Newton Constant, Entanglement Entropy and Black Holes

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VIA Talk

Plan of the talk:

- Historical introduction
- Newton constant
- Entanglement entropy
- Can entanglement entropy explain entropy of black holes?

Talk is based on arXiv: 1502.03758
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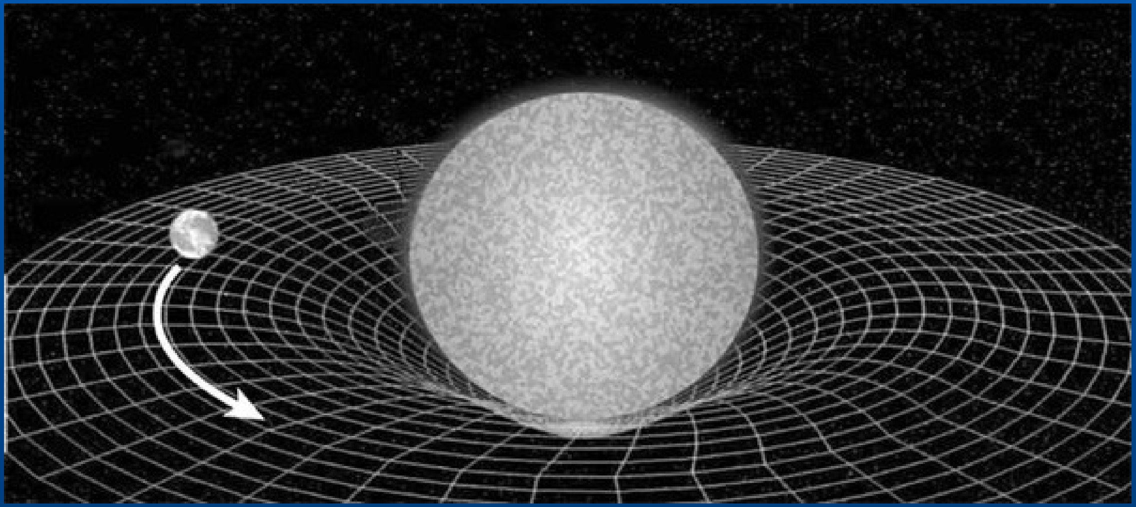
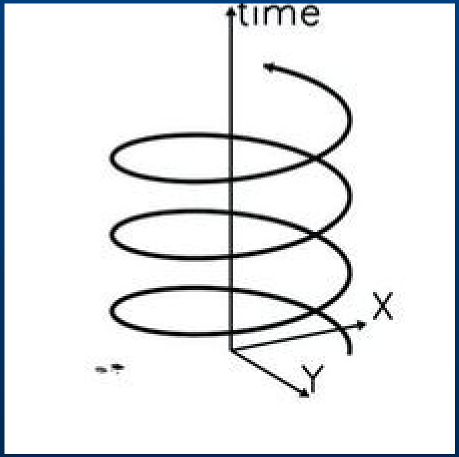
General Relativity (1915)

A. Einstein

D. Hilbert

Gravitational Force is Manifestation of Curved Space-time



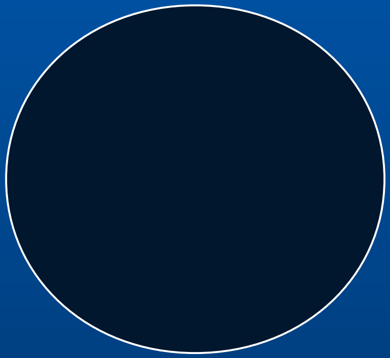


Solution with Spherical Symmetry

Describes space-time outside massive spherical Body of mass M and radius R

Karl Schwarzschild (1915)

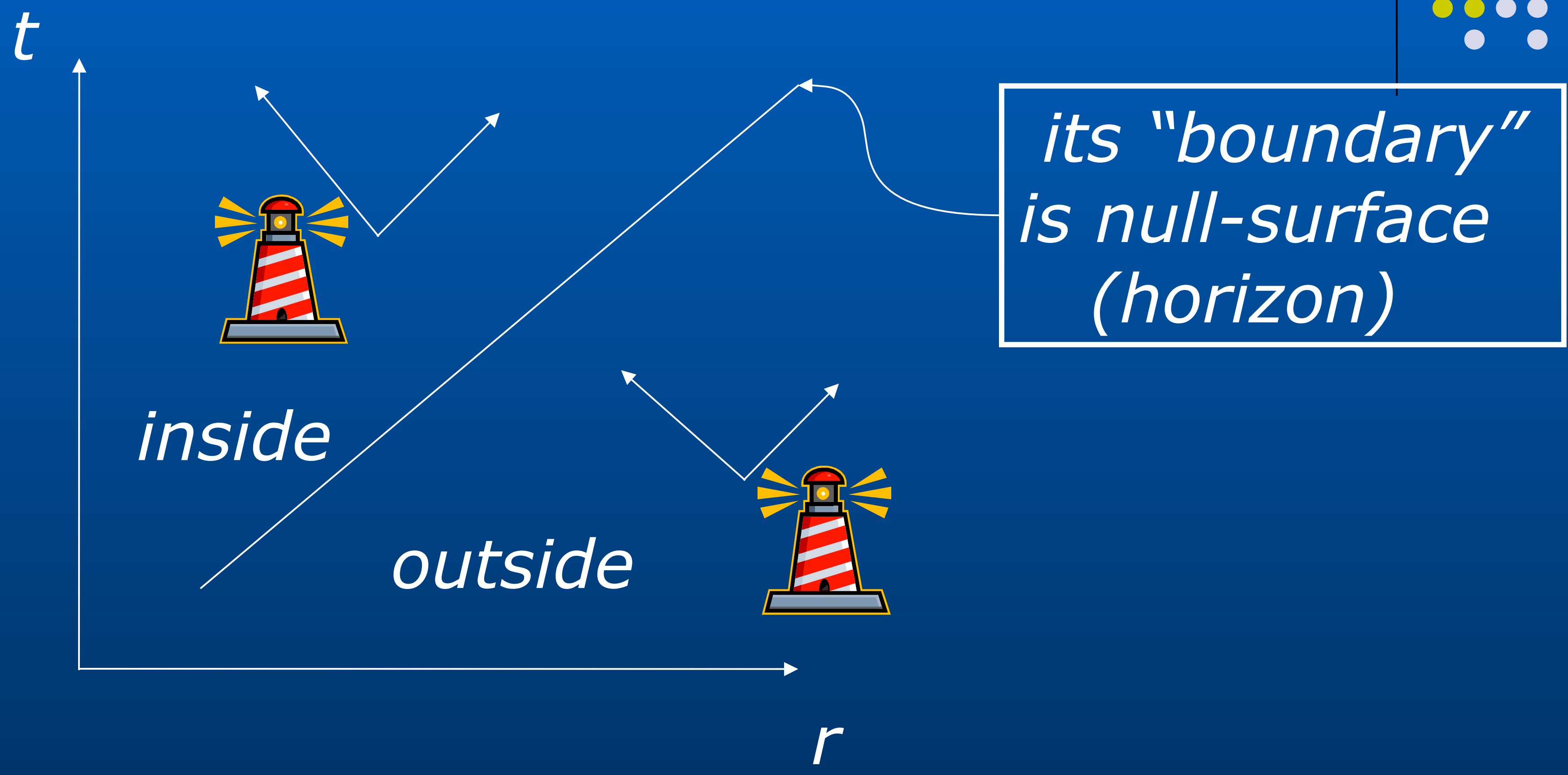
“Frozen Star”



M, R

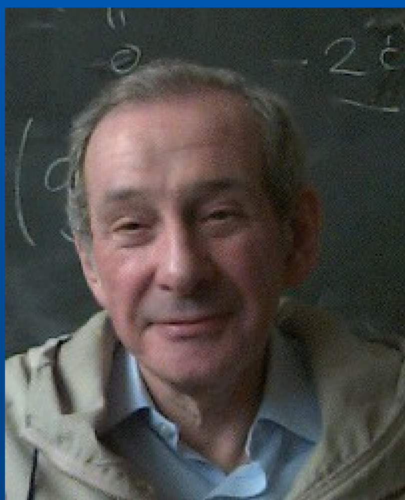
$$R < r_g = \frac{2GM}{c^2}$$

Why it is Black?

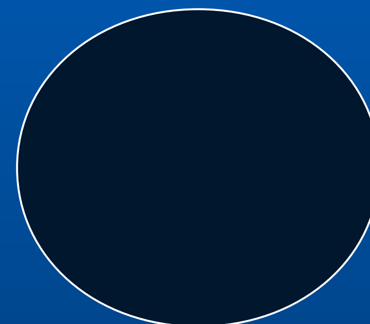


No light propagates to outside
If emitted from inside

No-hair for Black holes

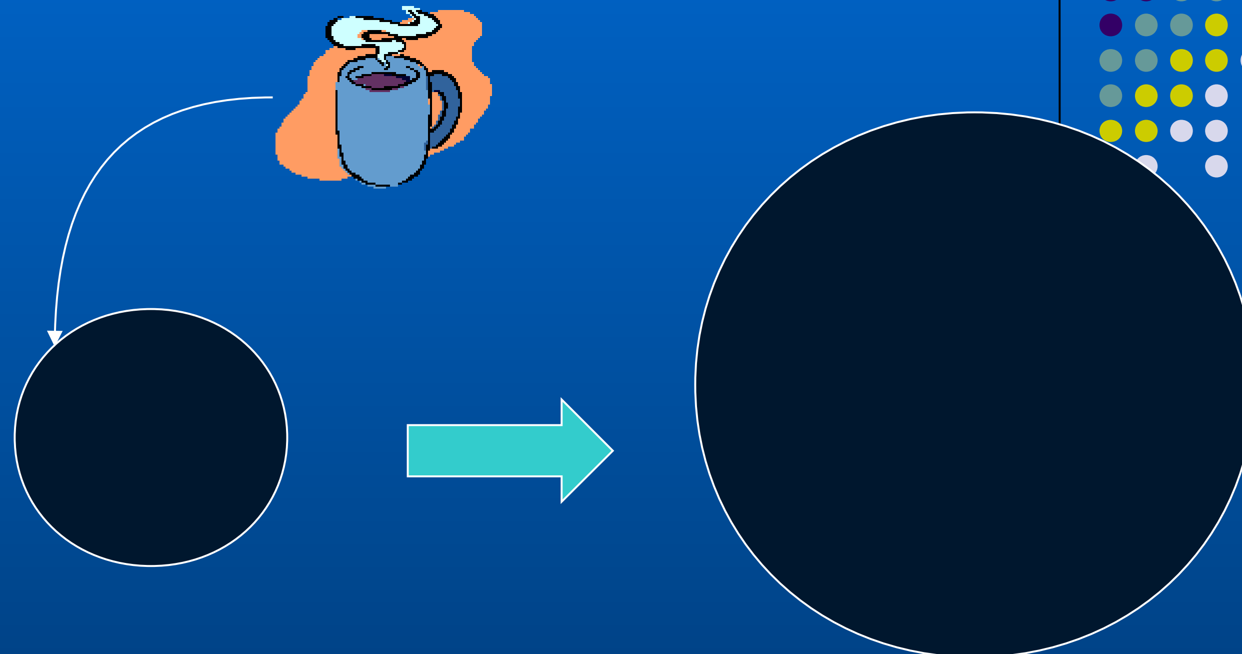


Werner Israel (1967)



Stationary Black Hole:
only parameters are
Q, M, J

Jacob Bekenstein
(1973)



Black hole has entropy proportional to area of horizon

Black Hole Emits Thermal Radiation

Stephen Hawking (1975)

Temperature $$T_H = \frac{hc^3}{8\pi GMk}$$

Entropy $$S_{BH} = \frac{Area}{4G}$$

Bekenstein-Hawking Entropy

$$S_{BH} = \frac{A(\Sigma)}{4G}$$

Bekenstein (73), Hawking (75)

$A(\Sigma)$ is area of horizon

G is Newton constant as it appears in classical mechanics

$$\Delta\phi = 4\pi G\rho$$

4 is important numerical factor

In order to explain BH entropy we have to ``explain'' S, A, G and 4

Newton constant

Quantum Effective Action : $$W_Q[g] = \sum_s \frac{(-)^{2s}}{2} \int_{\epsilon^2}^{\infty} \frac{d\tau}{\tau} e^{-\tau \Delta^{(s)}}$$

$$\Delta^{(s)} = -\nabla^2 \delta_{AB} + X^{(s)}_{AB}$$ covariant operator acting on spin-s field

Curvature expansion:

$$W_Q[g] = -\frac{1}{16\pi G(\epsilon)} \int R\sqrt{g} d^d x + O(curvature^2)$$

UV cut-off ϵ

Induced Newton Constant

$$\frac{1}{4G(\epsilon)} = \sum_s \frac{N_s}{(4\pi)^{\frac{d-2}{2}}(d-2)} \frac{1}{\epsilon^{d-2}} \left(\frac{\mathcal{D}_s(d)}{6} - c_{(s)}(d) \right)$$

$\mathcal{D}_s(d)$ is number of on-shell degrees of freedom

$$\mathcal{D}_{s=0}(d) = 1\,, \;\; \mathcal{D}_{s=1/2}(d) = \frac{2^{[d/2]}}{2}\,, \;\; \mathcal{D}_{s=1}(d) = d-2$$

$$\mathcal{D}_{s=3/2}(d) = (d-3)\,\frac{2^{[d/2]}}{2}\,, \;\; \mathcal{D}_{s=2}(d) = \frac{d(d-3)}{2}\,.$$

Contact terms

$$c^{(s)}(d) = \xi\,,\;\; c^{(s=1)}(d) = 1\,,\;\; c^{(s=2)}(d) = \frac{d^2 - d + 4}{2}$$

s=1: D. Kabat (95)
s=2: S.S. (2010)

also earlier work of Fradkin and Tseytlin (82)

Remarks

1. $G(\epsilon)$ is not positive in general (even if theory is unitary)

2. $1/G(\epsilon)$ is a difference of two (positive) contributions

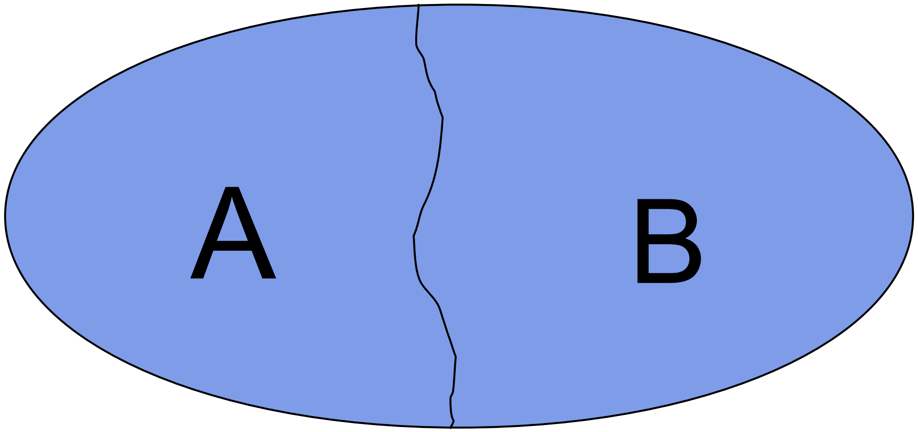
3. In some cases $1/G(\epsilon)$ may vanish (for instance for

   $\mathcal{N}=4$ SU(N) super-Yang-Mills)

4. Matter fields (fermions) contribute positively while
   mediators of interactions (bosons) contribute negatively

5. Why observed Newton constant is positive after all?

Entanglement entropy



Σ

$$|\psi(A,B)>$$

$$\rho(A,B)=|\psi><\psi|$$

$$\rho_A=Tr_B\,\rho(A,B)$$

$$S_A=-Tr_A\,\rho_A \ln \rho_A$$

Bombelli et el (86), Srednicki (93),
Frolov-Novikov (93)

Properties

- $S_A = S_B$ if $|\psi(A,B)>$ is pure state

$\Rightarrow$ S_A **depends on local geometry:**

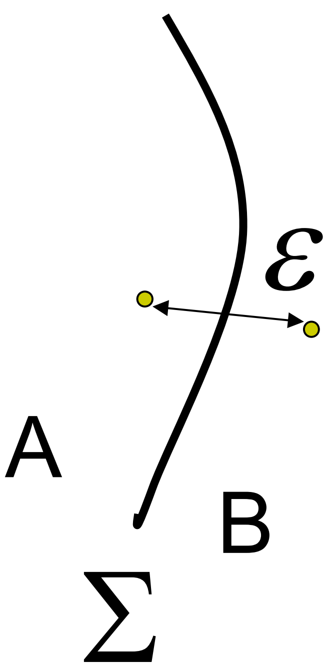
i) intrinsic or extrinsic geometry of Σ

ii) geometry of space-time near Σ

(modulo Gauss-Codazzi)

Properties

- S_A is non-zero due to short-distance correlations between A and B



$$< \phi(x), \phi(y) > \sim \frac{1}{|x-y|^{d-2}}$$

S_A depends on UV regulator $\mathcal{E}$

$$S \sim \frac{A(\Sigma)}{\epsilon^{d-2}}$$

Entanglement entropy of BH

Defined naturally for black holes (BH)

Reproduces universally area law

To leading order is same as in flat spacetime

Is a positive quantity due to physical degrees of freedom only

$$S_{ent} = \sum_s \frac{N_s}{(4\pi)^{\frac{d-2}{2}}(d-2)} \frac{1}{\epsilon^{d-2}} \frac{\mathcal{D}_s(d)}{6} A(\Sigma)$$

Can S_{BH} and S_{ent} be equal?

Suppose that there are only matter fermions (no bosons) so that contact terms vanish and assume that there is no bare (tree-level) Newton constant and entire newton constant is induced

$$1/G_{ren} = \sum_s \frac{N_s}{(4\pi)^{\frac{(d-2)}{2}}(d-2)\epsilon^{d-2}} \frac{\mathcal{D}_s(d)}{6}$$

Then two entropies are identical and UV cut-off defines Planck length

$$S_{BH} = \frac{A(\Sigma)}{4G_{ren}} = S_{ent}$$

If all fields (fermions and bosons) are present then contact terms non-vanishing

1. $$\frac{1}{4G(\epsilon)} = 1/G + \sum_s \frac{N_s}{(4\pi)^{\frac{d-2}{2}}(d-2)} \frac{1}{\epsilon^{d-2}} \left(\frac{\mathcal{D}_s(d)}{6} - c_{(s)}(d) \right)$$

G is bare Newton constant

2. $$S_{ent} = \sum_s \frac{N_s}{(4\pi)^{\frac{d-2}{2}}(d-2)} \frac{1}{\epsilon^{d-2}} \frac{\mathcal{D}_s(d)}{6} A(\Sigma)$$

3. $$S_{BH} = S_{ent}$$

Question: can we have 1. , 2. and 3. ?

Answer: yes !

Provided a consistency condition is satisfied

$$\frac{1}{4G} = \sum_s \frac{N_s}{(4\pi)^{\frac{d-2}{2}}(d-2)} \frac{1}{\epsilon^{d-2}} c_{(s)}(d)$$

So that

$$\frac{1}{4G_{ren}} = \sum_s \frac{N_s}{(4\pi)^{\frac{d-2}{2}}(d-2)} \frac{1}{\epsilon^{d-2}} \frac{\mathcal{D}_s(d)}{6}$$

Interesting consequence: relation between UV cut-off and Planck mass

$$N(\frac{1}{\epsilon})^{d-2} = M_{PL}^{d-2}$$

$$N = \sum_s N_s \mathcal{D}_s(d)$$ is effective number of species

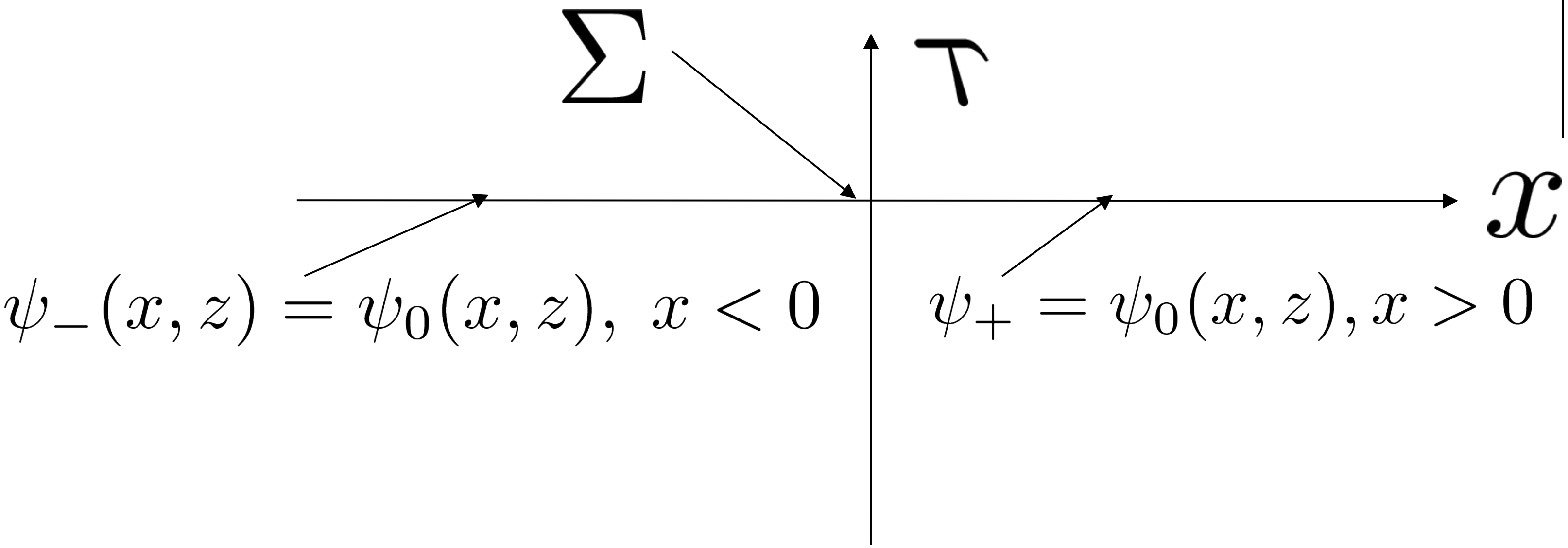
In agreement with earlier proposal of Dvali (2008)

Remarks

1. UV cut-off ϵ still defines the Planck scale
2. G_{ren} is positive !
3. Only physical degrees of freedom contribute to observed Newton constant and entropy

Thank you !

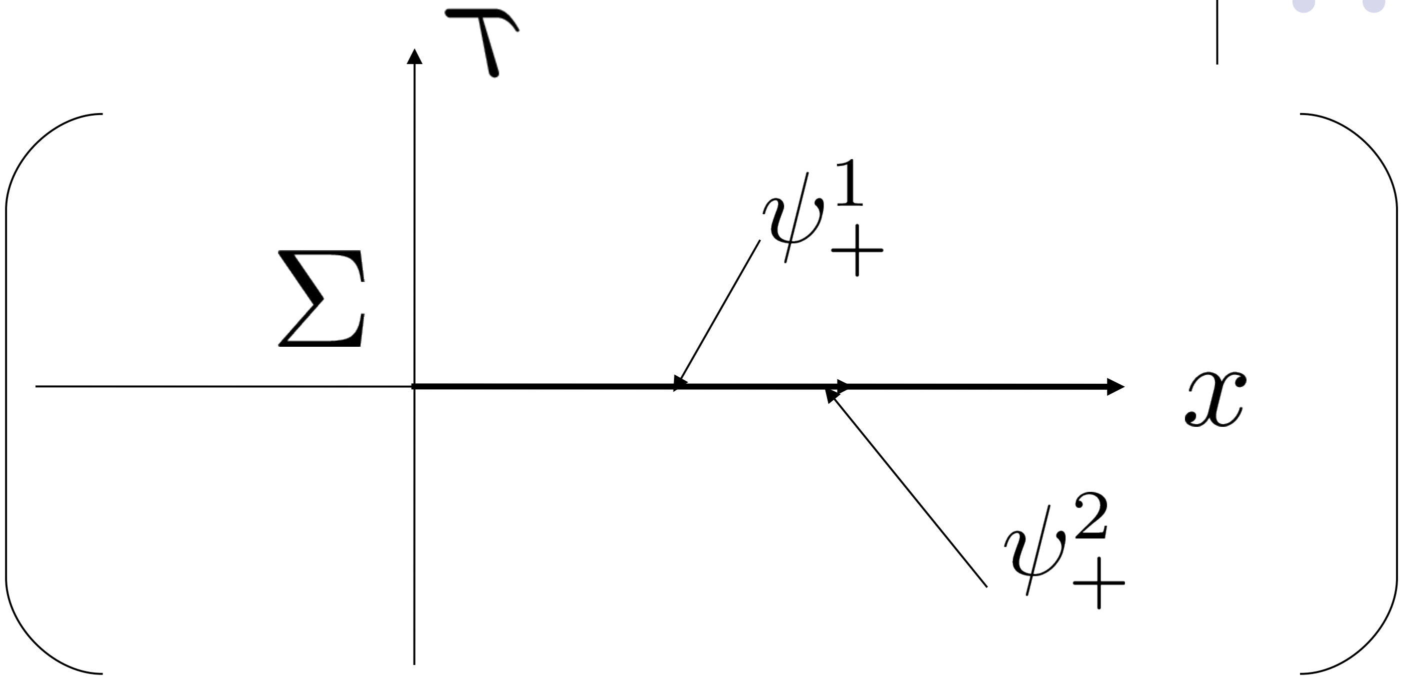
Replica method: wave function



$$\Psi[\psi_0(x,z)] = \int\limits_{\psi(X)|_{\tau=0}=\psi_0(x,z)} \mathcal{D}\psi\, e^{-W[\psi]}$$

Replica method: a reduced density matrix

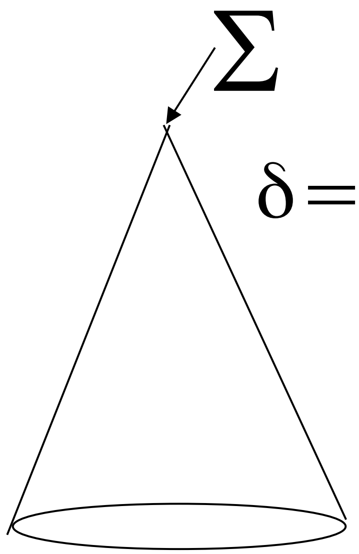
Path integral on



$$= \rho(\psi_+^1, \psi_+^2)$$

Replica Method: trace of density matrix

$$Tr\,\rho^n =$$ Path Integral on



$$\delta = 2\pi(1-n)$$

$$\%S_{ent} = -Tr\,\hat{\rho}\ln\hat{\rho} = -\left[\left(n\,\partial_n - 1\right)\ln Tr\,\rho^n\right]_{n=1}$$

Susskind (93), Callan-Wilczek (94)

Uniqueness of analytic continuation

$$n=1,2,..\to\alpha \quad , \quad \Re(\alpha)>1$$

Regularized trace of renormalized density matrix $\hat{\rho}=\dfrac{\rho}{Tr\,\rho}$ is bounded

$$|Tr_\varepsilon\hat{\rho}^\alpha|<1 \quad if \qquad \Re(\alpha>1)$$

Suppose we know $\quad Tr_\varepsilon\rho^n=Z_0(n) \quad for \quad \alpha=n \quad , \quad n=1,2,3,..$

The we can represent $\quad Z(\alpha)=Tr_\varepsilon\rho^\alpha \quad$ in the form

$$Z(\alpha)=Z_0(\alpha)+\sin(\pi\alpha)\,g(\alpha)$$

where $g(\alpha)$ is analytic and $\quad |g(\alpha=x+iy)|<e^{-\pi|y|}$

By Carlson's theorem $\qquad g(\alpha)\equiv 0$

Heat kernel and the Sommerfeld formula

$$(\partial_s + D) K(s, x, x') = 0$$
$$K(s=0, x, x') = \delta(x, x')$$

$2\pi\alpha$-periodic function from a 2π-periodic is constructed by using

$$K_\alpha(s, \phi, \phi') = K(s, \phi - \phi') + \frac{i}{4\pi\alpha} \int_\Gamma \cot\frac{w}{2\alpha} K(s, \phi - \phi' + w)\, dw$$

Sommerfeld (1897)

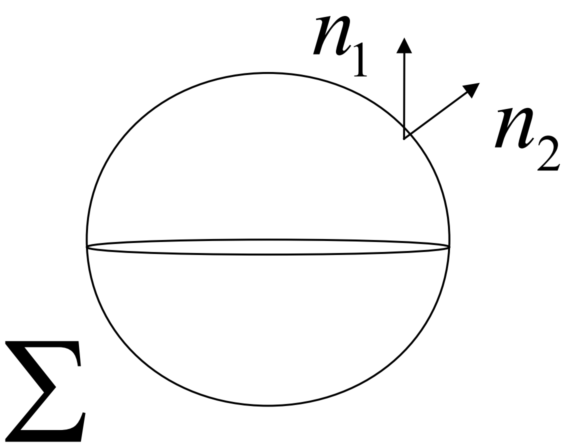
In presence of abelian symmetry $\phi \to \phi + w$

it is still a solution to heat equation

Useful mathematical tools (A): Riemann curvature and conical singularity

$$R^{\mu\nu}_{\alpha\beta} = R_{(reg)\alpha\beta}^{\;\;\mu\nu} + R_{(sing)\alpha\beta}^{\;\;\mu\nu}$$

$$R_{(sing)\alpha\beta}^{\;\;\mu\nu} = 2\pi(1-\alpha)[(n^{\mu}n_{\alpha})(n^{\nu}n_{\beta}) - (n^{\mu}n_{\beta})(n^{\nu}n_{\alpha})]\delta_{\Sigma}$$



$$(n^{\mu}n_{\alpha}) = n^{\mu}_{1}n^{1}_{\alpha} + n^{\mu}_{2}n^{2}_{\alpha}$$

Fursaev, SS (94)

A consequence: the Euler number for a manifold with cone singularity

$$\chi(M_\alpha)=\frac{1}{32\pi^2}\int_{M_\alpha/\Sigma}(R^2-4R_{\mu\nu}^2+R_{\mu\nu\alpha\beta}^2)+\sum_i(1-\alpha_i)\chi(\Sigma_i)$$

Fursaev, SS (94)

(Rediscovered by Atiyah, LeBrun (2012))

A special case is when M_α possesses a continuous Abelian isometry so that Σ_i are the fixed point sets of this isometry and $\alpha_i=\alpha$.

Then we arrive at a reduction formula $\chi(M)=\sum_i\chi(\Sigma_i)$

Example: singular surface of S_α^d $(d\geqslant 3)$ is S^{d-2} so that $\chi(S^d)=\chi(S^{d-2})$

Useful mathematical tools (B): Heat kernel method

$$W(\alpha) = -\frac{1}{2}\int_{\epsilon^2}^{\infty}\frac{ds}{s}\,\mathrm{Tr}K_{E_\alpha}(s)$$

$$\mathrm{Tr}K_{E_\alpha}(s) = \frac{1}{(4\pi s)^{\frac{d}{2}}}\sum_{n=0} a_n s^n$$

Coefficients in the expansion decompose on the bulk (regular) and the surface (singular) parts:

$$a_n = a_n^{reg} + a_n^{\Sigma}$$

Heat kernel method: regular terms in the expansion

Scalar field operator: $\mathcal{D} = -(\nabla^2 + X)$

$$a_0^{reg} = \int_{E_\alpha} 1 \quad , \quad a_1^{reg} = \int_{E_\alpha} (\frac{1}{6}\bar{R} + X)\,,$$

$$a_2^{reg} = \int_{E_\alpha} \left(\frac{1}{180}\bar{R}^2_{\mu\nu\alpha\beta} - \frac{1}{180}\bar{R}^2_{\mu\nu} + \frac{1}{6}\nabla^2(X + \frac{1}{5}\bar{R}) + \frac{1}{2}(X + \frac{1}{6}\bar{R})^2 \right)$$

These terms are proportional to α and do not contribute to the entropy

Heat kernel method: surface terms in the expansion

$$a_0^\Sigma = 0; \quad a_1^\Sigma = \frac{\pi}{3}\frac{(1-\alpha^2)}{\alpha}\int_\Sigma 1\,,$$

$$a_2^\Sigma = \frac{\pi}{3}\frac{(1-\alpha^2)}{\alpha}\int_\Sigma (\frac{1}{6}\bar{R} + X) - \frac{\pi}{180}\frac{(1-\alpha^4)}{\alpha^3}\int_\Sigma (\bar{R}_{ii} - 2\bar{R}_{ijij})\,,$$

where $\bar{R}_{ii} = \bar{R}_{\mu\nu} n_i^\mu n_i^\nu$ and $\bar{R}_{ijij} = \bar{R}_{\mu\nu\lambda\rho} n_i^\mu n_i^\lambda n_j^\nu n_j^\rho$

Fursaev (94)

Important remark:

These mathematical tools work only if there is abelian isometry in subspace orthogonal to entangling surface Σ .
This is not so for a surface (sphere, cylinder..) in flat Minkowski spacetime!

However: they work perfectly for Killing horizons!

Entanglement entropy of black holes

Wave function of black hole $\psi(\varphi_+, \varphi_-)$ is functional of modes

inside (φ_-) and modes outside (φ_+) black hole horizon

Barvinsky, Frolov and Zelnikov (94)

Partition function

$\mathrm{Tr}\rho^\alpha = e^{-W(\alpha)}$ is given by functional integral

over E_α , α —fold cover of Euclidean black hole instanton

(manifold with conical singularity at horizon)

Entanglement entropy of 4d black hole

Scalar field operator: $$\mathcal{D} = -(\nabla^2 + X)\,, \;\; X = -\xi \bar{R}$$

$$S_{d=4} = \frac{A(\Sigma)}{48\pi\epsilon^2} - \frac{1}{144\pi}\int_\Sigma \left(\bar{R}(1+6\xi) - \frac{1}{5}(\bar{R}_{ii} - 2\bar{R}_{ijij})\right)\ln\epsilon$$

S.S. (94)

Kerr-Newman black hole (m,a,q)

Entropy of a minimal scalar field, $\xi = 0$

$$S_{KN} = \frac{A(\Sigma)}{48\pi\epsilon^2} + \frac{1}{45}\left(1 - \frac{3q^2}{4r_+^2}(1 + \frac{(r_+^2+a^2)}{ar_+}\tan^{-1}(\frac{a}{r_+}))\right)\ln\frac{r_+}{\epsilon}$$

Mann, SS (96)

Horizon area $\quad A(\Sigma) = 4\pi(r_+^2 + a^2)$

$$r_+ = m + \sqrt{m^2 - a^2 - q^2}$$

Interesting limits:

- Schwarzschild black hole (q=a=0)

$$S_{Sch} = \frac{A(\Sigma)}{48\pi\epsilon^2} + \frac{1}{45}\ln\frac{r_+}{\epsilon}$$

- Extreme charged black hole (a=0, q=m)

$$S_{Ext} = \frac{A(\Sigma)}{48\pi\epsilon^2} - \frac{1}{90}\ln\frac{r_+}{\epsilon}$$

- Extreme Kerr black hole (q=0, a=m)

$$S_{Ext-Kerr} = \frac{A(\Sigma)}{48\pi\epsilon^2} + \frac{1}{45}\ln\frac{r_+}{\epsilon}$$

Renormalization

Bare gravitational action

$$W_{bare} = \int \left(-\frac{1}{16\pi G}(R + 2\Lambda) + c_1 R^2 + c_2 R^2_{\mu\nu} + c_3 R^2_{\alpha\beta\mu\nu} \right)$$

Black hole entropy $$S_{BH} = \frac{A_\Sigma}{4G} - 4\pi \int_\Sigma (2c_1 R + c_2 R_{ii} + 2c_3 R_{ijij})$$

Renormalization of entropy: $$S_{BH}(G, c_i) + S_{div}(\epsilon) = S_{BH}(G^{ren}, c_i^{ren})$$

Renormalization of action: $$\frac{1}{4G} + \frac{1}{48\pi\epsilon^2} = \frac{1}{4G^{ren}}$$

Susskind and Uglum (94), Jacobson (94), Fursaev and SS (94)

<u>The statement is valid for any field (fermionic and bosonic) except gauge fields (s=2 and s=1)</u>

Puzzle of non-minimal coupling

Non-minimal field operator $\mathcal{D} = -(\nabla^2 + X)\,,\ X = -\xi \bar{R}$

Renormalization of Newton constant

$$G^{-1}_{ren} = G^{-1} + \frac{1}{2\pi}(\frac{1}{6} - \xi)\frac{1}{\epsilon^2}$$

Entanglement entropy on Ricci flat metrics $\bar{R} = 0$
does not depend on ξ

$$S = \frac{A(\Sigma)}{48\pi\epsilon^2}$$

Gauge fields: s=1 and s=2

$$\frac{1}{4G_{ren}} = \frac{1}{4G} + \frac{1}{(4\pi)^{\frac{d-2}{2}}(d-2)}\left(\frac{D_s(d)}{6} - c_s(d)\right)\frac{1}{\varepsilon^{d-2}}$$

Spin s=1: $D_1(d) = d-2, \qquad c_1(d) = 1$

Spin s=2: $D_2(d) = \dfrac{d(d-3)}{2}, \qquad c_2(d) = \dfrac{(d^2 - d + 4)}{2}$

Entanglement Entropy: $$S = \frac{D_s(d)}{6(d-2)(4\pi)^{\frac{d-2}{2}}}\frac{A(\Sigma)}{\varepsilon^{d-2}}$$

Most intriguing question: can entanglement entropy account for entire BH entropy?

- A natural identification: UV cut-off at Planck scale

$$1/\epsilon = \Lambda \sim M_{PL} \quad \text{then} \quad S_{ent} \sim S_{BH}$$

- Do coefficients precisely agree?

- Entanglement entropy and induced gravity, problem of non-minimal coupling

Jacobson(94), Frolov et al. (96),
Hawking, Maldacena, Strominger (2000)

SOME OTHER DEVELOPMENTS

UV and IR modified theories

More general Lorentz invariant field operator $\mathcal{D} = F(\nabla^2)$

Examples:

(i) 4d brane in spacetime with compact fifth dimension

$$F(p^2) = \frac{p}{L}\tanh(Lp)$$

(ii) DGP model $F(p^2) = p^2 + m\sqrt{p^2}$

(iii) Non-commutative field theory $F(p^2) = p^2 + \dfrac{1}{\theta^2 p^2}$

(iv) UV modified theory $F(p^2) = p^2\, e^{p^2/\Lambda^2}$

Entropy in UV(IR) modified theories

Heat kernel on space with conical singularity

$$\mathrm{Tr} K_\alpha(s) = \frac{1}{(4\pi)^{d/2}}\left(\alpha V P_d(s) \;+\; \frac{\pi}{3\alpha^2}(1-\alpha^2)A(\Sigma)P_{d-2}(s) + ..\right)$$

Entanglement entropy

$$S = \frac{A(\Sigma)}{12\cdot(4\pi)^{(d-2)/2}}\int_{\epsilon^2}^{\infty}\frac{ds}{s}P_{d-2}(s)\,,$$

where

$$P_n(s) = \frac{2}{\Gamma(\frac{n}{2})}\int_0^\infty dp\, p^{n-1}\, e^{-sF(p^2)}$$

Nesterov, SS (2010)

Entropy in UV modified theories

(i) No matter how fast function $F(p^2)$ grows for large p entanglement entropy is always UV divergent

(ii) The area law and the statement on renormalization of entropy are valid for any $F(p^2)$

(iii) Example: $F(p^2) \simeq m^2 e^{\frac{p^2}{\Lambda^2}}$

$$S \simeq \frac{A(\Sigma)}{48\pi}\Lambda^2 \ln^2(\epsilon m)$$

Nesterov, SS (2010)

Entropy in non-Lorentz invariant theories

$$D = -\partial_t^2 + F\left(-\vec{\nabla}^2\right)$$

- there is no rotational symmetry in (r,t) plane

- only $2\pi n$ periodicity is allowed

- it is enough to compute entropy
$$\boxed{S = \frac{A(\Sigma)}{12(4\pi)^{(d-2)/2}} \int_{\varepsilon^2}^{\infty} \frac{ds}{s} P_{d-2}(s)}$$

$P_n(s)$ is the same as in Lorentz invariant case

Entropy in non-Lorentz invariant theories

Polynomial field operators: $F(-\vec{\nabla}^2) = m^{2(1-n)}(-\vec{\nabla}^2)^n$

Heat operator $\exp(-s\,D)$ is invariant under rescaling

$\vec{x} \to \lambda \vec{x} \;\; , \;\; t \to \lambda^n t \;\; , \;\; s \to \lambda^{2n} s$ and $\vec{x} \to \beta \vec{x} \;\; , \;\; m \to \beta^{n/(1-n)} m$

Structure of entanglement entropy is fixed by this invariance

$$S \sim \left(\frac{m^{n-1}}{\varepsilon}\right)^{\frac{d-2}{n}} A(\Sigma)$$

Logarithmic term in entropy of generic 4d CFT

Effective action

$$W_{CFT} = \frac{a_0}{\epsilon^4} + \frac{a_1}{\epsilon^2} + a_2 \ln \epsilon + w(g)\,, \;\; w(\lambda^2 g) = w(g) - a_2 \ln \lambda$$

A and B type conformal anomaly

$$a_2^{\rm bulk} = A E_{(4)} + B I_{(4)} \quad ,$$

$$E_{(4)} = \frac{1}{64}\int_E (R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} - 4R_{\mu\nu}R^{\mu\nu} + R^2) \quad ,$$

$$I_{(4)} = -\frac{1}{64}\int_E (R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} - 2R_{\mu\nu}R^{\mu\nu} + \frac{1}{3}R^2)$$

Duff (77)
Christensen, Duff (78)

Logarithmic term in entropy of generic 4d CFT

Entanglement entropy of arbitrary surface Σ

$$S_{(A,B)} = \frac{a_1^\Sigma}{\epsilon^2} + a_2^\Sigma \ln\epsilon + s(g) \ , \quad s(\lambda^2 g) = s(g) - a_2^\Sigma \ln\lambda$$

Surface anomaly
(combination of conformal symmetry and holographic interpretation)

$$a_2^\Sigma = A a_A^\Sigma + B a_B^\Sigma \ ,$$

$$a_A^\Sigma = \frac{\pi}{8}\int_\Sigma (R_{abab} - 2R_{aa} + R - \mathrm{Tr}k^2 + k_a k_a) = \frac{\pi}{8}\int_\Sigma R_\Sigma \ ,$$

$$a_B^\Sigma = -\frac{\pi}{8}\int_\Sigma (R_{abab} - R_{aa} + \frac{1}{3}R - (\mathrm{Tr}k^2 - \frac{1}{2}k_a k_a)) \ ,$$

SS (2008)

where k^a is extrinsic curvature of Σ (vanishes for black hole horizon)

Logarithmic term in entropy of generic 4d CFT: flat spacetime

$$S_{(A,B)} = \frac{A(\Sigma)}{4\pi\varepsilon^2} + \frac{\pi}{8}\int_\Sigma \left[A\, R_\Sigma + B\left(Tr\, k^2 - \frac{1}{2} k_a k^a \right) \right] \ln \varepsilon$$

$$R_\Sigma = k_a k^a - Tr\, k^2$$

Logarithmic term in entropy of generic 4d CFT: flat spacetime

Round sphere in Minkowski spacetime

$$S^{\text{sphere}}_{(A,B)} = \frac{A(\Sigma)}{4\pi\epsilon^2} + A\pi^2 \ln \frac{\epsilon}{a}$$

SS (2008)
Cassini-Huerta (2010)
Dowker (2010)

For a scalar field $A\pi^2 = \frac{1}{90}$

The logarithmic term is the same as for extreme black hole,

near-horizon region is $H_2 \times S_2$,

(Minkowski spacetime and $H_2 \times S_2$ are conformally related)

Logarithmic term in entropy of generic 4d CFT: flat spacetime

Cylinder in Minkowski spacetime

$$S^{cylinder}_{(A,B)} = \frac{A(\Sigma)}{4\pi\varepsilon^2} + B\frac{\pi^2}{8}\frac{L}{a}\ \ln(\varepsilon)$$

SS (2008)

For a scalar field numerically verified by Huerta (2012)

Logarithmic term in entropy of generic 4d CFT: black holes

Extreme charged black hole $s_{log}^{ext} = A\pi^2$

The Schwarzschild black hole $s_{log}^{sch} = (A - B)\pi^2$

Extreme Kerr black hole $s_{log}^{Ext-Kerr} = (A - B)\pi^2$

For a generic 4d CFT

$$A = \frac{1}{90\pi^2}(n_0 + 11n_{1/2} + 62n_1 + 0n_{3/2} + 0n_2) \ ,$$
$$B = \frac{1}{30\pi^2}(n_0 + 6n_{1/2} + 12n_1 - \frac{233}{6}n_{3/2} + \frac{424}{3}n_2)$$

Why log corrections are interesting?

- they are important at the final stage of evaporation

$$S = 4\pi G M^2 - \sigma \ln M$$

$$T^{-1} = 8\pi G M - \frac{\sigma}{M}$$

- consistency with microscopic calculation for extreme black holes

Banerjee, Gupta, Sen (2010)

Some open questions

- entanglement entropy in string theory

- non-minimal coupling (gauge fields)

- dynamical entangling surface (a brane?)

- ...

More work has to be done..