Black holes and stars in Horndeski theory

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Why modify gravity?

Why modify gravity?
- cosmological constant problems,
- non-renormalizability problem,
- benchmarks for testing General Relativity
- theoretical curiosity.

Many ways to modify gravity:
- f(R), scalar-tensor theories,
- Galileons, Horndeski (and beyond) theory, KGB, Fab-four,
- higher-dimensions,
- DGP,
- Horava, Khronometric
- massive gravity

- Most general scalar-tensor theory leading to equations of motions with no more than 2 derivatives;
- Cancellation of Lambda (Fab-Four), Self-tuning, Self-acceleration;
- Vainshtein mechanism

Scalar-tensor theories

Linear theory

Canonical kinetic term + quadratic mass:

$$S = \int d^4x \left(-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 \right)$$

$$\rightarrow \quad g^{\mu\nu}\nabla_\mu\nabla_\nu\phi + m^2\phi = 0$$

Linear partial differential equation of second order (one degree of freedom).
Need to specify two conditions, ϕ and $\dot{\phi}$

Non-linearity in potential term

Canonical kinetic term + arbitrary potential:

$$S = \int d^4x \left(-\frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi) \right)$$

$$\rightarrow \quad \nabla_\mu \nabla^\mu \phi + \frac{dV(\phi)}{d\phi} = 0$$

Non-linear partial differential equation of second order (inflationary models, quintessence).
However, because the kinetic term is canonical, the characteristic structure is the same.

Non-linearity in kinetic term

Armendariz-Picon, Damour, Mukhanov'99

Non-linear kinetic term?

- General Relativity
- QCD
- Hydrodynamics
- ...

$$X \equiv \frac{1}{2}\left(\partial_\mu \phi \partial^\mu \phi\right)$$

k-essence: $$\boxed{S = \int d^4x \ \mathcal{K}(X)}$$

Pure k-essence

k-essence as perfect fluid

$$S_\phi = \int d^4x \sqrt{-g}\, p(X)$$

$$X = \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi$$

Definitions:

$$u_\mu = \frac{\nabla_\mu \phi}{\sqrt{2X}}$$

(gradient of scalar field is timelike)

$$\varepsilon(X) = 2X p_{,X}(X) - p(X)$$

Stress tensor:

$$T_{\mu\nu} \equiv \frac{2}{\sqrt{-g}} \frac{\delta S_\phi}{\delta g^{\mu\nu}} = (\epsilon + p)\, u_\mu u_\nu - g_{\mu\nu} p$$

pure kinetic k-essence is perfect fluid !

Even more non-linear?

Monge-Ampere equaiton

Monge-Ampère equation

Monge'1784, Ampère'1820

$$A(u_{xx}u_{yy} - u_{xy}^2) + Bu_{xx} + Cu_{xy} + Du_{yy} + E = 0$$

- to find a surface with a prescribed Gaussian curvature
- optimizing transportation costs
-...

$$u_{xx}u_{yy} - u_{xy}^2$$

First galileon in history

Even more non-linear?

galileons, Horndeski

The most generic scalar-tensor theory in 4D, whose equations of motion contain no more than second derivatives

Horndeski'1974

$$S = \int d^4x \, F \left[g, \partial g, \partial^2 g, \varphi, \partial \varphi, \partial^2 \varphi \right]$$

$$\Big\downarrow ?$$

$$E[g, \partial g, \partial^2 g, \varphi, \partial \varphi, \partial^2 \varphi] = 0$$

Horndeski theory

Why no more than 2 derivatives in EOMs?

Ostrogradski ghost

Ostrogradski'1850

$$S = \int L(q, \dot{q}) dt \quad \rightarrow \quad \frac{dL}{dq} - \frac{d}{dt}\frac{dL}{d\dot{q}} = 0$$

$$S = \int L(q, \dot{q}, \ddot{q}) dt \quad \rightarrow \quad \frac{dL}{dq} - \frac{d}{dt}\frac{dL}{d\dot{q}} + \frac{d^2}{dt^2}\frac{dL}{d\ddot{q}} = 0$$

- Generically Hamiltonian is unbounded from below.

- New propagating degree of freedom appear. It is a ghost.

- Avoiding the theorem ?

Even more non-linear?

Universal equations

"Universal field equations"

Fairlie et al'1991

$$\mathcal{L}_n = F_n(\partial\varphi)W_{n-1}, \quad W_0 = 1$$
$$W_n = \mathcal{E}\mathcal{L}_n$$

$$\mathcal{L}_1 = (\partial\varphi)^2 \to W_1 = \Box\varphi \to$$
$$\mathcal{L}_2 = (\partial\varphi)^2\Box\varphi \to \mathcal{E}\mathcal{L}_2 = (\Box\varphi)^2 - (\nabla\nabla\varphi)^2$$

Galileons: flat case

first non-standard term

DGP: brane model of gravity

Dvali et al'00

Particular limit of the theory (decoupling limit) gives scalar field Lagrangian,

Luty et al'03

$$\mathcal{L}_{DGP} = -\frac{M_P^2}{4} h^{\mu\nu} \left(\mathcal{E}h\right)_{\mu\nu} - 3(\partial\pi)^2 - \frac{r_c^2}{M_P}(\partial\pi)^2\Box\pi + \frac{1}{2}h^{\mu\nu}T_{\mu\nu} + \frac{1}{M_P}\pi T$$

direct coupling to matter

$$(\Box\varphi)^2 - (\nabla\nabla\varphi)^2$$

Monge-Ampère type

Galileons: flat case

generalisation

Generalization of DGP scalar:
- direct coupling to matter
- Galilean symmetry
- up to second order derivatives in EOM

Nicolis et al'09

$$\mathcal{L}_\pi = \sum_{i=1}^{i=5} c_i \mathcal{L}_i,$$

$$\mathcal{L}_i \sim \pi^i$$

$$\begin{aligned}
\mathcal{L}_1 &= \pi, \\
\mathcal{L}_2 &= -\frac{1}{2}\partial_\mu \pi \partial^\mu \pi, \\
\mathcal{L}_3 &= -\frac{1}{2}\left(\partial\pi\right)^2 \Box\pi, \\
\mathcal{L}_4 &= -\frac{1}{4}(\Box\pi)^2 \partial_\mu\pi\partial^\mu\pi + \frac{1}{2}\Box\pi\partial_\mu\pi\partial_\nu\pi\partial^\mu\partial^\nu\pi + \ldots \\
\mathcal{L}_5 &= -\frac{1}{5}(\Box\pi)^3 \partial_\mu\pi\partial^\mu\pi + \frac{3}{5}(\Box\pi)^2\partial_\mu\pi\partial_\nu\pi\partial^\mu\partial^\nu\pi + \ldots
\end{aligned}$$

Galileons: flat case

equations of motion

Equations of motion (in flat space-time)

$$\begin{aligned}
\mathcal{E}_1 &= 1 \\
\mathcal{E}_2 &= \Box\pi \\
\mathcal{E}_3 &= (\Box\pi)^2 - (\partial_\mu\partial_\nu\pi)^2 \\
\mathcal{E}_4 &= (\Box\pi)^3 - 3\,\Box\pi(\partial_\mu\partial_\nu\pi)^2 + 2(\partial_\mu\partial_\nu\pi)^3 \\
\mathcal{E}_5 &= (\Box\pi)^4 - 6(\Box\pi)^2(\partial_\mu\partial_\nu\pi)^2 + 8\,\Box\pi(\partial_\mu\partial_\nu\pi)^3 + 3\big[(\partial_\mu\partial_\nu\pi)^2\big]^2 - 6(\partial_\mu\partial_\nu\pi)^4
\end{aligned}$$

Nonlinear second-order equations of motion !

No additional degree of freedom => no Ostrogradski ghost

Galileons: covariant case

Covariant Galileon: adding non-minimal scalar-matter coupling to flat Galileon.

Deffayet et al'09
+ many other works

Most general galileon Shift-symmetric action:

$$\mathcal{L}_2 = K\left(X\right)$$

$$\mathcal{L}_3 = G^{(3)}\left(X\right)\Box\varphi$$

$$\mathcal{L}_4 = G^{(4)}_{,X}(X)\left[(\Box\varphi)^2 - (\nabla\nabla\varphi)^2\right] + R\,G^{(4)}(X),$$

$$\mathcal{L}_5 = G^{(5)}_{,X}\left(X\right)\left[(\Box\varphi)^3 - 3\Box\varphi\left(\nabla\nabla\varphi\right)^2 + 2\left(\nabla\nabla\varphi\right)^3\right] - 6G_{\mu\nu}\nabla^{\mu}\nabla^{\nu}\varphi\,G^{(5)}\left(X\right)$$

Beyond Horndeski

Gleyzes, et al'14
+ many other works

There are extra Lagrangians which give
higher order equations of motion,
but the number of degrees of freedom does not increase !

Two extra pieces may be written as

Deffayet et al'15
Babichev et al'15

$$S_J^{\rm ext} = \int d^4x \sqrt{-g}\, F_J(\phi, X)\, G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi,$$
$$S_P^{\rm ext} = \int d^4x \sqrt{-g}\, F_P(\phi, X)\, P^{\alpha\beta\mu\nu}\nabla_\alpha\phi\nabla_\mu\phi\nabla_\beta\nabla_\nu\phi,$$

There are more of them.
Beyond beyond Horndeski

Langlois, Noui'15
Crisostomi, Koyama, Tasinato'16

Black holes

Black holes are bald (?)

- Gravitational collapse…
- Black holes eat or expel surrounding matter
- Their stationary phase is characterised by a limited number of charges
- No details about collapse
- Black holes are bald

No hair theorems/arguments dictate that adding degrees of freedom lead to trivial (General Relativity) or singular solutions.

E.g. in the standard scalar-tensor theories BH solutions are GR black holes with constant scalar.

Example of hairy black hole

BBMB solution

Bocharova et al'70, Bekenstein'74

Conformally coupled scalar field:

$$S[g_{\mu\nu},\phi] = \int \sqrt{-g}\left(\frac{R}{16\pi G} - \frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi - \frac{1}{12}R\phi^2\right) d^4x$$

Static spherically symmetric (nontrivial) solution:

$$ds^2 = -\left(1-\frac{m}{r}\right)^2 dt^2 + \frac{dr^2}{\left(1-\frac{m}{r}\right)^2} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

Secondary scalar hair:

$$\phi = \sqrt{\frac{3}{4\pi G}}\frac{m}{r-m}$$

NB. The geometry is of that of extremal RN.
The scalar field is unbounded at r=m

Shift-symmetric galileons

Arbitrary $G_2(X),\ G_2(X),\ G_4(X),\ G_5(X)$

Conserved current because of shift-symmetry: $$J^\mu = \frac{\delta S}{\delta(\partial_\mu \phi)}$$

No hair for galileon

Hui&Nicolis'12

Shift-symmetric galileon, with arbitrary $G_2(X),\ G_2(X),\ G_4(X),\ G_5(X)$
Assume that:

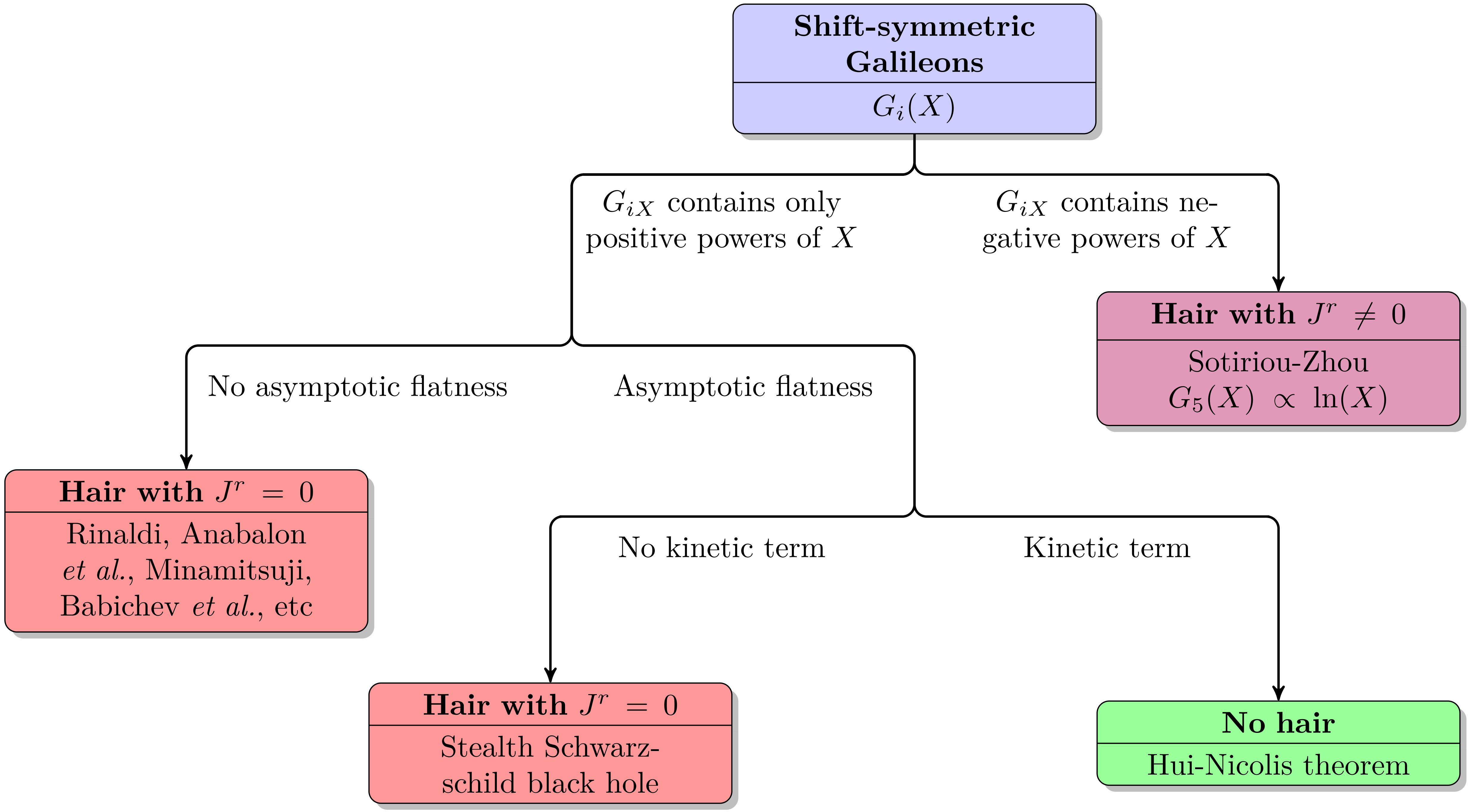
(i) spacetime and scalar field is static spherically symmetric,

$$ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2 \qquad \phi = \phi(r)$$

(ii) spacetime is asymptotically flat, and $\phi' \to 0$ as $r \to \infty$
and the norm of the current J^2 is finite (at the horizon)
(iii) there is a canonical kinetic term in the action and G_i are such that their derivatives $dG(X)_i/dX$ contain only positive or zero powers of X

A no-hair theorem then follows: the metric is Schwarzschild and the scalar field is constant

Plan to avoid no-hair theorem



Constructing hairs

Babichev, Charmousis'13

$$\boxed{ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2}$$

$$\mathrm{d}s^2 = -A(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{A(r)} + \rho(r)^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2).$$

$$\boxed{\phi = qt + \psi(r)}$$

Time-dependent scalar !

The only consistent solution for this ansatz is when $J^r = 0$

$$-qJ^r = \mathcal{E}_{tr} f$$

The norm of the current:

$$J^\mu J_\mu = -A(J^t)^2 + (J^r)^2/A,$$

The physical requirement of no-hair theorem is automatically satisfied by virtue of EOMs.

Explicit example

Babichev, Charmousis'13

$$\mathcal{L}^{\Lambda\mathrm{CGJ}} = R - \eta(\partial\phi)^2 + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 2\Lambda.$$

follows from general galileon with $G_4 = 1 + \beta X$ and $G_2 = -2\Lambda + 2\eta X.$

The general solution is given by the solution of the algebraic equation:

$$\boxed{(q\beta)^2\left(\kappa + \frac{r^2}{2\beta}\right)^2 - \left(2\kappa + (1 - 2\beta\Lambda)\frac{r^2}{2\beta}\right)k(r) + C_0 k^{3/2}(r) = 0,}$$

$$h(r) = -\frac{\mu}{r} + \frac{1}{\beta r}\int\frac{k(r)}{\kappa + \frac{r^2}{2\beta}}dr, \qquad f = \frac{(\kappa + \frac{r^2}{2\beta})^2\beta h}{k(r)},$$

$$\psi' = \pm\frac{\sqrt{r}}{h(\kappa + \frac{r^2}{2\beta})}\left(q^2(\kappa + \frac{r^2}{2\beta})h' - \frac{1 + 2\beta\Lambda}{4\beta^2}(h^2r^2)'\right)^{1/2}.$$

Explicit solutions

Asymptotically dS/AdS:

$$f = h = 1 - \frac{\mu}{r} - \frac{\Lambda_{\rm eff}}{3} r^2, \qquad \psi' = \pm \frac{q}{h}\sqrt{1-h}, \qquad \Lambda_{\rm eff} = -\frac{1}{2\beta}$$

Asymptotically static universe:

$$h = 1 - \frac{\mu}{r}, \quad f = \left(1 - \frac{\mu}{r}\right)\left(1 + \frac{\eta r^2}{\beta}\right) \qquad \psi' = \pm \frac{q}{h}\sqrt{\frac{\mu}{r(1+\frac{\eta}{\beta}r^2)}}$$

Asymptotically flat (no standard kinetic term)

$$f = h = 1 - \frac{\mu}{r} \qquad \psi' = \pm q\sqrt{\mu r}/(r-\mu).$$

Extension to other theories

The solutions are almost identical for the theory: Kobayashi, Tanahashi'14

$$\mathcal{L} = G_2(X) + G_4(X)R + G_{4X}[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2]$$

Beyond Horndeski: Babichev et al'15

$$\mathcal{L}^{\rm bH} = R + F_{\rm J}(X)G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi,$$

Cubic Galileon: Babichev et al'16

$$\mathcal{L} = \zeta\,(R - 2\,\Lambda) - \eta\,(\partial\phi)^2 + \gamma\,\Box\phi\,(\partial\phi)^2$$

Gauss-Bonnet term

Sotiriou, Zhou'13

$$\mathcal{L}^{\rm GB} = \frac{R}{2} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \alpha\phi\hat{G}.$$

follows from general galileon with $G_5 \propto \ln|X|$

The requirement iii) is violated!

$$\Box\phi + \alpha\hat{G} = 0.$$

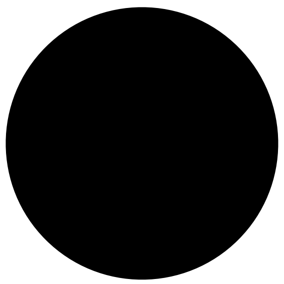
It is natural to have a non-trivial scalar

J^2 diverges at the horizon => violation of the condition ii) as well

Stars

Neutron stars with John

$$\mathcal{L} = R + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

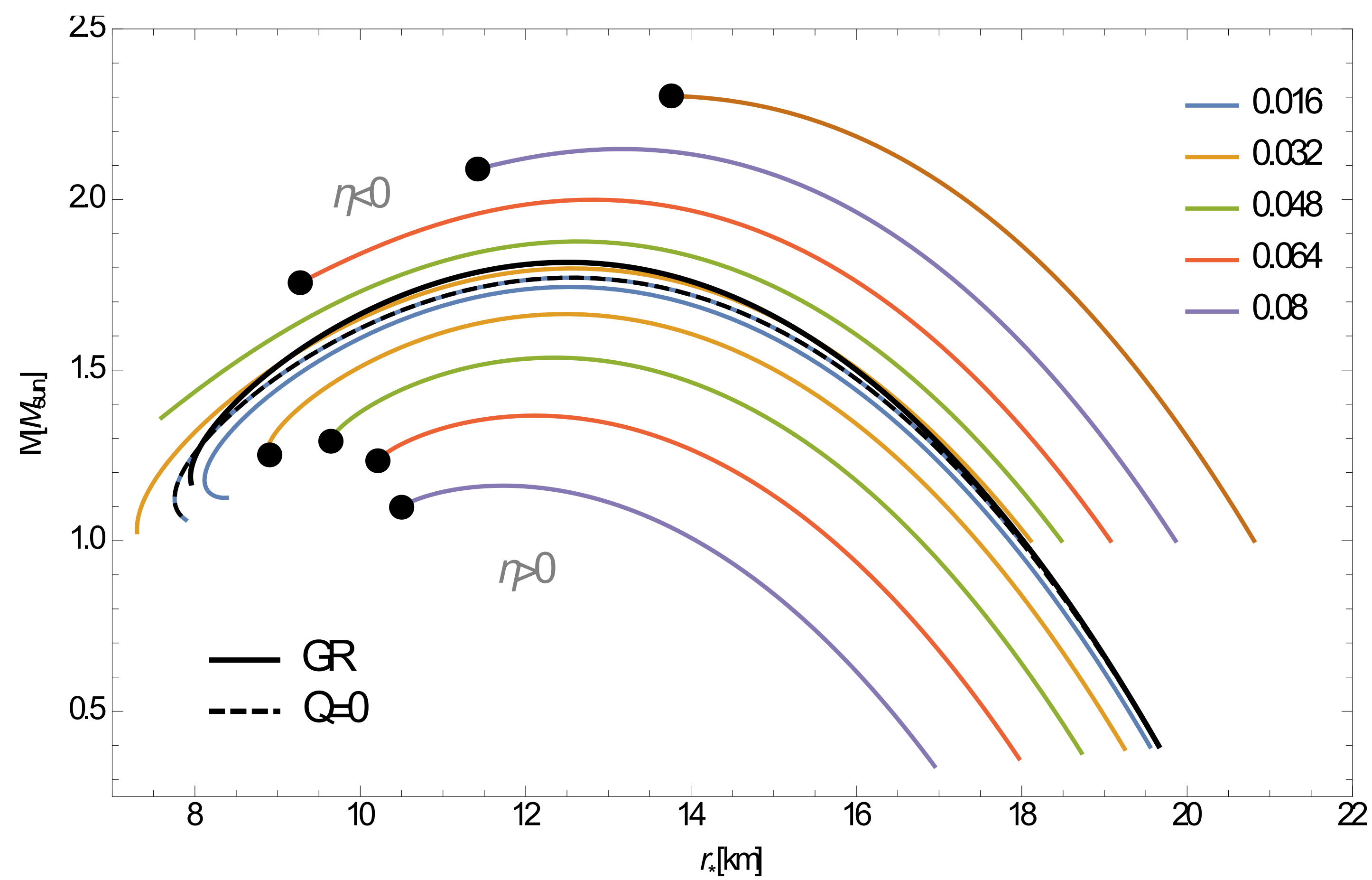


Stealth solution

$$\boxed{\phi = qt + \psi(r)}$$

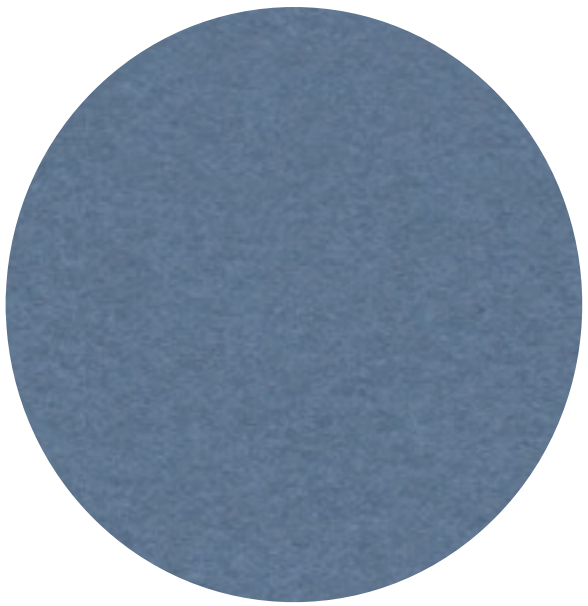
black hole -> neutron star ?
Outside the star the solution is the same as for stealth BH

Deviation from GR solution inside the star.



Neutron stars with John

$$\mathcal{L} = R + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

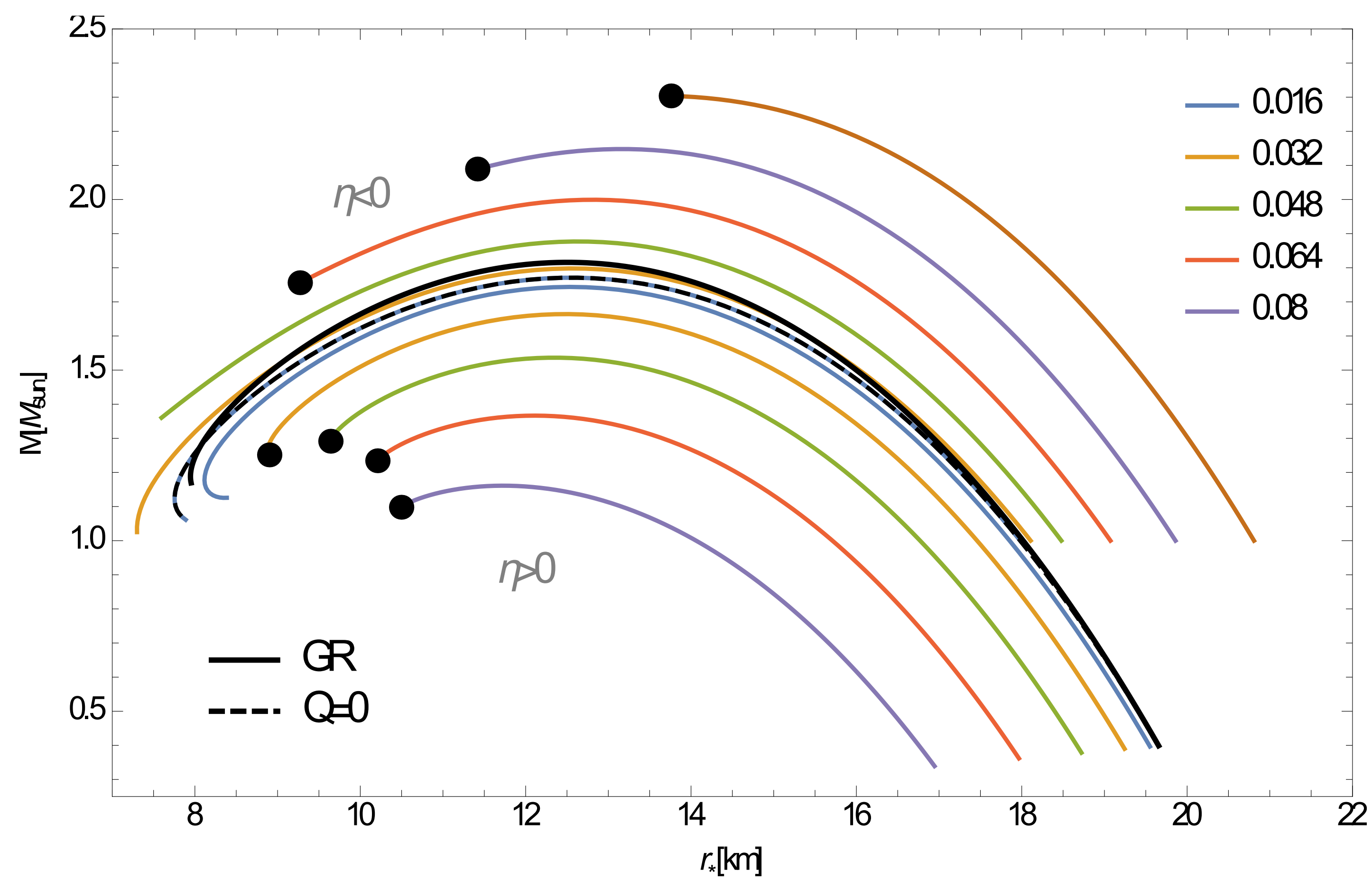


Stealth solution

$$\boxed{\phi = qt + \psi(r)}$$

black hole -> neutron star ?
Outside the star the solution is the same as for stealth BH

Deviation from GR solution inside the star.



Stars in beyond Horndeski theory

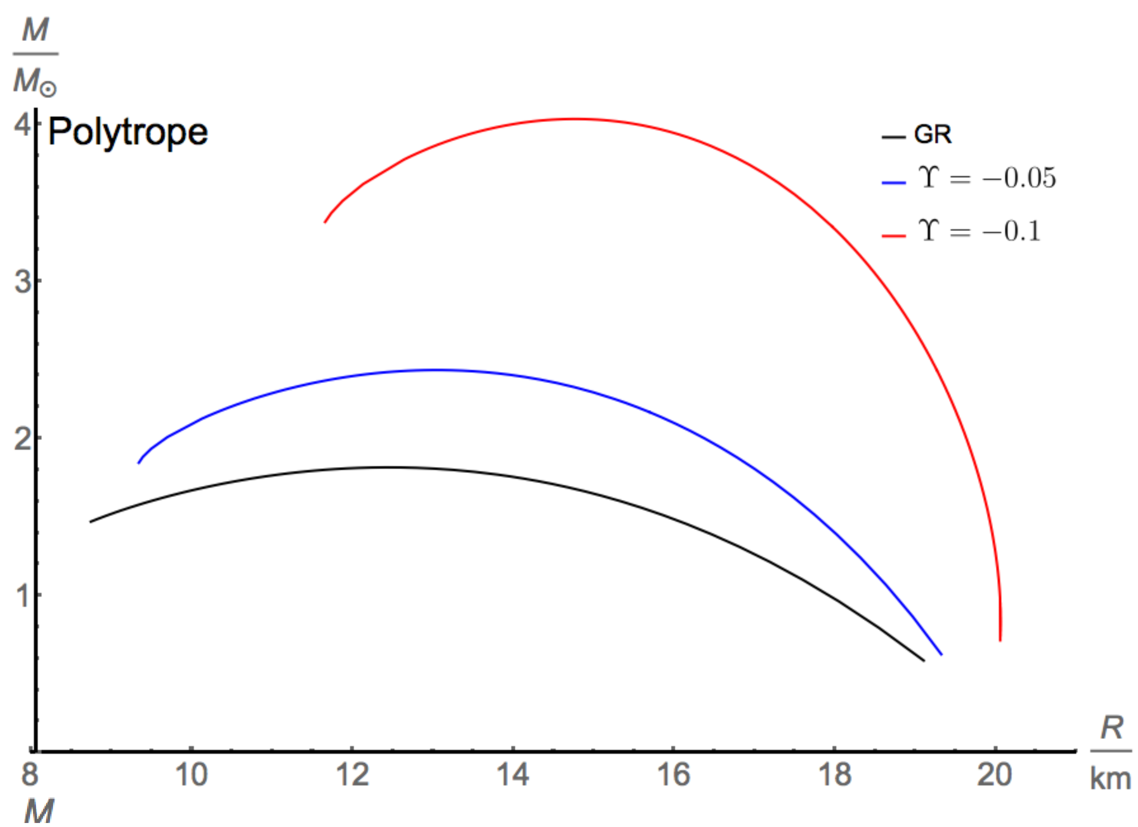
The Vainshtein mechanism is broken inside stars in beyond Horndeski theory

Stars in beyond Horndeski theory

An example of a relativistic star:

$$S = \int d^4x \sqrt{-g} \left[M_{\rm pl}^2 \left(\frac{R}{2} - \Lambda \right) - k_2 \mathcal{L}_2 + f_4 \mathcal{L}_{4,\rm bH} \right]$$

Outside the star : GR with an effective Lambda
Inside the star : deviation from GR



Conclusions

- Hairy black holes
- Non-trivial neutron stars
- Stability?
- Observational signatures?