial

PN comparison

Detweiler's redshift observable in Circular Schwarzschild

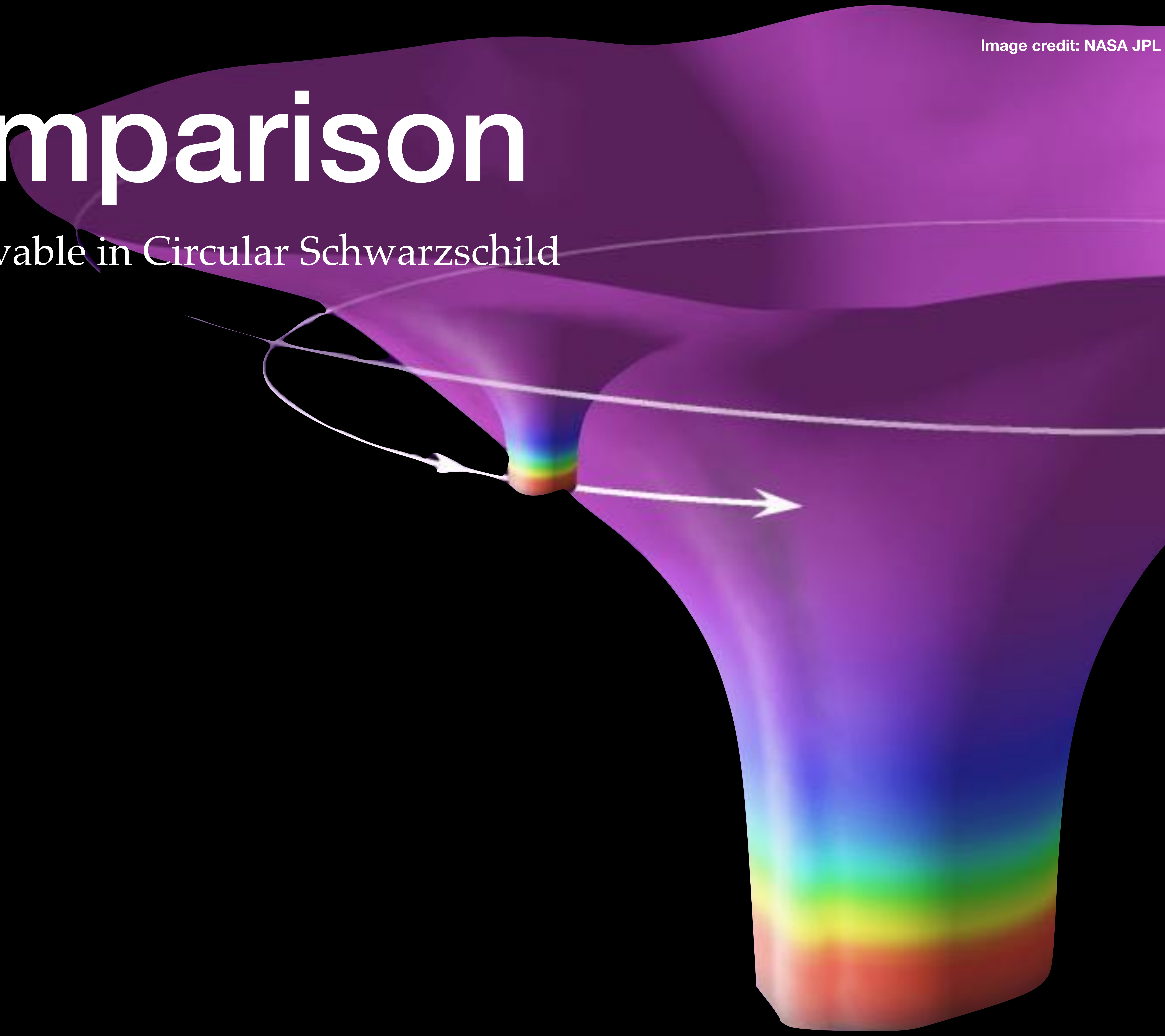

Image credit: NASA JPL

PN comparison

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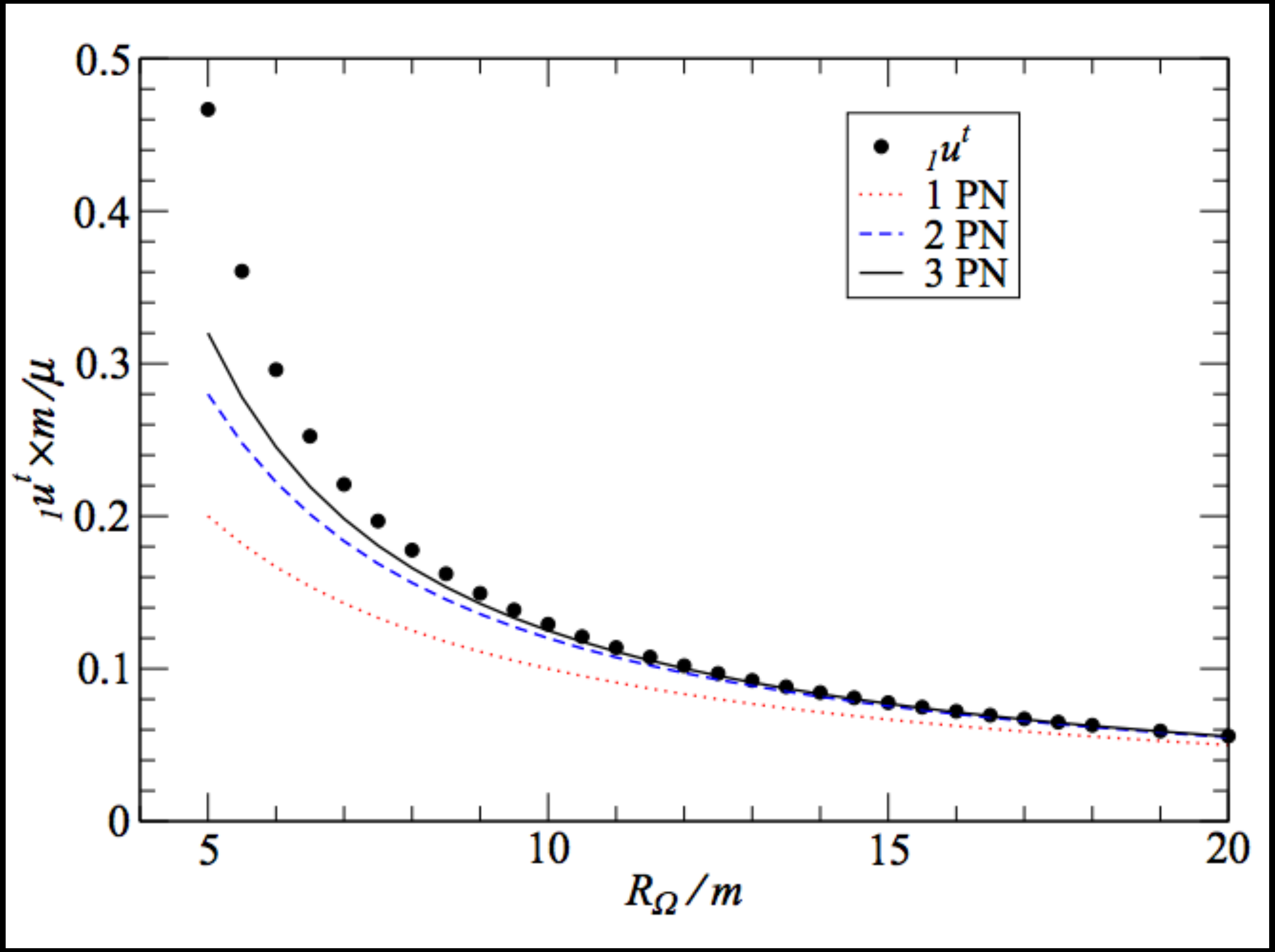
First gauge invariant quantity:



S. Detweiler, PRD77, 124026 (2008)

Image credit: NASA JPL

Image credit: NASA JPL

PN comparison

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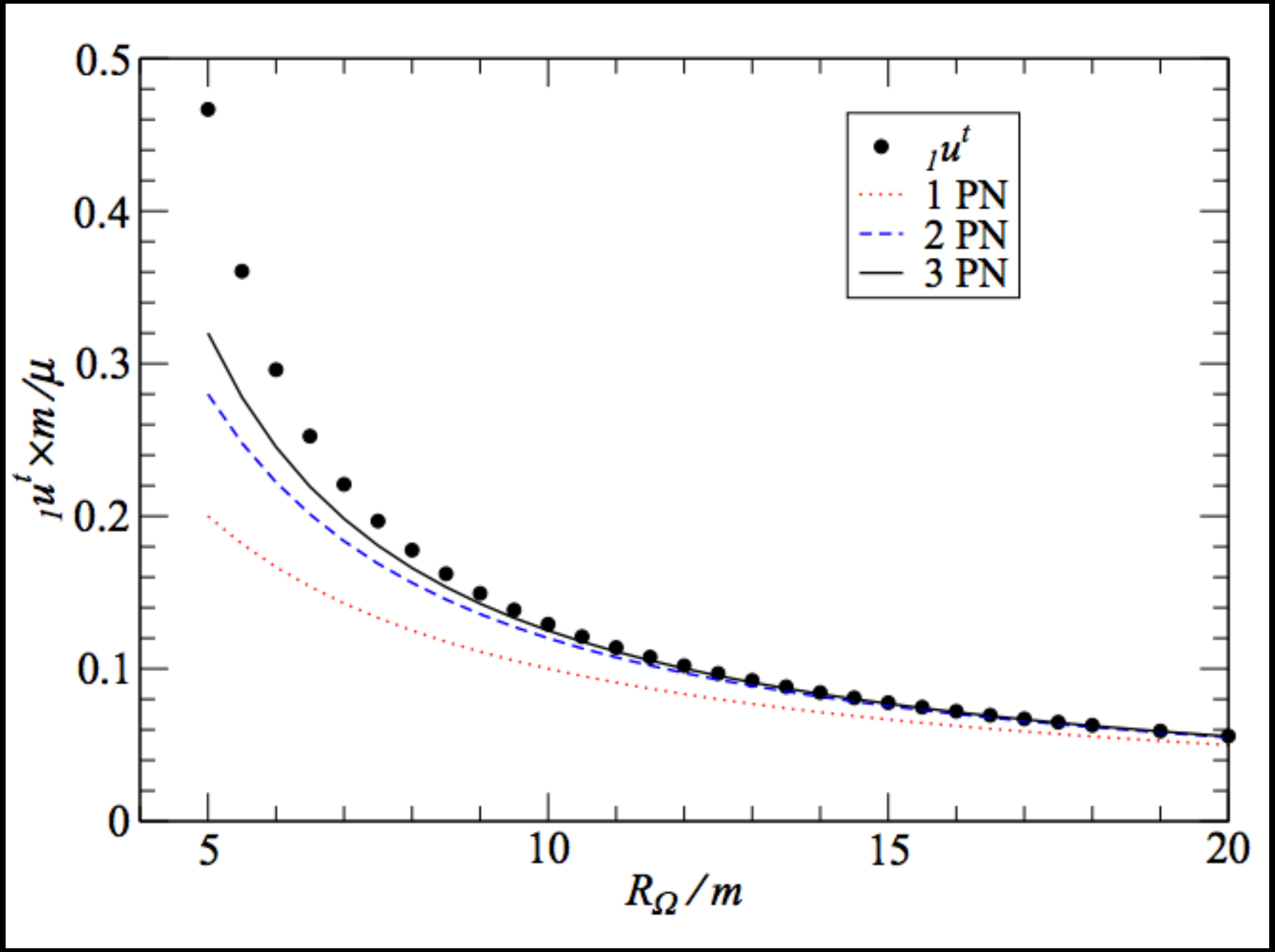
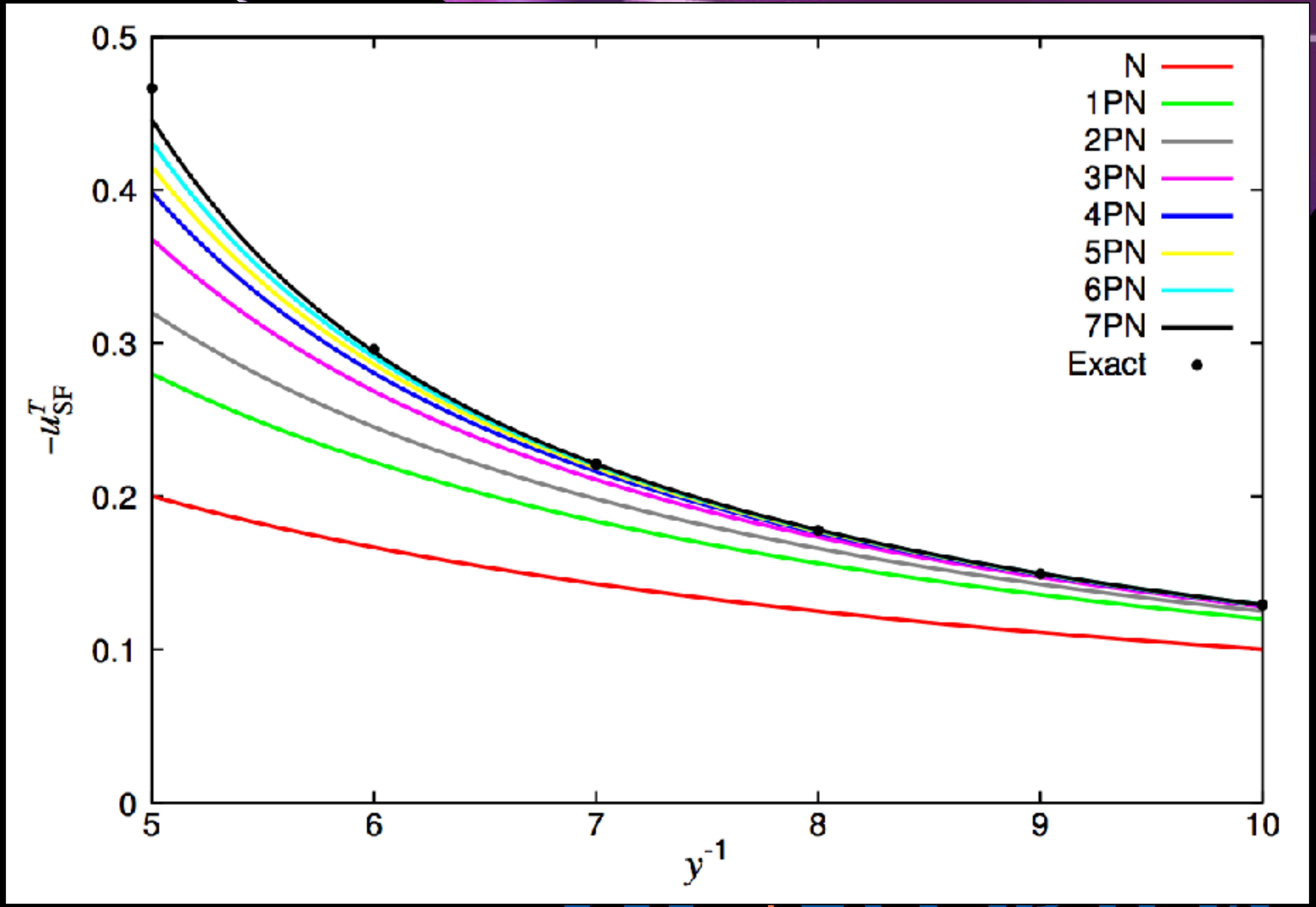


S. Detweiler, PRD77, 124026 (2008)

Reading off unknown PN coefficients:



L. Blanchet et al, PRD81, 084033 (2010)

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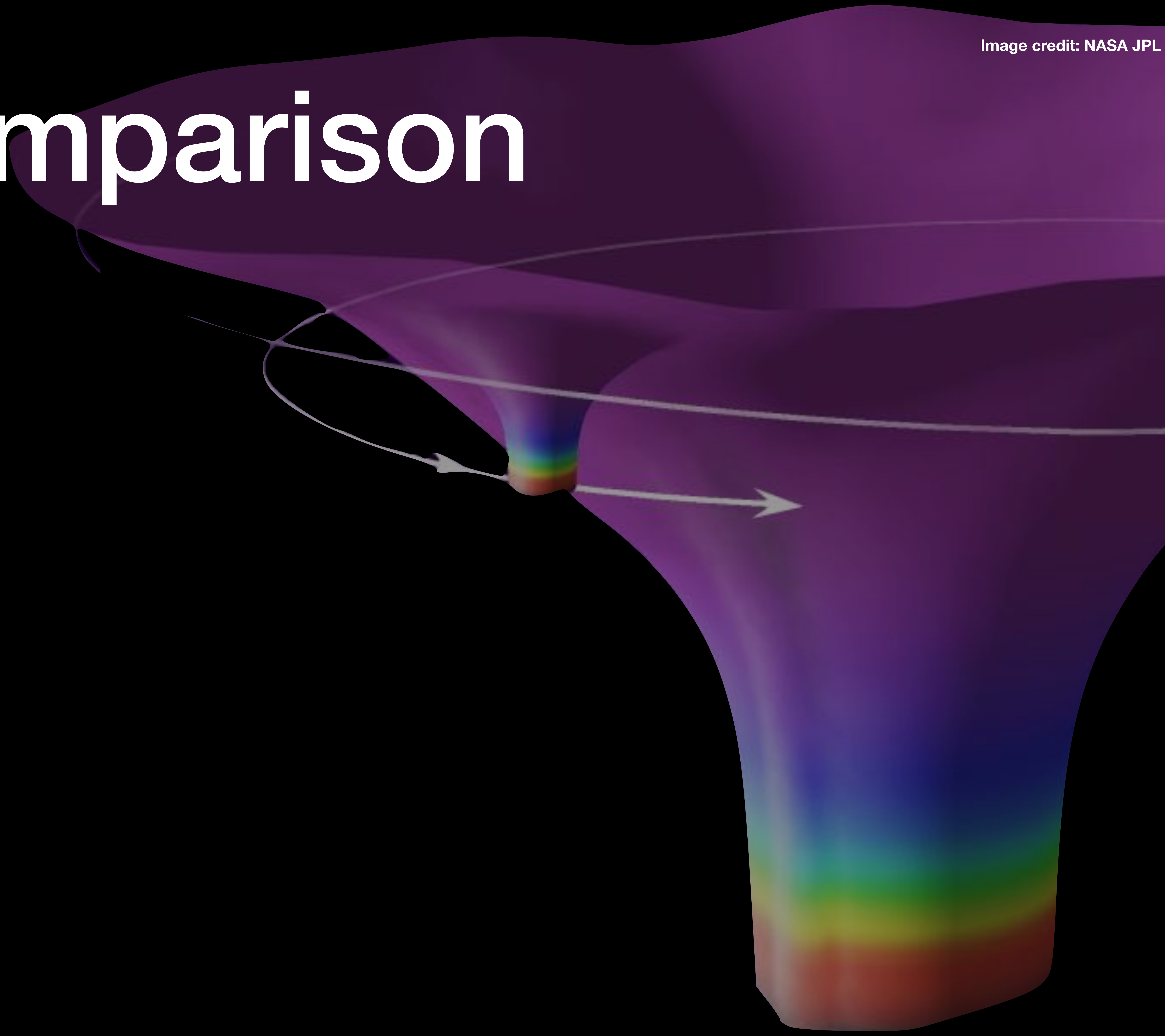
PN comparison

Image credit: NASA JPL



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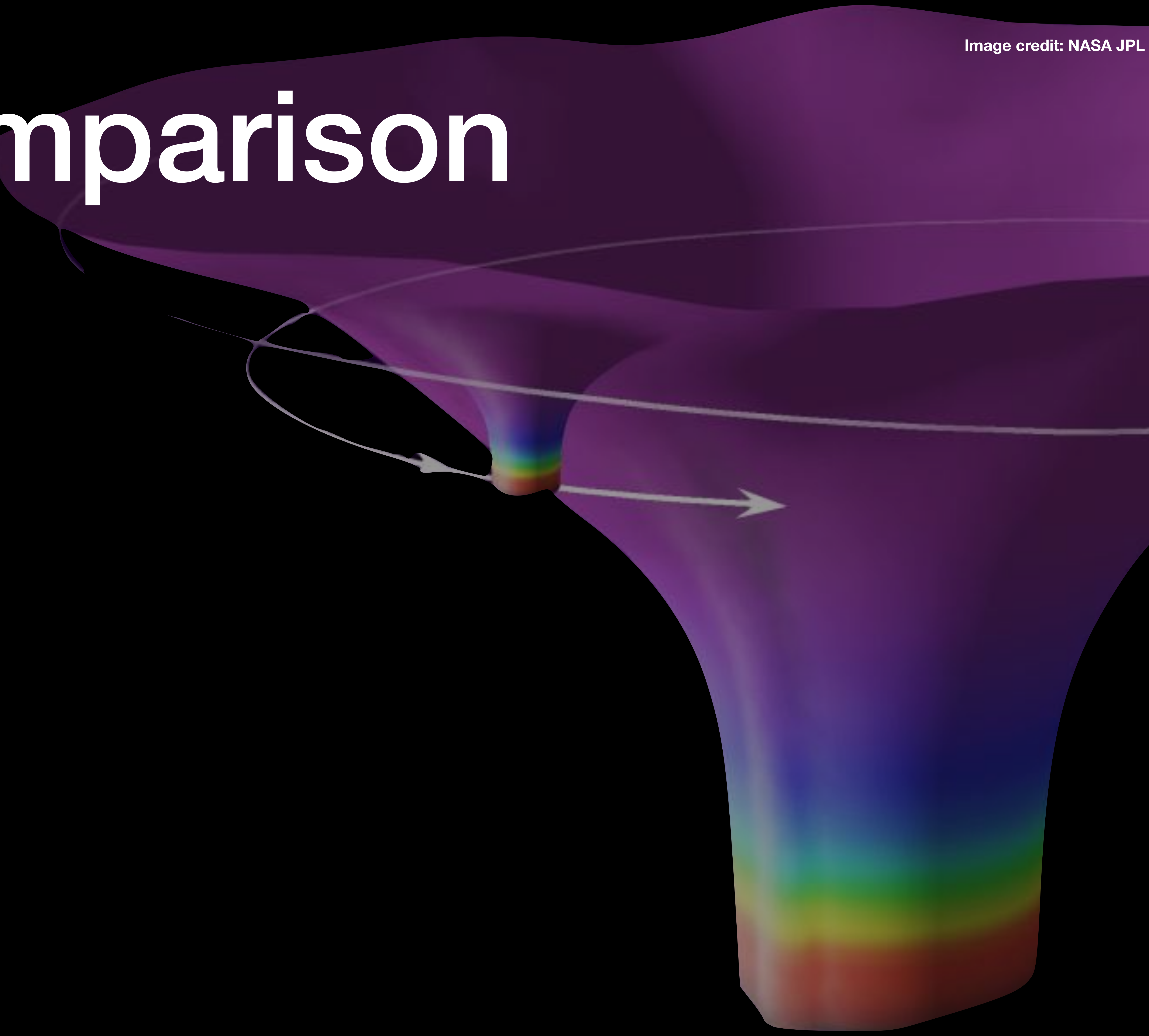
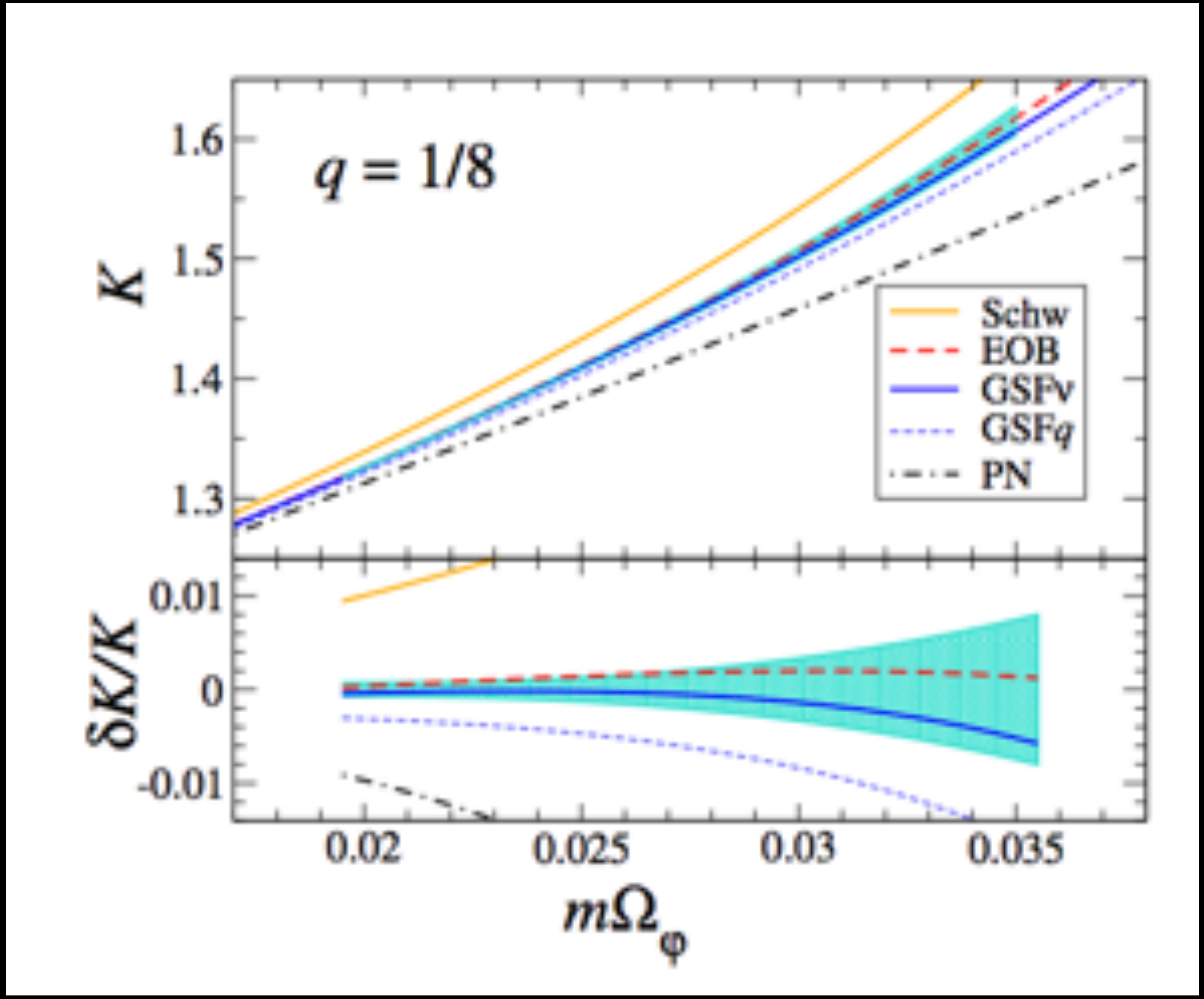
PN comparison

Image credit: NASA JPL



PN, GSF, NR comparison with symmetric mass: periastron advance in black hole binaries



A.LeTiec et al., PRL107, 141101 (2011)

PN comparison

Image credit: NASA JPL

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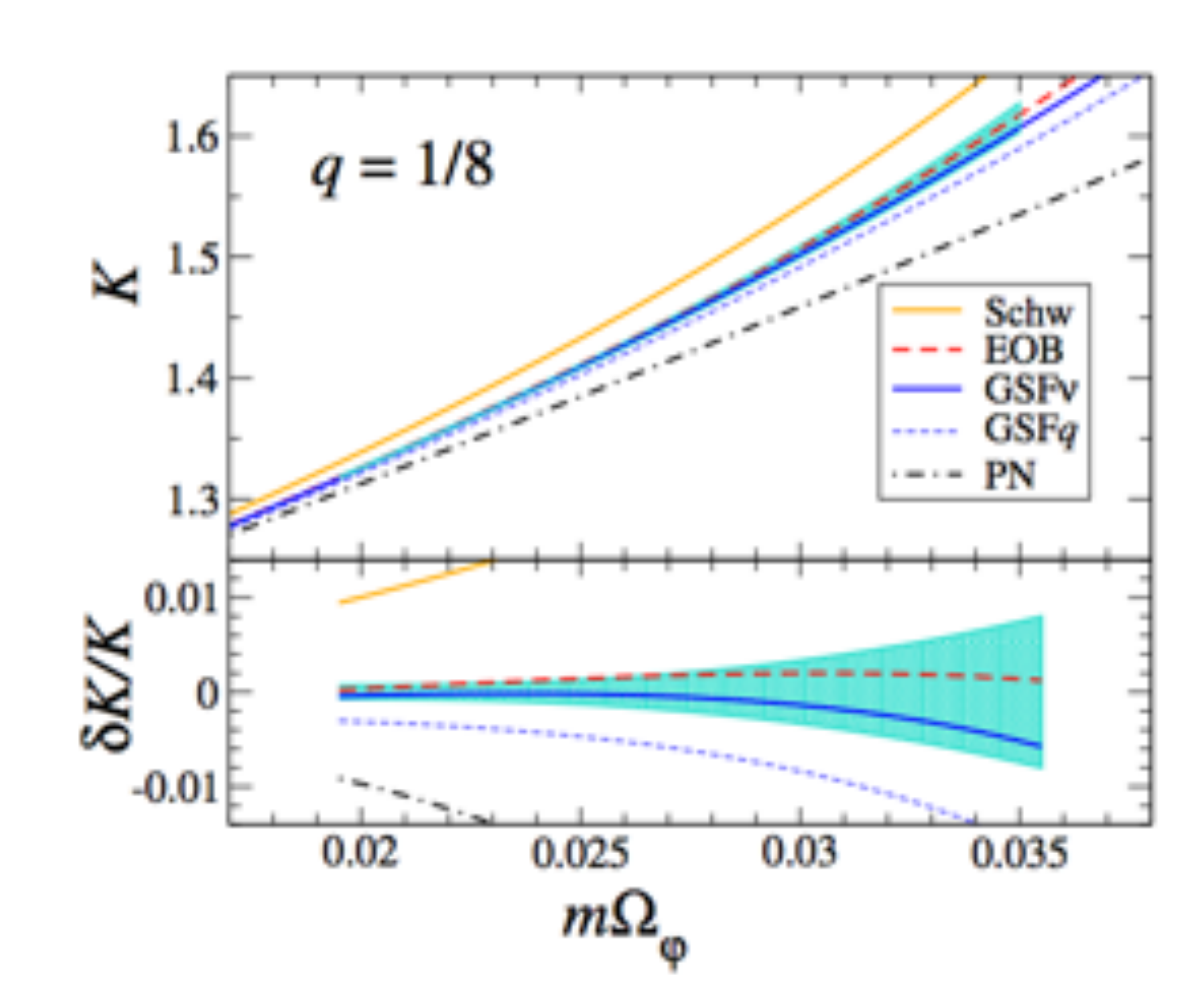
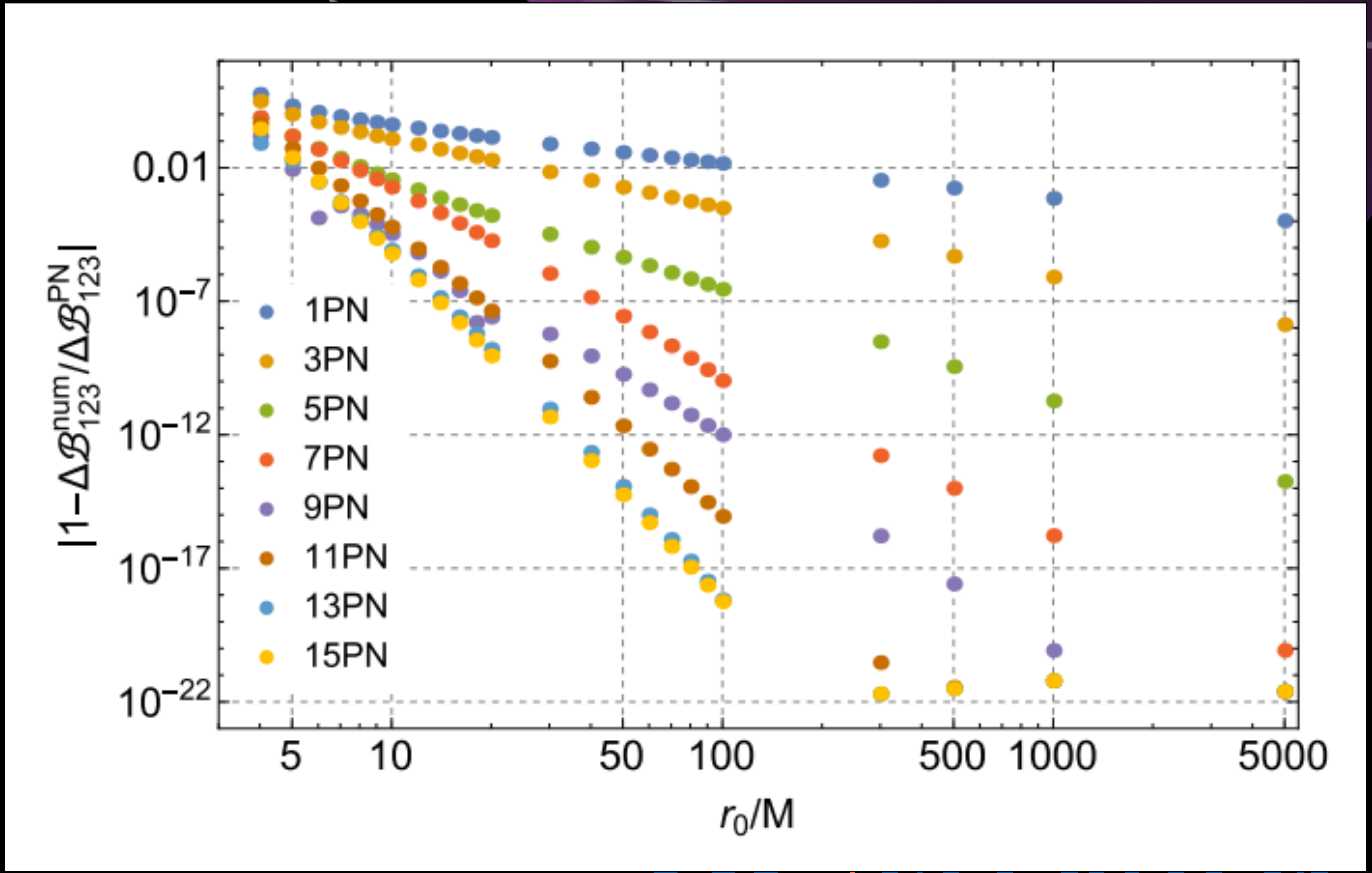


A.LeTiec et al., PRL107, 141101 (2011)

MST: Analytical method extended to 30PN: Octupolar Invariants



P.Nolan, C.Kavanagh et al., arxiv:1505.04447

Waveform missing ingredients

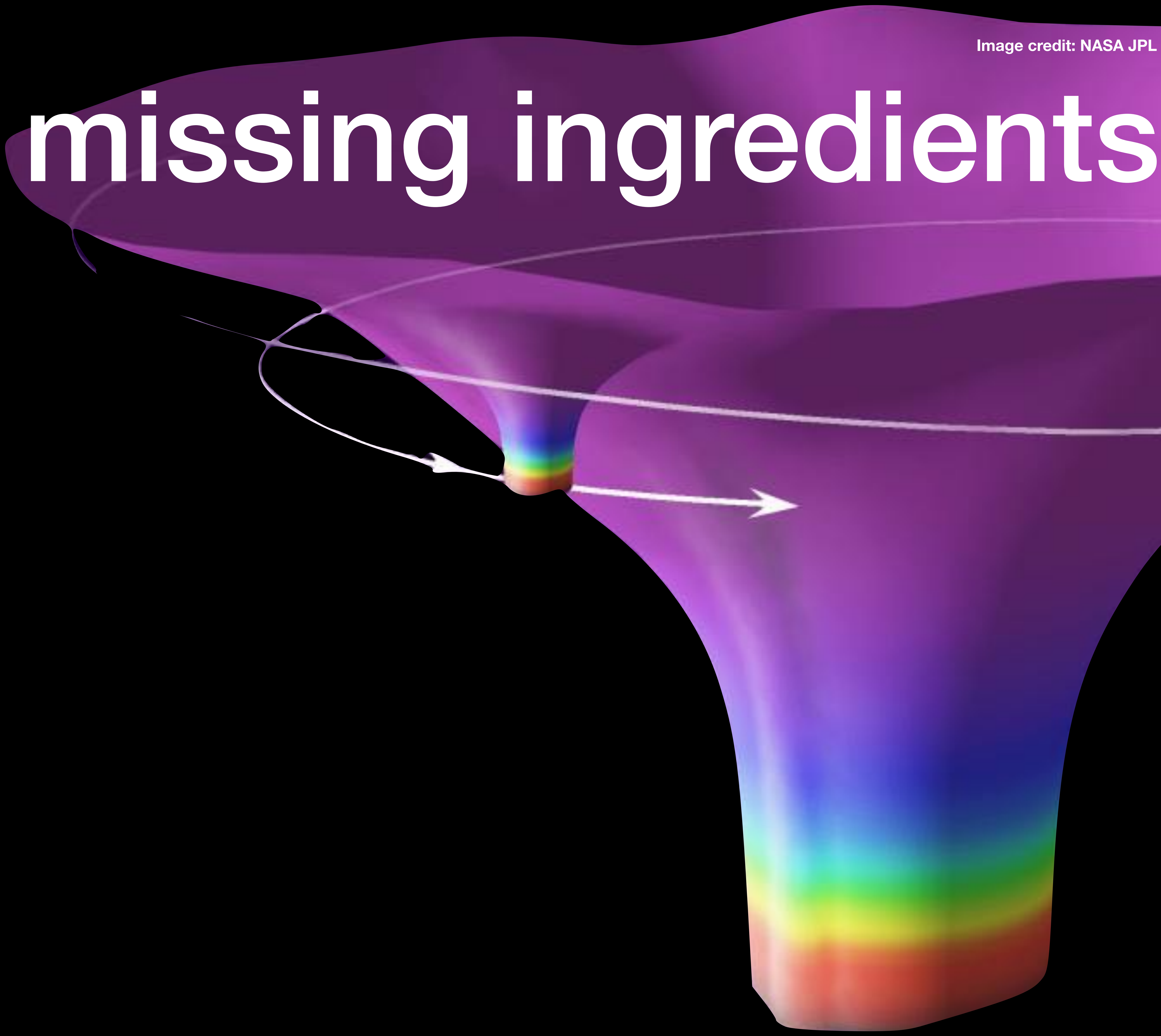


Image credit: NASA JPL

Waveform missing ingredients

- Second order

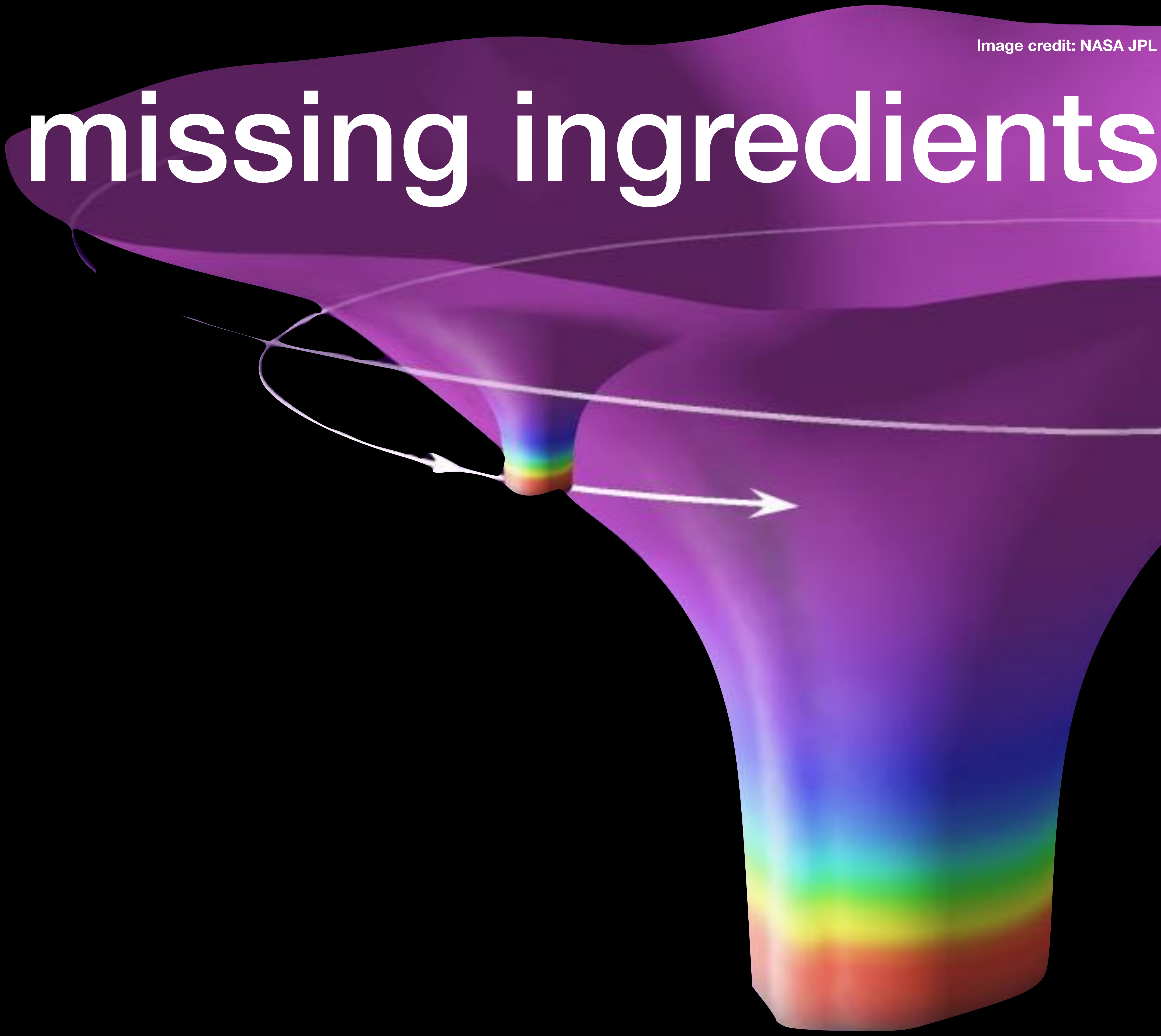


Image credit: NASA JPL

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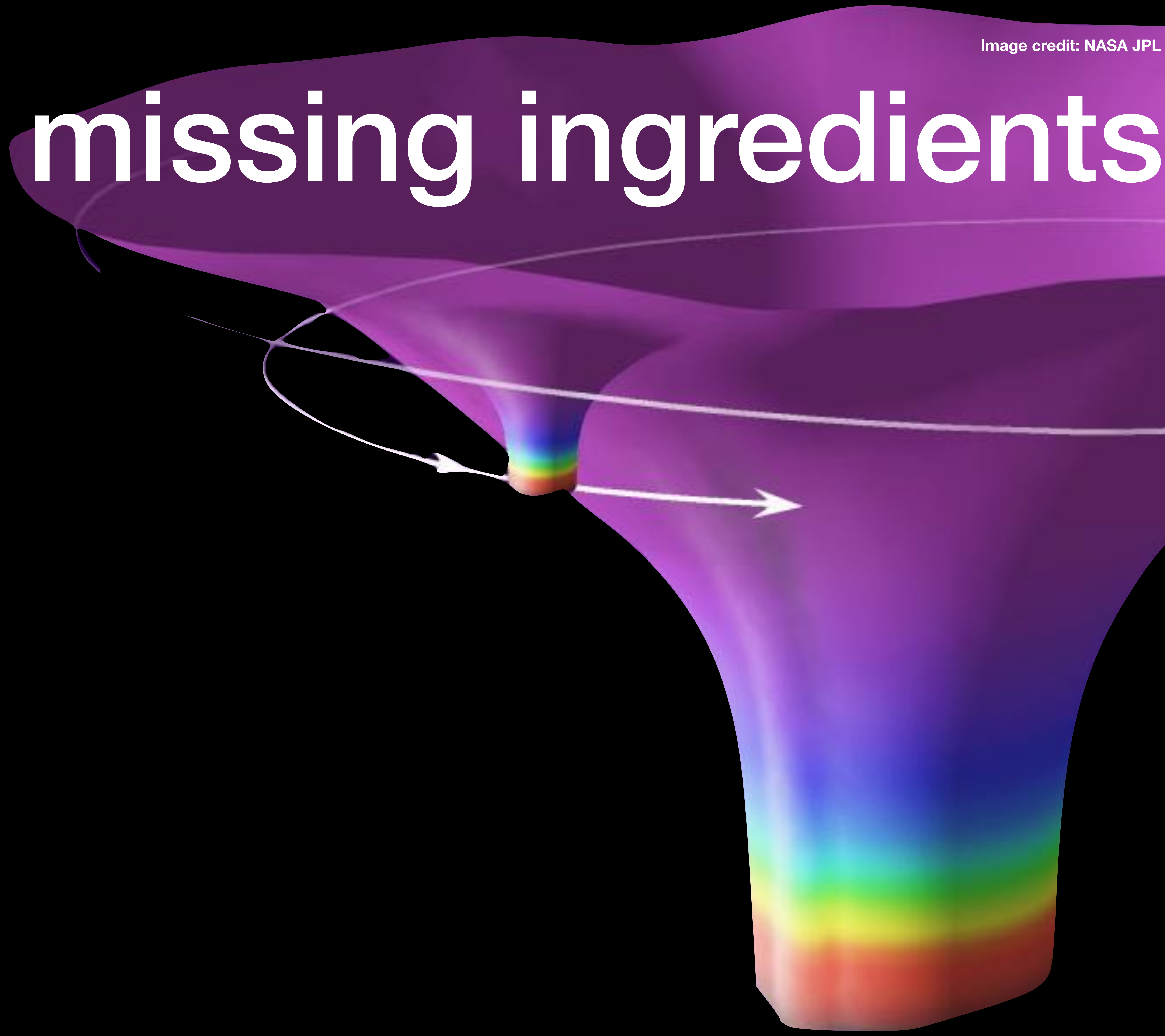

Image credit: NASA JPL

- Second order
    - Scalar case third order (C.Galley, 2010)

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Waveform missing ingredients

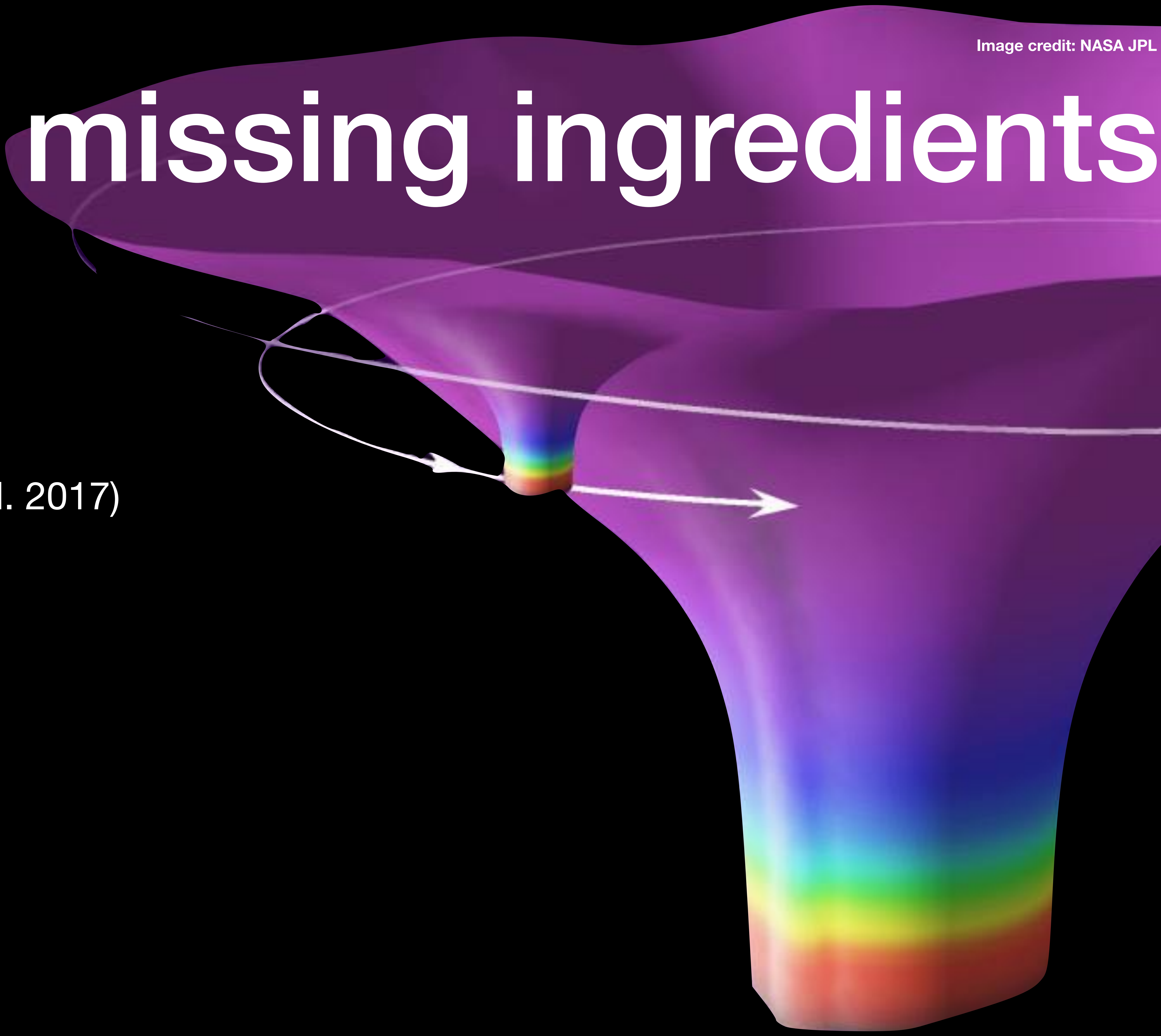

Image credit: NASA JPL

- Second order
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  - Electromagnetic second order (J.Moxon et al. 2017)

Waveform missing ingredients

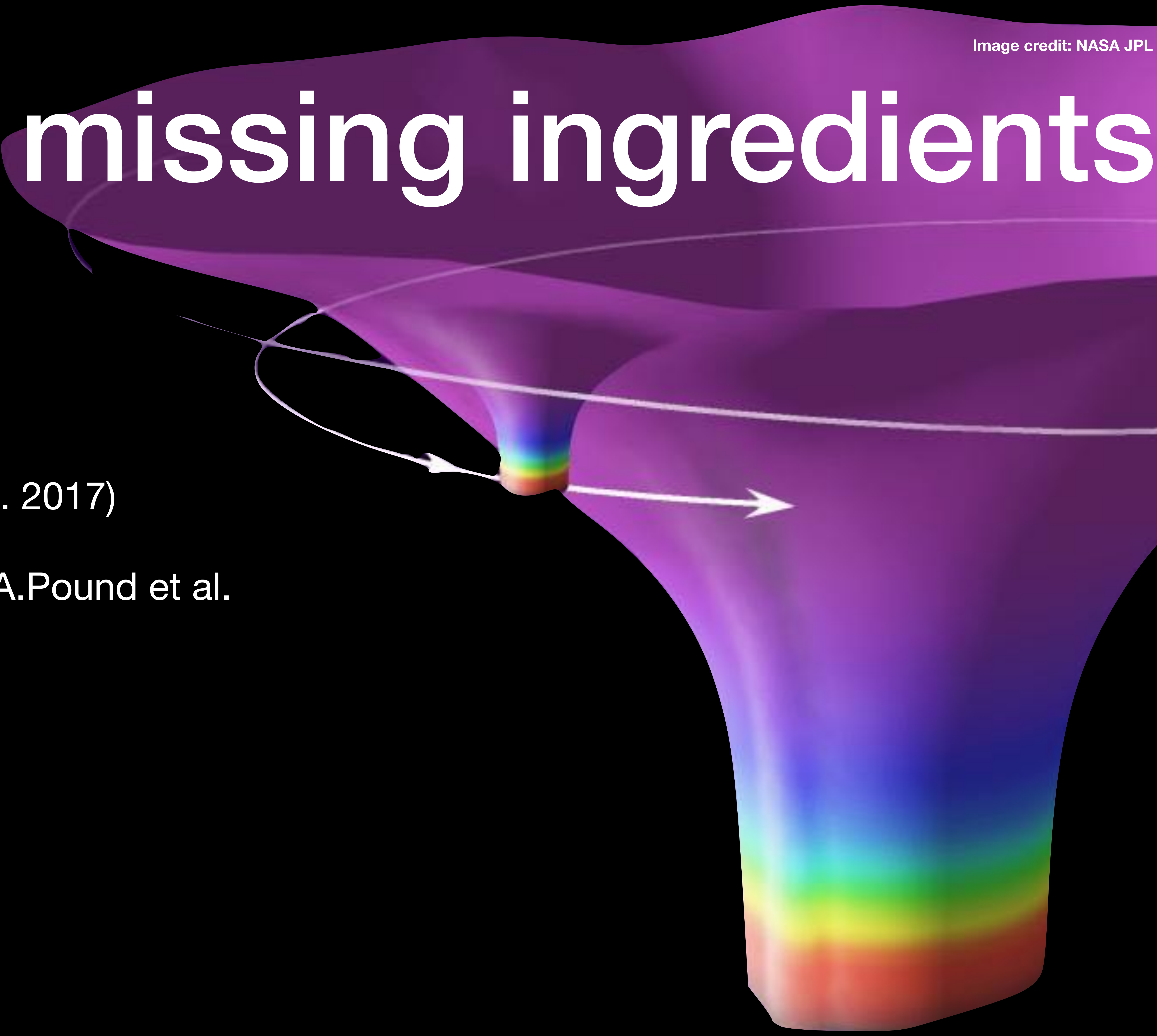

Image credit: NASA JPL

- Second order
    - Scalar case third order (C.Galley, 2010)
    - Electromagnetic second order (J.Moxon et al. 2017)
    - Ongoing work in gravitational second order (A.Pound et al. 2012-2017)

Image credit: NASA JPL

Waveform missing ingredients

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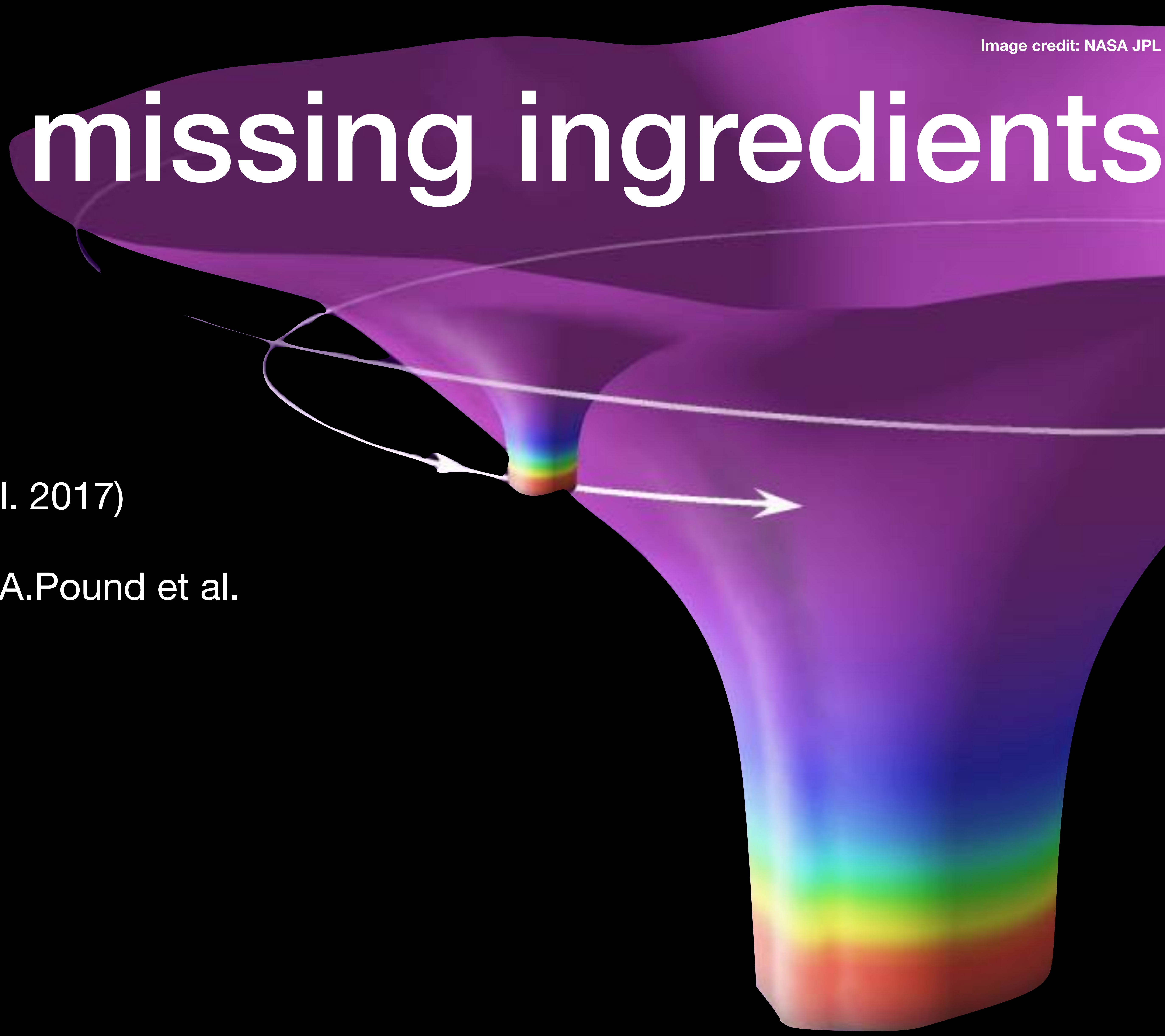
- Evolving the orbit



Waveform missing ingredients

Image credit: NASA JPL

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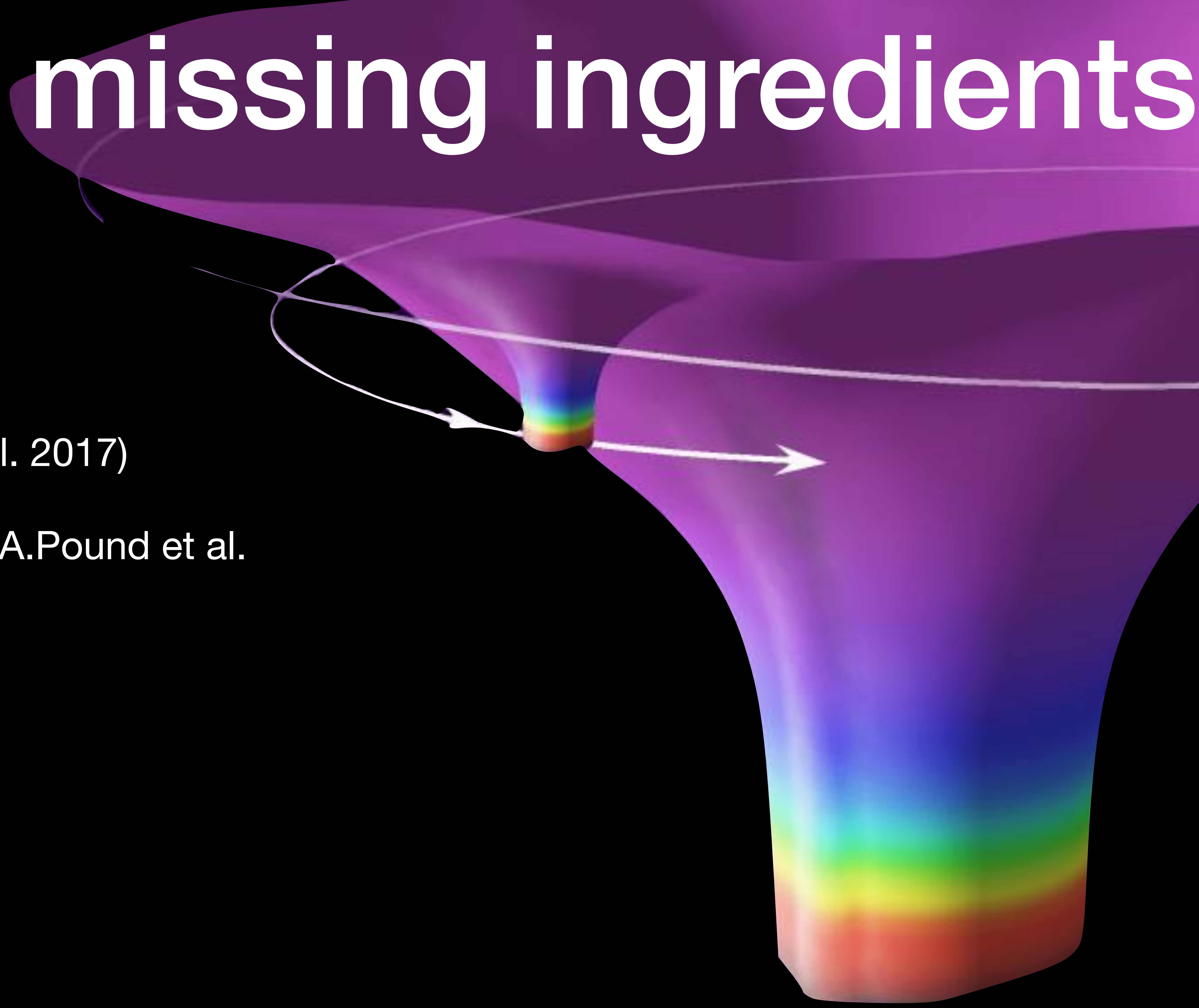

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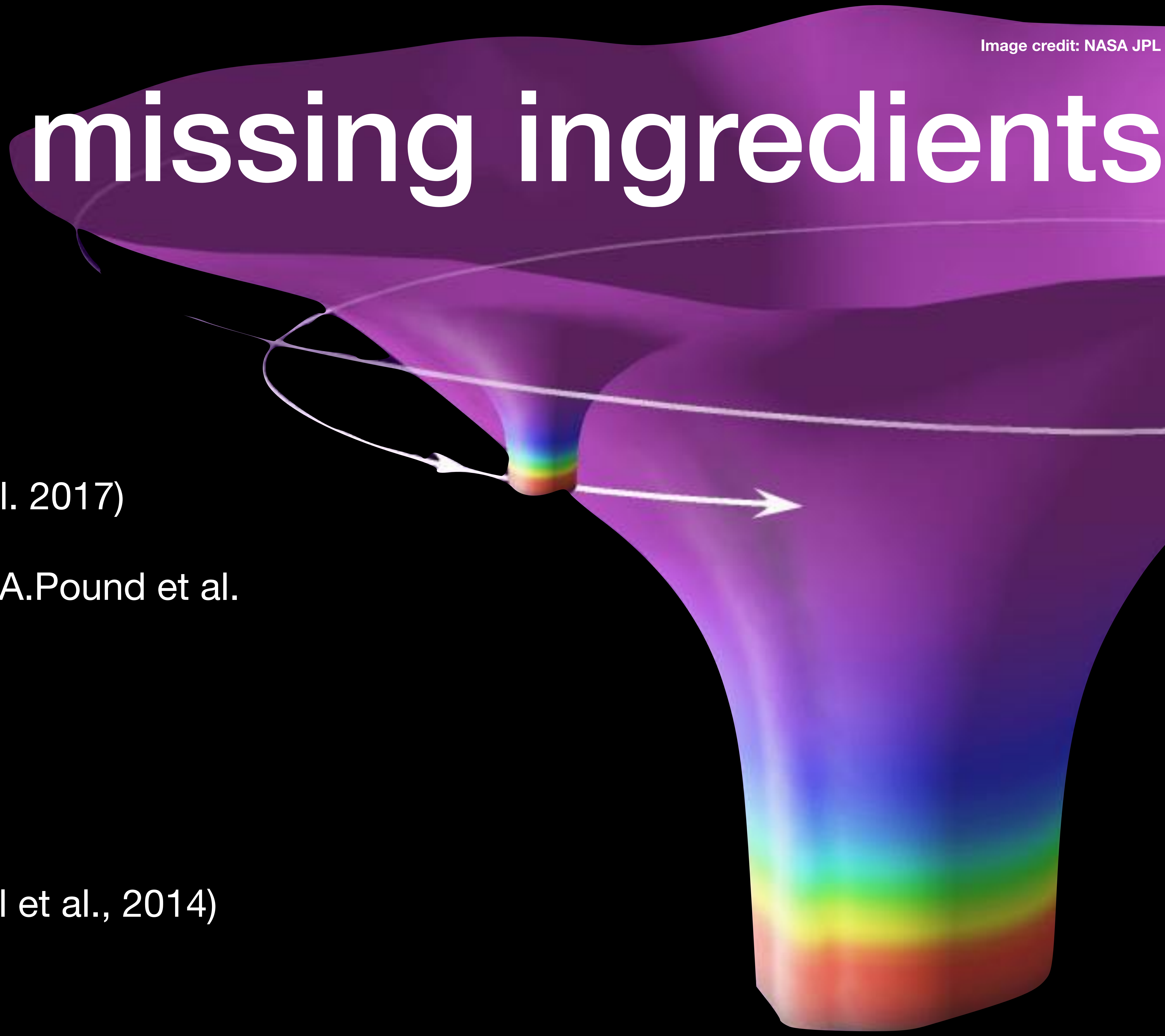
Waveform missing ingredients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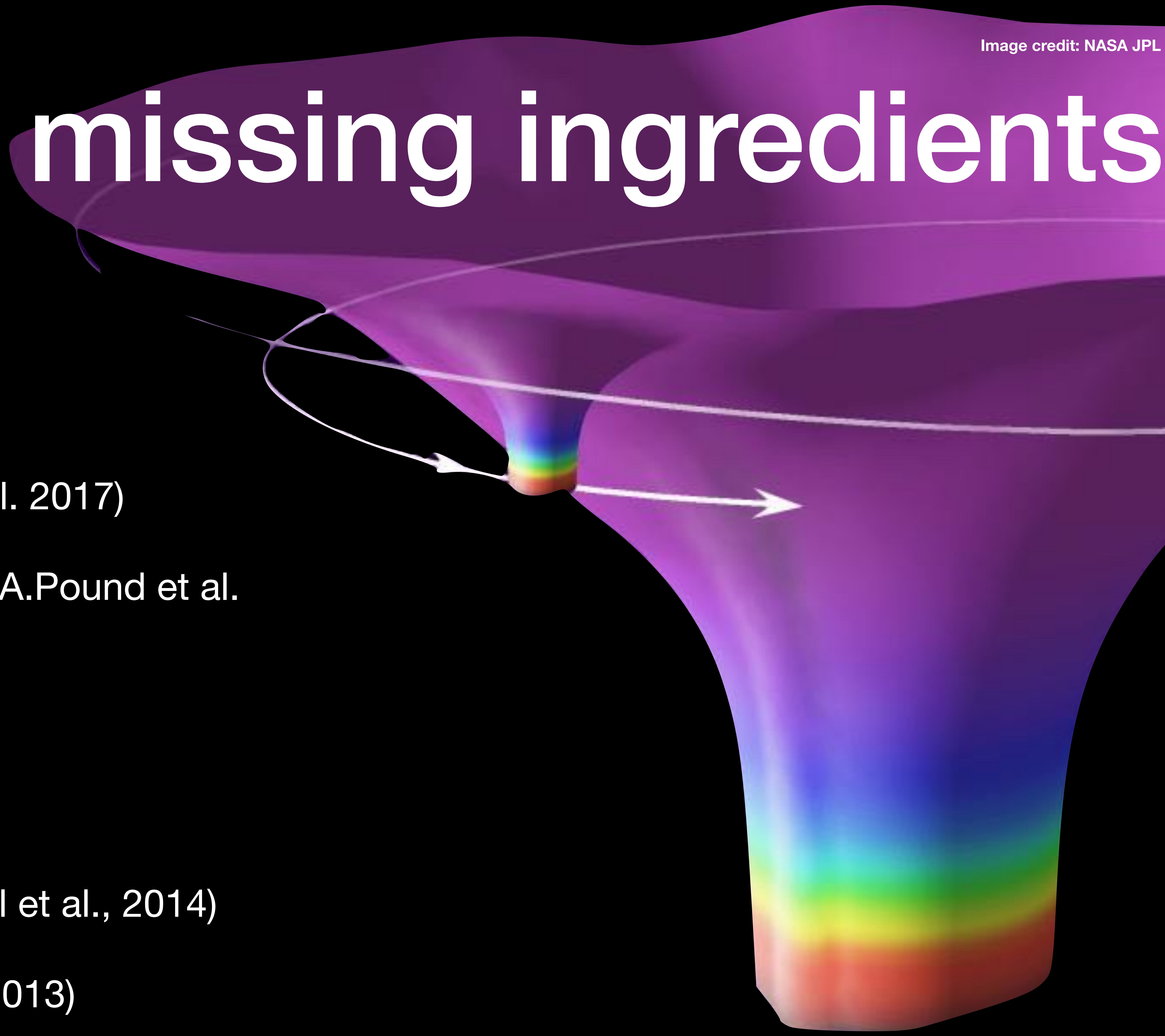

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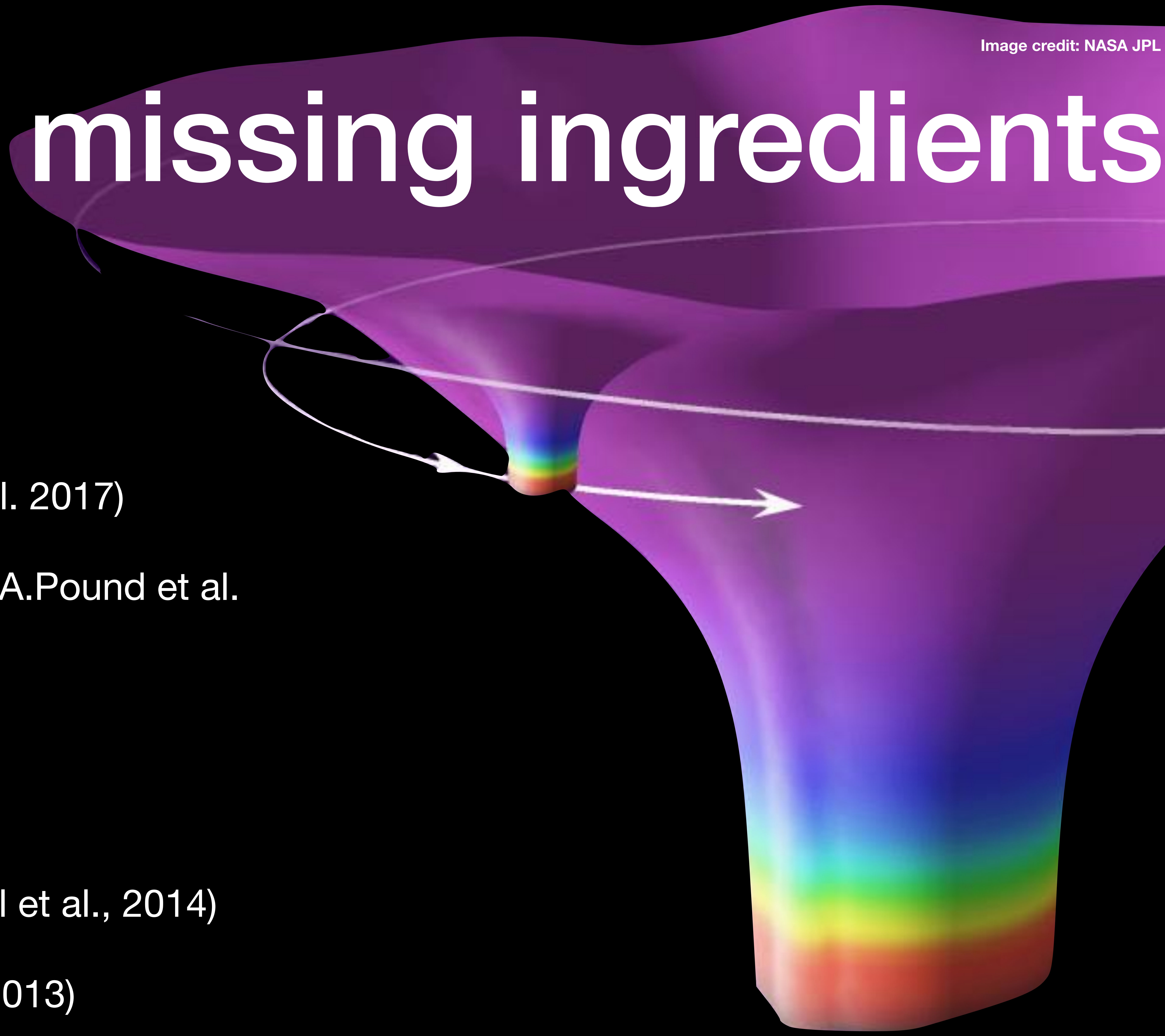
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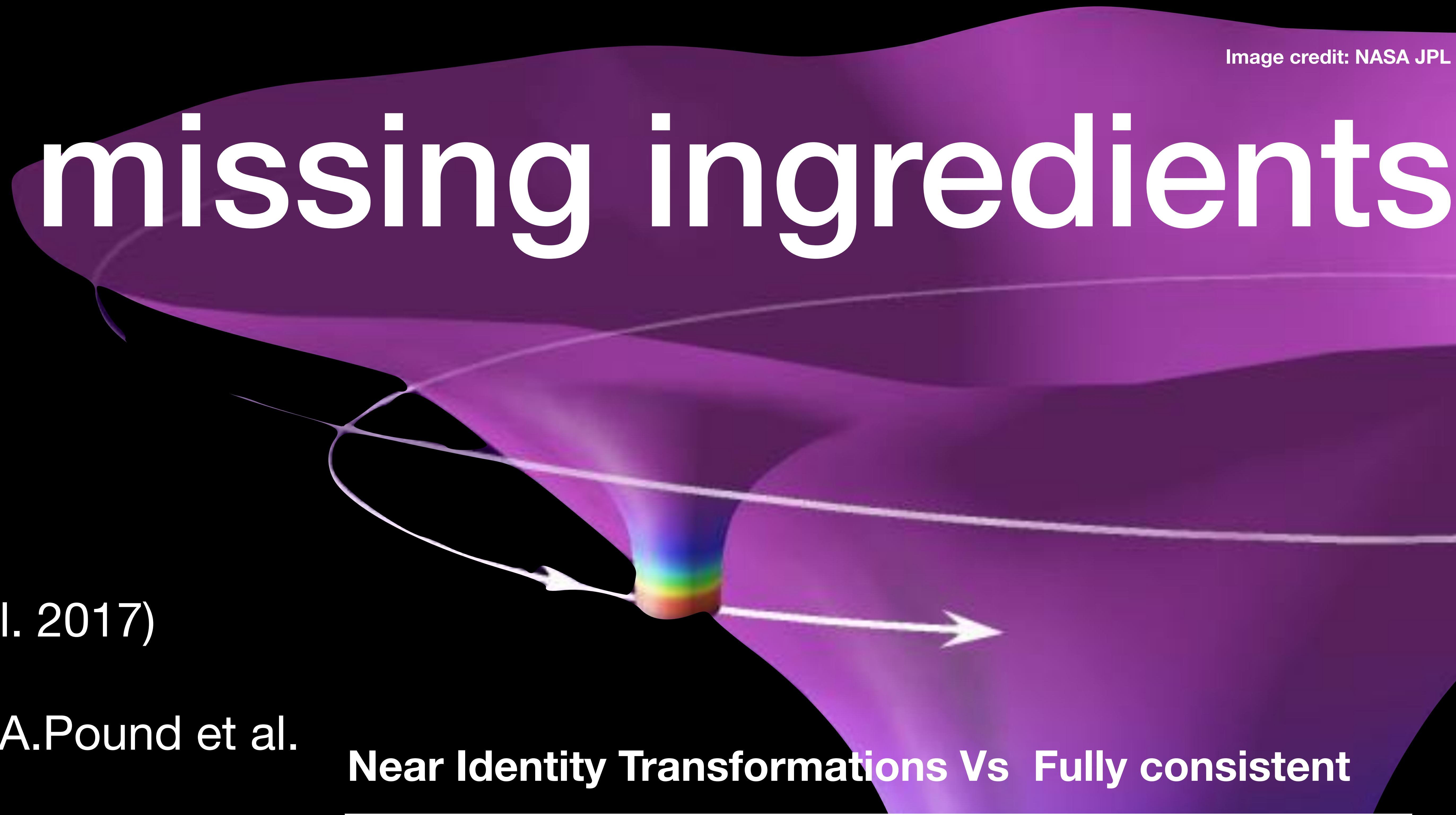
Waveform missing ingredients



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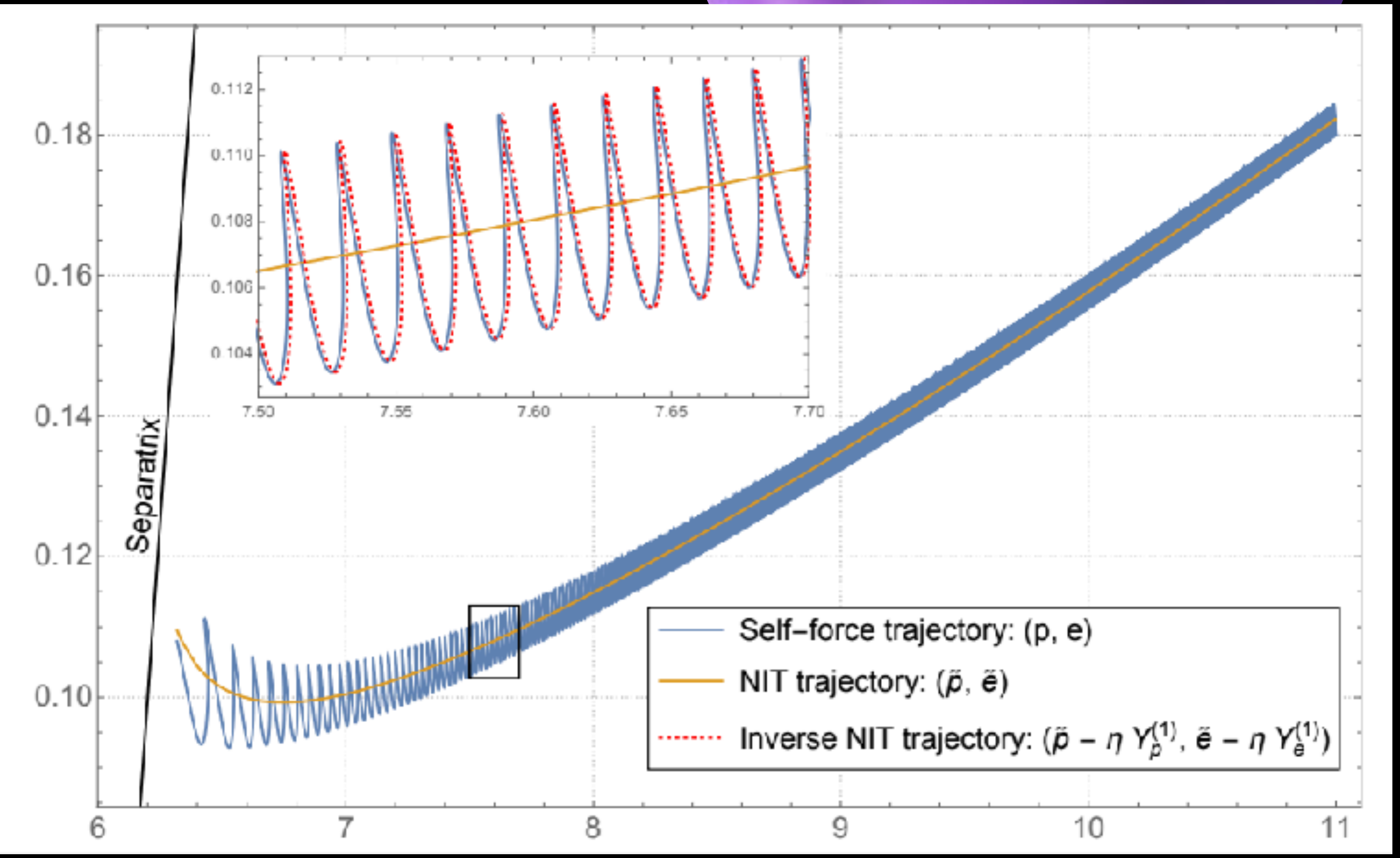
Near Identity Transformations Vs Fully consistent



Ongoing research: Waveforms

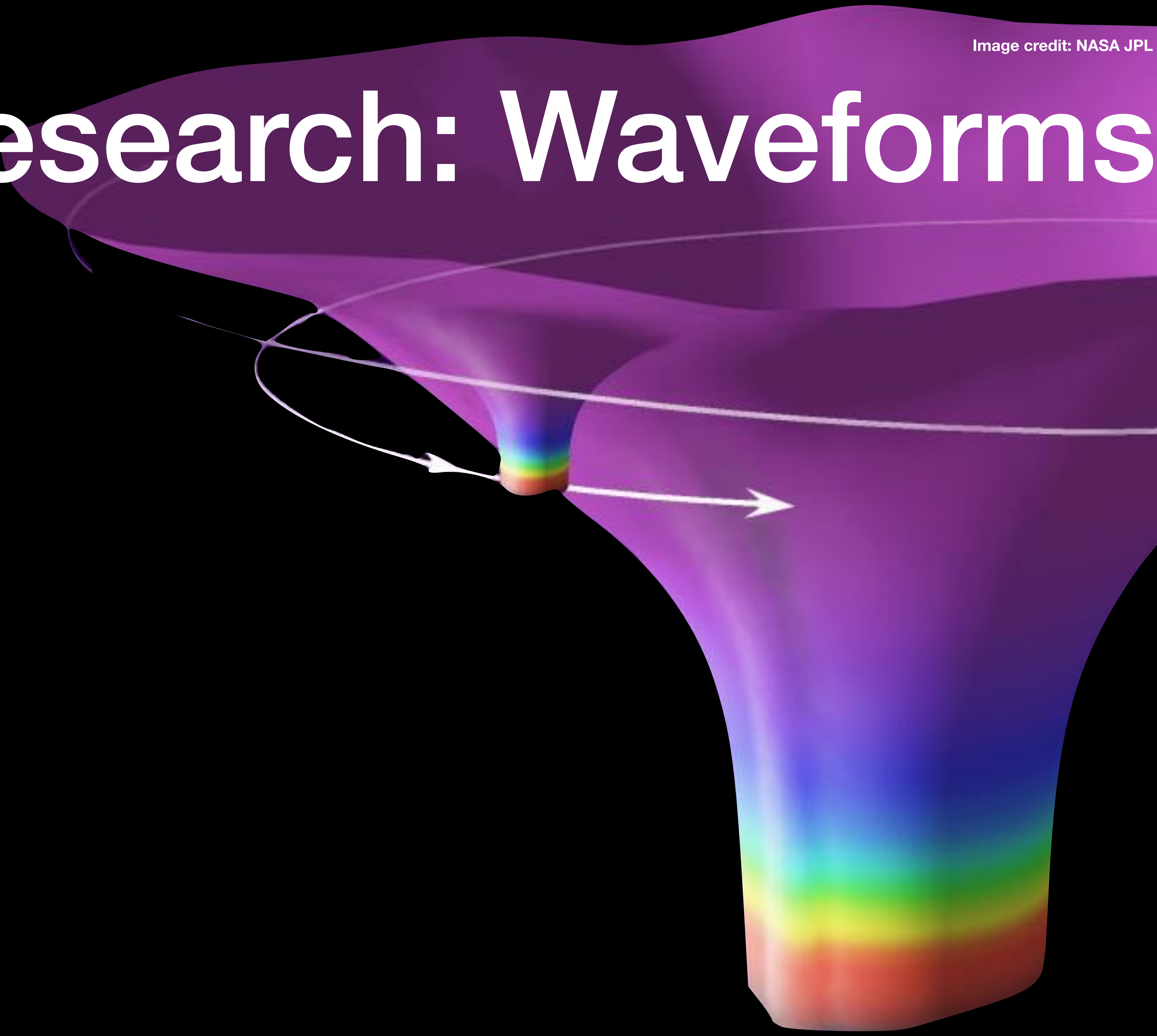
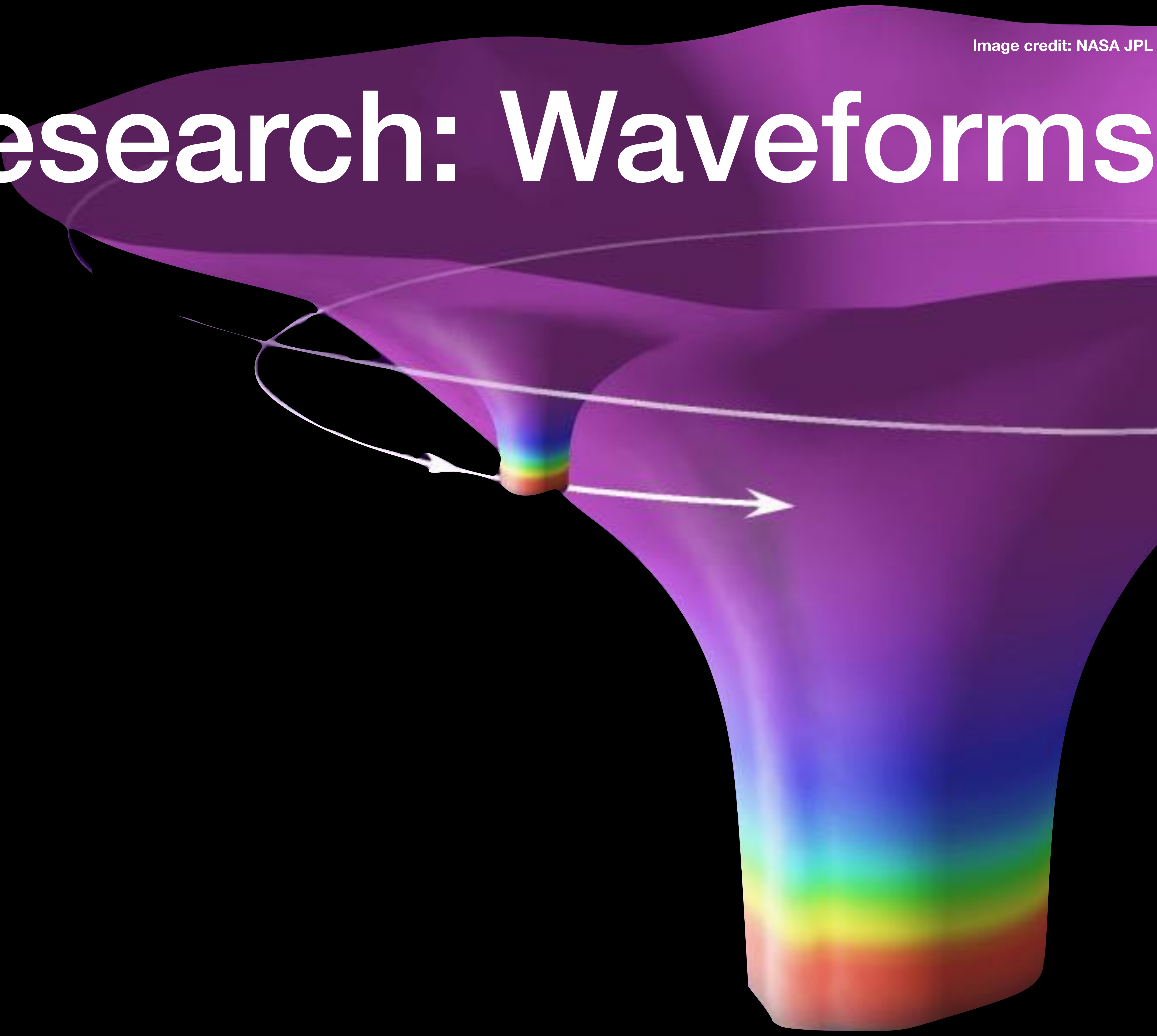


Image credit: NASA JPL

Ongoing research: Waveforms

Image credit: NASA JPL

- First order



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Ongoing research: Waveforms

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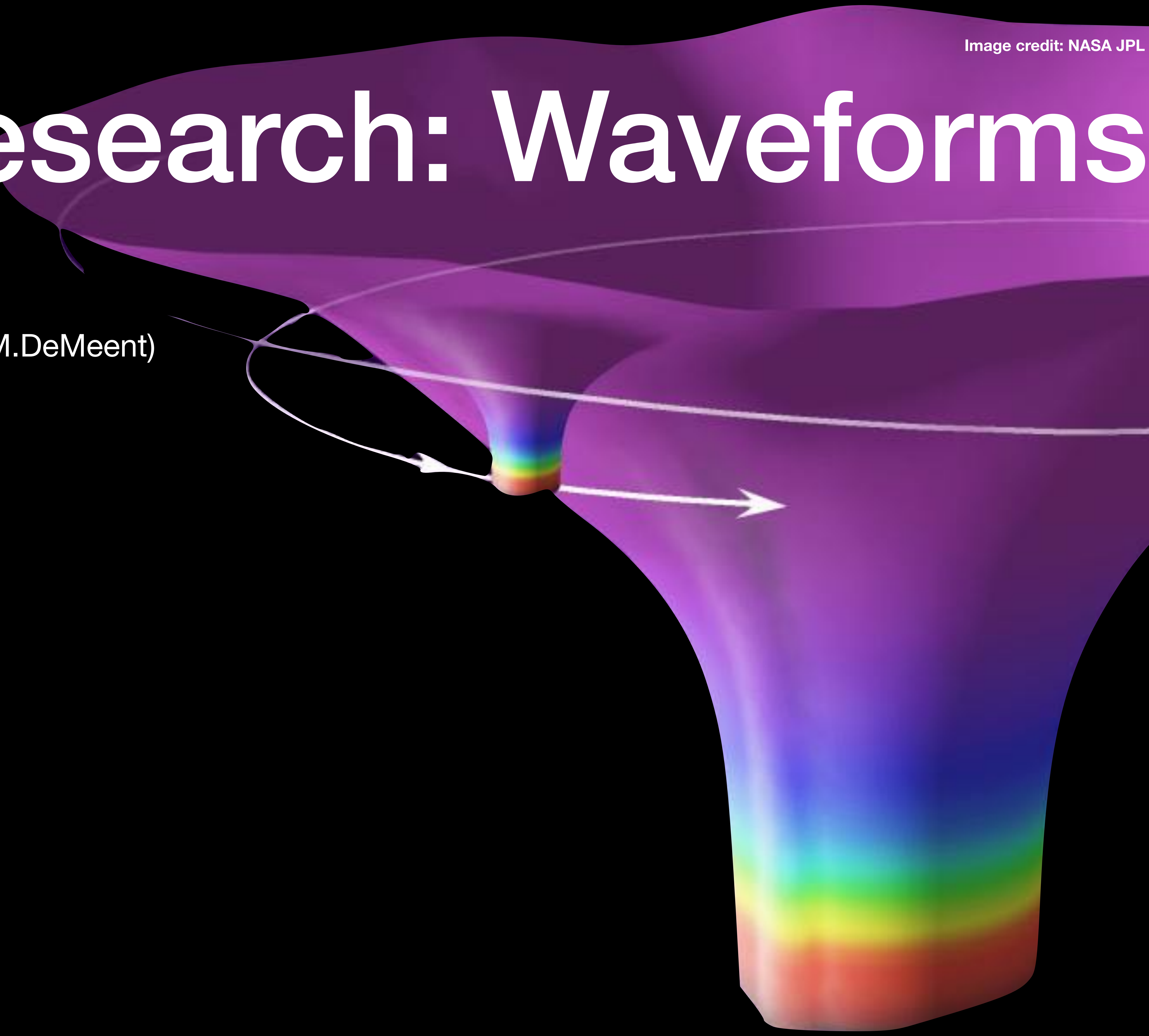

- First order
  - More accurate inclined eccentric Kerr models needed (M.DeMeent)

Ongoing research: Waveforms

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  - More accurate inclined eccentric Kerr models needed (M.DeMeent)
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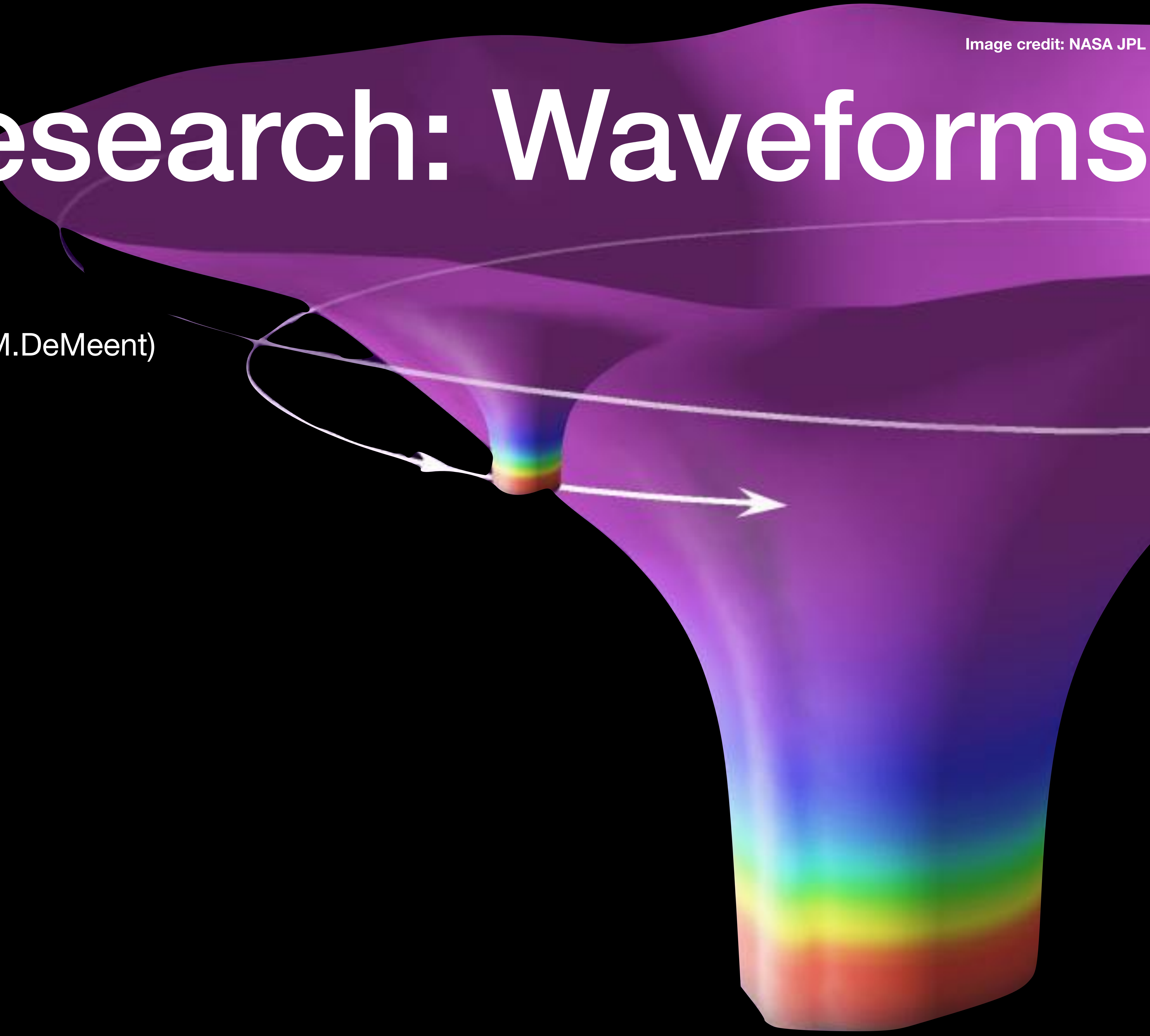

Image credit: NASA JPL

Ongoing research: Waveforms

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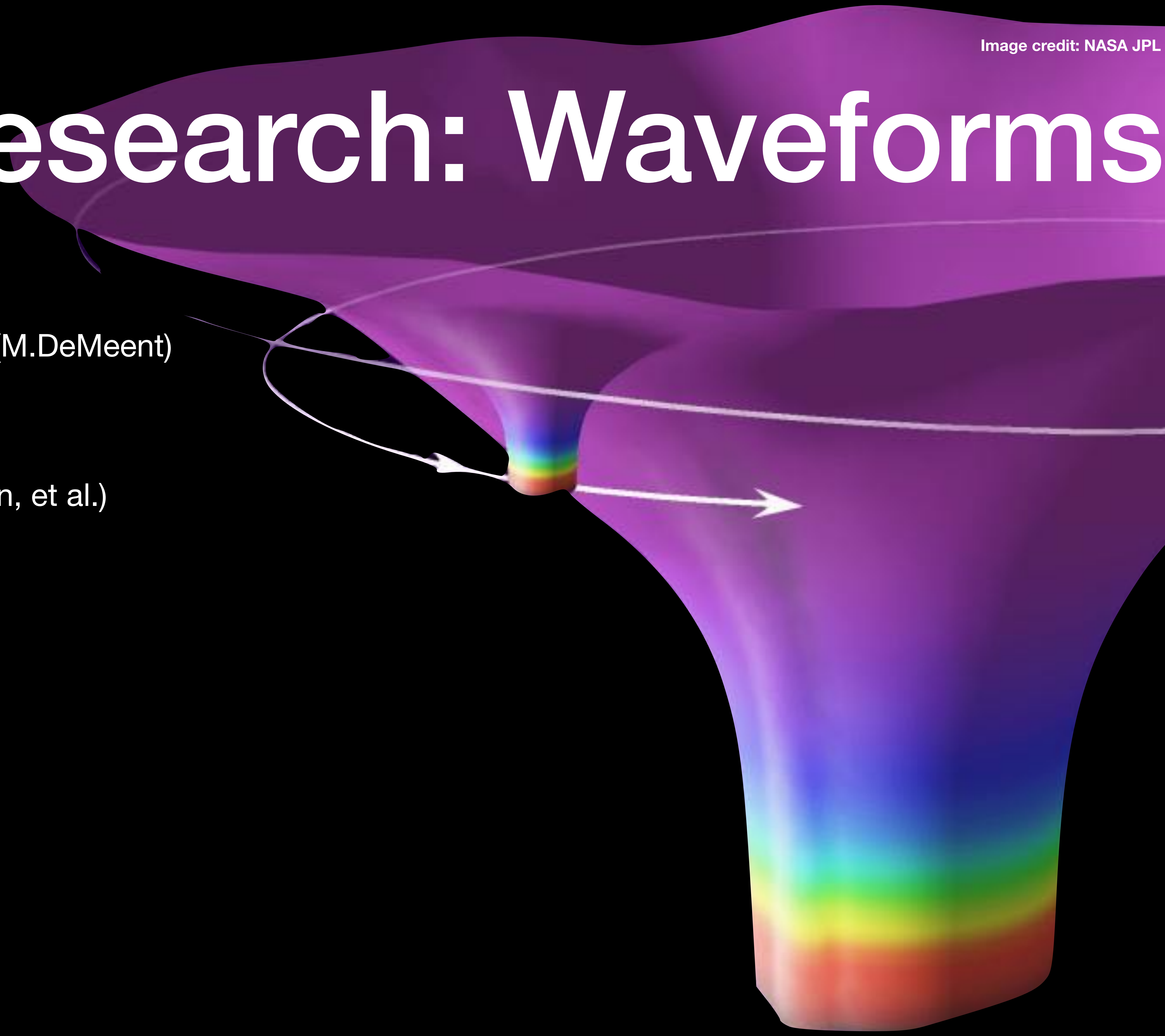

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Image credit: NASA JPL

Ongoing research: Waveforms

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- Second order

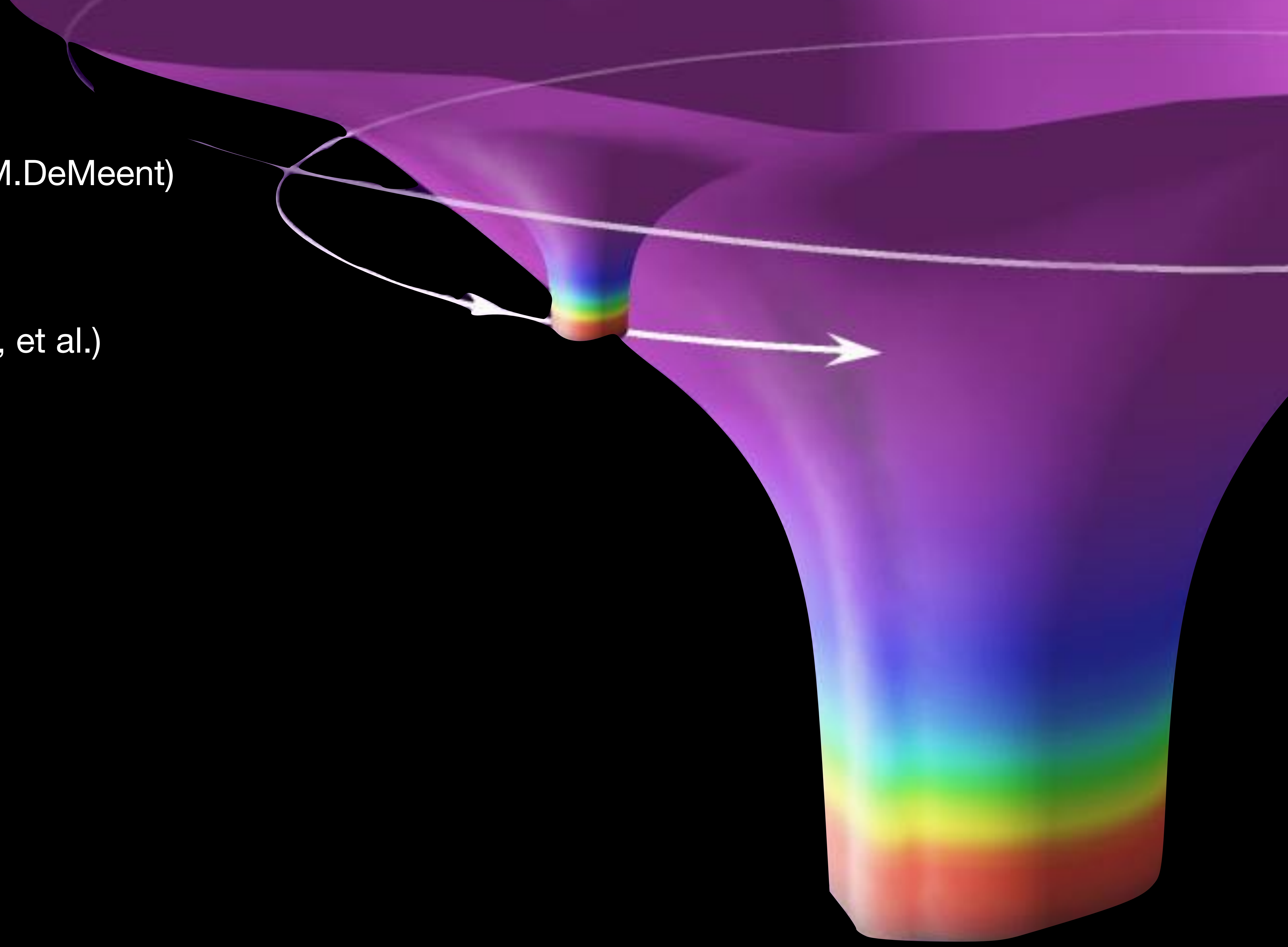

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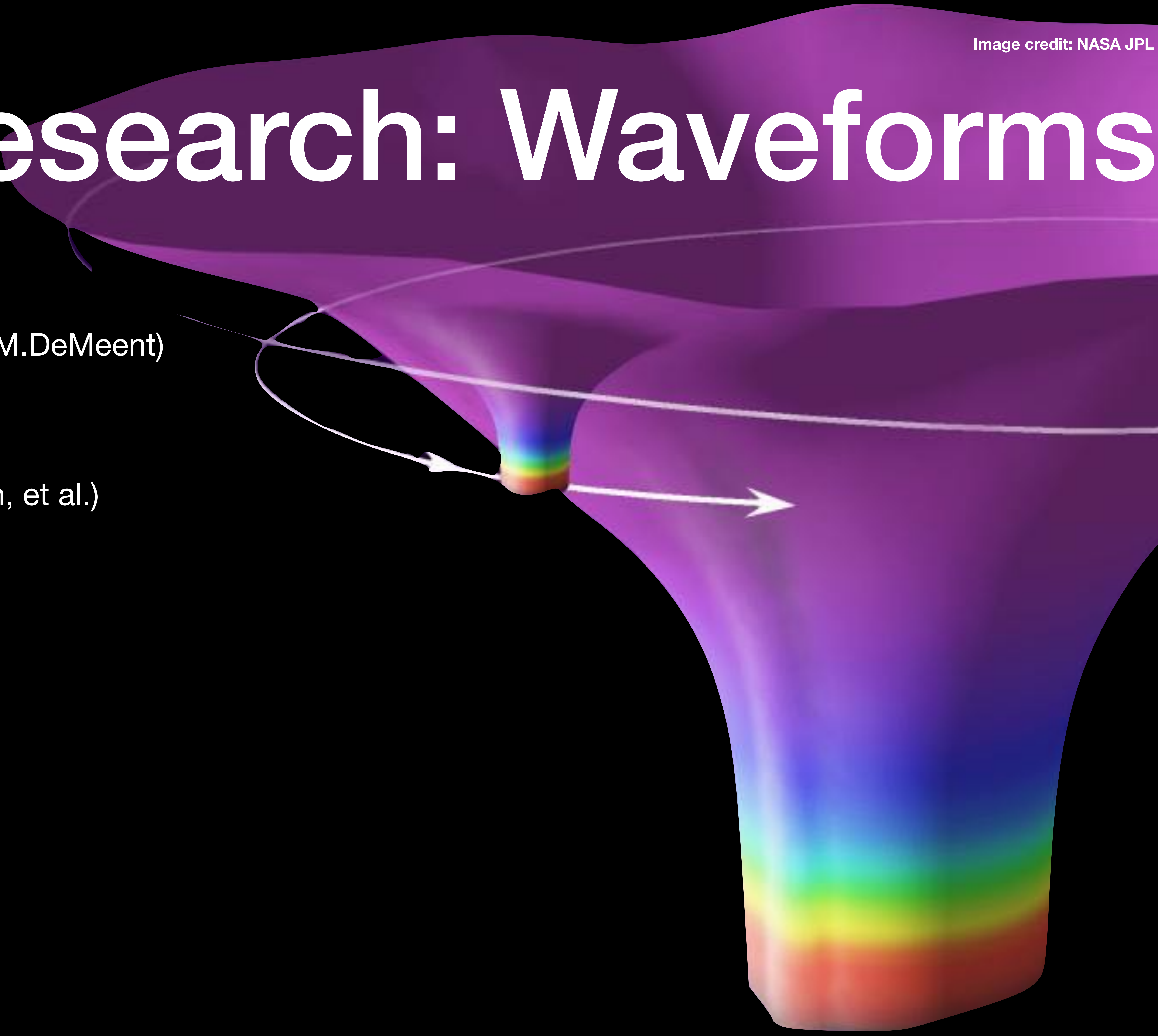
Image credit: NASA JPL

Ongoing research: Waveforms



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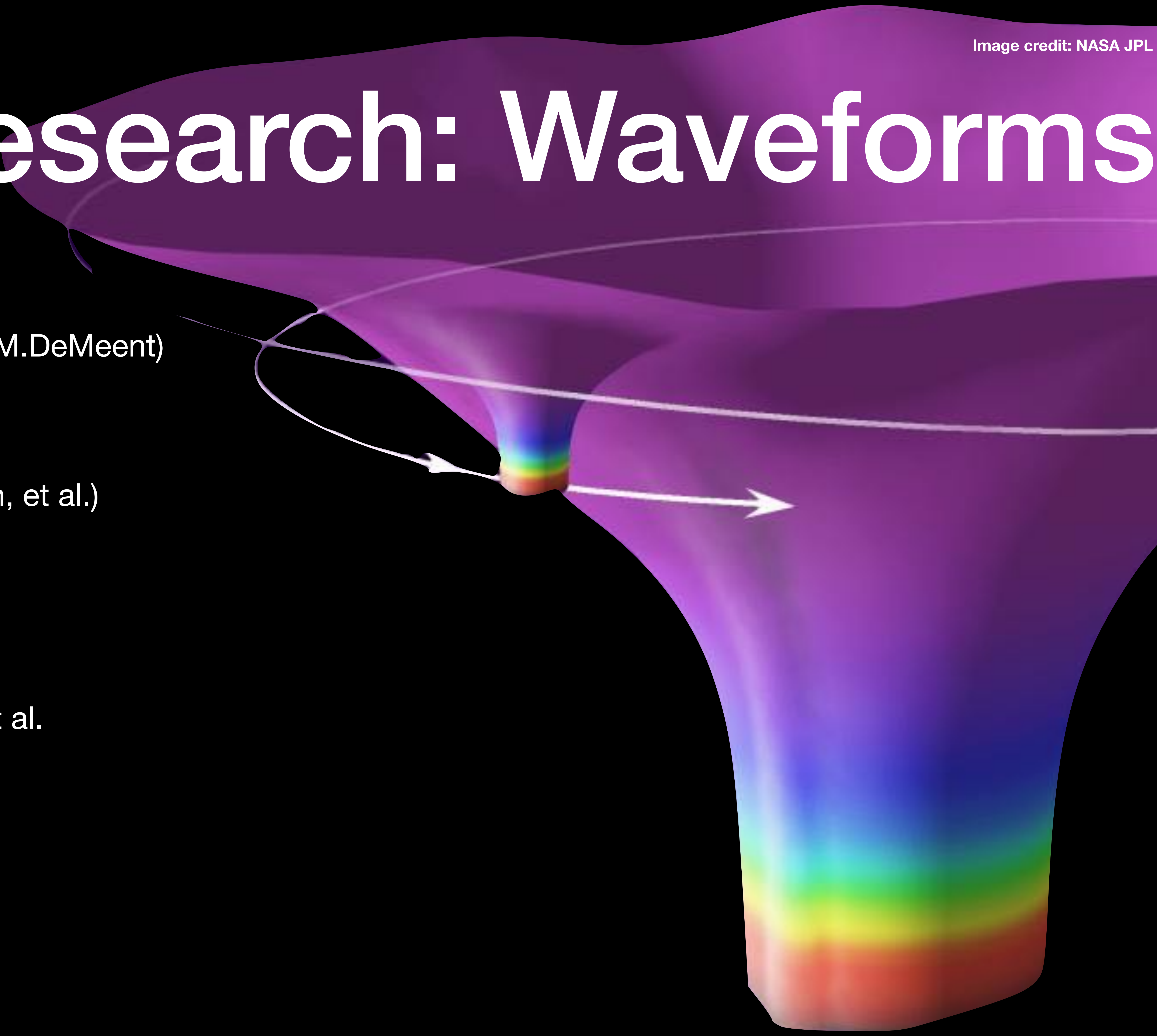
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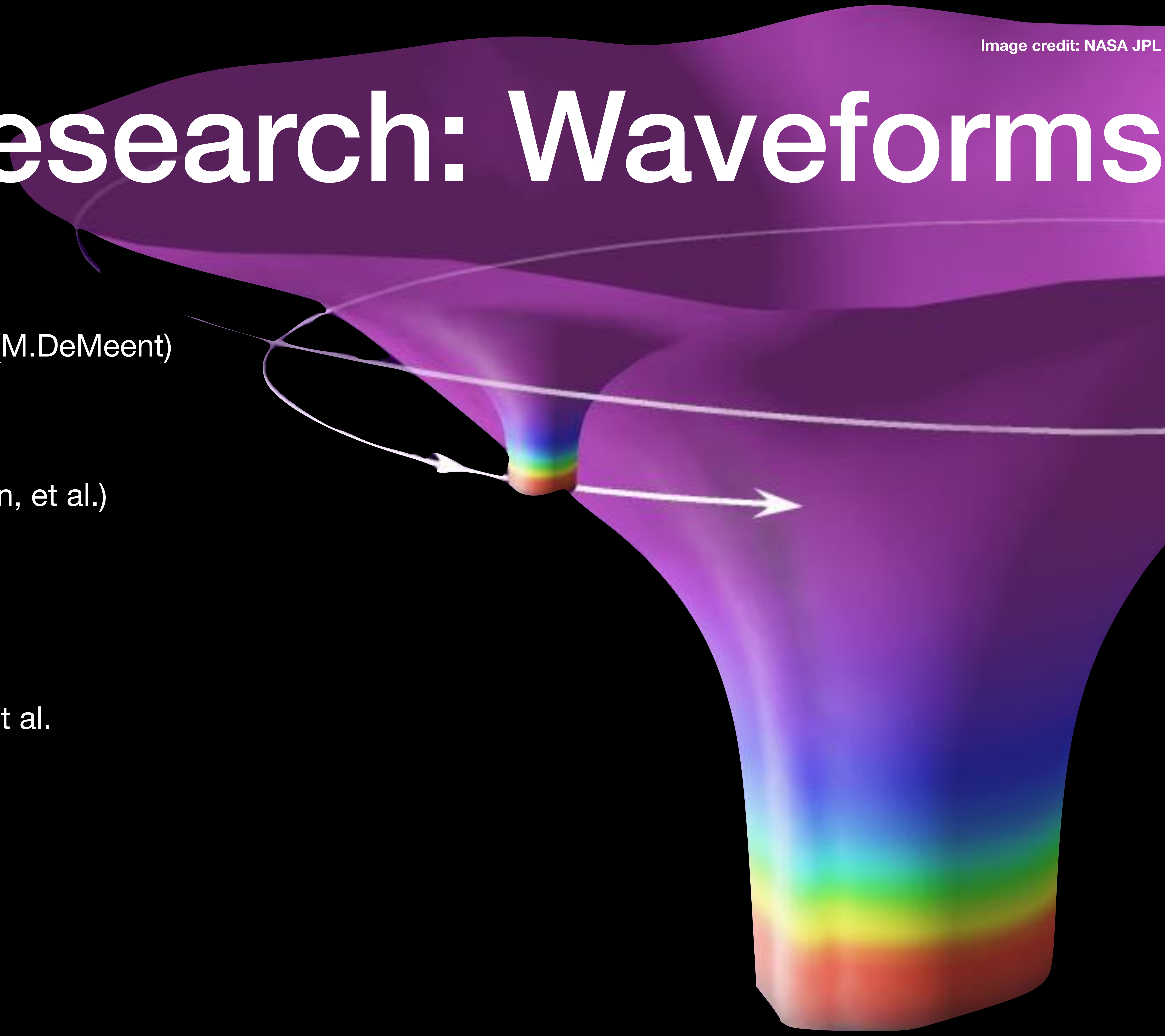

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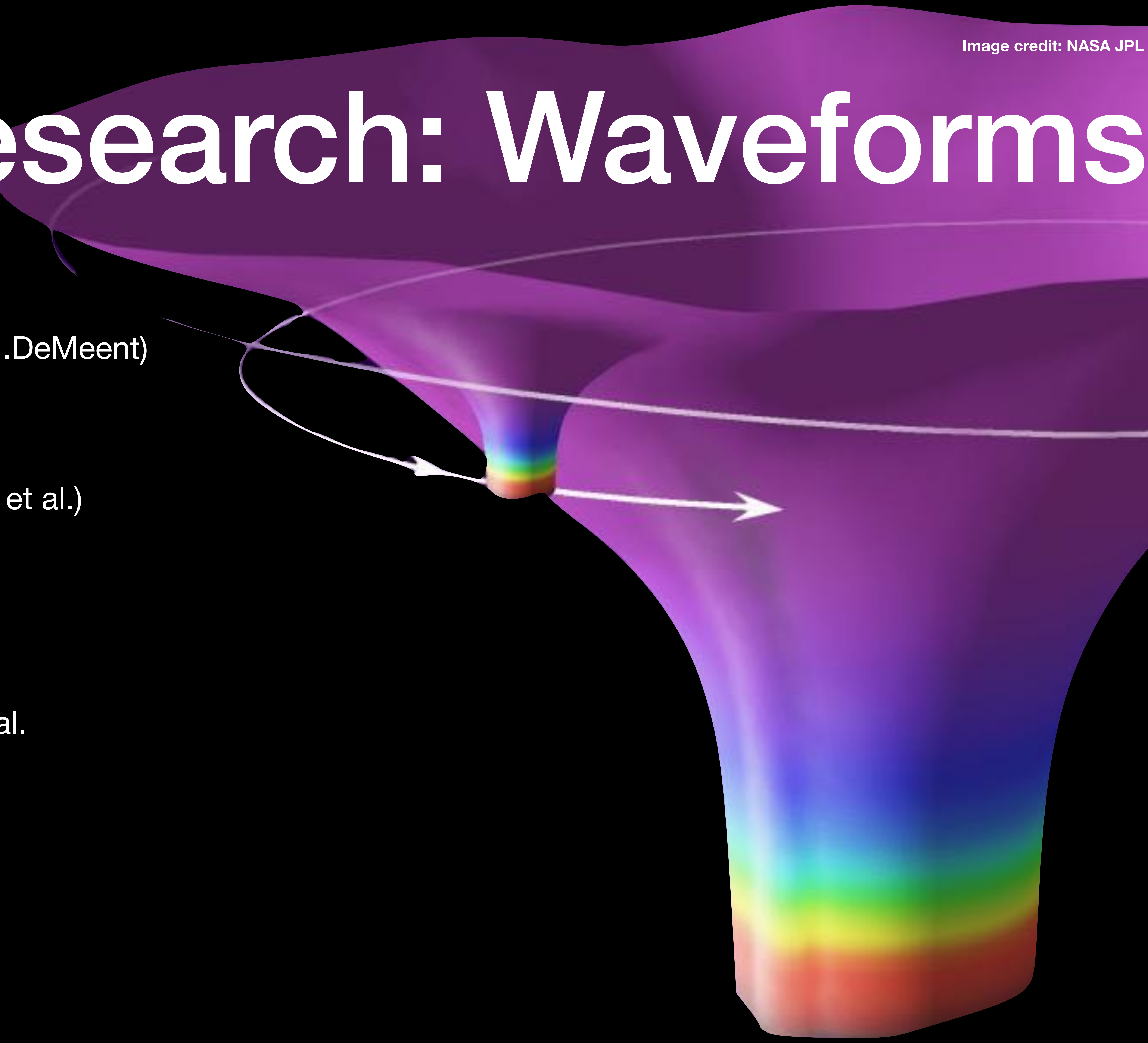
Ongoing research: Waveforms



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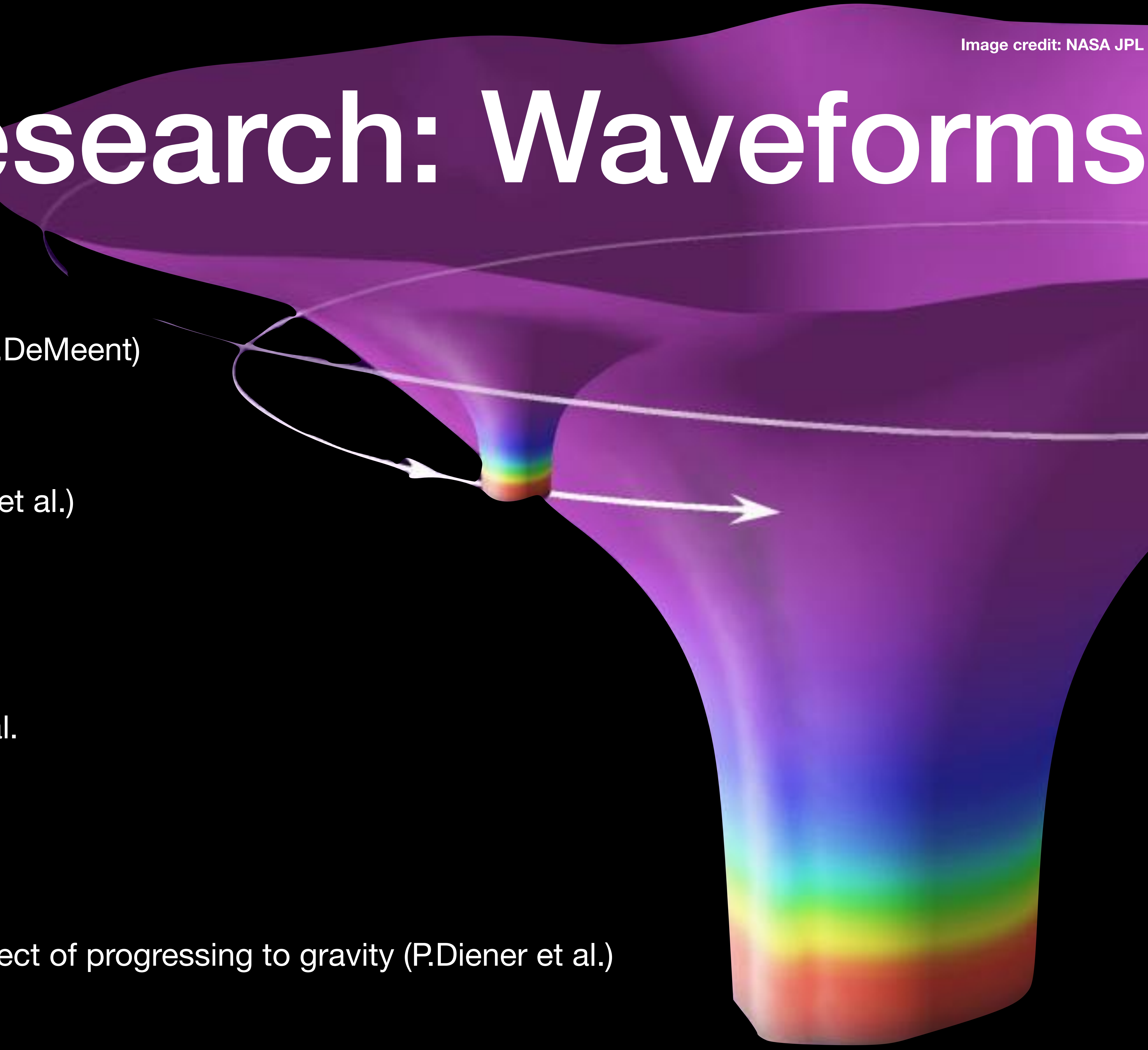

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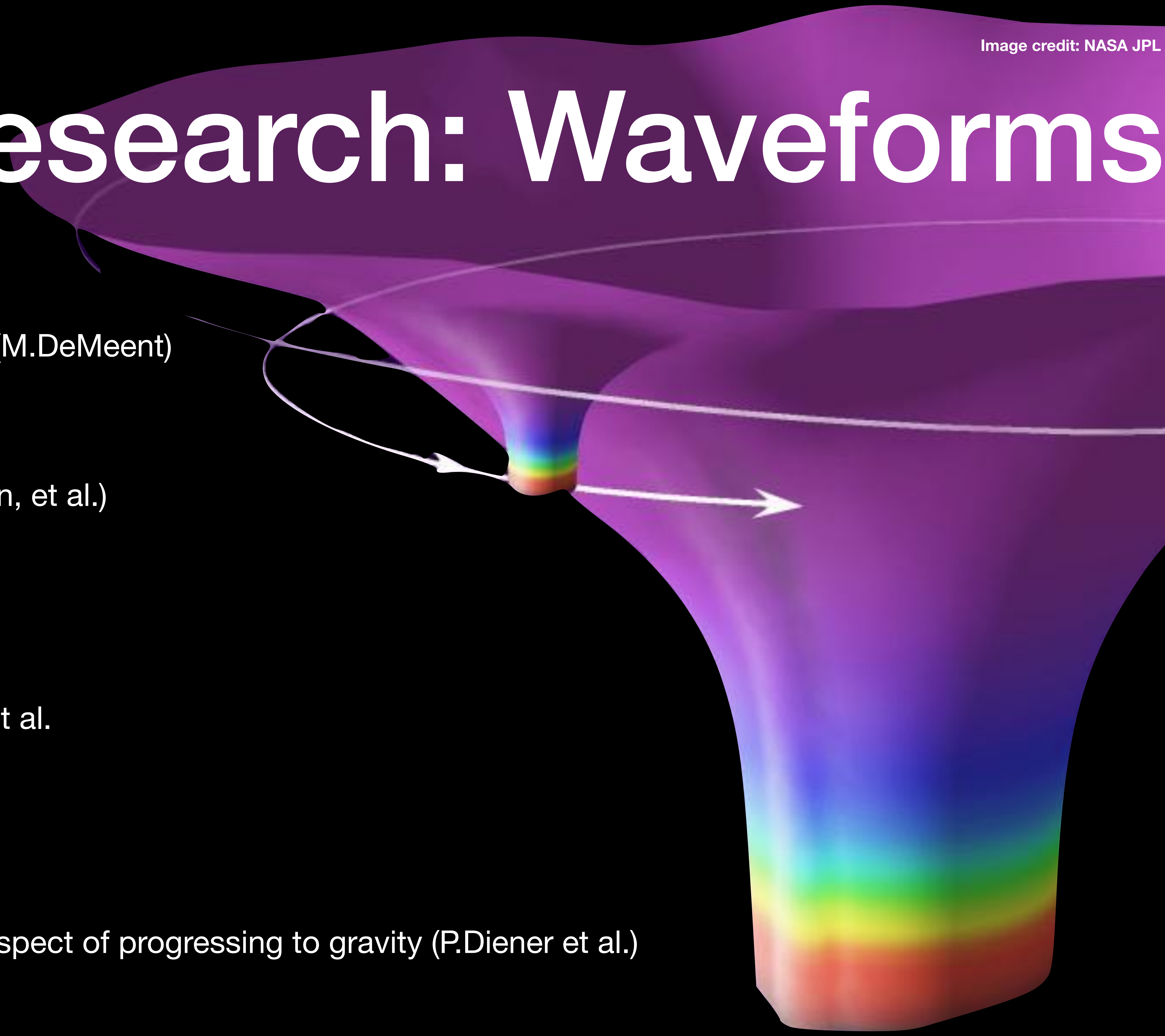
Ongoing research: Waveforms



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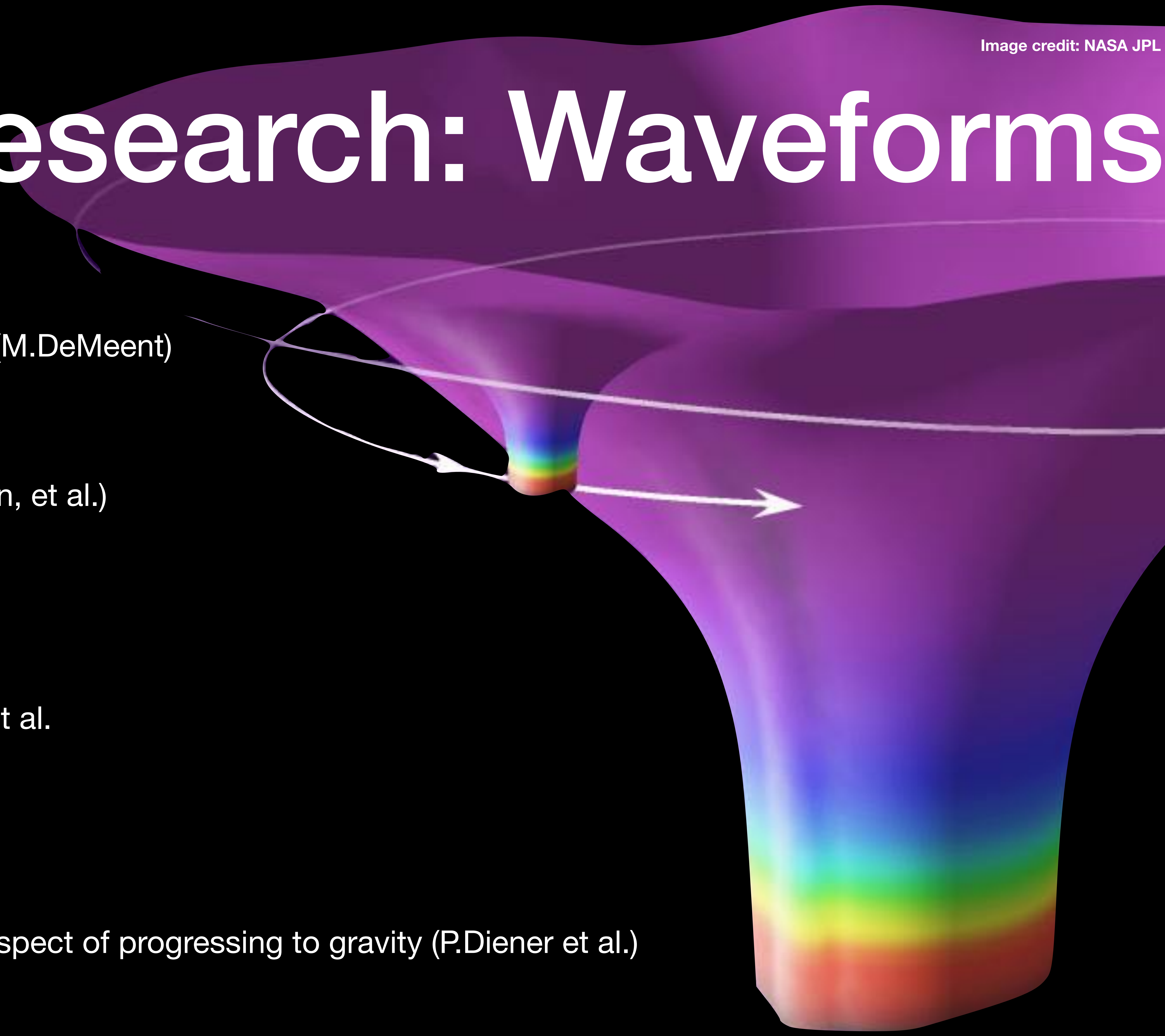
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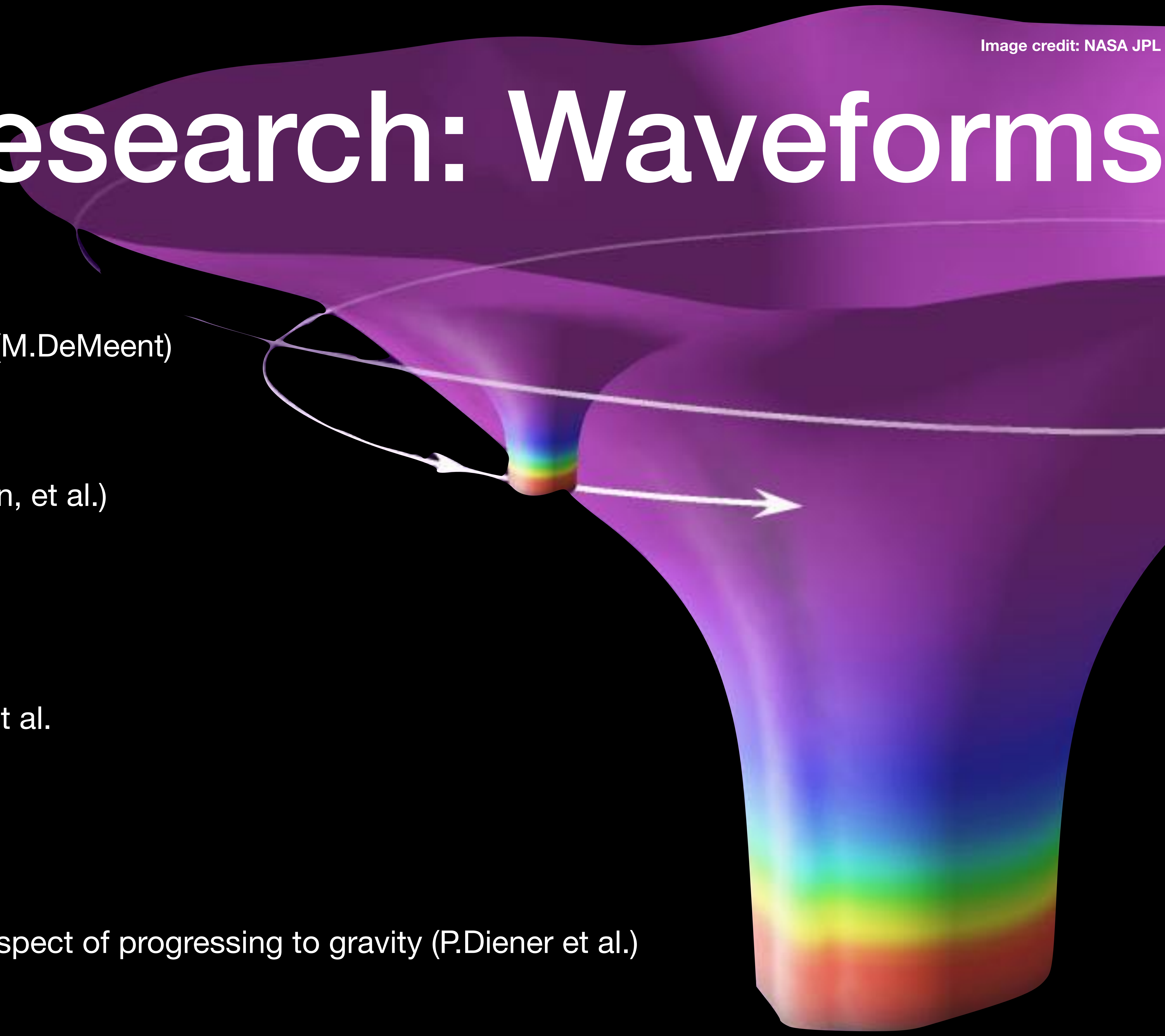
Ongoing research: Waveforms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  - NIT model similar to Kludge but can 'learn' from self consistent models

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