Brane world cosmology

Lecture from the course

"Introduction to Cosmoparticle Physics"

1. Different approaches of investigation

Fundamentally theoretic approach.

superstring U supergravity ⊂ M-theory (?)

D=10 D≤11

Phenomenological approach.

D=4+d

Studying possible manifestations of extra dimensions and possibilities of solutions of basic problems facing the M-theory (hierarchy of electroweak and Planck scales: 200GeV ←→ 10^{19}GeV, and others) by the example of simple (maybe inconsistent with fundamental theory) models.

Kaluza-Klein (KK) models

$$g_{MN} = \begin{pmatrix} g_{\mu\nu} & g_{\mu n} \\ g_{m\nu} & g_{mn} \end{pmatrix}$$

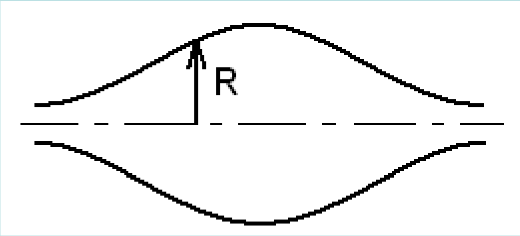
$M,N=0,\ldots D$

$\mu,\nu=0,1..3$

$m,n=\underbrace{4,\ldots D}_{d}$

Describes other interactions

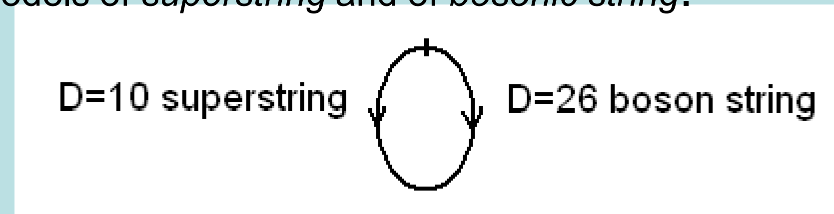
If extra d dimensions are compactified with R=const, then they are manifested in form of the KK-mass states for all particles, if $R\neq$const – in form of interaction.



Superstring theory, M-theory

Each species of particles is a certain vibration mode of a string – fundamental element of zero thickness and of Planckian length. Strings may be *closed* and *open*.

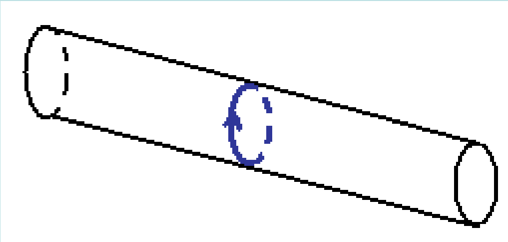
Most popular string model is a so called *heterotic string* (closed) – combines the models of *superstring* and of *bosonic string*.



Fixed gauge groups: $E_8 x E_8'$,   SO(32)

Each contains 248 bosons and 248 fermion fields

Possible consequence: existence of homotopically stable particle



Their mass is proportional to the radius of compactification

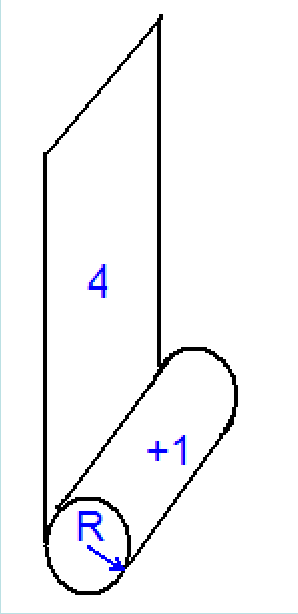
Superstring U supergravity ⊂ M-theory (?)

D=10   D≤11

2. Progress of phenomenological approach

1) Kaluza-Klein type models.

$$M_4 \times K_d$$



Solution of Klein-Gordon equation in 5 dimensions **(4 ordinary (x_μ, μ = 0,…3) + 1 cylindrical (z))** for massless field

$$\partial^2_{(5)}\phi = 0$$

gives

$$\phi = \exp(ip_\mu x^\mu)\exp\left(\frac{inz}{R}\right) \qquad n = 0, \pm 1, \pm 2, ..$$

Whence we have

$$p^2 = \frac{n^2}{R^2} \qquad n = 0, \pm 1, \pm 2, ..$$

From point of view 4d space-time a particle has got mass states – "KK-states"

$$m_n = \frac{|n|}{R}$$

Modern experiment restricts extra dimension manifestation so as

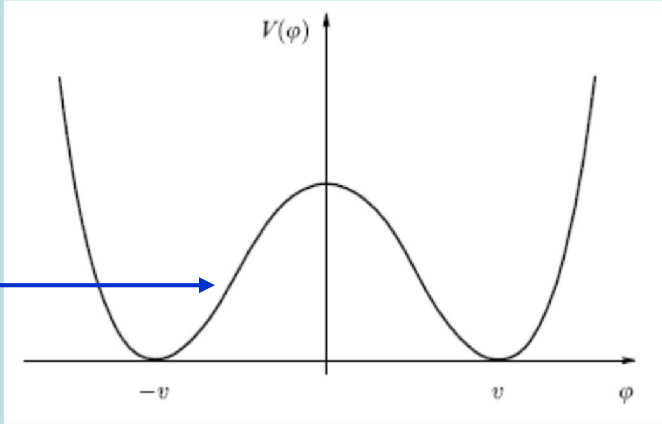
$1/R$ > a few × 100 GeV => ***R* < 10^{-17} cm,** i.e. extra dimension(s) should be very compact (with microscopic size).

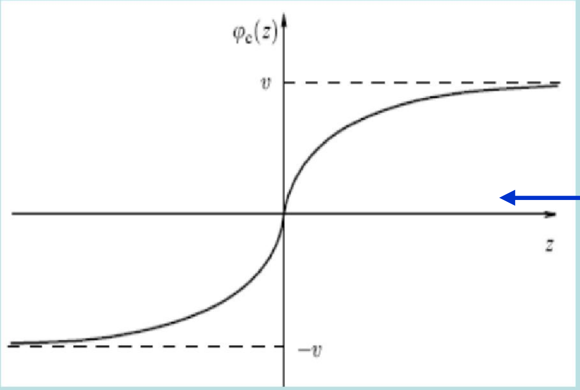
2. Progress of phenomenological approach

2) Capture of the fields by 4d manifold (brane)

The matter fields can be localized ("captured") within narrow hyper-surface ("brane"), while extra dimension(s) can be **large**. This can be realized with the help of scalar field interacting with usual matter field as demonstrated below.

Let a scalar field (φ) in 5 (infinite) dimensions has potential as shown.





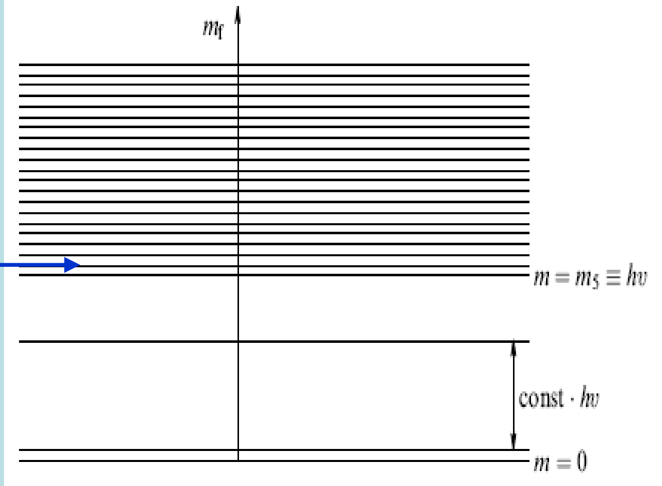
It leads to domain wall dividing 5d space-time on two parts with respect to one (z) of the dimensions.

2. Progress of phenomenological approach

2) Capture of the fields by 4d manifold (brane)

Let a fermion (ψ) has a Yukava coupling with given scalar field ($h\varphi\psi\psi$).

Then fermion gets mass states as shown on the picture, where zero-state turns to be localized (captured) near z=0 (domain wall), damping as exp(-$hv|z|$).



Mechanism of the capture of gauge fields must be more complicated.

Possible consequences:

1) $E \sim hv \quad \Rightarrow \quad e^+e^- \to nothing$ (products leave 4d space-time)

2) Decay of massive particles, e.g. DM.

2. Progress of phenomenological approach

3) ADD type models (or low scale gravity).

ADD – Arkani-Hamed, Dimopoulos, Dvali

$$\overbrace{M_4 \times K_d}^{\text{“bulk”}}$$

brane (M_4) compactified (K_d) with the size R (can be not small)

Effect of gravitation of brane in 4+d space-time is not taken into account.

Ordinary matter is considered to be captured on 4d brane (M_4) of the width $\Delta z \ll R$. Gravity, contrary to ordinary matter, propagates over all D=4+d space-time ($M_4 \mathrm{x} K_d$ – “bulk”).

2. Progress of phenomenological approach

3) ADD type models (or low scale gravity).

In given case the law of gravity has the form

$$V(r) \cong \begin{cases} \dfrac{m_1 m_2}{M_{\rm Pl}^{d+2}} \dfrac{1}{r^{d+1}}, & r << R \\ \dfrac{m_1 m_2}{M_{\rm Pl}^{d+2}} \dfrac{1}{R^d r}, & r >> R \end{cases} \Rightarrow \quad m_{\rm Pl}^2 = M_{\rm Pl}^{2+d} R^d$$

Effective 4d Planck mass (m_{Pl}) can be much larger than *true* "bulk" Planck mass (M_{Pl}) and the last one can be as small as electroweak scale (~1TeV) provided large enough *R*. That is hierarchy problem of particle physics is *re-formulated* in terms of large extra dimensions.

We have

$$R \sim M_{\rm Pl}^{-1} \left(\frac{m_{\rm Pl}}{M_{\rm Pl}} \right)^{2/d} \sim (\text{for } M_{\rm Pl} \sim 1\,\text{TeV}) \sim 10^{-17+32/d}\,\text{cm} \le R^{\rm exper} < 0.2\,\text{mm}$$

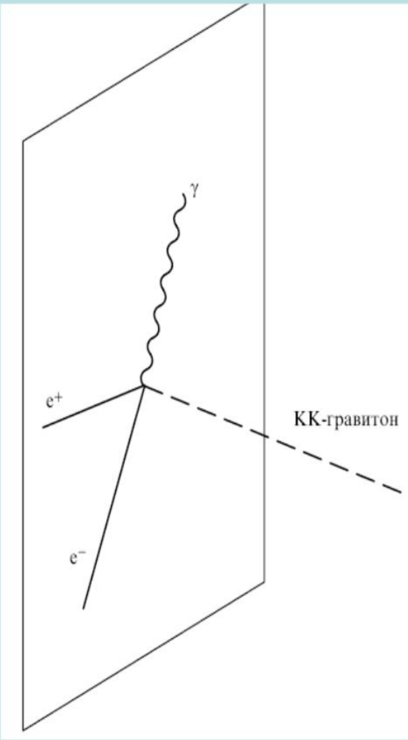
$$\left(0.2\,\text{mm} \approx \text{meV}^{-1}\right)$$

d=1 is excluded, while *d*≥2 is allowed.

2. Progress of phenomenological approach

3) ADD type models (or low scale gravity). Possible consequences

At $E \sim M_{Pl}$ there should be copious production of KK-gravitons with $m_n = n/R < E$ **($\Delta m = 1/R < 1meV$)**



$$e^+ e^- \to \gamma + nothing$$

$$\sigma \sim \frac{\alpha}{m_{\mathrm{Pl}}^2} N_{KK}(E), \qquad N_{KK}(E) = \left(\frac{E}{\Delta m} \right)^d$$ - number of KK-states with $m < E$.

Large $N_{KK}(E \sim M_{Pl})$ compensates small $1/m_{Pl}^2$, so $\sigma \sim \sigma_{e/m}$.

Miniblack hole formation at $E \sim M_{Pl}$.

2. Progress of phenomenological approach

3) ADD type models (or low scale gravity): Possible consequences

Problem of overproduction of KK-gravitons in the early Universe.

At $T \gg 1/R$ KK-gravitons are produced with the rate

$$\dot{N}_{G_{KK}} = n_a n_b \left\langle \sigma_{ab \to G_{KK} X} v_{ab} \right\rangle V \quad \Rightarrow \left\{ \begin{array}{c} V = N_\gamma / n_\gamma \\ n_{a,b,\gamma} \sim T^3 \\ dt \sim m_{\rm Pl} dT / T^3 \\ \langle \sigma v \rangle \sim N_{KK} m_{\rm Pl}^{-2} \end{array} \right\} \Rightarrow$$

$$n_{G_{KK}} = n_\gamma \int_{t(T=T_{\rm R})}^{t(T \sim m_{G_{KK}})} \frac{\dot{N}_{G_{KK}}}{N_\gamma} dt \sim n_\gamma \int_0^{T_{\rm R}} T^6 m_{\rm Pl}^{-2} \left(\frac{T}{\Delta m} \right)^d T^{-3} \frac{m_{\rm Pl} dT}{T^3} \sim \frac{T_{\rm R}}{m_{\rm Pl}} \left(\frac{T_{\rm R}}{\Delta m} \right)^d n_\gamma$$

$$\varepsilon_{G_{KK}} \sim (m_{KK} \sim T) \sim \frac{T_{\rm R}^2}{m_{\rm Pl}} \left(\frac{T_{\rm R}}{\Delta m} \right)^d n_\gamma$$

2. Progress of phenomenological approach

3) ADD type models (or low scale gravity): Possible consequences

For agreement with BBN one requires

$$\varepsilon_{G_{KK}} \sim \frac{T_R^2}{m_{Pl}}\left(\frac{T_R}{\Delta m}\right)^d n_\gamma < T_{n/p} n_\gamma$$

$$T_R \le M_{Pl}\left(\frac{T_{n/p}}{0.1 m_{Pl}}\right)^{\frac{1}{2+d}} \underset{\text{for } d=2\div6}{\sim} (0.6\cdot10^{-6} \div 2\cdot10^{-3}) M_{Pl} \quad \boxed{> T_{n/p} \approx 1\,\text{MeV}}$$

Conclusion is not changed in principle, if KK-gravitons are decayed before BBN.

Low T_R **(~MeV÷GeV)**, as obtained for M_{Pl}~1 TeV, puts forward the questions of baryosynthesis, inflation, dark matter origin in the early Universe.

M_{Pl}>>1 TeV (>10÷30 TeV) is preferable.

Satisfying to Ω(modern) in case of stable KK-gravitions intensifies constraints.

Constraint from SN1987: agreement with an observed neutrino signal requires

$$M_{Pl}(d=2) \ge 30\,\text{TeV}$$

Generally it agrees with constrains from cosmology.

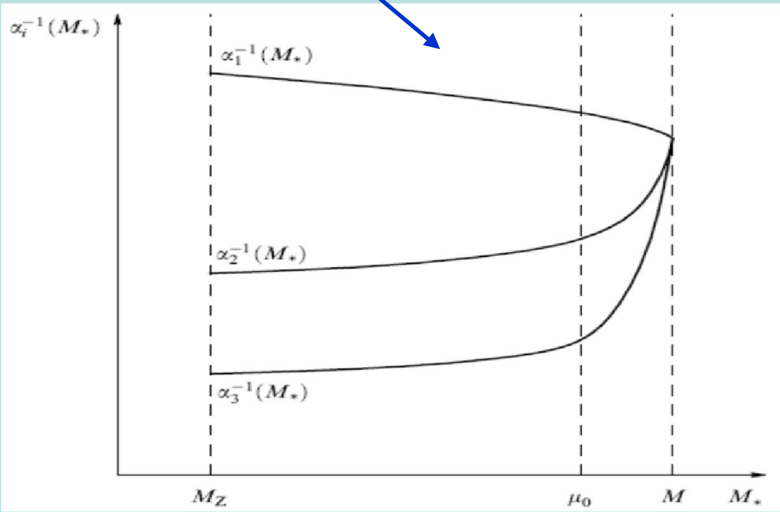
2. Progress of phenomenological approach

3) ADD type models (or low scale gravity): Possible consequences

Solution of small masses of known neutrinos due to sterile neutrino with KK-states and mixed with ordinary ones. It can be provided, as well as for a bulk Plank mass, by large extra dimensions.

$$m_\nu \sim \frac{vev}{(M_{\rm Pl}R)^{d/2}} \sim \frac{vev \cdot M_{\rm Pl}}{m_{\rm Pl}}, \qquad vev \sim m_W$$

All possible physics of Grand Unification should take place at $E \leq M_{\rm Pl}$. In this case gauge constants should run following another law: a power-like law can be suggested (logarithmic law changes onto power-like starting from some μ until M_{Pl}).



As a consequence of low energetic GUT physics, a problem of fast proton decay appears.

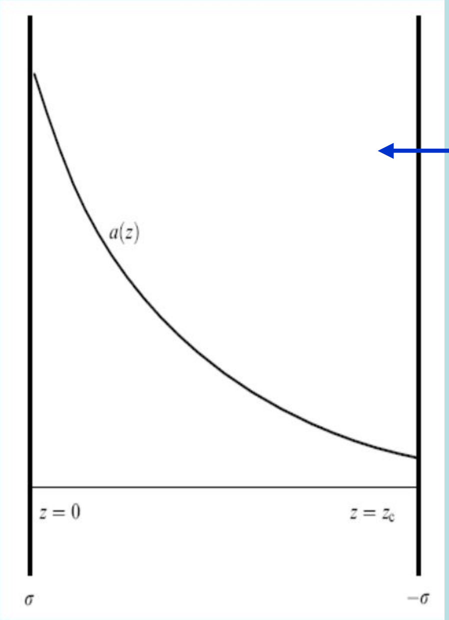
$$\tau_p \sim \frac{1}{\alpha_{\rm GUT}^2 m_p}\left(\frac{M_{\rm GUT}}{m_p}\right)^4$$

2. Progress of phenomenological approach

3) RS type models (or low scale gravity).

RS – Randall, Sundrum

There are attempts to realize idea of solving hierarchy problem due to extra dimensions taking properly into account effect of gravity in a bulk space.



In such configuration 4d gravity constant on the brane at $z=z_c$ (with negative tension, RS-1) turns out to be reduced exponentially as compared to a bulk (5d) gravity constant.

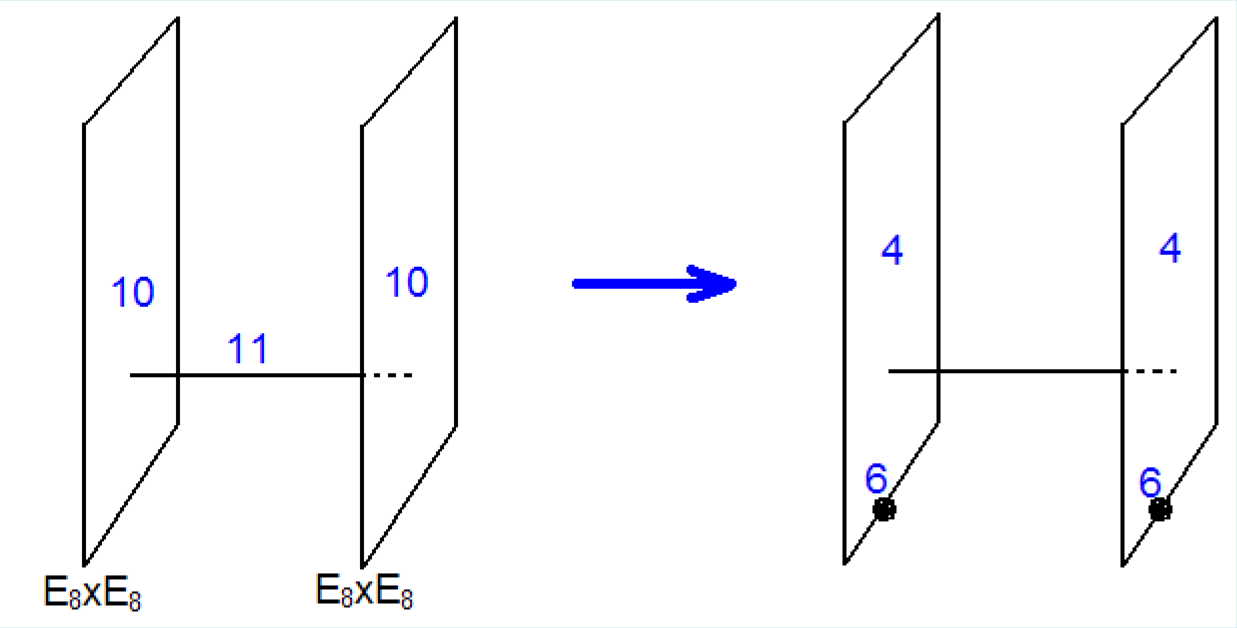
$$m_{\mathrm{Pl}} \sim \exp\left(kz_c\right) M_{\mathrm{Pl}}$$

k – curvature of the bulk space, can be $\sim M_{Pl}$. Note, 5d gravity constant has dimension here M_{Pl}^{-3}.

Lightest KK-gravitons has $m \sim k$ (from point of view of our brane) and interaction strength $\sim \sqrt{k / M_{\mathrm{Pl}}^3}$.

There are different variations of such kind model: model with our world on the $z=0$ brane (RS-2: in this case there is no exponential suppression of our 4d gravity constant with respect to 5d one, but electroweak symmetry breaking can be connected with the second brane ($z=z_c$) and in this manner relative exponential suppression can be reached), model with 3 branes, with intersecting branes.

3. Progress of fundamentally theoretic approach



Horava P, Witten E 1996 Nucl.Phys.B460 506; Nucl.Phys.B475 94

Remark: In principle, in string theory the tips of open strings can belong to the only brane – D-brane, which, in turn, could be 3-dimensional. However, it is not most likely to correspond to a realistic theory, and both different D-branes and closed strings (as in the case of heterotic string shown above) are considered.

4. Birth of brane world

There is no definite way of elaboration of brane cosmological model (including description of early Universe).

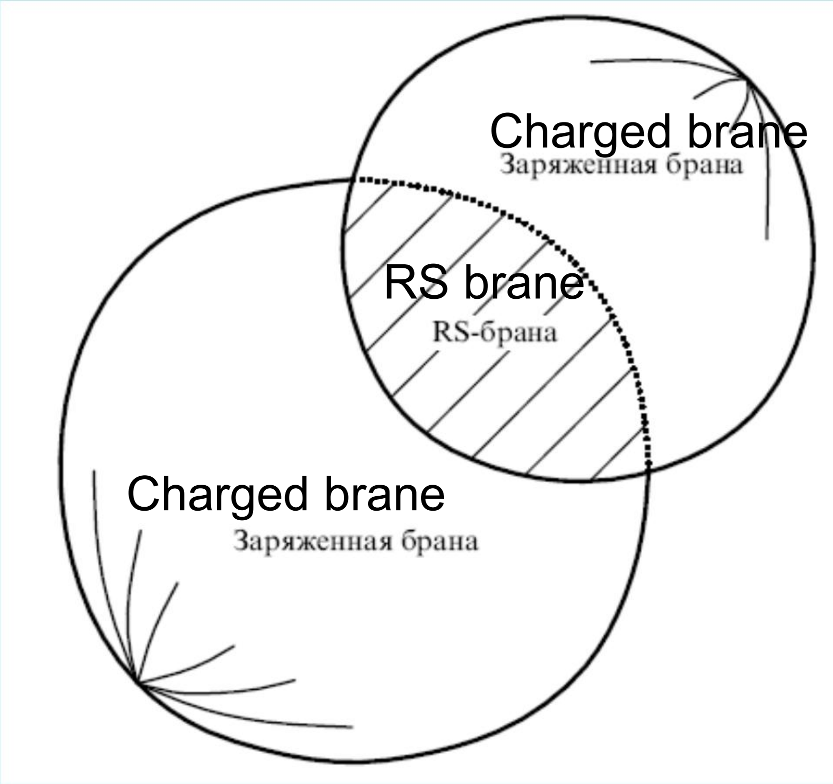
One of the scenario discussed is a *cyclic model* with colliding branes (Steinhardt P. J., Turok N. 2002 Science 296 1436; ibid 2002 Phys.Rev.D 65 126003) – alternative to inflationary models.

However, physics of brane collision is unclear.

4. Birth of brane world

A birth of brane(s) filled with matter and an empty interior bulk is a greater problem.

One possibility is illustrated:



Conclusions

- The extensions of Standard Model involve extra dimensions, which may not be small (compactified).

- In the case of infinite extra dimensions physics of the standard model can be localized on « branes », while gravity comes from the bulk space.

- Large extra dimensions can lead to significant effects of gravity at scales, much smaller, than 4D Planck scale.

- The test for extra dimensions is a challenge for cosmoparticle physics. Analysis of their signatures in astrophysical data and at accelerators can shed light on their existence.